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Sharing the Risk of Settlement Failure*

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ABSTRACT

Two policies toward payments-system risk are common, but superficially appear to be contradictory. One policy is to restrict the exposure to risk generated by one participant to other participants who are, by one measure or another, directly concerned with the risky participant. The other policy is to provide a "safety net," typically provided by government and funded by taxes collected from all participants and even from non-participants, to share losses due to "systemic risk." In this paper, we provide a model in which both of these policies can be constituents of an economically efficient regime of payments-risk management.

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1 Introduction

Large-value payments are typically made through continuing, multi-party, contractual, clearing and settlement arrangements. During the past several decades, there has been progressively increasing awareness of the importance of risk management in such arrangements. Because a very large loss can potentially be incurred if settlement of a payment fails, how such a loss would be shared should be a matter of substantial concern for the participants in an arrangement. Moreover, to the extent that complete contingent-claims markets do not exist for insurance against settement failures and that there are political pressures for governments or central banks to assume losses from such failures, management of settlement risk is also a public policy issue.

Some specific questions regarding risk management in a settlement arrangement are the following. If there is some risk of failure to settle a payment from one party to another, should the payment be settled through that arrangement? (For example, in a net-settlement arrangement, what is the level of risk at which a payment ought to be made instead through an alternative, real-time-gross-settlement, arrangement?) If so, then what considerations are relevant to determining whether third parties ought to share that risk? Are there conditions under which the general public or the central bank (in the case of a private arrangement) ought to bear some risk and, if so, what level of compensation would it be approprate for them to receive? If a third party possesses private information that would be of value in determining how best to settle a payment, how does the exposure of that party to the settlement risk affect the quality of information that the party chooses to provide? In this paper, we address these questions by analyzing a schematic, formal model of a settlement arrangement.

Settlement-arrangement designers, managers, and policy makers are well aware that the rules governing an arrangement can affect users' decisions about which transactions to make through the arrangement. Thus, to set the rules of an arrangement is implicitly to decide which payments will be settled through it, and which payments people will decide to settle in alternative ways. (In fact, rules governing an arrangement that lacks stringent risk controls are sometimes designed deliberately to make the arrangement infeasible or unattractive for use in making very large-value payments.) By modelling the cooperative setting of rules by participants in a settlement arrangement, and by participants in the economy as a whole, from this perspective, we are able to analyze welfare questions in a conceptually satisfactory way. Rather than taking that approach of specifying transactions exogenously as previous researchers have typically done, what we take to be exogenous are traders' utility functions, which we specify in a way that provides scope for welfare-improving transactions among some of the traders to occur. We also specify a settlement technology that imputes risks and costs to those potential transactions. Having specified the model in these terms, we are able to characterize the patterns of transactions that the traders would cooperatively choose to make.

This approach provides answers, for the class of model economies that we study, to the questions posed above. Not surprisingly, risk considerations play a role in determining which payments ought to be made. Under some conditions, even the general public (that is, traders who would not have transactions with the members of the settlement arrangement if risk were not present) ought to share settlement risk, as can happen in practice when a central bank serves as guarantor of a settlement arrangement. Private information regarding risk, even when it is possessed by a third party rather than by a direct party to a payment, is likely to be untruthfully reported unless the settlement arrangement is deliberately designed to elicit the truth. While these results about a schematic model economy are far from constituting definitive advice regarding actual settlement arrangements, we hope that this analysis may at least provide a helpful framework within which to think in an organized way about the issues involved in practical cases.

2 Modelling a transaction

Our first task is to formulate a model of a transaction that involves a risky asset transfer. The model should be rich enough to describe such a transaction recognizably, but simple enough to be analytically tractable.

Consider what sort of model could satisfy both the requirements of richness and simplicity. A transaction is a related set of asset transfers between traders. The assets involved might be either commodities or financial assets. An asset transfer involves two traders, the donor and the recipient, but a transaction can generally involve more than two traders. Therefore, at the very least, a model of a transaction involving a risky transfer should include three traders, so that a distinction can be drawn between a participant in the broad transaction and a participant (that is, the donor or the recipient) in the specific transfer where the risk occurs. In order for the third-party participant in the transaction—that is, the participant who is neither the donor nor the recipient of the risky transfer—to be essential to making a mutually beneficial transaction, there should be no "double coincidence of wants" between the donor and the receiver. This consideration suggests modelling the three participants as a "Wicksell triangle."

A good model ought to capture a distinction between two types of third party (or potential third party). A third party to the risky transfer in a Wicksell triangle might be intrinsically necessary in the sense that the donor and recipient of the risky transfer would have no double coincidence of wants, even if the transfer did not involve risk (that is, if the recipient would receive the expected value of the transfer with certainty). Alternatively, the riskiness of the transfer might impair a double coincidence of wants that would exist under certainty between the donor and the recipient, and the third party might be needed solely to restore that double coincidence by serving as a guarantor or insuror of the transfer. For characterizing the differences between the roles of these two types of third parties, a four-trader model (including both an intrinsic third party and a trader whose only involvement would be to share risk) can be useful. On the basis of these considerations, we will specify the set of traders to be $\{1, 2, \ldots, N\}$, where either $N = 3$ or $N = 4$. In either case, we will assume that trader 1 is essential to a mutually beneficial transaction but that trader 2 is the donor and trader 3 is the recipient of the risky transfer. When a four-trader economy is considered, the attributes of trader 4 will be specified in such a way that trader 4 can only participate in a risk-sharing capacity.

The risky transfer will be formalized in terms of a state space, Ω. An algebra of events (that is, subsets of Ω) is assumed to exist, and a probability measure Pr is defined on the algebra.

There is a distinguished event $S \subset \Omega$, with $0 < \Pr(S) < 1$. Assume that the risky transfer from trader 2 to trader 3 succeeds in S, and that it fails in the complementary event $F = \Omega \backslash S$. When we say that the transfer succeeds, we mean that trader 3 receives the entire quantity of the asset that is transferred. When we say that the transfer fails, we mean that the quantity of the asset that was intended to be transferred disappears irretrievably from the economy.[1]

Later, to analyze incentive issues, we will specify that trader 1 privately observes an event that is statistically relevant to the outcome of that risky transfer. This private information will be formalized in terms of events in the probability space Ω, but we defer presenting that formalization until it is needed.

Assume that each trader i has an endowment consisting solely of a type of commodity that only he possesses. We denote that type of commodity also by i. Intuitively, trader i is endowed with one unit of commodity of type i with certainty. In order to discuss state-contingent trade of these endowments, we adopt Arrow's convention that each type of commodity is a class of state-contingent commodities, one for each state in Ω. Thus the set of all commodities is $N \times \Omega$. Each trader i is endowed with one unit of commodity (i, ω) for every $\omega \in \Omega$.

A commodity bundle is represented by a measurable function $\gamma: N \times \Omega \to \mathbb{R}_+$. This definition is conventional, since $N \times \Omega$ is the commodity space.

Each trader's preference between commodity bundles conforms to expected utility. Trader i has a von Neumann-Morgenstern utility function $U^i: \mathbb{R}_+^N \to \mathbb{R} \cup \{-\infty\}$. Trader i's expected utility of consuming a commodity bundle γ is the expectation of the random variable $U^i(\vec{\gamma})$, where $\vec{\gamma}: \Omega \to \mathbb{R}_+^N$ is defined by

$$\forall \omega \in \Omega \quad \vec{\gamma}(\omega) = (\gamma(1, \omega), \ldots, \gamma(N, \omega)) \tag{1}$$

The sequence of economic activities in this economy is as follows.

Initially, before knowing whether the actual state of nature is in S or F, traders make an agreement for transfers of goods among them. The agreement among the traders is binding.

With one exception, the transfers are safe. That is, everything sent out reaches its intended recipient in its entirety and with certainty. The exception is the transfer of trader 2's endowment to trader 3. Recall that this transfer reaches trader 3 in its entirety in event S, but is completely and irretrievably lost in event F.

The traders also agree *ex ante* on a second round of transfers, to be made after the first transfers have been completed and the result of the risky transfer has become known. Thus the transfer to be made in the second round can be made contingent on which of the events S and F has occurred.[2]

[1]Failure of an actual transfer seldom involves such an irretrievable loss, although there are some contemporary examples and many historical examples of that type of failure.

[2]Strictly speaking, this sentence describes a different information structure from the preceding one. If traders can only distinguish between events S and F on the basis of observing the success or failure of a transfer, then they can not make any distinction unless a (non-zero) transfer has been attempted. To assume that they can make a state-contingent transfer in the second round even if no first-round transfer from 2 to 3 has been attempted neglects this limitation of their opportunity for inference. In the case where there is no private information, this ambiguity is harmless because risk-averse traders would not cooperatively choose to make a state-contingent transfer in the second round unless they had exposed themselves to settlement risk in the first round. How the ambiguity is resolved is important in the private-information case, though, and we will discuss this issue further when we analyze private information.

All second-round transfers, including the one from trader 2 to trader 3, are nonstochastic. However, second-round transfers are costly. Only a proportion $\rho < 1$ of the goods that a trader sends in the second round are received.[3]

Traders consume their stocks of goods after these two rounds of transfers have been completed. To simplify the characterization of traders' consumption resulting from settlement, we make two assumptions: that a trader is able to transfer only his own endowment good, and that only a few of the possible flows of those goods are feasible. Specifically a *round of transfers* is a vector ϕ. If $N = 3$, then $\phi \in \mathbb{R}^3_+$. If $N = 4$, then $\phi \in \mathbb{R}^5_+$. The coefficients of ϕ are interpreted as follows.

