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Competitive Equilibria with Limited Enforcement*

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ABSTRACT

Previous literature has shown that the study and characterization of constrained efficient allocations in economies with limited enforcement is useful to understand the limited risk sharing observed in many contexts, in particular between soveriegn countries. In this paper we show that these constrained efficient allocations arise as equilibria in an economy in which private agents behave competitively, taking as given a set of taxes. We then show that these taxes, which end up limiting risk sharing, arise as an equilibrium of a dynamic game between governments. Our decentralization is different from the existing ones proposed in the literature. We find it intuitively appealing and we think it goes farther than the existing literature in endogenizing the primitive forces that lead to a lack of risk sharing in equilibrium.

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Applied general equilibrium models have been proven useful in analyzing a variety of issues ranging, to name a few, from international business cycles to asset pricing. Many of these models assume the existence of complete asset markets that in turn implies complete risk sharing among agents in the economy. Complete risk sharing has implications that are often far from the data; to reconcile data and theory authors have developed models in which risk sharing is limited for a variety of reasons. One approach is to exogenously restrict the sets of tradable assets, another is to introduce a friction in the environment that endogenously limit the amount of achievable risk sharing.

For example in the international business cycle research Backus, Kehoe and Kydland (1992) presented a model with complete international asset markets and argued that the assumption generated "too much" international risk sharing and grossly counterfactual implications for international comovements of real variables. Baxter and Crucini (1995) explored a model in which the menu of international tradable assets is exogenously restricted to a single uncontingent bond and showed that the restriction did not help too much to reduce the gap between data and theory.

Kehoe and Perri (2002) examine a model in which limited risk sharing arises endogenously from the limited ability to enforce international credit arrangements between sovereign nations and find that this type of friction goes a long way in reconciling data and theory. This limited ability manifests itself in enforcement constraints which require that in each period and state, allocations can be enforced only if their value is greater than it would be if the country were excluded from all further intertemporal and interstate trade. This friction captures in a simple way the difficulties of enforcing contracts between sovereign nations that involve large transfers of resources which are backed only by promises to repay later.

This earlier work focuses on planning problems with enforcement constraints, namely the *constrained efficient allocations*, but does not analyze in details how these allocations can be decentralized.

In this paper we show that these allocations arise as equilibria of a dynamic game between governments with private agents acting competitively. In this game, private agents solve standard competitive equilibrium problems, while the government of each country can choose to prevent its citizens from repaying their outstanding international debts by taxing such repayments and, if there is capital, the government can tax capital income. We show that the allocations that solve the constrained planning problem can be supported as equilibria of this game if and only if they satisfy the enforcement constraints.

The main contribution of our work is to show how limited international risk sharing can endogenously arise in the equilibrium of an appropriately defined game with competitive private agents. As such our work builds on both to the work on international debt such as the studies of Eaton and Gersovitz (1981), Manuelli (1986), Kletzer and Wright (2000) together with those surveyed by Eaton and Fernandez (1995) as well as the literature on debt-constrained asset markets, particularly the work of Kehoe and Levine (1993,2000), Kocherlakota (1996), Ligon, Thomas and Worrall (2002), Alvarez and Jermann (2000) and Attanasio and Ríos-Rull (2000).

Our work goes beyond these literatures in several ways. In this literature, the equilibrium is modelled in one of two ways. In the international debt literature, private competitive agents are not explicitly modelled and instead a game is set up between two large agents, often thought of as the governments of the countries. In the debt-constrained asset market literature, private agents are explicitly modeled as competitive, but the constraints that private consumers face are not explicitly chosen by any agent as part of the equilibrium.

For example, in Kehoe and Levine (1993), the enforcement constraints are built directly into the commodity space. Alvarez and Jermann (2000) go the farthest and show how appropriately set constraints on debt can decentralize the constrained efficient allocations as a competitive equilibria. Although, even here, these debt constraints are not chosen by any agent, but rather it is shown that if these constraints are appropriately set then the allocations of interest can be decentralized. Jeske (2000) and Wright (2001) also analyze competitive equilibria with limited enforcement but focus on the case in the decision of repudiate the debt is taken by private agents and not by governments so the strategic element of default decision is not explicitly modelled.

In our work instead the debt taxes, which are the mechanism through which international risk sharing is limited, are derived endogenously as equilibria of a dynamic game between governments. We also show that decentralization with taxes is intuitively more appealing when countries have access to an investment technology.

We begin with a pure exchange economy with two countries and a representative consumer in both countries. We set up a planning problem with enforcement constraints and show how the resulting constrained efficient allocations can be characterized by a transition law for the ratio of marginal utilities of consumers across countries together with a resource constraint. We show that the constrained efficient allocations can be decentralized as either a competitive equilibrium with appropriately set debt constraints, as in Alvarez and Jermann (2000) or as a competitive equilibrium with debt taxes. In both notions of competitive equilibrium the frictions faced by private agents, namely the debt constraints or the debt taxes, while appropriately set, are exogenous.

We then go on to define a dynamic game in which the governments of the countries optimally choose the debt taxes as part of the equilibria, while private agents act competitively taking the debt taxes as exogenous. We show that any constrained efficient allocation can be supported as an equilibria of this dynamic game. In this sense, our economy is a standard competitive environment in which limited international risk sharing arises endogenously from the limited enforcement of international contracts and the strategic interactions between government.

We then add capital to the model, so that the economy is a standard two country growth model with enforcement constraints. We first show that the constrained efficient allocations cannot be decentralized with only the type of debt constraints used by Alvarez and Jermann (2000). The reason is that the Euler equation for capital accumulation is necessarily distorted away from the first best in the planning problem with enforcement constraints but with debt constraints alone there is no such distortion in the competitive equilibrium. If we add a savings constraint, as suggested by Seppala (1999), the constrained efficient allocations can be decentralized as competitive equilibria, but we find such a decentralization not intuitively appealing.

Finally, we show that if we allow for capital income taxes as well as debt taxes we can decentralize the constrained efficient allocations as competitive equilibria. It is then easy to show that any constrained efficient allocation can be supported as the equilibrium of a dynamic game in which governments choose both type of taxes.

1. Constrained Efficient Allocations

Consider the following deterministic pure exchange economy which is a special case of the stochastic pure exchange economy studied by Alvarez and Jermann (2000) and the stochastic production economy studied by Kehoe and Perri (2002).

Our theoretical world economy consists of two countries, $i = 1, 2$, each represented by a large number of identical, infinitely-lived consumers and a time-varying deterministic endowments of a single homogeneous good. The endowment of country i in period t is y_{it} while consumers in country i have utility, or *preferences,* of the form

$$(1) \qquad \sum_{t=0}^{\infty} \beta^t U(c_{it})$$

where c_{it} denotes consumption by consumers in country i at t and β denotes the discount factor. The *resource constraints* are given by

$$(2) \qquad c_{1t} + c_{2t} = y_{1t} + y_{2t}.$$

We assume that for $i = 1, 2$ and all $y_{it} \in [\underline{y}, \bar{y}]$ for some finite strictly positive constants $\bar{y}$ and $\bar{y}$.

