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**Repeated Insurance Relationships
in a Costly State Verification
Model: With an Application
to Deposit Insurance**

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ABSTRACT

We consider the problem of an insurer who enters into a repeated relationship with a set of risk averse agents in the presence of ex post verification costs. The insurer wishes to minimize the expected cost of providing these agents a certain expected utility level. We characterize the optimal contract between the insurer and the insured agents. We then apply the analysis to the provision of deposit insurance. Our results suggest—in a deposit insurance context—that it may be optimal to utilize the discount window early on, and to make deposit insurance payments only later, or not at all.

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1. Introduction

Many insurance arrangements possess two features that are rarely modeled simultaneously. First, insurers and insured agents typically enter into a repeated sequence of contractual arrangements, rather than just interacting once. And second, in a variety of circumstances, an insurer can engage in costly actions in order to verify the occurrence, the cause, and the actual cost of various adverse events that might be experienced by the agents he insures. Indeed, in many contexts—including the provision of deposit insurance to banks—a crucial issue is the amount of this verification activity to be undertaken by the insurer.

We construct a model in which a risk neutral insurer and a set of risk averse agents enter into a set of repeated insurance relationships. In particular, each risk averse agent receives a random income in each of two periods. In addition, the amount of income received by any agent is observed costlessly only by that agent. However, the insurer can observe any agent's income realization by incurring a fixed cost of $\gamma > 0$.¹

In this context we explore the nature of optimal insurance arrangements. We focus particularly on the following issues: (1) What is the optimal (cost minimizing) auditing or verification strategy on the part of the insurer? (2) What is the optimal (cost minimizing) insurance contract, and how is it related to the optimal verification strategy? (3) How is the optimal contract implemented? (4) And, under what circumstances should participation in an insurance program be mandatory?

These questions are of great importance in the context of a variety of insurance arrangements. But we think that they assume particular significance in the context of deposit insurance. For instance, it is commonly observed that banks are audited by regulatory agencies only relatively infrequently. This observation has led many to argue for more frequent bank audits, and for more rapid actions to reduce the implied obligations of the FDIC toward banks that are currently experiencing poor performance. But, when the FDIC is engaged in repeated interactions with the banks it insures, there are many strategies other than costly audits that can be used to induce truthful reporting or to deter moral hazard. Early on in the relationship between a bank and a deposit insurer, promises of enhanced (reduced) future payoffs for good (poor) current performance may well be sufficient to police bank behavior. Only later in the relationship is the usefulness of promised future rewards (or punishments)

insufficient to provide banks with appropriate incentives. Thus we find that, for appropriate levels of verification costs, lax initial auditing may indeed be the optimal strategy for a bank insurer, even when a bank reports poor current performance.

Secondly, banks are often offered a variety of instruments through which they can be (partially) insured. For example, in addition to deposit insurance, banks typically have access to a discount window, which may offer loans at either advantageous or disadvantageous rates of interest. Many existing analyses—for example, Diamond and Dybvig (1983)—suggest that the discount window and deposit insurance are redundant instruments.² However, we show that when auditing is sufficiently costly this need not be the case. Indeed, we describe circumstances under which discount window access and deposit insurance should be provided simultaneously, and banks with poor early (late) performance should be insured through the discount window (an explicit deposit insurance scheme). In addition, we indicate when it is optimal to operate only a discount window or only a deposit insurance program.

Finally, it is also the case that, at least in the U.S., most banks are not given the option of opting out of the existing deposit insurance program. Why should this be true? We state conditions under which the insurer's costs are reduced if deposit insurance participation is compulsory rather than voluntary. This suggests why it may be appropriate to force banks to obtain deposit insurance and/or discount window access.

To state our results a bit more formally, and to put them in a more general insurance context, we are able to obtain the following results under the assumption that insured agents have constant absolute risk aversion. First, in the second (last) period, the insurer should verify income reports iff the cost of doing so does not exceed some critical level, $\bar{\gamma}$. Only reports of low income levels should be audited. Moreover, when this auditing occurs, it is optimal for the insurer to provide complete insurance. On the other hand, if no auditing is undertaken, the insurer provides no insurance in the terminal period.

In the first period matters are substantially more complicated. Again, the insurer optimally verifies reported income levels iff the cost of doing so does not exceed some critical level, $\hat{\gamma}$. And, as before, only reports that income is low should be audited. We are also able to show that $\hat{\gamma} < \bar{\gamma}$ holds. Thus verification costs that are consistent with verifying low income reports in the second period may be excessively high for any monitoring to be optimal

in the first period. Or, put differently, the insurer may be more reluctant to monitor in early periods than he is in later periods.

When low income reports are audited in the first period they should also be audited in the second period, and the insurer should provide complete insurance *in each period*. When *no* auditing occurs in the first period, the insurer will provide partial, but incomplete insurance in that period. He may provide either complete insurance, or no insurance whatsoever in the second period. In particular, if $\gamma \in (\hat{\gamma}, \bar{\gamma})$, the insurer will do no verification in the first period and will provide partial insurance. Low income reports will trigger monitoring in the second period, and complete insurance will be provided at that date. On the other hand, if $\gamma > \bar{\gamma}$ holds, then high costs deter monitoring at any date. The insurer provides partial insurance in the first period; to render this incentive compatible a transfer occurs between the insurer and insured agents in the second period. The size of this transfer does *not* depend on an insured agent's second period income, but it does depend on whether or not a "claim" was filed in the first period. Agents with low first period income levels receive a smaller second period transfer than agents with high first period incomes.

Interestingly, whenever no auditing occurs in the first period, the optimal insurance contract can be regarded as involving a loan from the insurer to the insured agent in the event that a low income is received. This loan will typically be made at a "nonmarket" rate of interest.³

Finally, we ask how an optimal insurance contract is affected by the presence of a "voluntary participation constraint." Such a constraint prevents agents who report low first period income levels from being "punished" to such an extent that they prefer to withdraw from the insurance arrangement in the second period. Not surprisingly, it can be shown that either the ex ante expected utility of the risk averse agents may be reduced, or the expected cost of the insurer may be increased if the risk averse agents are given the option of withdrawing from an insurance agreement. This suggests why, under some circumstances, it may be optimal to compel agents to participate in an insurance program.

As we have already noted, we think that all of these results have interesting interpretations and applications in the context of deposit insurance. We now explore some of these implications more explicitly.

A. An Application to Deposit Insurance

When we apply our analysis to deposit insurance, we can think of a bank as a coalition of identical agents (following Townsend; 1978, 1983) that has invested in a known portfolio. The random income stream then represents a sequence of random returns on this portfolio. Under this interpretation of our model, the deposit insurer knows the probability distribution of returns on each bank's portfolio, and there are no moral hazard problems associated with deposit insurance. Six particularly interesting issues that arise with respect to deposit insurance are that (a) deposit insurance is often heavily subsidized, (b) deposit insurers are often criticized for inadequate supervision, (c) the recipients of deposit insurance also often have access to a discount window that extends credit at nonmarket rates, (d) use of the discount window may be "punished," and (e) for many banks, participation in a deposit insurance program is mandatory. Finally, (f) we can investigate when only deposit insurance or only the discount window should be utilized, as opposed to conditions under which both should coexist.

One question we can pose is, if there is a subsidy to be provided to banks, what is the optimal way to provide it? The answer to this question depends on the cost of "supervision" (state verification). If $\gamma \leq \hat{\gamma}$ holds, then the insurer should provide complete insurance in each period, and it should audit low income reports at each date. Any subsidy will be reflected in low "premiums." If $\gamma \in (\hat{\gamma}, \bar{\gamma}]$ holds, then the insurer should provide some insurance in the first period, but this insurance will be less than complete. No "oversight" activity should occur in the first period. In the second period complete insurance will be provided, and reports of low incomes will be verified. Thus the degree of "supervisory" activity should increase over time. Relatively lax initial auditing need not be suboptimal. Moreover, some subsidization of the banking system can optimally be provided in the form of low deposit insurance premiums. Finally, when $\gamma > \bar{\gamma}$ is satisfied, high monitoring costs render it inefficient to provide any deposit insurance. If the banking system is to be subsidized, it is efficient simply to make a lump-sum transfer, and not to couple the subsidy with a deposit insurance scheme.

When $\gamma > \hat{\gamma}$ holds, the insurer optimally provides partial insurance in the first period. As we show, this insurance can be provided by having the insurer make a loan in the event of a low first period income realization. The repayment of this loan should not (at least under

constant absolute risk aversion) be contingent on the second period income of the bank; thus, in effect, the insurer is a prior claimant on the assets of the bank. In addition, these loans should typically occur at nonmarket rates. They therefore resemble discount window loans, and it is optimal for the provider of these loans in the first period to be a prior claimant on the assets of the bank in the second period. This is often a point of contention between the Federal Reserve System and the FDIC, since the Fed is a prior claimant to depositors if it has extended a discount window loan. Despite FDIC complaints, our analysis indicates that this is a reasonable arrangement.

When should the discount window and deposit insurance be utilized simultaneously? Our analysis indicates that when $\gamma \in (\hat{\gamma}, \bar{\gamma}]$, the use of the discount window in the first period will be coupled with the use of deposit insurance in the second period; whereas when $\gamma > \bar{\gamma}$, the optimal way to provide “deposit insurance” is to make discount window loans in the first period, and to provide uncontingent transfers in the second period. The former result is particularly interesting, in that it suggests circumstances under which it is optimal to utilize both deposit insurance and the discount window to aid banks in distress.

Finally, most banks face compulsory participation in a deposit insurance program. We describe below conditions under which this is optimal. Indeed, if banks have the option of withdrawing from a deposit insurance program then, when $\gamma > \hat{\gamma}$ holds, this can increase the insurer’s expected cost of providing bank depositors with a particular expected utility level. It is therefore efficient to prevent banks from leaving the deposit insurance system of their own volition.

The remainder of the paper proceeds as follows. Section 2 outlines the formal problem to be analyzed, while Section 3 characterizes an optimal insurance contract under the assumption that each insured agent must receive a particular level of lifetime expected utility. Section 4 describes how this might be determined. Section 5 considers how the analysis must be modified when participation in an insurance program is or is not mandatory. Some concluding remarks are offered in Section 6.

2. The Model

A. Preliminaries

We consider a two-period model where one risk neutral agent—the “insurer”—provides insurance to a set of ex ante identical, risk averse agents.⁴ Throughout we let $t = 1, 2$ index the date.