1. ϕ_1 is the amount sent from trader 1 to trader 2;
2. ϕ_2 is the amount sent from trader 2 to trader 3;
3. ϕ_3 is the amount sent from trader 3 to trader 1;
4. ϕ_4 is the amount sent from trader 4 to trader 3;
5. ϕ_5 is the amount sent from trader 3 to trader 4.

As described above, either all, a proportion ρ, or none of the goods sent may be received. A *transaction* is a sequence $\tau = (\tau^1, \tau^S, \tau^F)$ of rounds of transfers. The elements τ^1, τ^S, and τ^F specify the initial round of transfers, the round of transfers in event S, and the round of transfers in event F, respectively.

A transaction is feasible if no trader is ever required to send a cumulative amount that would exceed his endowment. That is, transaction τ is feasible if

$$\forall i \;\; \tau_i^1 + \max\{\tau_i^S, \tau_i^F\} \leq 1. \tag{2}$$

Let $\mathcal{T}$ denote the set of feasible transactions.[4]

Now we provide an explicit definition of traders' consumptions resulting from a transaction. The following table specifies vectors z^1–z^5 in terms of which these consumptions are defined.

	$N = \{1,2,3\}$	$N = \{1,2,3,4\}$
z^1	$(1,0,0)$	$(1,0,0,0)$
z^2	$(0,1,0)$	$(0,1,0,0)$
z^3	$(0,0,1)$	$(0,0,1,0)$
z^4	$(0,0,0)$	$(0,0,0,1)$
z^5	$(0,0,0)$	$(0,0,1,0)$

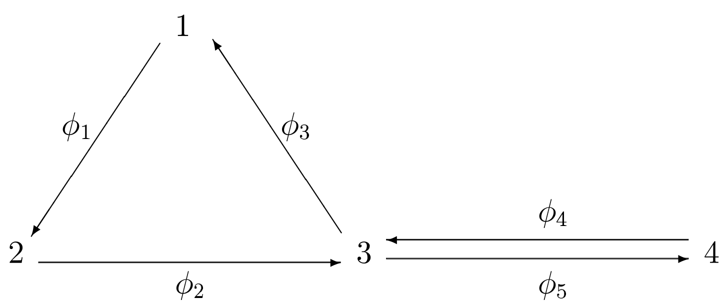
Furthermore, let χ_S and χ_F denote the indicator functions of S and F respectively, and define $\tau^\chi(\omega) = \chi_S(\omega)\tau^S + \chi_F(\omega)\tau^F$.

The consumption vector $c^i(\tau, \omega)$ that trader i receives in state ω as a consequence of transaction τ is as follows.

[3] This assumption, sometimes called "iceberg cost," can be viewed as a crude way of reflecting various intuitive considerations including time preference and exposure to business loss due to delayed availability of transferred funds.

[4] As noted in the footnote above, the informational constraint that, if $\tau_1^2 = 0$, then $\tau^S = \tau^F$, may or not be added to the definition of feasibility for a transaction. If all traders are risk averse, then the constraint is never binding when traders have common information.

Figure 1: Round of Transfers



$$
\begin{array}{rcl}
c^1(\tau,\omega) & = & (1-(\tau_1^1+\tau_1^\chi(\omega))z^1+(\tau_3^1+\rho\tau_3^\chi(\omega))z^3 \\
c^2(\tau,\omega) & = & (1-(\tau_2^1+\tau_2^\chi(\omega))z^2+(\tau_1^1+\rho\tau_1^\chi(\omega))z^1 \\
c^3(\tau,\omega) & = & (1-(\tau_3^1+\tau_3^\chi(\omega))z^3-(\tau_5^1+\tau_5^\chi(\omega))z^5) \\
 & & \quad +(\chi_S(\omega)\tau_2^1+\rho\tau_2^\chi(\omega))z^2+(\tau_4^1+\rho\tau_4^\chi(\omega))z^4 \\
c^4(\tau,\omega) & = & (1-(\tau_4^1+\tau_4^\chi(\omega))z^4+(\tau_5^1+\rho\tau_5^\chi(\omega))z^5
\end{array}
\tag{3}
$$

3 The core

We modify the core of an exchange economy to serve as the solution concept to characterize the set of mechanisms to which the traders might agree. A core allocation is one that can be obtained (according to (3)) by a feasible transaction, and such that no coalition of traders can implement another allocation that its members unanimously prefer–with at least one of them having a strict preference–by using an alternative transaction that is feasible for its members. Define a core transaction to be a feasible transaction from which a core allocation is obtained via (3).

To formalize the notion of unanimous preference within a coalition, for each nonempty $C \subseteq \{1,\ldots,N\}$, define $\theta \in \mathcal{T}$ to *C-dominate* $\tau \in \mathcal{T}$ if

$$\begin{aligned} &\forall i \in C \quad \mathsf{E}[U^i(c^i(\tau,\omega))] \leq \mathsf{E}[U^i(c^i(\theta,\omega))] \quad \text{and} \\ &\exists i \in C \quad \mathsf{E}[U^i(c^i(\tau,\omega))] < \mathsf{E}[U^i(c^i(\theta,\omega))]. \end{aligned} \tag{4}$$

Also define $\theta \in \mathcal{T}$ to be *feasible for* C if

$$\forall i \notin C \quad \forall \omega \quad c^i(\theta,\omega) = z^i \qquad \text{(No participation of other traders is required).} \tag{5}$$

Finally, define $\tau \in \mathcal{T}$ to be a *core transaction* if there exist no $C \subseteq N$ and $\theta \in \mathcal{T}$ such that θ is feasible for C and θ C-dominates τ.

Let us say that transaction τ is *individually rational* if it is weakly preferred to autarky by every $i \in N$, and that τ is *Pareto-undominated* if it is undominated for N.

Proposition 1 *Let each trader's utility function be locally nonsatiated, at all points, in his own endowment good. Then a feasible transaction τ is a core transaction if and only if the following conditions hold: τ is individually rational, Pareto-undominated, and not either $\{1,2,3\}$-dominated or $\{3,4\}$-dominated.*

Proof A core transaction must satisfy the conditions by definition. (It must be individually rational because autarky is feasible for each individual trader.) Conversely, suppose that a transaction $\tau \in \mathcal{T}$ is individually rational, Pareto-undominated, $\{1,2,3\}$-undominated, and $\{3,4\}$-undominated. The only coalitions C for which τ could be C-dominated without explicitly violating one of the three conditions are then

1. if $N = \{1,2,3,4\}$, then those coalitions consisting of trader 4 together with either trader 1 or trader 2; and
2. regardless of N, those coalitions to which exactly two members of $\{1,2,3\}$ belong.

Call these type-1 and type-2 coalitions respectively.

We will now show that no transaction θ can C-dominate any individually rational transaction τ for a coalition of either type 1 or type 2. C cannot be of type 1 because only the trivial transaction mechanism (that is, autarky) is feasible for $\{1,4\}$ or $\{2,4\}$. That is, θ cannot satisfy condition (4) for either of those two coalitions, since τ is individually rational.

Now suppose that C is of type 2. That is, either $C = \{a,b\} \subseteq \{1,2,3\}$, or else $C = D \cup \{4\}$ and $D = \{a,b\} \subseteq \{1,2,3\}$. In the former case, without loss of generality, $a = b+1 (\text{mod } 3)$. The only trade that can occur in any state of nature between these two traders is for a to receive some of b's endowment good. This must happen with positive probability, in order for (4) to be satisfied for a. In that case, though, (4) cannot be satisfied for b. The same argument applies in the latter case, unless b is trader 3 who receives some of the endowment good of trader 4 (and a is trader 1). If so, define ξ to be the mechanism that specifies the same transfers as θ between traders 3 and 4, but that specifies that 3 should give nothing to 1. Trader 3 strictly prefers ξ to θ, and trader 4 is indifferent between ξ and θ. Therefore, if θ were to C-dominate τ, then ξ would $\{3,4\}$-dominate τ, contrary to hypothesis. ∎

4 Analysis of a public information environment

It will be useful to carry through our analysis using specific utility functions to show why the preference and private information do matter in the settlement system.

