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GOVERNMENT DEBT AND TAXES

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Government Debt and Taxes

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1. Introduction

This chapter reinterprets the mathematics in the preceding chapter in order to model some aspects of government finance. By renaming variables, the household budget constraint of the last chapter is reinterpreted as the budget constraint facing a government that always abstains from inflationary finance (that is, a government that doesn't print currency to finance any of its expenditures). Section 2 of this chapter describes the restrictions that are placed on time series of government purchases and tax collections by the intertemporal government budget constraint. Intuitively, this constraint seems to place only mild restrictions on the time series because it is a restriction involving infinite sequences. There is always the possibility of satisfying the budget constraint, even while running large and persistent deficits, by intending to run large surpluses later. Section 2 gives a formal description of precisely how restrictive the intertemporal budget constraint is when government expenditures are covariance stationary.

Section 3 pursues the observation of Robert Barro [1979], that there is a formal analogy between the permanent income theory of consumption and a model of "tax smoothing." A simple linear-quadratic dynamic optimal tax model is formulated which is mathematically equivalent with the permanent income model of the last chapter. This model is one in which the sequence of net-of-interest government expenditures is taken as given, and as following a covariance stationary stochastic process. The idea is

that tax collections are distorting, and should be allocated over time to minimize the distortion. The model has the implication, stressed by Robert Barro, that in response to a large temporary increase in government expenditures, such as occurs during a short war, the government should raise taxes a little, run (potentially large) deficits during the war, and then run a large string of surpluses after the war.

2. Observable Implications of Present Value Balanced Budgets

Consider the flow version of the government budget constraint

$$(1) \quad B_{t+1} = R[B_t + g_t - \tau_t]$$

where R is the gross real rate of return on government one-period debt, g_t is the level of real government expenditures, τ_t is the level of real tax collections, and B_t is the stock of real government debt, due at time t , and denominated in units of time t goods. We assume that $R > 1$ and that R is constant over time. ^{1/} We assume that (g_t, τ_t) is a jointly covariance stationary stochastic process with Wold moving average representation ^{2/}

$$\begin{bmatrix} g_t \\ \tau_t \end{bmatrix} = \begin{bmatrix} \mu_g \\ \mu_\tau \end{bmatrix} + \left[\sum_{k=0}^{\infty} C_k L^k \right] \epsilon_t$$

or

$$(2) \quad \begin{bmatrix} g_t \\ \tau_t \end{bmatrix} = \begin{bmatrix} \mu_g \\ \mu_\tau \end{bmatrix} + \begin{bmatrix} c_1(L) \\ c_2(L) \end{bmatrix} \varepsilon_t.$$

Here, μ_g is the mean of g_t and μ_τ is the mean of τ_t . In (2), we assume that

$$\sum_{k=0}^{\infty} \text{tr } C_k C_k^T < +\infty.$$

We let

$$\begin{aligned} c_1(L) \\ c_2(L) \end{aligned} \equiv \sum_{k=0}^{\infty} C_k L^k;$$

where $c_1(L)$ and $c_2(L)$ are each 1×2 vector-polynomials in the lag operator L . In (2), $\{\varepsilon_t\}$ is a (2×1) vector white noise that is fundamental for (g_t, τ_t) ; that is,

$$C_0 \varepsilon_t = \begin{bmatrix} g_t - E g_t | \phi_{t-1} \\ \tau_t - E \tau_t | \phi_{t-1} \end{bmatrix}$$

where E is the linear least squares projection operator and ϕ_{t-1} is the information set $\{g_{t-1}, g_{t-2}, \dots, \tau_{t-1}, \tau_{t-2}, \dots\}$.

We impose on the solution to (1) the side condition that there exists an $M < +\infty$ such that

$$(3) \quad E_0 B_{t+1} < M$$

for all t . Condition (3) imposes an upper bound of M on the volume of government debt that is projected at time 0.

