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NATIONAL MONETARY POLICIES IN A WORLD ECONOMY: A ROLE FOR COOPERATION

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For several years, other countries have complained that a tight U.S. monetary policy has been adversely affecting their economies. Some have suggested--presumably, as at least a partial remedy--that the U.S. and other countries cooperate in choosing their monetary policies. In this paper, we present a qualitative model of a world economy that is consistent with these views. In it a tight monetary policy in one country can adversely affect the economies of other countries. And, as we explain below, the model implies that there is a role for cooperation among countries in choosing monetary policies.

The model we use is a many-country version of the single-country model described in Wallace (1984). It differs from other relatively simple multi-country models primarily in two ways. First, policies and their corresponding outcomes are described by equilibrium time paths, rather than by actions and outcomes at a single instant. Second, the outcomes under different policies are related explicitly to the well-being of people in different circumstances.

Since each country in the model is populated by groups that are affected differently by different monetary policies, we analyze the role of cooperation under the assumption that each country has a social welfare function. This function, the arguments of which are the utilities of individuals in the country, expresses how competing interests within a country are weighted in arriving at decisions. It is in terms of such social welfare functions that we find a role for cooperation in choosing monetary policies. In particular, we show that cooperation can produce a higher value of each country's social welfare function than results from noncooperation. Cooperation does not, however, make each person in each country better off.

We end up analyzing cooperation in a fairly special model. That suffices for making our point, that there is generally a role for cooperation, because the special model does not prejudice the results in favor of a role for cooperation. The use of a special model has the advantage of allowing us to keep the presentation relatively simple.[1/]

The main presentation is in four parts. In part one, we review the description of a single country found in Wallace (1984). In part two, we describe a world consisting of several countries and state the conditions for world equilibrium for arbitrary monetary policies. In part three, we study a special case in which the savings behavior of individuals in each country takes a particularly simple form. For that case, we describe in detail how one country is affected by other countries' monetary policies. We also show that a country cannot through its own monetary policy fully offset the effects on its residents of other countries' policies. Then, in part four, we analyze the role of cooperation in the special case. We show that if countries are identical, then noncooperation, which is modelled as a Nash equilibrium, gives rise in general to a lower value of each country's social welfare function than does cooperation.

I. Description of a Single Country

As noted above, for each country k, k = 1, 2, ..., K, we adopt the description used by Wallace (1983). We first describe private behavior in a single country and then the policy options of the government of a single country.

Private Demands and Supplies

Each country is populated by overlapping generations, the members of which live two periods. At each date t (where t is an integer) a new generation, generation t, appears. Its members are present in the economy at t (when they are young) and at t + 1 (when they are old). We assume that people do not move between countries, a standard assumption in models of international trade.

There is a single good at each date, the time t good, that is common to all countries.[2/] This good can be costlessly and instantly transported from any one country to any other country. There is, however, no production; time t good cannot be produced or used to produce time t' good where t' ≠ t.

Each member of generation t has preferences over bundles of time t and time t + 1 good, preferences that are representable by a utility function or an indifference curve map of the usual sort. Each such member also has an income stream or endowment consisting of some amount of the time t good and some amount of the time t + 1 good.

We assume that different generations are identical and that within each country there is a special kind of diversity within each generation. Each generation consists of two groups of people. Members of one group, called lenders (or savers), are identical and have preferences and endowments that lead them to want to lend (or save) at most rates of return. Members of the other group, called borrowers (or dissavers), are also identical and have preferences and endowments that lead them to want to borrow (or dissave) at most rates of return.

These assumptions imply that the competitive desired trades by the members of each group in country k can be described as functions of the terms of trade between the time t good and the time t + 1 good faced by the members of each group. We let $S^k(\ )$ denote the aggregate supply function or curve of the time t good (or desired saving or lending) on the part of the lender group of generation t in country k and let $D^k(\ )$ denote the aggregate demand curve for the time t good (or desired dissaving or borrowing) on the part of the borrower group for generation t in country k. In each case, the argument of the function is the intertemporal terms of trade which we express by the price of time t good in units of time t + 1 good (the gross real rate of return) faced by the members of the respective group. In general, as will we see, lenders, who are subject to a reserve requirement, face a different and lower rate of return than borrowers. Examples of possible $S^k(\ )$ and $D^k(\ )$ functions are shown in Figure 1. We assume that $D^k(\ )$ is downward sloping where it is positive.[3]

Since we will be describing how this economy evolves over time from the initial or current date which we label t = 1, in addition to the above description of the competitive behavior of the young of each generation in country k, we need a description of the behavior of the people who are in the second period of their lives at t = 1, the initial or current old. We assume that their preferences are such that they try to consume as much time 1 good as they can and that they are endowed with or start with some of the time 1 good and some nominally denominated assets. Their implied competitive behavior is very simple: they supply all their assets at any positive price in terms of the time 1 good.

The above assumptions imply the following simple relationship between prices, including rates of return, and the well-being of individuals in country k: the initial old are better off the more valuable are their nominally denominated assets at t = 1; lenders or savers in any generation t for t > 1 are better off the higher the rate of return they earn on savings; borrowers in any generation t for t > 1 are better off the lower the rate of return at which they can borrow. It is in terms of these relationships that we will describe how one country's policy affects other countries and how cooperation can improve welfare.

Government Policy

As in the one-country version of the model, we describe government policy in terms of sequences of the real government net-of-interest deficit and in terms of the way the real net-of-interest deficit is financed; the combination of additions to the monetary base and to interest-bearing government debt that finance the net-of-interest deficit.[4/] Consistent with this we write the cash flow constraint of country k's combined fiscal and monetary authority as

$$(1) \qquad G_t^k = p_t^k(H_{t+1}^k - H_t^k) + p_t^k(P_t^k B_{t+1}^k - B_t^k)$$

which must hold for all dates $t \geq 1$. Here G_t^k, measured in units of the time t good, is government k's real deficit net of interest payments. The first term on the right-hand side of equation (1) is the value in terms of time t good of government k's addition to its outstanding monetary base and the second term is the value of its addition to its outstanding debt, which consists of one-period, zero coupon (pure discount) bonds. Specifically, the variables on the right-hand side of equation (1) are defined in this way:

H_t^k = The country k monetary base (or high-powered money) that generation t - 1 starts with at time t.

p_t^k = The time t price of a unit of the country k monetary base in units of the time t consumption good ($1/p_t^k$ = the country k price level at time t).