This economy has, besides the resource constraints, enforcement constraints which require that at every point in time, each country prefers the allocation it receives relative to the allocation it could attain if it were in *autarky,* or self-sufficient, from then onward. These *enforcement constraints* are of the form

$$(3) \qquad \sum_{s=t}^{\infty} \beta^{s-t} U(c_{is}) \geq V_{it} = \sum_{s=t}^{\infty} \beta^{s-t} U(y_{is})$$

where V_{it} denotes the value of autarky from t onward, which is given by the value of utility in which consumers simply consume their endowment for t onward.

The constrained efficient allocations of this economy solve the planning problem of maximizing a weighted sum of the discounted utilities

$$(4) \qquad \max \left[\lambda_1 \sum_{t=0}^{\infty} \beta^t U(c_{1t}) + \lambda_2 \sum_{t=0}^{\infty} \beta^t U(c_{2t}) \right]$$

subject to the resource constraints (2) and the enforcement constraints (3) for $i = 1, 2$ and all t, where λ_1 and λ_2 are nonnegative initial weights.

An allocation $\{c_{1t}, c_{2t}\}_{t=0}^{\infty}$ is *constrained efficient* if it solves the planning problem for some nonnegative planning weights λ_1 and λ_2. We characterize these allocations as follows . Let $\beta^t \mu_{it}$ denote the multipliers on the enforcement constraints. Let $M_{it} = M_{it-1} + \mu_{it}$ and $M_{i,-1} = \lambda_i$. Then by grouping terms we can write the planning problem as

$$\max \sum_{t=0}^{\infty} \sum_{i} \beta^t [M_{it-1} U(c_{it}) + \mu_{it}(U(c_{it}) - V_{it}]$$

subject to the resource constraint (2). The first order conditions are summarized by

$$\frac{U'(c_{1t})}{U'(c_{2t})} = \frac{M_{2t}}{M_{1t}}$$

For notational simplicity, we use the normalized weight $z_t = M_{2t}/M_{1t}$ and the normalized multiplier $v_{it} = \mu_{it}/M_{it}$. Then the transition law for the z along with the first order conditions can be written as

$$(5) \qquad z_t = (\frac{1 - v_{1t}}{1 - v_{2t}}) z_{t-1}$$

$$(6) \qquad z_t = \frac{U'(c_{1t})}{U'(c_{2t})}$$

where $z_{-1} = \lambda_2/\lambda_1$.Thus, constrained efficient allocations are characterized by (5) and (6) along with the resource constraint and enforcement constraints for some sequence of relative weights z and multipliers v_i. Notice that, if in equilibrium some enforcement constraint is binding, the first order condition for relative consumption (6) is distorted away from those for the unconstrained efficient allocations in which $U'(c_{1t})/U'(c_{2t}) = \lambda_2/\lambda_1$ and the allocation will display less than perfect risk sharing.

We can get some intuition for how the binding pattern of the enforcement constraints is related to the allocations as follows. Combine (5) and (6) to give

$$(7) \qquad \frac{U'(c_{1t})}{U'(c_{2t})} = (\frac{1 - v_{1t}}{1 - v_{2t}})\frac{U'(c_{1t-1})}{U'(c_{2t-1})}$$

and realize that there are three possible binding patterns for the enforcement constraint: either country $1's$ constraint binds and country $2's$ constraint is slack $(v_{1t} > 0, v_{2t} = 0)$, country $2's$ constraint binds and country $1's$ constraint is slack $(v_{2t} > 0, v_{1t} = 0)$ or both countries's constraints are slack $(v_{1t} = v_{2t} = 0)$. If country $2's$ constraint binds in period t then

$$(8) \qquad \frac{U'(c_{1t})}{U'(c_{2t})} > \frac{U'(c_{1t-1})}{U'(c_{2t-1})}$$

so that the ratio of country $1's$ marginal utility to country $2's$ marginal utility increases relative to this ratio at $t-1$ with the reverse when country $1's$ constraint binds. If neither constraint binds then the ratio of marginal utilities stays the same. Of course, (8) also implies that if country $2's$ constraint binds then the consumer in country 1 have the higher intertemporal marginal rate of substitution from $t-1$ to t in that

$$(9) \qquad \frac{U'(c_{1t})}{U'(c_{1t-1})} > \frac{U'(c_{2t})}{U'(c_{2t-1})}$$

with the reverse when country $1's$ constraint binds.

This simple manipulation of (8) into (9) gives the intuition for the following lemma established by Alvarez and Jermann (2000).

Lemma 1. If $\{c_{1t}, c_{2t}\}$ is a constrained efficient allocation with

$$(10) \quad \sum_{s=t}^{\infty} \beta^{s-t} U(c_{js}) > \sum_{s=t}^{\infty} \beta^{s-t} U(y_{js})$$

then

$$(11) \quad \frac{U'(c_{jt+1})}{U'(c_{jt})} = \max_i \frac{U'(c_{it+1})}{U'(c_{it})}.$$

In words unconstrained consumers have the highest marginal rate of substitution. Alvarez and Jermann (2000) prove this using a simple variational argument, but for our purposes the algebra of (8) and (9) makes it obvious. We use this lemma when we construct asset prices for the decentralization with debt constraints. In that decentralization the asset prices are determined by the marginal rate of substitution for the unconstrained consumer(s) which, as this lemma shows, is the whatever marginal rate of substitution is the highest among the consumers.

We will be most interested in allocations for which the present value of the allocation, at the appropriately defined prices, is finite for each consumer. Letting $q_{0,t} = q_{0,1} q_{1,2} \dots q_{t-1,t}$ with

$$q_{t,t+1} = \max_i \frac{U'(c_{it+1})}{U'(c_{it})}$$

we say an allocation $\{c_{1t}, c_{2t}\}_{t=0}^{\infty}$ has *high implied interest rates* if for $i = 1, 2$,

$$(12) \quad \sum_{t=0}^{\infty} q_{0,t}(y_{1t} + y_{2t}) < \infty.$$

A. Decentralization with Debt Constraints

Here we consider decentralizing the constrained-efficient allocations with debt constraints along the lines of Alvarez and Jermann (2000). We will show that any constrained efficient allocation that has high implied interest rates can be decentralized as a competitive equilibrium with appropriately chosen debt constraints and initial assets.

