Business Cycle Modeling Without Pretending to Have Too Much A Priori Economic Theory

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Paper prepared for seminar on New Methods in Business Cycle Research, Federal Reserve Bank of Minneapolis, November 13-14, 1975. The views expressed herein are solely those of the authors and do not necessarily represent the views of the Federal Reserve Bank of Minneapolis or the Federal Reserve System. John Geweke adapted the maximum likelihood factor analysis algorithm for application to the frequency domain factory model and wrote a computer program for estimating and testing the one-index model. Paul Anderson extended that program to handle k noises and performed all the frequency domain calculations in this paper. Salih Neftci carried out the calculations for the observable index model. John Geweke's contribution in developing the factor analysis algorithm and in formulating the unobservable index model were enough for him to qualify as a coauthor of this paper.

This paper illustrates the application of a certain class of time series model to macroeconomics.[1] One motivation for this application is our suspicion that existing large-scale macroeconometric models represent, to an extent not admitted in the statistical theory applied to them, "measurement without theory."

In one sense, this idea is an extension of one put forward years ago by T. C. Liu [1960], when he argued that econometric models might, when only really reliable a priori restrictions were applied to them, turn out most often to be underidentified. Not only do we mistrust many of the zero-restrictions on coefficients in these models, we also consider to be unreliable both the restriction that their residuals be serially uncorrelated and the a priori classification of variables into strictly exogenous and endogenous categories. Thus, instead of Liu's conclusion that one ought to turn attention to direct estimation of reduced forms of these models, we conclude that one ought to consider estimation of general representations of the variables in the models as vector stochastic processes.

In part, our intention to explore alternatives to conventional structural macroeconometric models stems from our sympathy with Koopmans' judgments about the theoretical foundations of those models:

> In general the state of macroeconomic theory is unsatisfactory. There are too many reasonable alternatives among which presently available observations of aggregate time series cannot easily discriminate. A greater stock of relevant observations could be collected and brought to bear if the basic assumptions of dynamic

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[1] The same class of models we apply here may have other applications in economics and has, at least in part, appeared in other disciplines as well. See Priestley, Rao, and Tong (1974).

> economics were made about behavior of individual firms and consumers, and the implications then traced through to the aggregates, a task involving direct observation and model construction. There is also a need to introduce explicitly the random elements which reduce the reliability and degree of explicitness of prediction into the more distant future.

Now, just as when those words were written, very little of the a priori theory embodied in macroeconometric models is based explicitly on models of the behavior of individuals. Now, just as then, very little of the theory embodied in such models is explicitly stochastic. There is generally not even an attempt to justify the restrictions on serial correlation properties of residuals imposed in estimating such models on the basis of explicit economic theory. Many of the equations of such models, though formally identified by zero-restrictions on their coefficients, are, in fact, little more than attempts to capture certain statistical regularities in the sample period. The Phillips curve is a prime example of an empirical relationship that was initially incorporated in macroeconometric models without there first being a model of the individual behavior giving rise to the relationship. Another example is the common practice of using "capital utilization" indexes to adjust the measured capital stock before estimating an aggregate Cobb-Douglas production function. This practice is in spite of the fact that an optimizing firm with a Cobb-Douglas production function always uses all of its capital, and that no microtheory leading to an aggregate production function with utilization-adjusted capital has been put forward.[2/]

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[2]The public's expectations about future exogenous and endogenous variables are important arguments in many macroeconomic schedules including the Phillips curve, consumption schedule, investment

The fact that we question the assumptions ordinarily used in interpreting large econometric models does not mean that we necessarily regard the fitted equations themselves as useless. They probably do capture important statistical regularities, and in the empirical work reported below we aim at little more than this ourselves. The purpose of the kind of work we will be presenting is to explore the possibility that important statistical regularities are missed by existing largescale models,[3/] and also to see whether a class of models with a small intersection with the class of over-identified simultaneous equations models is capable of fitting the data approximately as well. This latter result would suggest that a good fit of standard models to the data should not be treated as strong evidence for the overidentifying restrictions they embody.

The models we estimate are certainly not "unrestricted" models. Even to explain the behavior of the main components of GNP, wages, prices, and unemployment, a model needs about ten equations, and many existing models contain several orders of magnitude more than that. Cyclical interactions among macroeconomic variables probably

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2 (continued)

schedule, and various asset demand schedules. In practice, most econometric models has posited that the public's expectations of a given variable are formed as distributed lags on the own variable itself, thus invoking the identifying restriction that the public ignores other variables in forming its forecasts. These restrictions are imposed in spite of the fact that the models themselves contain complicated dynamic interactions among variables that a priori lead one to suspect that it would be optimal to forecast a given variable by taking into account values of many other endogenous and exogenous variables. The zero identifying restrictions imposed on expectations generating mechanisms are thus not deduced from an appeal to optimizing behavior or any other economic theory we are aware of. Neither are the "unit sum" identifying restrictions that are usually imposed on expectations generators, as Lucas [1972] has emphasized.

[3]This seems pretty clear already, in fact, from the work by Nelson [1972] and Cooper and Nelson [1975].

commonly involve lags of eight or more quarters. A ten-equation, tenth-order autoregression of general form (ten lags of each of ten variables in each equation) leaves zero degrees of freedom, approximately, in U.S. postwar data.

Rather than reduce the dimensionality of our models by restricting particular equations a priori, as in the standard methodology, we proceed by imposing simplifying conditions which are symmetric in the variables. The intuition behind the particular restrictions we examine, leading to what we call "index" models, seems to us close to the intuition underlying the descriptive analysis of business cycles conducted by the National Bureau of Economic Research and described by Koopmans [1947] in his review of Burns and Mitchell as follows:

> The notion of a reference cycle itself implies the assumption of an essentially one-dimensional basic pattern of cyclical fluctuation, a background pattern around which the movements of individual variables are arranged in a manner dependent on their specific nature as well as on accidental circumstances. (There is a similarity here with Spearman's psychological hypothesis of a single mental factor common to all abilities.) This "one-dimensional" hypothesis may be a good first approximation, in the same sense in which the assumption of circular motion provides a good first approximation to the orbits of planets. It must be regarded, however, as an assumption of the "Kepler stage," based on observation of many series without reference to the underlying economic behavior of individuals.

We shall describe two related statistical models for representing the one-index (and more generally k-index) notion described by Koopmans. The first is an "unobservable index" model which is a natural counterpart of the standard factor analysis model alluded to by Koopmans in which the underlying factors are unobservable. The model is a frequency domain version of the factor analysis model and can be implemented by combining spectral analysis and factor analysis. The second model is an "observable index" model in which the underlying factors are observable.

Their attractiveness as statistical devices for restricting the dimensionality of vector time series models is not the only feature which draws us toward experimenting with index models. Certain theoretical macroeconomic models can be cast in index-model form. These include a class of models pioneered by Robert E. Lucas, Jr. [1975] as well as simple macroeconomic models which seem to us to reflect the pattern of quantitative thinking about the business cycle of many macroeconomists, "Keynesian" as well as "Monetarist". Thus, it would be a mistake to regard the techniques that we describe as being useful solely for pursuing measurement without theory. Economic models leading to index-model forms are discussed in more detail below.

The general form of index models

Index models all satisfy an equation of the form

1) $$y = a*z + u ,$$

where y the vector of observed dependent variables is $n\times 1$, u the vector of residuals is $n\times 1$, z the vector of indexes is $k\times 1$ with $k \ll n$, and a the vector of lag distributions relating z to y is therefore $n\times k$. In (A) all three of y, z, and u are stochastic processes, and the notation "*" stands for convolution, defined by $a*z(t) = \sum_{s=-\infty}^{\infty} a(s)z(t-s)$. We always take a "one-sided", i.e. $a(s) = 0$ for $s < 0$.

The kinds of economic theory which lead to index models do not in general contain implications about the properties of the residuals u other than that they should be small. Of course if there are no restrictions on the properties of u, any vector time series y can be written in the form (1) – and for arbitrary choice of z, a, and c. The expression (1) can simply be treated as the definition of u. However, by asserting that (1) "fits well", in the sense that the variance of each element u_i of u is small relative to the variance of the corresponding element y_i of y, regardless of how y_i is differenced or filtered [4], we obtain an hypothesis with content.

For empirical work, it is convenient to use still stronger hypotheses about the properties of u. If z is some linear combination $c*x$ of observable variables x (which may include current and past y's) then it is natural to hypothesize that (1) contains only current values of y, lagged values of y, and strictly exogenous variables, as is ordinarily assumed in modeling simultaneous equation systems. By this assumption we mean that any elements of the vector of observables x which are not lagged values of y are uncorrelated with u at all leads and lags. Further, it is natural to assume that (1) is complete in the sense that it determines current y uniquely from current and past values of x and u. Let us write x in two pieces, $\begin{bmatrix} x_0 \\ y \end{bmatrix}$, and divide c correspondingly into $[c_0\ c_1]$. Then substituting $c*x$ for z in (1) we obtain

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[4]This is equivalent to requiring that the i'th diagonal element of S_y be large relative to the i'th diagonal element of S_u at all frequencies. Consideration of the effect of time aggregation suggests that this "good fit" criterion should not be applied to the highest frequencies. We will not pursue this subtopic in this paper, though it is important for applications.

$$2) \qquad (I-a*c_1)*y = a*c_0*x_0 + u \quad .$$

The requirements we have imposed to this point amount to asserting that x_0 and u are uncorrelated at all leads and lags and that $(I-a*c_1)$ has an inverse under convolution.

If z is not a function of observable x's, it is natural to assume that z and u are orthogonal, i.e. that z and u are uncorrelated at all leads and lags.

Whether or not z is observable, identification requires further restrictions. We take as natural the one that individual elements u_i and u_j of the process u be orthogonal to one another, even though each u_i may itself be autocorrelated. This amounts to requiring that dependence on the indexes accounts for all the observed cross-relations among the series.

We have called the former of these two specifications the "observable index model", and the latter the "unobservable index model". Though the two specifications are in general distinct models, when either one "fits well", then both must fit well. This follows because as the variance of the residuals u_i in (1) shrinks relative to the variances of the index terms a_i*z , both types of specification amount to asserting that y differs only slightly from a singular process with rank equal to the length of the vector z .

To be more precise, suppose we write

$$3) \qquad S_y = \tilde{a}S_z\tilde{a}' + \tilde{a}S_{uz} + S_{uz}\tilde{a}' + S_u \ ,$$

where S_y and S_z are spectral density matrices, S_{uz} is the cross-spectral density matrix of u with z , and $\tilde{a}$ is the Fourier transform of a . Then if the model "fits well" in the sense we have been giving that phrase, S_u has its diagonal elements all small relative to the diagonal elements of $\tilde{a}S_z\tilde{a}'$. But this implies that $\tilde{a}S_{zu} + S_{uz}\tilde{a}'$ has small diagonal elements relative to the diagonal elements of $\tilde{a}S_z\tilde{a}'$ as well. Since in either type of index model we can normalize S_z to be the identity, we can always match the dominant $\tilde{a}S_z\tilde{a}'$ term using either type of index model. The differences between the two models will be in the "small" terms.

As will be illustrated in the section to follow, economic theory does not easily generate strict characterizations of the residuals in these models. Economic theories may, however, suggest that an index model with indexes of a certain nature should fit well. Because this kind of assertion does not effectively distinguish observable from unobservable index models, we will ourselves omit that distinction in the next section.

Economic Interpretation of Index Models

The NBER's framework for analysis of business cycles is perhaps the most prominent example of work in macroeconomics that fits comfortably within the index model framework, but it is not the only such example. In this section we give several examples of index models in macroeconomics.

To take the simplest example first, consider the following multiplier-accelerator model for determining GNP (Y) and its major components, consumption C, investment I, and government purchases, G:

$$\begin{aligned} Y(t) &= C(t) + I(t) + G(t) \\ C(t) &= b*Y(t) + u_1(t) \\ I(t) &= m*Y(t) + u_2(t) \\ G(t) &= r*Y(t) + u_3(t) \end{aligned} \tag{4}$$

Here b, m, and r are one-sided (on the past and present), square summable sequences, while $u_1(t)$, $u_2(t)$ and $u_3(t)$ are stochastic error processes. In the model (4), any subset of these variables (Y, G, C, I) forms a one-index model. (If all four variables are included, the presence of the national income identity makes the process singular.)

Note that because we interpret these equations as asserting a "good fit", they are not, like the equations of a standard simultaneous equations model, unaltered by changes in the choice of left-hand-side variable. The equations are to be interpreted as implying that the left-hand-side variable has substantially larger variance than the residual, and that interpretation may not remain viable if the equation is renormalized.

Any model which like (4) has a relatively small number of lagged or exogenous variables appearing in more than one equation is in the form of an index model. By this standard, many existing econometric business cycle models may not be very far from the form of an observable index model, if the number of indexes is taken fairly large (more than two or three).[5]

Now suppose we add to (4) a set of sectoral price equations,

5) $$p_j = f_j{*}P + g_j{*}Y + v_j \ , \quad j=1,\ldots,q$$

and a definition of the aggregate price index

6) $$p = \sum_{j=1}^{q} w_j p_j \quad .$$

The system formed by (5) and (6) asserts that the pattern of movement of sectoral prices is well explained by the history of aggregate output and an aggregate price index. The system (4), (5), with (6) substituted into (5), forms a two-index model. Furthermore, the subset of real variables explained by (4) involves only one index. Only by adding prices to the system do we incur the need for a second index.

Of course in reality the aggregate price level may well feed back into the determination of real variables. Let us examine what happens to this simple system when we include explicitly supply and demand for money and the possibility of interest rate effects on the real subsystem:

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[5] Of course many econometric models do have a rich supply of strictly exogenous variables -- expecially models of relatively small sectors of the economy. Such models might fall in the form (2), with y being only one component of x, but if such a model is identified by exclusion restrictions, without restrictions on lag length or serial correlation, it appears that it is unlikely to fit the form (2). This is a relatively subtle question whose detailed treatment we leave to another occasion.

$$M = k_1{*}P + k_2{*}Y + k_3{*}R + e_1 \quad \text{(demand for money)}$$

7) $$M = s_1{*}P + s_2{*}Y + s_3{*}R + e_2 \quad \text{(supply of money)}$$

$$I = m_1{*}Y + m_2{*}R + u_2 \quad \text{(replacing investment equation of (4)).}$$

Here the supply and demand for money equations are temporarily normalized on M, but our interpretation will depend heavily on which variables are in fact well explained by the money demand and supply interaction.

There are several ways our original simple two-index system, with one real and one nominal index, might be rationalized. If R (the interest rate) does not enter the investment equation ($m_2=0$), then supply and demand for money are just a pair of equations for recursively determining R and M, and can be omitted from the system. Alternatively, R might have very small variance, either because it is fixed by the supply equation (a pegged interest rate policy) or because it is fixed by the demand equation (a highly interest-elastic demand for money, or liquidity trap). Either of these situations in effect makes money supply passive relative to the real subsystem. In these cases, by merging the "small" term $m_2{*}R$ with u_2, we will preserve the one real, one nominal index structure.

In general, however, with m_2 non-zero the one real, one nominal index structure will not hold. We might, for example, solve the demand and supply of money for R in terms of Y and P. If the resulting equation fits well, we could use it to substitute an expression in terms of Y and P for R in the investment equation. We would thereby generate a two index model with a real and a nominal index, but it would no longer be true that the real sector of the model depended only on the real index. Another possibility is that the supply equation fixes M , subject to relatively small variance. If demand for money were interest-inelastic ($k_3 \doteq 0$), the supply and demand for money might then determine P as a function of Y . In that case we could substitute an expression in terms of Y for the nominal index P and obtain a one-index model.

One final possibility to note is that the money supply rule might fix the price level. Then P would effectively drop out of the system, but R would remain as a second index. We would have a two-index model, with one index being R , the other Y . A single index would explain the price vector, but two indexes would be required for the real subsystem.

This discussion could be elaborated further.[6/] We will arrest it here, observing what we have established so far -- that simple Keynesian models may take on an index-model form, that dichotomous models may take on a "one real, one nominal index" form, and that Keynesian models with interest-elastic investment do not suggest that a two-index model will show one real and one purely nominal index.

We now turn to models of the class constructed by Robert E. Lucas, which fit quite naturally into the index model framework, and predict a one-real index, one-nominal index pattern. Lucas's model substantially improves on the preceding models by providing an explicit behavioral interpretation of the model's dynamics. His model is "Keynesian" in the sense that it accounts for the presence of aggregate-

[6]We could, for example, add a supply of output system.

demand induced inflation-output or money-output correlations, but it is "monetarist" in the sense that it predicts the same one real index, one purely nominal index pattern that characterizes our dichotomous models and in its policy implications.

In Lucas's model, movements in aggregate demand interact with a stable structure of industry or market supply schedules to produce persistent fluctuations in real economic activity. These persistent fluctuations occur even though suppliers respond only to perceived movements in relative prices and form their perceptions rationally. The essential thing in Lucas's setup is the assumption that nominal aggregate demand is not immediately observable, though agents are assumed to understand its probability law. The notion that aggregate demand is not immediately observable is what gives the model the capacity to generate persistent (serially correlated) movements in real activity even where agents are rational.

A version of Lucas's model can be written

$$
\begin{aligned}
y_{it} &= c_i*(n_t-\hat{n}_t) + b_i*\varepsilon_{it}\ , & i=1,\ldots,N \\
P_{jt} &= d_j*(n_t-\hat{n}_t) + q\hat{n}_t + h_j*v_{jt} & j=1,\ldots,M
\end{aligned}
\tag{8}
$$

Here c_i, b_i, d_j, and H_j are each one-sided functions while q is a scalar. The y_i's are measures of real economic activity such as real output or employment in particular industries or aggregates of industries. The P_{jt}'s are prices of particular commodities or aggregates of commodities. The variate n_t is nominal aggregate demand, while $\hat{n}_t$ is the public's expectation of n_t formed as the linear least squares projection of n_t on some information set θ . According to the model, real variables respond only to the unexpected part of n_t , namely $n_t - \hat{n}_t$. A foreseen increase in n_t causes only the price variables to respond, leaving real quantities unaffected. The model thus incorporates the natural rate hypothesis. The variates ε_{it} and v_{jt} are second-order stationary random variables with properties to be specified shortly.

To complete the model, we must specify the information set θ. We assume that the public does not have current readings on the variate n_t, but does have readings on current and past values of an (sxl) vector x_t of variates correlated with the n process. The vector x_t may include n_{t-s} for s greater than some minimal "perception delay" $\delta \geq 1$. Furthermore, the public is assumed to know the cross-covariogram

$$E\{n_t \cdot x_{t-\tau}\} \qquad \tau = 0, \pm 1, \pm 2, \ldots,;$$

it also knows the first moments of the (n, x) process. The public forms $\hat{n}_t$ as the linear least squares projection of n_t on the spaced spanned by $\{x_t, x_{t-1}, \ldots\}$. We have the decomposition

$$9) \qquad n_t = \sum_{j=0}^{\infty} v_j x_{t-j} + u_t \equiv \hat{n}_t + u_t ,$$

where the v_j's are conformable to x_t and where by construction $E\, u_t x_{t-j} = 0$ for all $j \geq 0$; that is, the residuals in the least squares regressions are orthogonal to the regressors.