To this end, we study core transactions in some parametric versions of the economic environment defined above. We begin with a simple environment, where $N = \{1, 2, 3, 4\}$ and there is no private information. 1,2, and 3 are the essential parties and 4 is the stand-by party to transactions. We specify the traders' utilities as follows.

$$\begin{aligned} U^1(c) &= \ln(c_1 + \beta c_3) \\ U^2(c) &= \ln(c_2 + \beta c_1) \\ U^3(c) &= \ln(c_3 + \beta c_2 + \psi c_4) \\ U^4(c) &= \ln(c_4 + \varphi c_5) \\ &\quad \text{with } \beta > \max\{\sigma^{-1}, \rho^{-1}\}, 0 < \varphi\psi < 1. \end{aligned} \tag{6}$$

Here, goods received in trade are "better" substitutes for endowment goods for essential participants 1,2,3. Trader 4 considers trader 3's good to be a "worse" substitute for his own endowment good, and trader 3 considers 4's good to be a "worse" substitute for trader 2's good or even for his own endowment good. We assume that the transfer technology to satisfy $0 < \rho \leq \sigma < 1$ and $\sigma > 1/2$.[5]

By an abuse of notation, in this section we write $\mathsf{E}U^i(\tau)$ instead of $\mathsf{E}U^i\left(\psi_i(\tau, \omega)\right)$ for the expected utility of trader i for a given transaction τ. Hence,

$$\begin{aligned} \mathsf{E}U^i(\tau) &= \sigma \ln\left(1 - \tau_i^1 - \tau_i^S + a_{i-1}(\tau_{i-1}^1 + \rho\tau_{i-1}^S)\right) \\ &\quad + (1-\sigma)\ln\left(1 - \tau_i^1 - \tau_i^F + a_{i-1}(\tau_{i-1}^1 + \rho\tau_{i-1}^F)\right) \end{aligned} \tag{7}$$

for i =1,2,4 where $i - 1 = 3$ for $i = 1$ and $i - 1 = 5$ for $i = 4$, and $a_i = \beta$ for i =1,2,3, $a_4 = \psi$, and $a_5 = \varphi$. For $i = 3$, we have

$$\begin{aligned} \mathsf{E}U^3(\tau) &= \sigma \ln\left(1 - \tau_3^1 - \tau_3^S + \beta(\tau_2^1 + \rho\tau_2^S) - \tau_5^1 - \tau_5^S + \psi(\tau_4^1 + \rho\tau_4^S)\right) \\ &\quad + (1-\sigma)\ln\left(1 - \tau_3^1 - \tau_3^F + \beta\rho\tau_2^F - \tau_5^1 - \tau_5^F + \psi(\tau_4^1 + \rho\tau_4^F)\right). \end{aligned} \tag{8}$$

When a transacion τ is clear from the context, we may write for simplicity

$$\begin{aligned} C_i^S &= 1 - \tau_i^1 - \tau_i^S + a_{i-1}(\tau_{i-1}^1 + \rho\tau_{i-1}^S), \\ C_i^F &= 1 - \tau_i^1 - \tau_i^F + a_{i-1}(\tau_{i-1}^1 + \rho\tau_{i-1}^F) \end{aligned} \tag{9}$$

for $i = 1, 2, 4$ and

[5] The condition for a small transfer at the endowment allocation, using the safe technology, to increase the sum of utilities of the two traders is that $\beta\rho > 1$. Thus, the intuitive meaning of the latter assumption is that traders would have clear willingness to use the safe technology if it were the only transfer technology available.

$$\begin{aligned} C_3^S &= 1 - \tau_3^1 - \tau_3^S + \beta(\tau_2^1 + \rho\tau_2^S) - \tau_5^1 - \tau_5^S + \psi(\tau_4^1 + \rho\tau_4^S), \\ C_3^F &= 1 - \tau_3^1 - \tau_3^F + \beta\rho\tau_2^F - \tau_5^1 - \tau_5^F + \psi(\tau_4^1 + \rho\tau_4^F). \end{aligned} \tag{10}$$

C_i^A is interpreted as "real" consumption level of trader i in event A in the sense that it directly determines i's utility level in event A.

Throughout this section, then, we suppose that $N = \{1, 2, 3, 4\}$, and traders' utilities are parametrized by the equations (6). We begin by establishing a series of properties to be satisfied by core transactions that simplify the characterization of core transactions.

We first want to show that a transaction is in fact endogenously generated in our model. In other words we shall show that if $\tau = (\tau^1, \tau^S, \tau^F)$ is a transacition which is identically equal to zero, then τ is dominated. Thus, the following lemma will guarantee the nontriviality of considering endogenous transactions using the utility functions given by (6).

Lemma 1 *Let* $\tau = (\tau^1, \tau^S, \tau^F)$ *be a feasible transaction such that* $\min\{\epsilon_i \mid i = 1, \cdots, 4\} > 0$ *where*

$$\begin{aligned} \epsilon_i &= 1 - \tau_i^1 - \max\{\tau_i^S, \tau_i^F\} \text{ for } i = 1, 2, 4 \\ \epsilon_3 &= 1 - \tau_3^1 - \tau_5^1 - \max\{\tau_3^S + \tau_5^S, \tau_3^F + \tau_5^F\}. \end{aligned}$$

Assume that $\beta\rho \geq \sqrt[3]{2}$. *Then,* τ *is {1,2,3}-dominated. In particular, if* $\tau = 0$, *then* τ *is {1,2,3}-dominated. In other words, a transaction* τ *which is not {1,2,3}-dominated generates a transaction. Furthermore, at least one trader among the essential participants to transaction must be sending all of his endowment to others, i.e.,* $\min\{\epsilon_i \mid i = 1, 2, 3\} = 0$.

If the cost of a second round transfer $1 - \rho$ is no greater than the cost of risk $1 - \sigma$ of a second round transfer so that $\rho \geq \sigma$, then it is immediate that second round transfers dominate first round transfers. Now, assume that the second round transfer cost is higher than the cost of risk in transfer from 2 to 3 so that $0 < \rho < \sigma$. Then, it is clear that among those who do not face direct transfer risk, *i.e.*, i= 1,2,4,5, [6] second round transfers, should they be effected, involve transfers in the event F or S but not in both events.

Lemma 2 *Assume* $0 < \rho < \sigma$. *Let* τ *be a feasible transaction such that* $\tau_i^S \tau_i^F > 0$ *for some* $i \in \{1, 2, 4, 5\}$. *Then,* τ *is* $\{i, i+1\}$*-dominated* [7] *where, for* $i = 5, i+1$ *is understood to be* 4.

By the basic lemma 1 of a transaction, at least one trader among the essential participants {1,2,3} to transactions must be sending all of his endowment if the transaction is undominated. We shall show next that regardless of who is sending his entire endowment to other participants, trader 2 must be sending at least some of his endowment to trader 3.

[6] It is convenient to refer to trader 3 as trader $i = 5$ when we look at his transfer to trader 4.

[7] Here again, {4,5}-dominated means {4,3}-dominated.

Lemma 3 *Let τ be a feasible transaction which is individually rational and {1,2,3}-undominated. Then, trader 2 must be sending some of its endowments to trader 3, i.e.,*

$$\max\{\tau_2^1, \tau_2^S, \tau_2^F\} > 0.$$

We now wish to check some of the further details of undominated transactions. For this purpose we like to derive conditions under which a change in transfer from a trader or traders induces another feasible transaction that can dominate a given transaction.

Let $\epsilon, \eta_i^S, \eta_i^F$ be real numbers in a neighborhood of zero. For $i = 1, \cdots, 5$, and a real number t in a neighborhood of zero, define

$$\tau_{ti}^1 = \tau_i^1 + t\epsilon, \tau_{ti}^S = \tau_i^S + t\eta_i^S, \tau_{ti}^F = \tau_i^F + t\eta_i^F. \tag{11}$$

Let τ_t^i be a transaction such that [8]

$$\begin{aligned} \tau_{ti}^i &= (\tau_{ti}^1, \tau_{ti}^S, \tau_{ti}^F) \quad \text{and} \\ \tau_{tj}^i &= (\tau_j^1, \tau_j^S, \tau_j^F) \quad \text{for} \quad j \neq i \end{aligned} \tag{12}$$

so that in τ_t^i only trader i's transfers are changed.

For any ϵ (could be either positive or negative) in a neighborhood of zero, we set either η_i^S or η_i^F or both so as to make

$$\frac{d}{dt}\mathsf{E}U^i(\tau_t^i)(0) = 0. \tag{13}$$

We then look at a change in expected utility of trader $i+1$ resulting from a shift in transfer induced by ϵ, η_i^S, and η_i^F, where $i+1$ is 1 for $i = 3$, and 4 for $i = 5$. In other words, we change a transfer or transfers of a trader so as to keep his expected utility unchnaged, and see under what conditions the receiver's expected utility is increased. If that happens, the donor and the receiver can dominate the original transaction. We will check these conditions trader by trader.

4.1 Effects of a change in transfers of trader $i = 2$

We first establish properties concerning changes in transfers of trader $i = 2$.

Lemma 4 *Let τ be a feasible transaction. Then, for a change in transfers of trader 2 as defined by (11), (12), and (13), we have the following.*

1. *If we consider a simultaneous change in second round transfers in both events F and S by a same amount so that $\eta = \eta_2^F = \eta_2^S$, then:*

$$sgn\left[\frac{d}{dt}\mathsf{E}U^3(\tau_t^2)(0)\right] = sgn[\epsilon]sgn\left[\frac{C_3^F}{C_3^S} - r^{\sigma\rho}\right] \tag{14}$$

where

$$r^{\sigma\rho} = \left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\rho}{1-\rho}\right). \tag{15}$$

[8] Here, there is an abuse of notation again since, for i=1, we have $\tau_{t1}^1 = (\tau_{t1}^1, \tau_{t1}^S, \tau_{t1}^F)$. But no confusion should arise in the context of our arguments below.

