Interpreting Monetary Stabilization in a Growth Model With Credit Goods Production

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ABSTRACT

This paper is motivated by observations concerning the size of the banking sector and the growth rate of the economy before and after successful stabilizations of high inflations. The facts suggest that the relative size of the banking sector increases during a period of accelerating inflation and decreases immediately following a successful monetary stabilization. Furthermore, the GDP growth rate is lower during the high inflation period than after stabilization. The goal of this paper is to develop a monetary growth model which is qualitatively consistent with these observations. The model we use is a variant of the Lucas and Stokey (1987) model of cash and credit goods. The main innovation in our model is that while cash goods and credit goods are perfect substitutes in consumption we posit different technologies for their production. We show that the model's predictions on the impact of a permanent stabilization are consistent with the main real and monetary observations on high inflation countries.

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1. Introduction

Recent episodes of high inflations and successful stabilizations in Latin America and Israel have provided substantial evidence on the behavior of the main monetary economic aggregates during the high inflation period and in the post stabilization era (see, e.g., Bruno et. al. 1993). This recent evidence is roughly consistent with evidence on the monetary aggregates during the high inflation episodes in Europe during the 1920's that are described by Sargent (1982). There is less agreement on the evidence concerning the real consequences of the accelerating inflation and the abrupt stabilization programs on employment, unemployment and output. A particular debate followed Sargent's (1982) claim that the evidence rejects the hypothesis of a "Phillips Curve" trade-off between inflation and real output.

The evidence on Germany in the 1920's (Garber (1982) and Graham (1930)) indicates that the growth rate of industrial output was higher on average in the post stabilization period although a substantial sectoral shift in employment was associated with higher than average unemployment rates. Part of this sectoral shift was due to the increase in the banking sector during the period of accelerating inflation and the contraction of that sector after stabilization (Bresciani-Turroni (1937)).

Wicker (1986) provided substantial evidence on the unemployment in Austria, Hungary and Poland during the post stabilization period in the 1920's. In reference to the Hungarian case, he said that (1986, p.358):

"The most striking thing about these figures (incidence of trade union unemployment) is the extraordinary increase in the number of unemployed in the financial sector - 91.5 percent of the total net increase of 13,000. All of this increase can be attributed to the ending of hyperinflation which had increased substantially the money market as well as other operations of commercial banking."
Employment in the Hungarian industrial sector started to decrease prior to the stabilization of June 1924 while it was increasing in December of 1924. For Austria Wicker (1986) described a similar pattern where 10,000 workers in the banking sector lost their jobs immediately after stabilization. Real GNP decreased by one percent during the first year following the stabilization but increased by 7 and 10 percent in the following two years.

Bruno (1993, Table 1.2) claimed that the cross country evidence shows that countries with high inflation rates have significantly lower per capita GDP growth rates. Furthermore, in recent episodes of high inflation the per capita growth rate was lower during the high inflation period relative to the period with lower inflation.

Israel has experienced an accelerating inflation from 1980 until July 1985. At that date the Israeli government implemented a stabilization program which resulted in an abrupt drop of the annual inflation rate from a high of close to 500 percent to a low of 16–20 percent. During the high inflation period (1980–1985) the business-sector output in Israel rose at an annual rate of about one half that during the post stabilization period (1986–1990, see Bruno, table 2.1). Moreover, total factor productivity growth rate was zero before the stabilization and 2.6% per annum, after the stabilization (Bruno, table 2.1).⁴

Melnick (1993) provided evidence on the Israeli banking sector which we present in Table 1. It is clear that during the period of accelerating inflation from 1968 to 1985 the three input indicators – labor, automated machines and area of bank branches in Israel has a significant upward trend. Although the aggregate business-sector growth rate was higher in the post stabilization period, as we mentioned above, the banking sector experienced a clear reduction in inputs during the period that followed the end of high inflation (see Figure 1).⁵

In this paper we investigate a monetary growth model where credit services
are produced by labor and capital in order to facilitate trade in an alternative way to fiat money. The motivation for analyzing such a model are the observations described above concerning the size of the credit producing sector and the private sector output growth rate before and after monetary stabilizations. The observations may be summarized as follows. Moving from a regime of high monetary growth rate to one of a low monetary growth rate we observe the following real changes in the economy:

(i) the size of the credit (banking) sector decreases;
(ii) the real GDP growth rate increases.

In view of the above facts it seems that there should be an interest in providing an analytical model that combines the three variables that have a clear low frequency co-movement; the size of the banking sector, the growth rate of output, and the inflation rate. In this paper we develop a general equilibrium monetary growth model that incorporates a banking and credit sector and that can be calibrated using data from the NIP accounts. Potentially, one can use the model to analyze low frequency movements in inflation and the size of the banking and credit sectors.\(^6\)

The model is a variant of the Lucas and Stokey [1987] model of cash goods and credit goods. The main innovation in the present model relative to Lucas and Stokey [1987] is that unlike their model we posit that cash goods and credit goods are perfect substitutes in consumption and investment but differ in their production technologies.\(^7\) We think that this is sensible since gasoline is gasoline regardless of whether one pays for it with cash or cheque or credit card. The distinction ought to be traced to the additional resources in the form of the services of the banking sector or credit card companies that are required for the purchase of "credit gasoline" as opposed to "cash gasoline." That is, the production of a unit of credit gasoline
requires a unit of cash gasoline plus some credit services. In turn, the differences in the technology of producing cash versus credit gasoline ought to be reflected in different prices for credit gasoline and cash gasoline. This is indeed the case for gasoline at many gas stations around the U. S. at which different prices are listed depending on whether one pays for gas with cash or with credit.