In (1), we regard B_t as predetermined at time t , being entirely determined by past decisions about g_t and τ_t . Since B_{t+1} is then known at t , we can represent (1) in the equivalent form:

$$(4) \quad E_t B_{t+1} = R[B_t + g_t - \tau_t].$$

Subject to the condition (3), the solution to the difference equation (4) satisfies the "present value form" of the budget constraint:

$$(5) \quad E_t \sum_{j=0}^{\infty} R^{-j} \tau_{t+j} = B_t + E_t \sum_{j=0}^{\infty} R^{-j} g_{t+j}.$$

We now pose the following question: What restrictions on the time series for (g_t, τ_t) are imposed by the intertemporal budget constraint (5)? In other words, by observing time series of (g_t, τ_t) , can we determine whether (5) (or equivalently (1) and (2)) is being obeyed? Given representation (2), this question is well posed. The answer is that (5) imposes upon (2) the cross-equation restriction

$$(6) \quad c_1(R^{-1}) = c_2(R^{-1}),$$

and that μ_g and μ_τ are not restricted by (5).

To prove the equality asserted in (6), we argue as follows. Since B_t is bounded, is predetermined, and is a function solely of past values of g_t and τ_t , it has Wold representation of the form

$$(7) \quad B_t = \mu_B + c_3(L) \varepsilon_t.$$

Here, μ_B is a constant, $c_3(L) = \sum_{j=0}^{\infty} c_{3k} L^k$, and $c_{30} = 0$ because B_t is predetermined at t , and so is only a function of $\{g_{t-1}, g_{t-2}, \dots, \tau_{t-1}, \tau_{t-2}, \dots\}$. The Wiener-Kolmogorov prediction formula implies that

$$E_t B_{t+1} = \mu_B + \left(\frac{c_3(L)}{L}\right) \epsilon_t.$$

or

$$(8) \quad E_t B_{t+1} = \mu_B + [c_{31} + c_{32}L + \dots] \epsilon_t.$$

Substituting (7), (8), and (2) into (4) and noting that the result must hold for all realizations of ϵ_t gives

$$(9) \quad (c_{31}L + c_{32}L^2 + \dots) - R^{-1}(c_{31} + c_{32}L + \dots) = c_2(L) - c_1(L)$$

and

$$(10) \quad \mu_B - R^{-1}\mu_B = \mu_\tau - \mu_g.$$

Evaluating (9) at $L = R^{-1}$ makes the left side of the equation zero, implying our result:

$$(6) \quad c_2(R^{-1}) = c_1(R^{-1}).$$

Note that (10) can be rearranged to read

$$(11) \quad \mu_\tau = \mu_g + \mu_B[1 - R^{-1}].$$

Equation (11) states that the mean of tax collections μ_τ must equal the mean level of government expenditures plus the mean level of interest payments on the debt $[\mu_B(1 - R^{-1})]$. In other words, μ_τ and μ_g themselves are not restricted by (4) and (2), since a value of μ_B can always be found to satisfy (12).

The restriction (6) is interpretable in terms of the response of the system to innovations in (g_t, τ_t) . Writing out restriction (6), we have

$$(6') \quad \sum_{k=0}^{\infty} c_{2k} R^{-k} = \sum_{k=0}^{\infty} c_{1k} R^{-k}.$$

The $\{c_{jk}\}_{k=0}^{\infty}$ trace out the response of variable j ($j = 1$ for g , $j = 2$ for τ) to an innovation ε_t in (g_t, τ_t) . Restriction (6) states that the present value of these responses has to be equal for g and τ .