2. If the second round transfer in event F, τ_2^F, is held constant so that $\eta_2^F = 0$, then:

$$sgn\left[\frac{d}{dt}\mathsf{E}U^3(\tau_t^2)(0)\right] = sgn[\epsilon]sgn\left[\frac{C_2^F}{C_2^S} - r^{\sigma\rho}\right] . \tag{16}$$

3. If the second round transfer in event S, τ_2^S, is held constant so that $\eta_2^S = 0$, then:

$$\begin{aligned} sgn\left[\frac{d}{dt}\mathsf{E}U^3(\tau_t^2)(0)\right] &= sgn[\epsilon]sgn\left[\frac{C_3^F}{C_3^S} - \rho\left(\frac{1-\sigma}{\sigma}\right) - \rho\frac{C_2^F}{C_2^S}\right] \\ &= sgn[\epsilon]sgn\left[\frac{C_3^F}{C_3^S} - r_2^{\sigma\rho}\right] \end{aligned} \tag{17}$$

where

$$r_i^{\sigma\rho} = \left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\rho}{1-\rho R_i}\right), \tag{18}$$

$$R_i = \frac{C_i^F/C_i^S}{C_{i+1}^F/C_{i+1}^S} . \tag{19}$$

4. If the first round transfer, τ_2^1, is held constant so that $\epsilon = 0$, then:

$$\begin{aligned} sgn\left[\frac{d}{dt}\mathsf{E}U^3(\tau_t^2)(0)\right] &= sgn[\eta_2^S]sgn[1 - R_2] \\ &= sgn[\eta_2^F]sgn[R_2 - 1] . \end{aligned} \tag{20}$$

One may note that

$$\frac{\sigma C_3^F}{(1-\sigma)C_3^S} = \frac{\sigma(1/C_3^S)}{(1-\sigma)(1/C_3^F)}$$

is trader 3's expected marginal rate of substitution of consumption in event F for consumption in event S. Thus, $\rho/(1-\rho)$ is playing the role of relative price of consumption in event S, and (14) says for example that if trader 3's expected marginal rate of substitution of consumption in event F for consumption in event S exceeds the relative cost of consumption in event S, then an increase in first round transfer from 2 to 3 accompanied by a decrease in second round transfer by the same amount both in event S and F so as to keep trader 2's expected utility unchanged would increase trader 3's expected utility.

It is interesting to note that whether the first round transfer from trader 2 to trader 3 should be increased or not depends on trader 3's expected marginal rate of substitution of consumption in event F for consumption in event S whereas, by (16), the second round transfer in event S depends on the expected marginal rate of substitution of trader 2. On the other hand, by (17), the second round transfer in event F depends on the ratio of the expected marginal rates of substitution of both traders. By (20), these marginal rates are typically identical when there are positive second round transfers from 2 to 3 in both events.

One may summarize the results of lemma 4 in terms of real consumptions as follows.

Lemma 5 *Let τ be a feasible transaction which is not $\{2,3\}$-dominated. Then, the transfer of trader 2, $\tau_2 = (\tau_2^1, \tau_2^S, \tau_2^F)$ must satify the following properties.*

1. When one has $\tau_2^1, \tau_2^S, \tau_2^F > 0, \tau$ satisfies
$$\frac{C_2^F}{C_2^S} = \frac{C_3^F}{C_3^S} = r^{\sigma\rho}. \tag{21}$$

2. When one has $\tau_2^1 > 0, \tau_2^F > 0, \tau_2^S = 0, \tau$ satisfies
$$r^{\sigma\rho} \leq \frac{C_3^F}{C_3^S} \leq \frac{C_2^F}{C_2^S} \text{ , and } \quad \frac{C_3^F}{C_3^S} \leq r_2^{\sigma\rho}, \tag{22}$$
where in the last weak inequality, equality holds if $\tau_2^1 + \tau_2^F < 1$, in which case we have
$$\frac{C_2^F}{C_2^S} - \frac{C_3^F}{C_3^S} = \left(\frac{1-\rho}{\rho}\right)\left(\frac{C_3^F}{C_3^S} - r^{\sigma\rho}\right). \tag{23}$$
In particular, if $\dfrac{C_3^F}{C_3^S} = r^{\sigma\rho}$ holds, then
$$\frac{C_2^F}{C_2^S} = \frac{C_3^F}{C_3^S}.$$

3. When one has $\tau_2^1 > 0, \tau_2^S > 0, \tau_2^F = 0, \tau$ satisfies
$$\frac{C_2^F}{C_2^S} \leq r^{\sigma\rho} \leq \frac{C_3^F}{C_3^S}. \tag{24}$$

4. When one has $\tau_2^F > 0, \tau_2^S > 0, \tau_2^1 = 0, \tau$ satisfies the following.
   (a) If $\max\{\tau_2^F, \tau_2^S\} < 1$, then
   $$\frac{C_2^F}{C_2^S} = \frac{C_3^F}{C_3^S} \leq r^{\sigma\rho}.$$
   (b) If $\tau_2^F < 1$, then
   $$\frac{C_2^F}{C_2^S} \leq \frac{C_3^F}{C_3^S} \leq r^{\sigma\rho}.$$
   (c) If $\tau_2^S < 1$, then
   $$\frac{C_3^F}{C_3^S} \leq \frac{C_2^F}{C_2^S} \leq r^{\sigma\rho}.$$

4.2 Effects of a change in transfers of trader $i = 1$ or 3

We now proceed to check the properties of transfers from trader i=1 or 3.

Lemma 6 *Let τ be a feasible transaction. Then, for a change in transfers of trader i=1 or 3 as defined by (11), (12), and (13), we have the following.*

1. *If the second round transfer in event F, τ_i^F, is held constant so that $\eta_i^F = 0$, then:*

$$sgn\left[\frac{d}{dt}\mathsf{E}U^{i+1}(\tau_t^i)(0)\right] = sgn[\epsilon]sgn\left[\left(\frac{\sigma}{1-\sigma}\right)\left(\frac{1-\rho}{\rho}\right) - \left(\frac{C_i^S}{C_i^F} - \left(\frac{1}{\rho}\right)\frac{C_{i+1}^S}{C_{i+1}^F}\right)\right]$$
$$= sgn[\epsilon]sgn\left[\frac{C_{i+1}^F}{C_{i+1}^S} - r^{\sigma\rho}\left(\frac{1}{R_i} - \frac{1}{\rho}\right)\right] . \quad (25)$$

2. *If the second round transfer in event S, τ_i^S, is held constant so that $\eta_i^S = 0$, then:*

$$sgn\left[\frac{d}{dt}\mathsf{E}U^{i+1}(\tau_t^i)(0)\right] = sgn[\epsilon]sgn\left[\left(\frac{1-\sigma}{\sigma}\right)\left(\frac{1-\rho}{\rho}\right) - \left(\frac{C_i^F}{C_i^S} - \left(\frac{1}{\rho}\right)\frac{C_{i+1}^F}{C_{i+1}^S}\right)\right]$$
$$= sgn[\epsilon]sgn\left[\left(\frac{1-\sigma}{\sigma}\right)(1-\rho) - \frac{C_{i+1}^F}{C_{i+1}^S}(\rho R_i - 1)\right] . \quad (26)$$

3. *If the first round transfer, τ_i^1, is held constant so that $\epsilon = 0$, then:*

$$sgn\left[\frac{d}{dt}\mathsf{E}U^{i+1}(\tau_t^2)(0)\right] = sgn[\eta_i^F]sgn[R_i - 1]$$
$$= sgn[\eta_i^S]sgn[1 - R_i] . \quad (27)$$

There are two major differences between the results obtained in lemma 6 and lemma 4. One is that whether a change in the second round transfer in event F or S increases the expected utility of the recipient or not, each depends on the ratio of the expected marginal rates of substitution of both the donor and the recipient when the donor is $i = 1$ or 3, whereas in the previous case of donor being i=2, whether a change in the second round transfer in event S increases the expected utility of the recipient or not depends on the the expected marginal rate of substitution of the donor only. The other is that because of lemma 2 it does not make sense to consider a simultaneous change in the second round transfer in events F and S for traders i=1,3.

4.3 Effects of a change in transfers between traders $i = 3$ and 4

We now come to a consideration of transfers between traders 3 and 4. Here, the trader 3 is an essential participant to a transaction and is regarded to represent a tie between the settlement system and the outside party. The trader 4 is the stand-by party to a transaction. It might function as the central bank depending upon whether a transaction τ requires the trader 4 to effect transfer to 3 in event F.

