Federal Reserve Bank of Minneapolis
Research Department

# Designing Optimal Disability Insurance*

Mikhail Golosov and Aleh Tsyvinski

Working Paper 628

January 2003

ABSTRACT

In this paper we describe how to optimally design a disability insurance system. The key friction in the model is imperfectly observable disability. We solve a dynamic mechanism design problem and provide a theoretical and numerical characterization of the social optimum. We then propose a simple tax system that implements an optimal allocation as a competitive equilibrium. The tax system that we propose includes only taxes and transfers that are similar to those already present in the U.S. tax code: a savings tax and an asset-tested transfer program. Using a numerical simulation, we compare our optimal disability system to the current disability system. Our results suggest a significant welfare gain from switching to an optimal system.

*Golosov, Tsyvinski, University of Minnesota and Federal Reserve Bank of Minneapolis. We thank Daron Acemoglu, George-Marios Angeletos, Marco Bassetto, Hal Cole, Larry Jones, Patrick Kehoe, Robert Lucas, Jr., Lee Ohanian, Chris Phelan, Nancy Stokey, and especially V.V. Chari and Narayana Kocherlakota for their comments. 

## 1. Introduction

The Social Security Disability Insurance (SSDI) program is one of the largest social insurance programs in the United States. In 2001, the program provided income to more than 6 million individuals, which accounted for 14 percent of Social Security beneficiaries. The program cost $61 billion[1], constituted 15 percent of Social Security benefits, and amounted to 0.5 percent of gross domestic product (GDP). The size of the program far surpassed spending on unemployment insurance, food stamps, or any other similar program (SSA 2000). Perhaps not unrelatedly, disability is a common and significant shock to productivity. Except for SSDI few other options provide protection against disability risk. For example, only 25 percent of private-sector employees receive long-term disability coverage (SSA 2001).

In this paper we describe how to optimally design a disability insurance system. As in the classical work of Diamond and Mirrlees (1978, 1986), we assume it is difficult or impossible to know whether an individual is truly disabled and that disability is a permanent state. Given these assumptions, we solve a dynamic mechanism design problem and provide a partial theoretical and full numerical characterization of the social optimum. We then propose a simple tax system that implements an optimal allocation as a competitive equilibrium for an agent[2].

While there are multiple ways to implement an optimal allocation, the most obvious being a direct mechanism, the difficulty is in constructing a mechanism that includes only taxes modern governments currently use. The tax system that we propose includes only taxes

[1] The cost of cash benefits and Medicare and Medicaid health care benefits was about $101 billion in 1994 (Stoddard et al. 1998).

[2] By *implementation* we mean finding a tax system such that a solution to a competitive equilibrium problem coincides with the optimal solution.

and transfers that are similar to those already in the U.S. tax code: a savings tax and an asset-tested transfer program. We prove that a tax system consisting of a tax on savings accumulated while an agent is able and asset-tested history-dependent disability transfers an agent receives only if his assets are sufficiently low implements an optimal allocation. We show that asset-testing is a necessary component of implementation that allows control of individual savings.

We parameterize the model by solving a stylized version of the current social insurance system in the United States. We model the stylized insurance system as an integrated insurance system which is consistent with a view of the Social Security system as a joint disability and retirement insurance system. (See Diamond and Mirrlees (1978) and Mulligan and Sala-i-Martin (1999).)

We numerically evaluate features of the optimal allocations. We determine that consumption falls significantly after an individual becomes disabled, consumption of disabled and able depends on work history, and savings distortion is quantitatively significant.

In a calibrated model economy, we compare the welfare from the optimal system to that obtained under the current system. Our numerical results suggest a significant welfare gain from switching to a new system of social insurance (both disability and retirement) equal to 3.8 percent of consumption in each history. The welfare gain from the improvement of the disability insurance program is equal to 1 percent of consumption and is generated by providing better insurance against a permanent disability shock.

The key to our analysis is that we assume that disability is imperfectly observable. In practice, determining disability status proves to be very difficult. The statutory definition of disability as inability to engage in any "substantial gainful activity" (U.S. Department of

Health and Human Services 1988) is very broad. Multiple medical and vocational factors are taken into account when determining whether an individual is eligible for disability benefits. However, even determination of medical factors is often subjective. In 2001, a share of awards to applicants with difficult-to-verify criteria, such as mental disorders (mainly mental stress and excluding retardation) and a disease of the musculoskeletal system (typically back pain), constituted around 50 percent of total awards. In cases of multiple medical impairments, disability is even more difficult to determine. Vocational factors that are also very subjective accounted for 37 percent of SSDI awards in 1997. At the appeals stage the most often argued cause is pain. Administrative law judges place even smaller weight on medical evidence and consult independent medical experts only in 8 percent of the cases (U.S. GAO 1997).

The rest of the paper is structured as follows. In the next section, we describe the setup of the model. In Section 3, we provide a partial theoretical characterization of the optimum. In Section 4, we discuss implementation of the optimum. In Section 5, we calibrate the model. In Section 6, we provide a full quantitative characterization of the optimum. In Section 7, we discuss related literature. Finally, we conclude in Section 8.

## 2. Setup

In our model economy a unit measure of agents live for $T$ periods. In each period $t$ two types of goods exist a consumption-capital good and labor.

An agent in the economy has preferences defined over lifetime consumption and leisure which are represented by a separable utility function of the form

$$\sum_{t=1}^{T} \beta^{t-1}[u(c_t) + v(l_t)],\ 0 < \beta < 1,$$

where $c_t$ and $l_t$ denote the period $t$ consumption and labor of an agent.

Agents' skills evolve stochastically over time. In each period $t$ an agent's skill $\theta_t$ can take two values. A skill can be equal to zero, in which case an agent is disabled, or can be positive, in which case he is able. When an agent is able to work, he may have different values of his skill depending on age. We assume that disability is an absorbing state and that once disabled, an agent stays disabled for the rest of his life.

Without a loss of generality we can restrict our attention to very simple histories, and we need to keep track only of the agent's age and the age at which he became disabled. In period $t$, a history $h^t$ belongs to a set $\{(1, A), (2, A), ..., (j, A)\}$ where a number $j$ $(j \leq t)$ indicates the age at which an agent became disabled or a symbol $A$ that states that an agent is not disabled by period $t$. We denote the probability, as of period 1, of a particular history $h^t$ by $\mu(h^t)$.

The source of information friction in this economy is that the agent's disability status is imperfectly observable. We start by proving our results for unobservable disability and, in Section 6, discuss a case of imperfect disability. When an agent with skill $\theta_t$ works $l_t$ units of time, he produces $y_t = \theta_t l_t$ units of effective labor. Effective labor $y_t$ is observable, while the skill $\theta_t$ and labor $l_t$ are the agent's private information. The informational problem is also dynamic, because information about agent's types is gradually revealed to agents. An agent learns whether he is disabled at the beginning of period $t$ and not before.

We consider a setting in which the net interest rate $R$ and the wage $w$ are constant over time[3]. Since savings are observable we can restrict our attention to an aggregate intertemporal feasibility constraint rather than consider agent-specific constraints. An allocation of consumption and effective labor for each generation $(c, y)$ is *feasible* for a given level of initial

[3] The model can also be extended to the case of general equilibrium see Golosov and Tsyvinski (2002).

resources $T_0$ if and only if

$$\sum_{t=0}^{t=T}\sum_{h^t}\frac{1}{(1+R)^t}\mu(h^t)c_t(h^t)\leq\sum_{t=0}^{t=T}\sum_{h^t}\frac{1}{(1+R)^t}\mu(h^t)wy_t(h^t)+T_0.$$

Allocations must respect incentive-compatibility conditions because the age at which an agent becomes disabled is private information. Since the skill of disabled agents is equal to zero, only able agents can misreport their skills. Disabled agents cannot work and, therefore, cannot pretend to be able. In period $t$, an agent can report that he became disabled in period $j$ ($j \leq t$) or report that he is able.

Define for each period $t$ ($t \leq T$) a report of the agent's disability status to be:

$$\sigma_t : h^t \rightarrow \{j, A\},$$

where $j \in \{1, ..., t\}$.

An agent's reporting strategy up to period $t$ is a vector of period reports $\bar{\sigma}_t = \{\sigma_1, ..., \sigma_t\}$.

Let $\Sigma$ be the set of all possible reporting strategies, and define

$$W(:, (c, y)) : \Sigma \rightarrow R$$

$$W(\bar{\sigma}; (c, y)) = \sum_{t=1}^{T}\beta^{t-1}\sum[u(c_t(\bar{\sigma}_t(h^t))) + v(\frac{y_t(\bar{\sigma}_t(h^t))}{\theta_t})]\mu(h^t)$$

to be the utility from reporting $\{\sigma_1, ...\sigma_T\}$, given an allocation $(c, y)$.

Define $\{\sigma_1^*, ..., \sigma_T^*\}$ to be the truth-telling strategy if $(\sigma_t^*(h^t)) = h_t$ for all realizations of histories $h^t$. Then an allocation $(c, y)$ is *incentive-compatible* if

$$W(\{\sigma_1^*, ..., \sigma_T^*\}; (c, y)) \geq W(\{\sigma_1, ..., \sigma_T\}; (c, y)) \text{ for all } \sigma \in \Sigma.$$

An allocation which is incentive-compatible and feasible is said to be *incentive-feasible.*

A social planner maximizes the utility of the representative agent and solves the following programming problem, $P(T_0)$, for an arbitrary level $T_0$ of initial resources:

$P(T_0) = \max_{c,y} \sum_{t=1}^{T} \sum_{h^t} \beta^{t-1}[u(c_t(h^t)) + v(\frac{y_t(h^t)}{\theta_t})]\mu(h^t)$

subject to

$\sum_{t=0}^{t=T} \sum_{\theta^t} \frac{1}{(1+R)^t}\mu(h^t)c_t(h^t) \leq \sum_{t=0}^{t=T} \sum_{\theta^t} \frac{1}{(1+R)^t}\mu(h^t)wy_t(h^t) + T_0$

$W(\{\sigma_1^*, ..., \sigma_T^*\}; (c, y)) \geq W(\{\sigma_1, ..., \sigma_T\}; (c, y))$ for all $\sigma \in \Sigma$

$T_0$ given

$(c, y) \geq 0$ for all $t$.

In each period, a social planner chooses allocations of consumption and effective labor which depend on the agent's history, that is, on when an agent became disabled. A social planner chooses these allocations to maximize the lifetime expected utility of a representative agent where the expectation is taken with respect to all possible future histories of skill shocks. The constraint set of the problem is defined by feasibility and incentive-compatibility constraints. The feasibility constraint states that the discounted lifetime consumption does not exceed the discounted lifetime labor income of the agents plus initial transfers. The incentive-compatibility constraints state that in each period during an agent's life the expected utility from the truthful revelation of the disability status is higher than from any possible deviation.

## 3. Characterizing Pareto Optima

In this section we provide a partial theoretical characterization of the optimal allocation. We will prove that labor decisions are not distorted, but savings decisions are distorted. In Section 6, we provide a full numerical characterization of the optimal scheme.

### A. Features of the optimal contract

The assumption that disability is an absorbing state allows us to reduce the number of incentive constraints. In particular, for an agent who lives $T$ periods, the number of ex post incentive constraints is $T$ because disability is an absorbing state, and once an agent announces that he is disabled, he cannot claim to be able to work in the future.

Two types of margins characterize optimal allocations: a consumption-leisure margin and an intertemporal margin. Only able agents can work, and their consumption-leisure margin will not be distorted. This result is reminiscent of a result that in a static environment labor decisions of the highest skilled agent are undistorted (Mirrlees 1976).

PROPOSITION 1. *Suppose $(c^*, y^*)$ solves $P(T_0)$. Then for each period $t$, and for each history $h^t = A$:*

$$-v'(\frac{y_t^*(h^t)}{\theta_t}) = u'(c_t^*(h^t))w.$$

**Proof.** See the appendix. ■

The intertemporal margin, however, is distorted. An optimal solution has a wedge between a marginal rate of substitution and a marginal rate of transformation for able agents who face a non zero probability of becoming disabled. The savings wedge is positive because there is an adverse incentive effect of savings on incentives. To induce truthful revelation of an agent's type, the agent's consumption and labor allocations should be positively correlated. Higher savings have a wealth effect that decreases the incentive to work for a given schedule of consumption and labor. It is then optimal for society to deter savings. A savings wedge is positive as long as in the next period disability status cannot be perfectly predicted given the current history of the skills. There is no intertemporal wedge for disabled agents because their productivity in the next period is determined with certainty.

We can now adapt the proof of a more general result proven in Golosov, Kocherlakota, and Tsyvinski (2001) to provide a characterization of the intertemporal wedge at the optimal solution.

PROPOSITION 2. *Suppose* $(c^*, y^*)$ *solves* $P(T_0)$. *Then for each period* $t$

$$\beta(1+R) = E\{u'(c_t^*(h^t))/u'(c_{t+1}^*(h^{t+1}))|h^t\}.$$

**Proof.** See the appendix. ■

Proposition 2 states that at the optimal solution, consumption allocations satisfy the following "reciprocal" Euler equation:

$$E_t(1/z_{t+1}) = 1,$$

where $z_t = \beta(1+R)u'(c_t)/u'(c_{t-1})$.

This intertemporal first-order condition can be contrasted with the following standard first-order condition:

$$E_t z_{t+1} = 1.$$

If the skills are not perfectly predictable given the current history of skills, then due to Jensen's inequality the above first-order conditions are not equivalent, and we can prove the following lemma.

COROLLARY 1. *Suppose* $(c^*, y^*)$ *solves* $P(T_0)$. *Then if the probability of becoming disabled is interior* $0 < \mu(h^{t+1} = t+1|h^t) < 1$,

$$u'(c_t^*(h^t)) < \beta(1+R)E\{u'(c_{t+1}^*(h^{t+1}))|h^t\},$$

*otherwise* $u'(c_t^*(h^t)) = \beta(1+R)E\{u'(c_{t+1}^*(h^{t+1}))|h^t\}.$

**Proof.** Apply Jensen's inequality to the condition in Proposition 2. ■

If an able agent faces any uncertainty of whether he will be disabled then the expected marginal utility of investing in capital is higher than the marginal utility of current consumption. After an agent becomes disabled, all uncertainty is resolved, and there is no need to distort his intertemporal decision.

## 4. Implementation of the Optimum

In this section we describe a simple tax system that implements the optimum. We propose a mechanism that implements the optimal allocation and includes only taxes and transfers similar to those already in the U.S. tax code. We then describe a tax system that captures the essential features of the current disability system.

Since the only restrictions on the social planner's problem are incentive compatibility and feasibility, we implicitly allow a very large set of taxes. In fact, we allow any nonlinear taxes, including lump-sum taxes. Because of the generality of taxes, the social planner's allocation can be implemented in multiple ways, the most obvious of which is a direct mechanism. However, the direct mechanism and many other mechanisms may be too complex and may include taxes that have never been used in practice. We will address these concerns and propose a simple tax system that consists of an asset-tested disability transfers and savings taxes that are equal to the savings wedge in the social planner's problem. We will then argue that asset-testing is a necessary part of the implementation we propose, because they allow control of individual savings that make an incentive problem more pronounced.