It is useful to study how equality (6) can be attained as an application of the formula of Hansen and Sargent [] of Chapter #. Express equation (4) as

$$(12) \quad B_t = -d_t + R^{-1} E_t B_{t+1}$$

where $d_t \equiv g_t - \tau_t$. Thus, d_t is the net-of-interest government deficit. Let us use the compact notation

$$d_t = c_d(L) \varepsilon_t + \mu_d$$

where $c_d(L) \equiv c_1(L) - c_2(L)$ and $\mu_d = \mu_g - \mu_\tau$. Then application formula () of Chapter # to (12) gives

$$(13) \quad B_t = - \left\{ \frac{Lc_d(L) - R^{-1}c_d(R^{-1})}{L - R^{-1}} \right\} \varepsilon_t + \mu_B$$

or

$$B_t = c_3(L) \varepsilon_t + \mu_B,$$

where $c_3(L) \equiv - \{ [Lc_d(L) - R^{-1}c_d(R^{-1})] / [L - R^{-1}] \}$. We impose that B_t is predetermined at time t by requiring that $c_{30} = 0$, which is equivalent with imposing that $c_3(0) = 0$. Imposing $c_3(0) = 0$ in formula (13) for $c_3(L)$ gives

$$c_3(0) = c_d(R^{-1}).$$

We impose $c_3(0) = 0$ by requiring that $c_d(R^{-1}) = 0$, which is restriction (6).

With $c_d(R^{-1}) = 0$, (13) becomes ^{3/}

$$(14) \quad B_t = \mu_B - \left\{ \frac{Lc_d(L)}{L-R^{-1}} \right\} \varepsilon_t.$$

Expanding the polynomial in L by long division shows that the above equation is equivalent with

$$(15) \quad B_{t+1} = \mu_B + R\{c_0 + R[c_0 + R^{-1}c_1]L + \dots \\ + R^n[c_0 + R^{-1}c_1 + \dots + R^{-n}c_n]L^n + \dots\} \varepsilon_t \\ = \mu_B + R\left\{ \sum_{n=0}^{\infty} R^n \left(\sum_{j=0}^n c_j R^{-j} \right) L^n \right\} \varepsilon_t$$

where $c_d(L) \equiv \sum_{j=0}^{\infty} c_j L^j$. Thus, the response of B_{t+1} to the lagged innovation ε_{t-n} is equal to $R^{n+1}[c_0 + R^{-1}c_1 + \dots + R^{-n}c_n]$. The expected present value budget balance condition $c_d(R^{-1}) = 0$ is necessary in order that the response of real government debt to innovations in the deficit eventually damp out.

As an example of restriction (6), suppose that

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ 0 \end{bmatrix}$$

and that $c_1(L) = (\frac{1}{1-\rho_1 L}, 0)$, $|\rho_1| < 1$. Then (6) implies that $c_2(L) = (\frac{1}{1-\rho_2 L}, 0)$ where

$$\frac{1}{1-\rho_1 R^{-1}} = \frac{1}{1-\rho_2 R^{-1}}.$$

This last equality implies that $\rho_1 = \rho_2$. If g_t and τ_t are each first-order autoregressive processes, (6) implies that $\rho_1 = \rho_2$. Thus, under the first-order autoregressive specification, (6) is very restrictive.

However, (6) becomes progressively less restrictive as the order of the parameterization is increased. To take a second example, let

$$\varepsilon_t = \begin{bmatrix} \varepsilon_{1t} \\ 0 \end{bmatrix}, c_1(L) = \left(\frac{1}{1-\rho_1 L}, 0 \right), |\rho_1| < 1$$

and

$$c_2(L) = \left(\frac{1}{1-\delta_1 L - \delta_2 L^2}, 0 \right).$$

Here, τ_t is permitted to be second-order autoregressive while g_t must be first-order autoregressive. Then (6) implies that $\delta_1 + R^{-1}\delta_2 = \rho_1$. This is evidently a less restrictive condition than obtains when both processes are first-order autoregressive processes. It can be shown that (6) becomes less and less restrictive as we permit the dimensionality of the parameterization of $c(L)$ to increase. 4/