Lemma 7 *Let τ be a feasible transaction and consider a change in transfers between traders 3 and 4 as defined by a transaction τ_t satisfying*

$$\tau_{ti}^1 = \tau_i^1 + t\epsilon_i, \tau_{ti}^F = \tau_i^F + t\eta_i^F, \tau_{ti}^S = \tau_i^S \tag{28}$$

where $\epsilon_i = \eta_i^F = 0$ for i=1,2,3. Set ϵ_i and η_i^F for i=4,5 so that the expected utility of trader 4 remains unchanged at t=0. Then:

1. *If $\epsilon_i \neq 0$ and $\eta_i^F = 0$ for i=4,5, we have*

$$\frac{d}{dt}\mathsf{E}U^3(\tau_t)(0) = \left(\frac{\sigma}{C_4^F} + \frac{1-\sigma}{C_4^S}\right)(\varphi\psi - 1)\,\epsilon_5\ . \tag{29}$$

2. *If $\eta_4^F \neq 0, \epsilon_5 \neq 0$, and $\epsilon_4 = \eta_5^F = 0$, we have*

$$sgn\left[\frac{d}{dt}\mathsf{E}U^3(\tau_t)(0)\right] = sgn[\epsilon_5] sgn\left[\varphi\psi\rho\left(\frac{\sigma}{C_4^S} + \frac{1-\sigma}{C_4^F}\right) - \frac{1-\sigma}{C_4^F}\left(1 + \left(\frac{\sigma}{1-\sigma}\right)\frac{C_3^F}{C_3^S}\right)\right]. \tag{30}$$

Let us evaluate the terms inside the above bracket assuming that τ is a feasible transaction such that $\tau_i^1 = \tau_i^F = \tau_i^S = 0$ for $i = 4, 5$. Certainly one has $C_4^F = C_4^S = 1$. When τ is individually rational and not {2,3}-dominated, with $\tau_2^1, \tau_2^F, \tau_2^S$ all strictly positive, then by lemma 5 we have

$$\frac{C_3^F}{C_3^S} = \frac{C_2^F}{C_2^S} = r^{\sigma\rho} = \left(\frac{1-\sigma}{\sigma}\right)\left(\frac{\rho}{1-\rho}\right).$$

It thus follows that

$$sgn\left[\frac{d}{dt}\mathsf{E}U^3(\tau_t)(0)\right] = sgn[\epsilon_5] sgn\left[\varphi\psi - \frac{1-\sigma}{\rho(1-\rho)}\right].$$

Let us note the term $\dfrac{1-\sigma}{\rho(1-\rho)}$ in the above bracket. If $\sigma = \rho$, it is equal to $1/\rho$, and since we have $\varphi\psi < 1$, a transaction τ is not dominated even if $\tau_i^1 = \tau_i^F = \tau_i^S = 0$ for $i = 4, 5$. However, note that the denominater $\rho(1-\rho)$ of the term achieves its maximum at $\rho = 1/2$ with the maximum value 1/4. Therefore, one has

$$\min_\rho \left\{\frac{1-\sigma}{\rho(1-\rho)}\right\} = 4(1-\sigma).$$

It means that if the value of σ is close to 1, for example if $\sigma = 0.9$, then it is small enough, i.e. 0.4, and inside the bracket tends to become positive, in which case a transaction τ must specify $\tau_5^1 > 0$ and $\tau_4^F > 0$ if it is not {2,3}-dominated. The following lemma summarizes the above arguments.

Lemma 8 *Let τ be a feasible transaction which is not {2,3}-dominated.*

1. *If τ specifies a state non-contingent trade between traders 3 and 4, i.e., $\tau_5^1 > 0$ and $\tau_4^1 > 0$, then τ is {3,4}-dominated.*

2. *Assume $\varphi\psi > (1-\sigma)/\rho(1-\rho)$. If it is feasible to increase the state non-contingent transfer from 3 to 4 as well as the state contingent transfer from 4 to 3 in event F, then τ is {3,4}-dominated.*

4.4 Properties of undominated transactions

Using the properties we have derived so far, we will see how undominated transactions induce state contingent transfers and consumptions. For this purpose, we start from a transaction that has state non-contingent transfers and show how it needs to be changed in order for the transaction to be undominated.

We begin by a consideration of a feasible and individually rational transaction τ which is not state contingent, so that all the transfers are done at first round. In this case we have the following.

Proposition 2 *Let τ be a feasible individually rational transaction such that:*

1. *τ effects first round transfers only.*
2. *Traders 1,2, and 3 make positive transfers.*
3. *Traders 3 and 4 do not make transfers between them.*

Then:

1. *Even if trader 1 initiates a state contingent transfer by reducing state non-contingent tranfer so as to keep his own expected utility level unchanged, trader 2 will never gain in expected utility.* [9]
2. *If trader 2 initiates a second round transfer in event F by reducing state non-contingent tranfer so as to keep his own expected utility level unchanged, trader 3 can gain in expected utility provided that the sum of the first round transfers from trader 2 to trader 3 and trader 3 to trader 1 is large enough.*
3. *If trader 3 initiates a second round transfer in event S by reducing state non-contingent tranfer so as to keep his own expected utility level unchanged, trader 1 will gain in expected utility level provided that the sum of the first round transfers from trader 2 to 3 and trader 3 to 1 are sufficiently large.*
4. *Trader 3 will gain in expected utility if trader 3 makes a first round transfer to trader 4 while trader 4 makes a second round transfer to trader 3 in event F in such a manner to keep trader 4's expected utility unchanged, provided that the sum of the first round transfers from trader 2 to trader 3 and trader 3 to trader 1 is large enough, and if expected marginal rate of substitution between endowment goods of traders 3 and 4 is not too small relative to the iceberg-cost adjusted cost of risk so that $\varphi\psi > (1-\sigma)/\rho$.*

Thus, according to the proposition 2, if we start from a transaction which has only first round state non-contingent transfers, then there will be no incentives for traders 1 and 2 to initiate state contigent transfers between the two. But as regards to traders 2 and 3, they have incentives to initiate a state contingent transfer in event F from 2 to 3

[9]This is equivalent to stating that "Trader 1 will never gain in expected utility by initiating a state contingent transfer while keeping trader 2's expected utility level unchanged." Similar remarks apply to the susequent statements of this proposition.

provided the sum of the first round transfers from trader 2 to trader 3 and trader 3 to trader 1 is large enough to satisfy the following inequality.[10]

$$\tau_3^1 + \left(\frac{\beta\rho}{\sigma-\rho}\right)\tau_2^1 > 1 \tag{31}$$

For example, for parameter values $\sigma = 0.9, \rho = 0.8$, and $\beta = 1.6$, the coefficient of τ_2^1 in the inequality is 12.8 so that if the transfer from trader 2 exceeds $\frac{5}{64}$, regardless of the amount of transfer from trader 3 the inequality is satisfied. Thus, all those transactions satisfying (31) will be {2,3}-dominated by increasing the transfer from trader 2 in event F and decreasing the first round transfer from 2.

As to transfers from 3 to 1, when the given transaction is entirely state non-contingent, they have incentives to initiate a state contingent transfer in event S from 3 to 1 provided the sum of the first round transfers from trader 2 to 3 and trader 3 trader to 1 is large enough to satisfy the following inequality:

$$\tau_3^1 + \beta(1-\sigma)\left(\frac{1-\rho}{\rho}\right)\tau_2^1 > 1 \ ,$$

because τ will be {1,3}-dominated by increasing τ_3^S and decreasing τ_3^1 under this condition. For the parameter values we specied earlier, the condition above is met, for example, if $\tau_2^1 \geq 0.02$ regardless of the value of τ_3^1. Thus, we may expect at this stage that a core transaction τ to specify $\tau_3^S > 0$. [11] One might think at first that it is counter-intuitive to have $\tau_3^S > 0$. An economic intuition behind this is the following: as the first round transfer τ_2^1 fails in event F, the consumption of trader 3 in event F, C_3^F, is below the level of his consumption in event S, C_3^S. If the excess of trader 3's expected marginal rate of substitution of consumption in F for consumption in S over the iceberg-cost-adjusted trader 1's expected marginal rate of substitution of consumption in F for consumption in S exceeds relative cost of cousumption in event S and F, then it is mutually desirable for traders 3 and 1 to increase the transfer τ_3^S by decreasing the first round transfer τ_3^1. This adjustment cannot be done by changing the level of the first round transfer τ_3^1 alone or by changing the second round transfer in event F, τ_3^F, because one is required to increase C_3^F and τ_3^F cannot be decreased beyond zero to achieve this. Of course, it would be a different story if a given transaction τ has state contingent transfers. For example, if trader 2 sends a part of his endowment to 3 in event F and/or if trader 3 sends a part of his endowment in event F, then it is possible that τ would be {3,1}-dominated unless 3 does send a part of his endowment to 1 in event F.

For transfers between traders 3 and 4, we have seen in lemma 8 that if the transaction τ is not {2,3}-dominated, then there are incentives to initiate a state non-contingent transfer from 3 to 4 and a state contingent transfer from 4 to 3 in event F. Note, however, that in the statement of the proposition 2 above, a given transaction τ may be {2,3}-dominated. But even so, it turns out that there are incentives to initiate a state non-contingent transfer from 3 to 4 and a state contingent transfer from 4 to 3 in event F, provided that the sum of the first round transfers from trader 2 to trader 3 and trader

[10]For this inequality as well as other inequalities below, see the proof of the proposition 2 in the appendix.

[11]Later on we shall show that this need not be the case when traders do make state contingent transfers.

3 trader to 1 is large enough to satisfy the following inequality.

$$\frac{\beta\left(\varphi\psi-\frac{(1-\sigma)}{\rho}\right)}{(1/\rho)-\varphi\psi}\tau_2^1+\tau_3^1>1$$

In fact, traders 3 and 4 can dominate the state non-contingent transaction τ by initiating a state contingent transfer from 4 to 3 in event F together with a state non-contingent transfer τ_5^1 from 3 to 4 despite the fact that traders 3 and 4 do not mutually gain from trades in general. This shows that the trader 4 will participate in transaction only in risk-sharing capacity.

It may be of interest to note that although transfers are done in one direction only from a trader to another among essential participants, two rounds of transfers function as if there is an explicit means of payment or are barter trades between any two participants in the sense that an increase of a transfer in one round can be matched to a decrease of another transfer in the other round. This gives another sense in which the model in this paper can be said to represent a settlement network.