Aside from descriptive realism the major advantage of adopting the above alternative modeling strategy is that data can be brought to bear in calibrating the model and developing the model’s quantitative implications. We can use data on the value added, capital input, labor input, and the shares of capital and labor income in the credit and banking sectors to calibrate the technology for producing credit services which are needed for producing credit goods. This approach would not be possible if credit goods were treated simply as an additional argument of the utility function as in the model of Lucas and Stokey [1987].

As we will see the above modeling innovation not only enables us to capture the stylized facts mentioned earlier but also has new and interesting implications for the real effects of monetary policy and for the interrelationships among the money growth rate, the nominal and the real interest rates, the inflation rate, and the growth rate of real output. These implications arise because in this model the nominal interest rate has an allocative role in allocating resources between the goods producing and credit services producing sectors. This role arises because the nominal interest rate determines the wedge between the prices of cash goods and credit goods. Equivalently, the nominal interest rate determines the relative price of credit services and production goods. As a consequence, the nominal interest rate also influences factor rewards in the two sectors. Since one of these
factors is capital the nominal interest rate influences the real rate through its allocative role. This is a channel for the real effects of monetary policy that leads to an interesting interrelationships among the money growth rate, the nominal and the real interest rate, the inflation rate and the growth rate of real output. It is also the channel through which technological improvements in the provision of financial services, by making credit services, and, hence, credit goods relatively cheaper, affect the nominal interest rate and, thereby, currency velocity and inflation. 

Comparing steady states of an economy with high and low rates of monetary expansion shows that the model can interpret the main evidence on the banking sector and growth during and after the high inflation period. That is, a reduction in the monetary growth rate implies a decrease in the size of the banking sector and an increase in the GDP growth rate.

The impact of a surprise stabilization is analyzed using a log-linear approximation to the stochastic model around a deterministic steady state. We simulate the model in response to a negative shock to the money growth rate assuming that this shock has high persistence (is almost a random walk). We show that the model's predictions are consistent with observations on the result of stabilization. That is, the inflation rate, the nominal interest rate, the wage rate and velocity all go down immediately. The size of the credit sector is reduced but the growth rate of the sector goes up. The same is the case for real GNP and wages, while, as observed, consumption increases and the growth rate also goes up. The real interest rate increases as a result of the negative monetary shock.

The rest of this paper is organized as follows. In section 2 we describe the monetary growth model. Section 3 describes steady state determination. In section 4 we analyze the impact of a stabilization policy both by comparing steady states with high and low rates of monetary expansion as well as by analyzing the
impact of a surprise stabilization. Section 5 concludes with a discussion that summarizes the results of the model in relation to the facts of stabilization policies that were adopted in high inflation countries.

2. A Monetary Growth Model With Credit Goods Production

The model we develop is a competitive equilibrium model and is described in terms of the behavior of its three decision units, namely, the households, the producers and the government, and equilibrium conditions. We start by describing the behavior of the representative household.

The Representative Household

There is a representative infinitely lived household which has one unit of labor endowment available each period. The household consumes the amount $c_t$ of goods and supplies the exogenous amount $n$ ($n = 1$) of labor input in each period $t$. The household’s preferences are given by the following expected discounted sum of utility of consumption.

$$E_0\{\Sigma_{t=0}^{\infty} \beta^t U(c_t)\}, \quad 0 < \beta < 1. \quad (2.1)$$

The household purchases goods using cash ("cash goods") in the amount $c_{1t}$ and purchases goods using credit ("credit goods") in the amount $c_{2t}$ at the prices $p_{1t}$ and $p_{2t}$, respectively. As already mentioned cash goods and credit goods are perfect substitutes in consumption and investment. Therefore, the household divides total goods purchases $c_{1t} + c_{2t}$ into consumption in the amount $c_t$ and gross capital formation in the amount $k_{t+1} - (1-\delta)k_t$, where $k_t$
is the household's beginning of period \( t \) capital holding and \( \delta \in (0,1] \) is the depreciation rate of capital. This leads to the following constraint on the household's optimization.

\[
(2.2) \quad c_{1t} + c_{2t} \geq c_t + k_{t+1} - (1-\delta)k_t.
\]

In addition to capital, the household begins period \( t \) with \( m_t \) units of money and \( b_t \) units (in terms of face value) of nominal bonds. The household also receives nominal lump sum transfer payments from the government in the amount \( X_t \). As is usual in cash-in-advance (CIA) models of money there is a financial market in which the household can rearrange its portfolio of money and bonds. Once this is done the financial market closes and the goods markets (for purchasing cash goods and credit goods) open. In the cash goods market the household can purchase cash goods subject to the following CIA constraint.

\[
(2.3) \quad \frac{(m_t + X_t)}{p_{1t}} + \frac{b_t}{p_{1t}} - \frac{b_{t+1}}{[(1+R_t)p_{1t}]} \geq c_{1t}.
\]

In (2.3) \( R_t \) is the nominal interest rate from \( t \) to \( t+1 \). Note that the left side of (2.3) is the amount of cash the household has available after the close of the financial market.