At each date each risk averse agent receives a random income of θ^t . θ^t is drawn from a two element set, so that $\theta^t \in \{\theta_1, \theta_2\}$, $t = 1, 2$, with $0 \leq \theta_1 < \theta_2$. We let $\pi_i = \text{prob}(\theta^t = \theta_i)$, so that clearly $0 < \pi_i < 1$, $i = 1, 2$, and $\pi_1 + \pi_2 = 1$. Moreover, the income of any risk averse agent is an iid random variable (both over time, and across agents). We let $\bar{\theta} \equiv \sum \pi_i \theta_i$ denote the expected income of a representative risk averse agent in any period.

We also assume that the income level of any agent can be costlessly observed only by that agent. The insurer can observe the income of any specific individual in any period, but must incur a fixed cost of $\gamma > 0$ in order to do so. Thus we consider a standard costly state verification (CSV) model of the type originally developed by Townsend (1979), except that the CSV problem is repeated over time. To insulate the model from any dynamics other than those associated with the repeated CSV problem, we assume that the good cannot be stored between periods, and that agents have no “outside” saving or borrowing opportunities.

Let c^t , $t = 1, 2$, denote the consumption of a representative risk averse agent in period t . Then the period utility of that agent is $v(c^t)$. We assume that v is twice continuously differentiable, strictly increasing, and strictly concave. In addition, for most of the presentation, we will assume that v is of the CARA form⁵

$$v(c) = -e^{-\rho c} / \rho; \quad \rho > 0. \tag{1}$$

We will remark periodically on how the analysis must be modified if (1) is replaced by a more general utility function. Finally, the simplification obtained by focusing on CARA preferences also depends on allowing agents to have negative consumption levels, at least in principal. Thus, we do not restrict consumption to be nonnegative.

All agents are assumed to discount the future at the common rate $\beta \leq 1$. Thus a risk averse agent with a lifetime consumption stream of (c^1, c^2) receives the lifetime utility level

$$v(c^1) + \beta v(c^2).$$

The risk neutral agent enters into an insurance contract with each risk averse agent. Insurance contracts specify a set of contingencies under which the insurer will or will not verify the (reported) income level of an insured agent. In addition, contracts specify a set of transfers between the insurer and the insured agent which depend on the (reported) income history of the insured agent, as well as on whether or not verification of the state has occurred.

Following convention in the literature on problems of repeated private information, we assume that the insurer is committed in advance to deliver each insured agent a lifetime expected utility level of w . Thus the contractually specified transfers must yield each insured agent this expected utility. The objective of the insurer is to deliver the expected utility w to each agent at the lowest possible expected cost, in discounted present value terms. We discuss below how the expected utility level w might be determined in a market context.

Discussion

In order to apply this model to the provision of deposit insurance, we need to give some interpretation to the notion of a bank in this context. Following Townsend (1978, 1983) or Diamond and Dybvig (1983), we assume that a bank consists of a coalition of identical agents. Each of these agents can be viewed as having an initial endowment of one unit of “funds” in an initial period $t = 0$, at which point they do not wish to consume. We can also imagine that there is an investment technology that requires k units of funds (with k being an integer) in order to operate it. Once k units are invested, the return is θ^t in period t , per unit invested. All members of a given coalition see this return, but it cannot be observed costlessly by agents outside the coalition. In addition, each coalition naturally inherits the preferences of its representative member. Thus, as in Townsend (1978, 1983) or Diamond and Dybvig (1983), the behavior of any bank reflects the risk aversion of members of the coalition.

This leaves the following question: if there are N risk averse agents in the economy, with $N > k$, why don't banks diversify by investing in multiple projects? Here we can appeal to the answer given by Townsend (1978, 1983) as to why intermediary coalitions might have a finite size. If it is costly for agents to communicate and coordinate with one another (in

an ex ante sense), then the cost of adding additional agents and investment projects to a coalition might outweigh the potential gains from diversification. Hence there can be an optimal coalition size which does not permit diversification. However, if a coalition of k agents can deal with a single deposit insurer, appropriate insurance can be obtained, but again only at some cost, which here is related to the presence of the CSV problem. And, of course, in the context of deposit insurance, it is quite plausible to assume that the insurer can only observe a bank's investment return by engaging in a costly audit.

A second natural question concerns the potential presence of a moral hazard as well as a costly state verification problem in banking. Here, obviously, we abstract from the presence of moral hazard. This could be motivated by assuming that there is only a single investment opportunity open to any coalition, as would be true in an aggregative model of capital investment in the presence of a CSV problem. Such aggregative models appear in Williamson (1987), Bernanke and Gertler (1989), or Boyd and Smith (1997a, b).

B. Contracts

Insurance contracts have two components: (i) a strategy for verifying the state in each period, and (ii) a specification of state contingent transfers in each period. We begin by describing contractual terms at $t = 1$.

We abstract throughout from stochastic state verification.⁶ Thus, in the first period, an insurance contract specifies a set of states, $s(w)$, in which verification of the reported income level will occur.⁷ Clearly $s(w) \in \{\phi, \{\theta_1\}, \{\theta_2\}, \{\theta_1, \theta_2\}\}$.

With respect to contractually specified transfers, we adopt a standard recursive formulation.⁸ Let $s = 0$ if $s(w) = \phi$, let $s = 1$ if $s(w) = \{\theta_1\}$, let $s = 2$ if $s(w) = \{\theta_2\}$, and let $s = 3$ if $s(w) = \{\theta_1, \theta_2\}$. Then, in the first period, insurance contracts specify a payment *from* the insured agent *to* the insurer of $M_s(\theta_i, w)$, if the state verification strategy is s and the reported income level is $\theta^1 = \theta_i$. In addition, the contract specifies a "continuation expected utility level" of $u_s(\theta_i, w)$ if the state verification strategy is s , and the reported income level is $\theta^1 = \theta_i$. Thus insurance contracts at $t = 1$ specify a set of transfers and a set of future expected utilities to be delivered in period 2, as a function of the reported income level and the set of reported states that trigger state verification in period 1. Clearly there is a consistency

requirement that must be satisfied by the contractual transfers and continuation utilities. In particular, given a state verification strategy s , $M_s(\theta_i, w)$ and $u_s(\theta_i, w)$ must satisfy

$$\sum \pi_i \{v[\theta_i - M_s(\theta_i, w)] + \beta u_s(\theta_i, w)\} \geq w. \quad (2)$$

That is, the contract must deliver an expected utility of at least w to an insured agent.

At $t = 2$, the insurer must again adopt some strategy for verifying reported income levels. It is straightforward to show that the optimal monitoring strategy depends on events in period 1 only through the continuation utilities $u_s(\theta^1, w)$. Thus, suppose that some specific insured agent has been promised a continuation expected utility of u in period 2. Then an insurance contract specifies a set of states, $s(u)$, in which verification of the reported income level will occur at $t = 2$. As before, obviously $s(u) \in \{\phi, \{\theta_1\}, \{\theta_2\}, \{\theta_1, \theta_2\}\}$. And, again as before, let $s = 0$ if $s(u) = \phi$, let $s = 1$ if $s(u) = \{\theta_1\}$, let $s = 2$ if $s(u) = \{\theta_2\}$, and let $s = 3$ if $s(u) = \{\theta_1, \theta_2\}$. Then, in the second period, insurance contracts specify a payment *from* the insured agent *to* the insurer of $M_s(\theta_i, u)$ if the state verification strategy is s , if the agent reports an income level of $\theta^2 = \theta_i$, and if the agent has been promised an expected utility of u in period 2. Clearly the contractual transfers must result in the agent receiving an expected utility of at least u ; that is, given s , they must satisfy

$$\sum \pi_i v[\theta_i - M_s(\theta_i, u)] \geq u. \quad (3)$$

The objective of the insurer is to construct contracts that deliver an expected utility of w in a way that minimizes the expected discounted present value of the insurer's costs. We now proceed to describe the contracts that are optimal, from this perspective.

Discussion

In the context of the deposit insurance application discussed above, each bank has a net income of $\theta^1 - M_s(\theta^1, w)[\theta^2 - M_s(\theta^2, u_s(\theta^1, w))]$ per depositor in period 1 (2), *after* all transfers with the insurer have been completed. This income is then distributed by the bank in a pro rata manner among its depositors (coalition members). Thus banks behave in the manner described by Townsend (1978, 1983) or Diamond and Dybvig (1983).

3. Optimal Contracts

It is most convenient to characterize the optimal contract recursively. We therefore start with the optimal state verification strategy, and the optimal set of transfers at $t = 2$.

A. The Second Period

To begin, suppose that a specific risk averse agent has been promised an expected utility level of u in period 2. There are four possible ways that the insurer can proceed, associated with the four possible state verification strategies. We consider each in turn.

No State Verification

Suppose that $s(u) = \phi$. Then the insurer does not verify the state in period 2. In this case, the insurer must construct a set of transfers, $M_0(\theta_i, u)$, that satisfy three criteria. First equation (3) must be satisfied. Second, the absence of state verification means that the insurer is taking no overt actions to deter false reporting of the state. Therefore, the transfers promised *in each state* must be incentive compatible; that is, they must obey

$$v[\theta_1 - M_0(\theta_1, u)] \geq v[\theta_1 - M_0(\theta_2, u)] \quad (4)$$

$$v[\theta_2 - M_0(\theta_2, u)] \geq v[\theta_2 - M_0(\theta_1, u)]. \quad (5)$$

Finally, the transfers must minimize the insurer's expected cost of delivering the expected utility level u ; that is, $M_0(\theta_1, u)$ and $M_0(\theta_2, u)$ must be chosen to solve the problem

$$\max \sum \pi_i M_0(\theta_i, u) \quad (P_0)$$

subject to (3)–(5).⁹

Clearly, (4) and (5) imply that $M_0(\theta_1, u) = M_0(\theta_2, u) = M(u)$, where $M(u)$ satisfies

$$\sum \pi_i v[\theta_i - M(u)] \equiv u.$$

Moreover the cost to the insurer of providing the expected utility level u is obviously $-M(u)$, when no state verification occurs.

It clearly will be of value to know more about the properties of the function $M(u)$. These are stated in the following lemma.

LEMMA 1. (a) Suppose that v has the CARA form in (1). Then for all $u < 0$,

$$M(u) = \left[\ln \rho - \ln \left(\sum \pi_i e^{-\rho \theta_i} \right) + \ln(-u) \right] / \rho. \quad (6)$$

(b) Suppose that v is a general concave utility function. Define the function $c(u)$ by $v[c(u)] \equiv u$ and define the risk premium $R(x)$ in the conventional way;

$$\sum \pi_i v[x + (\theta_i - \bar{\theta})] \equiv v[x - R(x)].$$

Then $M(u) \equiv \bar{\theta} - c(u) - R[\bar{\theta} - M(u)]$. (c) In each case, $M(u)$ is a decreasing, concave function.