Given a transaction τ, a *net transfer gap* of trader i is defined to be

$$g_i(\tau)=1-\tau_i^1-\chi_{\{3\}}\tau_5^1-\max\{\tau_i^S+\chi_{\{3\}}\tau_5^S,\tau_i^F+\chi_{\{3\}}\tau_5^F\},$$

Net transfer gap $g_i(\tau)$ among the essential participants shows the maximal amount that trader i can further transfer to others, given a transaction τ. We also define *transfer gap* $\bar{g}_i(\tau)$ (among essential participants) by $\bar{g}_i(\tau)=g_i(\tau)$ for $i=1,2$ and

$$\bar{g}_3(\tau)=1-\tau_i^1-\max\{\tau_3^S,\tau_3^F\}\ .$$

Finally, we give two statements concerning core tranactions, assuming parameter values to satisfy

$$\begin{aligned}\varphi\psi &> \left(\frac{1-\sigma}{\rho}\right)\frac{1}{1-\rho}\ , \\ \beta\rho &> \sqrt[3]{2}\ ,\ \rho>\sqrt[3]{2}/2\ ,\end{aligned} \tag{32}$$

where the first inequality is satisfied, for example, if $\varphi\psi>0.63$ when $\sigma=0.9$ and $\rho=0.8$. It is also satisfied whenever we have $\varphi\psi>r^{\sigma\rho}$. The second and the third inequalities are to ensure that second round transfers are not too costly so that traders are induced to make such transfers.

Proposition 3 *A core transaction τ always specifies state contingent transfers. A typical core transaction τ specifies transfers such that:*

$$\begin{aligned}&\tau_i^1>0 \quad \textit{for } i=1,2,3,5,\ \tau_4^1=0\ ,\\ &\tau_1^F>0,\tau_2^F>0,\tau_3^F>0,\tau_4^F>0,\tau_5^F=0\ ,\\ &\tau_1^S=\tau_3^S=\tau_4^S=\tau_5^S=0,\tau_2^S>0\ ,\end{aligned}$$

in which case consumptions associated with the transaction are given by:

$$\begin{aligned}
\frac{C_2^F}{C_2^S} &= \frac{C_3^F}{C_3^S} = r^{\sigma\rho} \\
\frac{C_1^F}{C_1^S} &\geq \left(\frac{1-\sigma}{\sigma}\right)\frac{1}{1-\rho} \;>\; r^{\sigma\rho} \\
&\quad \textit{with equality holding when } g_1(\tau) > 0 \\
\frac{C_4^F}{C_4^S} &= \frac{1-\sigma}{\sigma}\left(\frac{1}{\varphi\psi\rho(1-\rho)} - 1\right) \;>\; \left(\frac{1-\sigma}{\sigma}\right)\frac{1}{1-\rho}\ .
\end{aligned}$$

We like to note the extent to which traders' consumptions that a typical core transaction induces are state contingent. Given a typical core transaction as in the beginning of the statement of the proposition above, for trader 2 and trader 3 the consumption level in event F relative to that in event S is $r^{\sigma\rho}$, which is less than 1 but approaches 1 as the value of ρ becomes closer to σ. This may be interpreted to say that the failure of receipt by trader 3 is compensated by other traders by the factor of $(\rho/(1-\rho)) - 1$. Trader 2 is as responsible as trader 3 for the loss as his relative consumtion level in event F is reuced to the level of trader 3. Trader 1 in turn compensates trader 2 but extent to which he joins in the compensation is less than that of trader 2 so that his relative consumtion in event F exceeds $r^{\sigma\rho}$. It is very instructive to note that trader 4 also participates in this compensation scheme but extent to which he does compensate trader 3 is much less than those of other traders in the sense that his relative consumtion level in event F is higher than those of all the essential participants.

Proposition 4 *Let τ be a core transaction. Then:*

1. *At least one essential participant must be sending all his endowment to other traders in some event. That is,*
$$(\exists i \in \{1,2,3\}) g_i(\tau) = 0.$$

2. *Suppose that trader 3 is not sending all of his endowment to other essential participants so that his transfer gap is positive, i.e., $\bar{g}_3(\tau) > 0$. Then, the transaction τ is a core transaction if and only if trader 3 is making a transfer to trader 4 either by the amount of his transfer gap or by the amount of "feasibility bound" given by*
$$v(\varphi\psi, \sigma, \rho, \beta) = \frac{(1-\sigma)\,(1-\varphi\psi\rho(1-\rho))}{\beta\,[\varphi\psi\rho(1-\rho) - (1-\sigma)]}\ ,$$
whichever is smaller, i.e.,
$$\tau_5^1 = \min\{\bar{g}_3(\tau), v(\varphi\psi, \sigma, \rho, \beta)\},$$
and trader 4 in turn is making a state contingent transfer in event F at most the amount given by
$$\tau_4^F = \left(\frac{\varphi\psi\rho(1-\rho) - (1-\sigma)}{\sigma\varphi\psi\rho(1-\rho)}\right)(1 + \beta\tau_5^1)\ . \tag{33}$$

The first part of the proposition 4 is a direct consequence of the lemma 1 and is due to our specification of preferences of essential participants that they prefer the endowment of another trader to his own. The second part comes from two things. One is that a core transaction in general specifies positive second round transfers in both events F and S as well as a positive first round state non-contingent transfer from trader 2 to trader 3. By (21) of lemma 5, this ensures trader 3's consumption in event F relative to that in event S, C_3^F/C_3^S, to be given by $r^{\sigma\rho}$. Second is that under this circumstance, given (32), the expected utility of both of the traders 3 and 4 can be increased whenever first round state non-contingent transfer from 3 to 4 and second round state contingent transfer in event F from 4 to 3 can be increased. For a first round state non-contingent transfer from 3 to 4, τ_5^1, the maximal amount that trader 4 would be just willing to send to 3 is given by the amount shown in (33).

5 Preliminary analysis of a private-information environment

In the general discussion above, we have contemplated that a third party within the coalition might have some private information, not possessed by either the payor or the payee, about the level of risk. In such a case, the information is potentially relevant to how the transaction should be conducted and even to how large a transaction ought to be undertaken.

If the privately informed third party were involved solely as the reporter of that information to the coalition, then there would be no problem about ensuring the truthfulness of the report. In particular, if compensation were required to induce reporting, that compensation could be made in the form of a flat fee. If the possessor of information functions in the payments process as an agent for one of the principals in the transaction, though, then there will generally be an issue of whether there is incentive for truthful reporting.[12] One might think, for example, that efficiency would generally require a payments coalition to penalize an information provider when a payment would fail without a warning of particularly risky circumstances having been given.

In this section we will show that there is indeed an incentive-compatability issue for the payments coalition to resolve, but that there is no simple generalization about how to resolve it. The incentive for truthful revelation of information depends on the pattern of risk sharing within the payments coalition, the differences in risk attitudes among coalition members, and the distribution of rents that is to be achieved by a *core transaction mechanism*, which generalizes the notion of a core transaction to a private-information

[12] This idea, that a dual role of privately informed members of a payments coalition is critical for understanding how the institutional design of a payment arrangement is related to the attainment of economic efficiency, has previously been studied by Rochet and Tirole. In their model, unlike the present one, traders' information can only be revealed through their trades, and not by making explicit reports. In many actual payments networks, the limited opportunities for traders to make explicit reports seem to fall between the absence of opportunity modelled by Rochet and Tirole and the completely adequate opportunity modelled here. When traders are required to set prior limits (which will not necessarily ever be binding in equilibrium) on their bilateral exposure to counterparties, for instance, their choices of which limits to set can be regarded as partially informative reports of their private information about those counterparties' riskiness.

environment. In fact, for the parametric environment that we study, some core mechanisms involve a binding incentive-compatibility constraint for truthful revelation that a transfer is likely to fail (that is, revelation of event L), while other core mechanisms for the same environment involve a binding constraint for truthful revelation that failure is unlikely (event H). As a practical matter, then, an implication of using the core transaction mechanisms as an equilibrium concept for payment arrangements is that supervisory authorities ought to accord substantial discretion to the governing body of a payments coalition to establish rules aimed at eliciting accurate information from members.

5.1 Generalizing the model to encompass private information

To model private information, suppose that an event that is statistically relevant to the outcome of that risky transfer will be privately observed by trader 1, who is not directly involved in the risky transfer but who is an essential participant in a mutually beneficial transaction among the traders. To consider the simplest case of nontrivial private information, suppose that trader 1 observes which element of the partition $\{H, L\}$ of Ω contains the true state of nature. Suppose that H and L satisfy

$$\Pr(S|L) < \Pr(S|H). \tag{34}$$

The agreement among traders regarding the structure of the transaction, described in section 2, is an *ex ante* agreement, made before trader 1 has received any information. However, trader 1 will observe H or L before the first round of transfers takes place. Thus it is natural for the agreement to specify that trader 1 will report what he observes, and that his report will determine which transaction to make. That is, the agreement among the traders specifies a *transaction mechanism* rather than a single transaction. Formally, a transaction mechanism is a mapping $\mu\colon \{\text{'H'}, \text{'L'}\} \to \mathcal{T}$.[13]

Transaction mechanism μ will elicit truthful reporting from trader 1 if the following incentive-compatibility condition is satisfied.

$$\forall A{\in}\mathcal{P} \quad \forall B{\in}\mathcal{P} \quad \mathsf{E}[U^1(c^1(\mu(B),\omega))|A] \leq \mathsf{E}[U^1(c^1(\mu(A),\omega))|A]. \tag{35}$$

Let $\mathcal{M}$ denote the set of incentive-compatible transaction mechanisms. We restrict attention to incentive-compatible mechanisms, as is justified by the revelation principle.[14] If $\mu \in \mathcal{M}$, then the resulting transaction τ and the consumption Γ^i for each trader $i \in N$ is defined as follows (with χ_A denoting the characteristic function of an event A).

$$\Gamma^i(\mu, \omega) = \sum_{A\in\mathcal{P}} \chi_A(\omega) c^i(\mu(A), \omega). \tag{36}$$

A core transaction mechanism can be defined in a way that is straightforwardly analogous to the definition of a core transaction.[15] Specifically, to formalize the notion of unanimous preference within a coalition, for each $C \subseteq N$, define $\nu \in \mathcal{M}$ to *C-dominate*

[13] 'H' and 'L' are names for the events H and L. They are linguistic reports of events, not events themselves (that is, subsets of Ω).