The household can use any excess cash leftover after the purchase of cash goods plus labor and capital income to purchase credit goods or accumulate cash for the next period. This leads to the following budget constraint.

\[
(2.4) \quad \frac{(m_t + X_t)}{p_{1t}} + \frac{b_t}{p_{1t}} - \frac{b_{t+1}}{[(1+R_t)p_{1t}]} - c_{1t} + v_t + r_t k_t \geq c_{2t} \frac{p_{2t}}{p_{1t}} + m_{t+1}/p_{1t}.
\]
In (2.4), \( w_t \) and \( r_t \) denote the wage and the rental on capital in units of the cash good.

The household's optimization problem now consists of maximizing (2.1) subject to the constraints (2.2)-(2.4) by choosing
\[
\{c_t, c_{1t}, c_{2t}, m_{t+1}, b_{t+1}, k_{t+1}\}_{t=0}^\infty
\]
for given laws of motion for
\[
\{p_{1t}, p_{2t}, x_t, r_t, w_t, r_t\}. 
\]

The Producer

There are two primary production sectors which use only the primary factors of production (capital and labor) and a third sector which uses only intermediate inputs. Each of these satisfy constant returns to scale in the inputs used in that sector. Individual producers in each sector maximize profits taking prices of outputs and inputs as given.\(^{10}\)

The first sector uses capital input in the amount \( K_{1t} \) and labor input in the amount \( n_{1t} \) to produce goods in the amount \( Y_t \) subject to the following constant returns to scale (in \( K_{1t}, n_{1t} \) and \( Y_t \)) production function.

\[(2.5) \quad Y_t = F(K_{1t}, \theta_{1t} K_{1t} n_{1t}). \]

The second sector uses capital input in the amount \( K_{2t} \) and labor input in the amount \( n_{2t} \) to produce "credit services" in the amount \( S_t \) subject to the following constant returns to scale (in \( K_{2t}, n_{2t} \) and \( S_t \)) production function.

\[(2.6) \quad S_t = G(K_{2t}, \theta_{2t} K_{2t} n_{2t}). \]
The output of goods $Y_t$ of the first sector can be used in two ways. Some part can be sold directly for cash. This part is denoted $c_{1t}$ and is referred to as "cash goods." The remaining part ($Y_t - c_{1t}$) is used as an intermediate input in the third sector along with the credit services produced by the second sector to produce "credit goods". This production relation in the third sector is of the following Leontief fixed coefficients type.

\[
(2.7) \quad c_{2t} = \min(Y_t - c_{1t}, S_t).
\]

The following special feature of the above technology should be noted. In the production of goods and credit services (2.5) and (2.6), there is an external effect on labor productivity arising from the aggregate capital stock in the economy. This turns the model into an endogenous growth model so that policies have growth as well as level effects. Thus, we can analyze the effects of a monetary stabilization on the real growth rate. Because of this endogenous growth feature the labor augmenting technology shocks $\theta_{1t}$ and $\theta_{2t}$ in (2.5) and (2.6), respectively, are assumed to follow stationary stochastic processes.

We will refer to the production side of this economy as a two sector economy where the two sectors are the goods producing sector (2.5) and the credit services producing sector (2.6).

\textit{Government}

In order to simplify the model we assume that the government sets the supply of bonds and money as follows.
(2.8a) \[ B_t = 0, \ t \geq 0, \]
(2.8b) \[ M_{t+1} = M_t + X_t = (1+x_t)M_t, \ t \geq 0. \]

In (2.8b), \( x_t \) is the money growth rate which is assumed to follow a stationary stochastic process which is independent of \( \{ \theta_{1t}, \theta_{2t} \} \).

**Equilibrium**

The following conditions (which are pretty self-explanatory) need to hold in equilibrium.

(2.9a) \[ K_{1t} + K_{2t} = K_t, \]
(2.9b) \[ n_{1t} + n_{2t} = 1, \]
(2.9c) \[ c_t + K_{t+1} - (1-\delta)K_t = c_{1t} + c_{2t} = Y_t = F(K_{1t}, \theta_{1t}K_t n_{1t}), \]
(2.9d) \[ c_{2t} = S_t = G(K_{2t}, \theta_{2t}K_t n_{2t}), \]
(2.9e) \[ k_t = K_t, \ K_0 \text{ given,} \]
(2.9f) \[ b_t = B_t = 0, \]
(2.9g) \[ m_t = M_t, \ M_0 \text{ given.} \]

In the next section we characterize the equilibrium for the above model and describe steady state determination.

3. Equilibrium and Steady State Determination

**Consumer Optimization**

The solution to the consumer's optimization problem is characterized by the following first order necessary conditions (FONCs).
(3.1a) \[ \frac{U_{c,t}}{(1+R_t)} = \beta E_t [p_{1t} U_{c,t+1}/P_{1,t+1}] \],

(3.1b) \[ U_{c,t} = \beta E_t [(1-\delta + r_{t+1}/(1+R_{t+1})) U_{c,t+1}] \],

(3.1c) \[ 1 + R_t = \frac{p_{2t}}{p_{1t}} \].

In the above equations \( U_{c,t} \) denotes the marginal utility of consumption at date \( t \). Condition (3.1a) is standard. Condition (3.1b) is slightly different from the traditional model, e.g. Cooley and Hansen (1989), in that it exhibits an inflation tax effect on the return to capital. In the steady state, changes in the money growth rate and, hence, in the inflation rate and the nominal interest rate affect the real return to capital. This effect is absent in the traditional model because investment is treated as a credit good. In our model cash goods may also be used for investment and this feature leads to an inflation tax effect on the return to capital and, hence, on the rate of investment. We think this feature of the model is sensible since there is no reason to suppose that candy must be paid for by cash whereas hammers can only be bought on credit.

