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Liquidity Traps and Monetary Policy: Managing a Credit Crunch*

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ABSTRACT

We study a model with heterogeneous producers that face collateral and cash-in-advance constraints. These two frictions give rise to a nontrivial financial market in a monetary economy. A tightening of the collateral constraint results in a recession generated by a credit crunch. The model can be used to study the effects on the main macroeconomic variables, and on the welfare of each individual of alternative monetary and fiscal policies following the credit crunch. The model reproduces several features of the recent financial crisis, such as the persistent negative real interest rates, the prolonged period at the zero bound for the nominal interest rate, and the collapse in investment and low inflation in spite of the very large increases in liquidity adopted by the government. The policy implications are in sharp contrast to the prevalent view in most central banks, which is based on the New Keynesian explanation of the liquidity trap.

Keywords: Liquidity trap; Credit crunch; Collateral constraints; Monetary policy, Ricardian equivalence

JEL classification: E44, E52, E58, E63

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1 Introduction

The year 2008 will long be remembered in the macroeconomics literature. This is so not only because of the massive shock that hit global financial markets, particularly the bankruptcy of Lehman Brothers and the collapse of the interbank market that immediately followed, but also because of the unusual and extraordinary response to the crisis, emanating from all major central banks. The reaction of the Federal Reserve is a clear example: it doubled its balance sheet in just three months—from \$800 billion on September 1 to \$1.6 trillion by December 1. Then the Fed kept on increasing its balance sheet to reach around \$3 trillion by the end of 2012. Similar measures were taken by the European Central Bank and other central banks in developed economies. Most macroeconomists would probably agree that the 2008-2013 period is, from the point of view of US macroeconomic theory and policy, among the most dramatic in the past 100 years, perhaps second only to the Great Depression.

Paradoxically, however, none of the models used by central banks in developed economies at the time were of any use in studying either the financial shock or the reaction of monetary policy. Those models ignored financial markets on the one hand and monetary aggregates on the other, for good reasons: by and large, big financial shocks have seemed to belong exclusively to emerging economies since the turbulent 1930s. We are not sure how to define emerging economies, but have always suspected that the term meant highly volatile financial markets. In light of this narrow definition, it seems that 2008 taught us, among other things, that we live in an emerging world.¹ In addition, monetary economics have developed, during the last two decades, around the central bank rhetoric of exclusively emphasizing the short-term nominal interest rate. Measures of liquidity or money were completely ignored as a stance of monetary policy. One reason was that the empirical relationships between monetary aggregates, interest rates, and prices, which remained stable for most of the 20th century, broke down in the midst of the banking deregulation that started in the 1980s.² Consequently, during times of financial distress, there is a need for general equilibrium models that can be used to study the effects of policies like those adopted in the United States since 2008.

The purpose of this paper is to provide one such model and to analyze the macroe-

¹See Diaz-Alejandro (1985) for an interesting view on the subject.

²For a detailed discussion, as well as a reinterpretation of the evidence that strongly favors the view of a stable “money demand” relationship, see Lucas and Nicolini (2013).

conomic effects of alternative policies. We study a model with heterogeneous entrepreneurs that face cash-in-advance constraints on purchases and collateral constraints on borrowing. The collateral constraints give rise to a nontrivial financial market. The cash-in-advance constraints give rise to a money market. We use this model to evaluate the effect of monetary and fiscal policies in the equilibrium allocation following a shock to financial intermediation. As we will show below, the collateral constraints imply that Ricardian equivalence does not hold. Thus, taxes and transfers will have allocative effects even if they are lump sum, so monetary policy cannot be studied in isolation as it can in representative agent models.

An essential role of financial markets is to reallocate capital from wealthy individuals with no profitable investment project (savers) to individuals with profitable projects and no wealth (investors). The efficiency of these markets determines the equilibrium allocation of physical capital across projects and therefore equilibrium intermediation and total output. A recent macroeconomic literature studies models of the financial sector with these properties, the key friction being an exogenous collateral constraint on investors. We borrow the model of financial markets from that literature.³ The equilibrium allocation critically depends on the nature of the collateral constraints; the tighter the constraints, the less efficient the allocation of capital and the lower are total factor productivity and output. A tightening of the collateral constraint creates disintermediation and a recession. We interpret this reduction in the ability of financial markets to properly perform the allocation of capital across projects as a negative financial shock.

The single modification we introduce into this basic model is a cash-in-advance constraint on households. Although we consider as an extension the case with nominal wage rigidity, in the benchmark model we assume prices and wages to be fully flexible, mostly to highlight the novel effects in the model. Monetary policy determines equilibrium inflation and nominal interest rates and has real effects. This is not only because of the usual well-understood distortionary effects of inflation in a cash-credit world, but, more important, because of the zero bound on nominal interest rates restriction that arises from optimizing the behavior of individuals in the model and the non-Ricardian nature of our model with financial frictions. The contribution of this paper is the analysis of the effect of monetary policy at the zero bound.

³We closely follow the work of Buera and Moll (2012), who apply to the study of business cycles the model originally developed by Moll (forthcoming) to analyze the role of credit markets in economic development. See Kiyotaki (1998) for an earlier version of a related framework.

One attractive feature of the financial sector model we use is that, if the shock to the collateral constraint that causes the recession is sufficiently large, the equilibrium real interest rate becomes negative for several periods.⁴ The reason is that savings must be reallocated to lower productivity entrepreneurs, but they will only be willing to do it for a lower interest rate. To put it differently, the “demand” for loans falls, which in turn pushes down the real interest rate in this non-Ricardian economy. Depending on what monetary and fiscal policy do, the bound on the nominal interest rate may become a bound on the real interest rate. For example, imagine a policy that targets, successfully, a constant price level: if inflation is zero, the Fisher equation, which is an equilibrium condition of the model, implies a zero lower bound on the real interest rate. At the heart of the mechanism discussed in the paper is the way in which the negative real interest rate interacts with the zero bound on nominal interest rates that arises in a monetary economy under alternative policies.

The qualitative properties of the recession generated by a tightening of the collateral constraint in the model are in line with some of the events that have unfolded since 2008, such as the persistent negative real interest rate, the sustained periods with an effective zero bound on nominal interest rates, and the substantial drop in investment. In addition, the model is consistent with the very large increases in liquidity while the zero bound binds. Thus, we argue, the model in the paper is a good candidate for interpreting the way in which monetary policy is affecting the economy nowadays. In addition, some—but not all!—features of this great contraction make it a (distant) cousin of the Great Depression of the 1930s. The Great Depression evolved in parallel to a major banking crisis, and the severity of the Depression was unique to US history. The role of monetary policy has also been at the center of the debate: for many, the unresponsive Federal Reserve played a key role in the unfolding of events during the Great Depression.⁵ A strongly held view attributes the reaction of the Fed in September 2008 to the lessons that Friedman and Schwartz draw from the Great Depression and attributes to the policy reaction the avoidance of an even major recession, an interpretation that is consistent with the results in our paper.⁶

We first study the case in which the monetary authority is unresponsive to the credit crunch. The model implies that the nominal interest rate will be at its zero lower bound

⁴This feature is specific to the credit crunch. If the recession is driven by an equivalent but exogenous negative productivity shock, the real interest rate remains positive, as we will show.

⁵See Friedman and Schwartz (1963).

⁶Claiming that he “prevented an economic catastrophe,” *Time* magazine named then-Chairman Bernanke Person of the Year on December 2009.

for a finite number of periods and that there will be a deflation on impact and higher inflation thereafter so that there is no arbitrage between money and bonds. If private bonds are indexed to the price level, the real effects are minor. On the contrary, if debt obligations are in nominal terms, the deflation strongly accentuates the recession well beyond the one generated by the credit crunch, due to a debt deflation problem. We then study active inflation-targeting policies for low values of the inflation target. In these cases, the deflation and the associated debt deflation problem are avoided by a very large increase in the supply of government liabilities that must accommodate the credit crunch. Was the different monetary policy recently adopted the reason why the Great Contraction was much less severe than the Great Depression? Our model suggests this may well be the case.⁷

The number of periods that the economy will be at the zero bound and the amount of liquidity that must be injected depend on the target for the rate of inflation. The evolution of output critically depends on this too. As we mentioned above, the interaction between the inflation target and the zero bound on nominal interest rates is the key to understanding the mechanism. Imagine, as before, that the target for inflation is zero. Thus, the Fisher equation plus the zero bound constraint imply that the real interest rate cannot be negative. This imposes a floor on how low the real interest rate can be. But for this to be an equilibrium, private savings must end up somewhere else: this is the role of government liabilities. In this heterogeneous credit-constrained agents model, debt policy does have an effect on equilibrium interest rates, even if taxes are lump sum. Thus, the issuance of government liabilities crowds out private investment.