The proposed tax system consists of three important features. The first feature is that disability transfers have to depend on the length of pre-disability work history. Two other features of the system are designed to control negative incentive effects of savings. Savings that were accumulated while an agent was able should be taxed, while savings accumulated by a disabled agent should not be taxed. Finally, disability transfers should be asset-tested, that is, paid only to agents who have assets below a pre-specified minimum. Even though some of these features are not currently in SSDI, all of the proposed taxes and transfers are already present in the U.S. tax code and in other social programs. Some welfare programs, such as Supplemental Security Income and Temporary Assistance for Needy Families, are asset-tested. The US tax code also includes taxes that are history dependent, for example, a capital gains tax.

This section is organized as follows. We start with a simple two-period example that clarifies the role of savings taxes and asset-testing in implementation. Then we formally prove that a simple class of taxes implements the optimal allocation. Finally, we describe a stylized version of the current U.S. disability insurance system.

### A. Simple example of implementation

We illustrate implementation of the solution to the social planner's problem with a simple example that will also clarify the role of asset-testing and the savings tax. We consider a setup in which agents live for two periods and are able in the first period of their lives. When an agent is able he has a skill $\theta_t = 1$. In the second period of his life, an agent will be able with probability $\mu_a$ and disabled with probability $(1 - \mu_a)$. Denote consumption of an able agent in period 1 by $c_1$, of an able agent in period 2 by $c_a$, and of a disabled agent in period 2 by $c_d$. Denote allocations of effective labor of able agents in periods 1 and 2 by $y_1$ and $y_a$, respectively.

Then the optimal allocation $(c^*, y^*) = \{(c_1^*, c_a^*, c_d^*), (y_1^*, y_a^*)\}$ is the one that maximizes the expected lifetime utility of an agent subject to feasibility and incentive compatibility.

$$\max_{(c,y)} u(c_1) + v(y_1) + \beta\mu_a[u(c_a) + v(y_a)] + \beta(1-\mu_a)u(c_d)$$

subject to

$$c_1 + \frac{\mu_a c_a}{1+R} + \frac{(1-\mu_a)c_a}{1+R} \leq wy_1 + \frac{\mu_a w y_a}{1+R}$$

$$u(c_a) + v(y_a) \geq u(c_d).$$

It is fairly straightforward to solve this problem. We take first-order conditions and rearrange them to obtain two features of the optimal allocation: intertemporal savings distortion and no distortion in labor decisions.

Intertemporal:

$$\frac{1}{u'(c_1^*)} = \frac{\mu_a}{(1+R)\beta u'(c_a^*)} + \frac{(1-\mu_a)}{(1+R)\beta u'(c_d^*)}.$$

Labor (intratemporal):

$$wu'(c_1^*) = -v'(y_1^*)$$

$$wu'(c_a^*) = -v'(y_a^*).$$

We claim that a simple tax system exists, consisting of a tax on capital and an asset-tested disability system that can implement the optimal allocation as a competitive equilibrium. The intuitive argument has two steps: designing taxes that implement the optimal allocation for an agent who chooses to tell the truth and then augmenting that system to ensure that truth-telling gives higher utility than claiming disability.

As a first step of implementation, we will solve a simpler problem of constructing a tax system that implements the optimal allocation as a solution to the problem of an agent who claims disability only if he is disabled (truth-telling agent). We will then add some other features to this tax system that will allow us to implement the optimal allocation as a competitive equilibrium.

Consider a tax system that consists of a savings tax $\tau$, a tax $T_1$ in period 1, and taxes $T_a$ on able agents and $T_d$ on disabled agents in period 2. Our goal is to construct these taxes such that the optimal allocation $(c^*, y^*)$ will be a solution to the problem of an agent who claims disability only when he is truly disabled. A truth-telling agent solves the following problem:

$$\max_{(c,y,k)} u(c_1) + v(y_1) + \beta\mu_a[u(c_a) + v(y_a)] + \beta(1 - \mu_a)u(c_d)$$

subject to

$$c_1 + k_2 = wy_1 + T_1$$

$$c_a = (1 + R(1 - \tau))k_2 + wy_a + T_a$$

$$c_d = (1 + R(1 - \tau))k_2 + T_d.$$

We construct the taxes as follows. We define the tax on savings $\tilde{\tau}$ such that the allocation $(c_1^*, c_a^*, c_d^*)$ satisfies the intertemporal first-order condition of the truth-telling agent:

$$\tilde{\tau} = \left( \frac{u'(c_1^*)}{\beta[\mu_a u'(c_a^*) + (1 - \mu_a)u'(c_d^*)]} - 1 \right) /R.$$

The only step left is to solve the system of linear equations to find taxes $\tilde{T}_1$, $\tilde{T}_a$, and $\tilde{T}_d$ and a level of capital $k_2^*$ such that the budget constraints are satisfied:

$$c_1^* + k_2^* = wy_1^* + \tilde{T}_1$$

$$c_a^* = (1 + R(1 - \tau))k_2^* + wy_a^* + \tilde{T}_a$$

$$c_d^* = (1 + R(1 - \tau))k_2^* + \tilde{T}_d.$$

This system of linear equations has more unknowns than equations, and thus there is no unique way to satisfy the budget constraints. One option for restricting the solutions would be to require that taxes on able agents are independent of their age, that is, to set $\tilde{T}_1 = \tilde{T}_a$.

The tax system we have constructed so far implements the optimum as a solution to the problem of the truth-telling agent. We will build on this system and add some features that are necessary to implement the solution to the social planner's problem as a competitive equilibrium. In contrast to the implementation we have described, in a competitive equilibrium, an agent has an option not to tell the truth and claim disability in period 2. To implement the optimal allocation we will need to construct a tax system that achieves two goals: an agent should prefer truth-telling and should then choose the same consumption, labor, and savings as in the optimal allocation.

We will show that adding asset-testing to the tax system constructed to implement the optimal allocation for a truth-telling agent will ensure that the agent will choose to tell the truth. The proposed tax system $\{\tau, \{T_a, T_d, \bar{k}\}\}$ consists of the same savings tax $\tau = \tilde{\tau}$ on capital as in the tax system used in the truth-telling problem and of the asset-tested disability program $\{T_a, T_d, \bar{k}\}$. The asset-tested disability insurance program consists of taxes $T_a$ and $T_d$, and an asset limit $\bar{k}$ defined as follows: if an agent claims disability and his savings are less than $k_2^*$, he receives $\tilde{T}_d$; otherwise, he receives $\tilde{T}_a$, where $k_2^*$, $\tilde{T}_d$, and $\tilde{T}_a$ are defined to be the same as in the problem of a truth-telling agent. We provide a formal proof later, but now we will discuss why both a savings tax and asset-testing are necessary elements of the tax system that implements the optimal allocation. Taking this tax system as given, an agent compares the utility of telling the truth to the utility from always claiming disability.

If an agent chooses to tell the truth, he then faces the following problem:

*Problem 1:*

$$\max_{(c,y,k)} u(c_1) + v(y_1) + \beta\mu_a[u(c_a) - v(y_a)] + \beta(1-\mu_a)u(c_d)$$

subject to

$$c_1 + k_2 = wy_1 + T_1$$

$$c_a = (1 + R(1-\tau))k_2 + wy_a + T_a$$

$$c_d = (1 + R(1-\tau))k_2 + T_d$$

where $\tau = \tilde{\tau}$; $T_1 = \tilde{T}_1$; $T_a = \tilde{T}_a$

$$\left\{ \begin{array}{l} T_d = \tilde{T}_d, \text{if } k_2 \leq k_2^* \\ T_d = \tilde{T}_a, \text{if } k_2 > k_2^* \end{array} \right\}.$$

If an agent chooses to claim disability regardless of true disability status, he solves the following program:

*Problem 2:*

$$\max_{(c,y,k)} u(c_1) + v(y_1) + \beta u(c_2)$$

subject to

$$c_1 + k_2 = wy_1 + T_1$$

$$c_d = (1 + R(1-\tau))k_2 + T_d$$

where $\tau = \tilde{\tau}$; $T_1 = \tilde{T}_1$; $T_a = \tilde{T}_a$

$$\left\{ \begin{array}{l} T_d = \tilde{T}_d, \text{if } k_2 \leq k_2^* \\ T_d = \tilde{T}_a, \text{if } k_2 > k_2^* \end{array} \right\}.$$

We claim that an asset-tested disability program and a savings tax are both necessary elements of implementation and proceed to illustrate why the optimal solution may not be implemented if one of these components is omitted.

***Necessity of asset testing***

Now suppose that the tax system includes a savings tax but that the disability system is not asset-tested. We will show that an agent will oversave and claim disability in period 2, regardless of his true disability status.

Consider a problem of an agent who claims that he is disabled in the second period and who faces a savings tax that is equal to the wedge and a disability system that is not asset-tested.

At the optimum the incentive compatibility constraint binds: $u(c_a^*) + v(y_a^*) = u(c_d^*)$

The value of the optimal solution is equal to the utility that an agent receives when he claims disability in the second period:

$$u(c_1^*) + v(y_1^*) + \beta u(c_d^*) = u(c_1^*) + \beta[\mu_a u(c_a^*) + (1-\mu_a)u(c_d^*)].$$

Consider a problem of an agent who claims disability in the second period:

$$\max_{c,y,k} u(c_1) + v(y_1) + \beta u(c_2)$$

subject to

$$c_1 + k_2 = wy_1 + \tilde{T}_1$$

$$c_2 = (1 + (1-\tilde{\tau})R)k_2 + \tilde{T}_d$$

We will show that an allocation of consumption $(c_1^*, c_d^*)$ is feasible for the agent but does not satisfy the intertemporal first order condition:

$$u'(c_1) = (1 + (1-\tilde{\tau})R)\beta u'(c_2)$$

The savings tax is defined to be equal to the wedge in the optimal problem:

$$(1 + R(1-\tilde{\tau}))\beta = \frac{u'(c_1^*)}{\mu_a u'(c_a^*) + (1-\mu_a)u'(c_d^*)}$$

But then substituting in the intertemporal first order condition we get:

$$u(c_1^*) \neq \frac{u'(c_1^*)}{\mu_a u'(c_a^*) + (1-\mu_a)u'(c_d^*)} u'(c_d^*)$$

An agent will choose some other allocation $(\tilde{c}_1, \tilde{c}_d)$ that satisfies the first order condition such that

$$u(\tilde{c}_1) + v(y_1^*) + \beta u(\tilde{c}_d) > u(c_1^*) + v(y_1^*) + \beta u(c_d^*) = u(c_1^*) + \beta[\mu_a u(c_a^*) + (1-\mu_a)u(c_d^*)].$$

This argument shows that the optimum $(c_1^*, c_a^*, c_d^*)$ cannot be implemented.

We have shown that a tax system that implements the optimal allocation necessarily includes both asset-testing and the savings tax. Unlike in the social planner's problem where all savings are controlled, agents can choose any level of capital. In the social planner's problem all individual savings are controlled. To implement the optimum, a tax system has to control this additional degree of freedom through an asset-tested disability insurance system.

**A tax system without a savings tax** Consider a tax system that is exactly the same as the one constructed above with the exception that there is no savings tax. We will show that in this case, even though an agent may tell the truth, he may choose a higher level of savings than in the optimal allocation. Compare two choices of savings behavior a truth-telling agent has available to him:

(1) Choose a level of savings higher than the asset limit $k_2^*$.

The agent's savings then are higher than the asset limit, and he is treated as able. That means that while solving *Problem 1,* he faces taxes of an able agent, $T_a = T_d = \tilde{T}_a$, no matter whether he is truly disabled or not.

(2) Save $k_2^*$.

The agent's savings are then lower than the asset limit. He is eligible to receive disability insurance. In *Problem 1* he faces the following taxes: $T_a = \tilde{T}_a$, $T_d = \tilde{T}_d$.

The utility from following strategy (1) may be larger than the utility from strategy (2). Recall that the level of savings $k_2^*$ satisfied the intertemporal first-order condition only for an agent who faced a positive savings tax. Faced with a zero savings tax, an agent may be better off by oversaving and facing lump-sum taxes of an able agent, $T_a = T_d = \tilde{T}_a$, than by saving a suboptimal $k_2^*$ and receiving disability $T_a = \tilde{T}_a$, $T_d = \tilde{T}_d$. If the utility from following strategy (1) is higher than the utility from following (2), then a truth-telling agent will choose an allocation different from the optimum, and $(c_1^*, c_a^*, c_d^*)$ can no longer be implemented. This intuition shows that the savings tax is a necessary part of the tax system implementing the planner's problem.

**B. Implementation: General case**

We proceed to formally prove how to construct a tax system that implements the optimal allocation. The tax system we consider includes three types of taxes: a savings tax and an asset-tested disability insurance system. All taxes may depend on the age at which an agent claims disability. Taxes also depend on the amount of assets that an agent has. In particular, an agent receives a disability transfer only if his assets are less than some pre-specified limit. If his assets are higher than that limit, he has to pay the taxes of an able agent.

Let's formally define what comprises an asset-tested disability insurance program. To simplify notation we define taxes for an agent of age $t$ who claims disability at age $j$, $j \leq t$, as $\tau_t^k(j)$ and $\tau_t(j)$. (If an agent has not claimed disability by age $t$, then $j = A$).

*Definition*: Let $j$ be the age at which an agent was awarded disability. An *asset-tested* disability insurance program is a collection of asset limits $\bar{k}_j$ and taxes $\tau_t(j)$: taxes on able agents $T_t^a$ and taxes on disabled agents $T_t^d(j)$ that satisfies the following requirements:

(1) *If an agent is able, he pays a tax for the able*: $\tau_t(j) = T_t^a$ *if* $j = A$.

(2) *If an agent has not received a disability transfer before age* $t$*, claims disability at age* $t$*, and has assets* $k_j$ *less than the asset limit* $\bar{k}_j$*, he receives a disability transfer*: *if* $j = t$ *and* $k_j \leq \bar{k}_j$*, then* $\tau_t(j) = T_t^d$.

(3) *If an agent has not received a disability transfer before age* $t$ *and claims disability at age* $t$ *but has assets* $k_j$ *larger than an asset limit* $\bar{k}_j$*, he pays taxes of an able agent*: *if* $j = t$ *and* $k_j > \bar{k}_j$*, then* $\tau_t(j) = T_t^a$.

(4) *If an agent has claimed disability at* $j$ *and has not worked since then, he receives a disability transfer: if* $t > j$*,* $(y_j, ..., y_t) = 0$*, and* $\tau_j(j) = T_j^d$*, then* $\tau_t(j) = T_t^d$.

Formally, the tax system that we want to consider $\{(\bar{k}_j, \tau(j)); \tau^k(j)\}$ consists of an asset-tested disability insurance system $(\bar{k}, \tau)$ and taxes on capital $\tau_k$.

Let's now define a competitive equilibrium for an agent who faces this tax system.