The proof of lemma 1 appears in Appendix A1. The function $M(u)$ describes how well the insurer can do without incurring any costs of state verification.

State Verification When $\theta^2 = \theta_1$.

We now consider what transpires when $s(u) = \{\theta_1\}$. In this case the insurer chooses a set of transfers, $M_1(\theta_i, u)$, that satisfy three conditions. First, equation (3) must hold. Second, the insurer will not verify the state when the insured agent reports an income level of θ_2 . Thus the agent must be deterred from reporting an income level of θ_2 when his true income level is θ_1 . In particular, (4) must hold. However, the fact that the insurer now verifies all reports that income is θ_1 implies that the insurer is freed from the necessity of confronting the incentive constraint (5). Of course, the cost of relaxing this constraint is the expected cost of verifying the state when $\theta^2 = \theta_1$; that is, $\pi_1 \gamma$. Finally, $M_1(\theta_1, u)$ and $M_1(\theta_2, u)$ must be chosen to minimize the insurer's expected cost of providing the expected utility level u , inclusive of the cost of state verification. Therefore $M_1(\theta_1, u)$ and $M_1(\theta_2, u)$ must be chosen to solve the problem

$$\max \sum \pi_i M_1(\theta_i, u) - \pi_1 \gamma \quad (P_1)$$

subject to (3) and (4). The solution to this problem is characterized in the following lemma.

LEMMA 2. (a) Suppose that $s(u) = \{\theta_1\}$. Then the optimal transfers $M_1(\theta_i, u)$ satisfy $M_1(\theta_i, u) = \theta_i - c(u)$; $i = 1, 2$. The maximized value of the insurer's objective function is $\bar{\theta} - \pi_1\gamma - c(u)$. (b) Suppose that preferences have the CARA form in (1). Then, for all $u < 0$, $c(u) = -[\ln \rho - \ln(-u)]/\rho$. The maximized value of the insurer's objective function is $\bar{\theta} - \pi_1\gamma + [\ln \rho + \ln(-u)]/\rho$. (c) In each case, the insurer's objective function is a decreasing, concave function of u .

The proof of lemma 2 appears in Appendix A2.

It is an immediate implication of lemmas 1 and 2 that the insurer will prefer to verify low income reports (rather than not to verify at all) iff

$$\bar{\theta} - \pi_1\gamma - c(u) \geq M(u) = \bar{\theta} - c(u) - R[\bar{\theta} - M(u)] \quad (7)$$

is satisfied. Clearly satisfaction of (7) is equivalent to the condition

$$R[\bar{\theta} - M(u)] \geq \pi_1\gamma. \quad (7')$$

Thus, the insurer will prefer to verify low reported income levels iff the expected cost of state verification is low enough relative to the risk premium that must be paid to yield an expected utility level of u when insurance provision is impossible (as is the case when no state verification occurs).

When agents have CARA preferences equation (7') takes a particularly simple form. The following result describes when the strategy $s(u) = \{\theta_1\}$ is preferred to $s(u) = \phi$.

PROPOSITION 1. *Suppose that v has the CARA form in (1). Then the strategy $s(u) = \{\theta_1\}$ is preferred to the strategy $s(u) = \phi$ iff*

$$\bar{\theta} + \left[\ln \left(\sum \pi_i e^{-\rho\theta_i} \right) \right] / \rho \equiv \pi_1\bar{\gamma} \geq \pi_1\gamma. \quad (8)$$

This result follows immediately from part (a) of lemma 1 and part (b) of lemma 2. It asserts that the preferred state verification strategy is independent of the promised expected utility level u when the risk premium, $R(x)$, is a constant function. It is this feature that renders

the CARA utility function so convenient.¹⁰

Proposition 1 implies that the strategy of verifying low income reports—and providing complete insurance—in the second period is preferred to the strategy of verifying no reports—and providing no insurance—iff the cost of verification does not exceed the critical value $\bar{\gamma}$. An important issue in many insurance contexts—including that of deposit insurance—is how does the optimal auditing strategy (how does $\bar{\gamma}$) depend on various characteristics of the insured agents' income streams? In a deposit insurance context, it is often argued that the greater the variability of a bank's income stream, the stronger is the case for auditing. This argument is, indeed, valid along some dimensions; but is not valid along others, as the following proposition shows.

PROPOSITION 2. (a) *The critical verification cost $\bar{\gamma}$ is increasing in $(\theta_2 - \theta_1)$.* (b) *The critical verification cost $\bar{\gamma}$ is decreasing in π_1 .*

The proof of proposition 2 appears in Appendix A3. The proposition asserts that the critical cost—below which monitoring in the second period is optimal—rises as $(\theta_2 - \theta_1)$ rises. Since an increase in $(\theta_2 - \theta_1)$ also increases the variance of an insured agent's income, part (a) of the proposition asserts a sense in which higher income variability strengthens the case for monitoring. However, part (b) of the proposition asserts that as the probability of receiving a low income level (π_1) increases, monitoring in the second period becomes a less attractive strategy. Since for low initial values of π_1 an incremental increase in that variable also increases the variance of income, it is therefore *not* true that any change that increases income variability also makes second period monitoring more attractive. Or, put otherwise, whether an increase in income variability strengthens the case for monitoring or not potentially depends on the source of the increase.

Other Verification Strategies

We now briefly consider the two remaining verification strategies for the second period; $s(u) = \{\theta_2\}$, and $s(u) = \{\theta_1, \theta_2\}$. We demonstrate that the former strategy is dominated by $s(u) = \phi$, while the latter is dominated by $s(u) = \{\theta_1\}$.

When $s(u) = \{\theta_2\}$, an insurance contract specifies a set of transfers, $M_2(\theta_i, u)$, that must satisfy three conditions. Clearly (3) must hold. In addition, the transfers must deter

an insured agent from falsely claiming a low income level when his income is high; thus (5) must hold. Finally, $M_2(\theta_1, u)$ and $M_2(\theta_2, u)$ must minimize the insurer's cost of delivering the expected utility level u , inclusive of the expected cost of state verification ($\pi_2\gamma$). Thus $M_2(\theta_1, u)$ and $M_2(\theta_2, u)$ are chosen to solve the problem

$$\max \sum \pi_i M_2(\theta_i, u) - \pi_2 \gamma \quad (\text{P}_2)$$

subject to (3) and (4). We now state the following result.

LEMMA 3. The maximized value of the objective in (P_0) exceeds the maximized value of the objective in (P_2) .

Proof. Equation (5) is equivalent to the requirement that $M_2(\theta_1, u) \geq M_2(\theta_2, u)$ hold. Then the solution to (P_2) sets $M_2(\theta_1, u) = M_2(\theta_2, u) = M(u)$, which coincides with the solution to the problem (P_0) . The result then follows from the observation that there are no monitoring costs associated with the strategy $s(u) = \phi$. ■

Thus it is never optimal to verify the state when $\theta^2 = \theta_2$.

It is also never optimal to verify all income reports. To see this note that, if $s(u) = \{\theta_1, \theta_2\}$, the insurer is freed from the necessity of satisfying any incentive compatibility requirements. Thus $M_3(\theta_1, u)$ and $M_3(\theta_2, u)$ should maximize $\sum \pi_i M_3(\theta_i, u) - \gamma$, subject to (3) alone. But clearly the solution to this problem has $\theta_i - M_3(\theta_i, u) = c(u), i = 1, 2$, which coincides with the solution to the problem (P_1) . But again the strategy $s(u) = \{\theta_1\}$ involves lower expected monitoring costs than the strategy $s(u) = \{\theta_1, \theta_2\}$.

Summary

In the second period, it is optimal either to engage in *no* state verification [$s(u) = \phi$] and to provide *no* insurance [$M_0(\theta_1, u) = M_0(\theta_2, u) = M(u)$], or to verify the state when reported income is low [$s(u) = \{\theta_1\}$], and to provide *complete* insurance [$\theta_i - M_1(\theta_i, u) = c(u)$]. The provision of no insurance in the absence of any state verification does not imply that *no* transfers are made in the second period; $M(u) = 0$ will hold only for $u = \sum \pi_i v(\theta_i)$.

Whether $s(u) = \phi$ or $s(u) = \{\theta_1\}$ is optimal depends on the value of u , *except* when

v is of the CARA form (1). In this case, it is optimal to verify low income reports iff $\gamma \leq \bar{\gamma}$, where $\bar{\gamma}$ is defined as in equation (8).

For future reference it will be useful to have a notation for the minimized expected cost of providing a risk averse agent the expected utility level u in the second period. If $s(u) = \phi$ [$s(u) = \{\theta_1\}$], we have shown that this cost is $-M(u) [c(u) - \bar{\theta} + \pi_1\gamma]$. We now define the function Q by

$$Q(u) \equiv \max\{\bar{\theta} - \pi_1\gamma - c(u), M(u)\}. \quad (9)$$

Then $-Q(u)$ gives the minimized expected cost of delivering the expected utility u at $t = 2$.

B. The First Period

In the first period, the insurer has promised each insured agent a lifetime expected utility of w . Then the insurer must choose a first period verification strategy, $s(w) \in \{\phi, \{\theta_1\}, \{\theta_2\}, \{\theta_1, \theta_2\}\}$, a first period set of state contingent transfers, $M_s(\theta_i, w)$, and a set of continuation utilities, $u_s(\theta_i, w)$, which minimize the expected cost of providing the expected utility level w . Thus these values must maximize $\sum \pi_i \{M_s(\theta_i, w) + \beta Q[u_s(\theta_i, w)]\}$, subject to (2) and a set of incentive constraints which depend on the verification strategy.

The function $Q(u)$ is depicted in Figure 1 (2) under the assumption that v displays strictly decreasing (increasing) absolute risk aversion. Clearly $Q(u)$ cannot be globally concave in either case; this makes the solution to the insurer's problem quite difficult to characterize analytically. However, $Q(u)$ is concave with constant absolute risk aversion, as the following result demonstrates.

LEMMA 4. Suppose that v is of the CARA form given in (1). Then

$$Q(u) = A + \ln(-u)/\rho \quad (10)$$

where A is defined by

$$A \equiv [(\ln \rho)/\rho] + \max\left\{\bar{\theta} - \pi_1\gamma, -\ln\left(\sum \pi_i e^{-\rho\theta_i}\right)/\rho\right\}.$$

The proof of lemma 4 is given in Appendix A4. In order to realize the gains associated with the insurer's objective function being concave, we henceforth focus on the case where the risk averse agents have CARA preferences.