In this economy the price of a claim to one unit of the consumption good in period $t+1$ in units of period t is denoted $q_{t,t+1}$ and a_{it+1} denotes the number such asset claims purchased by consumer i in period t. In this decentralization the representative consumer in country i chooses $\{c_{it}, a_{it+1}\}$ to solve

$$(13) \quad \max \sum_{t=0}^{\infty} \beta^t U(c_{it})$$

$$(14) \quad c_{it} + q_{t,t+1} a_{it+1} = y_{it} + a_{it}$$

$$a_{it+1} \geq B_{i,t+1}$$

with a_{i0} given where $B_{it+1} \leq 0$ specifies the lowest level of assets that the representative consumer in country i in period t is permitted to have. Thus,$\{B_{it+1}\}$ are a sequence of exogenous time-varying, country-specific debt constraints. A *competitive equilibrium with debt-constraints* $\{B_{1t+1}, B_{2t+1}\}$ together with initial assets a_{10} and a_{20} is a set of allocations $\{c_{1t}, c_{2t}\}$, asset holding $\{a_{1t+1}, a_{2t+1}\}$, and asset prices $\{q_{t,t+1}\}$ that i) for each i, $\{c_{it}, a_{it+1}\}$ solves (13) and ii) markets clear, so that

$$(15) \quad a_{1t+1} + a_{2t+1} = 0,$$

and (2) holds.

Letting $\beta^t \theta_t$ denote the multiplier on the debt constraint, the first order conditions are summarized by

$$(16) \quad q_{t,t+1} = \frac{\beta U'(c_{it+1})}{U'(c_{it})} + \frac{\theta_{it}}{U'(c_{it})}.$$

and the transversality condition

$$(17) \quad \lim_{t\to\infty} \beta^t U'(c_{it})[a_{it} - B_{it}] = 0.$$

Hence, the allocations and prices that constitute a competitive equilibrium are summarized by the resource constraint (2), the budget constraints (14), the first order conditions (16) with $\theta_{it} \geq 0$ and the transversality conditions (17).

Given a constrained efficient allocation $\{c_{1t}, c_{2t}\}$ with normalized multipliers $\{v_{1t}, v_{2t}\}$. We construct the asset prices, asset holdings and debt constraints that decentralize this allocation as follows. Let

$$(18) \quad q_{t,t+1} = \max_i \beta \frac{U'(c_{it+1})}{U'(c_{it})}$$

be the asset price and given this price and the allocations use (16) to define the multipliers θ_{it}. It is immediate that these multipliers have the right properties. If consumer i has the higher marginal rate of substitution so that $q_{t,t+1} = \beta U'(c_{it+1})/U'(c_{it})$ then $\theta_{it} = 0$. If consumer i has the lower marginal rate of substitution $U'(c_{it+1})/U'(c_{it}) < U'(c_{jt+1})/U'(c_{jt})$, then from (16) and (18) it follows that $\theta_{it} > 0$.

Using the transversality condition we can iterate on the consumer budget constraint to get an expression for the assets as

$$(19) \quad a_{it} = \sum_{s=t}^{\infty} q_{t,s}(c_{is} - y_{is})$$

where $q_{t,s} = q_{t,t+1}q_{t+1,t+2}\ldots q_{s-1,s}$. We set that initial assets $a_{i0} = \sum_{t=0}^{\infty} q_{0,t}(c_{it} - y_{it})$.

We set the debt constraints as follows. If the debt constraint binds for consumer i at t, so that $v_{it+1} > 0$, then we set the debt constraint $B_{it+1} = a_{it+1}$, so that the constrained consumer can borrow no more than his actual borrowing.

If the debt constraint is slack for consumer i at t, so that $v_{it} = 0$, then there are many ways to set the borrowing limit, all of which will be slack. The loosest is to set the limit equal to the present discounted value of future endowments, so $B_{it+1} = -\sum_{s=t+1}^{\infty} q_{t+1,s} y_{is}$. Alvarez and Jermann choose to set it according to the following counterfactual thought experiment. If at the constructed prices an unconstrained consumer happens to borrow exactly up to the limit in period t and then acts optimally from then on, this consumer will be indifferent between the proposed allocations and autarky. More formally, let $J_{it}(a_{it})$ denote the maximized value in (13) for some arbitrary level of initial assets, where have suppressed the dependence of this value on the current and future prices and debt constraints $\{q_{s,s+1}, B_{is+1}\}_{s=t}^{\infty}$. Then define debt constraints to be *not too tight* if the sequence $\{B_{it+1}\}$ satisfies

(20) $J_{it}(B_{it}) = V_{it}.$

Notice that (20) not only defines the debt constraints for the unconstrained consumer as we have discussed but applied to the constrained consumer it automatically implies that $B_{it+1} = a_{it+1}$.

To make our argument complete, we need to show that any constrained efficient allocation that satisfies the high implied interest rates condition (12) also satisfies the transversality condition (17). To see this note that with debt constraints that satisfy $B_{it+1} = -\sum_{s=t+1}^{\infty} q_{t+1,s} y_{is}$ for the unconstrained consumers and $B_{it+1} = a_{it+1}$ for the constrained consumer, from (19) it follows that $a_{it} - B_{it}$ is equal to $\sum_{s=t}^{\infty} q_{t,s} c_{is}$ for the unconstrained consumer and 0 for the constrained consumers. In either case, since c_{is} is nonnegative and satisfies the resource constraint, $a_{it} - B_{it} \leq \sum_{s=t}^{\infty} q_{t,s}(y_{1s} + y_{2s})$ and hence

$$
\begin{aligned}
(21) \qquad & \lim_{t\to\infty} \beta^t U'(c_{it})[a_{it} - B_{it}] \\
(22) \qquad \leq \; & U'(c_{i0}) \lim_{t\to\infty} \beta^t \frac{U'(c_{it})}{U'(c_{i0})} \sum_{s=t}^{\infty} q_{t,s}(y_{1s} + y_{2s}) \\
(23) \qquad \leq \; & U'(c_{i0}) \lim_{t\to\infty} \sum_{s=t}^{\infty} q_{0,s}(y_{1s} + y_{2s}) = 0
\end{aligned}
$$

where the inequality in (23) follows since, by construction, $q_{0,t} \geq \beta^t U'(c_{it})/U'(c_{i0})$, and the equality in (23) follows from the high implied interest rate condition.

From the construction it is immediate that the constrained efficient allocations $\{c_{1t}, c_{2t}\}$ together with the constructed asset positions $\{a_{1t}, a_{2t}\}$, debt prices $\{q_{t,t+1}\}$ and debt constraints $\{B_{1t+1}, B_{2t+1}\}$ form a competitive equilibrium with debt constraints. We have thus established the following.

Proposition 1. Any constrained efficient allocation that has high implied interest rates can be decentralized as a competitive equilibrium with debt constraints.