*Definition:* A tax system $\{(\bar{k}_j, \tau_t(j)); \tau_t^k(j)\}$, wages $w$, a rental rate of capital $r$, the age of claiming disability $\tilde{j}$ and allocations of consumption, labor supply, and savings $(\{\tilde{c}(h^t)\}, \{\tilde{y}(h^t)\}, \{\tilde{k}(h^t)\})$ constitute a *competitive equilibrium* if they solve the following problem:

$$\max_j \{\max_{c,y,k} E \sum_{i=0}^{T} \beta^{t-1}(u(c(h^t)) + v(\frac{y(h^t)}{\theta^t}))\}$$

subject to

$$c(h^t) + k(h^t) \leq wy(h^t) + R(1 - \tau_{t-1}^k(j-1))k(h^{t-1}) + k(h^{t-1}) + \tau_t(j)$$

where $\tau_{t-1}^{k}(j-1)$ denotes that taxes depend on history at age $t-1$,

The proposition that follows states the main result of this section. We will prove that we can construct a tax system consisting of a savings tax and an asset-tested disability insurance program such that the optimal allocation is implemented as a competitive equilibrium.

PROPOSITION 3. *For any optimal allocation* $(\{c^*\},\{y^*\})$ *that solves the social planner's problem, a tax system* $\{(\bar{k}_j,\tau_t(j));\tau_t^k(j)\}$ *exists, consisting of savings taxes* $\tau_t^k(j)$ *and an asset-tested disability insurance program* $(\tau_t(j),\{\bar{k}_t\})$ *such that the optimal allocation is a solution to the competitive equilibrium problem.*

**Proof.** See the appendix. ■

The proof of Proposition 3 includes the same steps as in the example that we discussed earlier. We start by constructing a tax system that implements the optimal allocation as a solution to the problem of a truth-telling agent. This tax system consists of a tax on capital that is equal to the intertemporal wedge in the optimal solution and of lump-sum taxes on able and disabled agents that satisfy the budget constraints. We then augment this tax system with asset-testing. We set the system of asset limits to be equal to the savings chosen by a truth-telling agent such that an agent can receive a disability transfer only if his assets are below the asset limit.

### C. Stylized version of a current U.S. disability insurance system

In this subsection we will describe a tax system that captures the prominent features of the current social insurance system in the United States.

We describe the most significant features of the integrated disability and retirement system. This modelling choice is consistent with a view of Social Security system as a joint disability and retirement insurance. In particular, we will analyze decisions of an agent who faces the probability of becoming disabled. An agent can save at a rate $r$ and has to pay a tax $\tau_k$ on savings. When an agent claims disability, he receives a disability transfer $T_d$ independent of the age when he claims disability. This disability insurance system is an example of the tax system that we considered before, though not necessarily implementing the optimum. In particular, a savings tax is the same each period, there is no tax on able agents, disability transfers are not history dependent, and there are no asset limits.

Faced with these taxes and transfers, an agent chooses an age $j$ of claiming disability. The higher the disability benefits, the earlier an agent applies for them, and an optimal $j$ decreases. The size of the disability system is equal to the number of people claiming disability multiplied by the size of the disability transfer. For each history $h^t$ for which an agent is able and does not claim disability ($h^t = A$), he consumes $c_t(h^t)$, saves $k_{t+1}(h^t)$, and works $y(h^t)$. An agent's income consists of the labor income $wy(h^t)$ and savings income $(1 + R(1 - \tau_k))k_t(h^{t-1})$ that is taxed at the rate $\tau_k$. If an agent is truly disabled ($\theta_t = 0$) or claims disability $h_t = j$, then an agent cannot work, and his income consists only of income from savings and a disability transfer $T_d$.

After an agent claims disability he no longer faces any uncertainty about being disabled in the future. Let's define how the probability of an agent's history changes if an agent claims disability at age $j$. Let $\mu^j$ be defined as follows:

$$\mu^j(h^t) = \mu(h^t) \qquad \text{if } t < j;$$

$$\mu^j(h^t) = 1 \qquad \text{if } t \geq j \text{ and } h^t = j;$$

$$\mu^j(h^t) = 0 \qquad \text{otherwise.}$$

Formally, an agent solves the following problem:

$$\max_j \max_{(c,y,k)} \sum_{t=1}^{t=T} \sum_{h^t} \mu^j(h^t)\beta^{t-1}\{u(c_t(h^t)) + v(\frac{y(h^t)}{\theta^t})\}$$

subject to

if $h^t = A$ and $t < j$

$$c_t(h^t) + k_{t+1}(h^t) = wy(h^t) + (1 + R(1 - \tau_k))k_t(h^{t-1})$$

for $\forall h_t$, for $h_t \geq h_{t-1}$, where $k_1(h^0) = 0$;

if $h_t \neq A$ or $t \geq j$

$$c_t(h^t) + k_{t+1}(h^t) = (1 + R(1 - \tau_k))k_t(h^{t-1}) + T_d.$$

In this setup, an agent has two motives for savings. One motive is insurance against disability risk. If an agent has not claimed disability (that is, his age is less than the application age $j$), an agent solves a standard incomplete markets problem. The other motive is that if an able agent's lifetime skill profile is not constant, then the agent saves, for example, in the anticipation of decreasing productivity at the end of the life cycle.

## 5. Parameterization

In this section we describe how we chose the parameters of the model.

### A. Parameterizing disability

We calibrate the probability of becoming disabled using the data from McNeil (1997) published by the U.S. Census Bureau. The data report the number of self-reported disabled people by age groups. The data are organized in five- and ten-year intervals[4]. We use the data to calculate a conditional probability of becoming disabled and extrapolate the data to

[4] These numbers are self-reported and hence need not reflect the true disability of these individuals. However, a number of papers showed that self-reported health characteristics are a very good approximation of the true health. For one of the recent papers, see Benitez-Silva et al. (2000).

one-year intervals by fitting an exponential function (Figure 1).

Table 1 reports shares of disabled people in our model by various age groups. We assume that 4 percent of the population is disabled at age 25, before entering the labor force. We compare the numbers we calculated to those reported in the Survey of Income and Program Participation (SIPP) and the Current Population Survey (CPS). The SIPP estimates the number of people with severe disabilities, and the CPS reports the number of people with work disabilities. The CPS does not have information about work disabilities of people who are over age 65.

**Table 1**

**Share of disabled population**

| Age groups | 25-34 | 35-44 | 45-54 | 55-64 | 65-74 |
|---|---|---|---|---|---|
| Model | 5.2% | 8.33% | 13.97% | 24.54% | 43.19% |
| CPS[a] | 5.5% | 9.1% | 13.2% | 23.1% | n.a. |
| SIPP[b] | 8.1% | | 13.9% | 24.2% | 30.7% |

[a]Stoddard et al. (1998)

[b]McNeil (1997)

**B. Main parameters**

We choose the parameters of the model such that a solution to the stylized version of a the social insurance system matches selected empirical observations. The utility function is chosen to be $ln(c) + aln(1 - l)$, where $a$ is relative disutility of labor. We assume that a period in the economy is one year and that each agent begins life at age 25 and lives to the age of 75 years[5].

We solve the model and find that a value of disutility of work $a = 1.5$ implies that average working hours constitute 30 percent of the time endowment. This number is consistent with the microeconomic evidence that households allocate about one-third of their discretionary time to market activities (Ghez and Becker 1975). We choose the interest rate in the economy to match an annual interest rate net of depreciation $R$ to be equal to 4.3 percent. Then we choose the discount factor $\beta = 0.958$ as a solution to $\frac{1}{\beta} = 1 + R$. The aggregate production function is a standard Cobb-Douglas production function with constant returns to scale $F(K, Y) = K^{\alpha}Y^{1-\alpha}$. In the production function, we set the capital share equal to $\alpha = 0.33$. We use the calibrated interest rate and the capital share to calculate the wage rate $w = 1.2243$.

We use a lifetime skill profile depicted in Figure 2 which we obtained by fitting a quadratic function in the data reported by Rios-Rull (1996). Agents achieve the highest skill level at around age 50. At age 50 they are 45 percent more productive then at age 25. After age 50 skills decline and reach the minimum at age 75, with skills being roughly equal to the skill level at age 25.

We choose disability transfers $T_d = 0.15w$ to be equal to 40 percent of the average labor income. We calculated this income replacement ratio using average wage indexing series and the formula for calculating the primary insurance amount (PIA) provided by the Social Security Administration. We assumed that average annual wage earnings in the economy were $32,000. Then we used the PIA formula to calculate the amount a person would get

[5] We also solved a version of the model that does not include retirement period from age 65 to 75. Results are very similar and reported in Golosov and Tsyvinski (2002).

when he is disabled. The PIA is calculated as 90 percent of the first $426 of the monthly earnings plus 32 percent of the amount above $426 up through $2,567 plus 15 percent of any amount exceeding $2,567. We chose tax on capital $\tau$ to be equal to 15 percent, the size of the income tax paid by an individual in the second-lowest income tax bracket.

We summarize the parameters of the model in Table 2.

**Table 2**

**Parameters of the model**

| $\alpha$ | $\beta$ | $a$ | $w$ | $R$ | $T_d$ | $\tau^k$ |
|---|---|---|---|---|---|---|
| 0.33 | 0.958 | 1.5 | 1.2243 | 4.3% | 0.19 | 0.15 |

## 6. Quantitative Results

The section is organized as follows. We first describe the results of calculations of a problem with a stylized social insurance system that we used to calibrate the model. We proceed to characterize a solution to the social planner's problem. We then evaluate the welfare gains of switching to an optimal social insurance system. Finally, we solve a social planner's problem for a case in which disability is imperfectly observable.

### A. Stylized social insurance system

In this subsection we report a numerical solution to the problem with a stylized disability insurance system. The nature of a decision problem of an agent is as follows. In each period an agent can be either able or disabled. If an agent is able, he faces uncertainty about becoming disabled in the following period. With some probability an agent remains able, and with a complementary probability an agent becomes disabled. If an agent is disabled he does not work and receives a disability transfer. An agent's strategy consists of two parts: at what age to claim disability and how much to consume, save, and work. An agent can insure himself against disability risk by accumulating savings. However, savings provide only imperfect insurance, because the return on savings is not contingent upon disability status.

In Figure 3 we report the results of computation of a stylized version of a disability insurance system. Agents choose to claim disability (retire) at age 62 ($j = 37$), which is consistent with the data on retirement, since disability insurance in our model also provides retirement benefits for older agents. We now proceed to discuss consumption and labor decisions of agents.

***Consumption profiles***

The uppermost line of the consumption graph represents consumption of agents who were able up to age 62 and then claimed disability benefits.

Consumption profiles have four distinct features:

(1) Consumption falls if an agent becomes disabled before age 62. Since there are no assets with a return contingent on disability, agents cannot fully insure against disability risk before age 62. For instance, consumption of an agent who became disabled at the age of 25 falls by 65 percent from 0.6 to 0.2. Consumption of a 55-year-old agent falls by the significantly smaller amount of 40 percent from 0.64 to 0.39.

(2) There is no drop of consumption at the age of retirement $j^* = 62$. An agent of age 62 applies for disability benefits regardless of whether he is truly disabled or not. After an agent chooses to claim disability, he no longer faces disability risk. Since the decision to claim disability is endogenous, an agent optimally saves capital to avoid a drop in consumption.

(3) The consumption profile decreases at a constant rate after an agent claims disability benefits. After an agent becomes disabled, he no longer faces uninsurable disability risk and can smooth his consumption by consuming capital income and disability transfers. Since a tax on savings is 15 percent, then the consumption of a disabled agent will decline at a constant rate.

(4) A consumption replacement ratio depends mainly on two factors: the shape of the skill profile and the time remaining to retirement. The closer an agent is to retirement, the better he can smooth his consumption, and the consumption replacement ratio increases. An agent with higher skills has higher earnings and a higher consumption replacement ratio. When agents retire, their replacement ratio is equal to one.

***Labor profiles***

The labor profile reflects the shape of the skill profile. Agents work the most around the age of 50 when they are most productive, then decrease their work hours. At the age of 62 agents claim disability and stop working.

**B. Optimal system**

In this subsection we numerically characterize an optimal disability insurance system. We describe a solution to the social planner's problem with the same age-dependent skills for able agents and the same present value of extra resources as in the simple disability insurance system.

A social planner seeks to provide insurance to disabled agents, while ensuring that able agents work and do not claim disability. A planner faces two types of trade-offs. The first trade-off is between intratemporal optimal insurance (making consumption of able and disabled agents constant) and the incentive compatibility constraint (allocating more consumption to able agents to induce them to work). The second trade-off is between intertemporal optimal insurance (making consumption constant across time) and the incentive compatibility constraint (allocating more consumption to able agents who have a longer work history). If disability were perfectly observable, then a planner would be able to achieve both intertemporal and intratemporal smoothing. In that case consumption of able and disabled agents would be equal and constant over time. However, we will see that the optimal allocation will differ significantly from the solution without informational friction.

***Quantitative results: Age-dependent skills***

We start the analysis of the optimal system by calculating a solution to the social planner's problem for the same parameters as in the stylized disability insurance case. The only additional parameter we have to determine is the amount of initial transfers in the optimal economy. Disability transfers constitute resources added to the stylized disability insurance economy, and capital tax revenues are resources subtracted from the economy. To perform a correct comparison with the stylized version of the U.S. disability insurance system, we must ensure that the optimal system has the same amount of resources as the benchmark model. We set the amount of initial resources $T_0$ in the social planner's problem equal to the present amount of the disability transfers in the benchmark system minus the present value of capital revenues.

**Consumption profiles** We report consumption profiles in Figure 4. The uppermost line represents the consumption of agents who were able all their lives.

Consumption profiles have three distinct features:

(1) There is a significant fall in consumption after an agent becomes disabled.

A difference in consumption of able and disabled agents is necessary to ensure that able agents do not deviate and claim disability. Consumption of a 24-year-old agent falls by 50 percent from 0.6 to 0.3, and consumption of a 60-year-old agent falls by approximately 35 percent from 0.68 to 0.44. The fall in consumption is large, especially taking into account that the probability of becoming disabled is very small. For example, the probability of becoming disabled is less than one-tenth of a percent at age 24 and is 2 percent at age 60. The fall in consumption is initially lower than in the benchmark model and larger for later periods in life. This happens because in the benchmark system, agents know that they will retire at age 62 and will not face any risks thereafter.

(2) Consumption of disabled and able agents depends on the length of working history and the skill profile of able agents.

Consumption of both able and disabled agents has to be history dependent to ensure that in the optimal solution current as well as future incentives to work are satisfied. Agents with a longer work history receive higher consumption both when they are able and after they become disabled. Consumption of an agent who becomes disabled at age 60 is almost 50 percent higher than consumption of an agent who becomes disabled at age 25. When an agent becomes disabled, he no longer faces an incentive problem. Since we chose the discount factor and the interest rate such that $\frac{1}{\beta} = 1 + R$, the agent's consumption after becoming disabled is constant. Able and disabled agents in the optimal system receive more consumption than their counterparts in the stylized disability insurance model.