The restriction that (6) imposes on a vector autoregression for (g_t, τ_t) can be expressed compactly. Let (2) have the alternative autoregressive representation

$$(16) \quad A(L) \begin{bmatrix} g_t \\ \tau_t \end{bmatrix} = \begin{bmatrix} \bar{\mu}_g \\ \bar{\mu}_\tau \end{bmatrix} + \varepsilon_t$$

where $\bar{\mu}_g, \bar{\mu}_\tau$ are constants, and where $A(L) = I - A_1L - \dots - A_nL^n$, where A_j is a 2×2 matrix. In (16), ϵ_t is the same innovation process that appears in (2). We assume that the zeros of $\det A(z)$ all exceed unity in absolute value. ^{5/} Representations (16) and (2) are linked by $A(L)^{-1} = C(L) \equiv \sum_{j=0}^{\infty} C_j L^j$. It follows that restriction (6) is equivalent with

$$(17) \quad (1, -1)A(R^{-1})^{-1} = (0, 0),$$

where $A(R^{-1})$ is the 2×2 matrix $I - A_1R^{-1} - \dots - A_nR^{-n}$.

Compared with the cross-equation restrictions characterizing many rational expectations models, ^{6/} the restriction $c_1(R^{-1}) = c_2(R^{-1})$ is a very weak one when $c_1(L)$ and $c_2(L)$ have high dimensional parameterizations. This stems from the weakness of the present value constraint (5) itself. This weakness relates informally to the fact that observers of governments frequently have a difficult time inferring from observed sequences for (g_t, τ_t) whether the budget is in balance in the present value sense. The government can always promise to run sufficient surpluses in the more or less distant future in order to balance the budget. The weakness of restriction (7) under high dimensional parameterizations of $c_1(L)$ and $c_2(L)$ is a precise way of formulating this weakness of the present value budget constraint, which obtains even under the restriction that (g_t, τ_t) is covariance stationary.

In the next section, we describe a simple model of government behavior which imposes additional cross-equation restrictions on the (g_t, τ_t) process.

3. Tax Smoothing

Robert Barro [1979] has pointed out the formal similarity of a particular dynamic optimal taxation problem to a model of consumer behavior. Barro attained a result, reminiscent of Hall's characterization of consumption, that taxes should follow a martingale. Here, we briefly describe the linear-quadratic optimal taxation problem which is equivalent to the permanent income model of consumption described in Chapter #. 7

The idea is that taxes are distorting, and that the government has an incentive to allocate taxes across time in order to minimize the distortion. The current period distortion at time t is modeled as equal to $[+u_1 \tau_t + \frac{u_2}{2} \tau_t^2]$ where u_1 and $u_2 > 0$, and where τ_t is total taxes collected at time t . 8 The government has to finance a given stream of government expenditures $\{g_t\}$, where g_t is an exogenous stochastic process. 9 The government's problem is to select a strategy for choosing $\{\tau_t, B_{t+1}\}_{t=0}^{\infty}$ in order to maximize

$$(17) \quad E \sum_{t=0}^{\infty} \beta^t [-u_1 \tau_t - \frac{u_2}{2} \tau_t^2] \quad 0 < \beta < 1$$

subject to the budget constraint

$$(18) \quad B_{t+1} = R[B_t + g_t - \tau_t] \quad R > 1$$

$$B_t < M < +\infty \quad \text{for all } t$$

B_0 given.

The government takes $\{g_t\}$ as a given process, and regards B_0 as a fixed initial condition. At time t , the government knows the values of $\{B_t, g_t, g_{t-1}, \dots\}$, but must forecast future values of g . We assume that $\beta R^2 > 1$ and $R > 1$.