Notice that because x_t does not in general contain all lagged n's, the least squares orthogonality condition does not imply that u is serially uncorrelated. Thus, u itself will in general be serially correlated, so that the model predicts aggregate-demand-induced, serially correlated movements in the y_i's even where $c_i(s) = 0$ for $s \neq 0$, all i. 7/

[7] Some economists have dismissed earlier versions of Lucas's natural-rate/rational-expectations models because they did not provide an endogenous explanation of how aggregate-demand-induced fluctuations in output could persist (e.g., Hall [1975]). If n_{t-s} for all $s \geq 1$ are included in x_t, the u's that appear in (9) (continued)

The system (8) is evidently in the form of a two-index model. Further, if we take one index to be $n_t - \hat{n}_t$, the real subsystem is by itself a one-index model. The second index is required only if we add prices to the system.[8/]

Now if we try to complete the specification of the system (8) so that it becomes exactly an "observable" or "unobservable" index model, we run into some difficulties. Since the model depends on economic agents' not being able to observe n_t, an unobservable-index framework is perhaps most natural. But recall that the restrictions imposed on this class of models include that the stochastic processes u and z be uncorrelated with each other. In the spirit of a rational expectations formulation, we ought to suppose that economic agents can observe the variables y and p which enter our model, and that these variables form a sub-vector of the vector x on which $\hat{n}_t$ is based. If this is so, it requires strong and arbitrary side restrictions to avoid the conclusion that $\hat{n}_t$ and u_t should be correlated. To justify the strict form of unobservable-index model which we fit below requires, in the context of Lucas's model, that u is a set of measurement errors bedeviling econometricians but not the public.

To make (8) an observable-index model, we must assume that econometricians can directly measure n_t, even though the public cannot. To justify this assumption we need to suppose either that the historical data on which model-fitting is based are not contemporaneously available to the public, or that

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7 continued are serially uncorrelated. By making the n's contemporaneously unobservable, Lucas achieved the restriction on information sets necessary to make serially correlated forecasting errors coexist with rational agents. Then nominal aggregate demand can generate serially correlated movements in outputs even though it is only the public's errors in forecasting nominal aggregate demand th cause outputs to respon .

[8] There is a possible exception worth noting. It is possible that $n_t - \hat{n}_t$ and $\hat{n}_t$ collapse to a single index. This could occur not only if forecasts are perfect ($\hat{n}_t = n_t$) but also if, e.g., forecasts of n_t are based on lagged values of n_t only.

to the extent they are available the public does not find it worthwhile to use them. These assumptions are of course as implausible a priori as those required to justify the unobservable-index formulation.

Finally in both specifications the requirement that the u_i's be mutually uncorrelated has no foundation in Lucas's theory.

Despite its explicit recognition of uncertainty in modeling behavior, Lucas's theory actually generates behavioral equations without residuals. As with most [9] macroeconomic theory then, we must tack on residuals to obtain empirically usable models, and the theory is silent about the nature of the residuals.

All of the economic models that we have studied here take as a primitive concept the notion of a one-dimensional "nominal aggregate demand" (or "reference cycle phase" in the jargon of the NBER). This section is intended to indicate how index models seem to be a natural statistical setting in which to study such macroeconomic models. However, none of the models studied here derives the existence of a one-dimensional driving process for the business cycle from more primitive assumptions. At this stage of development, the hypothesis that a low-order index model may fit the data well is thus in the category of an attractive empirical working hypothesis, with support in tradition, if not in logic.

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[9] One class of exceptions that we are aware of occurs where an exact model with no errors relates certain spot prices with forward prices. If the forward prices are "rational" linear least squares projections of future prices on a (large) information set Θ_t, but the economist models those expectations as "rational" linear least squares forecasts based on an information set Θ_t' that is strictly included in Θ_t, there emerges a set of strong orthogonality restrictions on the error in the structural equation. Shiller's work [1972] on the term structure is the original example from this class of setups; Fama's article [1975] is another such example. Notice how the argument hinges critically on having an <u>exact</u> theory to begin with.

Alternative characterizations of the models.

A vector stochastic process which is coveriance-stationary can be given the form of an unobservable-index model if and only if its spectral density (a matrix-valued function, the Fourier transform of the autocovariance function) can be written in the form

$$10) \qquad S_y = LL' + V ,$$

where S_y, L and V are all matrix-valued functions of frequency (ω) on $(-\pi, \pi)$, with L nxk and V diagonal with positive elements on the diagonal. That the unobservable-index-model form implies the representation (10) is not hard to see. Equation (10) follows directly from (3), the assumption that u and z are orthogonal (so $S_{uz} = 0$) and the fact that the positive definite matrix S_z appearing in (3) can be factored into the form $S_z = WW'$. Thus $L = \tilde{a}W$ and $V = S_u$. It is apparent from (10) that the separate components of $L = \tilde{a}W$ are not identified, so that to identify a we must make some arbitrary normalization of S_z. We take $S_z = I$.

Showing that the existence of a representation in the form (10) implies that y can be given an index-model representation is a somewhat subtler task, and will not be undertaken here. We cannot simply set $\tilde{a} = L$, because L may not be the Fourier transform of a one-sided function. Yet even if L is not the Fourier transform of a one-sided function, under certain regularity conditions a one-sided a exists such that $\tilde{a}\, \tilde{a}' = L\, L'$. In fact there are in general several such a's, and to identify a uniquely we require a further identifying restriction, namely that $a*z$ be the moving-average representation of the process $x = a*z$[10/].

The foregoing identification or normalization problems create serious practical difficulties in estimation of a. However, it is a great advantage of the unobservable-index formulation that, by estimating LL' without attempting to identify a, we can test the fit of the model without any need to impose the complicated identifying normalizations. The equation (10) is exactly the model of factor analysis, with the difference that the equation is a decomposition of the spectral density matrix at each frequency instead of being a decomposition of a single covariance matrix. Since estimates of S_y over frequency bands which are far enough apart are independent under their asymptotic distributions, we can apply the factor analysis model independently at each frequency. Except for slight complications arising from the fact that S_y is complex and conjugate-symmetric, not real and symmetric, estimation methods and statistical tests developed in the factor analysis literature carry over directly to the unobservable-index model.

A covariance-stationary vector stochastic process y[11/] has an observable-index representation, with $z = c*y$ for some c, if and only if its moving average representation can be written in the form

$$y = (I + \alpha*\gamma)*D*e \ , \tag{11}$$

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[10]If $x = a*z$, then given any one-sided square summable $k \times k$ function b such that $|\tilde{b}|^2 = I$, $x = a*b*b^{-1}*z$ and $b^{-1}*z$ has the identity as its spectral density matrix. By requiring that $a(o)z$ be the vector of one-step-ahead forecast errors in x, we fix q uniquely up to multiplication by a fixed unitary matrix, and $a*z$ becomes "the" working average representation of x. See Rozanov [1967] for a rigorous discussion of these notions.

[11]Strictly speaking we are considering only linearly regular processes (i.e. processes with no deficiencies to component). See Rozanov [1967] for a definition of linear regularity.

where α and γ' are each one-sided nxk matrix-valued functions, and D is a diagonal matrix-valued function. To see that (11) follows from our original specification (2), recall that we are now considering the case of no exogenous variables x , so that c_0 in (2) is empty, and c_1 and c are the same thing. We required that $(I-a*c_1)$ has a one-sided inverse under convolution, so that we can write

$$12) \qquad y = (I-a*c)^{-1}*u \quad .$$

The vector process u itself has a moving average representation of the form $u = D*e$, where e is a vector white noise process and D is a diagonal matrix-valued function. Substituting this representation into (12) yields

$$13) \qquad y = (I-a*c)^{-1}*D*e \ ,$$

which is the moving average representation of y.[12/]

Now the fact that (I-a*c) has a one-sided inverse implies that (1-c*a) also has a one-sided inverse.[13/] Then it is easy to verify that $(I-a*c)^{-1} = I + a*(1-c*a)^{-1}*c$. Substituting this expression for $(I-a*c)^{-1}$ in (13) gives us an expression exactly in the form (11), with $a = \alpha$ and $(1-c*a)^{-1}*c = \gamma$,[14/] namely

$$14) \qquad y = (I + a*(1-c*a)^{-1}*c)*D*e$$

[12]For the purist, this follows from the fact that current and past y and current and past u span the same Hilbert space, under the covariance inner product, and hence must have representations in terms of the same fundamental white noise.

[13]The Fourier transform of $1-c*a$, $1-\tilde{c}\tilde{a}$, is the determinant of $I-\tilde{a}\tilde{c}$, which in turn must be bounded away from zero in the lower half-plane for $I-a*c$ to have a one-sided inverse. But $1-\tilde{c}\tilde{a}$ bounded away from zero in the lower half-plane guarantees that $1-c*a$ has a one-sided inverse.

[14]Where y does not have an autoregressive representation, (11) may hold without the existence of any regression of the form (2). Since such cases can in a sense be approximated arbitrarily well by cases in which an equation like (2) does exist, it seems natural to include these cases as observable-index models.

From (11) we find the spectral density of y to be given by

$$15)\qquad S_y = |\tilde{D}|^2 + \tilde{\alpha}\tilde{\delta}\tilde{D} + \tilde{D}'\tilde{\delta}'\tilde{\alpha}' + \tilde{\alpha}\tilde{\delta}\tilde{D}\tilde{D}'\tilde{\delta}'\tilde{\alpha}' \quad .$$

Equation (15) asserts that y's spectral density is the sum of a diagonal matrix and a matrix of rank $2k$. Could it be then that observable-index models of rank k are equivalent to unobservable-index models of rank $2k$? The answer is no. If $\tilde{D}'\tilde{\delta}' \neq \tilde{\alpha}\lambda$, where λ is scalar, the singular matrix added to $|\tilde{D}|^2$ in (15) will generally have negative roots as well as positive roots. Even under the condition $\tilde{D}'\tilde{\delta}' = \tilde{\alpha}\lambda$, it can be shown that the unobservable-index models which can be generated from (15) are a very narrow class.[15]

An interesting question for further research arises here: Is there an attractive index model specification which would generate the general case of

$$16)\qquad S_y = V + M \quad ,$$

where V is diagonal with positive elements on the diagonal and M is an arbitrary (except for the requirement that S_y remain positive definite) conjugate-symmetric matrix of rank k? Such a general specification would probably allow use of the convenient factor-analytic-like methods which apply to the unobservable-index model, would cover both observable-index and unobservable-index models as special cases, and would probably avoid the all-too-common result that estimation of the unobservable-index model shows maximum likelihood at a point where V is singular.

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[15] Again, the reader must be referred elsewhere (Sims, 1975) for the detailed arguments. The gist of this argument is that if $\tilde{D}'\tilde{\delta}' = \tilde{\alpha}\lambda$, λ scalar, then $\delta * D$ is one-sided only under strong side conditions.

Causal Orderings in Index Models

In the degenerate case of $u = 0$ in (1) "causal orderings" in the sense of Granger can be characterized entirely in terms of the parameters a. In this case it is likely that many pairs of variables cannot be ordered. It is well known that " y does not cause x " in Granger's sense if and only if the linear least squares projection of y_t on x is a one-sided distributed lag.[16/] If a_i and a_j both have one-sided inverses under convolution, then $y_i = a_i * a_j^{-1} * y_j$ and $y_j = a_j * a_i^{-1} * y_i$. Thus each of the two variables is exogenous in a (perfectly fitting) onesided distributed lag regression with the other variable on the left, and no one-way ordering is possible. More generally a_i and/or a_j may not have one-sided inverses, in which case orderings may exist.

When we add error terms to the model, with the properties natural to the observable and unobservable cases, the a's no longer characterize causal orderings. The coefficients in the projection of y_i on some subset of variables Y included in the vector y, is given by $R_Y^{-1} * R_{Yy_i}$ where R_Y is the autocovariance function of Y and R_{Yy_i} is the cross-covariance function of Y with y_i, respectively. In the case of an unobservable index model, under the identifying assumption that z and u are orthogonal, one requires restrictions on the serial correlation properties of the u's, relating them to the a's, in order to restrict R_Y and R_{Yy_i} enough to generate a causal ordering. To the extent that the economics of the model is

[16] Y does not cause x in Granger's sense if, given values of all other variables in the system (including x) at times before t, knowledge of values of y at times before t cannot improve our forecast of $x(t)$. This notion is discussed in more detail in the paper by Sims in this volume.

embodied in its systematic component, economic characteristics of the model cannot imply a causal ordering.[17]

In the case of observable-index models with no exogenous variables, a certain limited class of causal orderings may be characterized by restrictions on a and c. It is well known that Granger causal orderings on linearly regular covariance-stationary vector processes are characterized by block triangularity conditions on the moving average representation. (See Sims (1972)). In particular, y_1 does not Granger-cause y_2 (y_2 is causally prior to y_1 in Granger's sense) if and only if in the joint moving average representation $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} * e$, A_{21} can be chosen to be zero. Looking now at the expression (14) for the moving average representation of an observable-index model, we see that if y is partitioned into $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and a and c are partitioned conformably, there are two simple conditions on a and c generating block triangularity with $A_{21} = 0$: $a_2 = 0$ or $c_1 = 0$ [18]. With $a_2 = 0$, the indexes do not affect y_2 , so that the elements of y_2 are mutually orthogonal. Further, any subvector of y_2 is causally prior to the remainder of the y vector, so that y_2 is not only causally prior as a block, but each element of y_2 is separately causally prior. With $c_1 = 0$, none of the elements of y_1 enter any of the indexes and y_1 can

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[17] John Geweke [1975b]has given a condition for exogeneity of y_1 in an unobservable-index system which, like the $a_1 = 0$ condition on an observable-index system discussed below, implies that all elements of y_1 are exogenous in all other equations of the system, including the other equations in the y_1-block. Geweke's condition also implies that the residuals from regressions of y_2 on y_1 form an unobservable-index model of the same order as the original model.

[18] The "c_1" here is the first element of the partition of c conformable to the partition of y into $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, not the "c_1" which appeared earlier when we discussed models containing exogenous variable x .

therefore be regarded as "passive". The elements of y_1 are related to each other only through their common dependence on y_2 .

Some further comparative properties of the models.

An unobservable-index model retains its form if a subset of elements of y is used in place of y itself. In fact, invariance of estimated a's and of fit of the model to omission of variables from the system is a property which might be used to test the unobservable-index specification. In an observable-index model on the other hand, only purely passive variables (y_j's with $c_j = 0$) can be ommitted from the model without invalidating the index-model specification.

An observable-index model of given order has twice as many independently specified lag distributions as an unobservable-index model of the same order, since the a parameters appear in corresponding positions in both models while the c parameters appear only in observable-index models. This might at first appear to conflict with the limiting equivalence of the two specifications, for the same order k , as perfect fit is approached. However, the paradox only reflects the fact that in the limit as a perfect fit is approached c is no longer identified, as the same estimate of z can be constructed from a variety of linear combinations of current and past y . Where the fit of the model is in fact very tight, one should either use the unobservable-index specification or impose a fixed form on c a priori.

If we estimate equation (2), passed through the filter D^{-1} , as a constrained autoregression, we obviously have an autoregressive representation of y immediately at hand. This is important for preparing forecasts and in some kinds of model-testing. Estimating the unobservable-index model does not lead directly to an autoregressive form, and is in this respect less convenient. Further, estimating (2) leads directly to estimates of a and of historical values of z, which is important for interpreting the model. Estimates of a and of historical z's are harder to obtain with unobservable index models. On the other hand, we have already noted that it is possible with the unobservable index model to test the fit of the model without estimating a or z, and this is much easier computationally than fitting the observable index model of corresponding order.

Observable-index and unobservable-index models are equivalent only in a narrow class of special cases. One case of this type is where some component of the vector u in (12) is identically zero, z is scalar, and the corresponding component of a has a one-sided inverse. Taking this special component of y to be y_1, we have then $z = a_1^{-1} * y_1$, making the model an observable-index model, but at the same time a degenerate case of an unobservable-index model. If u is a full rank process, the two kinds of model coincide only in a narrow class of cases, e.g. if $a_i(s) = \lambda_i a_1(s)$, all i, s, $c_i(s) = \lambda_i c_1(s)$, all i, s, and c_i has a one-sided inverse.[19]

[19] This result, together with some others characterizing the relation of index-models to standard simultaneous-equation models, is proved in Sims [1975].

Extimation and testing for the unobservable-index model

The unobservable index model can be estimated and tested by using suitable generalizations of the maximum likelihood method of estimating the standard factor analysis model, described by Joreskög [1967) and Lawley and Maxwell [1963][20]. Passing to the notation of Lawley and Maxwell, let

$$S_y(\omega) = C \ , \ \tilde{a}(\omega) = L \ ,$$
$$S_y = C = LL' + V = \tilde{a}\tilde{a}' + S_u$$

and remember that there is a 3-tuple (C, L, V) at each $\omega\varepsilon[0,\pi]$. Assuming that the (nxl) y_t process is normally distributed implies that $\tilde{y}(\omega)$ the finite-Fourier transform of y, evaluated at ω, has a complex normal probability distribution, asymptotically:

$$f(y;C) = \frac{1}{\pi^n|C|} \exp\ (-\tilde{y}(\omega)'C^{-1}\tilde{y}(\omega)).$$

Supposing that we have m independent observations on $\tilde{y}(\omega)$, say $\tilde{y},\ (\omega),\ldots,\tilde{y}_m(\omega)$ with common covariance matrix C, the likelihood function is

$$L(C;\tilde{y}_1,\ldots,\tilde{y}_m) = \frac{1}{\pi^{nm}|C|^m} \exp\ (-\sum_{i=1}^{m} \tilde{y}_i(\omega)'C^{-1}\tilde{y}_i(\omega))$$

with log likelihood

$$\text{(17)} \qquad \ln L(C;\tilde{y}_1,\ldots,\tilde{y}_m) = -m(n \ln \pi + \ln|C| + \operatorname{tr} SC^{-1})$$

where

$$S = \frac{1}{m} \sum_{i=1}^{m} \tilde{y}_i(\omega)\ \tilde{y}_i(\omega) \quad .$$

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[20] John Geweke [1975a]has shown how the computational techniques for the real factor analysis model can be adapted for application to the frequency domain factor analysis model. The computations reported below were made using Geweke's original one-index computer program amended by Paul A. Anderson to handle k indexes. See Geweke [1975a] for a more detailed discussion of the techniques described in the text.

Maximization of the log likelihood function (17) is equivalent to minimization of

$$\zeta = \ln|C| + \mathrm{tr}\ SC^{-1}$$

With C unrestricted, the maximum likelihood estimate of C is S. Under the frequency domain factor analysis model, estimation is carried out under the restriction $C = LL' + V$, so that the function minimized is

$$\text{(18)} \qquad \xi_f = \ln|LL'+V| + \mathrm{tr}\ S(LL' + V)^{-1} ,$$

where the minimization is with respect to L and V.