When they do, the verification strategy of the insurer in the second period is particularly simple; $s(u) = \phi$ [$s(u) = \{\theta_1\}$] iff $\gamma \geq (\leq) \bar{\gamma}$, where $\bar{\gamma}$ is as defined in (8). It remains to characterize the optimal auditing strategy in the first period. As before, this requires us to consider each possible verification strategy individually.

No State Verification in the First Period

Suppose that each insured agent has been promised the lifetime expected utility $w < 0$, and that—in the first period— $s(w) = \phi$. Then an optimal insurance contract specifies a set of transfers, $M_0(\theta_i, w)$, and a set of continuation utilities, $u_0(\theta_i, w)$, which satisfy three conditions. First the expected utility of an insured agent must equal w . Second, in each state an insured agent has no incentive to misrepresent his income. Third, the expected discounted present value of the cost of delivering the lifetime expected utility w is minimized. Therefore the values $M_0(\theta_i, w)$ and $u_0(\theta_i, w)$ must be chosen to solve the following problem:

$$\begin{aligned} U_0(w) &\equiv \max \sum \pi_i [M_0(\theta_i, w) + \beta Q[u_0(\theta_i, w)]] & (P'_0) \\ &\equiv \max \sum \pi_i [M_0(\theta_i, w) + \beta \{A + \ln[-u_0(\theta_i, w)]\}] \end{aligned}$$

subject to

$$\sum \pi_i \{-\exp[-\rho\theta_i + \rho M_0(\theta_i, w)]/\rho + \beta u_0(\theta_i, w)\} \geq w \quad (11)$$

$$\begin{aligned} &-\exp[-\rho\theta_1 + \rho M_0(\theta_1, w)]/\rho + \beta u_0(\theta_1, w) & (12) \\ \geq &-\exp[-\rho\theta_1 + \rho M_0(\theta_2, w)]/\rho + \beta u_0(\theta_2, w) \end{aligned}$$

$$\begin{aligned} &-\exp[-\rho\theta_2 + \rho M_0(\theta_2, w)]/\rho + \beta u_0(\theta_2, w) & (13) \\ \geq &-\exp[-\rho\theta_2 + \rho M_0(\theta_1, w)]/\rho + \beta u_0(\theta_1, w) \end{aligned}$$

and $u_0(\theta_i, w) \leq 0$. Equation (11) simply imposes that an insured agent's lifetime expected utility is at least w , and equations (12) and (13) are the incentive constraints, where each has been written out explicitly for the CARA utility function (1).

We now state the following result characterizing the solution to the problem (P'_0).

PROPOSITION 3. *The optimal contract with $s(w) = \phi$ has (a) $M_0(\theta_2, w) > M_0(\theta_1, w)$ and (b) $\theta_2 - M_0(\theta_2, w) > \theta_1 - M_0(\theta_1, w)$. Thus the contract provides some insurance against a low income realization, but this insurance is incomplete. (c) Define $m_i \equiv -\exp[\rho M_0(\theta_i, w)]/\rho w$ and $u_i \equiv u_0(\theta_i, w)/w$, and let*

$$\tilde{A} \equiv (1/\rho) \max \sum \pi_i [\ln m_i + \beta \ln u_i]$$

subject to ¹¹

$$\sum \pi_i [e^{-\rho\theta_i} m_i + \beta u_i] \leq 1 \tag{14}$$

$$e^{-\rho\theta_1} m_1 + \beta u_1 \leq e^{-\rho\theta_1} m_2 + \beta u_2 \tag{15}$$

$$e^{-\rho\theta_2} m_2 + \beta u_2 \leq e^{-\rho\theta_2} m_1 + \beta u_1 \tag{16}$$

and $u_i \geq 0$. Then

$$U_0(w) = \tilde{A} + (\ln \rho)/\rho + \beta A + [(1 + \beta)/\rho] \ln(-w). \tag{17}$$

The proof of proposition 3 appears in Appendix A5.

In the second period (or equivalently, in a static CSV insurance problem), the absence of state verification precludes the provision of any insurance (although it does not preclude uncontingent transfers). In a repeated CSV problem this is not the case; an insurer can prevent all agents from reporting a low income level and still provide some insurance in the first period by setting $u_0(\theta_2, w) > u_0(\theta_1, w)$. Thus agents who report a high income level make a larger net transfer to the insurer in period 1 than do agents who report a low income level [$M_0(\theta_2, w) > M_0(\theta_1, w)$], but they are rewarded for doing so by getting a high promised utility level in period 2 [$u_0(\theta_2, w) > u_0(\theta_1, w)$]. Of course this ability to provide some insurance at $t = 1$ comes at a cost; risk aversion implies that it is costly to set $u_0(\theta_2, w) > u_0(\theta_1, w)$.

Indeed, this is why complete insurance is not optimal: $u_0(\theta_2, w) - u_0(\theta_1, w)$ must be set at a sub-optimally high level in order to make complete insurance incentive compatible.

In order to compare the costs and benefits of verifying at least some income reports, the insurer must compare the cost of setting $u_0(\theta_2, w) > u_0(\theta_1, w)$ with the cost of monitoring in at least some states. We now investigate the relative magnitude of these costs.

State Verification When $\theta^1 = \theta_1$.

In this section, we suppose that $s(w) = \{\theta_1\}$. Then an optimal insurance contract specifies a set of transfers, $M_1(\theta_i, w)$, and a set of continuation utilities, $u_1(\theta_i, w)$, which satisfy three criteria. First, the contract yields an insured agent a lifetime expected utility no smaller than w . Second, insured agents are deterred from misrepresenting the state when $\theta^1 = \theta_1$. Third, the contract minimizes the expected discounted present value of the insurer's cost of providing the expected utility level w , inclusive of monitoring costs. Hence $M_1(\theta_i, w)$ and $u_1(\theta_i, w)$ must be chosen to solve the problem

$$U_1(w) \equiv \max \sum \pi_i \{M_1(\theta_i, w) + \beta Q[u_1(\theta_i, w)]\} - \pi_1 \gamma \quad (P'_1)$$

subject to (11), (12), and $u_1(\theta_i, w) \leq 0$. We now characterize the solution to (P'_1)

PROPOSITION 4. *The optimal contract with $s(w) = \{\theta_1\}$ has (a) $M_1(\theta_i, w) = \theta_i - c[w/(1 + \beta)]$, $i = 1, 2$, and (b) $u_1(\theta_i, w) = w/(1 + \beta)$, $i = 1, 2$. Thus complete insurance is provided. (c) The function U_1 is given by*

$$U_1(w) = \bar{\theta} - \pi_1 \gamma - [(1 + \beta)/\rho] \ln(1 + \beta) + (\ln \rho)/\rho + \beta A + [(1 + \beta)/\rho] \ln(-w). \quad (18)$$

The proof is given in Appendix A6. As in the static (second period) case, verifying the state when $\theta^1 = \theta_1$ enables the provision of complete insurance. Moreover, it is unnecessary to incur the cost of setting $u_1(\theta_2, w) \neq u_1(\theta_1, w)$ to deter false reporting of income levels.

It is now straightforward to compare the monitoring strategy $s(w) = \phi$ with the strategy $s(w) = \{\theta_1\}$.

PROPOSITION 5. (a) Define $\hat{\gamma}$ by

$$\pi_1 \hat{\gamma} \equiv \bar{\theta} - \bar{A} - [(1 + \beta)/\rho] \ln(1 + \beta). \quad (19)$$

Then $U_1(w) \geq U_0(w)$ holds iff $\gamma \leq \hat{\gamma}$. (b) $\hat{\gamma} < \bar{\gamma}$ hold. (c) $\partial \hat{\gamma} / \partial (\theta_2 - \theta_1) > 0$ holds.

Proposition 5 is proved in Appendix A7. The proposition asserts that the strategy of verifying low income reports is preferred to the strategy of no verification iff the monitoring cost does not exceed some critical level, $\hat{\gamma}$. The relative ranking of these strategies does *not* depend on the promised lifetime utility w .¹² Moreover, $\hat{\gamma} < \bar{\gamma}$ holds. Thus when $\gamma \in (\hat{\gamma}, \bar{\gamma})$, monitoring is expensive enough that it does not occur in the first period. At the same time, monitoring is sufficiently inexpensive that it does occur in the second period. Finally, $\hat{\gamma}$ increases as $(\theta_2 - \theta_1)$ increases. As we have noted previously, higher values of $(\theta_2 - \theta_1)$ correspond to greater income variability, other things equal. If income variability rises due to an increase in $(\theta_2 - \theta_1)$, this also increases the set of monitoring costs for which first period verification of low income reports is optimal.

As we demonstrate below, if $U_0(w) \geq (\leq) U_1(w)$, then the strategy $s(w) = \phi [s(w) = \{\theta_1\}]$ is optimal in period 1.¹³ Therefore there are three possibilities with respect to the optimal verification strategy on the part of the insurer.

Case 1. $\gamma \leq \hat{\gamma}$. In this case the insurer verifies a low income report whenever it occurs. The insurer provides complete insurance, and $\theta_i - M_1(\theta_i, w) = \theta_i - M_1[\theta_i, u_1(\theta_i, w)] = c[w/(1 + \beta)]$, since $u_1(\theta_i, w) = w/(1 + \beta)$; $i = 1, 2$. In other words, when monitoring costs are low, it is optimal simply to offer a sequence of static contracts with payments smoothed across periods.

Intuitively, a failure to offer complete insurance (at any date) involves the payment of a risk premium to the insured agent. When $\gamma \leq \hat{\gamma}$ holds, this payment is large relative to the expected cost of verifying low income reports (in every period). Therefore the insurer's expected costs are minimized by offering complete insurance. But incentive compatibility then obligates the insurer to engage in monitoring whenever low income levels are reported.

Case 2. $\gamma \in [\hat{\gamma}, \bar{\gamma}]$. Here the insurer engages in no monitoring in the first period. However, some insurance is provided since $M_0(\theta_2, w) > M_0(\theta_1, w)$ holds. In order for this to be incentive compatible, it is necessary to set $u_0(\theta_2, w) > u_0(\theta_1, w)$.

In the second period the insurer provides complete insurance, and audits in the event of a low reported income. Since $u_0(\theta_2, w) > u_0(\theta_1, w)$ holds, the net of transfer income of an agent with a high (low) reported income at $t = 1$ is $c[u_0(\theta_2, w)]$ ($c[u_0(\theta_1, w)]$) in period 2. Thus agents with high (low) first period incomes have high (low) second period incomes as well, again net of transfer.

This arrangement can be supported as follows. In the first period, agents who have high reported incomes make a payment to the insurer, while agents who have low first period incomes receive a loan. This is repaid in the second period. Moreover, the interest rate charged on the loan will not generally be a market rate.¹⁴ The loan repayment accounts for the low second period income, net of transfers, of agents who report $\theta^1 = \theta_1$.