This problem is mathematically equivalent to the one described in Section 2 of Chapter #. In the present problem, g_t plays the role of y_t in the previous model, τ_t plays the role of c_t , and B_{t+1} plays the role of A_{t+1} . It follows that all of the results of Chapter # have interpretations in terms of the present model. Thus, corresponding to (7) of Chapter #, we have

$$(19) \quad E_t \tau_{t+1} = \alpha + (\beta R)^{-1} \tau_t$$

where

$$\alpha = \frac{u_1 [1 - (\beta R)^{-1}]}{u_2}.$$

In the case that $(\beta R)^{-1} = 1$, (14) states that tax collections should follow a martingale regardless of the stochastic process followed by $\{g_t\}$. Corresponding to equation (9) of Chapter #, we have the representation for τ_t

$$(20) \quad \tau_t = \left(\frac{-\alpha}{R-1} \right) + \left(1 + \frac{1}{\beta R^2} \right) \left[\sum_{j=0}^{\infty} R^{-j} E_t g_{t+j} + B_t \right].$$

Equation (20) is a "permanent government expenditures theory" for tax collections.

We now pursue the analogy to the model in Chapter #. First, suppose that g_t has the representation

$$g_t = \bar{g} + \tilde{g}(L) \varepsilon_t, \quad \sum_{j=0}^{\infty} \tilde{g}_j^2 < +\infty$$

where $\epsilon_t = g_t - E_t g_t | 1$, g_{t-1} , g_{t-2} , Here, \bar{g} is the mean level of government expenditures, and ϵ_t is the innovation process to government expenditures. In the special case that $\beta R = 1$, we have, by analogy to equation (16) of Chapter #,

$$(21) \quad \tau_{t+1} - \tau_t = (1 - \frac{1}{R}) \tilde{g}(R^{-1}) \epsilon_{t+1}$$

and

$$(22) \quad (1-L)\tau_{t+1} = (1-R^{-1})\tilde{g}(R^{-1})\tilde{g}(L)^{-1}(y_{t+1}-\bar{y}).$$

Equation (22) gives the projection of taxes on government expenditures (that is, net-of-interest government expenditures). 10/

The special case considered by Muth was

$$(23) \quad \tilde{g}(L) = \frac{(1-bL)}{(1-L)} \quad 0 < b < 1.$$

As we saw in Chapter #, in this case (22) implies

$$\tau_{t+1} = \frac{(1-bR^{-1})}{1-bL} y_{t+1}.$$

Tracing out the responses of τ_t , B_t , and g_t to innovations ϵ_t gives

$$(24) \quad g_t = \epsilon_t + (1-b)[\epsilon_{t-1} + \epsilon_{t-2} + \dots]$$

$$(25) \quad \tau_t = (1-bR^{-1})[\epsilon_t + \epsilon_{t-1} + \epsilon_{t-2} + \dots]$$

$$(26) \quad B_{t+1} = b\epsilon_t + bR^{-1}[\epsilon_{t-1} + \epsilon_{t-2} + \dots].$$

Equation (20) reveals that a fraction $(1-b)$ of an innovation to g_t is "permanent." The effect of an innovation ϵ_t is to change

$(1-R^{-1}) \sum_{j=0}^{\infty} R^{-j} E_t g_{t+j}$ by the amount $(1-R^{-1})[1+(1-b)R^{-1}+(1-b)R^{-2} + \dots] \epsilon_t = (1-bR^{-1}) \epsilon_t$. Equation (25) states that an innovation to government expenditures should lead to a permanent change in taxes by the amount $(1-bR^{-1}) \epsilon_t$, which just equals the innovation to $(1-R^{-1}) \sum_{j=0}^{\infty} R^{-j} E_t g_{t+j}$.

For values of b close to 1, an innovation ϵ_t is mostly transitory (see (24)). According to (25), an innovation ϵ_t in government expenditures is accompanied by an innovation of only $(1-R^{-1}b) \epsilon_t$ in tax collections. However, this change in tax collections is permanent (see (25)). According to (26), an innovation ϵ_t in g_t leads to a large innovation of $b\epsilon_t$ in government borrowing B_{t+1} , $bR^{-1}\epsilon_t$ of which is "permanent" borrowing. These responses incorporate the fashion in which transitory increases in government purchases, as occur during a short war, are optimally financed by borrowing during the war while raising taxes "permanently."