The null hypothesis that k factors can account for the covariation of y at a given frequency (or band of frequencies) can be tested by using a likelihood ratio test. The relevant statistic is

$$\text{(19)} \qquad R = 2(\ell_1 - \ell_2)$$

where ℓ_1 is the value attained by the log-likelihood function unrestricted and ℓ_0 is the value attained by the log likelihood function under the k-index restriction. On the null hypothesis, R is distributed as chi-square with $\frac{1}{2}\{(n-k)^2 - (n+k)\}$ degrees of freedom. In practice a small sample correction suggested by Bartlett (see, e.g., Lawley and Maxwell [p. 23]) is used to adjust R.

It should be remembered that the chi-square tests are asymptotically valid only if there occur no boundary solutions in which over some band $V(\omega) = 0$ for some variable. We do encounter some such boundary solutions. Consequently, the formal test statistics should be interpreted with some circumspection.

In addition to the formal hypothesis test of the k-index model, it is useful to construct the coherence

$$\text{20)} \quad \mathrm{coh}_i(\omega) = \frac{[L(\omega)L'(\omega)]ii}{[C(\omega)]ii} \equiv \frac{\tilde{a}(\omega)\tilde{a}'(\omega))ii}{Sy(\omega)ii} \equiv \frac{Sy(\omega)ii-[S_u(\omega)]ii}{Sy(\omega)ii} .$$

which tells the proportion of the variance in y_i at frequency ω that can be accounted for by the k indexes. We also report the overall coherence defined by

$$21) \quad Tcoh_i = \frac{\sum_{\omega} \tilde{a}(\omega)\tilde{a}'(\omega)ii}{\sum_{\omega} Sy(\omega)ii} ,$$

where both $a(\omega)$ and $Sy(\omega)$ have been recolored by multiplying by the inverse Fourier transform of the filters used to whiten the variables. It is possible for the likelihood ratio test ( ) to call for rejection of a k-index model and yet for the model to explain a large proportion of the variance in some or all of the n y_i's . As we have noted, economic theories leading to index models seem to assert only that a one-index model will deliver "high" coherence for many interesting aggregate time series.

In practice, the tests of and summary statistics for the k-index model were calculated as follows. First the n variables in y_t were whitened by computing univariate autoregressions with linear trends included.[21] The residuals from these regressions were taken as the whitened values of y . For series of length T , the Fourier transform of the (nxl) whitened vector y_t ,

$$y(\omega_j) = \sum_{t=1}^{T} y_t e^{i\omega_j t} ,$$

was calculated at the frequencies

$$\omega_j = \frac{2\pi j}{T} , \quad -[\frac{T-1}{2}] \leq j \leq [T/2]$$

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[21]The procedure described here is asymptotically valid only if the order of the estimated autoregression in the first step is held fixed while sample size increases. If the estimated prewhitening autoregressions are richly parameterized, results are biased. Our prewhitening regressions were short, and re-estimates using standardized, arbitrarily chosen prewhitening filters on all series did not alter results.

where $[x]$ means the greatest integer less than or equal to x . Then across a band of m ω_j's , the cross spectral matrix of the whitened y's was estimated as

$$\hat{S}_y = \frac{1}{m} \sum_{j \in J} y(\omega_j)\, y'(\omega_j) \tag{22}$$

where J is the set of j's included in the band. For purposes of the formal likelihood ratio test of the k-index model, (22) was used to estimate S_y across a number of disjoint frequency bands. For each band, the maximum likelihood estimates of LL' and V' are obtained, and the likelihood ratio test (19) and coherence (20) computed. Where r nonoverlapping frequency bands are studied, the likelihood ratios at the different bands can be summed. Since it is the sum of r asymptotically independent $\chi^2(.5(n-k)^2-n-k)$ variates, the sum is asymptotically chi-square with $.5r[(n-k)^2-n-k]$ degrees of freedom. This summary statistic can be used to test the overall fit of the model.

By way of deriving a representation of the model in the time domain, the vector autoregressive representation for the y process implied by the k-index model can be derived as follows. First, calculate the Fourier transform $y(\omega)$ as above, and then smooth using a moving average across frequencies to estimate the cross spectral matrix S_y at a number of frequency points. (This differs from the above procedure used in testing the k-index model in that we now do not use nonoverlapping frequency bands. The asymptotic independence of the estimates of S_y at different bands, which is important for hypothesis testing, is lost at the gain of being able to estimate the cross spectral matrix at more frequencies.) Next, at each frequency calculate the maximum likelihood estimates of LL' and V for k indexes to obtain

$$\hat{S}_y^k = LL' + V \quad .$$

The estimate S_y^k is then "recolored" using the transfer functions implied by the filters used to whiten each y_{it}.

To obtain the matrix of cross covariances of y under the k-index restriction, we calculate the inverse Fourier transform of $\hat{S}_y^k$:

$$\underset{(n\times n)}{\hat{R}_y^k}(s) = \frac{1}{2\pi} \sum_{j\varepsilon J} \underset{(n\times n)}{S_y^k(\omega_j)} e^{-iw_j s}$$

where J indexes the set of frequencies at which the cross spectral matrix is calculated, and $\hat{R}_y^k(s)$ is an $(n\times n)$ matrix of estimated covariances at lag s under the k-index restriction. Using the elements of $\hat{R}_y^k(s)$ as estimates of the population covariances, the n vector autoregression can be calculated by entering the appropriate elements of $\hat{R}_y^k(s)$ in the usual formula for the projection of a random variable Z on a $(1\times b)$ random vector X:

$$P(Z|X) = \underset{1\times b}{X}\underset{b\times b}{[E(X'X)]^{-1}}\underset{(b\times 1)}{[EX'Z]} \quad :$$

We have not yet used this procedure to estimate vector autoregressions under the k-index hypothesis. We intend to use such vector autoregressions to generate forecasts and residuals. The procedure can be thought of as a way of estimating a vector autoregression under a restriction on the dimensionality of the parameter space. Since vector autoregressions of even low order typically have very many parameters, some such restriction seems useful in order to proceed with estimation.

Some Sample Coherences

By way of summarizing some of the raw facts we are seeking to account for, Table 1 reports coherences[22/] for pairs of variables among the following fourteen quarterly aggregates for the U.S. over the period 1950.I-1970.IV. Moody's Baa index (RBaa), the log of real GNP, the rate on 91-day Treasury bills (RTB), the log of the GNP deflator (P), the log of a straight-time wage index in manufacturing (w), the log of the money supply as measured by currency plus demand deposits (M1), the log of total federal and state and local government purchases (G), the federal and state local government deficit, the civilian unemployment rate (Un), the log of residential construction, the change in the log of the stock of inventories, plant and equipment investment (PL & EQPT), total consumption (cons), and corporate profits plus inventory valuation adjustment. Each series was prewhitened by computing an autoregression with five own lags with a linear trend and constant included. The residuals from those regressions were then used to compute the cross spectra.[23/] We

[22/] The coherence between series i and j at frequency w is defined as

$$\frac{|S_y(w)_{ij}|^2}{S_y(w)_{ii}\;\; S_y(w)_{jj}},$$

and is analogous to an R^2 statistic, telling the proportion of the variance in series i that can be accounted for by series j at the frequency w.

[23] This procedure biases the usual asymptotic distribution theory for the frequency domain estimates, unless the order of the initial autoregressions is reqarded as increasing with sample size much more slowly than the number of non-overlapping frequency bands at which spectral estimates are computed. Our separate fifth-order fitted univariate prewhitening filters are too flexible to make the required property of negligible sampling variability in the fitted univariate filters plausible. However, repetition of some of the main calculations with lower-order whitening filters, chosen a priori, showed no important alterations in the results. Had we been aware of this problem from the start, we certainly would have used something like fixed $(1-.956)^2$ prewhitening filter on all quarterly series, e.g., rather than filtered fifth-order autoregressions.

used Parzen's algorithm for estimating the cross spectrum as the Fourier transform of the cross covariogram. A Parzen window was used with 24 being the maximum lag used in the cross covariograms. For a sample size of 100 and this maximal lag, the use of the Parzen window implies the asymptotic confidence intervals around the coherence as summarized in Table 2. These were calculated using the method described by Jenkins and Watts [1968].

Many of the coherences in Table 1 are low, even at the business cycle frequencies. For example, the coherence of the GNP deflator with real GNP is low at the business cycle frequencies, never getting much above .3 at the business cycle frequencies. The coherences with money are interesting. In particular, notice that the coherence of money with some measures of real activity like unemployment and real GNP are substantially higher than are the coherences of money with the GNP deflator or the wage index.

Table 1a records the coherences between pairs of various monthly series we will be studying. Table 3 contains 95 percent confidence intervals for the coherences for the monthly data.

Overall, the coherences display some tendency to be highest at the low frequency components, perhaps giving some support to the concept of the business cycle as a set of correlated low frequency movements in a variety of aggregate variables. On the other hand, the coherences illustrate again Granger and Newbold's [1974] point that once own serial correlation is eliminated, economic time series are not all that highly correlated.

Estimated Unobservable Index Models

For quarterly time series extending over the period 1950I-1970IV, we have fit the unobservable index model to several subsets of the following macroeconomic variables: the official unemployment rate, real GNP, the GNP deflator, residential construction, plant and equipment investment, total consumption, inventory investment, corporate profits plus inventory valuation adjustment, an index of the straight time wage in manufacturing, and the money supply (currency plus demand deposits). Of these variables, the GNP deflator and wage index are nominal quantities; corporate profits is not deflated, but is a variable whose variance is probably very largely dominated by movements in its real component; the money supply is a potential contributor to variations in nominal aggregate demand; while the remaining variables are all deflated and thus are supposed to be measures of real economic activity.

The period consists of 89 quarterly observations. The filtered series were filled out with enough zeroes to bring the series up to 100 observations, so that periodogram ordinates were calculated at the 51 frequencies $w_j = 2\pi j/T$, j=0, 1, ..., 50. For the purpose of hypothesis testing,[24] the periodogram vector was averaged over the following four nonoverlapping bands: $w_j = 2\pi j/T$, j=1, ..., 11; j=12, ..., 23; j=27, ... 37; j=38, ..., 48. These four bands are centered at periodicities of 16 2/3 quarters, 5.88 quarters, 3.125 quarters, and 2.33 quarters, respectively. The first band ranges over periodicities of from 100 to 9.09 quarters, and thus is the band in which the frequencies composing the business cycle lie. We have omitted from the bands the seasonal periodicities of four and two quarters and also one frequency on either

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[24] Over a band of m periodogram ordinates at frequencies $w_j = 2\pi j/T$, we form S_y according to (11); i.e.,

$$\hat{S}_y = \frac{1}{m} \sum y(w_j)\, y'(w_j)$$

where $y(w_j)$ is the (9x1) vector of periodogram ordinates of the whitened y's at w_j. Since the rank of $y(w_j)\, y'(w_j)$ is one, the rank of $\hat{S}_y$ is at most m. Our computations require $\hat{S}_y$ to be invertible, which requires taking $m \geq 9$. This consideration explains why we have used only four nonoverlapping bands, since we have only 50 periodogram ordinates.

side of the seasonal. This accounts for the missing ordinates j=24, 25, 26, and j=49, 50.

Unobservable index models were fit to the five sets of variables listed in Table 4. Summaries of the results are contained in Tables 5 through 14. Set 1 includes six real variables (if we count corporate profits as a real variable) plus the GNP deflator. Since there is only one price variable, we might expect that this should be described well by a one-index model. The summary statistics in Tables 5 and 6 show that a one-index model fits pretty well. The over-all chi-square statistic attains of marginal significance value of .17 for the one-index model, though the marginal significance value is only .054 for the chi-square statistic at the business cycle frequencies. The coherences with the one-index are high at the business cycle frequencies for all variables except the GNP deflator, residential construction, and consumption. Its high coherence with GNP, unemployment, and corporate profits indicates that the one-index seems to be a measure of overall business activity. Notice that in the two-index model, the multiple coherence of GNP deflator remains low. Introducing the second index results in substantial increases in the multiple coherence for residential construction, plant and equipment investment, and real GNP, indicating that there seems to be a second real index. With the second index included, the marginal significance level at the business cycle frequencies climbs to .13.

Set 2 includes inventory investment in addition to the seven variables in Set 1. The one-index model performs almost as well as it did in Set 1, with marginal significance levels for the chi-square statistic being .045 at the business cycle frequencies and .12 overall. The coherences with the one index follow the same pattern for all variables as in Set 1, with inventory investment having a coherence of .5 overall with the one index. Adding a second index causes the marginal significance level to rise to .17 at the business cycle frequencies and causes substantial increases in the multiple coherences for the GNP deflator and inventory investment. The coherences for residential construction and consumption remain low. It seems that there is a nominal factor with which the GNP deflator and inventory investment are both correlated.

The third set adds the money-wage index to the variables in Set 2. A one-index model again does well with a marginal confidence level of .11 at the business cycle frequencies. The second index again raises the coherence for the GNP deflator and inventories, and to a much lesser extent the money-wage index. Notice that the money wage retains a low coherence even with two indexes included.

The fourth set includes the money supply along with the GNP deflator and a set of our real variables. Here a one-index model attains a marginal significance level of .037 at the business cycle frequencies. Actually, if the money supply is thought of as an important contributor to nominal aggregate demand, Lucas's model predicts that adding the money supply to a set of N real and one price variables will result in a deterioration in the adequacy of the one-unobservable-index model. That is because the money supply itself has predictable and unpredictable parts, is thus correlated with both the u and η of our equations (21) and (22) -- that is, it is correlated with both the real and the nominal factors. So a two-index model ought to be required. Notice that while the marginal significance level for the two-index model climbs to .11, the second index contributes to substantial increases in the multiple coherences for the GNP deflator and inventory investment, with a much more modest increase in the coherence for money. The money supply does not seem to be extremely highly correlated with our second "nominal" index. When a third index is added, however, the coherence for the money supply does rise drastically, as does the coherence for residential construction. There seems to be a credit-crunch index linking these two, perhaps reflecting the workings of interest rate ceilings.

Set 5 excludes inventory investment but includes money. Now a one-index model achieves a marginal significance of .109 at the business-cycle frequencies. Adding a second index leaves the multiple coherence of the GNP deflator low, but raises that for money and residential construction. This pattern is consistent with the results for Set 4.

It is noteworthy that a one-index model delivers high coherence for unemployment, GNP, plant and equipment investment, and corporate profits, and that the coherences for consumption and residential construction with the first index are quite low. This finding is consistent with casual observations that residential construction and consumption both behaved in a stabilizing or acyclical fashion during most post war business cycles.

We have also estimated index models for sets of monthly data extending over the period 1950.1-1970. Table 15 shows three sets of variables to be studied here. The data are average weekly hours, layoffs, manhours, the overall unemployment rate, the indsutrial production index, retail sales, net business formation, new orders for durables, an industrial materials price index, the wholesale price index, and the money supply (demand deposits plus currency). Of these variables, two are price indexes; one--the money supply--is a variate widely alleged to help determine nominal aggregate demand; retail sales and new orders for durables are undeflated and thus are nominal measures of activity; the remaining variables are deflated and so correspond to measures of real economic activity. The period consists of 26__ observations which we extended to 288 observations by filling out with zeroes. We calculated the periodogram ordinates at the 145 frequencies $2\pi j/T$, $j=0, 1, \ldots,$ 144. For the purpose of hypothesis testing the periodogram vector $y(w)$ of the whitened vector y_t was averaged over the following six nonoverlapping bands: $w_j=2\pi j/T$, $j=1, \ldots, 22$; $j=26, \ldots, 46$; $j=50, \ldots, 70$; $j=74, \ldots, 94$; $j=98, \ldots, 118$; $j=122, \ldots, 142$. These six bands are centered at periodicities of __, __, __, __, __, and __ months, respectively. The first band ranges over frequencies from __ months to __ months, and thus is the band composing the business cycle. We have omitted the seasonal periodicities and also several ordinates on each side of the seasonal periodicities. This accounts for the missing ordinates $j=23, 24, 25, 47, 48, 49, 71, 72, 73, 95, 96, 97, 119, 120,$ 121, 143, and 144.

The results are summarized in Tables 16-21. Set 1 includes all the variables except money. The one-index model bears very low marginal significance levels. However, it delivers high coherences for all of the real variables except business formation, moderate coherences for retail sales and new orders, and low coherences for the price indexes. Adding a second index raises the marginal significance levels, though they are still quite low. But adding the second index results in high multiple coherences for the two prices, and retail sales and new orders as well. The coherences for the other real variables remain about as they were with one-index. This pattern of coherences, with most real variables attaining high coherence with a single index, nominal variates attaining high coherence with the addition of a second index, is roughly consistent with the existence of neutral fluctuations in price level -- non-zero η_t in our version of the Lucas model.

Set 2 deletes the materials price index from Set 1. The pattern of results is identical with that of Set 1.

Set 3 adds money to the variables in Set 1. The pattern of results is the same as in Set 1, with money having low coherence with both the first and second indexes. As before, the second factor seems to be nominal one, but one with which money is not highly correlated.

Several features that run through these results are worth commenting on. First, there is the association of the GNP deflator and inventory investment with a second index in our quarterly models. As mentioned, this seems to be an anomaly if the second index is purely nominal.

Second, there is the very loose association of the money supply with both the real and nominal factors in both the quarterly and monthly results. This finding is probably surprising according to monistic theories of nominal aggregate demand. Yet the suspicion that the money supply is not a good measure of nominal aggregate demand -- and that probably no single good measure is available--is precisely the intuition underlying Lucas's model.

Third, there seems to be an index with which the money supply and residential construction are both coherent.

Fourth, there is low coherence of consumption and residential construction with the first index that accounts for most of the variation in measures of aggregate economic activity, such as real GNP and the unemployment rate.

The overall impression is one in which low order index models do fit the data well. Even according to the chi-square test statistics, one- or two-index models do a good job of describing the quarterly data. There does seem to be a tendency for the two main indexes in the fitted models to look like a "real business cycle" index and a "neutral price level change" index.