(3) The consumption replacement ratio (Figure 5) depends on the lifetime skill profile and the length of work history.

The optimal contract is designed to ensure that the most productive agents of ages 45 to 50 do not claim disability. That is one reason the replacement ratio initially decreases and then increases. The other reason the consumption eventually increases is that the social planner has to reward agents who have a longer work history. The replacement ratio decreases initially from 50 percent to 45 percent and then increases to 90 percent. The consumption replacement ratio increases more steeply under the benchmark system than under the optimal system.

**Savings distortions** An important feature of the model is that agents are allowed to save. The ability to save provides an additional disincentive to work compared to the model in which savings are not allowed. Savings allow extra insurance for an agent and decrease incentives to work. It is thus necessary for the optimal solution to discourage savings.

The savings distortion depends on two parameters: the length of the work history and the probability of becoming disabled. The longer the agent works, the “wealthier” he becomes, and the savings distortion must increase to deter agents from oversaving. The probability of becoming disabled increases for older agents, thus making their future consumption more unpredictable, and the savings distortion must also increase to deter them from claiming disability. Agents who have higher uncertainty about next period’s disability status have a higher savings distortion. The savings distortion (Figure 6) is quantitatively significant, especially given that the probability of becoming disabled is almost an order smaller than the size of the distortion. The distortion grows from 2.5 percent at age 25 to 9.5 percent at age 54, and decreases to 1.5 percent at age 75. The optimal contract places particular significance on satisfying the incentive constraints of the most skilled workers. The savings distortion for the most skilled workers is the largest.

**Labor** In the optimal solution all able agents work. Labor is decreasing with the length of the working history. A lower amount of required work is one of the instruments that ensure that the intertemporal and intratemporal incentive constraints are satisfied. For example, to induce an agent to work at the beginning of his life, the social planner may reward an agent with less work in the future. The amount that agents work (Figure 7) depends on two factors: productivity and the length of the work history. It is also efficient for the most productive agents to work. Labor supply is very similar to the lifetime skill profile. Agents who are 45 to 50 years old work the most, spending about 45 percent of their time at work. Younger and older people are less productive, and they work somewhere from 7 percent to 25 percent of the time. Under the optimal system agents work much more than under the benchmark system. The increase comes mostly from the fact that under the optimal system all able agents work. Under the benchmark system agents work only until the age of 62.

***Quantitative results: Age-independent skills***

This subsection describes the characteristics of a version of a social planner's problem in which the skills of able agents are constant across the lifetime and are equal to one. We are interested in analyzing this case because features of the optimal contract are no longer distorted by encouraging the most productive agents of ages 40 to 50 to work, and we can better understand the incentive effects of the optimal system. Figure 8 presents the results.

First, consumption of disabled agents and the replacement ratio are increasing at all ages because the social planner wants to reward agents who have longer working histories. Second, the savings distortion is increasing everywhere and is larger than the savings distortion with the age-dependent skill profile. The savings distortion is very similar in shape to the conditional probability of disability. Finally, labor is decreasing with the age of the agents.

### C. Welfare comparison

In this subsection we compare the welfare of a stylized disability insurance system with that of the optimal insurance system. Our numerical results indicate that there are significant welfare gains from switching to an optimal system. The principal reason that the size of the welfare gains is large is that the optimal system provides better insurance than the current system against a permanent skill shock, disability.

There are two sources of potential welfare gains from switching to the optimal contract. One is that the amount of resources is higher in the optimal system than in the benchmark model because all able agents work. The other is that in the optimal system agents are better insured from disability risk. In a stylized disability insurance system, asset returns are not contingent on disability realization, while in the optimal system a social planner can provide some of the state-contingent returns.

We use the following method to measure welfare gains from switching to an optimal system. Let $(c^o, y^o)$ be a solution to the social planner's problem and $(c^s, y^s)$ be a solution to the problem with a stylized disability insurance system. We find a constant $\gamma$ that proportionally reduces consumption for each history under the optimal system and produces the same lifetime utility as the lifetime utility in a stylized system.

Formally, we find $\gamma$ such that

$$E\sum_{t=1}^{T}\beta^{t-1}(u(c^{o}(h^{t}))+v(\frac{y^{o}(h^{t})}{\theta_{t}}))=E\sum_{t=1}^{T}\beta^{t-1}(u(\gamma c^{s}(h^{t}))+v(\frac{y^{s}(h^{t})}{\theta_{t}})).$$

The constant $\gamma$ that solves this equation is equal to 1.036, which implies that a switch to the optimal disability insurance system will increase consumption by 3.6 percent for each history. The increase of welfare of 3.6 percent of consumption is large because it represents the welfare gain from reforming both the disability and old age insurance program. If there were no informational friction then the welfare would increased by 5.4 percent. The optimal system achieves two-thirds of the welfare gain of the first-best system.

The welfare gain of 3.6 percent can be decomposed as follows. Under the optimal system all agents work, unlike the current system in which agents retire at age 62. Most of the extra resources that are equal to 5.1 percent is generated by the work of elderly and is distributed to disabled before age 62 (2.6 percent), able up to age 62 (1.5 percent) and retired (1 percent). In the optimal system, people who became disabled early win the most compared to the current system. It is optimal for the planner to transfer most of consumption generated by work of elderly to provide better insurance to those who experienced a permanent disability shock early in life.

We proceed to analyze an experiment of the welfare gain of only the disability insurance system. We calculate the welfare gain of the optimal system in which agents who are older than 62 are disabled and thus cannot work. This experiment allows to eliminate the welfare gain from providing incentives for the retired to work and can be interpreted as the welfare gain of the optimal disability insurance system. The numerical simulation of this model yield a welfare gain of 1 percent.

Two other papers evaluate the welfare implications of changes of disability benefits. Back-of-the-envelope calculations by Gruber (1996) for Canada and Bound and Burkhauser (1999) for the United States suggest a welfare gain of an increase in benefits. Bound, et al. (2002) find that under a moderate level of risk aversion an increase in benefits can lead to an increase in social welfare, especially for the low-income people. Unlike our calculations, these papers treat the degree of moral hazard as given and do not consider the welfare gains from switching to an optimal system.

**D. Tax system implementing the optimum**

We can use the optimal solution to compute the tax system implementing the optimal allocation. Since there are multiple ways to define the asset limits and transfers to able and disabled agents, we will have to make some assumptions that restrict the choice of instruments. In particular, we will require that the taxes for able agents be constant and independent of an agent's age. From the proof of Proposition 3 we know that the savings of the truth-telling agent will be equal to the asset limits of a mechanism that implements the optimum and that the savings tax that agents face is equal to the savings wedge in the optimal problem. Then the budget constraints of a truth-telling agent constitute a system of linear equations where the variables are transfers to disabled agents, a transfer to able agents, and capital levels. We can calculate and plot asset limits and transfers to disabled in Figures 9 and 10. Asset limits are eventually increasing because agents become wealthier as they accumulate more capital. That is also the reason that disability transfers are decreasing as agents receive a larger proportion of their income from savings. One interpretation of this system is that individuals who became disabled early in life receive large transfers, while those who become disabled later are supposed to supplement their lower disability transfers with savings accumulated while able.

### E. Optimal system: Imperfect observability

In this section we show how to modify the social planner's problem to include imperfect observability of disability. In practice, disability is partially unobservable, especially since there is no uniform and objective definition of disability. While it is possible to improve the quality of the screening process, evaluations of disability will probably always involve some subjective judgement, especially in the cases of multiple impairments, pain, or mental illness. (See Mashaw 1983 for further discussion of this issue.)

We incorporate imperfect monitoring in the problem in the following way. Assume that a social planner has an imperfect monitoring technology that has an award error of $p^m$. We model the cost of application as a percentage of a yearly consumption that is lost if an agent applies for disability benefits. In practice, the first step in determining the eligibility for disability benefits is whether an applicant "earns any sufficient gainful activity," that is earns more than \$780 per month. For an able individual, applying for disability benefits means forgoing wages he is currently earning for the duration of the application process, which is on average equal to six months. We chose application costs to be equal 15% of the yearly consumption. This cost is calculated from a loss of utility in consumption terms for an agent who does not work for 6 month and receives roughly \$5000 for half a year. We set $p^m = 50$ percent following Bound and Burkhauser (1999) and Bound, et al. (2002) to reflect the fact that, historically, half of individuals who apply for disability insurance are awarded benefits.

**Quantitative results** We report the results of computation of an optimal program with imperfect monitoring in Figure 11.

Introducing a monitoring technology implies that incentive constraints are relaxed, since deviators will be detected with the probability $p^m$ and receive only 85 percent of consumption. Optimal allocations with imperfect monitoring will be closer to the solution to the problem without informational frictions. Recall that in the problem with observable disability the consumption of able agents is equal to the consumption of disabled agents and is constant over time; the savings distortion is also equal to zero.

**Consumption profiles** Even though the fall in consumption after becoming disabled is significant, it is smaller than in the case of no monitoring. The consumption replacement ratio is higher than in the benchmark case and is explained by the fact that agents become more insured. For instance, the consumption of a 25-year-old agent falls by 40 percent, and the consumption of a 60-year-old agent falls by approximately 30 percent. The consumption fall is approximately three-fifth of the consumption fall for the case with no monitoring.

**Savings distortions** The savings distortion is lower, representing the fact that static as well as dynamic incentives are improved. The savings distortion remains quantitatively significant, but it is approximately a third of the size of the distortion in the case of unobservable disability. The distortion grows from 0.8 percent at age 25 to 2.5–3 percent around age 50, and decreases to almost zero at age 74.

**Labor profiles** The labor profile is higher than in the case with no monitoring. At the age of 25 agents spend around 27 percent of their time working, whereas at the age of 74 agents work 12 percent of their time.

**Welfare comparison** The optimal system with imperfect monitoring increases welfare by 4.2 percent of consumption. The increase in welfare is higher than that of the system where disability is not observable because a probability of detection relaxes the incentive compatibility constraints.

## 7. Related Literature

The first strand of literature motivates our focus on disability insurance as an important determinant of work decisions. Some of the important papers in this literature include Parsons (1980), Halpern and Hausman (1986), Bound (1989), Bound and Waidmann (1992), Gruber and Kubik (1997), Aarts, Burkhauser, and De Jong (1996), Gruber (2000), Autor and Duggan (2001). An excellent overview of the literature on disability programs is provided in Bound and Burkhauser (1999).

The second strand of the papers to which our paper is related is a classical analysis of Diamond and Mirrlees (1978, 1986) that studies a model similar to ours and provides insights on qualitative properties of optimal disability insurance systems. Our model builds on this analysis and extends it in two dimensions. First, we calibrate the model and deliver quantitative predictions on how to design an optimal system and improve current disability insurance. Second, we show how to implement an optimal system with a simple tax mechanism. We also show that an equal tax on savings of able and disabled individuals is not sufficient to implement the optimal system and that asset-testing is a necessary element of implementation. The second paper related to ours is a static analysis of disability insurance by Diamond and Sheshinski (1995), who consider a static mechanism design problem of optimal disability insurance. They also model disability as an imperfectly observable condition, but they study larger skill heterogeneity and consider two types of social insurance programs: welfare and disability.

This paper is also related to the literature on the optimal dynamic insurance in the presence of idiosyncratic shocks (see, among others, Green (1987), Thomas and Worrall (1990), Atkeson and Lucas (1992)) and, in particular, to our previous work on optimal capital taxation. In Golosov, Kocherlakota, and Tsyvinski (2001) we consider a general environment in which agent's skills are private information that follow an arbitrary stochastic process. We prove that for that in this general environment it is typically optimal to have a positive intertemporal wedge. In the present paper, we fully numerically characterize an optimal allocation of consumption and labor and determine that the savings wedge is quantitatively significance. We also show that the optimal allocations for such environments can be implemented with a "realistic" tax system.

Our paper is also related to a significant body of literature on designing unemployment insurance systems. (See, among others, Shavell and Weiss (1979), Hansen and Imrohohoroglu (1992), Wang and Williamson (1996), Hopenhayn and Nicolini (1997), and Acemoglu and Shimer (1999).) We argue that the optimal design of a disability insurance system is at least as important to understand as optimal unemployment insurance.

## 8. Conclusion

This paper constructs a dynamic mechanism design model of an optimal disability insurance system. The key informational friction in the model is that disability status is imperfectly observable. We characterize the optimal system theoretically and numerically, and we find that switching from the current to the optimal system may lead to significant welfare gains.

We also provide a method to implement the optimal system by using a simple tax system. The designed tax system is particularly attractive because it includes only taxes and transfers similar to those currently practiced in the United States. We argue that the proposed tax system must include two essential elements: a savings tax on able agents and an asset-tested disability transfer program.

Our paper also suggests new directions for the reform of the current disability insurance system. We determined that changing the structure of disability benefits can lead to significant welfare gains. While most of the current proposals to reform SSDI focus on improving administration of the program and on encouraging disabled people to return to work (see, for example, SSA 2001), very few researchers and policymakers have argued to change the structure of disability benefits. We argue that correctly designed disability benefits induce self-selection by reducing incentives to falsely claim disability while providing better insurance.

This paper suggests two feasible ways to improve the current disability benefits system. First, disability transfers should be age dependent. In the Netherlands, for example, the disability insurance system was recently significantly reformed[6]. One component of the reform

[6]The leniency of the Dutch disability system before the reform can be illustrated by the data on un-

is that disability benefits now increase with the age of disability up to age 50. Redesign of the program shifted the composition of diagnoses among newly disabled away from psychological and musculoskeletal disorders and significantly lowered disability rolls. (See Reno, Mashaw, and Gradison 1997.)

Second, savings decision of agents should be distorted. The first part of our proposal is to incorporate asset-testing into disability insurance. We believe that asset-testing can be successfully incorporated into the existing disability insurance system, since there are other social insurance programs in the United States that are asset-tested. The second necessary part of discouraging savings is a savings tax. Under our proposal, savings taxes are paid only on savings accumulated when an agent is able. More generally, savings accumulated by disabled may also be taxed, although at a different rate than the savings of able agents.

We have omitted two important features that we would like to explore in our future work. We have studied an economy in which there is a very limited heterogeneity in skills and where disability is permanent. In future work we would like to analyze a setup with several levels of unobservable skills, which will allow more precise choosing of the parameters of the model and may lead to interesting theoretical insights into the nature of optimal labor taxes in an economy with imperfectly observable stochastically changing skills and savings. Second, we would like to study an economy in which disability is not permanent. In that case, optimal disability benefits will also have to encourage individuals who recover from disability to leave disability rolls. This is an important extension given current interest in the proposed policy initiatives such as "Ticket to work" (See SSA 1998.) Both of these extensions are

employment provided by Organization for Economic Cooperation and Development. According to OECD, the regular rate of unemployment was 7 percent in 1994. Under the broad definition of unemployment that included disability and early retirement, the unemployment rate was 27 percent. (See de Jong 1997.)

interesting, but technically challenging. However, the magnitude of the welfare gains from switching to the optimal system gives us confidence that the forces we have captured in this paper are significant from both the theoretical and policy-making perspectives.