B. Decentralization with Debt Taxes

Here we discuss the how to decentralize the constrained efficient outcome as a competitive equilibrium with debt taxes. Here we show that if these debt taxes are appropriately chosen then the constrained efficient outcomes can be decentralized. In the next section we will allow the governments to purposefully choose these taxes.

In this economy the government of each country can tax payments made to foreigners and then rebate the proceeds in a lump sum fashion to its citizens. Except for these government policies, private markets function perfectly. We begin by setting up a competitive equilibrium with debt taxes. Consider the consumer and the government budget constraint for some arbitrarily given sequence of government policies and prices. Throughout we will focus on country 1, the notation for country 2 is analogous. It is convenient to define separate variables for saving and for borrowing. We let $s_{1t+1} \geq 0$ denote the savings or assets of a consumer in country 1, $b_{1t+1} \geq 0$ denote that consumer's borrowing or liabilities, and $\tau_{1t} \in [0,1]$ denote taxes levied by country 1 on payments from country 1 consumers to country 2 consumers. The problem for a consumer in country 1 is to maximize utility

$$\sum_{t=0}^{\infty} \beta^t U(c_{1t})$$

subject to the budget constraint

$$c_{1t} + p_{t,t+1}[s_{1t+1} - b_{1t+1}] = y_{1t} + (1 - \tau_{2t})s_{1t} - b_{1t} + T_{1t} \tag{24}$$

the nonnegativity constraints,

$$s_{it+1}, b_{it+1} \geq 0,$$

and bounds on debt $b_{1t+1} \leq \bar{b}$ where $\bar{b}$ is a large positive constant. Here $p_{t,t+1}$ is the price of a consumption at $t+1$ in units of date t, τ_{2t} is the tax rate by country 2 on payments s_{1t} that country 2 consumers make to country 1 consumers, T_{1t} are lump sum transfers from the government of country 1 to consumers in country 1. The initial assets s_{i0} and liabilities b_{i0} are given.

The government of country 1 chooses a tax rate on payments to foreigners τ_{1t} and rebates the revenues to the consumers of country 1 in a lump sum fashion, so that the government budget constraint in country 1 is $T_{1t} = \tau_{1t} b_{1t}$.

A *competitive equilibrium with debt taxes* $\{\tau_{1t}, \tau_{2t}\}_{t=0}^{\infty}$ together with initial assets and liabilities $\{s_{i0}, b_{i0}\}_{i=1,2}$ consists of an allocation $\{c_{1t}, c_{2t}\}_{t=0}^{\infty}$, assets $\{s_{1t+1}, s_{2t+1}\}_{t=0}^{\infty}$, liabilities $\{b_{1t+1}, b_{2t+1}\}_{t=0}^{\infty}$ and prices $\{p_{t,t+1}\}_{t=0}^{\infty}$ such that *i*) for each i, $\{c_{it}, s_{it+1}, b_{it+1}\}$ solves the consumer problem and *ii*) markets clear, so that

$$s_{1t+1} = b_{2t+1} \text{ and } b_{1t+1} = s_{2t+1}$$

and (2) holds.

To understand the consumer and the government budget constraints suppose that at time $t-1$ the representative consumer in country 1 lends $p_{t-1,t}s_{1t}$ at $t-1$ in exchange for a promise to receive, in period t, s_{1t} minus the taxes $\tau_{2t}s_{1t}$ levied by country 2 on repayments to country 1. Consumers in country 2 repay at total of $s_{1t} = b_{2t}$ with $(1-\tau_{2t})s_{1t}$ going to country 1 consumers and $\tau_{2t}s_{1t} = \tau_{2t}b_{2t}$ going to the government of country 2. The government of country 2 then redistributes its tax revenue in a lump sum fashion to its residents.

For brevity's sake from now on we let U'_{it} denote $U_{ct}(c_{it})$. Using this notation, the first order conditions for the consumer's problem are

$$(25) \quad p_{t,t+1}U'_{1t} \geq \beta U'_{1t+1}(1 - \tau_{2t+1})$$

with equality if $s_{1t+1} > 0$, so that country 1 is lending to country 2

$$(26) \quad p_{t,t+1}U'_{1t} \leq \beta U'_{1t+1}$$

with equality if $b_{1t+1} > 0$, so that country 2 is lending to country 1, where here and throughout we presume the debt constraint $b_{1t+1} \leq \bar{b}$ does not bind. The transversality condition is

$$(27) \quad \lim_{t\to\infty} \beta^t p_{t,t+1}U'_{1t}[s_{it+1} - b_{it+1} + \bar{b}] = 0.$$

We now show the following analog of Proposition 1.

Proposition 2. Any constrained efficient allocation that has high implied interest rates can be decentralized as a competitive equilibrium with debt taxes.

Proof. We decentralize a constrained efficient allocations with high implied interest rates as follows. We set the prices

$$(28) \quad p_{t,t+1} = \beta \min_i \frac{U'_{it+1}}{U'_{it}}$$

We set the taxes as follows. If at the given allocations $U'_{1t+1}/U'_{1t} \geq U'_{2t+1}/U'_{2t}$ we set $\tau_{1t+1} = 0$ and

$$(29) \quad 1 - \tau_{2t+1} = \frac{U'_{2t+1}/U'_{2t}}{U'_{1t+1}/U'_{1t}},$$

while if $U'_{1t+1}/U'_{1t} < U'_{2t+1}/U'_{2t}$, we set $\tau_{2t+1} = 0$ and

$$(30) \quad 1 - \tau_{1t+1} = \frac{U'_{1t+1}/U'_{1t}}{U'_{2t+1}/U'_{2t}}.$$

Notice, for later, that the constructed taxes lie between 0 and 1 and that

$$(31) \quad (1 - \tau_{1t+1})(1 - \tau_{2t+1}) = \frac{\min_i U'_{it+1}/U'_{it}}{\max_i U'_{it+1}/U'_{it}}$$

For the setting of assets and liabilities we set

$$(32) \quad s_{it+1} - b_{it+1} = \frac{\max_i U'_{it+1}/U'_{it}}{\min_i U'_{it+1}/U'_{it}} \sum_{s=t+1}^{\infty} q_{t+1,s}(y_{is} - c_{is}).$$

with $q_{t,s} = q_{t,t+1}q_{t+1,t+2} \ldots q_{s-1,s}$ and $q_{t,t+1} = \beta \max_i U'_{it+1}/U'_{it}$. If the right-side of (32) is nonnegative and we set $b_{it+1} = 0$ while if the right-side of (32) is negative, we set $s_{it+1} = 0$.