Representation (22) gives the projection of taxes on net-of-interest government expenditures. Proceeding by analogy with Section 6 of Chapter #, it is possible also to calculate the projection of τ_t on government expenditures gross of interest, g_{mt} . Defining $g_{mt} = (1-R^{-1})B_t + g_t$, we adopt the special stochastic structure

$$(27) \quad g_t = \bar{g} + \epsilon_t$$

where ϵ_t is a white noise with mean zero and constant variance. Under this specification, and $\beta R = 1$, it follows that

$$(28) \quad r_t = \frac{(1-\beta)}{(1-\beta L)} g_{mt},$$

as in Section 6 of Chapter #.

4. A Note on Government Expenditures on Capital Account

The preceding formulation assumes that all government expenditures are on "current account," in the sense that government expenditures g_t generate no prospects of future revenues for the government. We now indicate how parts of the preceding apparatus, in particular the government budget constraint, must be altered when some government expenditures are on "capital account." Capital account expenditures are defined as expenditures that lead to the accumulation of assets that yield a competitive rate of return to the government. Examples of capital account expenditures are government purchases of private capital, ^{11/} government expenditures on welfare and public education that increases the recipients' productivity by enough to increase the present value of subsequent tax collections by an amount equal to the government expenditure, and government loans to the private sector at the market rate of interest. Capital account expenditures are, by definition, self-amortizing and do not require current or subsequent taxation in order to finance them.

Let government owned "capital" be given by k_t at the beginning of period t . Government capital obeys the transition law $k_{t+1} = k_t + i_t$, where i_t is capital account government expenditures at period t . ^{12/} In period t , the government earns net

income on its capital in the amount $(R-1)k_t$. The government budget constraint (1) is modified to become

$$(29) \quad B_{t+1} = R[B_t + g_t + i_t - \tau_t - (R-1)k_t].$$

Using the transition law for k_t , we have that $i_t - (R-1)k_t = k_{t+1} - Rk_t$. Thus, the budget constraint (29) can be rewritten as

$$B_{t+1} = R\{B_t + g_t - \tau_t + k_{t+1} - Rk_t\}.$$

The stock form of this constraint is

$$(30) \quad E_t \sum_{j=0}^{\infty} R^{-j} \tau_{t+j} = B_t + E_t \sum_{j=0}^{\infty} R^{-j} g_{t+j} + E_t \sum_{j=0}^{\infty} R^{-j} [k_{t+j+1} - Rk_{t+j}].$$

Writing out the sum and cancelling common terms shows that

$$E_t \sum_{j=0}^{\infty} R^{-j} [k_{t+j+1} - Rk_{t+j}] = -Rk_t.$$

Substituting this into (30) gives

$$(31) \quad E_t \sum_{j=0}^{\infty} R^{-j} \tau_{t+j} = (B_t - Rk_t) + E_t \sum_{j=0}^{\infty} R^{-j} g_{t+j}.$$

Note that the term Rk_t equals the capitalized value of net interest earnings on the government's holdings of capital k_t :

$$\sum_{j=0}^{\infty} R^{-j} (R-1)k_t = Rk_t.$$

Thus, equation (31) states that the expected present value of tax collections equals the expected present value of government purchases on current account plus the government's initial net indebtedness, $B_t - Rk_t$. Note also that the stochastic process for i_t fails to appear directly in (31). Its failure to appear is a

reflection of the fact that capital account expenditures do not affect the prospects for present value budget balance.