What is entirely new is condition (3.1c) which is an arbitrage relation between the nominal interest rate and the price of the credit good relative to the cash good. It arises from the following consideration. A household can either purchase an extra unit of the cash good at price \( p_{1t} \) by borrowing in the financial market at the interest rate \( R_t \), thereby reducing its cash holding at \( t+1 \), or it can purchase an extra unit of the credit good at the price \( p_{2t} \) and reduce its cash holding at date \( t+1 \). Since cash goods and credit goods are perfect substitutes in consumption and investment the arbitrage relation (3.1c) must hold.

Thus, a key relation arising from consumer optimization is that the nominal interest rate equals the price of credit goods relative to cash goods.
This is the channel through which money growth affects the allocation of resources between the cash goods and credit services sectors (by affecting the price of credit goods relative to cash goods). It is also the channel through which technological improvements in the provision of credit services, by making credit goods relatively cheaper, affect the nominal interest rate and, thereby, currency velocity and inflation.

**Producer Optimization**

Let $p_{st}$ denote the price of credit services. Note that the price of goods $Y_t$ will equal the price of cash goods $(p_{1t})$ so long as cash goods are being produced. The following obvious conditions reflecting the equality of factor prices and marginal products characterize producer profit maximization.

\begin{align}
(3.2a) & \quad \nu_t = \theta_{1t} K_t F_{2t} = (p_{st}/p_{1t}) \theta_{2t} K_t G_{2t}, \\
(3.2b) & \quad r_t = P_{1t} = (p_{st}/p_{1t}) G_{1t}, \\
(3.2c) & \quad p_{2t} = p_{1t} + p_{st}.
\end{align}

Equations (3.1d) and (3.2c) imply the following relation between the nominal interest rate and the price of credit services relative to cash goods.

\begin{equation}
(3.3) \quad R_t = p_{st}/p_{1t}.
\end{equation}

**Steady State Analysis**

Using the above optimality conditions for the representative household and producer we now study the model’s non-stochastic steady state properties.
To do this we assume that the money growth rate $x_t$, and the technology shocks $\theta_{1t}$ and $\theta_{2t}$ are all constant over time. Further, the utility function is assumed to have the following form.

\begin{equation}
U(c) = \frac{(c^{1-\mu} - 1)}{(1-\mu)}, \mu > 0, \mu \neq 1,
= \ln(c) \quad \text{if } \mu = 1.
\end{equation}

The parameter $\mu$ is the risk aversion coefficient and utility is logarithmic in consumption when $\mu$ equals unity.

Let $g$ be the (endogenous) steady state growth rate of the economy, i.e., the common growth rate of $K_t$, $K_{1t}$, $K_{2t}$, $c_t$, $c_{1t}$, $c_{2t}$, $Y_t$, $S_t$, $w_t$. Using the CIA constraint (2.3) along with the supply of bonds and money given by (2.8) we can write the steady state inflation rate for cash goods as follows.

\begin{equation}
\pi_{1,t+1} \equiv \left(\frac{p_{1,t+1}}{p_{1,t}}\right) - 1 = \frac{1+x}{1+g} - 1.
\end{equation}

Using the above relationship together with the household's optimality conditions (3.1a) and (3.1b) we can derive the following equations for the nominal and the real interest rates.

\begin{align}
\left(3.6a\right) \quad 1 + R &= (1 + \rho)(1 + x)(1 + g)^{(\mu-1)}, \\
\left(3.6b\right) \quad r/(1+R) + (1 - \delta) &= (1 + \rho) (1 + g)^{\mu}.
\end{align}

where $\rho = (1-\beta)/\beta$ is the utility time preference rate.

Note that once the growth rate is determined, equations (3.5) and (3.6) determine the inflation rate, the real interest rate and the nominal interest rate. The growth rate $g$ is determined as follows. Equations (3.6) yield a relationship between
R and r which is determined by the money growth rate and the preference parameters. The production side of the economy determines a second relation between R and r, since R equals the relative price of the output of the two sectors and r is the return to capital which is one of the inputs. The growth rate g adjusts so that the relation between the nominal interest rate and the real interest rate implied by (3.6) is consistent with the relationship determined from the production side of the economy. A consequence of this feature of the model is that changes in the money growth rate (as well as changes in the technology parameters $\theta_1$ and $\theta_2$) have interesting level as well as growth effects. The relationships between the money growth rate, inflation rate, nominal interest rate, output growth rate and the real interest rate are different and more interesting than the usual Fisherian relationship.

As is clear from the above discussion a key relationship on which the properties of the model hinge is the relationship between the nominal interest rate (representing the relative price of the goods producing and the credit services producing sectors) and the real interest rate (representing the price of capital services) arising from the production side of the economy. This relationship, in turn, depends on the relative capital intensities of the two sectors. For instance, if the goods producing sector is more capital intensive, then an increase in the nominal interest rate will (assuming diversification) raise the wage rate and lower the rental to capital independently of factor supplies.

For the rest of this paper we consider the case where the goods producing sector is more capital intensive.\textsuperscript{11} We characterize the steady state of the economy using figures 2A–2C that show the production side of the economy. The production possibility frontier (PPF) between production goods and credit services (2A), the PPF between cash goods and credit goods (2B), the relation between the
factor returns \( r \) and \( w \) and the nominal interest rate \( R = p_g/p_1 \) (2C).