Policy can affect the real interest rate because Ricardian equivalence does not hold in models in which agents discount future flows with different rates, such as the one we explore. Thus, total government liabilities matter. At the zero bound, money and bonds are perfect substitutes, so monetary policy (increases in the total quantity of money) acts like debt policy.

But policy has an additional effect. In the model, a credit crunch generates a

⁷Friedman and Schwartz argued that the Fed should have substantially increased its balance sheet in order to avoid the deflation during the Great Depression. In 2002, Bernanke, then a Federal Reserve Board governor, said in a speech in a conference celebrating Friedman's 90th birthday, "I would like to say to Milton and Anna: Regarding the Great Depression. You're right, we did it. We're very sorry. But thanks to you, we won't do it again" (speech published in Milton Friedman and Anna Jacobson Schwartz, *The Great Contraction, 1929-1933*) (Princeton, NJ: Princeton University Press, 2008), 227.

recession because total factor productivity (TFP) falls. The reason, as we mentioned above, is that capital needs to be reallocated from high productivity entrepreneurs for which the collateral constraint binds, and therefore must be de-leveraged, to low productivity entrepreneurs for which the collateral constraint does not bind. As a result of the drop in productivity, output and investment fall at the same time that financial intermediation shrinks. Therefore, by keeping real interest rates high, an inflation-targeting policy leaves the most unproductive entrepreneurs out of production, thereby increasing average productivity. Thus, a target for inflation, if low, implies that the drop in productivity will be lower than in the real economy benchmark (i.e., there will be less reallocation of capital to low productivity workers), but the recession will be more prolonged (i.e., capital accumulation falls because of the crowding-out effect). If the target for inflation is higher, say 1%, then the effective lower bound on the real interest rate is -1%, lower than before. Thus, the amount of government liabilities that must be issued will be smaller, the crowding out will be smaller, but the drop in average productivity will be higher.

The model provides an interpretation of the events following 2009 that is different from the one provided by a branch of the literature that, using New Keynesian models, places a strong emphasis on the interaction between the zero bound constraint on nominal interest rates and price rigidities.⁸ This is also the dominant view of monetary policy at major central banks, including the Fed. According to this view, a shock—often associated with a shock to the efficiency of intermediation⁹—drove the natural real interest rate to negative values. The optimal monetary policy in those models is to set the nominal interest rate equal to the natural real interest rate. However, because of the zero bound, that is not possible. But it is optimal, unambiguously, to keep the nominal interest rate at the zero bound, as the Fed has been doing for over four years now. Furthermore, these models imply that it is unambiguously optimal to maintain the nominal interest rate at zero even after the negative shock reverts. This policy implication, called “forward guidance,” has dominated the policy decisions in the United States since 2008 and remains the conceptual framework that justifies the “exit strategy.”

On the contrary, the model we study stresses a different and novel trade-off between

⁸See Krugman (1998), Eggertsson and Woodford (2003), Christiano et al. (2011), Correia et al. (2013), and Werning (2011).

⁹See, for example, Curdia and Eggertsson (2009), Drautzburg and Uhlig (2011), and Galí et al. (2011).

ameliorating the initial recession and delaying the recovery. When the central bank chooses a lower inflation target, the liquidity trap lasts longer, the real interest rate is constrained to be higher, and therefore there is less reallocation of capital toward less productive, and previously inactive, entrepreneurs. The counterpart of the milder drop in TFP is a drop in investment due to the crowding out, leading to a substantial and persistent decline in the stock of capital and a slower recovery.

The paper proceeds as follows. In Section 2, we present the model and characterize the individual problems. In Section 3, we define an equilibrium and characterize the equilibrium dynamics for simple examples. In Section 4, we numerically solve the model under alternative monetary policies and discuss the results. We discuss the distribution of welfare consequences of alternative monetary policies in Section 5, and Section 6 concludes.

Related Literature We consider a monetary version of the model in Buera and Moll (2012), who apply to the study of business cycles the framework originally developed by Moll (forthcoming) to analyze the role of credit markets in economic development. Kiyotaki (1998) is an earlier example that focuses on a two-point distribution of shocks to entrepreneurial productivity. This framework is related to a long tradition that studies the role of firms' balance sheet in business cycles and during financial crises, including Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke et al. (1999), Cooley et al. (2004), Jermann and Quadrini (2012).¹⁰

Kiyotaki and Moore (2012) study a monetary economy where entrepreneurs face stochastic investment opportunities and friction to issue and resell equity on real assets. They also consider the aggregate effects of a shock to the ability to resell equity. In their environment, money is valuable provided that frictions to issue and resell equity are tight enough. They use their model to study the effect of open market operations that consist of the exchange of money for equity. Brunnermeier and Sannikov (2013) also study a monetary economy with financial frictions, emphasizing the endogenous determination of aggregate risk and the role of macroprudential policy. As in our model, a negative aggregate financial shock results in a deflation, although both of these papers consider environments where, for the relevant cases, the zero lower bound on the nominal interest rate is binding in every period.

¹⁰See Buera and Moll (2012) for a detailed discussion of the connection of the real version of our framework with related approaches in the literature.

Guerrieri and Lorenzoni (2011) also study a model where workers face idiosyncratic labor shocks. In their model, a credit crunch leads to an increase in the demand of bonds and therefore results in negative real rates. Although our model also generates a large drop in the real interest rate, the forces underlying this result are different. In our framework, the drop in the real interest rate is the consequence of a collapse in the ability of productive entrepreneurs to supply bonds (i.e., to borrow from the unproductive entrepreneurs and workers), as opposed to an increase in the demand for bonds by these agents. In our model, a credit crunch has an opposite, negative effect on investment.

2 The Model

In this section we describe the model, which closely follows the framework in Moll (forthcoming). The model’s attractive feature for our purposes is that it explicitly deals with heterogeneous agents that are subject to exogenous collateral constraints in a relatively tractable fashion. Tractability is obtained by restricting the analysis to specific function forms—log utility and Cobb-Douglas technologies—so the solution to the optimal problem of the consumers can be obtained in closed form and aggregates behave similarly to Solow’s growth model. This approach allows us to better understand the economics behind the simulations that we present.

We modify the original model by imposing a cash-in-advance constraint on the consumer’s decision problem so that we can determine the aggregate price level and the nominal interest rate. The analysis will be restricted to a perfect foresight economy in which starting at the steady state, all agents learn at time zero that, starting next period, the collateral constraint will be tightened for a finite number of periods.

2.1 Households

All agents have identical preferences, given by

$$\beta^t [\nu \log c_{1t}^j + (1 - \nu) \log c_{2t}^j], \tag{1}$$

where c_{1t}^j and c_{2t}^j are consumption of the cash good and of the credit good, for agent j at time t , and $\beta < 1$. Each agent also faces a cash-in-advance constraint,

$$c_{1t}^j \leq \frac{m_t^j}{p_t}, \quad (2)$$

where m_t^j is the beginning of period money holdings and p_t is the money price of consumption at time t .

The economy is inhabited by two classes of agents, a mass L of workers and a mass 1 of entrepreneurs, which are heterogeneous with respect to their productivity $z \in Z$. We assume that productivity is constant through their lifetime. We let $\Psi(z)$ be the measure of entrepreneurs of type z . Every period, each entrepreneur must choose whether to be an active entrepreneur in the following period (to operate a firm as a manager) or to be a passive one (and offer his wealth in the credit market). We now proceed to study the optimal decision problems of agents.

Entrepreneurs There are four state variables for each entrepreneur: her financial wealth (capital plus bonds), money holdings, the occupational choice (active or passive) made last period and her productivity. She must decide the labor demand if active, how much to consume of each good, whether to be active in the following period, and if so, how much capital to invest in her own firm. An entrepreneur's investment is constrained by her financial wealth at the end of the period a and the amount of bonds she can sell $-b$, $k \leq -b + a$, where we assume that the amount of bonds that can be sold are limited by a simple collateral constraint of the form

$$-b^j \leq \theta k^j \quad (3)$$

for some exogenously given $\theta \in [0, 1]$.

If the entrepreneur decides not to be active (i.e., to allocate zero capital to her own firm), then she invests all her nonmonetary wealth to purchase bonds.