Table 1 — GRAPHS OF COHERENCE OF ECONOMIC VARIABLES

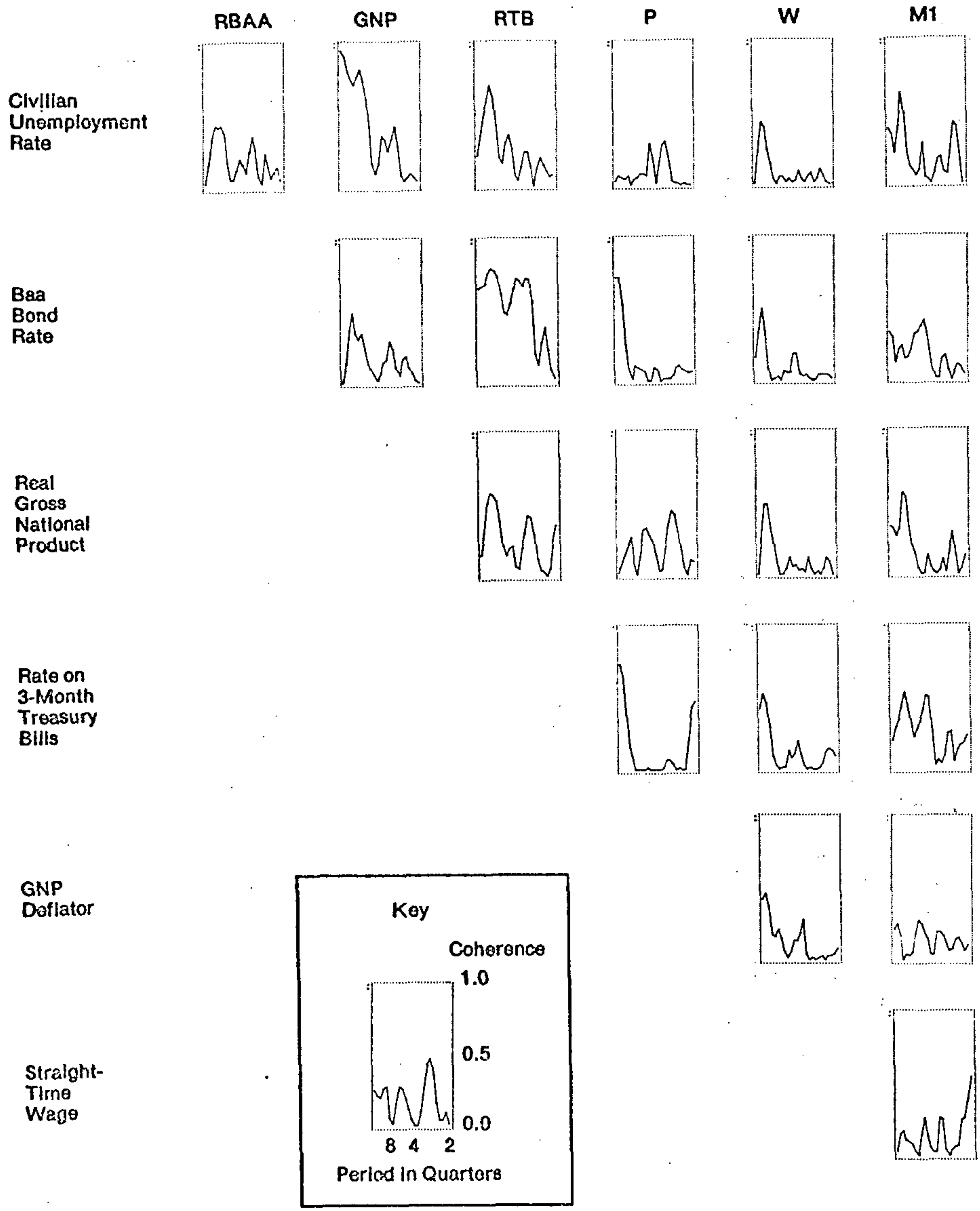

Table 1 (continued)

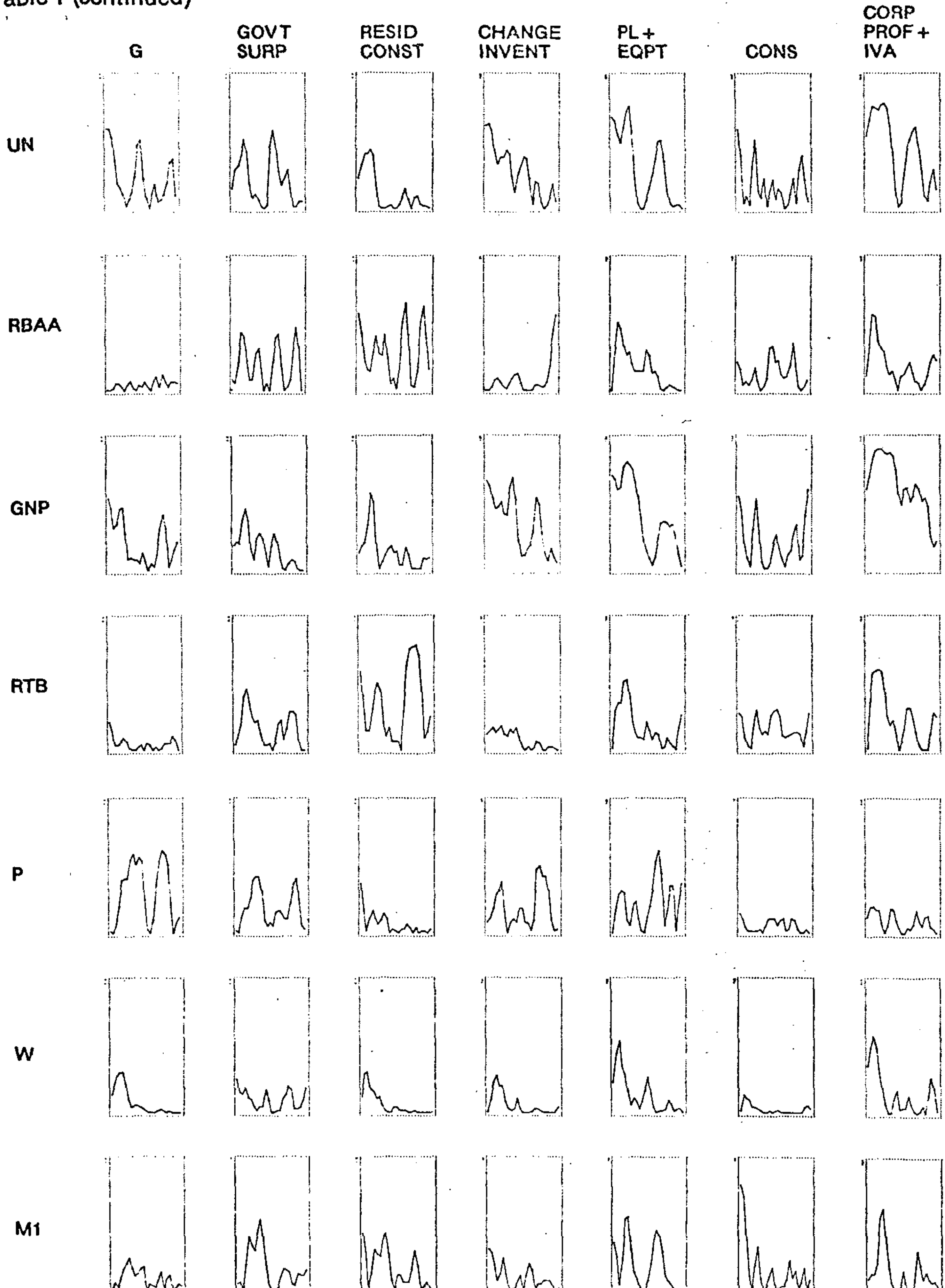

Table 1 (continued)

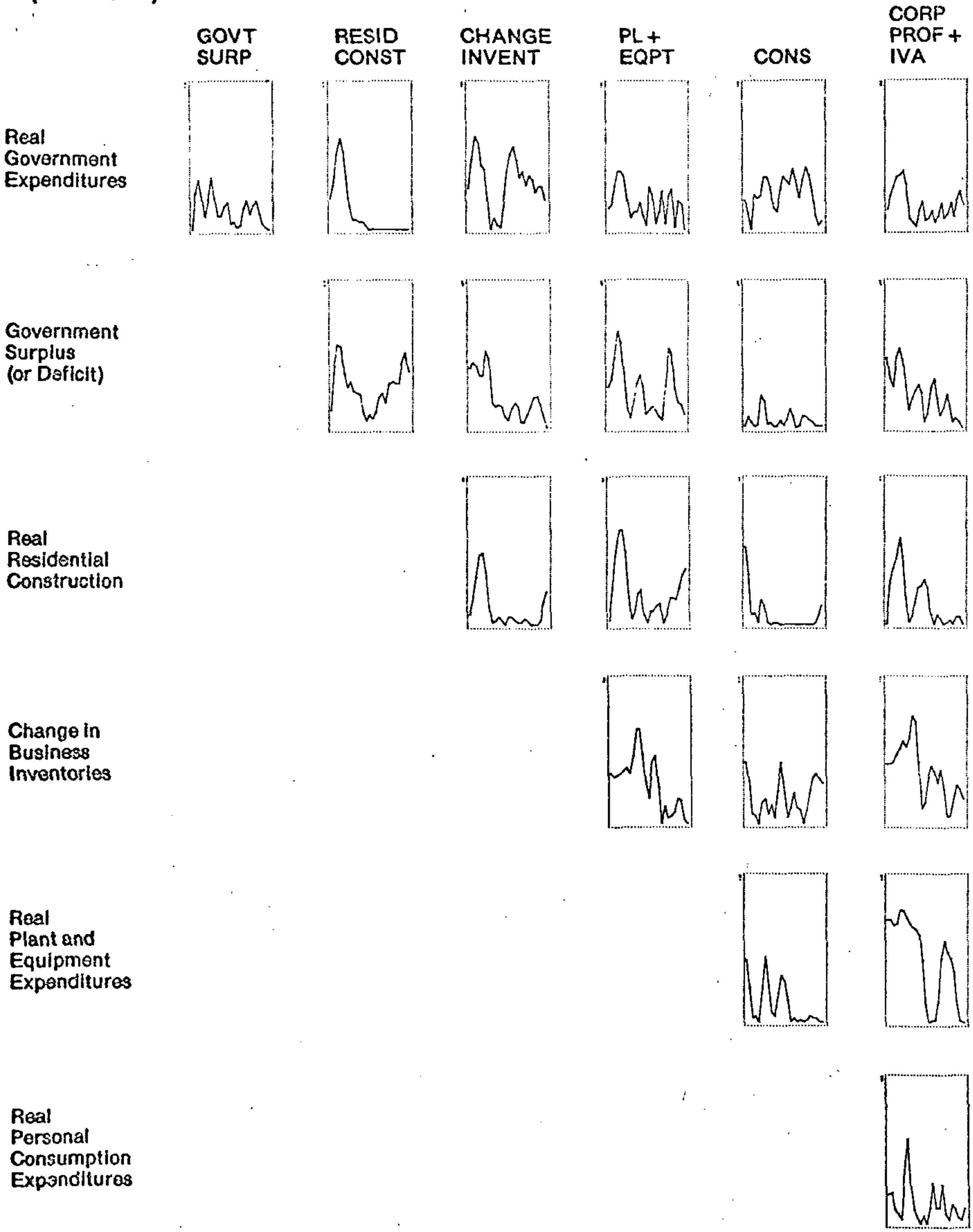

Table 1a — GRAPHS OF COHERENCE OF ECONOMIC VARIABLES

(Monthly Data)

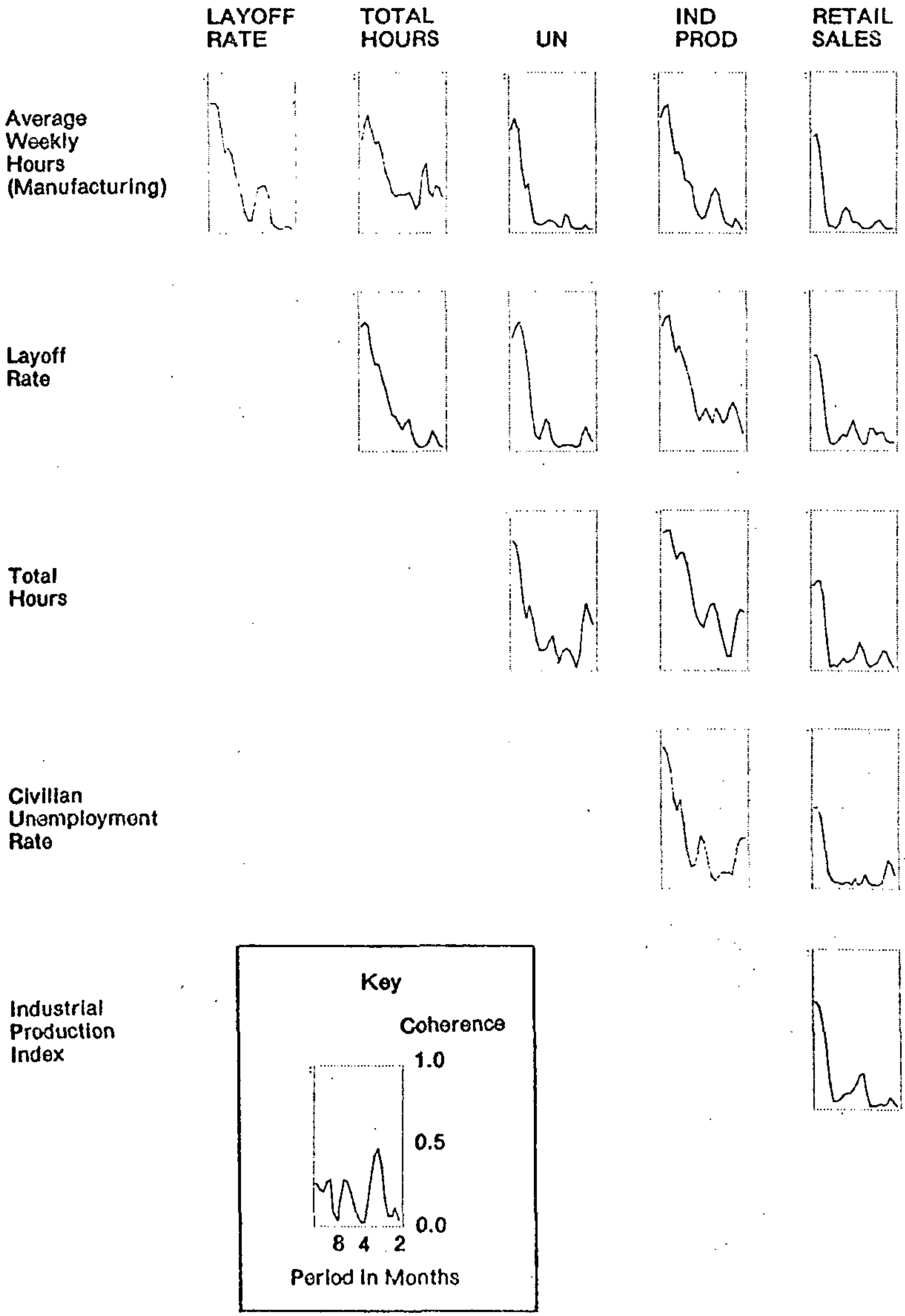

Table 1a (continued)

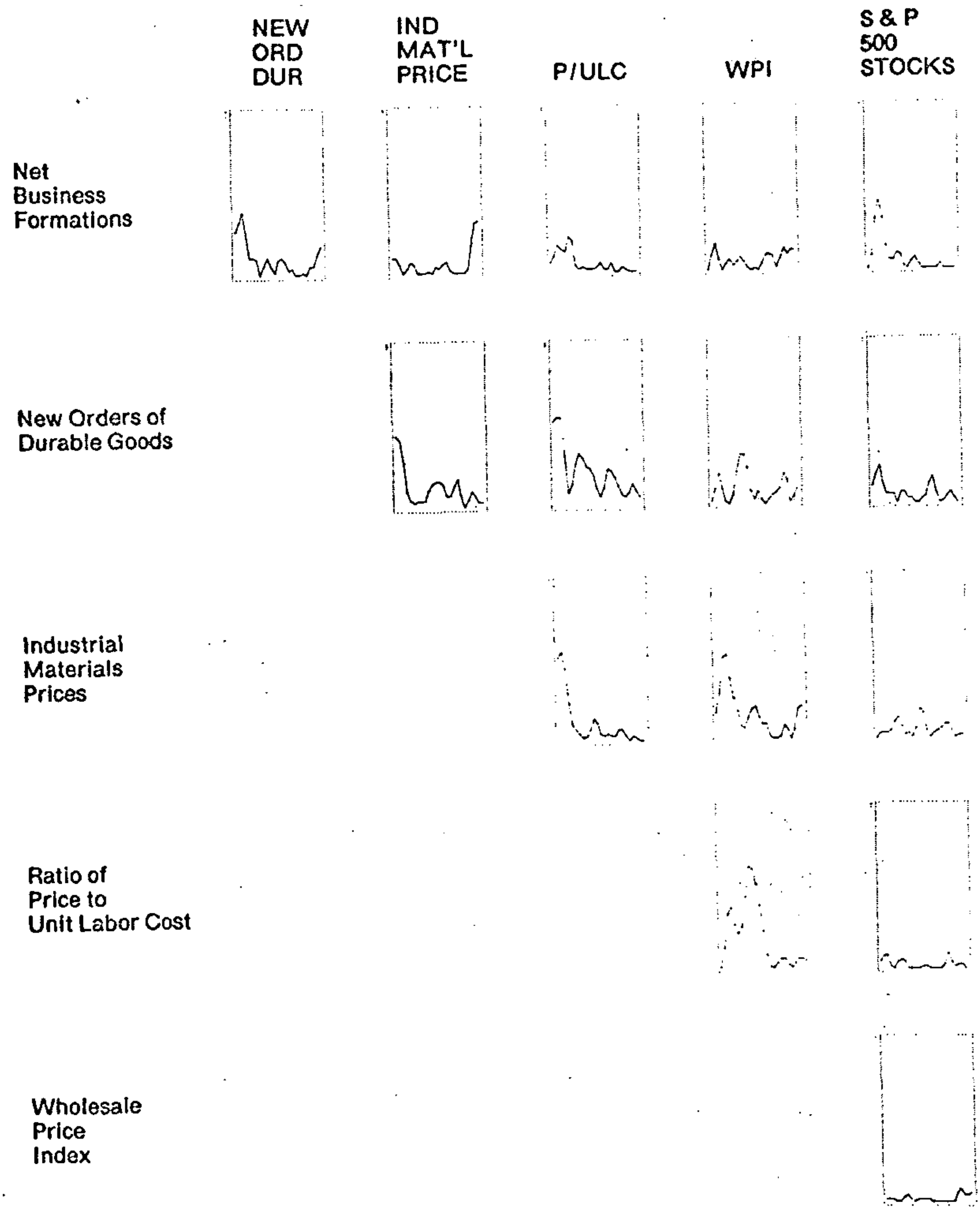

Table 1a (continued)