From 2A it is clear that a higher nominal interest rate implies a higher output of credit services relative to goods. When the goods producing sector is more capital intensive a decrease in the nominal interest rate will (assuming that both goods are produced) increase the real interest rate and decrease the wage rate independently of factor supplies (see figure 2C).

Figure 3 shows the determination of the nominal and the real interest rate in steady state. The downward sloping line is from the production side of the economy given in Figure 2C. The upward sloping line in these figures represents the relationship between the real and the nominal interest rates obtained by eliminating the growth rate \( g \) from equations (3.6). This relationship is characterized by the following equation,

\[
(3.7) \quad r = (1+R) \{ (1+p) [ \frac{(1+R)}{(1+x)(1+p)} ]^{\frac{\mu}{\mu-1}} - (1-\delta) \}.
\]

Assuming that the risk aversion coefficient \( \mu \) exceeds unity we get the upward sloping line in Figure 3. The intersection of the two lines determines the real and the nominal interest rates. Equations (3.5) and (3.6) then determine the real growth rate and the inflation rate. This completes the description of the steady state determination of the economy.

4. Permanent Stabilization

Using the above model we define a permanent stabilization as a reduction in the growth rate of money, \( x \). We discuss the impact of a reduction in the growth rate of money as a comparison between two steady states, where we start with a higher \( x \). Next we consider the case when the change in \( x \) is a shock to a highly
persistent process for the money growth rate. For the second part of the analysis we use a log-linear approximation around the non-stochastic steady state growth path of the economy.

**Steady State Stabilization**

Figure 3 shows the effects of a permanent change in the money growth rate, \( x \), on the nominal interest rate and the real interest rate. As can be seen from figure 3 a decrease in the money growth rate lowers the nominal interest rate and raises the real interest rate. Equation (3.6b) then shows that the real growth rate \( g \), increases. Figure 2C shows that the real wage decreases. From equation (3.5) it is clear that the inflation rate is reduced by more than the decrease in the money growth rate (since the real growth rate raises). Further, from (3.6a), the nominal interest rate decreases by less than the decrease in the money growth rate for the same reason.

The decrease in the nominal interest rate leads to a decrease in the output of credit services and credit goods and to an increase in the output of goods as well as cash goods (see figures 2A and 2B). This is accomplished by a movement of labor out of the credit services sector (which is labor intensive).

These features of the model are quite consistent with the observations on the growth of the credit services sector during periods of accelerating inflation and the decrease in the size of the sector following the stabilization. In addition, the results are consistent with the observations on the growth rate of output and the real interest rate before and after the stabilization date.

The impact on currency velocity (denoted \( v \)) can be seen from the following expression.
(4.1) \[ v_t = \frac{(p_{1t}c_{1t} + p_{2t}c_{2t})}{M_{t+1}} = 1 + \frac{p_{2t}c_{2t}}{p_{1t}c_{1t}} \]

= \frac{1 + (1 + r_t)c_{2t}}{c_{1t}}

where \((p_{1t}c_{1t} + p_{2t}c_{2t})\) is nominal GNP. The second equality in (4.1) is derived by using the CIA constraint (2.3) together with (2.8), (2.9g) and (3.1d). It is now easy to see that currency velocity must decrease with a reduction in the money growth rate, since the nominal interest rate as well as the relative output of credit goods go down.

Another aspect of this model is the effect of improvements in the technology of producing credit services and, thereby, credit goods. As can be seen in figure 2F an increase in \(\theta_2\) lowers the relationship between the real rate and the nominal rate. By combining this with the upward sloping line in figure 3, we see that an increase in \(\theta_2\) lowers the nominal interest rate, the real interest rate and the real growth rate, and raises the inflation rate. The relative output of credit services and credit goods goes up (see figure 2D) but the impact on currency velocity is ambiguous. These effects are consistent with the observation that technological improvements in credit arrangements have led to an increase in the quantity of credit services and credit goods and, by reducing the demand for currency, have resulted in an increase in inflation.

Stabilization Shock

The analysis of a stabilization shock requires a stochastic specification of the money growth rate rule together with a closed form solution for the equilibrium. For the solution we use a log-linear approximation around the steady state. In order to analyze the response of a high inflation economy to a shock in the money growth
rate we assume that the money growth rate is a first order Markov process that is close to a random walk. That is,

\[(4.2) \quad x_{t+1} = \rho_x x_t + (1-\rho_x) \bar{x} + e_t \]

where \(e_t\) is a white noise, \(\bar{x}\) is the mean money growth rate and \(\rho_x\) is close to unity (\(\rho_x = 0.99\)). The high inflation country has on average a monthly money growth rate of 10%. That is, we set the period to be one month and set \(\bar{x} = 0.10\).

We assume that the production functions \(F(.)\) and \(G(.)\) are both Cobb–Douglas with capital share parameters denoted by \(\alpha_1\) and \(\alpha_2\), respectively. Using data from the US we found (Aiyagari and Eckstein (1993)) that \(\alpha_1 = 0.41\) and \(\alpha_2 = 0.35\), and we use these values here.