## References


[1] Aarts, L., Burkhauser, R., and De Jong, P., eds., 1996, *Curing the Dutch Disease: An International Perspective on Disability Policy Reform*, Brookfield, VT: Avebury.

[2] Acemoglu D. and Shimer R., 1999, Efficient Unemployment Insurance, *Journal of Political Economy*, 107 (5), 893–928.

[3] Autor, D. H. and Duggan, M. G., 2001, The Rise in Disability Recipiency and the Decline in Unemployment, Working Paper No. 8336, National Bureau of Economic Research (forthcoming in *Quarterly Journal of Economics*).

[4] Atkeson, A. and Lucas, R. E., Jr., 1992, On Efficient Distribution with Private Information, *Review of Economic Studies*, 59, 427–453.

[5] Benitez-Silva, H., Buchinsky, M., Chan, H. M., Cheidvasser, S., and Rust, J., 2000, How Large Is the Bias in Self-Reported Disability?, Working Paper No. 7526, National Bureau of Economic Research.

[6] Bound, J., 1989, The Health and Earnings of Rejected Disability Insurance Applicants, *American Economic Review* 79(3), 482–503.

[7] Bound, J. and Burkhauser, R., 1999, Economic Analysis of Transfer Programs Targeted on People with Disabilities, *Handbooks of Labor Economics*, vol. 3C, pp. 3417–3528, Amsterdam; New York and Oxford: Elsevier Science, North-Holland.

[8] Bound, J., Cullen, J. B., Nichols, A., and Schmidt, L., 2002, The Welfare Implications of Increasing Disability Insurance Benefit Generosity, Working Paper No. 9155, National Bureau of Economic Research.

[9] Bound, J. and Waidmann, T., 1992, Disability Transfers, Self-Reported Health, and the Labor Force Attachment of Older Men: Evidence from the Historical Record, *Quarterly Journal of Economics* 107 (4), 1393–1419.

[10] Diamond, P. A., and Mirrlees, J. A., 1978, A Model of Social Insurance with Variable Retirement, *Journal of Public Economics* 10 (3), 295–336.

[11] Diamond, P. A. , and Mirrlees, J. A., 1986, Payroll-Tax Financed Social Insurance with Variable Retirement, *Scandinavian Journal of Economics* 88(1), 25–50.

[12] Diamond, P. A., and Sheshinski, E., 1995, Economic Aspects of Optimal Disability Benefits, *Journal of Public Economics* 57(1) , 1–23.

[13] de Jong, P. R., 1997, U.S. Disability from a European Perspective, in Reno, V., Mashaw, J., and Gradison, B., eds., *Disability: Challenges for Social Insurance, Health Care Financing & Labor Market Policy*, National Academy of Social Insurance, Brookings Press.

[14] Ghez, G. R. and Becker, G. S., 1975, *The Allocation of Time and Goods over the Life Cycle*, New York: Columbia University Press, 1975.

[15] Golosov, M. and Tsyvinski, A., 2002, Designing Optimal Disability Insurance: Technical Appendix.

[16] Golosov, M., Kocherlakota, N., and Tsyvinski, A., 2001, Optimal Indirect and Capital Taxation, Research Department Staff Report 293, Federal Reserve Bank of Minneapolis (forthcoming at *Review of Economic Studies*).

[17] Green, E., 1987, Lending and the Smoothing of Uninsurable Income, in *Contractual Arrangements for Intertemporal Trade*, ed. E. Prescott and N. Wallace, Minneapolis: University of Minnesota Press, 3–25.

[18] Gruber, J., 1996, Disability Insurance Benefits and Labor Supply, Working Paper No. 5866, National Bureau of Economic Research.

[19] Gruber, J., 2000, Disability Insurance Benefits and Labor Supply, *Journal of Political Economy* 108 (6), 1162–83.

[20] Gruber, J. and Kubik, J. D., 1997, Disability Insurance Rejection Rates and the Labor Supply of Older Workers, *Journal of Public Economics*, 64(1), 1–23.

[21] Halpern, J. and Hausman, J. A., 1986, Choice under Uncertainty: A Model of Applications for the Social Security Disability Insurance Program, *Journal of Public Economics*, 31(2), 131–161.

[22] Hansen G. D. and Imrohoroglu A., 1992, The Role of Unemployment Insurance in an Economy with Liquidity Constraints and Moral Hazard, *Journal of Political Economy* 100 (1), 118–142.

[23] Hopenhayn, H. A. and Nicolini, J. P., 1997, Optimal Unemployment Insurance, *Journal of Political Economy* 105 (2), 514–550.

[24] Mashaw, Jerry L., 1983, *Bureaucratic Justice: Managing Social Security Disability Claims*, New Haven, CT: Yale University Press.

[25] McNeil, J., 1997, Americans with Disabilities: 1994–95. Current Population Reports, Household Economic Studies P70-61, U.S. Department of Commerce, Bureau of the Census.

[26] Mirrlees, J. A., 1976, Optimal Tax Theory: a Synthesis, Journal of Public Economics, 6 (4), 327–358.

[27] Mulligan, C. B. and Sala-i-Martin, X., 1999, Social Security in Theory and Practice (II): Efficiency Theories, Narrative Theories, and Implications for Reform Working Paper No. 7119, National Bureau of Economic Research.

[28] Parsons, D. O., 1980. The Decline in Male Labor Force Participation, *Journal of Political Economy* 88 (1), 117–134.

[29] Parsons, D. O., 1991, Self-Screening in Targeted Public Transfer Programs, *Journal of Political Economy* 99 (4), 859–876.

[30] Reno, V., Mashaw, J., and Gradison, B., eds., 1997, *Disability: Challenges for Social Insurance, Health Care Financing & Labor Market Policy*, National Academy of Social Insurance, Brookings Press.

[31] Rios-Rull, J.-V., 1996, Life-Cycle Economies and Aggregate Fluctuations, *Review of Economic Studies* 63(3), 465–489.

[32] Shavell, S. and Weiss, L., 1979, The Optimal Payment of Unemployment Insurance over Time, *Journal of Political Economy* 87 (6), 1347–62.

[33] Stoddard, S., Jans, L., Ripple, J. and Kraus, L., 1998 *Chartbook on Work and Disability in the United States. An InfoUse Report.* Washington D.C.: U.S. National Institute on Disability and Rehabilitation Research.

[34] Thomas, J. and Worrall, T., 1990, Income Fluctuation and Asymmetric Information: An Example of a Repeated Principal-Agent Problem, *Journal of Economic Theory*, 51, 367–90.

[35] U.S. Social Security Administration (SSA), 2000, *Social Security Bulletin: Annual Statistical Supplement*.

[36] U.S. Social Security Administration (SSA), 1998, 2001, *Social Security Advisory Board Reports.*

[37] U.S. Department of Health and Human Services, 1988, *Social Security Handbook*, Tenth Edition.

[38] Wang, C. and Williamson, S. D., 1996, Unemployment Insurance with Moral Hazard in a Dynamic Economy, *Carnegie-Rochester Conference Series on Public Policy* 44, 1–41.

# 9. Appendix

In this appendix we provide formal proofs of Propositions 1–3.

## A. Characterizing Pareto Optimum

Here we will formally prove Propositions 1 and 2.

First, we want to simplify incentive compatibility constraints. There are only $T$ incentive constraints, and all of them bind. We want to rewrite them by substituting constraint at age $T$, into constraint at age $T-1$, and so on to period 1:

$$u(c_T(A)) + v(\frac{y_T(A)}{\theta_T}) = u(c_T(T)) + v(0).$$

Substitute this equality in the initial constraints of the agent who is $T-1$ years old:

$$u(c_{T-1}(A)) + v(\frac{y_{T-1}(A)}{\theta_{T-1}}) + \beta(u(c_T(T)) + v(0)) =$$

$$u(c_{T-1}(T-1)) + v(0) + \beta(u(c_T(T-1)) + v(0)).$$

We can repeat this procedure iteratively to get the following system of ex post incentive constraints:

$$u(c_t(A)) + v(\frac{y_t(A)}{\theta_t}) + \sum_{i=0}^{T-t} \beta^i\{u(c_{t+i}(t)) + v(0)\} = \sum_{i=0}^{T-t} \beta^i\{u(c_{t+i}(t)) + v(0)\} \text{ for all } \theta_t > 0.$$

We use these simplified incentive constraints in the social planner's problem and take the first-order conditions. Let $\lambda$ be the Lagrange multiplier on the feasibility constraint. Let $\eta_t$ be the Lagrange multiplier on the incentive compatibility constraint. To prove Propositions 1 and 2 we will need to find the following first-order conditions:

Consumption:

$$(1) \qquad c_t(A) : u'(c_t(A))[\mu(h_t = A)\beta^{t-1} + \eta_t] = \frac{\lambda}{(1+R)^{t-1}}\mu(h_t = A)$$

$$(2) \qquad c_t(t) : u'(c_t(t))[\mu(h_t = t)\beta^{t-1} - \eta_t + \beta\eta_{t-1}] = \frac{\lambda}{(1+R)^{t-1}}\mu(h_t = t)$$

with convention that $\eta_{-1} = 0$

Labor:

$$(3) \qquad v'(\frac{y_t(A)}{\theta_t})[\mu(h_t = A)\beta^{t-1} + \eta_t] = \frac{\lambda}{(1+R)^{t-1}}\mu(h_t = A)w$$

Equation (1) is a first-order condition with respect to the consumption of an able agent at age $t$. Equation (2) is a first-order condition with respect to the consumption at age $t$ of an agent who became disabled at age $t$.

**Consumption-labor is undistorted**

The proof of Proposition 1 follows from combining equations (1) and (3):

$$u'(c_t(A))w = v'(\frac{y_t(A)}{\theta_t}).$$

**Savings of able agents are distorted**

The proof of Proposition 2 is as follows.

Combine the first-order conditions for $c_{t+1}(A)$ and $c_{t+1}(t+1)$, that is, equations (1) and (2) evaluated at age $t+1$:

$$\beta[\mu(h_t = A)\beta^{t-1} + \eta_t] =$$
$$\frac{\lambda}{(1+R)^t u'(c_{t+1}(A))}\mu(h_{t+1} = A) + \frac{\lambda}{(1+R)^t u'(c_{t+1}(t))}\mu(h_{t+1} = t+1).$$

Substitute for the left-hand side from equation (1):

$$\frac{\lambda}{(1+R)^{t-1} u'(c_t(A))}\mu(h_t = A)\beta$$
$$= \frac{\lambda}{(1+R)^t u'(c_{t+1}(A))}\mu(h_{t+1} = A) + \frac{\lambda}{(1+R)^t u'(c_{t+1}(t))}\mu(h_{t+1} = t+1).$$

After simplification, the claim follows:

$$\frac{1}{u'(c_t(A))} = \frac{1}{\beta(1+R)}[\frac{1}{u'(c_{t+1}(A))}\frac{\mu(h_{t+1} = A)}{\mu(h_t = A)} + \frac{1}{u'(c_{t+1}(t))}\frac{\mu(h_{t+1} = t+1)}{\mu(h_t = A)}].$$

To show that savings of disabled agents are not distorted, take the first-order condition with respect to consumption at age $t+1$ of an agent who became disabled at age $t$:

$(2') \qquad c_{t+1}(t) : u'(c_{t+1}(t))\beta[\mu(h_t = t)\beta^{t-1} - \eta_t + \beta\eta_{t-1}] = \dfrac{\lambda}{(1+R)^t}\mu(h_t = t).$

Divide (2) by (2′) :

$u'(c_t(t)) = \beta(1+R)u'(c_{t+1}(t)).$

## B. Proof of Proposition 3

We start the proof by considering an auxiliary problem of a truth-telling agent who claims disability only when he is disabled. We will construct taxes such that the optimal solution is a also solution to the auxiliary problem. We will then augment this tax system with additional features that will implement the optimal solution as a competitive equilibrium.

Step I. Implementing the optimal allocation as a solution to a truth-telling agent.

We will consider a problem of an agent who claims disability only when he is truly disabled. Given taxes, an agent trades consumption, capital, and labor. This equilibrium will be useful later in constructing a competitive equilibrium. The truth-telling agent's problem is as follows:

$$\max_{(c,y,k)} E\sum_{t=1}^{T}\beta^{t-1}(u(c(h^t)) + v(\frac{y(h^t)}{\theta^t}))$$

subject to

$$c_t(h^t) + k_t(h^t) \leq wy(h^t) + R(1-\tau^k(h^{t-1}))k_{t-1}(h^{t-1}) + k_{t-1}(h^{t-1}) + \tau(h^t)$$

We want to construct taxes on savings and an asset-tested disability insurance system such that the optimal solution is also a solution to the truth-telling agent's problem. The intuition is as follows. Construct capital taxes such that the intertemporal first-order condition is satisfied, and then choose taxes on able and disabled agents in such a way as to satisfy an agent's budget constraint.

Define taxes on capital so that the wedge between the marginal rate of substitution and the marginal return on capital is equal to the optimal wedge:

$$(*) \qquad \tau_t^k(h^t) = \left( \frac{u'(c^*(h^t))}{\beta E\{u'(c^*(h^{t+1})|h^t\}} - 1 \right) / R.$$

Note that a disabled agent pays a tax on capital only in the first period of becoming disabled. After the first period the disabled agent pays no taxes.

Pick the remaining lump-sum taxes so that the budget constraints in the truth-telling agent's problem are satisfied and the consumption is equal to that in the social planner's problem $c(h^t) = c^*(h^t)$ for all $\theta^t$. This is always possible to do because the taxes may depend on the type and age of the agent.

With the taxes defined as above, the consumption-labor decision of the agent is not distorted, and the intertemporal wedge is exactly equal to the optimal wedge in the social planner's problem. Then $(\{c^*\}, \{y^*\})$ will satisfy all the first-order conditions of the agent's problem in the truth-telling equilibrium. Lump-sum taxes set the level of consumption equal to the level of consumption in the Pareto optimum and guarantee that budget constraints are satisfied, and $(\{c^*\}, \{y^*\})$ will indeed be the solution to the agent's problem.

Step II. Implementing the optimal solution.

The difference between the auxiliary problem of a truth-telling agent and the competitive equilibrium is that in the competitive equilibrium an agent can deviate from truth-telling and choose to claim disability even if he is not disabled. To implement the social planner's allocation as a competitive equilibrium, we need to devise a tax system that will ensure that an agent will not choose to deviate from truth-telling.

The proof of the implementation of the social planner's allocation as a competitive equilibrium is then as follows. Let $\{\bar{k}_t(h^t)\}$ be a sequence of savings that is a solution to the problem of the truth-telling agent with the previously constructed tax system. We impose the following restrictions on the disability transfers. An agent of age $i$ will be treated as an able agent in period $t$ if his capital stock in that period $k_t$ exceeds $\{\bar{k}_{t-1}(\theta^{t-1}(A))\}$. This requirement exists only in the first period when the agent claims disability insurance ("applies for it"). In all the subsequent periods his wealth is not monitored. Now we can show that a competitive equilibrium with asset-tested disability benefits is Pareto optimal.