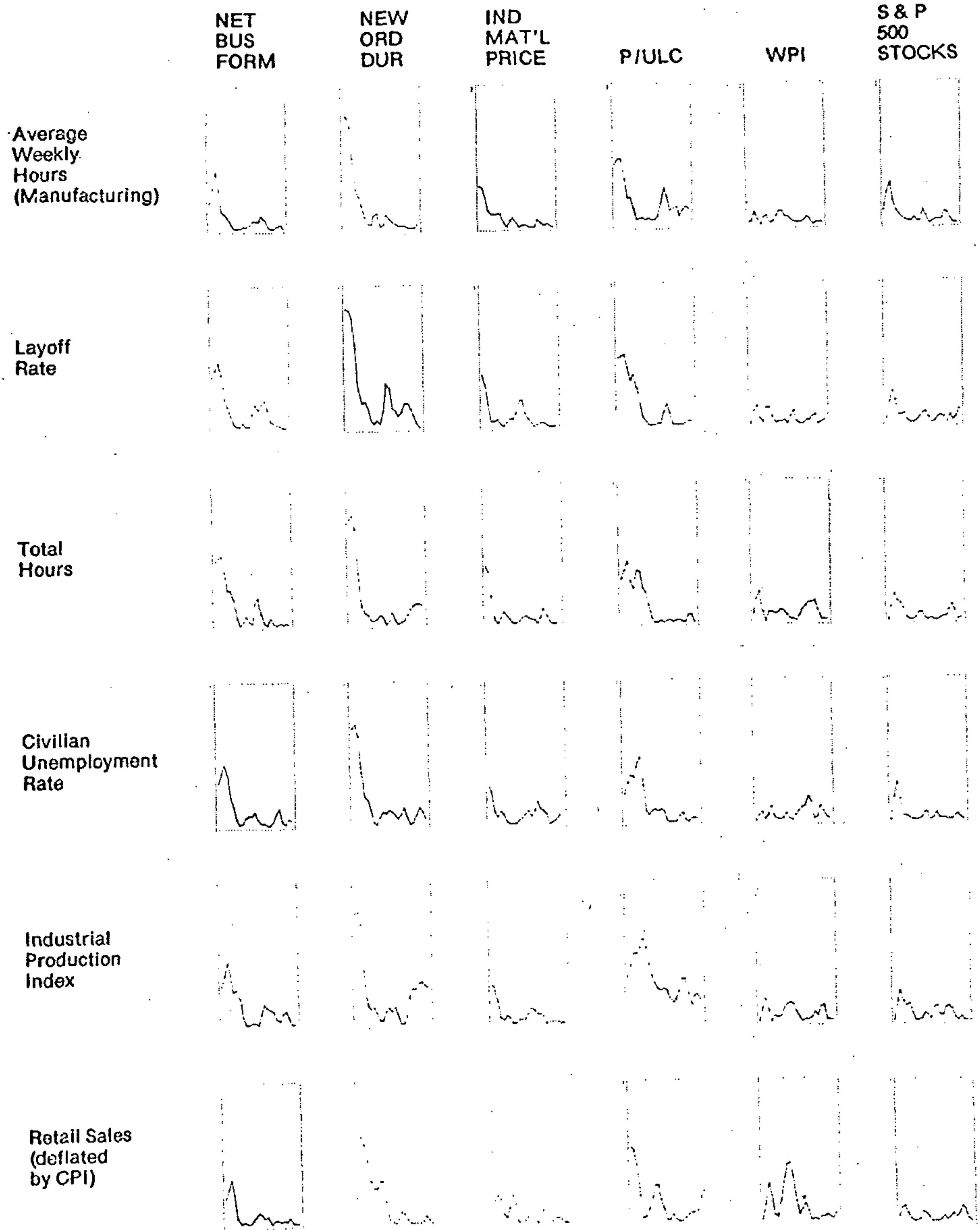

Table 2

Ninety-Five Percent Confidence Intervals with
89 Observations and 24 Frequency Points*

Coherence	Lower Limit	Upper Limit	Width
.050	0.000	.408	.408
.100	0.000	.482	.482
.150	0.000	.539	.539
.200	0.000	.586	.586
.250	.001	.628	.627
.300	.007	.665	.658
.350	.023	.699	.677
.400	.046	.731	.685
.450	.076	.760	.684
.500	.115	.788	.673
.550	.161	.813	.652
.600	.216	.838	.622
.650	.279	.861	.582
.700	.351	.884	.533
.750	.432	.905	.473
.800	.524	.925	.402
.850	.625	.945	.320
.900	.738	.964	.226
.950	.863	.982	.120

*Calculated for a Parzen window using the method described by Jenkins and Watts [ ].

Table 3

Ninety-Five Percent Confidence Intervals with
267 Observations and 24 Frequency Points*

Coherence	Lower Limit	Upper Limit
.050	0.000	.237
.100	.001	.313
.150	.011	.376
.200	.030	.431
.250	.057	.481
.300	.090	.527
.350	.128	.570
.400	.171	.612
.450	.219	.651
.500	.270	.688
.550	.326	.724
.600	.386	.758
.650	.449	.792
.700	.516	.824
.750	.588	.855
.800	.662	.886
.850	.741	.916
.900	.824	.944
.950	.910	.973

*Calculated for a Perzen window using method described by Jenkins and Watts.

Table 4

Quarterly Sets

Set	Un	GNP	GNP Defl.	Res. Constr.	Pl & E	Invent.	Cons.	Corp π	W	M1
1	X	X	X	X	X		X	X		
2	X	X	X	X	X	X	X	X		
3	X	X	X	X	X	X	X	X	X	
4	X	X	X	X	X	X	X	X		X
5	X	X	X	X	X		X	X		X

Table 5 Set 1

Bands(j)	One Noise Test Statistics $\chi^2(29)$	Marginal Significance Level	Two Noise Test Statistics $\chi^2(18)$	Marginal Significance Level
1-11	42.18	.054	24.78	.13
12-23	29.57	.436	10.58	.91
27-37	36.18	.168	15.08	.66
38-48	22.42	.802	10.02	.93
Overall Test	$\chi^2(116)=130.35$	.171	$\chi^2(72)=60.46$	.83

Periodogram ordinates were calculated at the angular frequencies = $w_j = \frac{2\pi j}{T}$, $T = 100$, $j = 0, 1, \ldots, 50$.

Periodicity of j^{th} frequency = $\frac{T}{j}$ quarters. The frequency bands correspond to the following periodicities:

j	Periodicity (quarters)
1-11	100-0.09
12-23	8.33-4.35
27-37	3.70-2.70
38-48	2.63-2.08

The seasonal frequencies $j = 25$, 50 and the adjacent frequencies $j = 24$, 26 and $j = 49$ were omitted from the bands used to compute the spectral decomposition and the k-noise-test statistics.

Table 6 Set 1

PROP OF VAR EXPLAINED BY 1 COMMON FACTORS

FREQUENCY	VAR. NO. 1 UNEMP RT	VAR. NO. 2 LREALGNP	VAR. NO. 3 LGNPDEFL	VAR. NO. 4 LRES CONST	VAR. NO. 5 LPL+EQPT	VAR. NO. 6 LCONS	VAR. NO. 7 LCORP+IVA
.1200PI	.81770	.88921	.24154	.32430	.72420	.11658	.94454
.3500PI	.45396	1.0000	.16788	.56390E-01	.57765	.70935E-01	.79205
.6400PI	.83507	.49642	.30774	.84224E-01	.40957	.51232E-02	.50045
.8600PI	.90837E-01	.25397	.27274	.16293	1.0000	.63993E-02	.25675
OVERALL	.73947	.88917	.24021	.26034	.70944	.80153E-01	.92681

PROP OF VAR EXPLAINED BY 2 COMMON FACTORS

FREQUENCY	VAR. NO. 1 UNEMP RT	VAR. NO. 2 LREALGNP	VAR. NO. 3 LGNPDEFL	VAR. NO. 4 LRES CONST	VAR. NO. 5 LPL+EQPT	VAR. NO. 6 LCONS	VAR. NO. 7 LCORP+IVA
.1200PI	.85074	1.0000	.23808	.87342	.79983	.26163	.92737
.3500PI	.67835	1.0000	.43217	.13697	.68604	.77124E-01	1.0000
.6400PI	.76618	.91367	.86246	.67923E-01	.60992	.57231E-01	.90491
.8600PI	.14456	1.0000	.44418	.25233	1.0000	.59540E-01	.53164
OVERALL	.94093	.99043	.24440	.68141	.79973	.15616	.92740

Table 7 Set 2

Bands(j)	One Noise Test Statistics $\chi^2(41)$	Marginal Significance Level	Two Noise Test Statistics $\chi^2(28)$	Marginal Significance Level
1-11	57.50	.045	34.75	.177
12-23	32.68	.820	13.64	.990
27-37	53.35	.094	27.04	.516
38-48	41.66	.442	25.60	.595
Overall Test	$\chi^2(164)=185.18$	.123	$\chi^2(112)=101.03$	.762

Periodogram ordinates were calculated at the angular frequencies = $w_j = \frac{2\pi j}{T}$, $T = 100$, $j = 0, 1, \ldots, 50$.

Periodicity of j^{th} frequency = $\frac{T}{j}$ quarters. The frequency bands correspond to the following periodicities:

j	Periodicity (quarters)
1-11	100-0.09
12-23	8.33-4.35
27-37	3.70-2.70
38-48	2.63-2.08

The seasonal frequencies $j = 25$, 50 and the adjacent frequencies $j = 24$, 26 and $j = 49$ were omitted from the bands used to compute the spectral decomposition and the k-noise-test statistics.

Table 8 Set 2

PROP OF VAR EXPLAINED BY 1 COMMON FACTORS

FREQUENCY	VAR. NO. 1 UNEMP RT	VAR. NO. 2 LREALGNP	VAR. NO. 3 LGNPDEFL	VAR. NO. 4 LRES CONST	VAR. NO. 5 DLINVENT	VAR. NO. 6 LPL+EQPT	VAR. NO. 7 LCONS	VAR. NO. 8 LCORP+IVA
.1200PI	.95626	.91461	.20507	.30391	.55972	.69431	.11685	.96595
.3500PI	.43913	.91497	.88705E-01	.78591E-01	.55806	.64910	.75154E-01	.9431[illegible]
.6400PI	1.0000	.35461	.25683	.12622	.11465	.37539	.45580E-02	.46993
.8600PI	.92673E-01	.25350	.29782	.18590	.36609E-01	1.0000	.65043E-02	.22449
OVERALL	.83612	.99948	.20276	.25370	.52337	.67597	.81836E-01	.94718

PROP OF VAR EXPLAINED BY 2 COMMON FACTORS

FREQUENCY	VAR. NO. 1 UNEMP RT	VAR. NO. 2 LREALGNP	VAR. NO. 3 LGNPDEFL	VAR. NO. 4 LRES CONST	VAR. NO. 5 DLINVENT	VAR. NO. 6 LPL+EQPT	VAR. NO. 7 LCONS	VAR. NO. 8 LCORP+IVA
.1200PI	.87623	.92712	.71014	.32363	1.0000	.78349	.14012	.95243
.3500PI	.67109	1.0000	.51421	.11839	.52386	.67319	.78747E-01	1.0000
.6400PI	.66518	.80074	.54113	.11393	.67594	1.0000	.31362E-01	.82358
.8600PI	.10198	.70758	.75048	.74495E-01	.62107	.74338	.14156	.59347
OVERALL	.86419	.92806	.70396	.26615	.95248	.78300	.10037	.95087

Table 9 Set 3

Bands(j)	One Noise Test Statistics $\chi^2(41)$	Marginal Significance Level	Two Noise Test Statistics $\chi^2(40)$	Marginal Significance Level
1-11	52.30	.111	35.99	.651
12-23	34.83	.740	23.27	.984
27-37	54.53	.077	42.55	.362
38-48	41.31	.457	29.24	.895
Overall Test	$\chi^2(164)=182.97$	.148	$\chi^2(160)=131.05$	.954

Periodogram ordinates were calculated at the angular frequencies = $w_j = \frac{2\pi j}{T}$, $T = 100$, $j = 0, 1, \ldots, 50$.

Periodicity of j^{th} frequency = $\frac{T}{j}$ quarters. The frequency bands correspond to the following periodicities:

j	Periodicity (quarters)
1-11	100-0.09
12-23	8.33-4.35
27-37	3.70-2.70
38-48	2.63-2.08

The seasonal frequencies $j = 25, 50$ and the adjacent frequencies $j = 24, 26$ and $j = 49$ were omitted from the bands used to compute the spectral decomposition and the k-noise-test statistics.

Table 10 Set 3

PROP OF VAR EXPLAINED BY 1 COMMON FACTORS

FREQUENCY	VAR. NO. 1 UNEMP RT	VAR. NO. 2 LREALGNP	VAR. NO. 3 LGNPDEFL	VAR. NO. 4 LSTWAGE	VAR. NO. 5 LRES CONST	VAR. NO. 6 DLINVENT	VAR. NO. 7 LPL.EQPT	VAR. NO. 8 LCONS	VAR. NO. 9 LCORP+IVA
.1200PI	.95429	.91293	.21349	.49345	.30571	.55583	.69092	.12119	.96565
.3500PI	.43603	.90748	.93453E-01	.21499E-01	.79944E-01	.55881	.64895	.74339E-01	.85031
.6400PI	1.0000	.35461	.25883	.61995E-01	.12622	.11466	.37541	.45539E-02	.46993
.8600PI	.92671E-01	.25349	.29779	.59709E-01	.18590	.36608E-01	1.0000	.65056E-02	.28450
OVERALL	.83411	.90672	.21074	.29931	.25526	.51997	.68200	.53451E-01	.94734

PROP OF VAR EXPLAINED BY 2 COMMON FACTORS

FREQUENCY	VAR. NO. 1 UNEMP RT	VAR. NO. 2 LREALGNP	VAR. NO. 3 LGNPDEFL	VAR. NO. 4 LSTWAGE	VAR. NO. 5 LRES CONST	VAR. NO. 6 DLINVENT	VAR. NO. 7 LPL.EQPT	VAR. NO. 8 LCONS	VAR. NO. 9 LCORP+IVA
.1200PI	.98154	.92156	.79672	.51979	.31529	.93457	.78094	.15839	.95514
.3500PI	.57109	1.0000	.51424	.17146	.11337	.52387	.67319	.78742E-01	1.0000
.6400PI	.56663	.80190	.54200	.45065E-01	.11406	.67508	1.0000	.31029E-01	.82121
.8600PI	.11172	.61068	.59023	.59193E-01	.20059	.69017	1.0000	.20215	.46342
OVERALL	.85932	.92244	.78675	.38847	.26673	.89689	.78218	.10361	.95256

Table 11 Set 4

Bands(j)	One Noise Test Statistics $\chi^2(55)$	Marginal Significance Level	Two Noise Test Statistics $\chi^2(40)$	Marginal Significance Level	Three Noise Test Statistics $\chi^2(27)$	Marginal Significance Level
1-11	75.08	.037	51.06	.113	24.81	.585
12-23	40.21	.933	20.71	.995	11.16	.997
27-37	60.84	.274	30.09	.873	16.03	.952
38-48	60.96	.270	41.90	.388	28.92	.365
Overall Test	$\chi^2(220) = 237.08$	.204	$\chi^2(160)=143.76$	.817	$\chi^2(108)=80.92$	.976

Periodogram ordinates were calculated at the angular frequencies $= w_j = \frac{2\pi j}{T}$, $T = 100$, $j = 0, 1, \ldots, 50$.

Periodicity of j^{th} frequency $= \frac{T}{j}$ quarters. The frequency bands correspond to the following periodicities:

j	Periodicity (quarters)
1-11	100-0.09
12-23	8.33-4.35
27-37	3.70-2.70
38-48	2.63-2.08

The seasonal frequencies $j = 25, 50$ and the adjacent frequencies $j = 24, 26$ and $j = 49$ were omitted from the bands used to compute the spectral decomposition and the k-noise-test statistics.

Table 12 Set 4

PROP OF VAR EXPLAINED BY 1 COMMON FACTORS

FREQUENCY	VAR. NO. 1 UNEMP.RT	VAR. NO. 2 LREALGNP	VAR. NO. 3 LGNPDEFL	VAR. NO. 4 LM1	VAR. NO. 5 LRES CONST	VAR. NO. 6 DLINVENT	VAR. NO. 7 LPL.EQPT	VAR. NO. 8 LCONS	VAR. NO. 9 LCORP+IVA
.1200PI	.84977	.93613	.21773	.28168	.26281	.56252	.68492	.12651	.89845
.3500PI	.43639	.91708	.97035E-01	.51568E-01	.80182E-01	.55580	.64565	.73225E-01	.87726
.6400PI	.65832	.73002	.24552	.73328E-01	.28865E-01	.21120	.28036	.10498E-01	.67750
.8600PI	.82680E-01	.25349	.29783	.33570E-01	.18589	.36609E-01	1.0000	.65040E-02	.25051
OVERALL	.82831	.93171	.21507	.27275	.21453	.53089	.67432	.85918E-01	.89032

PROP OF VAR EXPLAINED BY 2 COMMON FACTORS

FREQUENCY	VAR. NO. 1 UNEMP RT	VAR. NO. 2 LREALGNP	VAR. NO. 3 LGNPDEFL	VAR. NO. 4 LM1	VAR. NO. 5 LRES CONST	VAR. NO. 6 DLINVENT	VAR. NO. 7 LPL.EQPT	VAR. NO. 8 LCONS	VAR. NO. 9 LCORP+IVA
.1200PI	.87520	.93926	1.0000	.37270	.26820	.84758	.73196	.17193	.51112
.3500PI	.67835	1.0000	.43216	.79475E-01	.13698	.53886	.68603	.77118E-01	1.0000
.6400PI	.65357	.81562	.54733	.37229	.11366	.69574	1.0000	.22553E-01	.84345
.8600PI	.89870E-01	.72358	1.0000	.19129	.50046E-01	.51306	.56072	.72594[illegible]	.66555
OVERALL	.86352	.93980	.98189	.36246	.22583	.83306	.77563	.11191	.91369

Table 13 Set 5

Bands(j)	One Noise Test Statistics $\chi^2(41)$	Marginal Significance Level	Two Noise Test Statistics $\chi^2(28)$	Marginal Significance Level	Three Noise Test Statistics $\chi^2(17)$	Marginal Significance Level
1-11	52.43	.109	29.45	.390	15.51	.558
12-23	35.64	.707	16.61	.956	8.01	.966
27-37	43.61	.361	21.50	.804	10.11	.899
38-48	41.62	.444	27.06	.515	19.35	.309
Overall Test	$\chi^2(164)=173.29$	.294	$\chi^2(112)=94.62$	.881	$\chi^2(68)=52.98$	.910

Periodogram ordinates were calculated at the angular frequencies $= w_j = \frac{2\pi j}{T}$, $T = 100$, $j = 0, 1, \ldots, 50$.

Periodicity of j^{th} frequency $= \frac{T}{j}$ quarters. The frequency bands correspond to the following periodicities:

j	Periodicity (quarters)
1-11	100-0.09
12-23	8.33-4.35
27-37	3.70-2.70
38-48	2.63-2.08

The seasonal frequencies $j = 25, 50$ and the adjacent frequencies $j = 24, 26$ and $j = 49$ were omitted from the bands used to compute the spectral decomposition and the k-noise-test statistics.