The specification of the utility function has already been described in the previous section. We assume that the risk aversion coefficient \(\mu = 3\). The monthly depreciation rate of capital \(\delta\) is assumed to be 0.006 (annual 8% depreciation) and the monthly growth rate \(g\) is set to 0.0016 (annual growth rate of 2%). We use data on \(y/k_1\) to set \(r = \alpha_1 \frac{Y}{k_1}\) and this yield a value of \(r = 0.015\). We then set \(R\) using equation (3.6). Eliminating \((1+\rho)\) between (3.6a) and (3.6b) we obtain

\[(4.5) \quad \frac{r}{(1+R)} + 1 - \delta = (1+R) (1+g) / (1+x), \]

and we solve this equation for \(R\) and get a value of \(R = 0.11\).

The amount of time that consumers spend working is normalized to unity. We assume that for a high inflation country the value added share of credit services in GNP (denoted by \(\phi\)) is 4%. Note that \(\phi = p_8 S / (p_1 Y + p_8 S) = RS / (Y + RS) = Rc_2 / [c_1 + (1+R)c_2]\). The steady state values of some key variables in the model have been calculated using the steady state characterization of the economy in the
following way. The efficiency conditions (3.2a) and (3.2b) are used to calculate factor allocations and yield the following values.\textsuperscript{13}

\begin{align*}
(4.6a) \quad \frac{k_1}{k} & = 0.985, \text{ and } \frac{k_2}{k} = 0.015, \\
(4.5b) \quad n_1 & = 0.975, \text{ and } n_2 = 0.025, \\
\text{The shares of credit and cash goods in total goods are calculated using the above values of } \phi \text{ and } R, \text{ and yield the following values.}
\end{align*}

\begin{align*}
(4.7) \quad \frac{c_2}{c_1+c_2} & = 0.815, \quad \frac{c_1}{c_1+c_2} = 0.185.
\end{align*}

The above values imply a value for the ratio of credit goods to cash goods. This is used in (4.1) to calculate velocity and yields the following value.\textsuperscript{14}

\begin{align*}
(4.8) \quad v & = 1.25.
\end{align*}

Lastly, the steady state form of the resource constraint (2.9c) is used to calculate the share of consumption in total goods as follows.

\begin{align*}
(4.9) \quad \frac{c}{Y} & = 1 - (g+\delta) \left(\frac{k}{k_1}\right) \left(\frac{k_1}{Y}\right) = 0.77.
\end{align*}

In computing the impulse response dynamics of our endogenous growth model it is important to distinguish between level effects and growth effects of a transitory shock to the money growth rate. The variables we look at are divided into those that grow and those that are stationary. These are listed below as "growth variables" in set \(A_t\) and "level variables" in set \(B_t\).
(4.10a) \[ A_t = \{K_t, K_{1t}, K_{2t}, C_t, C_{1t}, C_{2t}, Y_t, W_t, P_{1t}, P_{2t}\}, \text{ (growth variables)}, \]
(4.10b) \[ B_t = \{\ln(n_{1t}), \ln(n_{2t}), R_t, r_t, v_t\}, \text{ (level variables)}. \]

Define the growth rate of a growth variable \( a_t \) by \( g_{at} = \ln a_t - \ln a_{t-1} \). Suppose that the shock hits the model economy at date 1 and let \( g^* \) be the steady state growth rate of this variable before the shock hits. Then, for each growth variable, we report the level effect at date 1 which is given by \( (g_{a,1} - g^*) \) and the growth rate effects for \( t > 1 \) which are given by \( (g_{a,t} - g^*)_t \).

Note that the aggregate capital stock \( K_t \) is predetermined at \( t \), so that it is not subject to a level effect but is only subject to a growth rate effect. Further, note that the level effect at date 1 should be interpreted as the percentage deviation from what the level of the variable would have been in the absence of the shock. This follows because \( g_{a,1} - g^* = [\ln a_1 - \ln a_0] - [\ln((1+g^*_0)a_0) - \ln a_0] = \ln a_1 - \ln((1+g^*_0)a_0) \). Moreover, the growth rate effect for \( t > 1 \) is given as a deviation from the steady state growth rate. Since we use a value of \( \rho_x \) that is close to one we can interpret the sign of the growth rate deviation as the direction of the impact of a permanent unexpected change in the growth rate of the money supply.

The level effect on a level variable \( b_t \in B_t \) is defined as \( (b_t - b^*) \) where \( b^* \) is the steady state level of this variable. Therefore, the effects on labor input in the goods producing sector \( (n_{1t}) \) and labor input in the credit services sector \( (n_{2t}) \) should be interpreted as percentage deviations from their steady state values. As we explained above, the signs of these changes should be interpreted as the signs of the impacts of a permanent change in the money growth rate.

Table 2 presents the impact of a negative shock to the money growth rate with a serial correlation of 0.99, that is, the shock is very close to being permanent. The dynamics of the response of all variables is exactly that of the shock so that the
persistence of the monetary shock implies persistence of each variable in the direction
of the initial response. The first column in table 2 presents the signs of the level
effects on variables in group A and the effects on all variables in group B. The
second column presents the signs of the growth rate effects on variables in group
A.

The immediate effect of a surprise reduction in the growth rate of money is as
we expect from the model. That is, the nominal interest rate decreases leading to a
reduction in credit goods production and the velocity of money. The inflation rate
obviously goes down but by more than the nominal interest rate so that the real
interest rate is going up. These responses are consistent with the wide observations
on these variables during the post stabilization period. In addition, the growth rate of
all real variables goes up, so that consumption level and growth rate are higher as
we usually observe in response to stabilization. Hence, the reduction in inflation has
a real impact on the long run path of the economy.