We assume that the technology available to entrepreneurs of type z is a function of capital and labor

$$y = (zk)^{1-\alpha} l^\alpha.$$

This technology implies that revenues of an entrepreneur net of labor payments is a linear function of the capital stock, ρzk , where $\rho = \alpha((1 - \alpha)/w)^{(1-\alpha)/\alpha}$ is the return to the effective units of capital zk , and w denotes the real wage. Thus, the end of period investment and leverage choice of entrepreneurs with ability z solves the following linear problem:

$$\begin{aligned} \max_{k,d} \quad & \rho zk + (1 - \delta)k + (1 + r)b \\ & k \leq a - b, \\ & -b \leq \theta k, \end{aligned}$$

where r is the real interest rate. Denoting the maximum leverage by $\lambda = 1/(1 - \theta)$, it is straightforward to show that the optimal capital and leverage choices are given by the following policy rules, with a simple threshold property¹¹

$$k(z, a) = \begin{cases} \lambda a, & z \geq \hat{z} \\ 0, & z < \hat{z} \end{cases}, \quad b(z, a) = \begin{cases} -(\lambda - 1)a, & z \geq \hat{z} \\ a, & z < \hat{z}, \end{cases}$$

where \hat{z} solves

$$\rho \hat{z} = r + \delta. \tag{4}$$

Given entrepreneurs' optimal investment and leverage decisions, they would face a linear return to their nonmonetary wealth that is a simple function of their productivity

$$R(z) = \begin{cases} 1 + r, & z < \hat{z} \\ \lambda(\rho z - r - \delta) + 1 + r, & z \geq \hat{z}. \end{cases} \tag{5}$$

Given these definitions, the budget constraint of entrepreneur j , with net worth a_t^j and productivity z^j , will be given by

$$c_{1t}^j + c_{2t}^j + a_{t+1}^j + \frac{m_{t+1}^j}{p_t} = R_t(z^j)a_t^j + \frac{m_t^j}{p_t} - T_t(z^j), \tag{6}$$

where we allow lump-sum taxes (transfers if negative) to be a function of the (exoge-

¹¹See Buera and Moll (2012) for the details of these derivations.

nous) productivity of entrepreneurs.

Note that these budget constraints imply that agents choose, at t , money balances m_{t+1}^i for next period, as the cash-in-advance constraints (2) make clear. Thus, we are adopting the timing convention of Svensson (1985), in which goods markets open in the morning and asset markets open in the afternoon. Thus, agents buy cash goods at time t with the money holdings they acquired at the end of period $t - 1$. Similarly, production by entrepreneurs at time t is done with capital goods accumulated at the end of period $t - 1$. An advantage of this timing for our purposes is that it treats all asset accumulation decisions symmetrically, using the standard timing from capital theory.¹²

Workers Workers are all identical and are endowed with a unit of time that they inelastically supply to the labor market. Thus, their budget constraints are given by

$$c_{1t}^W + c_{2t}^W + a_{t+1}^W + \frac{m_{t+1}^W}{p_t} = (1 + r_t)a_t^W + w_t + \frac{m_t^W}{p_t} - T_t^W, \quad (7)$$

where a_{t+1}^W and m_{t+1}^W are real financial assets and nominal money holdings chosen at time t , and T_t^W are lump-sum taxes paid to the government. If $T_t^W < 0$, these represent transfers from the government to workers. We impose on workers a nonborrowing constraint, so $a_t^W \geq 0$ for all t .¹³

2.2 Demographics

The decision rules of entrepreneurs imply that the wealth of active entrepreneurs increases over time, while that of inactive entrepreneurs converges to zero. Thus, each active entrepreneur saves away from the collateral constraint asymptotically. In order for the model to have a nondegenerate asymptotic distribution of wealth across productivity types, we assume that a fraction $1 - \gamma$ of entrepreneurs depart for Nirvana and are replaced by an equal number of new entrepreneurs. The productivity z of the new entrepreneurs is drawn from the same distribution $\Psi(z)$, i.i.d. across entrepreneurs and over time. We assume that there are no annuity markets and that each new entrepreneur inherits the assets of a randomly drawn departed entrepreneur. Agents do

¹²This assumption implies that unexpected changes in the price level have welfare effects, since agents cannot replenish cash balances until the end of the period.

¹³This is a natural constraint to impose. It is equivalent to impose on workers the same collateral constraints entrepreneurs face, since workers will never decide to hold capital in equilibrium.

not care about future generations, so if we let $\hat{\beta}$ be the pure discounting factor, they discount the future with the compound factor $\beta = \hat{\beta}\gamma$, which is the one we used above.

2.3 The Government

In every period the government chooses the money supply M_{t+1} , issues one-period bonds B_{t+1} , and uses type-specific lump-sum taxes (subsidies) $T_t(z)$ and T_t^W . Government policies are constrained by a sequence of period-by-period budget constraints:

$$B_{t+1} - (1 + r_t)B_t + \frac{M_{t+1}}{p_t} - \frac{M_t}{p_t} + \int T_t(z)\Psi(dz) + T_t^W = 0, \quad t \geq 0. \quad (8)$$

We denote by T_t the total tax receipts of the government:

$$T_t = \int T_t(z)\Psi(dz) + T_t^W.$$

In representative agent models, monetary policy can be executed via lump-sum taxes and transfers that, because those models satisfy Ricardian equivalence, are neutral. However, in this model, Ricardian equivalence will not hold for two related reasons. First, agents face different rates of return to their wealth. Thus, the present value of a given sequence of taxes and transfers differs across agents. Second, lump-sum taxes and transfers will redistribute wealth in general, and these redistributions do affect aggregate allocations, due to the presence of the collateral constraints. In the numerical sections, we will be explicit regarding the type of transfers we consider and the effect they have on the equilibrium allocation.

2.4 Optimality Conditions

The optimal problem of agents is to maximize (1) subject to (2) and (6) for entrepreneurs or (7) for workers. Note that the only difference between the two budget constraints is that entrepreneurs have no labor income. For workers, as for inactive entrepreneurs, the real return to their nonmonetary wealth equals $1 + r_t$. In what follows, to save on notation, we drop the index for individual entrepreneurs j unless strictly necessary. Since this is a key aspect of the model, we first briefly explain the zero bound equilibrium restriction on the nominal interest rate that arises from the

agent's optimization problem.¹⁴ Then, we discuss the other first-order conditions.

In this economy, gross savings (demand for bonds) come from inactive entrepreneurs and, potentially, from workers. Note that the return on holding financial assets for these agents is $R_t(z) = (1 + r_t)$, while the return on holding money—ignoring the liquidity services—is given by p_t/p_{t+1} . Thus, if there is intermediation in equilibrium, the return on holding money cannot be higher than the return on holding financial assets. If we define the nominal return as $(1 + r_t) \frac{p_t}{p_{t-1}}$, then for intermediation to be nonzero in equilibrium, the zero bound constraint

$$(1 + r_t) \frac{p_t}{p_{t-1}} - 1 \geq 0 \tag{9}$$

must hold for all t .

The first-order conditions of the household's problem imply the standard Euler equation and intratemporal optimality condition between cash and credit goods:

$$\frac{1}{\beta} \frac{c_{2t+1}}{c_{2t}} = R_{t+1}(z), \quad t \geq 0, \tag{10}$$

$$\frac{\nu}{1 - \nu} \frac{c_{2t+1}}{c_{1t+1}} = R_{t+1}(z) \frac{p_{t+1}}{p_t}, \quad t \geq 1. \tag{11}$$

Solving forward the period budget constraint (6), using the optimal conditions (10) and (11) for all periods, and assuming that the cash-in-advance constraint is binding at the beginning of period $t = 0$, we obtain the solutions for consumption of the credit good and financial assets for agents that face a strictly positive opportunity cost of

¹⁴Formal details are available from the authors upon request.

money in period $t + 1$,¹⁵

$$\begin{aligned} c_{2t} &= \frac{(1-\nu)(1-\beta)}{1-\nu(1-\beta)} \left[R_t(z)a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \right] \\ a_{t+1} &= \beta \left[R_t(z)a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)}. \end{aligned}$$

These equations always characterize the solution for active entrepreneurs even when nominal interest rates are zero. The reason is that for them, the opportunity cost of holding money is given by $R_t(z)p_{t+1}/p_t > (1+r_t)p_{t+1}/p_t \geq 1$, where the last inequality follows from (9). The solution also characterizes the optimal behavior of inactive entrepreneurs, as long as $(1+r_t)p_{t+1}/p_t - 1 > 0$.