Table 14 Set 5

PROP OF VAR EXPLAINED BY 1 COMMON FACTORS

FREQUENCY	VAR. NO. 1 UNEMP RT	VAR. NO. 2 LREALGNP	VAR. NO. 3 LGNPDEFL	VAR. NO. 4 LM1	VAR. NO. 5 LRES CONST	VAR. NO. 6 LPL+EQPT	VAR. NO. 7 LCONS	VAR. NO. 8 LCORP+IVA
.1200PI	.83822	.91639	.23431	.25449	.28543	.70722	.12038	.91980
.3500PI	.45395	1.0000	.16787	.69507E-01	.56381E-01	.57764	.70932E-01	.79905
.6400PI	.75700	.50521	.36431	.14484	.67427E-01	.47242	.47456E-02	.48187
.8600PI	.82665E-01	.25351	.29787	.33569E-01	.18588	1.0000	.65047E-02	.2505[illegible]
OVERALL	.81858	.91425	.23373	.24741	.23243	.69559	.81819E-01	.99355

PROP OF VAR EXPLAINED BY 2 COMMON FACTORS

FREQUENCY	VAR. NO. 1 UNEMP RT	VAR. NO. 2 LREALGNP	VAR. NO. 3 LGNPDEFL	VAR. NO. 4 LM1	VAR. NO. 5 LRES CONST	VAR. NO. 6 LPL+EQPT	VAR. NO. 7 LCONS	VAR. NO. 8 LCORP+IVA
.1200PI	.84435	1.0000	.26982	.56276	.79835	.80009	.27533	.94096
.3500PI	.67835	1.0000	.43216	.79478E-01	.13698	.68604	.77120E-01	1.0000
.6400PI	.74450	.93233	.84894	.23120	.73562E-01	.65586	.41908E-01	.78071
.8600PI	.14456	1.0000	.44418	.86347E-01	.25293	1.0000	.59537E-01	.53165
OVERALL	.83479	.99955	.27886	.54436	.62315	.79118	.15997	.93969

PROP OF VAR EXPLAINED BY 3 COMMON FACTORS

FREQUENCY	VAR. NO. 1 UNEMP RT	VAR. NO. 2 LREALGNP	VAR. NO. 3 LGNPDEFL	VAR. NO. 4 LM1	VAR. NO. 5 LRES CONST	VAR. NO. 6 LPL+EQPT	VAR. NO. 7 LCONS	VAR. NO. 8 LCORP+IVA
.1200PI	1.0000	1.0000	.54655	.59964	.55447	.69913	.63453	1.0000
.3500PI	.69136	1.0000	.51369	.34822	1.0000	.71095	.98297E-01	1.0000
.6400PI	1.0000	1.0000	.82925	.42424	.58275	.76315	.20500E-01	.73892
.8600PI	1.0000	.43301	1.0000	.43923	.40954	1.0000	.22161	.26940
OVERALL	.98582	.99800	.54943	.59018	.60567	.70312	.32860	.99211

Table 15

Monthly Sets

Set	Av. Hrs.	Layoff	Manhour	Unempl.	Ind. Prod.	Ret. Sales	Net Bus. Form	New Ord. Dur.	Ind. Mat. Prices	Wholesale	M
1	X	X	X	X	X	X	X	X	X	X	
2	X	X	X	X	X	X	X	X		X	
3	X	X	X	X	X	X	X	X	X	X	X

Table 16 Set 1

Bands(j)	One-Index Test Statistics $\chi^2(71)$	Marginal Significance Level	Two-Index Test Statistics $\chi^2(54)$	Marginal Significance Level	Three-Index Test Statistics $\chi^2(39)$	Marginal Significance Level
1-22	155.47	.000	83.85	.006	51.01	.094
26-46	98.63	.017	59.82	.273	35.80	.617
50-10	107.37	.003	64.71	.151	42.69	.316
74-94	85.24	.119	59.50	.282	36.08	.604
98-118	79.17	.237	57.22	.357	38.67	.485
122-142	65.67	.657	45.16	.799	29.89	.853
Overall Test	$\chi^2(426)=591.55$	.000	$\chi^2(324)=370.26$	.039	$\chi^2(234)=234.13$	.485

Periodogram ordinates were calculated at the angular frequencies $w_j = 2\pi j/T$, $T = 288$, $j = 0, 1, \ldots, 144$.

Periodicity of j^{th} frequency = T/j quarters. Seasonal frequencies and adjacent frequencies were omitted from bands used to compile test statistics.

Table 17 Set 1

PROP OF VAR EXPLAINED BY 1 COMMON FACTORS

FREQUENCY	VAR. NO. 1 AVGWKLYHRS	VAR. NO. 2 LAYOFFRATE	VAR. NO. 3 MANHOURS	VAR. NO. 4 UNEMPL RT	VAR. NO. 5 INDPRODIDX	VAR. NO. 6 RETAILSALS	VAR. NO. 7 NETBUSFORM	VAR. NO. 8 NEWORD DUR	VAR. NO. 9 IND MAT PR	VAR. NO. 10 WHOL PRICE
.0799PI	.94084	.92115	.87804	.79208	.95075	.56149	.43804	.77588	.20288	.20778
.2500PI	.54534	.67273	.80993	.37793	.87106	.13299E-01	.16990	.15677	.42414E-01	.93317E-01
.4167PI	.27818	.42267	.49796	.29293	.71040	.17470	.13971E-01	.50803E-01	.48740E-01	.12402
.5833PI	.35461	.36571	.62279	.38091E-01	.62251	.17354	.20400	.17774	.54436E-01	.12252E-01
.7500PI	.25605	.28431	.26279	.86679E-01	.50029	.90212E-02	.23898	.88047E-01	.10449E-01	.20011
.9167PI	.11727	.19518	.70478	.59106	.49724	.19984	.25637E-11	.25597	.94551E-02	.78097E-01
OVERALL	.76038	.83151	.96415	.76713	.94093	.41007	.42085	.62755	.17754	.20480

PROP OF VAR EXPLAINED BY 2 COMMON FACTORS

FREQUENCY	VAR. NO. 1 AVGWKLYHRS	VAR. NO. 2 LAYOFFRATE	VAR. NO. 3 MANHOURS	VAR. NO. 4 UNEMPL RT	VAR. NO. 5 INDPRODIDX	VAR. NO. 6 RETAILSALS	VAR. NO. 7 NETBUSFORM	VAR. NO. 8 NEWORD DUR	VAR. NO. 9 IND MAT PR	VAR. NO. 10 WHOL PRICE
.0799PI	.84779	.93287	.90928	.86731	.95451	.68993	.47217	.90096	.70500	.71223
.2500PI	.56387	.70221	.79531	.39395	.91993	1.0000	.20347	.45111	.26889	.23549
.4167PI	.57626	.41694	.56099	.36479	.64525	.45196	.79293E-01	.47126	.22689	.80563
.5833PI	.39773	.45217	.64597	.87559E-01	.61257	.37838	.20470	.17796	.22693	1.0000
.7500PI	.74976	.21916	.48542	.77607E-01	.38445	.15954E-01	.16090	1.0000	.14985	.13073
.9167PI	.19657	.19844	.72515	.59566	.56530	.23527	.31954	.77497	.63944	.19931
OVERALL	.79180	.84435	.89529	.83931	.94583	.68117	.45621	.43246	.63037	.79532

PROP OF VAR EXPLAINED BY 3 COMMON FACTORS

FREQUENCY	VAR. NO. 1 AVGWKLYHRS	VAR. NO. 2 LAYOFFRATE	VAR. NO. 3 MANHOURS	VAR. NO. 4 UNEMPL RT	VAR. NO. 5 INDPRODIDX	VAR. NO. 6 RETAILSALS	VAR. NO. 7 NETBUSFORM	VAR. NO. 8 NEWORD DUR	VAR. NO. 9 IND MAT PR	VAR. NO. 10 WHOL PRICE
.0799PI	.96120	.92994	.93413	1.0000	1.0000	.73751	.55744	.91797	.56440	.56336
.2500PI	1.0000	.74709	.79968	.43519	1.0000	1.0000	.26053	.49750	.30011	.30734
.4167PI	1.0000	.63172	.56055	.57894	.61129	.52789	.20319	.43930	.34376	.79269
.5833PI	.43723	.59252	.70497	.26009	.65369	.33341	.29311	1.0000	.32502	.83810
.7500PI	1.0000	.40153	.44502	1.0000	.53264	.30482E-01	.29394	.51091	.19946	.24610
.9167PI	.57799	.26623	.90323	.73973	.54401	.24043	.49994	.45211	.44292	.19806
OVERALL	.83657	.86424	.92046	.97761	.99218	.72280	.54210	.95759	.65268	.65967

Table 18 Set 2

Bands(j)	One-Index Test Statistics $\chi^2(55)$	Marginal Significance Level	Two-Index Test Statistics $\chi^2(40)$	Marginal Significance Level	Three-Index Test Statistics $\chi^2(27)$	Marginal Significance Level
1-22	113.56	.000	63.12	.011	31.26	.260
26-46	78.38	.021	47.45	.195	24.68	.592
50-10	89.10	.002	51.35	.108	31.01	.271
74-94	61.14	.265	7.06	1.00	21.16	.779
98-118	63.65	.198	43.13	.339	25.08	.570
122-142	51.46	.611	36.64	.622	23.14	.678
Overall Test	$\chi^2(330)=457.30$	.000	$\chi^2(240)=248.74$	.336	$\chi^2(162)=156.33$	.611

Periodogram ordinates were calculated at the angular frequencies $w_j = 2\pi j/T$, $T = 288$, $j = 0, 1, \ldots, 144$.

Periodicity of j^{th} frequency = T/j quarters. Seasonal frequencies and adjacent frequencies were omitted from bands used to compile test statistics.

Table 19 Set 2

PROP OF VAR EXPLAINED BY 1 COMMON FACTORS

FREQUENCY	VAR. NO. 1 AVGWKLYHRS	VAR. NO. 2 LAYOFFRATE	VAR. NO. 3 MANHOURS	VAR. NO. 4 UNEMPL RT	VAR. NO. 5 INDPRODIDX	VAR. NO. 6 RETAILSALS	VAR. NO. 7 NETBUSFORM	VAR. NO. 8 NEWORD DUR	VAR. NO. 9 WHOL PRICE
.0799PI	.83919	.92051	.87924	.79357	.95342	.55766	.43916	.72964	.20570
.2500PI	.53807	.67297	.80762	.39013	.87675	.14164E-01	.16938	.15545	.89879E-01
.4167PI	.30254	.41493	.51683	.29495	.70426	.16995	.14729E-01	.44337E-01	.11022
.5833PI	.37571	.35343	.63846	.34376E-01	.60317	.17526	.21282	.13145	.75342E-02
.7500PI	.24252	.29759	.24583	.87233E-01	.52391	.85097E-02	.23298	.19265	.19476
.9167PI	.11429	.19774	.69574	.58649	.50712	.19823	.28618E-01	.25301	.78817E-01
OVERALL	.75982	.83045	.86545	.76847	.94358	.40709	.42195	.62201	.22258

PROP OF VAR EXPLAINED BY 2 COMMON FACTORS

FREQUENCY	VAR. NO. 1 AVGWKLYHRS	VAR. NO. 2 LAYOFFRATE	VAR. NO. 3 MANHOURS	VAR. NO. 4 UNEMPL RT	VAR. NO. 5 INDPRODIDX	VAR. NO. 6 RETAILSALS	VAR. NO. 7 NETBUSFORM	VAR. NO. 8 NEWORD DUR	VAR. NO. 9 WHOL PRICE
.0799PI	.84741	.93069	.92133	.94196	.96345	.70112	.46290	.93489	.51144
.2500PI	.55849	.70170	.79331	.39571	.92394	1.0000	.20252	.45153	.23336
.4167PI	.55675	.42691	.57869	.36793	.64409	.47115	.89872E-01	.44379	.80101
.5833PI	.44522	1.0000	1.0000	.53329E-01	.45393	.14279	.24988	.32664	.14227
.7500PI	1.0000	.29605	.41580	.17896	.53230	.14221E-01	.17711	.44500	.20732
.9167PI	.35972	.33981	.92394	.54995	.58327	.19320	.49193E-01	.41256	.59289E-01
OVERALL	.80126	.86342	.91123	.81602	.95377	.68529	.44621	.84437	.50603

PROP OF VAR EXPLAINED BY 3 COMMON FACTORS

FREQUENCY	VAR. NO. 1 AVGWKLYHRS	VAR. NO. 2 LAYOFFRATE	VAR. NO. 3 MANHOURS	VAR. NO. 4 UNEMPL RT	VAR. NO. 5 INDPRODIDX	VAR. NO. 6 RETAILSALS	VAR. NO. 7 NETBUSFORM	VAR. NO. 8 NEWORD DUR	VAR. NO. 9 WHOL PRICE
.0799PI	.86312	.93413	.93502	1.0000	1.0000	.74117	.56151	.92959	.59569
.2500PI	.77346	.81923	.78211	.43054	1.0000	.76654	.23756	.66047	.48396
.4167PI	.92740	.42766	.62933	1.0000	.56959	.45220	.42352	.51170	.70023
.5833PI	.50105	.59393	.73146	.26128	.63056	.40465	.28474	1.0000	.68921
.7500PI	1.0000	1.0000	.52620	.39024	.46598	.15381	.29587	.38523	.33586
.9167PI	.39689	.40224	1.0000	.58161	.62708	.17891	1.0000	.45655	.17214
OVERALL	.83823	.88263	.92349	.97517	.99172	.68432	.55106	.87480	.59435

2-1

Table 20 Set 3

Bands(j)	One-Index Test Statistics $\chi^2(89)$	Marginal Significance Level	Two-Index Test Statistics $\chi^2(70)$	Marginal Significance Level	Three-Index Test Statistics $\chi^2(53)$	Marginal Significance Level
1-22	181.07	.000	117.45	.000	84.17	.004
26-46	109.39	.070	70.87	.448	44.94	.777
50-10	124.92	.007	82.55	.145	57.14	.324
74-94	91.76	.399	67.05	.578	40.30	.900
98-118	85.55	.584	63.35	.700	42.67	.844
122-142	86.40	.558	63.64	.691	43.03	.834
Overall Test	$\chi^2(534)=679.08$	.000	$\chi^2(420)=464.91$	.064	$\chi^2(318)=312.26$	.580

Periodogram ordinates were calculated at the angular frequencies $w_j = 2\pi j/T$, $T = 288$, $j = 0, 1, \ldots, 144$.

Periodicity of j^{th} frequency = T/j quarters. Seasonal frequencies and adjacent frequencies were omitted from bands used to compile test statistics.

Table 21 Set 3

PROP OF VAR EXPLAINED BY 1 COMMON FACTORS

FREQUENCY	VAR. NO. 1 AVGWKLYHRS	VAR. NO. 2 LAYOFFRATE	VAR. NO. 3 MANHOURS	VAR. NO. 4 UNEMPL RT	VAR. NO. 5 INDPRODIDX
.0799PI	.94699	.92631	.96969	.79473	.94926
.2500PI	.54235	.57234	.41362	.37339	.87723
.4167PI	.27999	.42474	.43471	.27942	.71479
.5833PI	.36327	.32437	.42552	.37742E-01	.50169
.7500PI	.25647	.22420	.25967	.88499E-01	.50676
.9167PI	.11452	.20209	.59576	.54044	.50871
OVERALL	.75559	.43473	.35525	.74035	.93970

FREQUENCY	VAR. NO. 6 RETAILSALS	VAR. NO. 7 NETBUSFORM	VAR. NO. 8 NEWORD DUR	VAR. NO. 9 INV MAT RJ	VAR. NO. 10 WHOL PRICE	VAR. NO. 11 LMI
.0799PI	.59041	.43063	.79652	.19925	.23691	.17226
.2500PI	.11343E-01	.16076	.14234	.41340E-01	.91277E-01	.52316E-01
.4167PI	.17805	.14240E-01	.32622E-01	.51935E-01	.12423	.25743E-01
.5833PI	.15792	.20789	.14379	[illegible]	.13410E-01	.46601E-01
.7500PI	.52286E-02	.24065	.40445E-01	.49100E-02	.19449	.44346E-02
.9167PI	.15253	.29437E-01	.26835	.90927E-02	.27531E-01	.34414E-01
OVERALL	.45769	.42168	.67352	.13335	.26346	.15700

PROP OF VAR EXPLAINED BY 2 COMMON FACTORS

FREQUENCY	VAR. NO. 1 AVGWKLYHRS	VAR. NO. 2 LAYOFFRATE	VAR. NO. 3 MANHOURS	VAR. NO. 4 UNEMPL RT	VAR. NO. 5 INDPRODIDX
.0799PI	.95415	.93753	.90353	.93268	.94947
.2500PI	.55909	.71072	.73775	.39662	.91159
.4167PI	.58193	.47290	.55967	.36298	.64996
.5833PI	.41273	.47347	.64640	.87963E-01	.58710
.7500PI	.20474	.22959	.45577	.79704E-01	.41759
.9167PI	.14684	.20702	.71203	.59067	.56735
OVERALL	.79494	.85041	.33329	.95391	.93947

FREQUENCY	VAR. NO. 6 RETAILSALS	VAR. NO. 7 NETBUSFORM	VAR. NO. 8 NEWORD DUR	VAR. NO. 9 INV MAT RJ	VAR. NO. 10 WHOL PRICE	VAR. NO. 11 LMI
.0799PI	.60273	.47489	.37410	.74962	.63474	.25612
.2500PI	1.0000	.19059	.60341	.33245	.22466	.53596E-01
.4167PI	.44517	.80562E-01	.65527	.23372	.75533	.66204E-01
.5833PI	.33732	.21142	.19823	.22445	.57773	.45483E-01
.7500PI	.20948E-01	.16769	1.0000	.14329	.11459	.41846E-01
.9167PI	.22277	.28852	.33055	.74201	.19430	.73491E-01
OVERALL	.68602	.45853	.64361	.73482	.49173	.20026

PROP OF VAR EXPLAINED BY 3 COMMON FACTORS

FREQUENCY	VAR. NO. 1 AVGWKLYHRS	VAR. NO. 2 LAYOFFRATE	VAR. NO. 3 MANHOURS	VAR. NO. 4 UNEMPL RT	VAR. NO. 5 INDPRODIDX
.0799PI	.87435	.94121	.91762	1.0000	.97231
.2500PI	.71396	.46021	.76522	.44692	.99289
.4167PI	.96614	.39411	.56595	1.0000	.55892
.5833PI	.59390	1.0000	1.0000	.12741	.48134
.7500PI	1.0000	.17254	.51311	1.0000	.37519
.9167PI	.20225	1.0000	.76459	.60691	.59208
OVERALL	[illegible]	.49311	.39684	.94042	.96416

FREQUENCY	VAR. NO. 6 RETAILSALS	VAR. NO. 7 NETBUSFORM	VAR. NO. 8 NEWORD DUR	VAR. NO. 9 INV MAT RJ	VAR. NO. 10 WHOL PRICE	VAR. NO. 11 LMI
.0799PI	.69571	.59718	.96444	.73927	.72174	.41342
.2500PI	.89535	.19966	.62579	.50279	.54741	.15225
.4167PI	.39693	.34968	.57848	.31047	.65552	.20228
.5833PI	.14624	.39311	.62674	1.0000	.27215	.97719E-01
.7500PI	.31542E-01	.17530	1.0000	.24541	.29960	.50682E-01
.9167PI	.20183	.29792	.37185	.97194	.21119	.34612
OVERALL	.65527	.57704	[illegible]	.72965	.71653	.63311

3-M

Example of an application of observable-index models.

In the example we are about to discuss, an observable-index model is fit to a five-variable system of quarterly data on money (M1), a price index, a "demand-pressure" variable (ratio of unfilled orders for durable goods to shipments), the unemployment rate, and wage index.[25] The sample period is 1949III - 1971IV, deliberately chosen to allow a substantial period of out-of-sample projections.