We can see that the constructed prices, taxes, assets and liabilities are a competitive equilibrium with taxes as follows. To check the constructed prices, notice that in equilibrium at any date t, either country 1 is lending to country 2, so that $s_{1t+1} = b_{2t+1} > 0$ and $s_{2t+1} = b_{1t+1} = 0$ and

$$(33) \quad p_{t,t+1} = \beta \frac{U'_{1t+1}}{U'_{1t}}(1-\tau_{2t+1}) = \beta \frac{U'_{2t+1}}{U'_{2t}}$$

, or country 2 is lending to country 1 so that $s_{2t+1} = b_{1t+1} > 0$ and $s_{1t+1} = b_{2t+1} = 0$ and

$$(34) \quad p_{t,t+1} = \beta \frac{U'_{1t+1}}{U'_{1t}} = \beta \frac{U'_{2t+1}}{U'_{2t}}(1-\tau_{1t+1})$$

or neither is lending so that $s_{1t+1} = b_{2t+1} = s_{2t+1} = b_{1t+1} = 0$ so that

$$(35) \quad \beta \max_i \frac{U'_{it+1}}{U'_{it}} \leq p_{t,t+1} \leq \beta \min_i \frac{U'_{it+1}}{U'_{it}}$$

When either (33) or (34) hold it is clear that $p_{t,t+1}$ satisfies (28) while if (35) holds the price $p_{t,t+1}$ can take on a range of values one of which is given by (28). By inspection the constructed taxes satisfy (33)-(35).

To check the constructed assets and liabilities, substitute the government budget constraint $T_{1t} = \tau_{2t} b_{1t,}$ so that

$$(36) \quad c_{1t} + p_{t,t+1}(s_{1t+1} - b_{1t+1}) = (1-\tau_{1t})s_{1t} - (1-\tau_{2t})b_{1t} + y_{1t}$$

where if $s_{1t} > 0$, $b_{1t} = 0$, and $(1-\tau_{1t})(1-\tau_{2t}) = (1-\tau_{1t})$, while if $s_{1t} = 0$ and $b_{1t} \geq 0$ then $(1-\tau_{1t})(1-\tau_{2t}) = (1-\tau_{2t})$ and hence, in general, we can write (36) as

$$(37) \quad c_{1t} + p_{t,t+1}(s_{1t+1} - b_{1t+1}) = (1-\tau_{1t})(1-\tau_{2t})(s_{1t} - b_{1t}) + y_{1t}.$$

Using the transversality condition we can iterate on (37) to obtain

$$(38) \quad s_{1t+1} - b_{1t+1} = \frac{1}{(1-\tau_{1t+1})(1-\tau_{2t+1})} \sum_{s=t+1}^{\infty} \rho_{t+1,s}(y_{1s} - c_{1s})$$

where $\rho_{t,s} = \rho_{t,t}\rho_{t,t+1}\dots\rho_{s-1,s}$, $\rho_{t,t} = 1$ and

$$\rho_{t,t+1} = \frac{p_{t,t+1}}{(1-\tau_{1t})(1-\tau_{2t})}.$$

Using (28) and (31) it is clear that $\rho_{t,t+1} = q_{t,t+1}$ and hence $\rho_{t,s} = q_{t,s}$ and using this relation along with (31) in (38) reduces to (32). The final step is to show that at the constructed allocations, if the high implied interest rate condition (12) holds then the transversality condition (27) holds. Notice first that

$$\bar{b} \lim_{t\to\infty} \beta^t p_{t,t+1} U'_{1t} = 0. \tag{39}$$

To see this note that since $p_{t,t+1}$ satisfies (28), $p_{t,t+1}U'_{1t} \leq U'_{1t+1}$, while the high implied interest rate condition implies

$$0 = \lim_{t\to\infty} \sum_{s=t}^{\infty} q_{0,t}(y_{1s}+y_{2s}) \geq \underline{y} \lim_{t\to\infty} \sum_{s=t}^{\infty} q_{0,t} \geq \underline{y} \lim_{t\to\infty} q_{0,t} \geq \frac{\underline{y}}{U'(c_{i0})} \lim_{t\to\infty} \beta^t U'_{1t} \geq 0$$

so $\lim_{t\to\infty} \beta^t U'_{1t} = 0$. Since $p_{t,t+1}$ satisfies (28), $p_{t,t+1}U'_{1t} \leq U'_{1t+1}$ and hence (39) holds. Thus, we need only show $\lim_{t\to\infty} \beta^t p_{t,t+1} U'_{1t}[s_{1t+1} - b_{1t+1}] = 0$. But,

$$\lim_{t\to\infty} \beta^t p_{t,t+1} U'_{1t}[s_{1t+1} - b_{1t+1}] =$$

$$= \lim_{t\to\infty} \beta^t U'_{1t} \max_i \left(\frac{U'_{it+1}}{U'_{it}}\right) \sum_{s=t+1}^{\infty} q_{t+1,s}(y_{is} - c_{is}) \tag{40}$$

$$= \frac{U'_{10}}{\beta} \lim_{t\to\infty} \beta^t \frac{U'_{1t}}{U'_{10}} \sum_{s=t+1}^{\infty} q_{t,s}(y_{is} - c_{is}) \tag{41}$$

$$\leq \frac{U'_{10}}{\beta} \lim_{t\to\infty} \sum_{s=t+1}^{\infty} q_{0,s}(y_{is} - c_{is}) \tag{42}$$

$$\leq \frac{U'_{10}}{\beta} \lim_{t\to\infty} \sum_{s=t+1}^{\infty} q_{0,s}(y_{1s} + y_{2s}) = 0 \tag{43}$$

Where (40) follows from (28) and (32), (41) from the definition of $q_{t,t+1}$, (42) follows by the definition of $q_{0,s}$, and the inequality in (43) follows from the resource constraint while the equality follows from (12). *Q.E.D.*

Although we have shown that debt taxes or borrowing constraints can be used to decentralize the same allocation there is an important difference in how prices and interest rates are defined in the two decentralizations. In the debt constraints economy the interest rate $(1/q_{t,t+1})$ is given by the marginal rate of substitution of the agent whose enforcement constraint is not binding, while in the economy with debt taxes the interest rate $(1/p_{t,t+1})$ is given by the marginal rate of substitution of the agent whose enforcement constraint is binding. So, in general, the decentralization with debt taxes will produce "higher" interest rates than the decentralization with borrowing constraint. At an intuitive level, in the decentralization with debt constraints, interest rates are low and the debt constraint is needed to prevent the agent with the binding enforcement constraint from borrowing "too much". Conversely in the decentralization with debt taxes interest rate are high and taxes are needed to prevent the agent whose enforcement constraint is not binding from saving "too much".

One implication of these different decentralizations how interest rates respond to enforcement frictions. If we start from a frictionless economy and add enforcement problems we have the following: In the debt-constraint decentralization, the interest rate falls relative to the frictionless economy while in the debt-tax decentralization the interest rate rises relative the frictionless economy. We find this feature of the debt tax decentralization somewhat appealing.