The solution for inactive entrepreneurs in periods in which the nominal interest rate is zero, $(1+r_t)p_{t+1}/p_t - 1 = 0$, is

$$a_{t+1} + \frac{m_{t+1}}{p_t} - \frac{m_{t+1}^T}{p_t} = \beta \left[R_t(z)a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)},$$

where

$$\frac{m_{t+1}^T}{p_t} = \frac{\nu(1-\beta)\beta}{1-\nu(1-\beta)} \left[R_t(z)a_t - \sum_{j=0}^{\infty} \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \right] \quad (12)$$

are the real money balances that will be used for transaction purposes in period $t + 1$. Thus, $m_{t+1}/p_t - m_{t+1}^T/p_t \geq 0$ are the excess real money balances, hoarded from period t to $t + 1$.

The optimal plan for workers is slightly more involved, since their income is non-

¹⁵Note that it could be possible that initial money holdings are so large for an active entrepreneur that the cash-in-advance constraint will not be binding in the first period. This case will not be relevant provided initial real cash balances are small enough; that is,

$$\frac{m_0}{p_0} \leq \frac{\nu(1-\beta)}{1-\nu(1-\beta)} \left[R_0(z)a_0 - \sum_{j=0}^{\infty} \frac{T_j(z)}{\prod_{s=1}^j R_s(z)} \right].$$

If this condition is not satisfied, then the optimal policy for period $t = 0$ is to consume a fraction $\nu(1-\beta)$ and $(1-\nu)(1-\beta)$ of the present value of the wealth, inclusive of the initial real money balances, in cash and credit goods. Similarly, the nonmonetary wealth and real money holdings at the end of the first period are functions of the present value of the wealth, inclusive of the initial real money balances.

homogeneous in their net worth and they will tend to face binding borrowing constraints in finite time. In particular, as long as the $(1 + r_\infty)\beta < 1$, as will be the case in the equilibria we will discuss, where r_∞ is the real interest rate in the steady state, workers drive their wealth to zero in finite time and are effectively hand-to-mouth consumers in the long run. That is, for sufficiently large t ,

$$c_{2,t}^W = \frac{(1 - \nu)(w_t - T_t^W)}{1 - \nu(1 - \beta)} \text{ and } c_{1,t+1}^W = \frac{m_{t+1}^W}{p_{t+1}} = \frac{\nu(w_t - T_t^W)}{1 - \nu(1 - \beta)} \frac{\beta p_t}{p_{t+1}}.$$

Along a transition, workers may accumulate assets for a finite number of periods. This would typically be the case if they expect a future drop in their wages—as in the credit crunch we consider—or they receive a temporarily large transfer, $T_t^W < 0$.

3 Equilibrium

Given sequences of government policies $\{M_t, B_t, T_t(z), T_t^W\}_{t=0}^\infty$ and collateral constraints $\{\theta_t\}_{t=0}^\infty$ an equilibrium is given by sequences of prices $\{r_t, w_t, p_t\}_{t=0}^\infty$ and corresponding quantities such that:

- Entrepreneurs and workers maximize, taking as given prices and policies,
- The government budget constraint is satisfied, and
- Bond, labor, and money markets clear:

$$\int b_{t+1}^j dj + b_t^W + B_{t+1} = 0, \quad \int l_t^j dj = 1, \quad \int m_t^j dj + m_t^W = M_t, \quad \text{for all } t.$$

To illustrate the mechanics of the model, we first provide a partial characterization of the equilibrium dynamics of the economy for the case in which the zero lower bound is never binding, $1 + r_{t+1} > p_t/p_{t+1}$ for all t , workers are hand-to-mouth, $a_t^W = 0$ for all t , and the share of cash goods is arbitrarily small, $\nu \approx 0$. Second, we discuss some properties of the model when the zero bound constraint binds. Finally, we study a very special case for which we can obtain closed-form solutions.

3.1 Equilibrium Away from the Zero Bound

Let $\Phi_t(z)$ be the measure of wealth held by entrepreneurs of productivity z at time t . Integrating the production function of all active entrepreneurs, equilibrium output is given by a Cobb-Douglas function of aggregate capital K_t , aggregate labor L , and aggregate productivity Z_t ,

$$Y_t = Z_t K_t^\alpha L^{1-\alpha}, \quad (13)$$

where aggregate productivity is given by the wealth-weighted average of the productivity of active entrepreneurs, $z \geq \hat{z}_t$,

$$Z_t = \left(\frac{\int_{\hat{z}_t}^{\infty} z \Phi_t(dz)}{\int_{\hat{z}_t}^{\infty} \Phi_t(dz)} \right)^\alpha. \quad (14)$$

Note that Z_t is an increasing function of the cutoff \hat{z}_t and a function of the wealth measure $\Phi_t(z)$. In turn, given the capital stock at $t + 1$, which we discuss below, the evolution of the wealth measure is given by

$$\begin{aligned} \Phi_{t+1}(z) = & \gamma \left[\beta \left[R_t(z) \Phi_t(z) - \sum_{j=0}^{\infty} \frac{T_{t+j}(z) \Psi(z)}{\prod_{s=1}^j R_{t+s}(z)} \right] + \sum_{j=1}^{\infty} \frac{T_{t+j}(z) \Psi(z)}{\prod_{s=1}^j R_{t+s}(z)} \right] \\ & + (1 - \gamma) \Psi(z) (K_{t+1} + B_{t+1}), \end{aligned} \quad (15)$$

where the first term on the right-hand side reflects the decision rules of the γ fraction of entrepreneurs that remain alive, and the second reflects the exogenous allocation of assets of departed entrepreneurs among the new generation.

Then, given the (exogenous) value for λ_{t+1} and the wealth measure $\Phi_{t+1}(z)$, the cutoff for next period is determined by the bond market clearing condition

$$\int_0^{\hat{z}_{t+1}} \Phi_{t+1}(dz) = (\lambda_{t+1} - 1) \int_{\hat{z}_{t+1}}^{\infty} \Phi_{t+1}(dz) + B_{t+1}. \quad (16)$$

To obtain the evolution of aggregate capital, we integrate over the individual decisions and use the market clearing conditions. It results in a linear function of aggregate output, the initial capital stock, and the aggregate of the (individual-specific) present

value of taxes,

$$\begin{aligned}
K_{t+1} + B_{t+1} &= \beta [\alpha Y_t + (1 - \delta)K_t + (1 + r_t)B_t] - \beta \int_0^\infty \sum_{j=0}^\infty \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \Psi(dz) \\
&\quad + \int_0^\infty \sum_{j=1}^\infty \frac{T_{t+j}(dz)}{\prod_{s=1}^j R_{t+s}(z)}. \tag{17}
\end{aligned}$$

Solving forward the government budget constraint (8), using that $\nu \approx 0$, and substituting into (17),

$$\begin{aligned}
K_{t+1} &= \beta [\alpha Y_t + (1 - \delta)K_t] + (1 - \beta) \int_0^\infty \sum_{j=1}^\infty \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \Psi(dz) \\
&\quad - (1 - \beta) \int_0^\infty \sum_{j=1}^\infty \frac{T_{t+j}(z) \Psi(dz)}{\prod_{s=1}^j (1 + r_{t+s})} \\
&\quad + \beta L T_t^W - (1 - \beta) L \sum_{j=1}^\infty \frac{T_{t+j}^W}{\prod_{s=1}^j (1 + r_{t+s})}. \tag{18}
\end{aligned}$$

The first term gives the evolution of aggregate capital in an economy without taxes. In this case, aggregate capital in period $t + 1$ is a linear function of aggregate output and the initial level of aggregate capital. The evolution of aggregate capital in this case is equal to the accumulation decision of a representative entrepreneur (Moll, forthcoming; Buera and Moll, 2012). The second term captures the effect of alternative paths for taxes, discounted using the type-specific return to their nonmonetary wealth, while the last term is the present value of taxes from the perspective of the government. For instance, consider the case in which the government increases lump-sum transfers to entrepreneurs in period t , financing them with an increase in government debt and, therefore, with an increase in the present value of future lump-sum taxes. In this case, future taxes will be discounted more heavily by active entrepreneurs, implying that the last term is bigger than the second. Thus, this policy results in lower aggregate capital in period $t + 1$.