The equation actually estimated is obtained by inverting (13) to yield

$$D^{-1}*(I-a*c)*y = e \tag{23}$$

We have taken $c(s) = 0$ for $s > 2$, $D^{-1}(s) = 0$ for $s > 2$, and $D^{-1}*a(s) = 0$ for $s > 3$. These are just limits on lengths of lag of the type necessary in any dynamic modeling. They make (23) a constrained fifth order autoregression. To keep the estimation process relatively simple, we take $a(0) = 0$, though as we shall see the data seem not to support this convenient assumption. We have used only one-index versions of the model. Obviously a and c can be multiplied and divided by the same constant without

[25] Precise definitions and sources for the data are as follows: M: Currency plus demand deposits, FRB data bank series #94, (Source: Business Statistics, 1973). Price, P: Implicit price deflation for nonterm business product, FRB series #156 demand pressure. C: (unfilled orders for durable goods)/(total shipments). (Source: Business Statistics, 1973). Unemployment, U: unemployment rate (total), FRB series #150. Wage, W: employee compensation is nonfarm business product, FRB series #251 (Series description: Business Conditions Digest, June, 1972). The latter four series were originally chosen as rough approximations to four series appearing in the "price" and "wage" equations of the FRB model. Particularly in the case of C, this approximation was even rougher than intended, as the corresponding variable in the FRB model is (unfilled orders of producers durables)/(shipments of producer's durables).

there is a non-trivial subclass of index models which cannot be normalized this way. However normalizations which do not, like this one, bring in unwanted restrictions, are difficult to implement.[26]

Some of the conclusions developed in the model seem solid, in part because they are non-controversial. For example, as one would have expected on the basis of the work by Nelson [1972], and Cooper and Nelson [1975], but perhaps not on the basis of Pierce's recent work [1977],[27] there are significant cross-relations among these five series, and they are of economically plausible form. Also, the restrictions implicit in the one-dimensional unobservable-index form, which reduce the 125 parameters of the 5-variable general fifth order autoregression to 42, are not strongly in conflict with the data.

On the other hand, the model appears without "coaching" in the form of a priori constraints to generate conclusions with interesting economic interpretations. Money is strictly exogenous relative to the rest of the system. "Phillips curve" relations between wage or prices and unemployment arise largely from the common response of these variables to money. Money affects unemployment fairly promptly,

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[26] By requiring that there be a one-sided $k \times n$ g such that $g*a$ is the identity and that $c*y$ be serially uncorrelated, or equivalently, that $a(o)c*y$ be the one-step-ahead forecast error (innovation) in $a*c*y$, we would fix a and c up to multiplication by a fixed $k \times k$ unitary matrix. This normalization would avoid unwanted restrictions, but appears difficult to implement.

[27] Nelson [1972] and Cooper and Nelson [1975] show that for some series univariate autoregressions provide better out-of-sample projections than multivariate models of the standard type, but there are some series for which standard multivariate models do provide better out-of-sample projections. Pierce examines all possible bivariate relations among a group of financial sector variables. Though there are significant relations among many of Pierce's series, he emphasizes that the number of pairings of series within this sector for which no statistically significant relation is detectable is unexpectedly high.

and the effect then decays over the course of two years. "Surprise" changes in prices or wages reduce unemployment, but only for about a year. Prices and wages respond more slowly and permanently to money. These conclusions have a monetarist ring, but the length of the lag in the response of real variables in the system to innovations ("surprise changes") in money appears to leave more room for discretionary monetary policy than is implied by some recent classical rational expectations macroeconomic models.

This latter set of conclusions is discussed in this paper only to show that results from "non-structural" models of this type may be open to some interesting economic interpretations. They are illustrative of a methodological point, and are not meant to be treated as firmly established empirical results, for several reasons. Most important of these reasons is the fact that some obvious experiments on the list of variables included in the model have not been carried out. One might suspect, for example, that the strong effects of money on real variables in this system, and money's exogeneity as well, might not persist in a system which included GNP. A comparison (discussed below) of this five-variable system with an observable-index model which omits money from the system illustrates how important the variable-list can be in interpreting results from these systems.

Another reason for not treating the empirical results as firmly established is the fact that some tests for specification error of general form accept the null hypothesis of correct specification only in the somewhat uncomfortable 5-10% range of marginal significance levels. And finally this system is estimated using seasonally adjusted data. without special measures of the type we ordinarily employ[28] to take account of this source of possible bias.

Table 1 displays the estimated $D^{-1}*a$, c, and D lag distributions for (3), together with their asymptotic standard errors.[29]

While it is difficult to tell much about the dynamics of the estimated system from Table 22 directly, one can reach some conclusions by looking for zeroes in the table. Thus, the strongly significant $D(s)$ estimates indicate that every residual in (23) is serially correlated. The fact that some a and c coefficients are more than twice their standard errors indicates that there are statistically significant cross-variable effects in the data. One can also make some inferences about which variables would be plausibly treated as exogenous in the system by looking for statistically insignificant a's. From this table it would appear plausible that money, unemployment, and demand pressure are all exogenous, in the sense that feedback from other variables into them is statistically insignificant. However, before reaching a conclusion on this it is important to see (as we shall below) how much feedback into these variables from others is implied by the point estimates.

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[28] Note that for the unobservable-index models we have been able conveniently to exclude seasonal bands from the data, which should minimize seasonal bias.

[29] Estimates were obtained by maximum likelihood, conditional on the observations on y for the five initial periods 1948II - 1949II. Though this is not strictly a maximum likelihood procedure (it ignores information about parameters available in the initial observations), it is asymptotically equivalent to maximum likelihood. Natural logarithms were taken of all variables and linear trends then removed by least squares for each variable before the observable-index model was fit.

Variables for which the corresponding row of c vanishes are "passive" -- they may be affected by other variables in the system, but their own residual does not feed back into the determination of other variables. It appears from the table that a null hypothesis of passivity might be accepted for price and demand pressure.

The reasonableness and possible economic mechanisms of the model's dynamics can be assessed by examining the model's response to "innovations" in each of the five variables. The innovation in an element of a vector stochastic process is the difference between the element's current value and the best forecast of the current value available last period -- the one-step ahead forecast error.[30] Thus Panel A of Table 23, e.g., displays the response of the estimated system to a unit upward "surprise" in the money variable. Because the system implies that residuals are serially correlated, the initial-period unit surprise in money generates a sustained smooth rise and slow fall in money, rather than a quick return to zero. One could of course trace out instead the system response to a unit disturbance in money with an immediate return to zero or with the disturbance fixed indefinitely at the unit level, but these would give less reliable pictures of the dynamics. What Table 23 displays are responses to <u>typical</u> patterns of deviation from trend for each variable. For money it is clear that a unit deviation from trend followed by immediate return to the trend value would be atypical. Since such a pattern of behavior for money is rare or non-existent in the historical period,

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[30] The notion of an "innovation", we should note, is tied to theory based only on first and second moments, or else to an assumption of normality. In general it is not true that the only information about y_t available from observing previous values of y_s concerns the mean of y_t: thus in a general stochastic system one could not do what we do here: discuss the response to a "shock" without reference to the initial state of the system.

the model's tracing of the effects of such a pattern is likely to be unreliable.[31]

To pick out one interesting pattern of results, note that Panels B and E of Table 23 show that surprise increases in price or wage generate a response in unemployment of the type which might be predicted by a rational expectations theory of the Phillips curve: an initial drop in unemployment, followed a year later by a rise in unemployment above trend of roughly the same order of magnitude. The year-long persistence of the initial effect is perhaps greater than would be expected on the basis of the strictest classical rational expectations models, but it is certainly weaker than would be suggested by policy discussions that assume that the vertical Phillips curve is always five or more years in the future. Furthermore, Panel D shows that an unemployment innovation has no damping effect on prices or wages (what effect it has is positive). This could mean that surprise changes in unemployment reflect supply-side influences not related to the standard Phillips curve mechanism. For example, unemployment innovations might reflect shifts in composition of the labor force or adjustment to downward shifts in supply of non-labor inputs.

Going back to Panel A, however, we see a pattern of covariation much more consistent with the existence of an exploitable Phillips curve. An upward innovation in money generates a long-sustained drop in unemployment accompanied by an even longer-sustained rise in prices and wages, leaving the real wage roughly constant. At least over this

[31] This point is a special case of a generally applicable point: any kind of statistical model can give unreliable projections for inputs of historically unprecedented form. The point has been made before by others, but bears repetition.

sample period, the model suggests that expansionary monetary policy did produce sustained decreases in unemployment together with sustained rises in wages and prices. Of course any reasonable modeling of expectation-formation is likely to suggest, as does the rational expectations formulation, that the form of the response to policy depends on the nature of the policy, so that Panel A might not be a reliable tool for policy projection if policies ended up systematically different from what they were in the sample period. Nonetheless, the persistent effect of money-innovations in Panel A definitely implies either that expectations are not rational, that there are important sources of lags other than information delays, or that the model estimated here is very mistaken.

Now to cast the proper amount of doubt on these interpretations right away, consider Table 24, which reports results analogous to those of Table 2, Panel E for a model fit to the same sample period but excluding the money variable from the system. Comparing Table 24 with Panel E of Table 23, we see that the deviation from trend in the wage itself generated by a wage innovation is more rapidly damped in the five-variable system, that the effect of the wage on C, the demand pressure variable, is much larger in the four-variable system, and that the "expectational Phillips curve" behavior shown in Panel E is not present in Table 24. In fact, in results not displayed here, one can see that no innovation in the four variable model generates the kind of persistent negative covariation in wage and unemployment which appears in Panel A of Table 23.

From the point of view of the larger model, it is easy to explain the large differences between the two sets of results -- the "innovations" in the smaller model are not subject to the same economic interpretation as those in the larger model because a substantial part of the "forecast errors" in the smaller model are predictable from knowledge of past values of the money variable. (The sum of squared residuals for wage, e.g., is 30% smaller for the five-variable system). This is only an illustration of the theoretical point made earlier, that innovations and the system's typical responses to them will not remain fixed under changes in the list of variables unless all non-passive variables remain in the system. Clearly in this case money is non-passive. However, it seems quite likely that in a system which included some direct measure of aggregate current activity, such as GNP, that measure would not be passive, and the results of Table 23 could undergo substantial changes.

To assess the amount of cross-dependence in the system, it is useful to ask what proportion of the variance of k-step-ahead forecast errors in one variable is accounted for by innovations in each of the other variables, allowing k to take on different values. As k approaches infinity, the variance of the k-step ahead forecast error approaches the variance of the series itself. Table 25 reports these computations.[32] Over a one-year horizon, each variable is explained primarily by its own innovation, though the wage has substantial contributions from prices and unemployment. Over a four-year horizon, the bulk of the explanation for price and wage movements has shifted to other variables, the leading role being played by money, though unemployment contributes non-negligible explanation as well.

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[32] The coefficients in Table 23 are the coefficients of the moving average representation of y. The numbers in Table 25 are obtained by taking sums of squares of the coefficients in Table 23 over the relevant horizon, weighting each panel by the variance of the corresponding innovations.

The two real variables, unemployment and demand pressure, are explained primarily by their own innovations over all horizons, with some non-negligible explanatory power at time horizons greater than a year attributed to other variables. Money at all time horizons is explained almost entirely (more than 97 per cent of variance) by its own innovations, which is to say that it is sharply causally prior in Granger's sense.

In light of Table 25, it might be interesting to test the hypothesis that money is exogenous, that unemployment is exogenous, and that wages and prices are passive. Only the first of these has been tested. The test is executed by estimating the model subject to the constraint that the row of a corresponding to money is zero, then using the computed constrained likelihood maximum to generate a likelihood ratio test. The test statistic, asymptotically distributed as $\chi^2(3)$, is .63, which corresponds to a marginal significance level greater than .50.

It is also interesting to test the very strong constraint on the system that all elements of a and c be zero. In this form, the system becomes a set of univariate 3rd-order autoregressions, so that no cross-variable effects are allowed. For this null hypothesis, the likelihood ratio statistic is 44.34 and is asymptotically distributed as $\chi^2(27)$. The hypothesis is therefore rejected at a marginal significance level between .01 and .02.

We might ask whether an unconstrained autoregression of the same order as our equation (V) (fifth-order in this case) fits substantially better than the index model. The likelihood ratio test statistic for this hypothesis is 89.7 with 73 degrees of freedom. This allows rejection of the index-model constraint at a marginal significance level of about .09. This latter result probably should leave us willing to use the index-model, but should make us a little uncomfortable about doing so.

The form of index-model we have fit requires that $a(0)$ be zero, i.e. that z have no contemporaneous effects on y, so that current innovations are uncorrelated. The general form of index model makes no such restriction, and is only slightly more complicated to fit. Table 28 shows the matrix of cross-correlations among the residuals from the fitted model. Treating 90 times the sum of squares of off-diagonal elements in that matrix as $\chi^2(10)$ yields a test statistic of 16.9, whose marginal significance level is about 8%. However the row of correlations corresponding to the wage is clearly large, and the test statistic for that row alone would be 14.17, which as a $\chi^2(4)$ statistic has a marginal significance level of less than .01. Since the non-zero covariances are concentrated in a single row, there is some prospect that they have a form which could be accomodated by a one-dimensional observable-index model with $a(0) \neq 0$, but it seems quite unlikely that the model actually fit to the data is correct in assuming no contemporaneous correlations. the system were re-estimated allowing non-zero $a(0)$. However, significance tests might be quite different with this change in specification.

Three types of test for the stability of the model over time were carried out. In one, the sample was split between 1959 and 1960 and the model fit separately to each half-sample. The likelihood-ratio test for the null hypothesis that both halves of the sample were the same was 28.30, which is asymptotically $\chi^2(42)$.[33] In another test, the model was used to predict the period immediately following the sample period (i.e., beginning in 1972.I.). It appears that 1972.I was an unusual quarter, at least from this model's perspective: the residual for the money supply was more than six standard deviations, and that for the wage was 3.8 standard deviations. Since, as we have seen, it appears that the model ought to allow positive contemporaneous correlation in wage and money residuals, these two bad residuals probably reflect the same phenomenon, a dramatic shift in the pattern of behavior of money, which the model projects as a pure autoregression. Whether or not the structure of the model conditional on money shifted remains an open question.

It is also of some interest to look at projections made far out of the sample period, to see how badly the model behaves in the recent period of recession. As can be seen from Table 26, the model predicts, using data through 1975.I, an unemployment rate peaking at 9.2% in the first quarter of 1976, and price deflation beginning in the first quarter of 1976. Part of the reason for this forecast appears to be that money is predicted to be expanded at only a 2 per cent annual rate during 1975. Inserting actual data for money in the second quarter

[33] However, residual variances appear to be larger in the earlier part of the sample, which probably biases the sample-split test in favor of the null hypothesis.

of 1975 (but not for other variables) results in the projections in Table 27. Now the unemployment rate peaks at 9.1% in the 1975II and III, and the price index remains roughly constant through 1977I. Whether one regards these projections as bad enough to cast doubt on the usefulness of models of this type, or instead as surprisingly reasonable for a model applied without refitting to a period so far outside of its sample period, is a matter of judgment. Probably that judgment ought to be reserved, in any case, until similar exercises can be carried out with a variable list correcting some of the glaring omissions from this model's list of variables.

Table 22

ESTIMATED COEFFICIENTS FOR OBSERVABLE-INDEX MODEL

S	C(s) 1	2	3	$D^{-1}*a(s)$ 0	1	2
M	1.0000	0.0	0.0	.0018	.0092	.0063
				(.018)	(.024)	(.013)
P	4.4136	-2.9456	-2.1460	.0552	-.0084	-.0090
	(3.06)	(2.40)	(2.16)	(.0189)	(.0170)	(.0116)
C	.1795	.0847	-.1535	.2763	-.1824	-.0359
	(.150)	(.252)	(.169)	(.154)	(.221)	(.131)
U	.3031	-.3991	.2344	-.3546	.1265	.2285
	(.125)	(.184)	(.120)	(.246)	(.303)	(.238)
W	5.3265	.6989	-4.9039	.0561	.0264	-.0289
	(2.97)	(2.03)	(2.45)	(.0240)	(.0202)	(.0158)

D(s) (diagonal elements)

	1	2	3
M	-1.866	1.124	-.247
	(.114)	(.208)	(.115)
P	-.871	.021	-.076
	(.211)	(.272)	(.158)
C	-1.310	.181	.218
	(.113)	(.193)	(.108)
U	-1.532	.768	-.138
	(.114)	(.195)	(.115)
W	-.766	.209	-.311
	(.194)	(.185)	(.229)

NOTE: Standard errors in parenthesis (from asymptotic distribution). Here, as in all tables of this section, M is money, P is price, C is demand pressure, U is unemployment, and W is wage. For precise definitions see footnote 25 in the text.