2. Endogenizing the Debt Taxes

In our decentralizations we have used the constrained efficient allocations to construct the appropriate debt constraints or debt taxes that decentralize the given allocations, but we do not have much of a story about where these constraints or taxes come from. Here we provide a story for how the constructed debt taxes may come out of an equilibrium of a dynamic game with both government behavior and consumer behavior endogenous.

We set up this game as follows. In each period the consumers and the government can vary their decisions, depending on the history of government policies up to the time the decision is made. We let $\pi_t = (\tau_{1t}, \tau_{2t})$ denote the two governments' policies in period t. At the beginning of period t, the government of each country chooses a current policy as a function of the history of past government policies $h_{t-1} = (\pi_0, ..., \pi_{t-1})$ together with a contingency plan for setting future policies for all possible future histories. Let $\sigma_{it}(h_{t-1})$ denote the period t tax on debt repayments chosen by the government of country i when faced with history h_{t-1}. After the governments sets the current policies, consumers and firms make their decisions. Faced with the history $h_t = (h_{t-1}, \pi_t)$ consumers in country i choose their time t consumption and new assets and liabilities denoted $f_{it}(h_t) = (c_{it}(h_t),\ s_{it}(h_{it}), b_{it}(h_t))$. The prices are a function of the history and are denoted $p_{t,t+1}(h_t)$. Let $\sigma = (\sigma_1, \sigma_2)$ where σ_i denotes the infinite sequence of functions (σ_{it}), and use similar notation for the other variables.

For some given initial assets and liabilities, a *sustainable equilibrium* is a triple (σ, f, p) such that i) for $i = 1, 2$, for every history h_t, the consumer allocations $f_{is}(h_s)$ for $s = t, ..,$

solve

$$\max \sum_{s=t}^{\infty} \beta^{s-t} U(c_{is})$$

subject to

$$\begin{aligned} & c_{1s} + p_{s,s+1}(h_s)(s_{1s+1} - b_{1s+1}) \\ = \; & y_{1s} + (1 - \tau_{2s}(h_{s-1})s_{1s}(h_{s-1})) - b_{1s}(h_{s-1}) + T_{1s}(h_{s-1}) \end{aligned}$$

where the future histories policies and prices are given by induced from h_t, by σ and p in the obvious fashion. That is, $h_{t+1} = (h_t, \sigma_{t+1}(h_t))$, $h_{t+2} = (h_t,\sigma_{t+1}(h_t),\sigma_{t+2}(h_t, \sigma_{t+1}(h_t))$, and given these induced future histories, the policies and prices are given by $\sigma_s(h_{s-1})$ and $p_s(h_s), ii)$ for every history h_t, the goods and bond markets clear and the government budget constraint holds from $s = t, ...,$ so that

$$c_{1s}(h_s) + c_{2s}(h_s) = y_{1s} + y_{2s}$$

as well as $s_{1s}(h_s) = b_{2s}(h_s)$, $s_{2s}(h_s) = b_{1s}(h_s)$ and $T_{1s}(h_{s-1}) \equiv \tau_{1s}(h_{s-1})b_{1s}(h_{s-1})$, where the future histories h_s are induced by σ in the obvious fashion, $iii)$ for every history h_{t-1}, the government of $1's$ policies from t onwards, σ_{1s} for all $s \geq t$ solve

$$\max \sum_{s=t}^{\infty} \beta^{s-t} U(c_{1s}(h'_{s-1}))$$

where $h'_t = (h_{t-1}; \sigma'_{1t}(h_{t-1}), \sigma_{2t}(h_{t-1})$, and $h'_{t+1} = (h'_t; \sigma'_{1t}(h'_t), \sigma_{2t}(h'_t))$ and so on. A similar condition holds for the government of 2.

Notice that in the definition, we require that both the consumers and the government act optimally for every history of policies—even for histories not induced by the governments' policy plans. This requirement is analogous to the requirement of perfection in a game. In this definition the consumers act competitively in that they take current policies and prices and the evolution of future histories as unaffected by their actions. The governments are not competitive. The government of country 1, for example, takes the allocation rules f_1 and f_2, the price function p and the policies plan of the government of 2, σ_2 as given. But the government of 1 realizes that it can affect outcomes both directly through having the consumer face a different tax on payments to foreigners and indirectly through affecting the evolution of the future history, and thus affecting the policies chosen by the other government, the allocations chosen by the consumers, and the prices.

Recall that a sustainable equilibrium (σ, f, p) is a sequence of functions that specify policies, allocations and prices for all possible histories. Thus, when we start from the null history at date 0, a sustainable equilibrium induces a particular sequence of policies, allocations, and prices that we denote by (π, x, p). We call this the *outcome* induced by the sustainable equilibrium. In what follows we adapt the work of Chari and Kehoe (1990, 1993), which builds on the work of Abreu (1988), to characterize these outcomes.

We first construct a sustainable equilibrium that we call the *autarky equilibrium*. We then characterize the allocations that can be induced by reverting to this autarky equilibrium after deviations. We define the autarky allocations rules f^a, price rule p^a and policy plans σ^a starting from some given initial assets and liabilities as follows. The policy plan $\sigma^a_{it}(h_{t-1}) = 1$, for all i and t. The autarky allocation rules and price rules are defined as follows. Given any history h_t, the autarky prices $p^a_{t,t+1}(h_t)$ and allocations $(c^a_{it}(h_t), s^a_{it+1}(h_t), b^a_{it+1}(h_t))$ are defined as follows. The prices of debt and the quantities of assets and liabilities are identically zero, so $p^a_{t,t+1}(h_t) = s^a_{it+1}(h_t) = b^a_{it+1}(h_t) = 0$ while the autarky allocations are given by $c^a_{it}(h_t) = y_{it}$.

The utility of autarky for consumer i in period t is V_{it}.

We now show characterize the outcomes that can be sustained by a set of plans called the $revert - to - autarky$ plans which are defined as follows. For an arbitrary sequence of policies, allocations, and prices (π, x, p), these plans specify continuation with the candidate sequences (π, x, p) as long as the specified policies have been chosen in the past; otherwise, they specify revert to the autarky plans (σ^a, f^a, p^a). We then have

Proposition 3. An arbitrary triple of sequences (π, x, p) can be sustained by the revert-to-autarky plans if and only if (i) the sequence is a competitive equilibrium with debt taxes, and (ii) for $i = 1, 2$, for every t, the following inequality holds:

$$(44) \quad \sum_{s=t}^{\infty} \beta^{s-t} U(c_{is}) \geq V_{it},$$

Proof. Suppose, first, that (π, x, p) can be sustained by the revert-to-autarky plans, that is, suppose the associated revert-to-autarky plans (σ, f, p) constitute a sustainable equilibrium. From the definition of a sustainable equilibrium, consumer optimality requires that x maximize consumer welfare at date 0. This requirement together market clearing ensures that this sequence is a competitive equilibrium at date 0. Next we claim that (44) holds for all i and t. Note that a feasible policy for the government of i at t is to choose the autarky policies for all $s \geq t$ by taxing repayments to foreigners at rate 1. This policy will lead to a continuation utility of V_{it}^a and hence optimality of government policy ensures (44).