B_t^k = The nominal face value, in terms of the country k monetary base, of the country k maturing government bonds owned by members of generation t - 1 at t.

P_t^k = The price at time t, in terms of the country k monetary base, of a bond which pays one unit of the country k monetary base at t + 1. (The country k nominal interest rate at t is $(1/P_t^k) - 1$.)

To insure that the monetary base of country k has value in equilibrium and to insure that there is a possibility that bonds bear nominal interest in equilibrium ($P_t^k < 1$), we assume that country k imposes (and is able to costlessly enforce) the following kind of reserve requirement on its residents. Any resident of country k that saves a positive amount must hold a fraction λ^k of such saving in the form of country k base money. As will be explained further below, this requirement implies that the gross real rate of return faced by country k lenders at date t is the following weighted average, $\bar{r}_t^k = \lambda^k R_t^k + (1-\lambda^k)r_t$, where R_t^k is the gross real rate of return on the country k monetary base, namely, p_{t+1}^k/p_t^k, and r_t is the gross real rate of return on loans, there being a single real return on loans in all countries as described below.

As discussed more fully in Wallace (1984), this reserve requirement is intended to capture in a simple way the role played by legal restrictions on private borrowing and lending in actual economies. Taken literally, it is an accurate description of an economy in which all private lending or saving by residents must take the form of bank or financial intermediary accounts against which there is a reserve requirement in the form of each country's base money and in which such institutions are in other respects free to hold assets in any form they want. If these institutions operate competitively and costlessly, then the rate they pay on their liabilities, their deposits, is a weighted average of the rate they earn on reserves and the rate they earn on loans, the weighted average described above as that facing private lenders.

This completes the description of the typical country in our model of many countries. We now turn to a description of how countries interact in our model of a world economy.

II. World Equilibrium Taking Policies as Given

Before turning to a formal description of the conditions for equilibrium in a world consisting of K countries, each of which satisfies the assumptions made above, we want to describe some conditions on prices and interest rates, arbitrage conditions, that are implicit in the description of individual trading opportunities that we gave above. The first involves arbitrage between goods and monies and the second involves arbitrage among securities.

We noted above that we are assuming that the single good in our world economy can be costlessly transported between countries. The first arbitrage condition--called purchasing power parity--is that the prices of the monies of any two countries in terms of the good and the exchange rate between the two monies are such that there are no gains from the following set of transactions: selling goods in country k, using the resulting country k money to buy country k' money, and using the country k' money to buy the good in country k'. As the reader can verify, the condition that no gain be made from such transactions is that the exchange rate, the price of country k money in units of country k' money at date t, be equal to the following ratio of prices of monies, $p_t^k/p_t^{k'}$.

The second arbitrage condition is that interest rates facing borrowers in the K countries of our world economy are such as to imply the same terms of trade at any date t between the time t good and the time t + 1 good. It is implied by the assumption that anyone in any country can borrow and lend in any other country subject only to the reserve requirement on those doing positive saving. Under this assumption, we cannot have an equilibrium in which the real return on loans in one country exceeds that in another country because in such a situation no saver would want to hold securities bearing the lower return (a demand which is consistent with reserve requirements) and every borrower would want to borrow at the lower rate. Together, these imply an excess supply of the securities bearing the lower return and an excess demand for those bearing the higher return, which, of course, cannot be an equilibrium. Thus, the assumption that individuals and governments can borrow and lend anywhere subject only to the reserve requirements described above implies that there is a single real rate of return on loans in our world economy. In the notation introduced above, it implies a single rate of return r_t.

For prices, including interest rates and exchange rates, that satisfy these arbitrage conditions, the real trading opportunities facing individuals are those described earlier; essentially, trading present consumption for future consumption or, equivalently, trading present consumption for securities which are promises to future consumption.

With these preliminaries out of the way, we are ready to describe what we mean by a competitive, perfect foresight, equilibrium for this world economy. Competitive means that people treat prices as beyond their control when they choose quantities. Perfect foresight in our context means that anticipated rates of return on assets equal actual or realized rates of return or, more particularly, that at each date t the young correctly anticipate the price of the monies of the different countries in terms of goods at the next date. Equilibrium means that all markets clear at each date. From now on, we will refer to a competitive, perfect foresight equilibrium as simply an equilibrium. We first set out and describe equilibrium conditions generally. Then we describe the special conditions for an equilibrium in which real variables are unchanging over time, an equilibrium we call a stationary equilibrium. The latter are the conditions which we will use subsequently.

The definition of equilibrium that we give below is valid only for values of r_t for which the borrowers of each country actually borrow, or, more precisely, only for values of r_t for which $D^k(r_t) > 0$ for every k. The following notation allows us to state this condition concisely. Let $\tilde{r}^k$ be such that $D^k(\tilde{r}^k) = 0$ and let $\tilde{r}$ be the smallest of the $\tilde{r}^k$, $k = 1, 2, \ldots, K$. Then $D^k(r) > 0$ for all k if $r < \tilde{r}$. This condition appears as part of the following definition of an equilibrium.

<u>Definition</u>. Given each country's reserve requirements, λ^k, its initial nominal indebtedness including its base money, $H_1^k + B_1^k$, a total which is assumed positive, and sequences for its net-of-interest real deficit, G_t^k, and a sequence for its base money, H_{t+1}^k, an equilibrium consists of a sequence for r_t satisfying $r_t < \tilde{r}$ and sequences for each country for p_t^k, P_t^k, R_t^k, $\bar{r}_t^k$, and B_{t+1}^k that for all $t > 1$ satisfy equation (1), the cash flow constraint for each country and

(2) $$\sum_k [S^k(\bar{r}_t^k) - D^k(r_t)] = \sum_k [p_t^k(H_{t+1}^k + P_t^k B_{t+1}^k)]$$

(3) $$\bar{r}_t^k = \lambda^k R_t^k + (1-\lambda^k) r_t$$

(4) $$R_t^k = p_{t+1}^k / p_t^k$$

(5) $$r_t = p_{t+1}^k / p_t^k P_t^k$$

(6) $$r_t \geq R_t^k$$

(7) $$p_t^k H_{t+1}^k \geq \lambda^k S^k(\bar{r}_t^k)$$

where equations (3)-(7) must hold for each country k and where for each country k either (6) or (7) must hold at equality.