5. Concluding Remarks

In this paper we developed a monetary growth model designed to address
observations concerning changes over time in the relative size of the credit
and banking sector, growth rate of the economy, and co-movements among
inflation, currency velocity and the relative size of the credit and banking
sector. The model contains a new and hitherto unexplored channel for the long
run and the short run effects of changes in the money growth rate that can be
interpreted as part of stabilization programs. This channel arises because while
cash goods and credit goods are perfect substitutes in consumption and
investment we posit different technologies for their production.

Because of this feature the nominal interest rate has an allocative role
in our model since it determines the price of credit goods relative to cash goods. Due to this same feature the nominal interest rate also determines factor returns, i.e., the real wage and the real interest rate. As a consequence, the interrelationships among the money growth rate, inflation rate, nominal interest rate, real interest rate, and the real growth rate are different from those that arise in the usual monetary growth models. In particular, we can provide an economic interpretation for the observations on the size of the banking sector and the economic growth rate in economies which experienced a transition from high inflation to low inflation due to a monetary stabilization program. In particular, we show that a permanent decrease in the growth rate of money increases the growth rate of output, decreases the share of the credit and banking sector in GNP, increases the real interest rate and decreases inflation and the nominal interest rate.

So far we have provided a qualitative explanation for the observations. In future work we plan to extend the model quantitatively to analyze the business cycle implications during the period of stabilization. Furthermore, we are interested in exploring the extent to which technological innovations in the production of credit services and credit goods contribute to movements in inflation, interest rates, currency velocity and output.
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Table 1


<table>
<thead>
<tr>
<th>Year</th>
<th>Employment in banks as % of total</th>
<th>Teller machines per 100 people</th>
<th>Area of banks per 1000 people</th>
<th>Inflation (annual %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1968</td>
<td>1.4</td>
<td>0.00</td>
<td>75.8</td>
<td>1.9</td>
</tr>
<tr>
<td>1969</td>
<td>1.5</td>
<td>0.00</td>
<td>74.4</td>
<td>3.9</td>
</tr>
<tr>
<td>1970</td>
<td>1.5</td>
<td>0.00</td>
<td>74.5</td>
<td>10.1</td>
</tr>
<tr>
<td>1971</td>
<td>1.6</td>
<td>0.00</td>
<td>74.4</td>
<td>13.4</td>
</tr>
<tr>
<td>1972</td>
<td>1.7</td>
<td>0.00</td>
<td>73.3</td>
<td>12.4</td>
</tr>
<tr>
<td>1973</td>
<td>1.8</td>
<td>0.00</td>
<td>73.5</td>
<td>26.4</td>
</tr>
<tr>
<td>1974</td>
<td>1.9</td>
<td>0.00</td>
<td>73.4</td>
<td>56.2</td>
</tr>
<tr>
<td>1975</td>
<td>2.0</td>
<td>0.00</td>
<td>76.4</td>
<td>23.5</td>
</tr>
<tr>
<td>1976</td>
<td>2.1</td>
<td>0.00</td>
<td>76.6</td>
<td>38.0</td>
</tr>
<tr>
<td>1977</td>
<td>2.2</td>
<td>0.02</td>
<td>77.1</td>
<td>42.5</td>
</tr>
<tr>
<td>1978</td>
<td>2.3</td>
<td>0.03</td>
<td>77.3</td>
<td>48.1</td>
</tr>
<tr>
<td>1979</td>
<td>2.5</td>
<td>0.04</td>
<td>77.4</td>
<td>111.4</td>
</tr>
<tr>
<td>1980</td>
<td>2.7</td>
<td>0.06</td>
<td>84.2</td>
<td>133.0</td>
</tr>
<tr>
<td>1981</td>
<td>2.7</td>
<td>0.08</td>
<td>92.1</td>
<td>101.5</td>
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<tr>
<td>1982</td>
<td>2.8</td>
<td>0.10</td>
<td>98.7</td>
<td>131.5</td>
</tr>
<tr>
<td>1983</td>
<td>2.8</td>
<td>0.12</td>
<td>103.2</td>
<td>190.7</td>
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<td>1984</td>
<td>2.7</td>
<td>0.12</td>
<td>103.6</td>
<td>444.9</td>
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<tr>
<td>1985</td>
<td>2.5</td>
<td>0.12</td>
<td>105.6</td>
<td>185.2</td>
</tr>
<tr>
<td>1986</td>
<td>2.4</td>
<td>0.13</td>
<td>103.1</td>
<td>19.6</td>
</tr>
<tr>
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<td>0.13</td>
<td>101.5</td>
<td>16.1</td>
</tr>
<tr>
<td>1988</td>
<td>2.1</td>
<td>0.13</td>
<td>99.1</td>
<td>16.4</td>
</tr>
<tr>
<td>1989</td>
<td>2.1</td>
<td>0.13</td>
<td>100.8</td>
<td>20.7</td>
</tr>
</tbody>
</table>

* Sources: Annual Statistics of Israel’s Banking system
  Supervisor of Banks, Bank of Israel
  Central Bureau of Statistics
  (Taken from Melnick (1993))
Table 2
Response to a Negative Shock in The Money Growth Rate
(deviations from steady state)