Now suppose that some arbitrary triple of sequences (π, x, p) satisfies (i), and (ii). We show that the associated revert-to-autarky plans constitute a sustainable equilibrium. Consider, first, histories for which there have been no deviations from π up until t. Since (π, x, p) is a competitive equilibrium at date 0, x is optimal for consumers at date 0 given π and

p, and thus the continuation of x is optimal for consumers when faced with the continuation of π and p. In terms of government optimality, consider the situation of the government of country 1. If it deviates at date t, the consumers in both countries and the government of country 2 will revert to the autarky allocation rules and autarky policy plans from time t onward. Under these allocation rules foreigners will never lend to the domestic consumers, regardless of the policies chosen by the government of 1. Thus, the best the government of 1 can obtain is the value of autarky from then on given by the right-hand side of (44). Given the assumed inequality, then sticking to the specified plan is optimal.

Consider, next, histories for which there has been a deviation from π before t. Clearly the autarky plans from then on are sustainable. From the consumer's point of view, given that no debt will be repaid it is not optimal to lend. The price of debt is zero since the value to a foreigner of a promise to pay one unit tomorrow, net of taxes equal to one unit, is worthless. Thus, the consumer is indifferent between any amounts to borrow or lend because all have value 0 and all pay 0. From the government's point of view, given that the foreign government never allows its citizens to repay their debts, regardless of the domestic government's actions, it is optimal to prevent its the domestic agents from repaying their foreign debt. *Q.E.D.*

Combining Propositions 2 and 3 we immediately obtain the following proposition.

Proposition 4. Any constrained efficient allocation is the outcome of a sustainable equilibrium.

3. Adding capital

We now explore how our results change when we move from a pure exchange economy to a growth model with capital. We first show in a constrained efficient allocation if the enforcement constraints bind then the Euler equation for capital is distorted. This result implies that a competitive equilibrium with debt constraints alone cannot decentralize such an allocation. But, if in addition to the debt constraints, we allow for constraints that limit the amount of capital that can be accumulated, namely *capital constraints*, then we can decentralize the constrained efficient allocations. We argue that such capital constraints are not necessarily intuitively appealing and turn to our preferred decentralization. In that decentralization a combination of debt taxes and capital taxes can decentralize the constrained efficient allocations. Finally, we sketch out how these taxes may endogenously arise in a dynamic game similar to that in the previous section. .

We modify our pure exchange economy in several ways. The preferences are the same as before. The resource constraint is now

$$(45) \quad c_{1t} + c_{2t} + k_{1t+1} + k_{2t+1} = A_{1t} f(k_{1t}) + A_{2t} f(k_{2t}) + (1-\delta)(k_{1t} + k_{2t})$$

with k_{i0} given, where k_{it+1} is the capital stock chosen in period t for use in production at $t+1$, $f(k)$ is a standard production function that is increasing, concave, and continuously differentiable and satisfies the standard Inada conditions, and A_{it} are country-specific deterministic exogenous shocks, and δ is the depreciation rate. The enforcement constraints are now

$$(46) \quad \sum_{s=t}^{\infty} \beta^{s-t} U(c_{is}) \geq V_{it}(k_{it})$$

where

$$(47) \quad V_{it}(k_{it}) = \max \sum_{t=0}^{\infty} \beta^t U(c_{it})$$

$$(48) \quad c_{it} + k_{it+1} = A_{it} f(k_{it}) + (1-\delta) k_{it}$$

Notice that the problem with (financial) autarky reduces to that of a planning problem of a closed economy growth model. Notice also that the value of utility under autarky in period t depends on the amount of capital located in country i in that period, namely k_{it}. The derivatives of this value, namely $V'(k_{it})$, will be the root problem behind why the equilibrium with debt constraints alone cannot decentralize the constrained efficient allocations.

The constrained efficient allocations of this economy solve the planning problem of maximizing a weighted sum of the discounted utilities

$$\text{(49)} \quad \max \left[\lambda_1 \sum_{t=0}^{\infty} \beta^t U(c_{1t}) + \lambda_2 \sum_{t=0}^{\infty} \beta^t U(c_{2t}) \right]$$

subject to the resource constraints (45) and the enforcement constraints (46) for $i = 1, 2$ and all t, where λ_1 and λ_2 are nonnegative initial weights.

An allocation $\{c_{1t}, c_{2t}, k_{1t+1}, k_{2t+1}\}_{t=0}^{\infty}$ is *constrained efficient* if it solves the planning problem for some nonnegative weights λ_1 and λ_2. Let $\beta^t \mu_{it}$ denote the multipliers on the enforcement constraints. Let $M_{it} = M_{it-1} + \mu_{it}$and $M_{i,-1} = \lambda_i$. Then by grouping terms we can write the planning problem as

$$\max \sum_{t=0}^{\infty} \sum_{i} \beta^t [M_{it-1} U(c_{it}) + \mu_{it}(U(c_{it}) - V_{it}(k_{it})]$$

subject to (45). The first order conditions are summarized by

$$\frac{U'(c_{1t})}{U'(c_{2t})} = \frac{M_{2t}}{M_{1t}}$$

$$U(c_{it}) = \beta \{ \frac{M_{it+1}}{M_{it}} U'(c_{it+1})[F'(k_{it+1}) + 1 - \delta] - \frac{\mu_{it+1}}{M_{it}} V'(k_{it+1}) \}$$

Rewriting these using $z_t = M_{2t}/M_{1t}$ and $v_{it} = \mu_{it}/M_{it}$ gives that the transition law for the z along with the first order conditions can be written as (5), (6) and

$$\text{(50)} \quad U(c_{it}) = \beta \{ \frac{U'(c_{it+1})}{1 - v_{it+1}} [F'(k_{it+1}) + 1 - \delta] - \frac{v_{it+1}}{1 - v_{it+1}} V'(k_{it+1}) \}$$

where $z_{-1} = \lambda_2/\lambda_1$. Equation (50) is the Euler equation for capital accumulation in the economy with enforcement constraints. It is distorted away from the familiar Euler equation from the standard growth model that would arise in the absence of such constraints, namely,

$$(51) \quad U(c_{it}) = \beta U'(c_{it+1})[F'(k_{it+1}) + 1 - \delta].$$

Notice that if v_{it+1} were equal to zero then (50) would reduce to (51).