In a deposit insurance context, this observation has the following implication. When $\gamma \in [\hat{\gamma}, \bar{\gamma}]$, an optimal insurance arrangement can be supported by offering banks access to the discount window at $t = 1$. At $t = 2$, of course, the discount window will not operate, and “distressed” banks will receive deposit insurance payments. Indeed, all banks will receive complete insurance in the second period, but of course banks with outstanding loans repay them, thereby reducing their net second period income. Hence both deposit insurance and discount window loans should be provided; these are not redundant instruments. Note also that the provider of discount window loans should have a claim on bank assets in the second period that is prior to that of depositors.¹⁵

Case 3. $\gamma \geq \bar{\gamma}$. Here the cost of monitoring is so high that the insurer never verifies a reported income level. Even so it is feasible to provide some insurance in the first period, as again $M_0(\theta_2, w) > M_0(\theta_1, w)$. And, as before, $u_0(\theta_2, w) > u_0(\theta_1, w)$ also holds.

In the second period agents with high (low) reported first period incomes make an uncontingent payment of $M_0[u_0(\theta_2, w)]$ ($M_0[u_0(\theta_1, w)]$). Thus agents who benefit from insurance in the first period receive a reduced second period transfer. Again this arrangement can be supported by having the insurer make a loan (typically at a nonmarket rate) to agents with low first period income levels. However, here no insurance is provided in the second period. Thus, in a deposit insurance context, our analysis suggests that it is optimal to offer only discount window services—and not to offer deposit insurance—when the costs of state verification are sufficiently high.

To summarize, when our analysis is applied to deposit insurance, it indicates that—for low monitoring costs ($\gamma \leq \hat{\gamma}$)—it is unnecessary to operate a discount window. Deposit insurance is provided in each period, and insurance is complete. For intermediate levels of monitoring costs ($\gamma \in [\hat{\gamma}, \bar{\gamma}]$), “distressed” banks should have access to the discount window when they are relatively “young”; “older” banks with low income levels should be covered by deposit insurance. Hence both deposit insurance and the discount window should be utilized. However, when monitoring costs are high ($\gamma \geq \bar{\gamma}$), it is optimal not to offer deposit insurance, and to utilize the discount window alone.

Discussion

Why might it be optimal *not* to monitor in the event of low reported income levels at $t = 1$ and, at the same time, be optimal to monitor in the event of low reported income levels at $t = 2$? The answer is that, at $t = 1$, it is possible to provide some insurance in an incentive compatible way without engaging in state verification. Indeed, by setting $u_0(\theta_2, w) > u_0(\theta_1, w)$ it is possible to raise $M_0(\theta_2, w)$ above $M_0(\theta_1, w)$. Of course it is costly to have $u_0(\theta_2, w) - u_0(\theta_1, w)$ be too large; at an optimum this cost must be balanced against the risk premium that is required when risk averse agents receive less than complete insurance. However, in the second period promises of future payoffs cannot be used to control the incentive problems affecting insured agents. Thus, the only mechanism for addressing these problems is the use of costly monitoring. As a result, the cost of providing a particular expected utility level is necessarily (weakly) lower in the first than in the second period.

When $\gamma \in (\hat{\gamma}, \bar{\gamma})$ holds, risk averse agents receive more complete insurance in the second than in the first period. On the other hand, when $\gamma > \bar{\gamma}$, this pattern is reversed. Thus, depending on the costs of monitoring an optimum may involve the level of insurance provision either increasing or decreasing over time.

C. Other Verification Strategies

As in the static case, the verification strategies $s(w) = \{\theta_2\}$ and $s(w) = \{\theta_1, \theta_2\}$ are never optimal. Here we briefly sketch the arguments establishing this fact.

To begin, consider the first period monitoring strategy $s(w) = \{\theta_2\}$. Here an insurance contract calls for a set of first period transfers, $M_2(\theta_i, w)$ and $u_2(\theta_i, w)$, that satisfy (11). In

addition, the insurer must prevent misreporting of the state when $\theta^1 = \theta_2$; hence $M_2(\theta_i, w)$ and $u_2(\theta_i, w)$ must satisfy (13). Finally, the insurer's expected cost, in discounted present value terms, of providing the expected utility level w must be minimized, where costs include the expected cost of state verification $\pi_2\gamma$. Thus $M_2(\theta_i, w)$ and $u_2(\theta_i, w)$ must be chosen to solve the following problem:

$$U_2(w) \equiv \max \sum \pi_i \{M_2(\theta_i, w) + \beta Q[u_2(\theta_i, w)]\} - \pi_2\gamma \quad (\text{P}'_2)$$

subject to (11), (13), and $u_2(\theta_i, w) \leq 0$. Lemma 5 characterizes the solution to (P'₂):

LEMMA 5. (a) The values $M_2(\theta_i, w)$ and $u_2(\theta_i, w)$ solving (P'₂) are identical to the values $M_0(\theta_i, w)$ and $u_0(\theta_i, w)$ solving (P'₀). (b) $U_2(w)$ satisfies $U_2(w) \equiv U_0(w) - \pi_2\gamma$. Thus the verification strategy $s(w) = \{\theta_2\}$ is always inferior to the strategy $s(w) = \phi$.

The proof of lemma 5 appears in Appendix A8.

Finally, consider the monitoring strategy $s(w) = \{\theta_1, \theta_2\}$. Under this strategy $M_3(\theta_i, w)$ and $u_3(\theta_i, w)$ must satisfy (11), and the insurer confronts a first period verification cost of γ . However, there are no incentive constraints confronting the insurer. Hence $M_3(\theta_i, w)$ and $u_3(\theta_i, w)$ are chosen to solve the following problem:

$$U_3(w) \equiv \max \sum \pi_i \{M_3(\theta_i, w) + \beta Q[u_3(\theta_i, w)]\} - \gamma \quad (\text{P}'_3)$$

subject to (11) and $u_3(\theta_i, w) \leq 0$. The solution to (P'₃) sets $M_3(\theta_i, w) = \theta_i - c[w/(1+\beta)]$ and, in addition, sets $u_3(\theta_i, w) = w/(1+\beta)$. It therefore coincides with the solution to the problem (P'₁). Clearly, then, $U_3(w) = U_1(w) - \pi_2\gamma$, and the strategy $s(w) = \{\theta_1, \theta_2\}$ is inferior to the strategy $s(w) = \{\theta_1\}$.¹⁶

4. The Determination of Expected Utility Levels

The analysis we have conducted thus far applies equally to the provision of insurance by the government, or to the provision of insurance by the private sector. Even in the context of deposit insurance, there is no reason that this insurance cannot be provided by the private sector, so long as (a) it has adequate resources to make the necessary payments, and (b) the

government is not subsidizing deposit insurance. Of course if either of these conditions fails to obtain, deposit insurance will be supplied by the government.

In this section we briefly describe how the lifetime expected utility levels, w , received by risk averse agents might be determined in a market context. This obviously depends on the level of competition among insurers; we discuss the two polar extremes of free entry and monopoly. In addition, under some interpretations of the analysis—such as the government provision of deposit insurance—no market forces would be at work. We describe some restrictions on w that might apply in this case.

A. Free Entry

Suppose that there is free entry into insurance provision. Then w is determined as follows. Define the function $U(w)$ by $U(w) \equiv \max\{U_0(w), U_1(w)\}$. Then $U(w)$ gives the (maximized) expected discounted present value of the insurer's net income, given that he is providing the expected utility level w , inclusive of expected monitoring costs. With free entry, the equilibrium expected utility level, denoted w^* , is uniquely defined by $U(w^*) = 0$.

Of course w^* must be such that risk averse agents prefer to enter into insurance contracts, rather than to receive the uninsured income stream $\{\theta^1, \theta^2\}$. In the absence of any insurance, the lifetime expected utility level of a risk averse agent would be $\bar{w} \equiv (1 + \beta) \sum \pi_i v(\theta_i)$. Therefore we want to verify that $w^* \geq \bar{w}$ holds.

Now clearly it is feasible for an insurer to set $s(\bar{w}) = \phi$, and to offer the contract $M_0(\theta_i, \bar{w}) \equiv 0$, $u_0(\theta_i, \bar{w}) \equiv \bar{w}/(1 + \beta)$, and $M_0[\theta_i, u_0(\theta_i, \bar{w})] \equiv 0$ at the expected utility level \bar{w} . Moreover, this attains an (expected) net income level of zero for the insurer. It follows that $U_0(\bar{w}) \geq 0$ holds, as does $U(\bar{w}) \geq U_0(\bar{w}) \geq 0$. Since U is a strictly decreasing function, therefore $w^* \geq \bar{w}$ is satisfied. There is therefore a unique equilibrium with free entry.

B. Monopoly

A monopoly insurer can extract all of the surplus associated with the existence of an insurance agreement. Therefore, under a monopoly arrangement, the insurer offers each risk averse agent the lifetime expected utility level \bar{w} . The argument just given establishes that this earns nonnegative expected profits for the insurer, since $U(\bar{w}) \geq 0$ holds.

C. A Government Insurer

So long as participation in an insurance program is not mandatory, voluntary participation dictates that a government insurer must set $w \geq \bar{w}$. However, it is no longer the case that $U(w) \geq 0$ must hold in order for the insurer to participate in the arrangement. If $U(w) < 0$ obtains, obviously the government provides a net subsidy to participants in the insurance program.

In some contexts, such as deposit insurance, one might take the view that the government would perform a supervisory function over banks, even in the absence of any insurance provision. Then there is no net subsidy associated with insurance provision alone if $U(w)$ is not less than the cost that would be associated with pure supervision. In other words, if insurance provision is coupled with costly supervision that would necessarily occur in any event, then $U(w) < 0$ can hold without subsidizing insurance. There is, obviously, an argument for coupling these functions.

5. Mandatory Participation

In many insurance contexts, participation in an insurance scheme is mandatory. A particular example of this arises in deposit insurance, where most banks are compelled to be members of the FDIC. We now ask, are there any justifications for compelling banks (or agents more generally) to participate in an insurance program?

In order to answer this question, we begin by considering the opposite situation, in which risk averse agents are free to withdraw from an insurance arrangement whenever it is in their advantage to do so. Then the constraint $u_s(\theta_i, w) \geq \bar{w}$ must be appended to the problems (P'_s) , $s = 0, 1, 2, 3$ in Section 3.B. Since $w \geq (1 + \beta)\bar{w}$, clearly this does not affect the solution to the problem (P'_1) , and it also does not affect the relative ranking of the strategies $s(w) = \phi$ and $s(w) = \{\theta_2\}$, or of $s(w) = \{\theta_1\}$ and $s(w) = \{\theta_1, \theta_2\}$. However, adding a constraint to the problem (P'_0) cannot raise $U_0(w)$, and it strictly lowers $U_0(w)$ if the constraint binds. Thus a voluntary participation constraint enlarges the set of parameter values for which it is optimal to provide complete insurance at $t = 1$, and to verify reports of low income levels.