Finally, we describe the determination of the price level. In the previous derivations, in particular, to obtain (18), we have used that $\nu \approx 0$, and therefore, the money market clearing condition is not necessarily well defined.¹⁶ More generally, given monetary and

¹⁶To determine the price level in the cashless limit, we need to assume that as $M_{t+1, \nu} \rightarrow 0$,

fiscal policy, the price level is given by the equilibrium condition in the money market,

$$\frac{M_{t+1}}{p_t} = \frac{\nu(1-\beta)\beta}{1-\nu(1-\beta)} \left[\alpha Y_t + (1-\delta)K_t + (1+r_t)B_t - \int_0^\infty \sum_{j=0}^\infty \frac{T_{t+j}(z)}{\prod_{s=1}^j R_{t+s}(z)} \Psi(dz) \right]. \quad (19)$$

The nominal interest rate is obtained from the intertemporal condition of inactive entrepreneurs,

$$\frac{1}{\beta} \frac{c_{2t+1}}{c_{2t}} = \frac{1+i_{t+1}}{\frac{p_{t+1}}{p_t}} = 1+r_{t+1}. \quad (20)$$

Note that, except for the well-known Sargent-Wallace initial price level indeterminacy result, we can think of monetary policy as sequences of money supplies, $\{M_t\}_{t=0}^\infty$, or sequences of nominal interest rates, $\{i_t\}_{t=0}^\infty$. We will think of policy as one of the two sequences and therefore abstract from the implementability problem.¹⁷

There are two important margins in this economy. The first is the allocation of capital across entrepreneurs, which is dictated by the collateral constraints and which determines measured TFP (see (14)). The second is the evolution of aggregate capital over time, which, in the absence of taxes, behaves as in Solow's model (see (18) and set $T_{t+j}(z) = 0$). Clearly, fiscal policy has aggregate implications: the net supply of bonds affects (16) and taxes affect (18). However, monetary policy does not, since none of those equations depend on nominal variables. Monetary policy does have effects, since it distorts the margin between cash and credit goods, but in a fashion that resembles the effects of monetary policy in a representative agent economy. This is the case only if, as assumed above, the zero bound does not bind.

3.2 Equilibrium at the Zero Bound

In periods in which the zero bound binds, successful inflation-targeting policies will affect equilibrium quantities if the target for inflation is tight enough. The reason is that, by successfully controlling inflation, monetary policy, together with the zero bound on nominal interest rates, can impose a bound on real interest rates. To see

$M_{t+1}/\nu \rightarrow \tilde{M}_{t+1} > 0$. See details in Section 3.3.3.

¹⁷Because we use log utility, there is a unique solution for prices, given the sequence $\{M_t\}_{t=0}^\infty$.

this, use (20) and the zero bound to write

$$1 + i_t = \frac{p_t}{p_{t-1}} (1 + r_t) \geq 1, \text{ so } r_t \geq \frac{p_{t-1} - p_t}{p_t} = - \left(\frac{\pi_t}{1 + \pi_t} \right), \quad (21)$$

where π_t is the inflation rate.

Imagine now an economy with zero net supply of bonds that enters a credit crunch, generated by a drop in maximum leverage, λ_t . Equation (16) implies that the threshold \hat{z}_{t+1} has to go down, to reduce the left-hand side and increase the right-hand side so as to restore the equilibrium. This drop in the gross supply of private bonds will reduce the real interest rate, so the marginal entrepreneurs that were lending capital now start borrowing until market equilibrium is restored (i.e., the net supply of bonds is zero). If the credit crunch is large enough, the equilibrium real interest rate may become negative. If inflation is not high enough, the bound (21) may be binding. Imagine, for instance, the case of inflation targeting with a target equal to zero. Then, the real interest rate cannot become negative.

What will an equilibrium look like? In order to support the zero inflation policy, the government needs to inject enough liquidity so that the net supply of money (or bonds, since they are perfect substitutes at the zero bound) goes up to the point where conditions (16) and (21) are jointly satisfied. This policy will have implications for the equilibrium cutoff \hat{z}_{t+1} . In addition, as can be seen in (17), the injection of liquidity (increases in B_{t+1}) affects capital accumulation. Thus, at the zero bound, the level of inflation chosen by the central bank, if low enough, can affect the two relevant margins in the economy. To further explore these implications in this general model, we need to solve it numerically. But before doing that, we now present a particular (very special) case that can be analytically solved and analyzed, which we find very useful in isolating and understanding some of the mechanisms of the model.

3.3 A Simple Case with a Closed-Form Solution

An interesting feature of dynamic models with collateral constraints, such as the one we analyze, is the interaction between the credit constraints and the endogenous savings decisions. This interaction generates dynamics that imply long-run effects that are very different from the ones obtained on impact, precisely through the endogenous decisions agents make over time to save away from those constraints. A complication is that the endogenous wealth distribution becomes a relevant state variable, and it

becomes impossible to obtain analytical results.

It is possible, however, to obtain closed-form solutions if we shut down that endogenous evolution of the wealth distribution. Some of the effects that the simulations of the general model exhibit are also present in this simplified version, where they are easier to understand. We now proceed to discuss that example.

Consider the case in which $\gamma \rightarrow 0$ (but $\hat{\beta} \rightarrow \infty$, so as to keep $\beta = \hat{\beta}\gamma$ constant). So, equation (15) becomes

$$\Phi_{t+1}(z) = \Psi(z)(K_{t+1} + B_{t+1}).$$

Thus, in the limit agents live for only a period, but the saving decisions are not modified. In addition, since it simplifies the algebra, we let z be uniform in $[0, 1]$. Then, the equilibrium condition for the credit market (16) becomes

$$\hat{z}_{t+1} = \theta_{t+1} + (1 - \theta_{t+1})b_{t+1},$$

where $b_t = \frac{B_t}{K_t + B_t}$, and the value of TFP in equation (14) becomes

$$Z_t = \left(\frac{1 + \theta_t + (1 - \theta_t)b_t}{2} \right)^\alpha. \quad (22)$$

Finally, we normalize $L = 1$ so the law of motion for capital in this special case is given by

$$\begin{aligned} K_{t+1} = & \beta \left[\alpha \left(\frac{1 + \theta_t + (1 - \theta_t)b_t}{2} \right)^\alpha K_t^\alpha + (1 - \delta)K_t \right] \\ & + (1 - \beta) \left[\int_0^\infty \sum_{j=1}^\infty \frac{T_{t+j}}{\prod_{s=1}^j R_{t+s}(z)} dz - B_{t+1} \right]. \end{aligned} \quad (23)$$

Note that given sequences of policies and collateral constraints, this equation fully describes the dynamics of capital.

The economy behaves as it does in Solow's growth model, except that the collateral constraint as well as the fiscal policy both matter. The collateral constraint matters because it affects aggregate TFP. Policy matters because the model, as we show in detail below, does not exhibit Ricardian equivalence. Finally, the real interest rate is given by

$$\delta + r_{t+1} = \alpha \frac{\theta_{t+1} + (1 - \theta_{t+1}) b_{t+1}}{Z_{t+1}^{\frac{1-\alpha}{\alpha}} K_{t+1}^{1-\alpha}}.$$

In addition, the constraint (21) must be satisfied.

In order to gain understanding of some of the effects on equilibrium outcomes of changes in the collateral constraint and on the effects of monetary and fiscal policy, we now solve several simple exercises. In the first two exercises, we solve for the real economy, where $\nu = 0$ and focus the discussion on the evolution of real variables. We first set all transfers to zero and study the effect of a credit crunch: an anticipated drop in θ_t that lasts several periods and then goes back to its steady state value. We show that total factor productivity, output, and capital accumulation drop, so the effect of output is persistent. We also show that if the credit crunch is large enough, the real interest rate becomes negative. We then keep θ_t constant and study the effect of debt-financed transfers to show the effect of an increase in the outside supply of bonds in the equilibrium. We show that debt issuance crowds out private investment but increases total factor productivity, so the effect on output is ambiguous. In addition, debt issuance increases the real interest rate. Finally, we consider the cashless limit and study the behavior of the price level following a credit crunch where the real interest rate becomes negative and the zero bound on the nominal interest rate becomes binding. We show that if the central bank does not change the nominal quantity of money, a deflation follows.

3.3.1 The Effect of a Credit Crunch

To isolate the effect of a credit crunch, we set $b_t = 0$ and $T_t(z) = T_t^w = 0$. In this case,

$$\hat{z}_t = \theta_t \text{ and } Z_t = \left(\frac{1 + \theta_t}{2} \right)^\alpha.$$

Given the level of capital, the interest rate is given by

$$\delta + r_t = \frac{\theta_t}{(1 + \theta_t)^{1-\alpha}} \frac{2^{1-\alpha} \alpha}{K_t^{1-\alpha}}, \quad (24)$$

which implies that the real interest rate falls with θ_t . This drop in the real interest rate is what provides the incentives to the less efficient entrepreneurs to enter until the

credit market clears, thereby reducing TFP.¹⁸

The law of motion for capital is given by

$$K_{t+1} = \beta \left[\alpha \left(\frac{1 + \theta_t}{2} \right)^\alpha K_t^\alpha + (1 - \delta)K_t \right]. \quad (25)$$

We model a temporary credit crunch as

$$\begin{aligned} \theta_0 &= \theta^{ss} \\ \theta_t &= \theta_l < \theta^{ss} \text{ for } t = 1, 2, \dots, T \\ \theta_t &= \theta^{ss} \text{ for } t > T \end{aligned}$$

and assume that all agents have perfect foresight. The effect on capital is identical to a temporary drop in TFP in Solow's model: capital does not change on impact, but starts going down until T . Then, it starts going up to the steady state. The interest rate drops on impact, since the ratio $\frac{\theta_{t+1}}{(1+\theta_{t+1})^{1-\alpha}}$ is increasing on θ_{t+1} . Note that

$$\delta + r_1 = \frac{\theta_l}{(1 + \theta_l)^{1-\alpha}} \frac{2^{1-\alpha} \alpha}{K_{ss}^{1-\alpha}},$$

so the real interest rate will be negative if θ_l is low enough.