Set taxes on capital as in $(*)$ the auxiliary problem:

$$\tau_t^k(h^t) = \left(\frac{u'(c^*(h^t))}{\beta E\{u'(c^*(h^{t+1})|h^t\}} - 1\right)/R.$$

The tax on capital in period $t$ is the same for an able agent and for the agent who declared disability in period $t$ and is equal to the wedge in the solution to the social planner's problem. An agent who became disabled before age $j$ does not face a capital tax. Set savings limits to the capital of the able $k_t^*(\theta^t)$ in the solution to the truth-telling agent's problem: $\bar{k}_t = k_{t-1}^*(h^{t-1})$, where $h^{t-1} = A$.

We need to show that no agent will want to apply for disability benefits unless he is disabled. The proof consists of two parts. First, we will show that an able agent who does not claim disability in that period will always want to choose the same allocation (in particular, the level of capital) as in the truth-telling agent's problem This implies that if an agent claims disability at age $j$, then he will have exactly the same amount of capital as in the truth-telling equilibrium. Then given the same level of capital at the onset of disability, it is suboptimal for an able agent to claim disability because of the incentive compatibility of the social planner's problem.

1) We will show that an able agent who claims that he is able at age $t$ does not have an incentive to oversave; that is, in each period $t$ he chooses exactly the amount of capital as in *the* solution to the truth-telling agent's problem $k_{t-1}^{*}(h^{t-1})$, where $h^{t-1} > 0$.

Let's consider a period right before period $j$ when an able agent wants to claim disability. If he saves more than $\bar{k}_t$, then next period he will receive a transfer $T_t^a$ which is less than $T_t^d$. Moreover, $\bar{k}_t$ was an optimal level of savings under the truth-telling equilibrium. So, next period he receives strictly less resources and does not gain anything by oversaving. If an agent chooses $\bar{k}_t$ in the last period, then up to period $t$ he faces the same problem as under the truth-telling equilibrium. His allocations are the same as those in the truth-telling agent's problem.

2) We want to show that an agent does not want to claim disability when he is able and then oversave.

Suppose that an agent of age $t$ lies and applies for disability insurance while being able. The first part of the proof implies that in each period $s$ up to period $t$ an agent will have at most $\bar{k}_{s-1}$ units of capital. As we have shown the maximization problem of the deviating agent up to period $t$ is identical to the problem which the truth-telling agent solves for the first $t-1$ period of his life. Then the allocations up to period $t$ are the same as if he did not plan to deviate in period $t$.

In period $t$, when an agent lies, his problem will be identical to that of the truth-telling agent who became disabled at age $t$, and therefore his allocation from age $t$ on will be identical to the allocations of that agent. An incentive compatibility of the social planner's problem guarantees that utility of this deviation cannot be greater than utility from continuing to work as an able agent.

Finally, from the definition of the disability insurance system, an agent will not want to work after he claimed disability, because he will be treated as an able agent for the rest of his life.

We have proved that this asset-tested disability insurance system preserves incentive compatibility and implements the allocations of a social planner's problem. ■