If the strategy $s(w) = \phi$ is optimal in the presence of a voluntary participation con-

straint, and if the constraint binds, then $u_0(\theta_1, w) = \bar{w}$ will hold. This constraint attenuates the ability of the insurer to provide insurance in the first period, since an ability to raise $M_0(\theta_2, w)$ above $M_0(\theta_1, w)$ depends on the ability to raise $u_0(\theta_2, w)$ sufficiently above $u_0(\theta_1, w)$. Moreover, since the existence of a binding voluntary participation constraint strictly reduces $U_0(w)$, we have the following implication. With free entry into insurance provision, the presence of a binding voluntary participation constraint must (weakly) lower the lifetime expected utility of insured agents. The claim follows from the fact that the equilibrium expected utility level w^* satisfies $U(w^*) = U_0(w^*) = 0$. Thus, in a competitive insurance market, insured agents do not benefit by having the option of withdrawing from their insurance arrangements. Similarly, with a government insurer committed to a particular ex ante expected utility level, w , the presence of a binding voluntary participation constraint must (weakly) raise the expected discounted present value of the insurer's costs. In a deposit insurance context, this suggests why it might be optimal for the government to make participation in a deposit insurance program mandatory for banks.

6. Conclusions

We have examined the problem of a risk neutral insurer who enters into a repeated relationship with a set of risk averse agents. This insurer can verify the income levels of these agents by incurring a fixed cost. In this context we have characterized the optimal auditing strategy on the part of the insurer, as well as how this strategy is related to the insurance contracts that he offers. We have demonstrated that an optimum either has the insurer providing complete or no insurance in the second period; in the former case he must verify reports of low income levels. Which strategy is optimal depends on the magnitude of the insurer's cost of verification, on the risk aversion of the insured agents, and on the amount of risk that they face.

In the first period the insurer may or may not verify reports of low incomes as well. If it is optimal to do so, all risk averse agents receive complete insurance in all periods, and low income reports are necessarily audited in the second period. If it is not optimal to engage in state verification in the first period, then partial insurance is provided at that date. Either complete insurance or no insurance may be provided in the second period. In the former

case, the insurer engages in some state verification. Moreover, an optimal insurance contract can be supported by having the insurer make some loans in the first period. These loans will typically be made at nonmarket rates.

In a deposit insurance context these results can be interpreted as follows. (1) The case for early or frequent auditing of banks is strong only when the cost of auditing is relatively low. When it is not, it is more efficient to use promised future rewards and punishments in order to provide banks with appropriate incentives. (2) When the costs of state verification are moderate or high [$\gamma \in (\hat{\gamma}, \bar{\gamma})$], the deposit insurer will want to provide only partial insurance in the first period. This insurance can be provided by offering banks access to a discount window. (3) For large levels of monitoring costs “deposit insurance” should be provided only through the use of the discount window. For intermediate levels of monitoring costs, both the discount window and conventional deposit insurance should be utilized, while for low levels of monitoring costs the discount window plays no role in an optimal insurance arrangement. Finally, we have shown why it may be optimal to compel banks to participate in a deposit insurance program.

Of course these results have been obtained under several strong assumptions: (i) agents have CARA utility functions, (ii) there are two dates, and (iii) there are two possible return states for any insured agent. The first assumption is definitely important to our results. Relaxing it is a natural topic for future investigation, although clearly such an investigation will have to have a significant numerical component. The second and third assumptions are of far less importance. We now briefly indicate how their relaxation would affect our analysis.

When there are T periods, with $2 \leq T < \infty$, then the results will be qualitatively very similar to those we report here. In particular, if monitoring costs are low, the insurer monitors low income reports and provides complete insurance at all dates. For intermediate levels of monitoring costs the insurer engages in *no* state verification for all $t \leq t^*$,¹⁷ and then verifies all reports of low incomes for $t = t^* + 1, \dots, T$. Partial insurance is provided prior to $t^* + 1$ (this can be done by extending loans at appropriate rates of interest), while complete insurance is provided for $t \geq t^* + 1$. Finally, for high levels of monitoring costs, the insurer never engages in state verification. Partial insurance is provided in all periods prior to T .

When $T = 2$ but the number of possible return states exceeds two, matters become

substantially more complicated. In particular, it seems that every possible combination of return states is a potential candidate for the set of possible verification states.¹⁸ Thus, for more than two return states, the number of possible verification strategies proliferates at a rapid rate. However, there are a few general things that can be said. First, with CARA utility, the optimal verification strategy in the second period will continue to be independent of the promised expected utility level in that period. As a result, Q will remain a concave function. Second, it will still be true that complete insurance will be provided in both periods when verification costs are low enough, and that no verification will take place in any period when verification costs are too high. In the latter circumstance partial insurance will be provided in the first period, while no insurance will be provided in the second. Third, it is still true that partial insurance can be provided through the discount window, although an optimal arrangement will generally have the property that the loan quantity extended and the interest rate charged will depend on the income level of the insured agent.

In addition to the three assumptions just discussed, it is important to note that we have abstracted from a number of features that are highly relevant to the provision of deposit insurance. One is the potential presence of serial correlation in the income of the insured agent. In the context of deposit insurance, it is often argued that the early and frequent auditing of banks—along with rapid action in the case of troubled banks (“prompt corrective action”)—is important in order to reduce the expected discounted present value of future losses to the FDIC. Implicit in this argument is the (natural) notion that bank portfolio performance displays positive serial correlation.

When there is serial correlation in the income of the insured agent, matters become substantially more complicated. However, it is possible to show that for intermediate levels of monitoring costs—that is, for monitoring costs consistent with no (some) first (second) period state verification, agents who report low first period incomes will *not* obtain complete insurance in the second period. Indeed, an optimal insurance arrangement for this pattern of state verification requires that agents be “punished” for reporting low first period incomes. Typically this punishment will include incomplete insurance provision in the second period, *except* when there is no serial correlation in income. Interestingly, such an arrangement cannot be supported through the use of a discount window alone; at a minimum a discount

window loan must be accompanied by some coinsurance of deposits in the second period. However, a complete analysis of the case of serial correlation must remain a topic for future investigation.

A second obvious feature of deposit insurance that we have abstracted from is the presence of moral hazard problems. In future work we hope to integrate a moral hazard problem into the current analysis. However, we do believe that our main conclusions will remain intact even in the presence of moral hazard. For example, unless monitoring costs are low, it will still be true that in early periods promises of expected future rewards and punishments can be used effectively to present banks with appropriate incentives. It is only in later periods that these incentive schemes cannot be used, potentially rendering auditing necessary. Thus we think that our conclusions about the relationship between verification costs and optimal auditing strategies will continue to be valid in the presence of moral hazard. Moreover, in the absence of auditing partial insurance (in early periods) will still be called for. Obviously this can be provided through the discount window. It seems, then, that our focus on costly state verification does raise the same issues that will be relevant for an analysis of moral hazard.

A final issue on which our analysis sheds some light is the following: is there a role for the government in the provision of deposit insurance? Our analysis suggests that deposit insurance can easily be provided by the private sector, *if* the following conditions are satisfied. (a) Private insurers have adequate resources to provide full insurance (as they would, for instance, if perfect diversification was possible). (b) It is not difficult for the market to enforce the optimal penalties associated with reports of low first period income levels. (c) It is not difficult to enforce compulsory participation in a private deposit insurance program. (d) Deposit insurance is not used as a device for subsidizing the banking system. When these conditions hold, private sector insurance provision is perfectly feasible. If any of them are violated, this strengthens these case for deposit insurance provision by the government.

Notes

¹Thus this is a repeated version of the costly state verification environment introduced by Townsend (1979). Note that we assume that agents take no actions to influence the probability distribution governing their income levels; we therefore abstract from considerations of moral hazard. Repeated moral hazard problems are examined by Phelan and Townsend (1991), Phelan (1994), and Wang (1997). We also assume that the probability distribution of each agent's income is known by the insurer, thereby abstracting from problems of adverse selection.

²See Williamson (1995) for an important exception to this statement. Williamson considers a model where the provision of deposit insurance *and* discount window services can be optimal in the presence of branching restrictions. This is the case because there may be a shortage of collateral suitable for use at the discount window. In our model there are neither branching restrictions, nor is collateral required at the discount window.

³Townsend (1982), in a model of a two period insurance relationship without costly state verification, also considered the possibility that insurance contracts might involve the extension of a loan. He showed that the extension of a loan by itself, with the insurer charging a market rate on the loan, could not be optimal. That is also the case here. However, a loan at nonmarket rates may be optimal when $\gamma \geq \bar{\gamma}$.

⁴It is not important for our purposes that these agents are identical in all respects, *ex ante*. However, this does economize on notation. Also, it is not important to the analysis whether the set of risk averse agents is large or small. We can readily consider the situation where the set of insured agents is finite, so long as we assume that the insurer has enough assets to provide complete insurance for all possible configurations of asset returns. In the context of deposit insurance—where seigniorage income is always a potential source of revenue—this seems very reasonable.

Finally, we note that the assumption of two periods is not at all essential to the results, so long as the insurer has a finite horizon. We elaborate on this point in the conclusion.

⁵A number of studies of economies with repeated private information problems have, at least in part, assumed CARA utility. See, for instance, Green (1987), Atkeson and Lucas (1992), Phelan (1994), and Wang (1997).

⁶As is well-known, stochastic state verification is associated with a number of contractual features that seem not to be observed in practice. For example, in a static insurance context, Mookherjee and Png (1989) show that the optimal contract associated with stochastic monitoring pays a “bonus” to an agent who is monitored and found to have reported truthfully. Thus agents who are monitored do better than otherwise identical agents who are not monitored. This does not seem to be a feature of many actual insurance contracts.

In a finite horizon model, these features must carry over, at least in the terminal period. We abstract from stochastic monitoring in order to avoid these implications. In addition, in a calibrated model of borrowing and lending with risk neutral agents, Boyd and Smith (1994) have suggested the gains from stochastic monitoring are likely to be small.

⁷We will focus throughout on contracts that induce truthful revelation of the state. Thus we do not distinguish notationally between the reported and the actual state.

⁸The use of this formulation is justified in Green (1987), Spear and Srivastava (1987), and Atkeson and Lucas (1992).