[14] Myerson (1991) provides an exposition of incentive compatability and the revelation principle.

[15] Allen (undated) has previously used incentive-compatible mechanisms in this way to define a core equilibrium concept for environments with private information.

$\mu \in \mathcal{M}$ if

$$\begin{aligned} &\forall i \in C \quad \mathsf{E}[U^i(\Gamma^i(\mu,\omega))] \leq \mathsf{E}[U^i(\Gamma^i(\nu,\omega))] \quad \text{and} \\ &\exists i \in C \quad \mathsf{E}[U^i(\Gamma^i(\mu,\omega))] < \mathsf{E}[U^i(\Gamma^i(\nu,\omega))]. \end{aligned} \tag{37}$$

Also define $\nu \in \mathcal{M}$ to be *feasible for* C if

$$\begin{array}{rl} \forall i \notin C \quad \forall \omega \quad \Gamma^i(\nu,\omega) = z^i & \text{(No participation of other traders is required)} \\ \text{(35) holds if} \quad 1 \in C & \text{(Incentive compatability).} \end{array} \tag{38}$$

Finally, define $\mu \in \mathcal{M}$ to be a *core transaction mechanism* if there exist no $C \subseteq N$ and ν such that ν is feasible for C and ν C-dominates μ.

With the core of a transaction mechanism so defined, proposition 1 has the following, straightforward generalization.

Proposition 5 *Let each trader's utility function be locally nonsatiated, at all points, in his own endowment goods. Then $\mu \in \mathcal{M}$ is a core transaction mechanism if and only if the following conditions hold: μ is individually rational, Pareto-undominated, and optimal for payments-system participants, and μ is not $\{3,4\}$-dominated.*

5.2 A parametric environment with accurate private information

Consider the three-trader environment in which trader 1 receives a private signal about the success or failure of a transfer from trader 2 to trader 3. To simplify this preliminary analysis, we assume that success and failure have equal probability, and that trader 1's signal is perfectly accurate, That is, we assume that $\Pr(H) = \Pr(L) = 1/2$, $\Pr(S|H) = 1$, and $\Pr(S|L) = 0$.

In this section, we work with piecewise-linear utility functions for the traders. Their utilities will be defined in terms of parameters δ and ϵ, which are assumed to satisfy $0 < \epsilon < \delta < 1/4$. Utility functions are defined in terms of the following functions on the nonnegative real numbers.

$$\begin{aligned} V(x) &= \min\{1, x\} + \epsilon \max\{0, x-1\}; \\ W(x) &= \min\{1/2, x\} + \epsilon \max\{0, x-1/2\}. \end{aligned} \tag{39}$$

Define the agents' utilities as

$$\begin{aligned} U^1(c) &= c_1 + c_3; \\ U^2(c) &= V(c_1 + c_2); \\ U^3(c) &= W(c_2) + \delta c_3. \end{aligned} \tag{40}$$

Because trader 1's utility function is linear and information is perfectly accurate, the incentive-compatability constraint reduces to the following two equations. The first and second equations state that trader 1 has incentive to report truthfully in event H and L, respectively.

$$
\begin{aligned}
&(\mu_3^1(H) + \mu_3^S(H)) - (\mu_1^1(H) + \mu_1^S(H)) \geq (\mu_3^1(L) + \mu_3^S(L)) - (\mu_1^1(L) + \mu_1^S(L)) \\
&(\mu_3^1(L) + \mu_3^F(L)) - (\mu_1^1(L) + \mu_1^F(L)) \geq (\mu_3^1(H) + \mu_3^F(H)) - (\mu_1^1(H) + \mu_1^F(H))
\end{aligned}
\tag{41}
$$

It has been proved that, in the three-trader environment, a transfer mechanism is in the core if and only if it implements a Pareto efficient (subject to both technological and incentive constraints) allocation that is individually rational for each trader. An allocation that maximizes a weighted sum of traders' expected utilities, with all weights strictly positive, is Pareto efficient. Therefore, for $\alpha \in \mathbb{R}^3_{++}$, consider

$$
U(\mu, \alpha) = \sum_{i=1}^{3} \alpha_i \mathsf{E} U^i(\Gamma_i(\mu, \omega)). \tag{42}
$$

Regarding the transfer technology, suppose that

$$
\rho = 1/2. \tag{43}
$$

5.3 IC constraint can bind in H

Let α satisfy the following conditions.

$$
\begin{aligned}
\delta\alpha_3 < \alpha_1 < \alpha_2 < \alpha_3; \\
\rho\alpha_3 = \alpha_3/2 < \alpha_2.
\end{aligned}
\tag{44}
$$

(For example, if $\epsilon = 1/10$ and $\delta = 2/10$, then $\alpha = (2, 3, 4)$ satisfies the inequalities (44).)

Consider a transfer, contingent on an announcement of trader 1's information that is assumed for the moment to be truthful, that would maximize U for this value of α. It must have the following features, where c_j^i denotes the amount of consumption by trader i of the endowment good of trader j.

- If trader 1 announces event H, then the following transfers should be made in the first round. Trader 3 should send his entire endowment to trader 1, because $\delta\alpha_3 < \alpha_1$. Trader 2 should send half a unit to trader 3, because $\delta\alpha_3 < \alpha_2 < \alpha_3$. After having made that transfer, $c_2^2 = 1/2$. Therefore trader 1 should send half a unit to Trader 2, so that $c_1^2 + c_2^2 = 1$, because $\epsilon\alpha_2 < \delta\alpha_3 < \alpha_1 < \alpha_2$.

- If trader 1 announces event L, then the following transfers should be made. In this case, also, trader 3 should send his entire endowment to trader 1 in the first round, because $\delta\alpha_3 < \alpha_1$. However, trader 2 should send nothing to trader 3 in either round. Such a transfer would fail in the first round by the assumption that trader 1 announces the truth, and it should not be made in the second round because $\rho\alpha_3 < \alpha_2$. Thus, since the condition that $c_1^2 + c_2^2 \geq 1$ will be satisfied, trader 1 should send nothing to trader 2, because $\epsilon\alpha_2 < \alpha_1$.

- Since trader 1 has perfectly accurate information, transfers in round 1 can be optimized contingent on whether a transfer from trader 2 to trader 3 would actually succeed or fail. Therefore. no further transfer is needed in round 2. Since making a transfer in round 2 would incur an "iceberg cost" that is avoidable in round 1, no transfer should be made in round 2 after the transfers described above have been made in round 1.

A transaction mechanism that would provide this contingent allocation, if trader 1 were to report truthfully, is specified by

$$\tilde{\mu} = \Big(\tilde{\mu}(H), \tilde{\mu}(L)\Big) = \bigg(\Big((1/2, 1/2, 1)(0,0,0)(0,0,0)\Big), \Big((0,0,1)(0,0,0)(0,0,0)\Big)\bigg). \quad (45)$$

This mechanism is individually rational for all traders, but it is not incentive compatible. Specifically, trader 1 always has incentive to announce event L, regardless of his actual observation, in order to avoid having to give up half of his endowment.

In order to make the mechanism incentive compatible, it must be modified so that, if the trader 1 observes event H, then he will consume no more (total of goods 1 and 3) by misrepresenting his observation as L than by truthfully announcing H. This can be accomplished while retaining the same equilibrium allocation as $\tilde{\mu}$ would provide, by defining transaction mechanism μ to be identical to $\tilde{\mu}$ except that $\mu_1^S(L) = 1$. That is, a core transaction mechanism that maximizes $U(\mu, \alpha)$ is

$$\mu = \Big(\mu(H), \mu(L)\Big) = \bigg(\Big((1/2, 1/2, 1)(0,0,0)(0,0,0)\Big), \Big((0,0,1)(1,0,0)(0,0,0)\Big)\bigg). \quad (46)$$

In summary, care has to be taken to specify a core transfer mechanism in a way that achieves incentive compatability, but having to impose the incentive-compatability constraint need not make any trader worse off than he would be in the core allocation of an environment where all traders could observe $\mathcal{P}_2$ directly. In this sense, incentive-compatability is not a binding constraint. Later in this paper, we will establish that incentive-compatability can be a binding constraint when the information of trader 1 is less than perfectly accurate.