Equation (2) says that world net private saving--the sum over countries of each country's saving supplied at the weighted average of the return on its base money and the return on securities less each country's private borrowing--must equal the total value, a total over countries, of government liabilities. Equations (5), (6) and (7) define the returns facing savers and borrowers in each country and contain our perfect foresight assumptions--namely, that the returns that determine choices at t match the actual returns. Note that (5) implies that the ratio of a country's gross inflation rate, p_t^k/p_{t+1}^k, to its gross nominal interest rate, $1/P_t^k$, is the same for all countries. Inequalities (6) and (7) and the accompanying proviso are related to the reserve requirement. Inequality (6) says that the return on loans is at least as great as that on base money of each country. If it were not, then unlimited gains could be made by borrowing and using the proceeds to acquire base money, activities which would not violate the reserve requirement. That being so, no equilibrium can violate (6). Inequality (7) expresses the reserve requirement: the value of country k base money must be at least as great as the reserve requirement times gross saving of the residents of country k. The proviso arises in this way. If $r_t > R_t^k$ then wealth maximization implies that country k residents and everyone else hold no more of country k base money than the minimum required, which is to say that (7) holds at equality. Alternatively, if the value of country k's base money exceeds the minimum required to be held $(p_t^k H_{t+1}^k > \lambda^k S^k(\bar{r}_t^k))$, then wealth maximization implies that the return on country k base money is as great as the return on securities, which is (6) at equality.[5/]

Instead of trying to study all possible equilibria for this economy for arbitrary sequences for government policies, we study a limited class of policies and of potential equilibria under those policies. We study only policies for which each country's net-of-interest deficit is a constant, $G_t^k = G^k$ and each country's ratio of government bonds to base money is a constant, $B_{t+1}^k/H_{t+1}^k = \beta^k$. For such policies, we attempt to describe only those equilibria for which all real variables are constant over time, equilibria we call stationary equilibria. For such policies, we formally define a stationary equilibrium as follows.

<u>Definition</u>. Given λ^k, $H_1^k + B_1^k > 0$, G^k and β^k for each k, a stationary equilibrium consists of a scalar $r < \tilde{r}$ and of scalars R^k, $\bar{r}^k$, h^k, b^k, and p_1^k, for each k, where h^k denotes a constant

real value of the country k monetary base, $p_t^k H_{t+1}^k$, and b^k denotes a constant real value of the government bonds of country k, $(p_t^k P_t^k B_{t+1}^k)$, that satisfy

(8) $$G^k = (1-R^k)h^k + (1-r)b^k$$

(9) $$\sum_k [S^k(\bar{r}^k)-D^k(r)] = \sum_k (h^k+b^k)$$

(10) $$\bar{r}^k = \lambda^k R^k + (1-\lambda^k)r$$

(11) $$r \geq R^k$$

(12) $$h^k \geq \lambda^k S(\bar{r}^k)$$

(13) $$G^k = h^k + b^k - p_1^k(H_1^k+B_1^k)$$

where either (11) or (12) must hold with equality.

Note that equation (8) is the stationary version of the country k cash flow constraint, equation (1), and that (13) comes from that constraint for the first date, t = 1. For constant real sequences, this definition of an equilibrium and the earlier one are equivalent.

Below we make assumptions that imply that stationary equilibria are necessarily binding stationary equilibria, ones for which (12) holds at equality. We focus on binding stationary equilibria because we suspect that those are the relevant cases for the current world economy.[6] Our approach to studying binding stationary equilibria is to solve equations (8)-(10) and equation (12) at equality for the h^k, b^k and the rates of return, and then to verify that the implied solution satisfies equation (11). If it does, and if that solution also implies a positive p_1^k using equation (13), then it is a valid solution.

If equilibria are binding, we can reduce equations (8)-(10) and (12) (at equality), 3K + 1 equations, to K + 1 equations in K + 1 unknowns, r and the R^k. From the definitions of h^k and b^k, we have

$$(14) \qquad b^k/h^k = \beta^k P_t^k$$

Since, by (4) and (5), $P_t^k = R_t^k/r_t$, a constant in a stationary equilibrium, we can rewrite (14) as

$$(15) \qquad b^k = h^k \beta^k R^k/r$$

Then, upon substituting the right-hand side of (15) and (12) at equality into (8) and (9) we have, respectively,

$$(16) \qquad G^k = \lambda^k S^k(\bar{r}^k)[(1-R^k)+(1-r)\beta^k R^k/r]$$

$$(17) \qquad \sum_k [S^k(\bar{r}^k)-D^k(r)] = \sum_k [\lambda^k S^k(\bar{r}^k)(1+\beta^k R^k/r)]$$

If we use (10) to replace $\bar{r}^k$ by the weighted average of R^k and r then the resulting versions of equations (16) and (17) are the K + 1 equations in the K + 1 unknowns, r and R^k for each k, that we referred to above. Moreover, as noted above, if the solution for these K + 1 equations satisfies (11) and is such that (13) can be solved for a positive p_1^k for each k, then the solution is a valid binding equilibrium.

III. World Equilibrium in a Special Case

Since (16) and (17) are complicated equations for general functions $S^k(\ )$ and $D^k(\ )$, we will study in detail only a special case of the model, one in which each $S^k(\ )$ function is a constant, denoted S^k, which does not depend on the return, $\bar{r}^k$. 7/ This case is easy to study because for it, as we now show, equations (16) and (17) can be rewritten as a set of completely recursive equations, equations which can be solved one at a time.