A. Decentralization with Debt Constraints

We now show that a competitive equilibrium with debt constraints alone cannot decentralize the constrained efficient allocations, except for the trivial case in which the enforcement constraints never bind.

Consider an economy with two countries $i = 1, 2$ each of which has a representative consumer. Each consumer owns his production unit. Only that consumer can work with that unit but the consumer can borrow and lend from anyone else, subject to some time varying borrowing constraints. The representative consumer in country i solves the problem

$$(52) \quad \max \sum_{t=0}^{\infty} \beta^t U(c_{it})$$

$$(53) \quad c_{it} + q_{t,t+1} a_{it+1} + k_{it+1} = f(k_{it}) + (1-\delta) k_{it} + a_{it}$$

$$(54) \quad a_{it+1} \geq B_{it+1}$$

where B_{it+1} is an exogenous time-varying, agent-specific borrowing constraint. (We can imagine that $k_{it} \geq 0$, but when f satisfies the Inada conditions this will never bind, so we ignore it.) Letting $\beta^t \theta_{it}$ denote the multiplier on the borrowing constraint we can summarize the first order conditions by

$$q_{t,t+1} = \frac{\beta U'(c_{it+1})}{U'(c_{it})} + \frac{\theta_{it}}{U'(c_{it})}$$

(55) $U'(c_{it}) = \beta U'(c_{it+1})[f'(k_{it+1}) + (1-\delta)]$

with $\theta_t \geq 0$.

Proposition 5. If the enforcement constraint ever binds in the constrained efficient allocation, then the constrained efficient allocations cannot be decentralized as an equilibrium with debt constraints.

Proof. By construction the normalized multiplier $v_{it+1} \in [0,1]$. Suppose, by way of contradiction, that the enforcement constraint binds in some period t, so that v_{it+1} is strictly positive. Since U' and F' are positive and V' is negative then the right side of (50), namely

$$\frac{U'(c_{it+1})}{1-v_{it+1}}[F'(k_{it+1}) + 1 - \delta] - \frac{v_{it+1}}{1-v_{it+1}}V'(k_{it+1})$$

is strictly larger than the right side (55). Thus if (50) holds at the constrained efficient allocation, the Euler equation in the decentralized equilibrium (55) cannot also hold. *Q.E.D.*

Imagine that in addition to the borrowing constraint we add a capital constraint, namely a time-varying constraint on the amount of capital that can be saved of the form

(56) $k_{it+1} \leq D_{it+1}$

where $\{D_{it+1}\}$ is a sequence of constants. It should be fairly obvious that if we choose these constants appropriately then we can decentralize the constrained efficient allocations. To see this imagine adding (56) to the consumer's problem (52). Letting $\beta^t \lambda_{it}$ denote the multiplier on (56) then the first order condition for capital (55) is changed to

(57) $U'(c_{it}) = \beta U'(c_{it+1})[f'(k_{it+1}) + (1-\delta)] - \lambda_{it}.$

With the appropriate choice of the capital constraints, the multiplier λ_{it} can be set so that (57) coincides with (50). (For an alternative approach that arrives at the same conclusion see Seppala (1999).)

B. Decentralization with Taxes

Consider now decentralizing the constrained efficient outcomes as a competitive equilibrium with taxes on capital income together with taxes on debt. With these two taxes we can both mimic the distorted first order conditions that define the constrained efficient outcomes.

The problem for a consumer in country 1 is to maximize utility

$$\sum_{t=0}^{\infty} \beta^t U(c_{1t})$$

subject to the budget constraint

$$(58) \quad c_{1t} + p_{t,t+1}[s_{1t+1} - b_{1t+1}] + k_{it+1} = w_{1t} + (1 - \tau_{2t})s_{1t} - b_{1t} + R_{it}k_{it} + w_{1t} + T_{1t}$$

and the nonnegativity constraints $s_{it+1}, b_{it+1} \geq 0$ and s_{i0}, b_{i0} and k_{i0} are given. Here $R_{it} = 1 + [1 - \eta_{it}][r_{it} - \delta]$ is the gross return on capital after taxes and depreciation, r_{it} is the before tax return and η_{it} is the tax on capital income net of depreciation $(r_{it} - \delta)$. We can summarize firm behavior by conditions for rental rates and wage rates

$$(59) \quad r_{it} = f'(k_{it}) \text{ and } w_{it} = f(k_{it}) - k_{it}f'(k_{it})$$

A *competitive equilibrium with taxes* $\{\tau_{1t}, \tau_{2t}, \eta_{1t}, \eta_{2t}\}_{t=0}^{\infty}$ together with initial assets, liabilities and capital stocks $\{s_{i0}, b_{i0}, k_{i0}\}_{i=1,2}$ consists of allocations $\{c_{1t}, c_{2t}, k_{1t+1}, k_{2t+1}\}_{t=0}^{\infty}$, assets $\{s_{1t+1,}s_{2t+1}\}_{t=0}^{\infty}$, liabilities$\{b_{1t+1,}b_{2t+1}\}_{t=0}^{\infty}$ and prices $\{p_{t,t+1}, r_{it}, w_{it}\}_{t=0}^{\infty}$ such that i) for each i, $\{c_{it}, s_{it+1}, b_{it+1}, k_{it+1}\}$ solves the consumer problem and $ii)$ markets clear, so that

$$s_{1t+1} = b_{2t+1} \text{ and } b_{1t+1} = s_{2t+1}$$

and (45) hold.

The construction of the debt prices, debt prices, assets and liabilities is nearly identical to that for the pure exchange economy. The rental rates and wage rates are given by (59) while the tax on capital income is backed out from the Euler equation

$$U'(c_{it}) = \beta U'(c_{it+1})[1 + (1 - \eta_{it+1})(F_{kit} - \delta)].$$

The following proposition is then immediate.

Proposition 6. Any allocation that satisfies the resource constraint and has high implied interest rates can be decentralized as a competitive equilibrium with debt and capital taxes.

It is straightforward to show that any constrained efficient outcome is the outcome of a suitably defined sustainable equilibrium.

4. Conclusion

We have proposed a new decentralization of constrained efficient allocations in which the forces that give rise to the limited risk sharing are more explicitly modeled than in the existing literature. The decentralization is intuitively appealing when applied international risk sharing problems for economies with capital and limited commitment. In a similar vein it may be possible to similarly model the forces that limit risk-sharing in other decentralizations. For example, it should be possible to describe an equilibrium in which the borrowing constraints studied by Alvarez and Jermann (2000) are explicitly chosen by financial intermediaries in an appropriately defined dynamic game.

In this paper we have focused on the deterministic case to economize on notation but all the results analyzed in the paper immediately generalize to a stochastic economy, provided that debt constraints, capital constraints and taxes can be state-contingent.

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