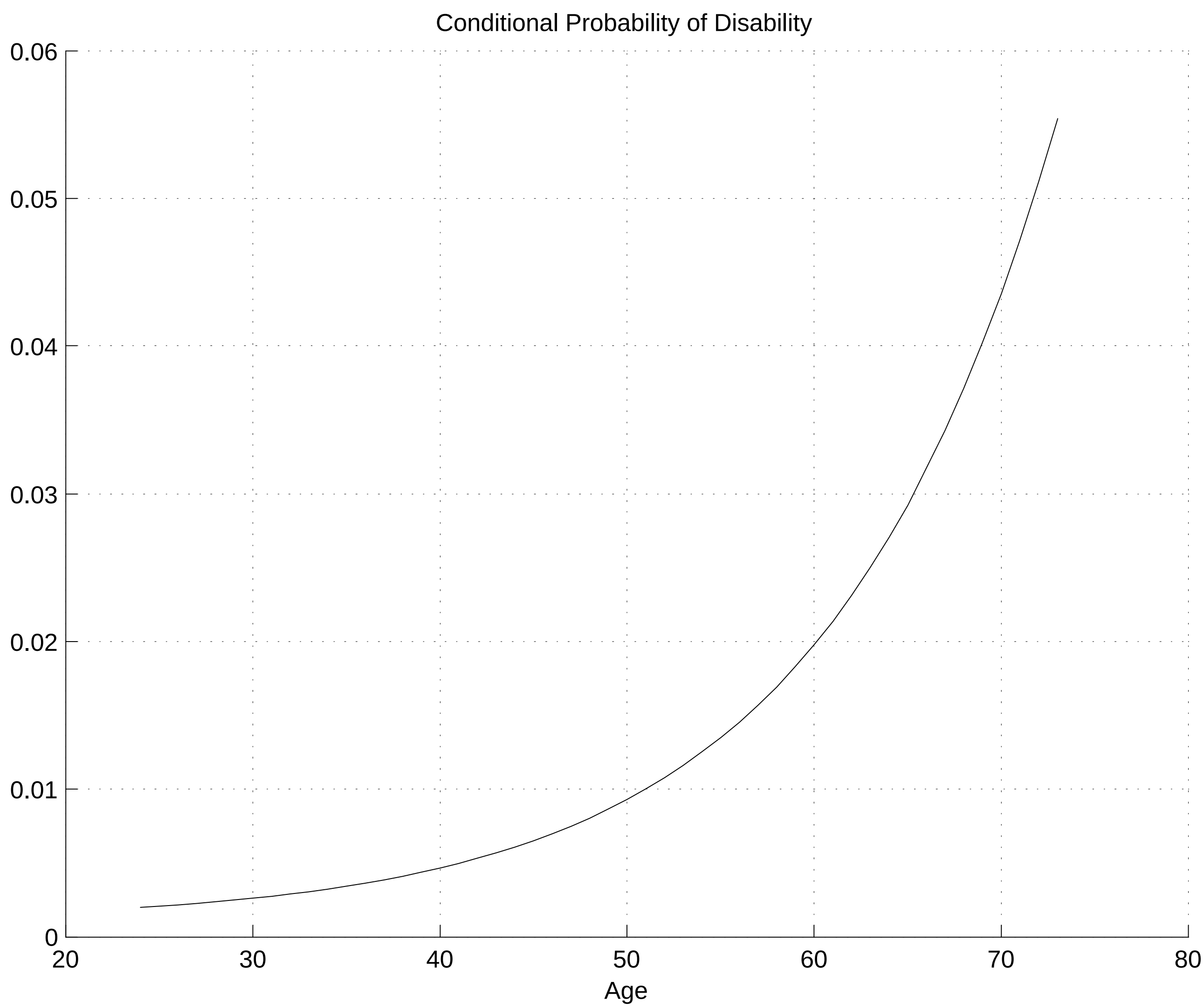


Figure 1: Conditional probability of disability

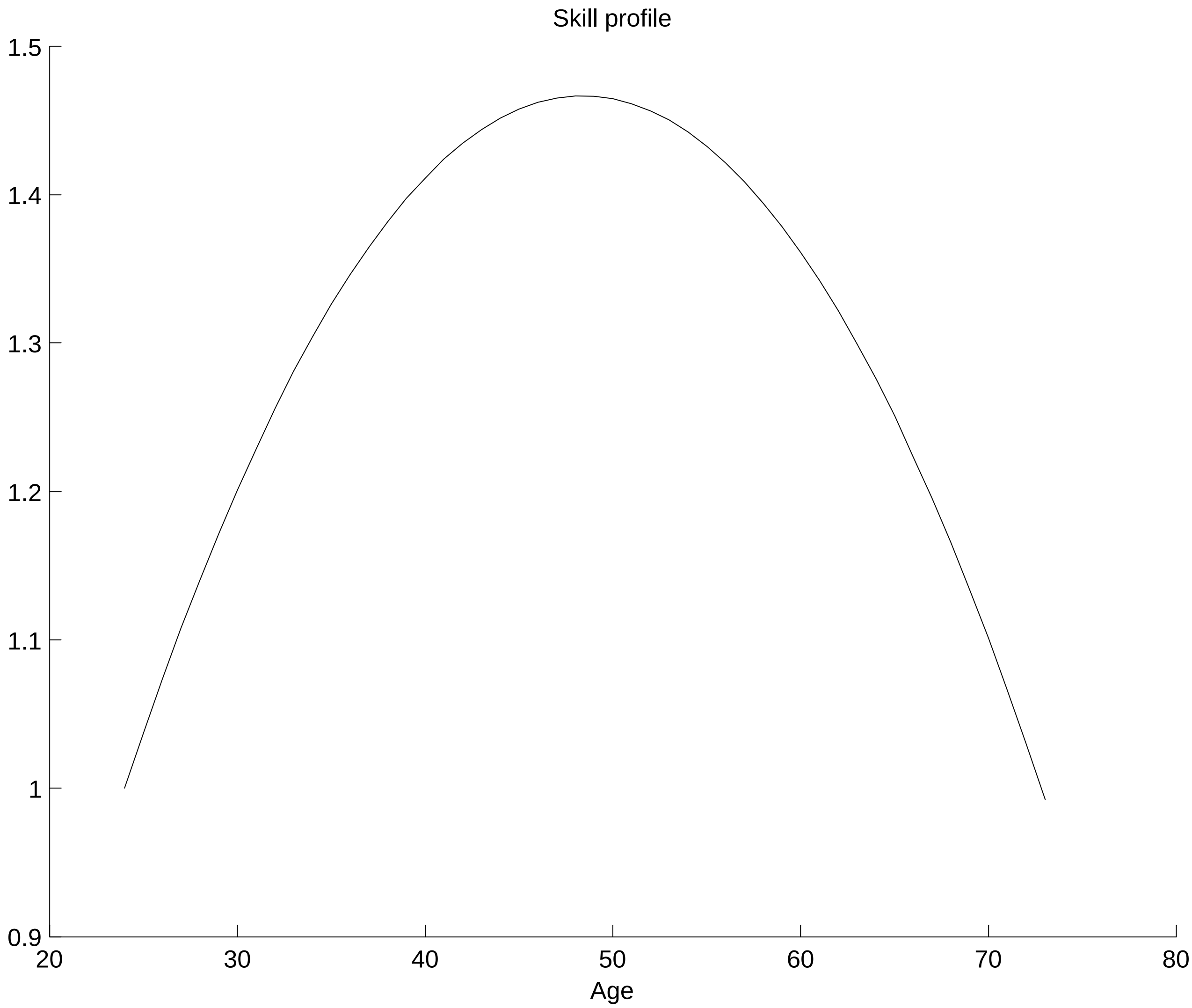


Figure 2: Lifetime skill profile

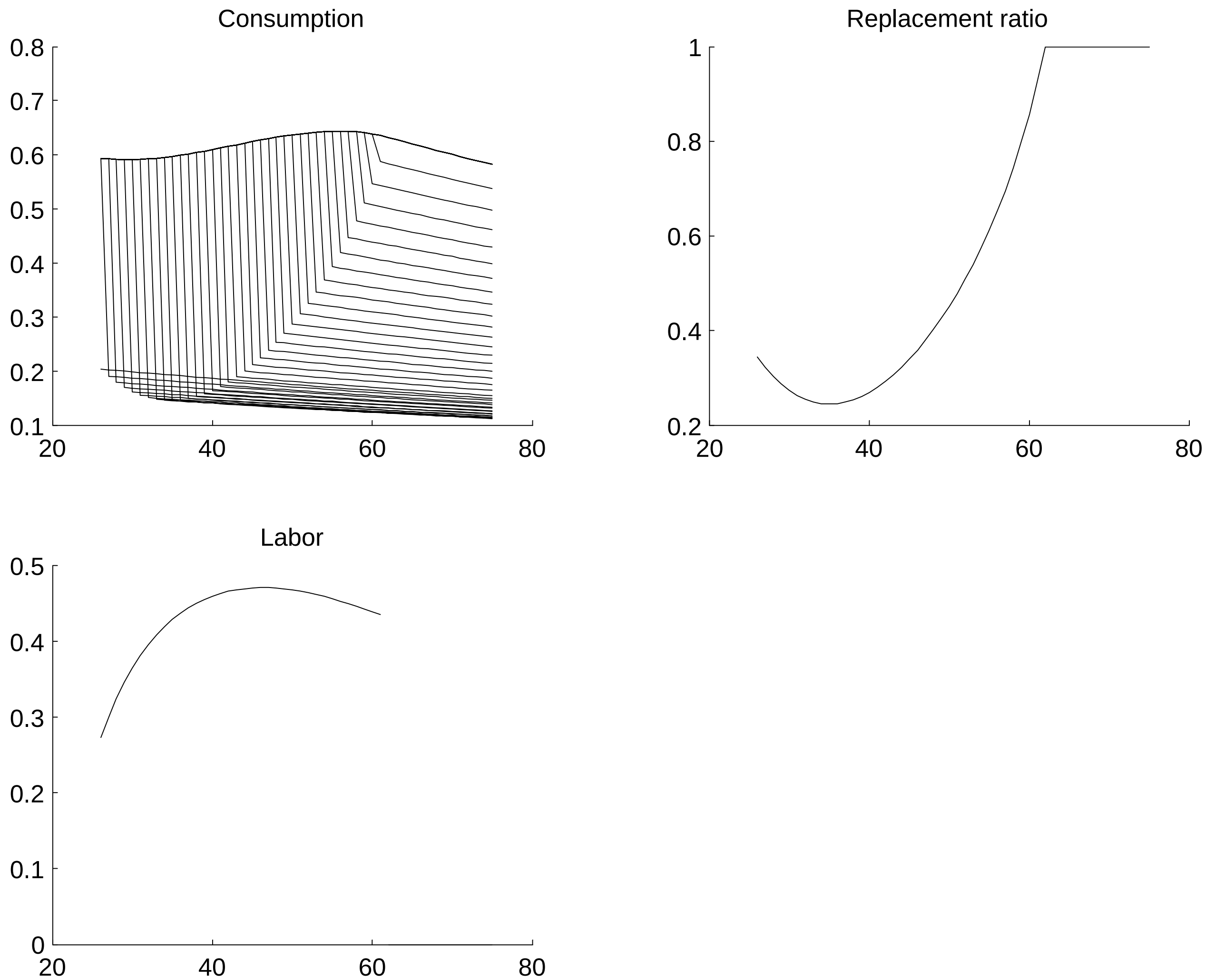


Figure 3: Stylized DI

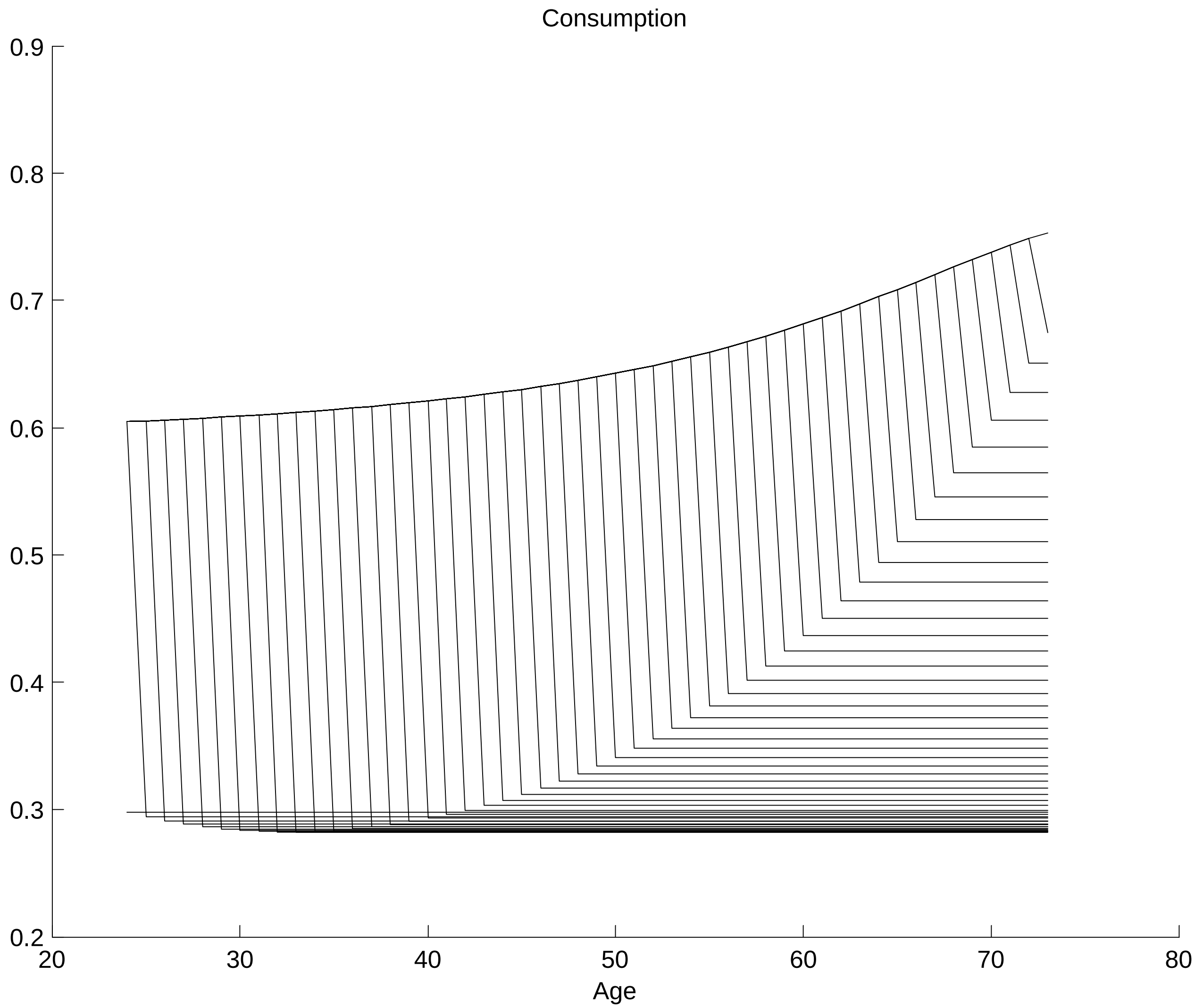


Figure 4: Optimal system: consumption profiles

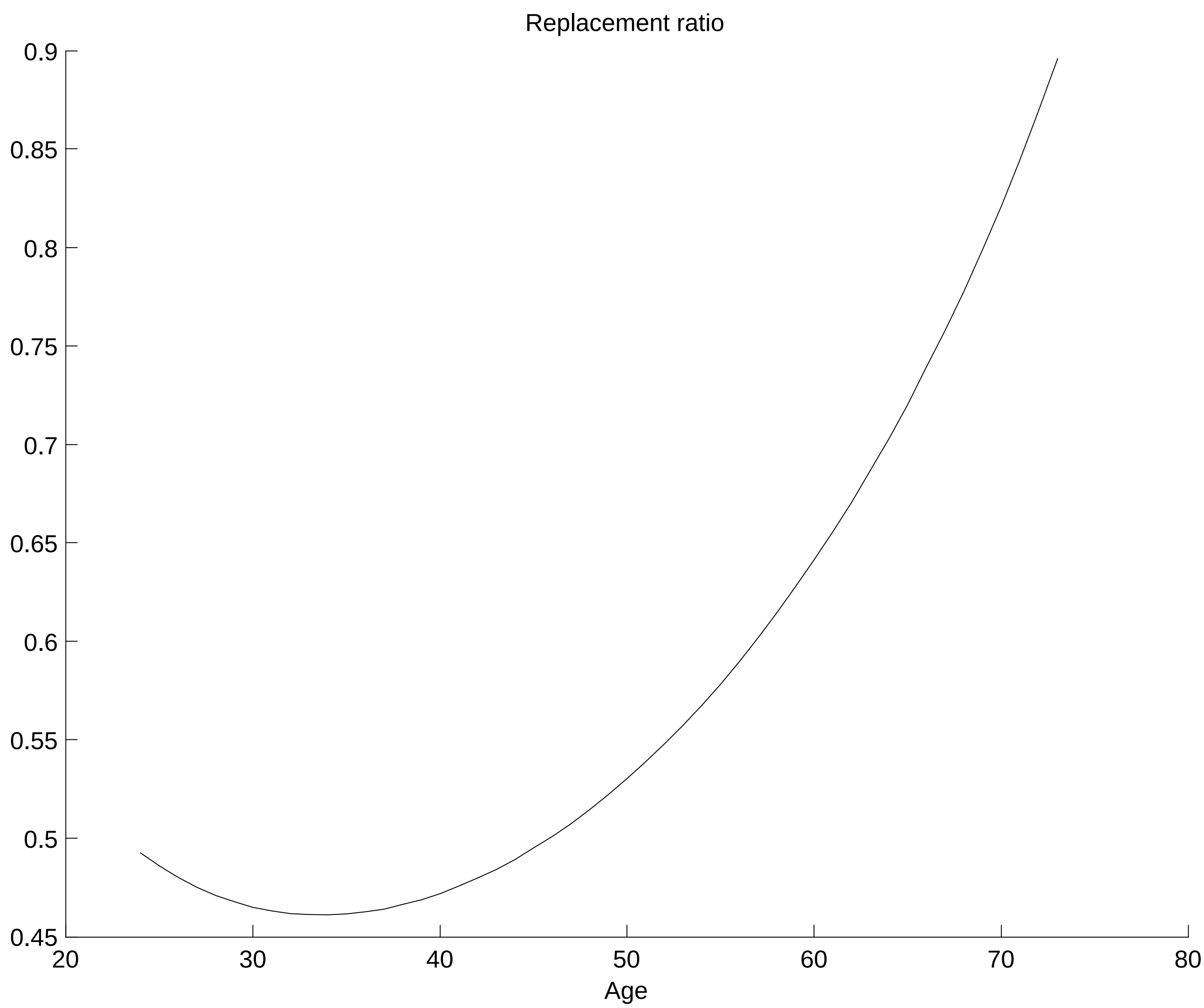


Figure 5: Optimal system: consumption replacement ratio

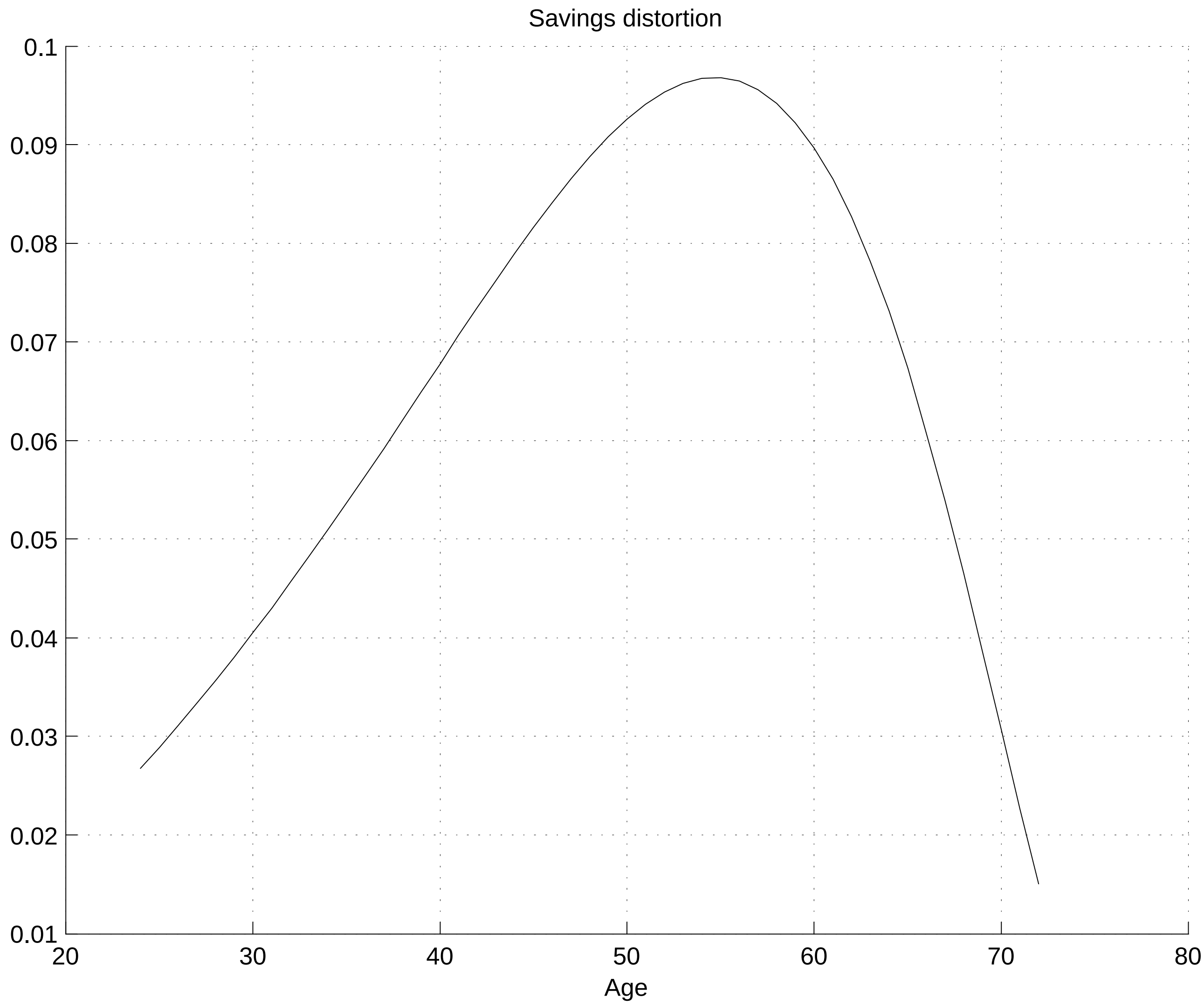