We begin by solving equation (16) for R^k/r, obtaining

$$(18) \qquad R^k/r = (1-G^k/\lambda^k S^k)/[r(1+\beta^k)-\beta^k]$$

Solving (16) in this way is valid if $r(1+\beta^k) - \beta^k \neq 0$. Below we present conditions that insure that the implied solution for r is such that this holds. Then, if we substitute the right-hand side (RHS) of (18) into the RHS of (17) and at the same time impose the constant saving assumption, we can write the result as

$$(19) \qquad E(r) = F(r,\beta)$$

where

$$E(r) \equiv \sum_k [(1-\lambda^k)S^k - D^k(r)]$$

$$F(r,\beta) \equiv \sum_k \{(\lambda^k S^k - G^k)\beta^k/[r(1+\beta^k)-\beta^k]\}$$

and where $\beta = (\beta^1, \beta^2, \ldots, \beta^K)$. Note that $E(r)$, an increasing function of r, is the world private excess demand for securities if the reserve requirement is binding in every country. The function $F(r,\beta)$ can be interpreted as the supply of securities by all the governments, a supply implied by the stationary versions of their cash-flow constraints, bindingness of all the reserve requirements, and the choices of government portfolios, the β^k. If equation (19), which contains only one unknown, r, can be solved, then its solution can be used in equation (18) to find R^k. It can also be used to find p_1^k, the country k initial value of base money, from the following equation,

$$(20) \qquad p_1^k = (\lambda^k S^k - G^k) r (1+\beta^k) / [r(1+\beta^k) - \beta^k]$$

Equation (20) is obtained from (13)--with $H_1^k + B_1^k = 1$--by substituting for b^k and h^k from (15) and (12) at equality and for R^k/r from (18).[8/]

The propositions we want to establish, mainly about solutions to equations (18)-(20), are implied by the following assumptions:

A.1 $\qquad \lambda^k S^k > G^k \geq 0$ and $\beta^k > -1$ for all k

A.2 $\qquad \sum_k [S^k - D^k(1)] < 0$

A.3 $\qquad \tilde{r} > 1$ and $E(\tilde{r}) > [\sum_k (\lambda^k S^k - G^k)]/(\tilde{r}-1)$

A.4 $\qquad rD'(r)/D(r) < -1$ where $D(r) \equiv \sum_k D^k(r)$

The first part of A.1 places bounds on the net-of-interest deficit; the upper bound is such that the deficit can be financed with $\beta^k = 0$; the lower bound says that net-of-interest, the budget is not in surplus. The second part of A.1 limits ratios of government debt to base money to those that keep the sum of the monetary base and the face value of government debt positive. Assumption A.2 says that net private saving is negative at $r \leq 1$, namely at negative and zero real interest rates. These two assumptions have the following consequence.

<u>Proposition 1</u>. Under A.1 and A.2, any stationary equilibrium has $r > 1$ and is a binding equilibrium.

(Proofs of this and succeeding propositions appear in Appendix A.)

Assumption A.3 assures that we get a binding equilibrium with $r < \tilde{r}$. It assures that no matter how large are the β^k--that is, that no matter how tight monetary policies are--there is an equilibrium with $r < \tilde{r}$. 9/ Note that if countries are identical, so that, among other things $D^k(\tilde{r}) = 0$ for all k, then assumption A.3 is implied by the simple condition $(1-\lambda) > \lambda/(\tilde{r}-1)$.

<u>Proposition 2</u>. Under A.1-A.3, there exists a binding equilibrium with $r < \tilde{r}$.

Proposition 2 leaves open the possibility that there are several solutions to equation (19) and, hence, several binding equilibria with $r < \tilde{r}$. The next proposition shows that the elasticity condition A.4 rules out this possibility.

<u>Proposition 3</u>. Under A.1-A.3 and either A.4 or the existence of an equilibrium with $F(r,\beta) \geq 0$, equation (19) has a unique solution with $r < \tilde{r}$.

The arguments in the proofs of Propositions 2 and 3 imply that the functions $E(r)$ and $F(r,\beta)$ are essentially as shown in Figure 2. In particular, under the assumptions of Proposition 3, we can define the unique value of $r < \tilde{r}$ that satisfies (19) as a function of β, say

$$(21) \qquad r = \phi(\beta)$$

Then, by direct substitutions into (18) and (20), we get the corresponding unique solutions for R^k and p_1^k. We now use these solutions to describe how one country is affected by another country's monetary policy.

The dependence of r on each of the β^k can be described using Figure 2. It is straightforward to verify that the function F is increasing in each of the β^k.[10/] Thus, the greater is any β^k, the higher is the function F in Figure 2 and, therefore, the greater is r. In other words, the greater the ratio of government bonds to base money in any one country, the greater the real rate of return, r.[11/] That result and (18) and (20) imply that a tightening in another country's monetary policy has the following effects on country k.[12/]

Effects in Country k of a Tightening in Another Country's Monetary Policy

	If government is debtor, $\beta^k > 0$	If government is creditor, $\beta^k < 0$
Gross real interest rate (r)	Increases	Increases
Gross nominal Interest Rate ($r/R^k = 1/P^k$)	Increases	Increases
Gross inflation Rate ($1/R^k = p_t^k/p_{t+1}^k$)	Increases	Decreases
Price Level ($1/p_1^k$)	Increases	Decreases

It follows immediately that if the government of country k is a creditor ($\beta^k < 0$), a tightening in the monetary policy of another country increases the welfare of country k's initial old (p_1^k increases), increases the welfare of country k's savers (R^k and r increase), and decreases the welfare of country k's borrowers (r increases). On the other hand, if the government of country k is a debtor ($\beta^k > 0$), the welfare of its initial old decreases (p_1^k decreases), the welfare of its savers can either increase or decrease (R^k decreases while r increases), and the welfare of its borrowers decreases (r increases).

Such effects of another country's monetary policy would not be grounds for complaint if country k could simply offset all those effects by its choice of monetary policy. It is straightforward to show, however, that it cannot. Thus, even if it were able to decrease β^k by enough to offset the effect on r and, hence, on its borrowers, then it follows from (20) that p_1^k would decrease, which would hurt its initial old.[13] This is one sense in which our model implies that a tighter monetary policy in one country has adverse effects on other countries.