⁹Notice that we also impose no nonnegativity constraints on the income of the insurer. We thus de facto assume that the insurer either has adequate assets to provide essentially any level of transfers contemplated, or that the insurer can raise any resources required. The latter is probably a reasonable way to treat a government that provides deposit insurance, for example.

¹⁰If $R'(x) < (>)0$ holds, then (7') is satisfied iff $u \leq (\geq)w^*$, where w^* satisfies $R[\bar{\theta} - M(w^*)] \equiv \pi_1\gamma$. Thus the strategy $s(u) = \{\theta_1\}$ is preferred to the strategy $s(u) = \phi$ iff u is sufficiently low (high).

¹¹Clearly (14), (15), and (16) are simply (11), (12), and (13) rewritten using our transformed variables.

¹²When absolute risk aversion is not constant, the relative ranking of the strategies $s(w) = \phi$ and $s(w) = \{\theta_1\}$ will depend on w . With decreasing absolute risk aversion, it is easy to show that there is a critical value $\hat{w} < w^*$ such that $s(w) = \phi$ is preferred to $s(w) = \{\theta_1\}$ iff $w \geq \hat{w}$. Thus high promised expected utility levels render the “no verification” strategy superior to the “verify low income reports” strategy.

¹³In other words, it is never optimal to set $s(w) = \{\theta_2\}$ or $s(w) = \{\theta_1, \theta_2\}$.

¹⁴That is, if the risk averse agents could borrow as much as they wanted at the interest rate charged, they would generally want to borrow an amount different from the size of the loan they actually obtain.

¹⁵Note that the use of a discount window loan is only one among many possible devices for supporting an optimum. However, in CSV environments it is standard usage to refer to contracts that call for uncontingent future payments (in the absence of state verification) as debt contracts. Thus, according to the standard use of the term in the literature, the optimal insurance arrangement in this case calls for a “loan” in the first period, if the bank reports a low income level.

¹⁶With CARA preferences, the relative ranking of the strategies $s(u) = \phi$ and $s(u) = \{\theta_1\}$ in the second period is independent of u . Then, if $\gamma \leq (\geq) \bar{\gamma}$, there are only two possible configurations of verification strategies: (i) verify low income reports in both (only the first) period(s), and (ii) verify low income reports only in the first (in neither) period.

When absolute risk aversion is not constant, the choice of $u_s(\theta_i, w)$ determines whether or not low income reports are verified at $t = 2$. Thus whether or not verification occurs at $t = 2$ potentially depends on the realization of θ^1 . Obviously this proliferates the possibilities with respect to verification strategies. It is quite difficult to rank the various possible outcomes.

¹⁷We might also note that t^* is increasing in the cost of monitoring.

¹⁸Except that it is never necessary to verify the return in every state.

Appendix

A1. Proof of Lemma 1

(a) When v has the CARA form given in (1), $M(u)$ is defined by

$$-\rho^{-1} \sum \pi_i \exp\{\rho[M(u) - \theta_i]\} \equiv u. \quad (\text{A1})$$

Rearranging terms in (A1) gives equation (6).

(b) The definitions of $c(u)$ and $M(u)$ imply that

$$\begin{aligned} v[c(u)] &\equiv u \equiv \sum \pi_i v[\theta_i - M(u)] \equiv \sum \pi_i v[\bar{\theta} - M(u) + (\theta_i - \bar{\theta})] \\ &\equiv v\{\bar{\theta} - M(u) - R[\bar{\theta} - M(u)]\}. \end{aligned} \quad (\text{A2})$$

It is then immediate that $M(u) \equiv \bar{\theta} - c(u) - R[\bar{\theta} - M(u)]$.

(c) Differentiating

$$\sum \pi_i v[\theta_i - M(u)] \equiv u$$

with respect to u yields

$$M'(u) = - \left\{ \sum \pi_i v'[\theta_i - M(u)] \right\}^{-1} < 0. \quad (\text{A3})$$

Differentiating (A3) with respect to u , we obtain

$$M''(u) = -M'(u) \sum \pi_i v''[\theta_i - M(u)] / \left\{ \sum \pi_i v'[\theta_i - M(u)] \right\}^2 < 0. \quad (\text{A4})$$

This establishes the result. ■

A2. Proof of Lemma 2

(a) Consider the problem $\max \sum \pi_i M_1(\theta_i, u) - \pi_1 \gamma$ subject to (3). (This is problem (P₁) with the incentive constraint (4) omitted.) The solution to this problem sets $\theta_1 - M_1(\theta_1, u) = \theta_2 - M_1(\theta_2, u) = c(u)$. Since $M_1(\theta_2, u) > M_1(\theta_1, u)$ holds, (4) is satisfied. Hence this is also

the solution to (P₁). Moreover

$$\sum \pi_i M_1(\theta_i, u) - \pi_1 \gamma = \sum \pi_i [\theta_i - c(u)] - \pi_1 \gamma = \bar{\theta} - \pi_1 \gamma - c(u).$$

(b) With the CARA preferences in (1), $c(u)$ is defined by

$$-e^{-\rho c(u)}/\rho \equiv u. \tag{A5}$$

Rearranging terms in (A5) gives $c(u) = -[\ln \rho - \ln(-u)]/\rho$ as claimed.

(c) Differentiating $v[c(u)] \equiv u$ with respect to u yields $c'(u) = v'[c(u)]^{-1} > 0$. Differentiating again yields $c''(u) = -v''[c(u)]/v'[c(u)] > 0$. The claim then follows from part (a). ■

A3. Proof of Proposition 2

Rearranging terms in equation (8) yields

$$\bar{\gamma} \equiv [(1 - \pi_1)(\theta_2 - \theta_1)/\pi_1] + (\rho\pi_1)^{-1} \ln [\pi_1 + (1 - \pi_1)e^{-\rho(\theta_2 - \theta_1)}].$$

Then clearly

$$\partial \bar{\gamma} / \partial (\theta_2 - \theta_1) = (1 - \pi_1) / [\pi_1 + (1 - \pi_1)e^{-\rho(\theta_2 - \theta_1)}] > 0,$$

establishing part (a) of the proposition.

For part (b), it is straightforward to show that $\partial \bar{\gamma} / \partial \pi_1 < 0$ holds iff

$$\pi_1 [1 - e^{-\rho(\theta_2 - \theta_1)}] < [\pi_1 + (1 - \pi_1)e^{-\rho(\theta_2 - \theta_1)}] \ln [1 - \pi_1 + \pi_1 e^{\rho(\theta_2 - \theta_1)}]$$

is satisfied.

Now define the variables x and y by $x \equiv e^{-\rho(\theta_2 - \theta_1)}$ and $y \equiv x/\pi_1(1-x)$. Then obviously $x \in (0, 1)$ and $y > 0$, and to establish the proposition we need only show that

$$G(y) \equiv (1 + y)^{-1} - \ln[(1 + y)/y] < 0.$$

But clearly

$$\lim_{y \rightarrow \infty} G(y) = 0$$

and $G'(y) = 1/y(1+y)^2 > 0$ are both satisfied. Thus $G(y) < 0, \forall y > 0$ holds, completing the proof. ■

A4. Proof of Lemma 4

As noted in equation (9), $Q(u) \equiv \max\{\bar{\theta} - \pi_1 \gamma - c(u), M(u)\}$. The result then follows from equation (6), and part (b) of lemma 2. ■

A5. Proof of Proposition 3

As in the proposition, define the values m_i and u_i by

$$m_i \equiv -\exp[\rho M_0(\theta_i, w)]/\rho w \quad (\text{A6})$$

$$u_i \equiv u_0(\theta_i, w)/w \quad (\text{A7})$$

with $w < 0$. Then the problem (P'_0) is easily shown to be equivalent to the problem of choosing m_i and $u_i, i = 1, 2$, to solve

$$\max \sum \pi_i (\ln m_i + \beta \ln u_i) + (\ln \rho)/\rho + \beta A + [(1 + \beta)/\rho] \ln(-w) \quad (\tilde{P}'_0)$$

subject to (14)–(16) and $u_i \geq 0, i = 1, 2$. It is then immediate that

$$U_0(w) = \tilde{A} + (\ln \rho)/\rho + \beta A + [(1 + \beta)/\rho] \ln(-w).$$

We now characterize the solution to the problem (\tilde{P}'_0). We begin with several lemmas.

LEMMA A1. The solution to (\tilde{P}'_0) satisfies $m_2 \geq m_1$.

Proof. The constraints (15) and (16) imply that

$$e^{-\rho \theta_2} (m_1 - m_2) \geq \beta (u_2 - u_1) \geq e^{-\rho \theta_1} (m_1 - m_2). \quad (\text{A8})$$

The result then follows from $e^{-\rho\theta_1} > e^{-\rho\theta_2}$. ■

LEMMA A2. The constraint (16) binds in the problem (\tilde{P}'_0) .

Proof. Suppose not. Then it is easily verified that the solution to (\tilde{P}'_0) must have $u_1 = u_2$ and $e^{-\rho\theta_1}m_1 = e^{-\rho\theta_2}m_2$. But then (16) is violated, establishing the result. ■

LEMMA A3. The constraint (15) does not bind in the problem (\tilde{P}'_0) .

Proof. Lemma A2 establishes that the constraint (16) is binding. Therefore

$$e^{-\rho\theta_2}(m_2 - m_1) = \beta(u_1 - u_2) \quad (\text{A9})$$

holds. Moreover, the constraint (15) can be rewritten as

$$e^{-\rho\theta_1}(m_2 - m_1) \geq \beta(u_1 - u_2). \quad (\text{A10})$$

Lemma A1 asserts that $m_2 \geq m_1$ holds, while $e^{-\rho\theta_1} > e^{-\rho\theta_2}$. Thus satisfaction of (A9) implies the satisfaction of (A10). ■

We can now transform the problem (\tilde{P}'_0) as follows. Define the variable x by $x = m_2 - m_1$. Then satisfaction of (A9) requires that

$$e^{-\rho\theta_2}x/\beta = u_1 - u_2. \quad (\text{A11})$$

Solving (A11) for u_2 , and substituting the result into (14), we can write that constraint as a function of m_1 and u_1 alone:

$$1 \geq m_1 \left(\sum \pi_i e^{-\rho\theta_i} \right) + \beta u_1 \equiv \psi m_1 + \beta u_1. \quad (\text{A12})$$

In addition, we can write the objective function in (\tilde{P}'_0) as

$$\pi_1 \ln m_1 + \pi_2 \ln (m_1 + x) + \beta \pi_1 \ln u_1 + \beta \pi_2 \ln [u_1 - (e^{-\rho\theta_2}x/\beta)].$$

We now choose values m_1 , x , and u_1 to maximize this expression subject to (A12). The first order conditions associated with this problem can be manipulated to yield

$$(1 - \pi_1) m_1 (1 - \psi m_1) (\psi e^{\rho \theta_2} - 1) = \pi_1 (m_1 + x) [1 - \psi m_1 (1 + \beta)] \quad (\text{A13})$$

$$(1 + \beta) e^{-\rho \theta_2} x = 1 - (\beta e^{-\rho \theta_2} + \psi) m_1 \quad (\text{A14})$$

$$u_1 = (1 - \psi m_1) / \beta. \quad (\text{A15})$$

We now state the following result.