Figure 6: Optimal system: savings wedge

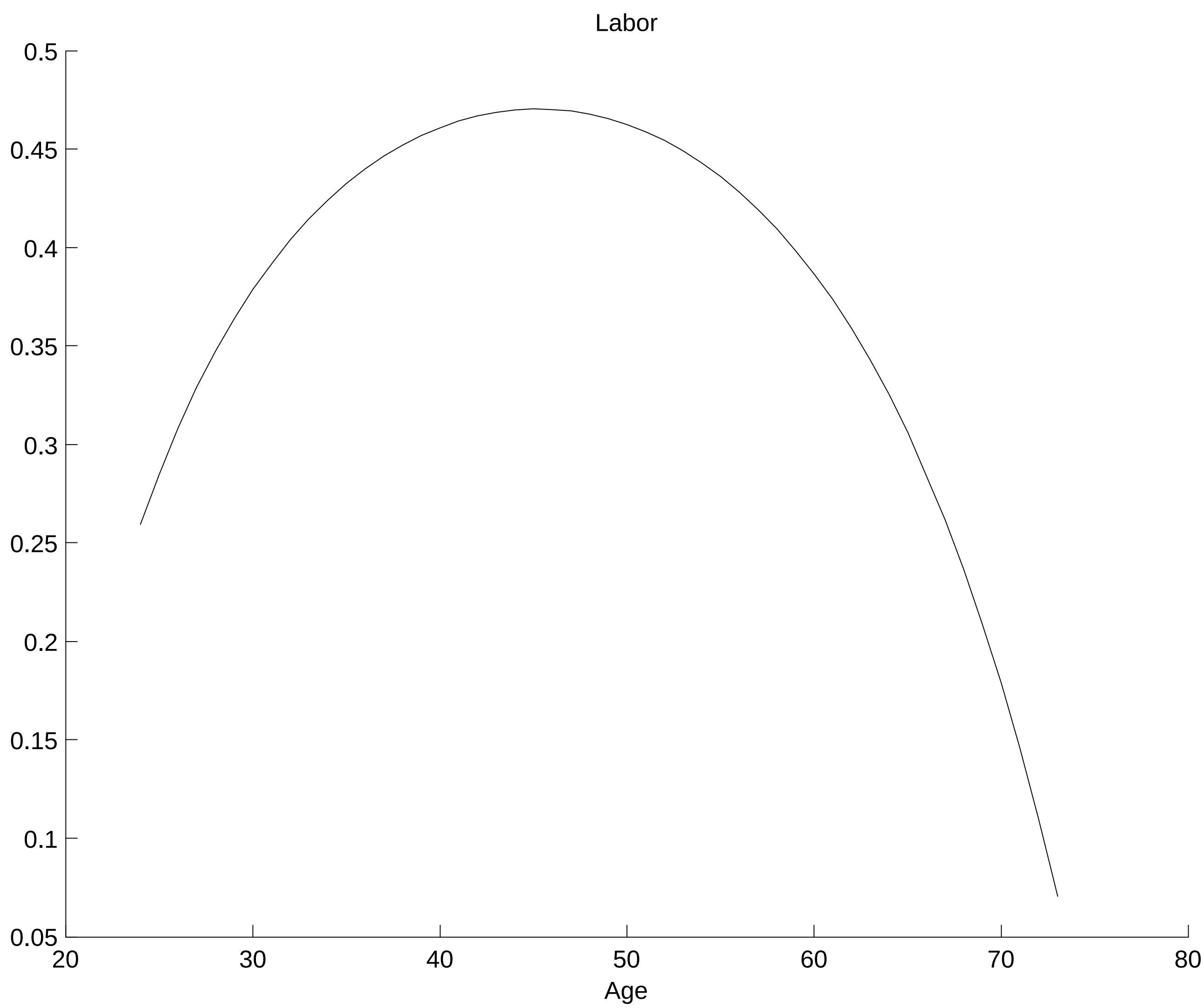


Figure 7: Optimal system: labor profile

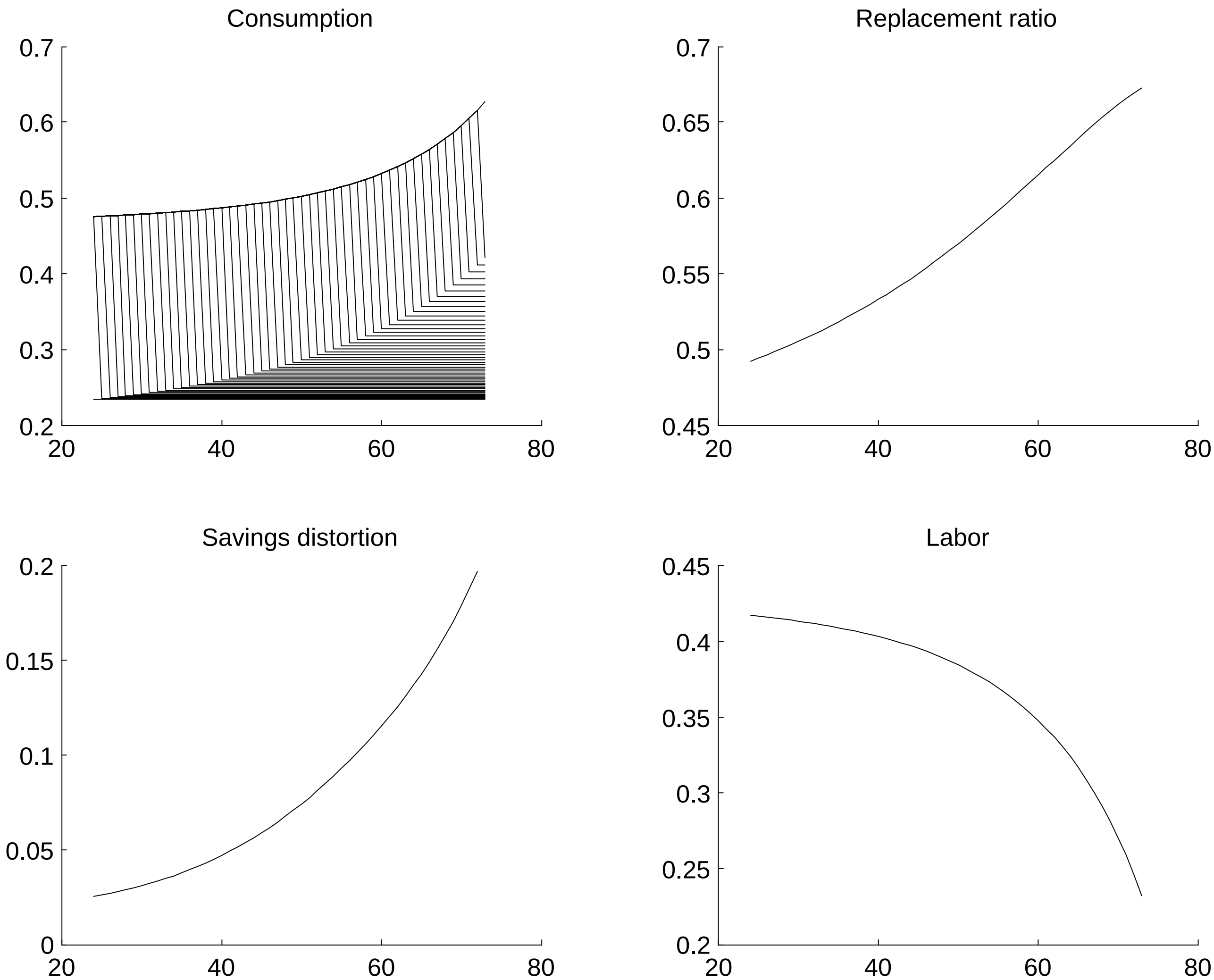


Figure 8: Optimal system: flat skill profile

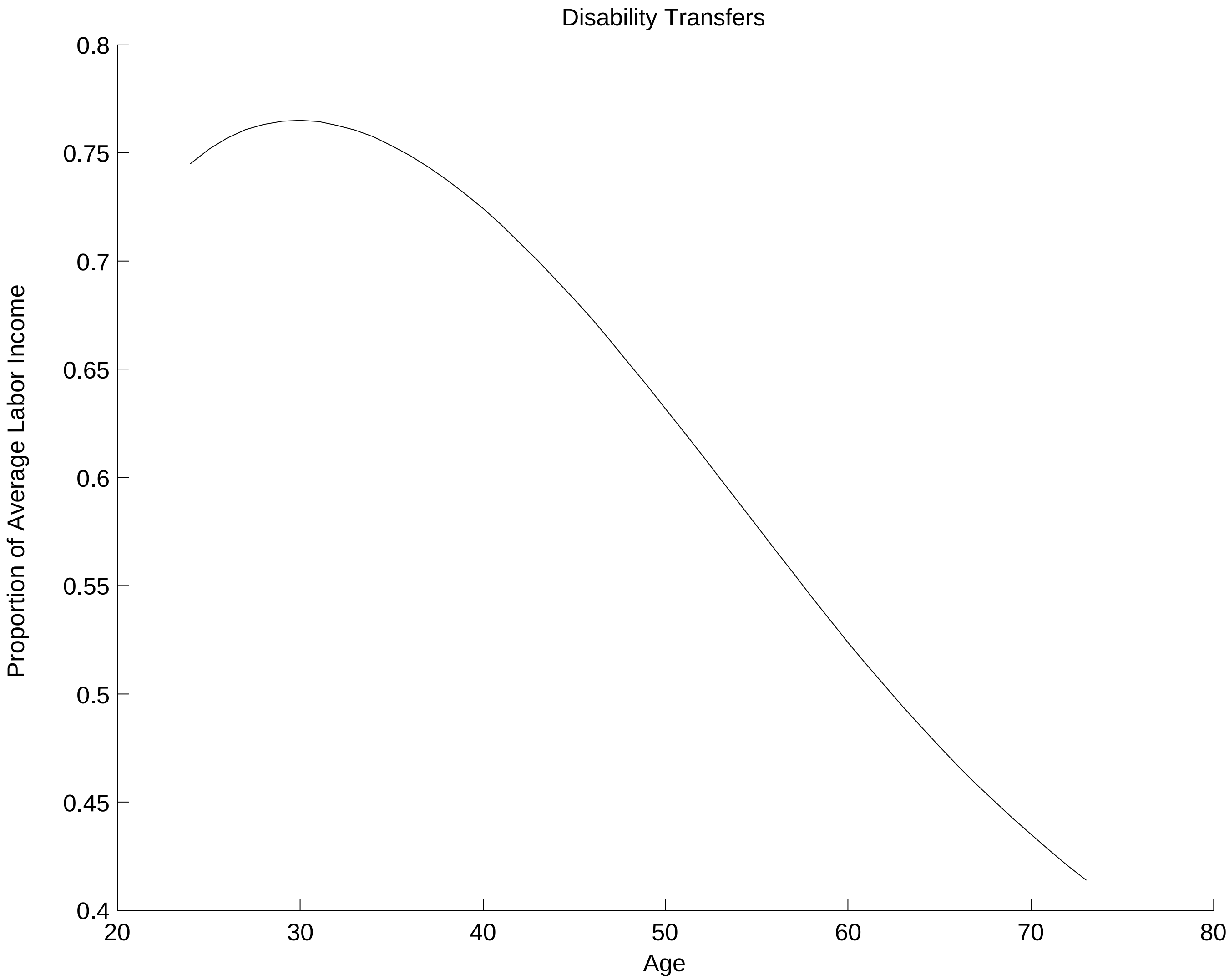


Figure 9: Tax system: disability transfers

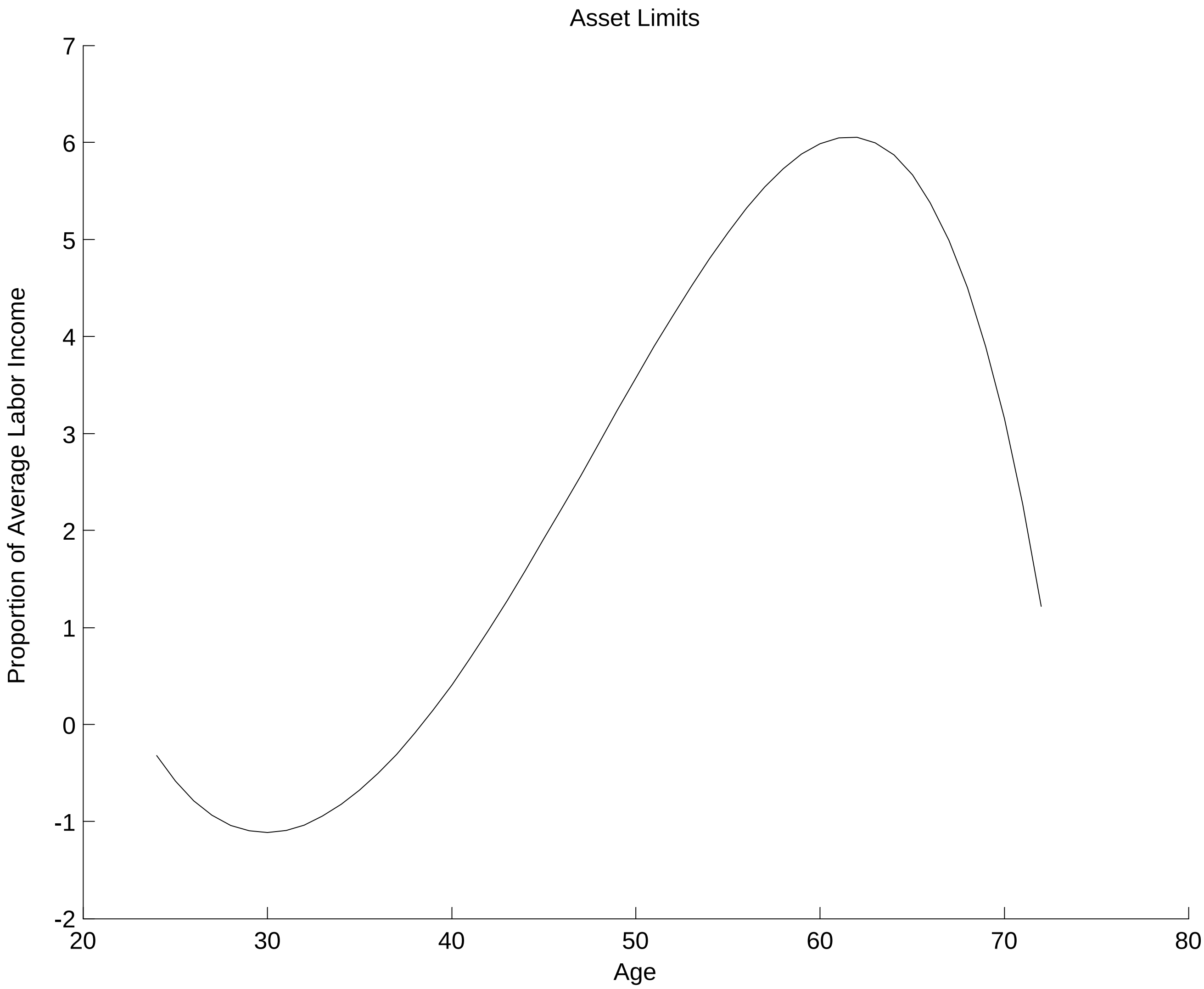


Figure 10: Tax system: asset limits

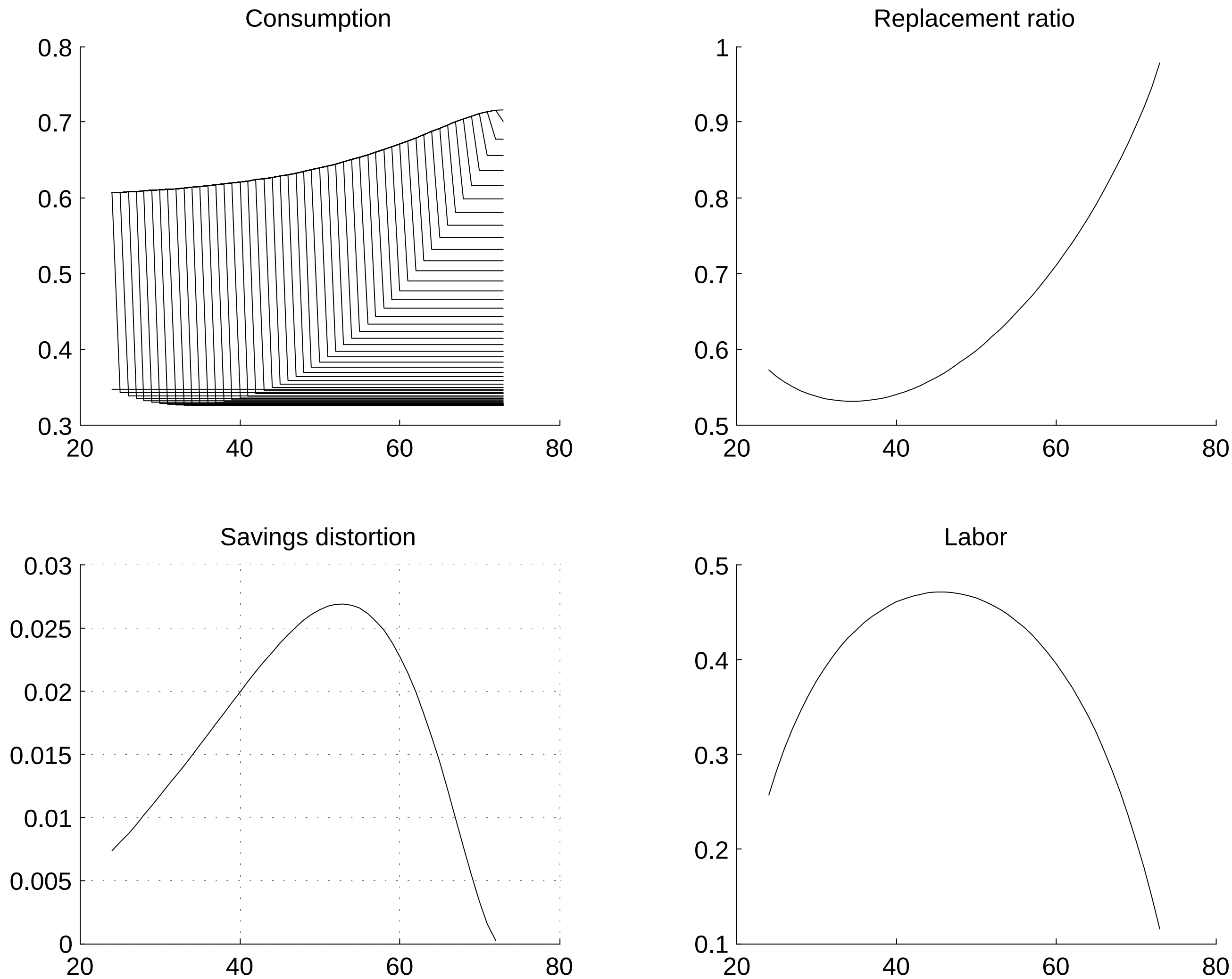


Figure 11: Optimal system: imperfect monitoring