IV. Noncooperation versus Cooperation in Choosing Monetary Policies

As shown in the last section, countries interact in our model in the sense that one country's choice of a monetary policy affects residents of other countries. Here we consider whether and in what sense that interaction implies a role for cooperation among countries in choosing monetary policies, the β^k. We address this question by comparing what happens if there is cooperation in choosing monetary policies with what happens if there is not. We make the comparison using the following definitions of not cooperating and of cooperating.

Not cooperating will mean that each country k chooses monetary policy, β^k, to maximize its social welfare function taking as given the monetary policies of all the other countries, β^j for all $j \neq k$.[14] The outcome of noncooperating will be described by a vector $\hat{\beta} = (\hat{\beta}^1, \hat{\beta}^2 \ldots \hat{\beta}^K)$ that simultaneously satisfies these conditions for all countries. In the terminology of game theory, such a vector is called a Nash equilibrium. Cooperating will mean that all the β^k are chosen to maximize a weighted average of the social welfare functions of the individual countries. We will show that these definitions and our model imply that there is scope for cooperation in the sense that cooperation can produce a higher value of every country's social welfare function than does not cooperating.

Our first task is to describe the social welfare function of a country. Since we are considering only stationary equilibria in which all generations of country k savers and all generations of borrowers experience the same consumption bundles, the country k social welfare function can be expressed as a function of three arguments--the welfare of a country k initial old person, that of a country k saver and that of a country k borrower. Moreover, since current old persons are better off the higher is p_1^k, since savers are better off the higher is $\bar{r}^k = \lambda^k R^k + (1-\lambda^k)r$, and since borrowers are better off the lower is r, we can express the country k welfare function as a function of those three variables, namely as a function $u^k[p_1^k, (\lambda^k R^k+(1-\lambda^k)r, r]$ where u^k is increasing in p_1^k and $\bar{r}^k$, decreasing in r, and is in other respects like an ordinary utility function.[15/]

The next step is to express social welfare for country k in terms of the monetary policy parameters, the vector $\beta = (\beta^1, \beta^2, \ldots, \beta^K)$. This is done by substituting the solutions for r, R^k and p_1^k we found above, in the last section, into the expression for u^k. Thus, using (21), $r = \phi(\beta)$, we write the solution for R^k (see (18)) as

$$(22) \qquad R^k = \phi_2^k(\beta) \equiv \phi(\beta)(1-G^k/\lambda^k S^k)/[\phi(\beta)(1+\beta^k)-\beta^k]$$

and that for p_1^k (see (20)) as

$$(23) \qquad p_1^k = \phi_3^k(\beta) \equiv \phi(\beta)(\lambda^k S^k-G^k)(1+\beta^k)/[\phi(\beta)(1+\beta^k)-\beta^k]$$

Then we let

$$(24) \qquad V^k(\beta^k, \beta^{)k(}) \equiv u^k\{\phi_3^k(\beta), \lambda^k\phi_2^k(\beta)+(1-\lambda^k)\phi(\beta), \phi(\beta)\}$$

where $\beta^{)k(} = (\beta^1, \beta^2, \ldots, \beta^{k-1}, \beta^{k+1}, \ldots \beta^K)$, the vector β with β^k excluded.

As noted above, we are assuming that what happens under noncooperation is described by a Nash equilibrium with each country k choosing β^k, taking $\beta^{)k(}$ as given. Formally, then, the noncooperative solution is a vector $(\hat{\beta}^1, \hat{\beta}^2, \ldots \hat{\beta}^K)$ such that for each k, $\beta^k = \hat{\beta}^k$ maximizes $V^k(\beta^k, \hat{\beta}^{)k(})$. Under cooperation we assume that a vector β is chosen to maximize $\sum w^k V^k(\beta^k, \beta^{)k(})$, where w^k is the weight given to the country k social welfare function and where the summation is over the K countries.

We appraise the noncooperative solution under the assumption that the world economy consists of identical countries. If there is a role for cooperation in a world of identical countries, then very generally there is a role for cooperation in a world of dissimilar countries.

<u>Proposition 4</u>. If countries are identical and A.1-A.4 hold, then there generally exist monetary policies that imply a higher value of the common social welfare function V than is implied by any noncooperative solution with a common value of β^k for all k (any symmetric Nash equilibrium).

The proof in the Appendix establishes that the (total) derivative of $V(\beta, \beta(1_{K-1}))$ (where 1_{K-1} denotes a K - 1 element vector of all whose elements are unity) with respect to β evaluated at $\beta = \hat{\beta}$, the symmetric Nash equilibrium, is given by

$$(25) \qquad dV(\beta, \beta(1_{K-1}))/d\beta = -\{\lambda\hat{R}/[\hat{r}(1+\hat{\beta})-\hat{\beta}]\}[Su_1-(\hat{r}-1)u_2)]$$

where u_i stands for the partial derivative of u with respect to its ith argument and variables with "^"s over them denote symmetric Nash equilibrium values. Since the RHS of (25) is in general not zero, even though the terms in squared brackets tend to offset each other, there are common values of β^k which dominate in terms of social welfare functions what happens under a symmetric Nash equilibrium. Since such alternative values could be chosen under cooperation, this establishes a role for cooperation.

This role for cooperation is implied by the fact that a single country taking other countries' policies as given faces different tradeoffs among triplets $(p_1,\bar{r},r)$, the arguments of u, than are faced by all countries acting jointly and varying all the β^k together. These differences also help explain why the right-hand side of (25) can be either positive or negative, which means that there is no general presumption about whether cooperation would lead to tighter or easier monetary policies.

In Figure 3, we display the way two of the pair-wise tradeoffs among $(p_1,\bar{r},r)$ are ordered at a symmetric Nash equilibrium, the point at which the tradeoff curves cross. In each case the curve labeled C (for cooperative) represents the tradeoff implied by varying all the β^k together, while the curve labeled N (for noncooperative) represents the tradeoff faced by a single country varying β^k with β^j for $j \neq k$ fixed at $\hat{\beta}$. In each case, the u curve is a kind of indifference curve. In Figure 3a, the u curve depicts all combinations of $\bar{r}$ and r, with p_1 held fixed at its Nash equilibrium value, that imply a value for the social welfare function, u, equal to its value at the Nash equilibrium, $\hat{u}$. In Figure 3b, the u curve is defined analogously, but with the roles of p_1 and $\bar{r}$ interchanged.