LEMMA A4. At an optimum, (a) $m_2 > m_1$ and (b) $m_2 < m_1 e^{\rho(\theta_2 - \theta_1)}$ hold.

Proof. (a) Clearly an optimum has $m_1 + x > 0$ and $u_1 > 0$. Equation (A15) then implies that $1 - \psi m_1 > 0$ holds. Moreover,

$$\psi e^{\rho \theta_2} \equiv \pi_1 e^{\rho(\theta_2 - \theta_1)} + \pi_2 > \pi_1 + \pi_2 = 1.$$

Then it is immediate from (A13) that $1 > \psi m_1 (1 + \beta)$.

We now note that—from equation (A14)— $x > 0$ holds iff

$$(1 - \psi m_1) (\psi e^{\rho \theta_2} - 1) > -[1 - \psi m_1 (1 + \beta)].$$

Our previous observations imply that this condition is necessarily satisfied. Thus $x \equiv m_2 - m_1 > 0$.

To establish part (b), we suppose to the contrary that

$$m_2 \geq m_1 e^{\rho(\theta_2 - \theta_1)} \quad (\text{A16})$$

holds at an optimum, and derive a contradiction. To do so, note that (A16) is equivalent to

$$x \equiv m_2 - m_1 \geq m_1 [e^{\rho(\theta_2 - \theta_1)} - 1]. \quad (\text{A16}')$$

It is straightforward but tedious to show that equations (A13) and (A16') imply the following

relation:

$$\beta\psi m_1 / (1 - \psi m_1) \geq \pi_1 + (1 - \pi_1)e^{-\rho(\theta_2 - \theta_1)}.$$

It is also possible to show that equations (A15) and (A16') imply that

$$\beta\psi e^{\rho\theta_2} / [(1 + \beta)e^{\rho(\theta_2 - \theta_1)} - 1] \geq \beta\psi m_1 / (1 - \psi m_1).$$

We then have our desired contradiction if

$$\pi_1 (1 - \pi_1) e^{-\rho(\theta_2 - \theta_1)} > \beta\psi e^{\rho\theta_2} / [(1 + \beta)e^{\rho(\theta_2 - \theta_1)} - 1]. \quad (\text{A17})$$

But (A17) is easily shown to be equivalent to

$$\pi_1 [e^{\rho(\theta_2 - \theta_1)} - 1] + (1 - \pi_1) [1 - e^{-\rho(\theta_2 - \theta_1)}] > 0,$$

a condition which is obviously satisfied. This establishes the lemma. ■

We can now prove parts (a) and (b) of proposition 3. Since $w < 0$, $m_2 > m_1$ is equivalent to $M_0(\theta_2, w) > M_0(\theta_1, w)$. In addition, $\theta_2 - M_0(\theta_2, w) > \theta_1 - M_0(\theta_1, w)$ is easily shown to be equivalent to $m_2 < m_1 e^{\rho(\theta_2 - \theta_1)}$. That this condition holds was established in lemma A4. This completes the proof of proposition 3. ■

A6. Proof of Proposition 4

Consider the less constrained problem

$$\max \sum \pi_i \{M_1(\theta_i, w) + \beta Q[u_1(\theta_i, w)]\} - \pi_1 \gamma$$

subject to (11) and $u_1(\theta_i, w) \leq 0$. The solution to this problem sets $M_1(\theta_i, w) = \theta_i - c[w/(1 + \beta)]$ and $u_1(\theta_i, w) = w/(1 + \beta)$, $i = 1, 2$. Clearly this satisfies (12). Thus this is also the solution to the problem (P'_1). This establishes parts (a) and (b) of the proposition.

(c) Obviously

$$\begin{aligned}
U_1(w) &= \sum \pi_i \{M_1(\theta_i, w) + \beta Q[u_1(\theta_i, w)]\} - \pi_1 \gamma \\
&= \sum \pi_i \theta_i - c[w/(1 + \beta)] + \beta Q[w/(1 + \beta)] - \pi_1 \gamma \\
&= \bar{\theta} - \pi_1 \gamma - c[w/(1 + \beta)] + \beta A + (\beta/\rho) \ln[-w/(1 + \beta)] \\
&= \bar{\theta} - \pi_1 \gamma - [(1 + \beta)/\rho] \ln(1 + \beta) + (\ln \rho)/\rho + \beta A \\
&\quad + [(1 + \beta)/\rho] \ln(-w)
\end{aligned}$$

where the last inequality follows from part (b) of lemma 2. This establishes the proposition. ■

A7. Proof of Proposition 5

(a) A comparison of equations (17) and (18) indicates that $U_1(w) \geq U_0(w)$ holds iff

$$\begin{aligned}
&\bar{\theta} - \pi_1 \gamma - [(1 + \beta)/\rho] \ln(1 + \beta) \\
&\quad + (\ln \rho)/\rho + \beta A + [(1 + \beta)/\rho] \ln(-w) \\
&\geq \tilde{A} + (\ln \rho)/\rho + \beta A + [(1 + \beta)/\rho] \ln(-w)
\end{aligned} \tag{A18}$$

is satisfied. Clearly (A18) reduces to

$$\pi_1 \gamma \leq \bar{\theta} - \tilde{A} - [(1 + \beta)/\rho] \ln(1 + \beta) \equiv \pi_1 \hat{\gamma}. \tag{A18'}$$

This establishes part (a) of the proposition.

(b) A comparison of equations (A18') and (8) indicates that $\hat{\gamma} < \bar{\gamma}$ holds iff

$$\left[\ln \left(\sum \pi_i e^{-\rho \theta_i} \right) \right] / \rho > -\tilde{A} - [(1 + \beta)/\rho] \ln(1 + \beta). \tag{A19}$$

Now it is feasible in the problem that defines \tilde{A} (see proposition 3) to set $u_1 = u_2 = (1 + \beta)^{-1}$ and $m_1 = m_2 = 1/(1 + \beta) \left(\sum \pi_i e^{-\rho \theta_i} \right)$, while lemma A4 establishes that this is strictly suboptimal. Thus

$$\tilde{A} > - \left[\ln \left(\sum \pi_i e^{-\rho \theta_i} \right) \right] + [(1 + \beta)/\rho] \ln(1 + \beta) \tag{A20}$$

holds. But (A20) implies that (A19) is satisfied. This establishes part (b) of the proposition.

(c) Define the variables \tilde{m}_i and \tilde{u}_i , $i = 1, 2$, by $\tilde{m}_i \equiv m_i e^{-\rho\theta_i}$ and $\tilde{u}_i = \beta u_i$. Define \tilde{B} by

$$\tilde{B} = \max_{m_i, u_i} \sum \pi_i [\ln \tilde{m}_i + \beta \ln \tilde{u}_i]$$

subject to

$$\tilde{m}_2 + \tilde{u}_2 = \tilde{m}_1 e^{-\rho(\theta_2 - \theta_1)} + \tilde{u}_1$$

$$\sum \pi_i (\tilde{m}_i + \tilde{u}_i) \leq 1.$$

Then it is readily verified that

$$\hat{\gamma} \equiv \left[\beta \ln \beta - (1 + \beta) \ln(1 + \beta) - \tilde{B} \right] / \rho \pi_1,$$

and clearly $\partial \hat{\gamma} / \partial (\theta_2 - \theta_1) = -(\rho \pi_1)^{-1} \partial \tilde{B} / \partial (\theta_2 - \theta_1) > 0$. This completes the proof of part (c). ■

A8. Proof of Lemma 5

Part (a) follows immediately from the observation that only the constraint (16) [(13)] is binding in the problem (\tilde{P}'_0) [(P'_0)]. Part (b) is then obvious. ■

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Figure 1. The Function $Q(u)$; $R' < 0$

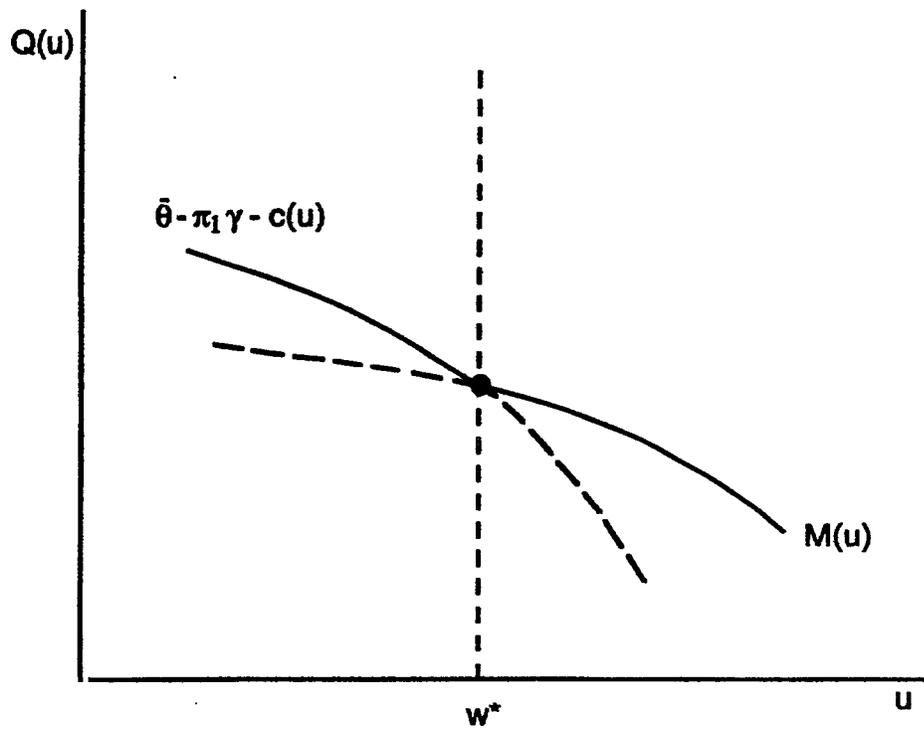


Figure 2. The Function $Q(u)$; $R' > 0$