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>LEVEL EFFECT</th>
<th>GROWTH RATE EFFECT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_t$</td>
<td>NA</td>
<td>+</td>
</tr>
<tr>
<td>$K_{1t}$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$K_{2t}$</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$c_t$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$c_{1t}$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$c_{2t}$</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$Y_t$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$w_t$</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>$p_{1t}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$p_{2t}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\ln(n_{1t})$</td>
<td>+</td>
<td>NA</td>
</tr>
<tr>
<td>$\ln(n_{2t})$</td>
<td>-</td>
<td>NA</td>
</tr>
<tr>
<td>$R_t$</td>
<td>-</td>
<td>NA</td>
</tr>
<tr>
<td>$r_t$</td>
<td>+</td>
<td>NA</td>
</tr>
<tr>
<td>$v_t$</td>
<td>-</td>
<td>NA</td>
</tr>
</tbody>
</table>

*NA – not applicable


**FOOTNOTES**

1. The main evidence includes the following: (i) average inflation accelerated towards the stabilization date and then abruptly fell to a small positive rate; (ii) velocity (real balances) increases (decrease) towards the stabilization date and then decreases (increase). After stabilization the level of velocity (real balances) is higher (lower) than the level before the high inflation period; (iii) seignorage and government deficit seem to have almost no trend during the high inflation period and both are reduced to zero during the post stabilization period; (iv) during the period after stabilization ex-post real interests rate are extremely high (see, e.g., Bental and Eckstein (1989), Vegh (1992) and Bruno (1993) among many others).

2. Bruno (1993) and Vegh (1992) provided a comprehensive discussion of the facts concerning the recent episodes and the 1920's experiences of high inflations and stabilizations. They distinguish between short inflation episodes, such as those of the 1920's and Bolivia, vis-a-vis chronic episodes, such as Israel and Argentina. Vegh agrees (Bruno does not agree) with Sargent's claims concerning the facts about the real consequences of high inflation stabilizations.

3. Garber (1982) and Wicker (1986) provided evidence on sectoral shifts of employment and some delayed unemployment in Germany, Austria and Hungary in the 1920's. They claim that their evidence contradicts the main claim of Sargent (1982) that there is no evidence of a stable trade-off between inflation and unemployment ("Phillips-Curve"). However, Sargent (1982), following Graham (1930), cites the evidence on short run sectoral shifts of employment and capital but emphasized Graham's (1930) conclusion that aggregate industrial output in Germany increased in the post stabilization period and was decreasing during the period of accelerating inflation.

4. Leiderman (1993, p.7) concludes that the data "does not conform well with the
notion of a short-run tradeoff between inflation and unemployment".  

Melnick and Yashiv (this volume) make a similar point about the relationship between the banking sector in Israel and the inflation process. We do not have direct evidence on the banking sectors in Argentina and Bolivia during the high inflation period and the post stabilization era. However, casual evidence supports the pattern we reported above for the other countries.

Wicker (1986) claimed that the fact that we observe substantial sectoral shifts in employment, and movements in and out of unemployment in the post stabilization period is inconsistent with the rational expectations hypothesis. The model developed here makes it clear that this claim is incorrect. In fact, Sargent (1982), at the end of the introduction, points out that sectoral shift of inputs as a result of the stabilization should be expected in a general equilibrium rational expectations model.

Gillman (1993) followed the same approach to analyze the welfare cost of inflation. Ireland (1994) developed a growth model with transaction costs to analyze the trend in $M_1$ and $M_2$ velocities and the welfare cost of inflation. (See also Lacker and Shreft (1993) and Marquis and Raffett (1993)).

Thus, the technology of providing credit services is one of the determinants of the nominal interest rate. This feature of our model is similar to that of Bryant and Wallace (1979).

Another advantage of our modeling strategy is that we do not arbitrarily designate consumption goods as cash goods and investment goods as credit goods as in, for example, Cooley and Hansen (1989). Both cash goods and as well as credit goods may be used for consumption and/or investment.

The following specification is related to that in Fischer (1983).

Empirically, the production function of financial services is more labor intensive than
the aggregate production function.

12In the case that the capital intensities in production are identical for the two sectors, the PPF is flat and the price of financial services and the nominal interest rate are determined by technology. The downward sloping line in Figure 3 is vertical, and, hence, changes in the money growth rate affect the real interest rate and the growth rate, but not the nominal interest rate. In this case the model becomes similar to that of Bryant and Wallace (1979).

13Equation (3.2a) can be written as: \((1-\alpha_1)Y/n_1 = R (1-\alpha_2)S/n_2\). Using the definition of \(\phi\), this implies \(n_1 = 1/[1+(1-\alpha_2)\phi/\{(1-\alpha_1)(1-\phi)\}]\). Further, \(n_2 = 1 - n_1\). Equation (3.2b) can be written as \(\alpha_1 Y/k_1 = R\alpha_2 S/k_2\). Using the definition of \(\phi\), this implies \(k_1/k = 1/[1+\alpha_2\phi/\{\alpha_1(1-\phi)\}]\). Further, \(k_2/k = 1-k_1/k\).

14Note that velocity, \(v = (p_{1t}c_{1t} + p_{2t}c_{2t})/M_{t+1} = (p_{1t}c_{2t} + p_{2t}c_{2t})/p_{1t}c_{1t} = 1 + (1+R)c_2/c_1\).
FIGURE 1

TOTAL NUMBER OF BANK ACCOUNTS
ISRAEL, ALL TYPES

ACCOUNTS (Millions)

Figure 2

2a) 

2b) 

2c) 

2d) 

2e) 

2f)