Figure 3 is best thought of as obtained from the following opportunity or attainable sets and indifference surfaces in three dimensional space, a space with axes labeled, p_1, $\bar{r}$ and r, respectively. One opportunity set is obtained by tracing out all the triplets $(p_1,\bar{r},r)$ implied by different common values of the β^k. Since we get one triplet for each common value of β^k, the set of such triplets is a curve in three dimensional space. (This could be called the symmetric cooperative opportunity curve.) Another opportunity set is implied by tracing out all the triplets implied by different values of β^k holding fixed β^j at $\hat{\beta}$ for $j \neq k$. This is another curve in three dimensional space. It depicts the opportunities facing a single country given that the monetary policies of all the other countries are held fixed at $\hat{\beta}$. (This could be called a noncooperative opportunity curve.) As we have defined them, these two curves intersect at the point corresponding to the Nash equilibrium. We also want to put into our three dimensional space indifference surfaces or contours of the function u--each surface or contour representing all triplets $(p_1,\bar{r},r)$ that imply a single value of u. In particular, we want to focus on the indifference surface defined by $u = \hat{u}$. An implication of $\hat{\beta}$ being a Nash equilibrium is that this particular indifference surface and the noncooperative opportunity curve as defined above are tangent at $(\hat{p}_1,\hat{\bar{r}},\hat{r})$.

Figure 3 is obtained from these three dimensional constructs as follows. Imagine a plane through the Nash equilibrium point perpendicular to the p_1 axis. The intersection of this plane and the indifference surface defined by $u = \hat{u}$ is the u curve of Figure 3a. The N curve of Figure 3a is the projection of the noncooperative opportunity curve on this plane, while the C curve is the projection of the symmetric cooperative opportunity curve on it. (The projection of a curve onto a plane is the intersection of the plane with perpendiculars dropped from the curve onto the plane.) Figure 3b is obtained analogously. The tangencies depicted in Figure 3 are implied by the tangency between the noncooperative opportunities curve and the indifference surface in the three dimensional space.

The fact that the N and C curves in Figure 3 are not tangent where they cross is synonomous with noncoincidence of the noncooperative and cooperative opportunity curves in the neighborhood of where they intersect in the three dimensional space. This noncoincidence occurs because all the β^k enter symmetrically in determining r, but enter asymmetrically in determining both R and p_1. The symmetry implies that the effect on r of varying one of the β^k holding the others constant is equal to 1/K times the effect on r of varying all of them. Given that, the N and C curves in Figure 3 would be tangent only if the effects on R and p_1 of varying one of the β^k were also 1/K times the corresponding effects of varying all the β^k. That, however, is not the case because the own-country policy appears separately in (18) and (20), while the other β^k appear only by way of r.

As established in Appendix A, the noncooperative and cooperative tradeoffs between p_1 and r (see Figure 3b) are such that the price of achieving a given increase in p_1 is lower in terms of the implied increase in r when a country acts on its own than when it acts jointly with all others. This effect by itself tends to make higher social welfare achievable at lower r--that is, through lower β^k's which are easier monetary policies. If it were the case that the welfare of savers did not appear in the social welfare function (so that the partial derivative of u with respect to $\bar{r}$ were zero), then this effect would determine that higher social welfare could be obtained by cooperatively lowering all the β^k from $\hat{\beta}$. Note that this shows up in equation (25), whose right-hand side would be negative if u_2 were zero.

Figure 3a displays the opposite tendency. As established in Appendix A, the noncooperative and cooperative tradeoffs between $\bar{r}$ and r are such that the price of achieving a given increase in $\bar{r}$ is higher in terms of the implied increase in r when a country acts on its own than when it acts jointly with all other countries. This effect tends to make higher social welfare achievable by increasing all the β^k from $\hat{\beta}$--that is, by tighter monetary policy which raises r. Consistent with this, the right-hand side of (25) would be positive if the social welfare function did not depend on the wealth of the initial old through p_1; that is, if u_1 were zero.

Note that while the offsetting effects displayed in Figures 3a and 3b and on the right-hand side of equation (25) could offset each other exactly, that does not generally happen. In terms of our description of the preference contours and opportunity curves in the three dimensional space it happens only if the cooperative opportunity curve, despite not being coincident at the Nash equilibrium with the noncooperative opportunity curve, is tangent to the $u = \hat{u}$ indifference surface. Note also that whether cooperatively chosen policy is tighter or easier than the outcome under noncooperation, it is able to produce a higher value of the common social welfare function only by aiding some groups at the expense of others. As noted in the introduction, in this model cooperation cannot benefit everyone.

Finally, we want to call attention to the fact that we have not proved existence of a symmetric Nash equilibrium. Although it is easy to make assumptions about V which guarantee such existence--namely, that for each possible common value for monetary policy chosen by other countries, country k has a unique best policy which is neither indefinitely easy (in the direction of -1) nor indefinitely tight (in the direction of ∞) and which depends in a continuous way on the policy chosen by other countries--it is difficult to make appealing assumptions about the structure and the social welfare function, u^k, that imply these conditions.[16] Given our general theme that there is scope for cooperation in models like ours, we do not find this difficulty disturbing. Nonexistence of a noncooperative equilibrium is certainly not an argument for noncooperation.

Footnotes

1/In particular, the special model should make our presentation accessible to undergraduate students of economics--at least those whose background includes intermediate microeconomic theory and some calculus.

2/Under well-known conditions, the single time t good can be interpreted as a composite good. See, for example, Kareken and Wallace [1981], page 210.

3/If the arguments of borrowers' utility functions are normal goods, then $D^k(\ )$ is downward sloping where it is positive. For a more detailed description of the derivation of these functions, see Wallace [1984] or the section on the derivation of demand in any intermediate price theory text.