5. Conclusions

The case for "tax smoothing" based on the model of Section 3 depends on some special features of the problem confronting the tax authority. ^{13/} The objective function has been given a form which simplifies the problem. In particular, the "current return function" $[-u_1 \tau_t - u_2 \tau_t^2]$ is specified to depend on current tax collections, and not on expected future tax collections. This is a critical simplification for reasons indicated in Section # of Chapter #. When private agents face genuinely dynamic optimum problems, their current decisions, and therefore measures of the current distortions caused by taxation, will generally depend on the entire path of current and future taxes. A framework which accommodates these effects would induce important differences in both the mathematical structure of the government's problem and the substantive nature of the results. In Chapter #, we shall return to an optimal taxation problem that incorporates such dynamic effects of future taxes on current distortions.

Footnotes

1/For extensive discussions of the government budget constraint in systems in which the real interest rate is not assumed constant, see Lucas and Stokey [1983] and Sargent [1986].

2/Using an argument in Hansen and Sargent [1980], it can be shown that all that is required for the arguments in this section to go through is that (g_t, τ_t) be a process of mean exponential order less than R . This weak assumption would require minor modifications of the argument in several places, but would still imply restriction (6).

3/Note that formally the polynomial in L in (14) equals the polynomial $(1-RL)^{-1}Rc_d(L)$ that would be obtained by writing (1) as $(1-RL)B_{t+1} = Rd_t$, then inverting the polynomial $(1-RL)$ to solve for B_{t+1} . The coefficients in the polynomial $(1-RL)^{-1}Rc_d(L)$ diverge in positive powers of L unless $c_d(R^{-1}) = 0$. (Technically, the condition $c_d(R^{-1}) = 0$ places a zero in the numerator at $L = R^{-1}$ which cancels out the explosive effects of the pole at $L = R^{-1}$ contributed by the denominator.)

4/If we permit $c(L)$ to have an infinite dimensional parameterization, (6) is not restrictive in a precise sense described by Sims [1972]. In the mean square norm defined by Sims, it can be shown that the set of stochastic processes of the form (2) obeying $c_1(R^{-1}) = c_2(R^{-1})$ is dense in the space of all stochastic processes of the form (2).

5/Actually, all that we really require is that the zeros of $\det A(z)$ exceed R^{-1} in absolute value, though this weaker

condition would necessitate some technical caveats at various points in our argument.

6/ Examples of which are described in Chapters XI, XII, and XIII.

7/ In a heuristic way, such a tax smoothing model might be used to rationalize the large deficits in the government budget created during the Reagan administration. The argument would be that the advent of the Reagan administration signaled a large negative innovation in the expected present value of government expenditures, with substantial expenditure reductions to occur only in the more or less distant future. The optimal response to this innovation is to reduce taxes immediately and to run a string of government deficits while waiting for expenditures to be reduced.

8/ It is important that the distortion at time t is assumed not to depend on expected future tax collections.

9/ The model could be extended to have the government choose a $\{g_t\}_{t=0}^{\infty}$ stochastic process, say, by adding to the current period return a quadratic function in g_t and a random shock ε_t . This would not alter the key substantive implications of the model in the text.

10/ Note that representation (21) obeys the restriction (6). Formally, (21) shows that when $c_1(L) = \tilde{g}(L)$, $c_2(L) = (1-L)^{-1}(1-R^{-1})\tilde{g}(R^{-1})$. These imply that $c_1(R^{-1}) = c_2(R^{-1}) = \tilde{g}(R^{-1})$. This merely verifies that solution (21) builds in budget balance in the sense of expected present values.

11/This assumes that the products of government-owned enterprises are priced correctly. If the government-owned enterprise charges too low a price, then the purchasing of that enterprise is only partly a "capital account" item. Notice that it is not durability of the purchased good alone that determines whether it is a capital account expenditure. Rather, the decisive aspect is the prospect for sufficient future income to the government stemming from the purchase.

12/We assume that i_t is a stochastic process such that k_t is of mean exponential order less than R .

13/For more general models of the problem confronting the tax authority, see Lucas and Stokey [1983] and Turnovsky and Brock [1980].

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