Then, between periods 2 and T , where θ remains low, the interest rate goes up as capital goes down, approaching what would be the new steady state if the change were permanent. At time $T + 1$, the interest rate jumps up because of the direct effect of θ_t , overshooting the steady state value since the capital stock is below the steady state, and then it goes down as the capital stock recovers. These dynamics are illustrated for the case of a smoother version of this shock in Figure 2.

3.3.2 The Effect of Policy

We consider the case in which taxes and transfers are made to all entrepreneurs, independently of their type (so $T(z)_t = T_t$) and in which there are no taxes or transfers to workers (so $T_t^w = 0$).¹⁹ First, note that (22) trivially implies that total factor produc-

¹⁸The drop in the real interest rate as the collateral constraint falls enough is a general feature of the model, which does not depend on the particular simplifying assumptions used in this example. In general, $r_t = \hat{z}_t / \mathbf{E}[z|z \geq \hat{z}_t]^{1-\alpha} K_t^{\alpha-1} - \delta$, which tends to $-\delta$ as θ_t , and therefore \hat{z}_t , converges to zero.

¹⁹We solve the case in which workers are also taxed in Appendix B, where, to keep analytical tractability, we also assume that workers are hand to mouth. In our simulations we solve for the

tivity is increasing on the ratio of debt to total assets. We assume that the economy starts at the steady state and the policy we study is given by

$$B_0 = 0, \quad B_1 = -T_0 = B > 0 \quad \text{and} \quad (1 + r_1)T_0 + T_1 = 0,$$

so $B_t = 0$ for $t \geq 2$. Given this policy, the law of motion for capital (23) becomes

$$K_1 = K_{ss} - (1 - \beta)B \left[1 - \int_0^1 \frac{(1 + r_1)}{R_1(z)} dz \right]. \quad (26)$$

When $\theta = 1$, $R_1(z) = (1 + r_1)$ for all z (and $\hat{z} = 1$), then

$$(1 + r_1) \int_0^1 \frac{1}{R_1(z)} dz = 1$$

and Ricardian equivalence holds. But when $\theta \in (0, 1)$, from (5) it follows that $R_1(z) > (1 + r_1)$ for $z > \hat{z}$ (and $\hat{z} \in (0, 1)$), then

$$0 < \int_0^1 \frac{(1 + r_1)}{R_1(z)} dz < 1.$$

Thus, the level of capital is lower than the steady state for any positive level of debt.²⁰

Thus, starting at $B = 0$, as debt increases, total factor productivity goes up as seen from (22), but capital goes down, so the net effect on output is ambiguous.²¹ Finally, note that the interest rate on period 1 is given by

$$\delta + r_1 = \frac{\theta + b_1(1 - \theta)}{(1 + \theta + b_1(1 - \theta))^{1-\alpha}} \frac{\alpha 2^{1-\alpha}}{K_1^{1-\alpha}},$$

where the first term is increasing on b_1 , so the interest rate will be higher than in the steady state.

To summarize, a credit crunch and an increase in debt have opposite effects on total factor productivity and on the real interest rate: while the credit crunch reduces both, the increases in debt increase both. On the contrary, both the credit crunch and the debt increase reinforce each other in that they reduce capital accumulation. As

general case in which workers can hold bonds.

²⁰The opposite is true if the government lends such that $B < 0$.

²¹The relationship between government debt and aggregate capital is nonmonotonic. In particular, one can show that as $B \rightarrow \infty$, aggregate capital converges to the steady state value in an economy with $\theta = 1$.

increases in outside liquidity (bonds plus money at the zero bound) dampen the drop in the real interest rate, they will be effective at achieving a target for inflation when a credit crunch implies

$$(1 + r_t)(1 + \pi^*) < 1.$$

Doing so also implies a lower drop in TFP (a higher threshold) but a larger drop in capital. The net effect on output is in general ambiguous, but it can be shown to be positive in the neighborhood of $B = 0$.²²

This trade-off will be present in our simulations of the general model that allows for rich dynamics of the wealth distribution and uses alternative functional forms for the distribution of z .

3.3.3 Deflation Follows Passive Policy

We want to discuss the behavior of the price level following a credit crunch that drives the real interest rate into negative territory and such that the zero bound constraint binds during at least one period. The characterization is simpler in the case in which the zero bound is binding for only one period. As it turns out, if the parameters satisfy certain properties, this will indeed be the case. Thus, as we explain in the example, we will make two assumptions that parameters must satisfy for the equilibrium to be such that the zero bound binds only in one period. Under these conditions, we then explain why deflation will be the result of a credit crunch if policy does not respond.

The cashless limit We consider the limiting case of the cashless economy (i.e., $\nu \rightarrow 0$). In this case, the distortions associated with a positive nominal interest rate do not affect the real allocation.²³ In taking the limit, though, we also let nominal money balances shrink at the same rate so we can still meaningfully determine the equilibrium price level. The details follow.

When the cash-in-advance constraint is binding, the first-order condition is

$$p_t c_t^1 = \frac{\nu}{1 - \nu} c_t^2 p_{t-1} \frac{1}{R_t(z)}.$$

²²In the case in which workers are the only individuals being taxed and subsidized, the net effect on output is negative. The analysis of the net effect on output is presented in Appendix B.

²³In the general case, nonnegligible money balances crowd out capital and ameliorate the drop in the real interest rate and in total factor productivity.

We define $\bar{m}_t = \frac{m_t}{\nu}$, so $p_t c_t^1 = m_t = \bar{m}_t \nu$. Replacing above and taking the limit when $\nu \rightarrow 0$, we obtain

$$\bar{m}_t = c_t^2 p_{t-1} \frac{1}{R_t(z)}.$$

Finally, using the optimal rule for the credit good specialized for the limiting case

$$c_t^2 = (1 - \beta) R_t(z) k_t$$

and aggregating over all agents, we obtain

$$\bar{M}_t = (1 - \beta) K_t p_{t-1}. \quad (27)$$

Because of the cashless limit and since debt and transfers are all zero, the real variables follow the solution described in (25) and (24), irrespectively of the evolution of the price level. However, the price level does depend on the behavior of real variables, so it is useful to obtain some explicit solutions.

If we let $\beta \equiv (1 + \rho)^{-1}$, $\rho > 0$, the steady state is given by

$$K_{ss} = \left(\frac{\alpha}{\rho + \delta} \right)^{\frac{1}{1-\alpha}} \left(\frac{1 + \theta_{ss}}{2} \right)^{\frac{\alpha}{1-\alpha}} \quad (28)$$

and

$$\left(\frac{2\theta_{ss}}{1 + \theta_{ss}} \right) (\rho + \delta) = r_{ss} + \delta. \quad (29)$$

We assume that

$$\frac{2\theta_{ss}}{(1 + \theta_{ss})} > \beta, \quad (30)$$

which implies that the real interest rate is positive in the steady state.²⁴ During the

²⁴The necessary condition for positive interest rates in the steady state, $\frac{2\theta_{ss}}{(1 + \theta_{ss})} > \frac{\delta}{\rho + \delta}$, is weaker. The stronger condition that we assume will also imply that the zero bound on nominal interest rates binds one period at most and simplifies the example.

credit crunch, at $t = 1$ the real interest rate is

$$\delta + r_1 = (\rho + \delta) \frac{2\theta_l}{(1 + \theta_l)} \left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha,$$

which is negative as long as

$$\frac{2\theta_l}{(1 + \theta_l)} \left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha < \frac{\delta}{(\rho + \delta)}. \quad (31)$$

Clearly, there exists a value for $\theta_l \in (0, \theta_{ss})$ such that this constraint is satisfied.