5.4 What if S could not be distinguished from F without a non-zero transfer being made?

The foregoing discussion has assumed that the events S and F could be distinguished even if no transfer were attempted (that is, if a zero transfer were specified). The alternative assumption, that the informed traders can only learn about these events through the actual success or failure of a non-zero transfer, may be thought to be more widely applicable. Under this assumption, incentive compatability can be achieved either by requiring trader 1 to transfer half of his endowment to trader 2 in L, or else by having trader 3 retain half of his endowment rather than transferring it to trader 1 in H. (Because the environment is piecewise linear, combinations of these two modifications do not have to be considered.) Because $\epsilon < \delta$, having trader 3 retain half of his endowment achieves the higher value of U. That is, $U(\mu, \alpha)$ is maximized by defining μ so that

- Trader 1 sends one unit to Trader 2, and trader 2 sends half a unit to trader 3, in event H;
- Trader 1 sends nothing to Trader 2, and trader 2 sends nothing to trader 3, in event L;
- Trader 3 sends one unit to trader 1 in H, but only half a unit in L.

5.5 IC constraint can bind in L

Now change α so that making transfer from trader 2 to trader 3 even in event L is appropriate to maximize the α-weighted sum of utilities. That is, let α satisfy the following conditions.

$$\delta\alpha_3 < \alpha_1 < \alpha_2 < \rho\alpha_3 = \alpha_3/2. \tag{47}$$

(For example, if $\epsilon = 1/10$ and $\delta = 2/10$, then $\alpha = (2, 3, 8)$ satisfies the inequalities (47).)

The transaction mechanism that maximizes U for this value of α has the following characteristics.

- Trader 1 sends half a unit to Trader 2, and trader 2 sends half a unit to trader 3, in event H.
- Trader 1 sends one unit to Trader 2, and trader 2 sends one unit to trader 3, in event L. The transfer from trader 2 to trader 3 is made in the second transfer period, so trader 3 only receives half a unit.
- Trader 3 always sends one unit to trader 1.

Again, this mechanism is individually rational for all traders but is not incentive compatible. Specifically, trader 1 always has incentive to announce event H, regardless of his actual observation, so that he will be required to transfer only half of his endownent, rather than all of it.

In order to make the mechanism incentive compatible, it must be modified so that trader 3 sends only half a unit to trader 1 in event H, but sends a full unit in event L. (Again, reducing the amount transferred from trader 3 to trader 1 achieves a higher value of U than increasing the amount transferred from trader 1 to trader 2 would achieve.)This modified mechanism is the core mechanism corresponding to choice of μ to maximize $U(\mu, \alpha)$. That is, the core mechanism has the following characteristics.

- Trader 1 sends half a unit to Trader 2, and trader 2 sends half a unit to trader 3, in event H.
- Trader 1 sends one unit to Trader 2, and trader 2 sends one unit to trader 3, in event L. The transfer from trader 2 to trader 3 is made in the second transfer period, so trader 3 only receives half a unit.
- Trader 3 sends half a unit to trader 1 in event H, and a full unit in event L.

6 Conclusion

The rules in a settlement system must encourage the participants to take optimal degrees of risk in accordance with their attitude towards risk. If some participants have socially useful private information, the rules in a settlement system must be constructed such that it does not give participants adverse incentives to mask their information. Policy makers can achieve this objective if they think about the rules in a settlement system as a mechanism design problem. If policy makers ignore those points and introduce new rules into a settlement system, the equilibrium allocation of goods might be distorted.

If we regard the crucial issue in the settlement system as efficient risk sharing among the participants in the presence of private information, this view allows policy makers to consider the rules governing a settlement system as a kind of social safety net. Efficient risk sharing under the default of banks might involve the transfer of resources from the agents who normally are not directly involved with the settlement network. The central bank plays such a role as a lender of last resort, by transferring the resources of the general public into the banking sector during a period of financial panic.

Appendix: Understanding central banks as a risk-sharing device

The discussion in this paper is based on the view that standard microeconomic theory could be useful to analyze the settlement network. We try to understand the flow of funds between the parties as an endogenous phenomenon. We emphasis that both private information and preference could play independent roles to determine the optimal risk sharing in equilibrium. In principle, the approach could be useful to design regulations that avoid unintended distortions by giving due consideration to the incentives and private information of participants.

This strategy is consistent with the other recent contributions to the microeconomics of banking that emphasize the anlysis of formal models (See Dewatripont and Tirole (1994), Freixas and Rochet (1997)). In this appendix, we justify the practical relevance of this approach by examining several heuristic examples that support our argument.

We will discuss, in turn, (i) the role of central bank as a lender of last resort, and (ii) historical examples of barter trade that exactly match our model.

The role of central bank

Throughout this paper, we argue that the fourth trader who serves as a lender of last resort, can be viewed as a central bank only if there are no gains from trade with this agent in the absence of risk. We argue that the fourth trader should be treated as the general public if there is gains from trade without risk.

More formally, trader 4, who obtains goods at the usual time and sends goods to trader 3, looks like a central bank if $\beta\gamma < 1$. Note that the fact that this inequality does not hold implies that a unit of exchange of goods between trader 3 and trader 4 is Pareto improving, hence it is no wonder that trader 4 transfers his own goods to trader 3. The implication of our model is that since the inequality does not hold ex ante, there is no gain from trade in the absence of risk. The fact that the marginal utility of trader 3, which becomes substantially high given the shipment failure from trader 2, induces trader 4 to serve as a lender of last resort, because trader 4 sends goods to trader 3 only under the situation of settlement failure. The fact that trader 3 sends some goods to trader 4 without shipment failure could be understood that trader 3 pays some fee in order to obtain insurance against shipment failure.

We stress the view that the fourth trader plays the role of central bank, but it is not different from the usual traders intrinsically, and that there is no particular reason to believe there are gains from trade between the central bank and others without risk consideration. We further argue that if there were a risk in a settlement network, without central bank, then a private institution would serve such a role. Our view is based on the U.S. history of banking.

For example, take agent 2 as a New York bank, agent 3 as a Boston bank, and agent 1 as a Philadelphia bank. Consider shipment failure of a bank as a default of the bank. Safer shipments take time as banks inspect the quality of bank notes. Note that the fourth trader would act as a clearing house, supervising banks and financing its supervision activity with a membership fee. This suggests that if there is no central bank, then there would be quasi-central banks by private arrangement. (See Gorton and Mullineaux (1987)). This consideration justifies why our model captures some of the important aspects of the free banking area documented by King (1983) and the emergence of central banks. Such interpretation suggests that our model is even consistent with the recent view that regards the role of Federal Reserve as the supplier of settlement services among private netting arrangements once we regard trader 1,2 and 3 as private clearing networks (See Summers and Gilbert (1996)).

Note that Green (1997) shows that by allowing traders to issue so-called novation securities, in equilibrium, both the initial securities and the novation securities will trade at face value in a monetary economy á la Freeman (1996a, b). This means the risk induced by trading-opportunity uncertainty will be fully insured, and efficiency will be attained. The novation securities bear a striking resemblance to the clearinghouse loan certificates that were issued during those episodes in the absence of a central bank. Those certificates, and the central banking role played by U.S. clearinghouses at that time generally, are described by Timberlake (1984).

Loss sharing rule in historical economies

Strictly speaking, we have discussed the implication of triangular and barter trades with risky delivery of goods.

Given these limitations, one way of viewing our model is as a somewhat realistic model of shipping insurance and other loss-sharing arrangements in historical economies. Early modern Europe and feudal Japan provide such examples. Regarding Europe, Lopez and Raymond (1955, p.259) reprint Genoese documents of 1191 and 1192, in which a merchant pays a premium to a shipper who puts up security for the successful delivery of the merchant's goods. Moreover, some examples of insurance contracts involving third-party underwriters can be found as early as the fourteenth century. In such a contract, the underwriters are supposed to purchase for a certain price a certain amount of goods from a merchant, but that the contract is to be void if the goods arrive safely at a certain port.

Feudal Japanese sea law, dating back to at least 723 AC, also pertained to an economy that exemplified the general features of our model.[16] Most sea transportation was commissioned by government. Specifically, local government officials (owners of the cargo) hired sailors to ship goods from their regions to the central government as feudal tax payments.[17] Sometimes bad weather forced the sailors to jettison the cargo in order to

[16] The following discussion is based on Takeda (1992) and Toyoda and Kota (1970).

[17] Even after taxes became payable in money, goods continued to be transported to the capital by sea in order to be sold to raise the tax money. The common law of sea transportation did not change very much from the one in the eighth century, judging from the one of the oldest written sea laws, called "Kaisen-Shikimoku," which dates from 1223.

stay afloat. In that event, the central government asked the sailors and the local government to pay 40% and 60% of the damage of the cargo respectively. (That is, the central government tried to induce the sailors not to abandon too much cargo intentionally by introducing this rule.) However, if the ship sank or more than half of the sailors were drowned despite having jettisoned the cargo, then central government did not ask them (that is, either surviving sailors or drowned sailors' heirs) to pay indemnity. These provisions were evidently designed to induce sailors to take appropriate actions contingent on the severity of the weather at sea—a situation regarding which they possessed private information relative to the senders and receivers of their cargo.[18]

As well as serving as a fairly realistic model of such historical arrangements, it also serves as a more schematic model of loss-sharing arrangements adopted by various payments systems today. It is noteworthy that the model explains these arrangements in terms of standard concepts of insurance theory, without having to invoke unproved assertions about a special, ill-defined kind of "systemic" risk.

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[18] Presumably the provisions also served to prevent sailors from engaging in intentional fraud—a phenomenon from which our model abstracts—by surreptitiously stopping en route to sell the cargo and then claiming to have jettisoned it in a storm.

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