4/In our monetary policy experiments, real government net-of-interest deficits and the underlying real government expenditure and tax streams are held fixed.

5/Two conditions in this definition depend on the restriction $r_t < \tilde{r}$. Without it, the argument of $D^k(\ )$ in (2) is not necessarily r_t and without it the RHS of (7) would have to be $\lambda^k S^k(\bar{r}_t^k) + \lambda^k \max[0,-D^k(\bar{r}_t^k)]$.

6/One significant feature of nonbinding equilibria is perfect substitution among the monetary bases of the different countries. See Kareken and Wallace [1981] for a discussion of the consequences of such substitution.

7/If lenders have a utility function of the Cobb-Douglas form and if their endowment is entirely in the form of income when they are young, then $S^k(\ )$ is a constant fraction of that income (and does not depend on any rate of return).

8/Setting $H_1^k + B_1^k = 1$ for all k saves space and is innocuous. It amounts to no more than choosing monetary units of the different countries in a particular way.

9/It is easy to produce examples of economies with $\tilde{r}$ as large as we want. For example, if every borrower in every country has a Cobb-Douglas utility function which weights consumption when young and when old equally and has the same lifetime income pattern, say, w_1^b when young and w_2^b when old, then $\tilde{r} = w_2^b/w_1^b$.

10/Note that $\partial F(r,\beta)/\partial\beta^k = (\lambda^k S^k - G^k)r/[r(1+\beta^k)-\beta^k]^2 > 0$.

11/By the implicit function theorem, $\partial\phi/\partial\beta^k = (\partial F/\partial\beta^k)/[\partial E/\partial r - \partial F/\partial r] > 0$.

12/Let $z^k \equiv (\lambda^k S^k - G^k)/[r(1+\beta^k)-\beta^k] > 0$. Then from (18) we have $d(R^k/r)/d\beta^j = (\partial\phi/\partial\beta^j)z^k(1+\beta^k)/\lambda^k S^k < 0$ and $dR^k/d\beta^j = -(\partial\phi/\partial\beta^j)z^k\beta^k/\lambda^k S^k$, and from (20) we have $dp_1^k/d\beta^j = -(\partial\phi/\partial\beta^j)z^k\beta^k(1+\beta^k)$ where $j \neq k$.

13/From (20), the partial derivative of p_1^k with respect to β^k holding r constant is $r(\lambda^k S^k - G^k)/[r(1+\beta^k)-\beta^k]^2 > 0$.

14/If there are many similar countries so that each is a small part of the world economy, then taking other countries' monetary policies as given is approximately the same as taking the world interest rate, r, as given, as unaffected by the choice of β^k.

15/In particular, we assume that the upper contour sets of u^k are strictly convex.

Appendix A

Proof of Proposition 1. By (12) and $\beta^k > -1$, $h^k + b^k > 0$. But, then, by (9) and A.2, r must exceed unity. To see that a nonbinding stationary equilibrium cannot exist, note that $R^k = r > 1$ and $h^k + b^k > 0$ imply that the RHS of (8) is negative, which contradicts A.1.

Proof of Proposition 2. We will show that equation (19) has a solution with $r \in (1,\tilde{r})$. That and A.1 will imply immediately that the RHS of (18) is positive and less than unity ($0 < R^k < r$) and that the RHS of (20) is positive ($p_1^k > 0$).

Since $E(r)$ and $F(r,\beta)$ are continuous functions of r (for a fixed β), to show that (19) has a solution with $r \in (1,\tilde{r})$ it suffices to show that $E(1) < F(1,\beta)$ and that $E(\tilde{r}) > F(\tilde{r},\beta)$.

We have $E(1) = \sum[S^k-D^k(1)] - \sum\lambda^k S^k < -\sum\lambda^k S^k$, the inequality being a consequence of A.2. We also have $F(1,\beta) = \sum(\lambda^k S^k-G^k)\beta^k > -\sum(\lambda^k S^k-G^k) > -\sum\lambda^k S^k$, both inequalities being consequences of A.1. Thus, $E(1) < F(1,\beta)$.

Since $F(r,\beta)$ is increasing in β^k for each k, $F(\tilde{r},\beta)$ is less than the limit of $F(\tilde{r},\beta)$ as $\beta^k \to \infty$ for every k. This limit is $[\sum_k(\lambda^k S^k-G^k)]/(\tilde{r}-1)$. Therefore assumption A.3 implies $E(\tilde{r}) > F(\tilde{r},\beta)$.

Proof of Proposition 3. Since this is obviously true if $\beta^k = 0$ for all k, we proceed under the assumption that $\beta^k \neq 0$ for at least some k. Letting $f(r) \equiv F(r,\beta)$, we first establish that

(i) $f'(r) < -(1/r)f(r)$

Note that $f'(r) = -\sum x_k(r)y_k(r)$, where $x_k(r) = (\lambda^k S^k - G^k)\beta^k/[r(1+\beta^k)-\beta^k]$ and $y_k(r) = 1/[r-\beta^k/(1+\beta^k)]$. We also have that if $\beta^k > 0$, then $x_k(r) > 0$ and $y_k(r) < 1/r$; while if $\beta^k < 0$, then $x_k(r) < 0$ and $y_k(r) > 1/r$. Therefore $-f'(r) = \sum x_k(r)y_k(r) > (1/r)\sum x_k(r) = (1/r)f(r)$. Thus we have (i).

Inequality (i) says that $f(r) \equiv F(r,\beta)$ is downward sloping wherever F is not negative. It follows that if there is a solution to (19) where $f > 0$, say at r_+, then there is no other solution. There cannot be a solution at $r > r_+$ because E(r) is increasing and f can never get to a higher value than $f(r_+)$ without violating (i). There cannot be a solution with $r < r_+$ because then f could never get to a value as great as $f(r_+)$ without violating (i). (Note that we get an equilibrium where $F > 0$ if enough of the β^k are positive. Thus, if $\beta^k > 0$ for all k, then we have a unique solution to (19) without appeal to A.4.)