The conditions that determine the price level Let $\bar{M}_{t+1} = \bar{M}$ and assume that in equilibrium $i_t > 0$ for $t \geq 2$. Then, using (27) we obtain that for all $t \geq 2$

$$1 + i_t = (1 + r_t) \frac{p_t}{p_{t-1}} = (1 + r_t) \frac{K_t}{K_{t+1}}.$$

Note that the real interest rate is positive, but there is deflation. It is possible to show, however, that under assumption (30) the deflation is not enough to make the nominal interest rate negative.

Lemma 1: *Given assumption (30), $i_t > 0$ for $t \geq 2$.*

Proof: See Appendix A.

Since all cash-in-advance constraints are binding from $t \geq 2$,

$$p_t = \frac{\bar{M}}{(1 - \beta)K_{t+1}} \text{ for } t \geq 1.$$

We now show that under certain conditions on the parameters, the zero bound is binding at $t = 1$.

Lemma 2: *If $\delta > \rho$, then there exists a $\tilde{\theta}_l > 0$ such that $i_1 = 0$ for all $\theta_l \in (0, \tilde{\theta}_l]$.*

Proof: See Appendix A.

Finally, we show that if the economy starts at the steady state, at time zero, when agents learn there is a credit crunch, the equilibrium price level must be strictly below its steady state value.

Lemma 3: *Under the assumptions of Lemmas 1 and 2, $p_0 < p_{ss}$ for all $\theta_t \in (0, \tilde{\theta}_t)$.*

Proof: See Appendix A.

The intuition for this result is standard. The credit crunch drives the real interest rate below zero to the point at which the zero bound is reached. At this point, there is an excess demand for money as a “store of value.” That excess demand is, of course, real rather than nominal. As the nominal quantity is fixed by policy, the demand pressure results in deflation. The excess demand for money as a store of value will be positive until future inflation is high enough such that the return on money is the same as the return on bonds. As the quantity of money is constant, the price level in the steady state must be the same as the initial price level. The initial deflation allows for future inflation along the path, required for the arbitrage condition to hold, with a zero “long-run” inflation. The resulting deflation is similar to the one that would arise away from the zero bound if output expands and the nominal quantity of money remains fixed, but in this case is caused by a shock that affects the market for bonds.

4 Numerical Examples

In this section, we numerically solve the model to illustrate the way in which monetary policy interacts with the credit crunch. In these numerical examples we allow for endogenous dynamics of the distribution of wealth shares across entrepreneurs, more general distributions of productivity levels, forward-looking workers, a monetary economy away from the cashless limit, nominal debt contracts, and nominal wage frictions. As before, for all the experiments we consider, we start the economy at the steady state and assume that in the first period, agents learn that there will be a deterministic credit crunch. All other parameters are kept constant. Given this credit crunch, we consider two different scenarios for monetary policy. In the first one, we illustrate the interactions between real and nominal variables as a result purely of the credit crunch, in the absence of a policy response. In the second scenario, we assume that monetary and

fiscal policies are such that inflation is kept low and constant at targeted values that are consistent with the typical mandates of central banks. To achieve the desired target, monetary policy must be active, and the equilibrium outcome will depend on the accompanying debt, tax, and transfer policies. Thus, we consider alternative lump-sum tax and subsidy schemes. We compare, in all cases, the evolution of the equilibrium with a benchmark case in a real economy with no government (i.e., one in which we set the parameter $\nu = 0$ and $T(z)_t = T_t^W = M_{t+1} = 0$). In the real economy there is no money, so the zero lower bound is not a relevant consideration. The assumptions that allow us to obtain a relatively simple characterization of the individual’s problem and aggregation (e.g., log utility and individual Cobb-Douglas technologies) make the model less suitable for a full quantitative analysis. For instance, since the model does not have a well-defined size distribution of entrepreneurs, we cannot use moments on the size distribution of establishments or firms to calibrate the distribution of productivities or to match the leverage of the economy. Therefore, the numerical examples that follow should be seen as theoretical illustrations of the model mechanisms.

The model has very few parameters. We set the time period to a quarter. On the production side, we set the capital share in output $\alpha = 1/3$ and the depreciation rate $\delta = 1 - (1 - 0.07)^{1/4}$. For preferences and the demographic structure we set the relative importance of the cash good $\nu = 0.5$, and the discount factor $\beta = 0.986$ to match a quarterly interest rate of 0.005. We choose the survival rate $(1 - \gamma) = 0.9^{1/4}$ to imply a 10% yearly exit rate of entrepreneurs and set the leverage parameter $\theta = 0.75$, which is consistent with data on credit to real assets in Buera et al. (2014). The distribution of productivity z is assumed to be lognormal(0, 1).

4.1 Real Benchmark

As a benchmark, we first present the effects of a credit crunch in a real economy and no policy, so $\nu = 0$, and $B_t = T_t = 0$. The results in this section closely follow those in Buera and Moll (2012).

In the top left panel of Figure 1, we show the evolution of the exogenous driving force of the credit crunch, the debt to capital ratio θ_t . We also show the evolution of total factor productivity, Z_t , as well as the capital stock and the real interest rate during a credit crunch (solid line), and compare them with the evolution of these variables following an “equivalent” exogenous TFP shock (dashed line).²⁵

²⁵Specifically, we feed into the model an unanticipated exogenous TFP shock that replicates the

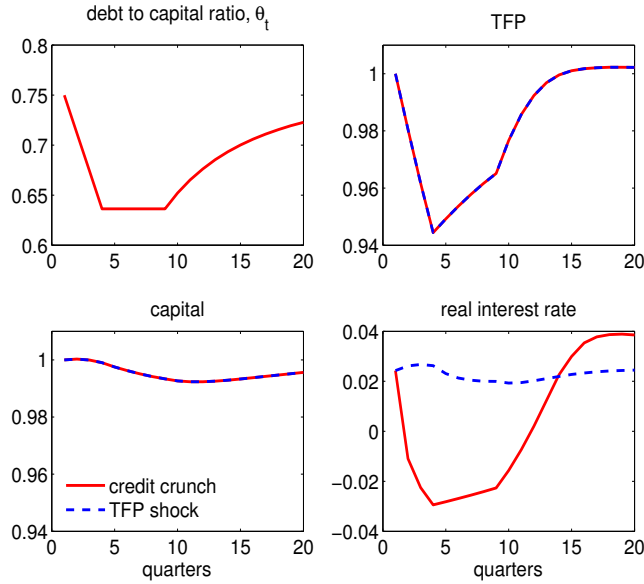


Figure 1: Aggregate Implications of a Credit Crunch: Real Benchmark

As in the simple example, the immediate effect of the credit crunch is to reduce the amount of bonds that active entrepreneurs can issue, so they will manage a lower amount of capital. But as the capital stock is given, some of it will be reallocated to previously inactive, less productive entrepreneurs. This immediately lowers total factor productivity (top right panel) and therefore output (not shown). But for those entrepreneurs to find it optimal to manage capital, the real interest rate has to go down (bottom right panel). The lower output implies that there are fewer resources for investment, and therefore the capital stock drops below its steady state level (bottom left panel), which further implies a reduction in output. Relative to the simple example discussed in Section 3.3.1, in this case TFP starts recovering before the drop in the collateral constraint reverts. This is because the fraction of aggregate wealth owned by productive entrepreneurs increases as their profitability is boosted with the fall in factor prices, the wage, and the interest rate.

As shown in Buera and Moll (2012), the change in aggregate variables, with the exception of the interest rate, is the same in response to a credit crunch or to the corresponding exogenous TFP shock. The drop in the interest rate is substantially

evolution of the endogenous TFP during a credit crunch and then solve for the capital stock and the real interest rate.

more pronounced following a credit crunch, compared with the case of an exogenous TFP shock. As the supply of bonds by productive entrepreneurs is further constrained during a credit crunch, the equilibrium interest rate must drop to clear the bond market. This force is not present in a contraction that is driven by an exogenous decline in TFP. We would like to stress the effect of the credit crunch on the real interest rate. The New Keynesian literature on the zero bound, which represents the dominant view, assumes shocks to the discount factor in order to generate a negative “natural” rate of interest.²⁶ Although our model also generates a large drop in the real interest rate, the forces underlying this result are different. In our numerical exercises we choose a credit crunch—the values for θ_t —such that the equilibrium exhibits a negative real interest rate for two years and such that it averages an annualized value of -2% , a value that was suggested in the literature mentioned above.

4.2 Nonresponsive Monetary Policy

We now show the equilibrium of the monetary model assuming that policy does not respond to the shock, so the quantity of money does not change. Note that although we focus on the case of money rules, in an equilibrium, given a money rule, we obtain a unique sequence of interest rates. One could therefore think of policies as setting those same interest rates.²⁷ Since there is no change in monetary policy, we do not need to change transfers either. We consider an economy with no public debt and therefore no taxes or transfers, $B_t = T_t = 0$ all t .