TABLE 23

PANEL A: RESPONSE TO M-INNOVATION

M	P	C	U	W
1.00000	0	0	0	0
1.86756	.552243E-01	.276326	-.354558	.561099E-01
2.37377	.169601	.829494	-1.25084	.201376
2.59924	.339065	1.67842	-2.53859	.400492
2.67452	.542271	2.67947	-3.86706	.625785
2.69110	.748299	3.64837	-4.80432	.844017
2.69315	.936546	4.43647	-5.08681	1.02987
2.69438	1.09315	4.93866	-4.68149	1.17304
2.69410	1.21354	5.12586	-3.74081	1.27179
2.68821	1.30078	5.02904	-2.52148	1.33282
2.67409	1.36164	4.71810	-1.28489	1.36652
2.65162	1.40449	4.28048	-.231861	1.38337
2.62269	1.43672	3.79826	.527468	1.39202
2.58990	1.46356	3.33538	.972899	1.39792
2.55571	1.48781	2.93167	1.15092	1.40345
2.52178	1.51014	2.60363	1.14372	1.40863
2.48889	1.52988	2.34993	1.03972	1.41214
2.45702	1.54581	2.15839	.912223	1.41229
2.42570	1.55679	2.01279	.808672	1.40777
2.39431	1.56218	1.89787	.749441	1.39798
2.36230	1.56198	1.80217	.733274	1.38312
2.32933	1.55670	1.71881	.745905	1.36399
2.29534	1.54725	1.64481	.768707	1.34171
2.26047	1.53461	1.57971	.785351	1.31743
2.22502	1.51973	1.52403	.785535	1.29217
2.18930	1.50331	1.47814	.765908	1.26662
2.15360	1.48587	1.44154	.728899	1.24119
2.11812	1.46766	1.41282	.680466	1.21604
2.08301	1.44878	1.38983	.627654	1.19114
2.04829	1.42926	1.37015	.576652	1.16642
2.01397	1.40908	1.35153	.531641	1.14178
1.97999	1.38825	1.33225	.494471	1.11717
1.94634	1.36682	1.31124	.464978	1.09261
1.91298	1.34490	1.28813	.441663	1.06818
1.87992	1.32261	1.26308	.422456	1.04398
1.84716	1.30010	1.23662	.405363	1.02013
1.81475	1.27748	1.20942	.388885	.996721
1.78271	1.25490	1.18216	.372177	.973848
1.75108	1.23243	1.15537	.355015	.951559
1.71990	1.21014	1.12943	.337615	.929879
1.68918	1.18809	1.10455	.320427	.908811

TABLE 23

PANEL B, RESPONSE TO P-INNOVATION

M	P	C	U	W
0	1.00000	0	0	0
.791559E-02	1.11485	1.21959	-1.56487	.247646
.606155E-01	1.08097	1.63405	-2.91981	.477195
.108296	1.05291	1.82783	-2.67637	.392896
.138470	.968710	1.49446	-1.32761	.286463
.140922	.800668	.685554	.706975	.159412
.113161	.639412	-.154905	2.49400	.402123E-02
.703096E-01	.489348	-.972026	3.48942	-.117350
.220393E-01	.359928	-1.58694	3.67236	-.210211
-.226576E-01	.264621	-1.93006	3.16522	-.267149
-.578414E-01	.196772	-2.02508	2.25569	-.291058
-.821944E-01	.151696	-1.90911	1.23720	-.294861
-.965659E-01	.121295	-1.66223	.331784	-.286692
-.103666	.975331E-01	-1.35948	-.322769	-.275759
-.106680	.756044E-01	-1.06003	-.689424	-.267501
-.108297	.525280E-01	-.804160	-.803430	-.263753
-.110371	.278297E-01	-.607836	-.741111	-.264349
-.113718	.254803E-02	-.470287	-.590334	-.267320
-.118317	-.217231E-01	-.380013	-.425531	-.270367
-.123629	-.433733E-01	-.321277	-.295881	-.271569
-.128912	-.613375E-01	-.279734	-.222694	-.269743
-.133501	-.752298E-01	-.245048	-.204215	-.264618
-.136960	-.852801E-01	-.211830	-.224444	-.256623
-.139131	-.921527E-01	-.178915	-.262150	-.246616
-.140101	-.966928E-01	-.147782	-.298133	-.235579
-.140112	-.997172E-01	-.120913	-.319492	-.224374
-.139470	-.101872	-.100434	-.320879	-.213605
-.138459	-.103569	-.873483E-01	-.303494	-.203583
-.137294	-.105000	-.813380E-01	-.272769	-.194362
-.136098	-.106187	-.810007E-01	-.235777	-.185826
-.134914	-.107061	-.843266E-01	-.199057	-.177782
-.133724	-.107525	-.892156E-01	-.167260	-.170039
-.132483	-.107508	-.938938E-01	-.142669	-.162459
-.131141	-.106986	-.971562E-01	-.125428	-.154977
-.129663	-.105990	-.984274E-01	-.114191	-.147593
-.128034	-.104591	-.976776E-01	-.106908	-.140357
-.126263	-.102882	-.952556E-01	-.101520	-.133339
-.124373	-.100958	-.916985E-01	-.964211E-01	-.126607
-.122397	-.989019E-01	-.875662E-01	-.906675E-01	-.120212
-.120368	-.967784E-01	-.833281E-01	-.839600E-01	-.114183
-.118315	-.946291E-01	-.793077E-01	-.764834E-01	-.108524

TABLE 23

PANEL C, RESPONSE TO C-INNOVATION

M	P	C	U	W
0	0	1.00000	0	0
.321870E-03	.991105E-02	1.35926	-.636321E-01	.100700E-01
.297275E-02	.296066E-01	1.67924	-.219148	.352959E-01
.871385E-02	.493503E-01	1.79947	-.379318	.597527E-01
.163247E-01	.696378E-01	1.82072	-.484600	.786313E-01
.248720E-01	.837810E-01	1.72056	-.487868	.925300E-01
.324084E-01	.913948E-01	1.53831	-.382980	.972032E-01
.379340E-01	.928746E-01	1.29402	-.208662	.949112E-01
.409484E-01	.887169E-01	1.01554	-.805685E-02	.871062E-01
.414764E-01	.810265E-01	.731576	.175040	.758169E-01
.400349E-01	.714771E-01	.464634	.310086	.634851E-01
.372892E-01	.615941E-01	.232670	.384104	.516170E-01
.339308E-01	.525013E-01	.461379E-01	.398479	.413023E-01
.305283E-01	.447212E-01	-.915946E-01	.365734	.329734E-01
.274524E-01	.384078E-01	-.182624	.303437	.265530E-01
.248838E-01	.334084E-01	-.233186	.229163	.217288E-01
.228436E-01	.294299E-01	-.251550	.157116	.180642E-01
.212530E-01	.261713E-01	-.246440	.963207E-01	.151652E-01
.199917E-01	.233866E-01	-.225874	.506326E-01	.127481E-01
.189418E-01	.209242E-01	-.196448	.198031E-01	.106512E-01
.180150E-01	.187189E-01	-.163133	.104356E-02	.881976E-02
.171614E-01	.167645E-01	-.129369	-.939927E-02	.726159E-02
.163650E-01	.150824E-01	-.973427E-01	-.150492E-01	.600603E-02
.156313E-01	.136922E-01	-.683322E-01	-.185052E-01	.507185E-02
.149746E-01	.125939E-01	-.430110E-01	-.212324E-01	.444946E-02
.144066E-01	.117626E-01	-.216775E-01	-.237139E-01	.409874E-02
.139304E-01	.111518E-01	-.439884E-02	-.257914E-01	.395684E-02
.135386E-01	.107030E-01	.891726E-02	-.270433E-01	.395126E-02
.132154E-01	.103568E-01	.184927E-01	-.270904E-01	.401313E-02
.129405E-01	.100618E-01	.246677E-01	-.257750E-01	.408715E-02
.126934E-01	.978031E-02	.278871E-01	-.232112E-01	.413667E-02
.124569E-01	.949035E-02	.286759E-01	-.197363E-01	.414399E-02
.122190E-01	.918382E-02	.276024E-01	-.158074E-01	.410706E-02
.119736E-01	.886292E-02	.252354E-01	-.118889E-01	.403434E-02
.117195E-01	.853581E-02	.221020E-01	-.836135E-02	.393934E-02
.114591E-01	.821262E-02	.186539E-01	-.546965E-02	.383604E-02
.111964E-01	.790254E-02	.152463E-01	-.331214E-02	.373593E-02
.109360E-01	.761212E-02	.121306E-01	-.186171E-02	.364659E-02
.106817E-01	.734485E-02	.946020E-02	-.100556E-02	.357165E-02
.104363E-01	.710150E-02	.730431E-02	-.589829E-03	.351158E-02
.102011E-01	.688090E-02	.566782E-02	-.458132E-03	.346486E-02

TABLE 23

PANEL D, RESPONSE TO U-INNOVATION

M	P	C	U	W
0	0	0	1.00000	0
.543538E-03	.167367E-01	.837454E-01	1.42485	.170051E-01
.416858E-02	.237779E-01	.113177	1.37782	.329648E-01
.784493E-02	.356234E-01	.182321	1.12510	.386304E-01
.126645E-01	.467067E-01	.226381	.830604	.514356E-01
.175277E-01	.557450E-01	.261836	.597658	.612729E-01
.221675E-01	.646178E-01	.285918	.430310	.684775E-01
.264796E-01	.700831E-01	.285921	.332310	.730793E-01
.299020E-01	.725649E-01	.267111	.293678	.734332E-01
.322605E-01	.723510E-01	.230079	.292541	.710233E-01
.334738E-01	.698156E-01	.180720	.309450	.664646E-01
.336437E-01	.659700E-01	.126756	.325643	.607055E-01
.330373E-01	.614615E-01	.741729E-01	.329213	.547280E-01
.319391E-01	.568769E-01	.283543E-01	.315725	.490285E-01
.306219E-01	.525940E-01	-.772480E-02	.286307	.439649E-01
.292888E-01	.487443E-01	-.331409E-01	.246112	.396153E-01
.280562E-01	.453402E-01	-.485053E-01	.201571	.358911E-01
.269678E-01	.422935E-01	-.555785E-01	.158528	.326491E-01
.260120E-01	.394908E-01	-.565239E-01	.121123	.297262E-01
.251492E-01	.368390E-01	-.534657E-01	.913430E-01	.270030E-01
.243346E-01	.342800E-01	-.481770E-01	.692929E-01	.244161E-01
.235332E-01	.317985E-01	-.419281E-01	.537771E-01	.219518E-01
.227266E-01	.294113E-01	-.355123E-01	.429866E-01	.196332E-01
.219133E-01	.271511E-01	-.293396E-01	.351016E-01	.174975E-01
.211038E-01	.250524E-01	-.235711E-01	.286823E-01	.155784E-01
.203140E-01	.231399E-01	-.182446E-01	.228383E-01	.138953E-01
.195600E-01	.214234E-01	-.133622E-01	.172089E-01	.124483E-01
.188532E-01	.198974E-01	-.893961E-02	.118223E-01	.112206E-01
.181991E-01	.185448E-01	-.501759E-02	.691228E-02	.101834E-01
.175971E-01	.173417E-01	-.164991E-02	.274897E-02	.930329E-02
.170423E-01	.162631E-01	.111773E-02	-.478399E-03	.854797E-02
.165272E-01	.152869E-01	.326735E-02	-.271613E-02	.789047E-02
.160443E-01	.143957E-01	.481968E-02	-.404541E-02	.731095E-02
.155870E-01	.135774E-01	.583452E-02	-.464484E-02	.679661E-02
.151506E-01	.128249E-01	.640219E-02	-.473760E-02	.634013E-02
.147325E-01	.121343E-01	.662966E-02	-.454129E-02	.593768E-02
.143316E-01	.115030E-01	.662557E-02	-.423194E-02	.558695E-02
.139477E-01	.109292E-01	.648752E-02	-.392666E-02	.528558E-02
.135812E-01	.104101E-01	.629376E-02	-.368340E-02	.503044E-02
.132324E-01	.994236E-02	.609965E-02	-.351269E-02	.481729E-02
.129011E-01	.952167E-02	.593845E-02	-.339513E-02	.464112E-02

TABLE 23

PANEL E, RESPONSE TO W-INNOVATION

M	P	C	U	W
0	0	0	0	1.00000
.955277E-02	.294150	1.47184	-1.88853	1.06502
.797787E-01	.618521	2.99295	-4.83368	1.16093
.179014	.639203	3.20041	-5.19508	1.23821
.241897	.646159	3.24748	-3.48007	1.10951
.277055	.602542	2.63519	-1.15073	.994578
.279746	.505729	1.75280	1.17706	.839809
.258503	.418936	.871346	2.76746	.688323
.227554	.337783	.623034E-01	3.46577	.572524
.194738	.278914	-.506431	3.39026	.482717
.167669	.244717	-.828145	2.75846	.426733
.149400	.227668	-.930560	1.88350	.393854
.139709	.223060	-.865494	1.01703	.373508
.136770	.222715	-.710760	.330424	.358437
.137581	.221182	-.526438	-.977540E-01	.341474
.139394	.215637	-.357694	-.276532	.320138
.142212	.205097	-.228790	-.265809	.294319
.138993	.190670	-.144207	-.147849	.265578
.135593	.174213	-.970100E-01	-.259799E-03	.236462
.130539	.157715	-.740322E-01	.121175	.209204
.124619	.142786	-.619202E-01	.187636	.185393
.118632	.130312	-.507636E-01	.195113	.165717
.113183	.120505	-.352815E-01	.156717	.150030
.108594	.113037	-.146489E-01	.935117E-01	.137643
.104919	.107280	.893782E-02	.266111E-01	.127617
.102013	.102542	.319614E-01	-.281616E-01	.119038
.996314E-01	.982381E-01	.510590E-01	-.625037E-01	.111197
.975147E-01	.939832E-01	.640093E-01	-.753016E-01	.103668
.954551E-01	.896085E-01	.700976E-01	-.706653E-01	.962908E-01
.933261E-01	.851214E-01	.699895E-01	-.553272E-01	.891029E-01
.910846E-01	.806391E-01	.652907E-01	-.362516E-01	.822492E-01
.887519E-01	.763176E-01	.579960E-01	-.189544E-01	.758959E-01
.863869E-01	.722965E-01	.499978E-01	-.670207E-02	.701706E-01
.847583E-01	.686661E-01	.427514E-01	-.497575E-03	.651323E-01
.818245E-01	.654569E-01	.371276E-01	.402849E-03	.607685E-01
.797225E-01	.626479E-01	.334304E-01	-.226506E-02	.570116E-01
.777649E-01	.601838E-01	.315254E-01	-.651010E-02	.537630E-01
.759444E-01	.579952E-01	.310161E-01	-.106371E-01	.509182E-01
.742413E-01	.560154E-01	.314137E-01	-.135453E-01	.483845E-01
.726313E-01	.541913E-01	.322673E-01	-.147870E-01	.460920E-01
.710915E-01	.524877E-01	.332384E-01	-.144533E-01	.439949E-01

TABLE 24

FOUR-VARIABLE SYSTEM, RESPONSE TO W-INNOVATION

P	C	U	W
0	0	0	1.00000
.277556	1.59455	.615850	1.20865
.769403	5.81136	-.692737E-01	1.42941
1.16946	8.40311	-.937248E-01	1.64529
1.45628	10.1786	.539820	1.87622
1.62536	10.2403	1.87452	2.01863
1.72778	9.56666	3.27649	2.08155
1.78190	8.19065	4.47926	2.06479
1.80251	6.58246	5.27552	2.00027
1.79030	4.78742	5.70824	1.89941
1.74982	3.01139	5.82468	1.77658
1.68415	1.31001	5.71546	1.63665
1.59878	-.207673	5.43651	1.48652
1.49795	-1.50608	5.04375	1.33026
1.38586	-2.54764	4.57475	1.17260
1.26562	-3.33498	4.06416	1.01658
1.14006	-3.87705	3.53632	.864914
1.01149	-4.20057	3.01081	.719247
.882048	-4.33475	2.50031	.580891
.753550	-4.31399	2.01367	.450663
.627643	-4.17114	1.55603	.329185
.505754	-3.93788	1.13057	.216855
.389138	-3.64191	.738854	.113960
.278870	-3.30707	.381620	.206745E-01
.175865	-2.95260	.590121E-01	-.628988E-01
.808708E-01	-2.59354	-.229123	-.136733
-.552364E-02	-2.24095	-.483163	-.200868
-.828915E-01	-1.90254	-.703617	-.255417
-.150964	-1.58324	-.891142	-.300567
-.209627	-1.28587	-1.04654	-.336589
-.258912	-1.01164	-1.17081	-.363833
-.298990	-.760702	-1.26515	-.382731
-.330156	-.532528	-1.33098	-.393793
-.352817	-.326243	-1.36996	-.397595
-.367476	-.140837	-1.38399	-.394775
-.374718	.246844E-01	-1.37515	-.386018
-.375190	.171224	-1.34572	-.372041
-.369584	.299562	-1.29809	-.353581
-.358624	.410369	-1.23476	-.331382
-.343047	.504231	-1.15826	-.306179
-.323592	.581700	-1.07112	-.278685

TABLE 25

PROPORTION OF VARIANCE OF k-PERIOD AHEAD FORECAST
EXPLAINED BY INNOVATION IN ROW VARIABLE

k		M	P	C	U	W
4Q	M	.997	.021	.017	.007	.053
	P	.001	.854	.024	.022	.159
	C	.000	.021	.915	.007	.057
	U	.001	.052	.022	.943	.133
	W	.001	.052	.022	.021	.598
8Q	M	.984	.171	.067	.060	.277
	P	.002	.530	.015	.035	.058
	C	.003	.080	.840	.024	.125
	U	.007	.165	.059	.860	.258
	W	.003	.053	.019	.022	.282
16Q	M	.973	.452	.154	.066	.520
	P	.002	.240	.034	.054	.044
	C	.005	.069	.725	.039	.075
	U	.018	.211	.071	.814	.236
	W	.002	.028	.016	.028	.124
24Q	M	.973	.606	.171	.068	.637
	P	.002	.158	.034	.054	.045
	C	.005	.047	.709	.039	.053
	U	.018	.168	.071	.812	.178
	W	.002	.021	.015	.027	.087

TABLE 26

PROJECTIONS FROM 1975.I INITIAL CONDITIONS

	M	P	C	U	W
1975 II	285.4	173.2	1.638	9.1	177.7
III	287.0	173.8	1.634	9.1	179.5
IV	288.7	173.7	1.595	9.2	180.0
1976 I	290.1	173.3	1.533	9.2	180.5
II	291.2	172.7	1.466	9.2	181.0
III	292.0	172.2	1.411	8.8	181.6
IV	292.8	171.8	1.371	8.2	182.5
1977 I	293.6	171.6	1.346	7.6	183.5

TABLE 27

PROJECTIONS FROM 1975 II INITIAL CONDITIONS FOR M,

1975.I INITIAL CONDITIONS FOR OTHER VARIABLES

	M	P	C	U	W
1975 II	290.4	173.2	1.638	9.1	177.7
III	296.6	173.9	1.644	9.1	179.7
IV	301.0	174.1	1.626	9.0	180.9
1976 I	303.7	174.0	1.594	8.8	182.1
II	305.2	173.8	1.561	8.5	183.6
III	306.2	173.7	1.538	8.0	185.2
IV	307.0	173.7	1.522	7.5	186.9
1977 I	307.8	173.9	1.512	7.0	188.6

TABLE 28

CORRELATIONS AMONG RESIDUALS

	M	P	C	U	W
M	1.0				
P	.103	1.0			
C	-.051	.035	1.0		
U	-.099	.009	.076	1.0	
W	.157	.218	-.186	-.225	1.0

References

Cooper, J. P., and C. R. Nelson (1975), "The Ex Ante Prediction Performance of the St. Louis and FRB-MIT-PENN Econometric Models and Some Results on Composite Predictors," Journal of Money, Credit, and Banking (February).

Fama, Eugene F. (1975), "Short-Term Interest Rates as Predictors of Inflation", American Economic Review, 65 (June) 269-283.

Geweke, John B. (1975a), "Estimation and Hypothesis Testing in the Frequency Domain Factor Analysis Model." Unpublished manuscript. Minneapolis, Minnesota: University of Minnesota.

_________, (1975b), unpublished manuscript, University of Wisconsin.

Granger, C.W.J., and P. Newbold (1974), "Spurious Regressions in Econometrics," Journal of Econometrics, 2, No. 2, (July), 205-210.

Hall, Robert E. (1975), "The Rigidity of Wages and the Persistence of Unemployment", Brookings Papers on Economic Activity, No. 2, 301-350.

Jenkins, G. M., and H. Watts. Spectral Analysis and Its Applications. Holden-Day, 1968.

Joreskog, K. G. (1967), "Some Contributions to Maximum Likelihood Factor Analysis," Psychometrika, 32, No. 4 (December).

Kendall, M. G. A Course in Multivariate Analysis. London: 1961.

Koopmans, T. C. (1947), "Measurement Without Theory," Review of Economics and Statistics.

_________, Scientific Papers of Tjalling C. Koopmans. Ed. by M. Beckmann, C. F. Christ, and M. Nerlove. New York: Springer-Verlag, 1970, 375-376.

Lawley, D. N., and A. E. Maxwell. Factor Analysis as a Statistical Method. London: 1963.

Liu, Ta-Chung (1960), "Underidentification, Structural Estimation, and Forecasting," Econometrica, 28, No. 4, (October).

Lucas, Robert E., Jr. (1972), "Expectations and the Neutrality of Money," Journal of Economic Theory, 4, (April), 103-124.

_________, (1973), "Some International Evidence on Output-Inflation Tradeoffs," American Economic Review, 63, (June).

_________, (1975), "An Equilibrium Model of the Business Cycle," Journal of Political Economy, 83, 1113-1144.