It is for the case when there is no solution with $F > 0$ that we need A.4. Since $E(1) < F(1,\beta) \equiv f(1)$, uniqueness is implied if $f'(r) < E'(r)$ at any solution.

At any solution, we have the following string of inequalities:

$$f'(r) < -(1/r)f(r) = -(1/r)E(r) = -(1/r)\sum(1-\lambda^k)S^k + (1/r)D(r) < -(1/r)\sum(1-\lambda^k)S^k - D'(r) < -D'(r) = E'(r)$$

The first inequality is (i); the second (an equality) uses the assumption that we are at a solution, the third (an equality) uses the definition of E(r); while the fourth uses A.4 and the fact that $D(r) > 0$ at any $r < \tilde{r}$.

Proof of Proposition 4. We show that the derivative of $V(\beta,\beta(1_{K-1}))$ w.r.t. β is in general different from zero when it is evaluated at a symmetric Nash equilibrium, $\hat{\beta}$, which satisfies the first-order condition $V_1(\hat{\beta},\hat{\beta}(1_{K-1})) = 0$.

Since $dV(\beta,\beta(1_{K-1}))/d\beta = V_1(\beta,\beta(1_{K-1})) + (K-1)V_{)(}(\beta,\beta(1_{K-1}))$, where $V_{)(}$ denotes the partial derivative of V w.r.t. any argument other than the first and since $V_1(\hat{\beta},\hat{\beta}(1_{K-1})) = 0$, our task is to derive an expression for $V_{)(}(\beta,\beta(1_{K-1}))$ and to evaluate it at $\beta = \hat{\beta}$.

From (24)

(i) $$V_{)(}(\beta,\beta(1_{K-1})) = (u_1)(\partial\phi_3^k/\partial\beta^j) + (u_2)\lambda(\partial\phi_2^k/\partial\beta^j) + [(1-\lambda)(u_2)+(u_3)](\partial\phi/\partial\beta^j)$$

(ii) $$V_1(\beta,\beta(1_{K-1})) = (u_1)(\partial\phi_3^k/\partial\beta^k) + (u_2)\lambda(\partial\phi_2^k/\partial\beta^k) + [(1-\lambda)(u_2)+(u_3)](\partial\phi/\partial\beta^k)$$

where u_i stands for the partial derivative of u w.r.t. its ith argument and where the partial derivatives of ϕ, ϕ_2^k, and ϕ_3^k are to be computed from (21), (22) and (23), respectively.

At a symmetric Nash equilibrium, the RHS of (ii) is zero and $\partial\phi/\partial\beta^k = \partial\phi/\partial\beta^j$. These imply, by subsitution from (ii) into (i), that

(iii) $V_{)(}(\hat{\beta},\hat{\beta}(1_{K-1})) = (u_1)[(\partial\phi_3^k/\partial\beta^j)-(\partial\phi_3^k/\partial\beta^k)]$

$$+ (u_2)\lambda[(\partial\phi_2^k/\partial\beta^j)-(\partial\phi_2^k/\partial\beta^k)]$$

Since, by (22) and (23),

(iv) $(\partial\phi_3^k/\partial\beta^j) - (\partial\phi_3^k/\partial\beta^k) = -\lambda SR/[r(1+\beta)-\beta]$

(v) $(\partial\phi_2^k/\partial\beta^j) - (\partial\phi_2^k/\partial\beta^k) = R(r-1)/[r(1+\beta)-\beta]$

we have

(vi) $V_{)(}(\hat{\beta},\hat{\beta}(1_{K-1})) = -\{\lambda\hat{R}/[\hat{r}(1+\hat{\beta})-\hat{\beta}]\}[(u_1)S-(u_2)(\hat{r}-1)]$

Although the terms in square brackets on the RHS of (vi) have *opposite signs, in general they are not of equal magnitudes*. Thus, $V_{)(}(\hat{\beta},\hat{\beta}(1_{K-1}))$ is in general not zero.

<u>Cooperative and Noncooperative Tradeoffs</u> (for the case of identical countries). Write (18) as $R^k = g(\beta^k,r)$ and (20) as $p_1^k = h(\beta^k,r)$. Also, let $dr/d\beta^k = \partial\phi(\beta)/\partial\beta^k$ and $dr/d\beta = \sum_k\partial\phi(\beta)/\partial\beta^k$. If these derivatives are evaluated at $\beta^k = \beta$ for all k, then $dr/d\beta = K dr/d\beta^k$, which is used below.

We begin by finding the tradeoffs between R^k and r. We have

(i) $dR^k/d\beta^k = g_1 + g_2(dr/d\beta^k)$

where g_i is the partial derivative of g w.r.t. its ith argument. Therefore

(ii) $(dR^k/dr)_N = (dR^k/d\beta^k)/(dr/d\beta^k) = g_1/(dr/d\beta^k) + g_2$

where N denotes noncooperative (holding $\beta^j = \hat{\beta}$ for $j \neq k$). Also,

(iii) $dR^k/d\beta = g_1 + g_2 dr/d\beta$

and, therefore,

(iv) $(dR^k/dr)_C \equiv (dR^k/d\beta)(dr/d\beta) = g_1/(dr/d\beta) + g_2$

where C denotes cooperative (varying all the β^k together). Therefore,

(v) $(dR^k/dr)_C - (dR^k/dr)_N = -(K-1)g_1/(dr/d\beta^k) > 0$

since $dr/d\beta^k > 0$ and, from (18), $g_1 = -(1-G/\lambda S)(r-1)/[r(1+\beta^k)-\beta^k]^2 < 0$.

Then since $\bar{r}^k = \lambda R^k + (1-\lambda)r$, it follows immediately that $(d\bar{r}^k/dr)_C - (d\bar{r}^k/dr)_N$ is λ times the RHS of (v).

Finally, in exactly the same way as (v) was obtained, we get

(vi) $(dp_1^k/dr)_C - (dp_1^k/dr)_N = -(K-1)h_1/(dr/d\beta^k) < 0$

The inequality in (vi) is a consequence of $h_1 = (\lambda S-G)r/[r(1+\beta^k)-\beta^k]^2 > 0$ (see (20)).