As shown in Figure 1, a credit crunch results in a large decline in the return of real assets. In a monetary economy, the return of real assets cannot be lower than the return of money. If they are the same, the economy is at the zero lower bound. If at the zero bound there is a further tightening of the collateral constraint, there will be an excess demand for “store of value,” leading in equilibrium to the hoarding of real money balances by inactive entrepreneurs, in excess of the ones needed for transaction purposes. Since the supply of money is held fixed in this exercise, the price level must drop initially so that, in equilibrium, the supply of *real* balances meets the excess

²⁶This discount factor shock is meant to be a reduced form of a shock to financial intermediation; see Curdia and Woodford (2010).

²⁷If one were to think of policy as setting a sequence of interest rates, the issue of price level determination should be addressed. The literature has adopted two alternative routes: the Taylor principle or the fiscal theory of the price level. We abstract from those implementation issues in this paper.

demand of real balances of inactive entrepreneurs and the return on money is the same as the return on bonds.

The responses of the main variables in the nominal economy with a fixed money supply are illustrated in Figure 2. As discussed above and illustrated in the bottom right panel, there is a large deflation on impact and positive inflation afterward as the supply of bonds by productive entrepreneurs recovers and the excess demand for real balances slowly reverts to zero. The initial deflation increases the value of the money balances at the beginning of the initial period, thereby crowding out investment. The larger decline in investment relative to the real benchmark is illustrated in the lower left panel of Figure 2. The lower value for investment implies that the recession is deeper, but the overall effect on output is small.²⁸

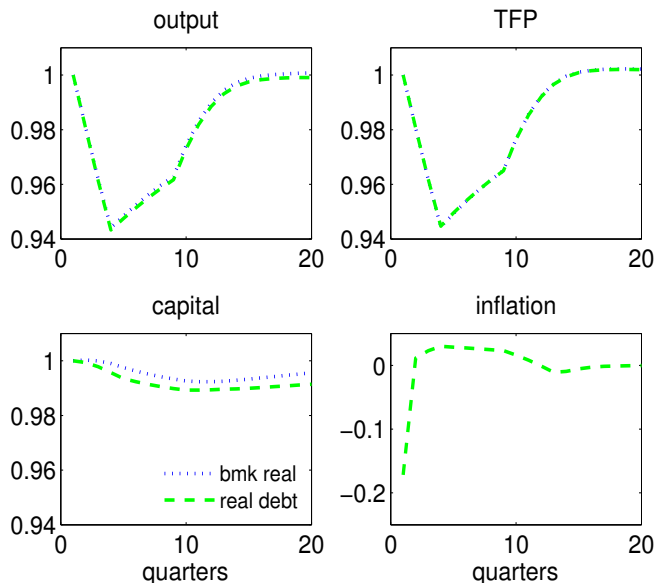


Figure 2: Aggregate Implications of a Credit Crunch: Constant Money

In the context of the model, this unexpected shock has relatively minor consequences. However, it suggests that a potential problem may arise to the extent that debt instruments are nominal obligations or if there is downward wage rigidity as the New Keynesian models assume.²⁹ We now explore these two extensions.

²⁸Capital will be one-third of a percentage point lower, but only one-third of the decline in the capital stock translates to output.

²⁹This “debt deflation” problem has been mentioned as one of the possible costs of deflations before, particularly in reference to the Great Depression (Fisher, 1933).

4.2.1 Nominal Bonds

We now solve the model assuming that entrepreneurs only issue nominal bonds. As before, the real value of bond issuance is restricted by the collateral constraint in (3).

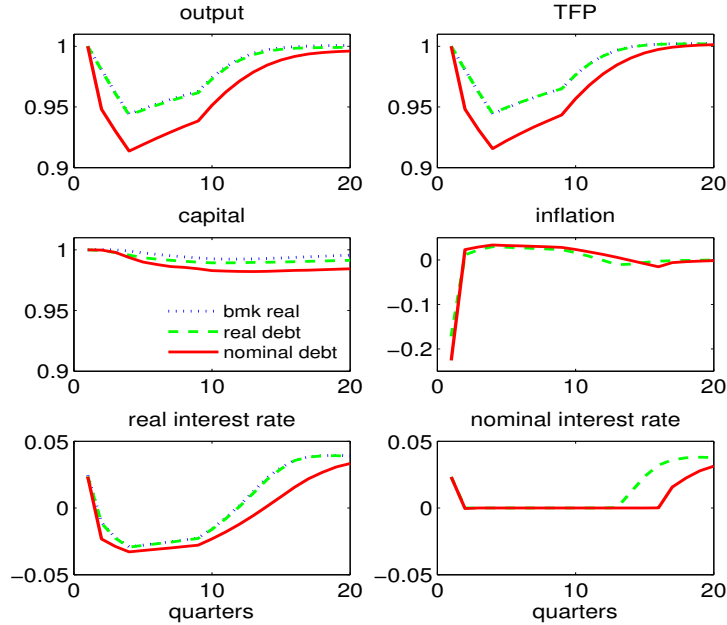


Figure 3: Aggregate Implications of a Credit Crunch: Nominal Debt

The results, which are dramatically different, are depicted in Figure 3, which also plots both the benchmark (dotted line) and the case of indexed debt (dashed line). The recession is deeper and more persistent, driven mainly by a sharper decline in TFP. The intuition for the dramatic effect of the debt deflation is simple: the initial deflation implies a large redistribution from high productivity, leveraged entrepreneurs toward bondholders, who are inactive, unproductive entrepreneurs. The ability of productive entrepreneurs to invest is now hampered by both the tightening of collateral constraints and the decline of their net worth. As a consequence, there needs to be a larger decline in the real interest rate so that in equilibrium more capital is reallocated from productive to unproductive entrepreneurs (bottom left panel), which results in a larger deflation and a nominal interest rate that remains at zero for longer (center and bottom left panels).

This example shows that the initial deflation can be very costly in terms of output

and could provide motivation for policy interventions to stabilize the price level and output. An alternative motivation is given by the existence of nominal price rigidities.

4.2.2 Sticky Wages

We consider in this section an extension of the benchmark model with nominal wage rigidities, a case that can be easily implemented following the New Keynesian tradition. In particular, we consider workers that are grouped into households with a continuum of members supplying differentiated labor inputs. Each member of the household is monopolistically competitive and gets to revise the wage in any given period with a constant probability as in Calvo (1983). To simplify the solution of the model, in this extension we consider the cashless limit discussed in Section 3.3.3.³⁰

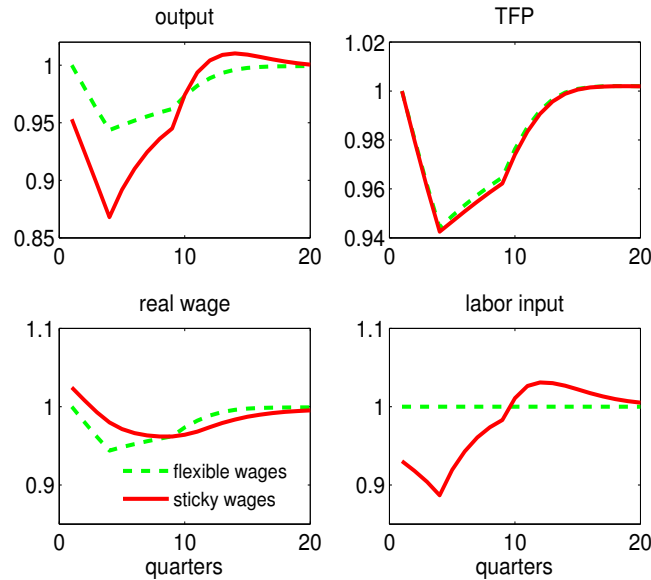


Figure 4: Aggregate Implications of a Credit Crunch: Sticky Wages

Figure 4 shows the evolution of the economy following an unanticipated credit crunch for cases with flexible wages (dashed line) and rigid wages (solid line). In both examples we assume that bonds are indexed to the price of consumption and the supply of money remains constant. Thus, the case with flexible wages (dashed line)

³⁰A detailed and totally standard description of this extension is provided in Appendix C. The parametrization and calibration of this version of the model closely follows the one in Correia et al. (2013).

corresponds to the case labeled “real debt” in Figures 2 and 3, with the caveat that in this section we consider the cashless limit.

With rigid wages, the initial deflation causes an increase in the real wage and a sharp decline in the labor input. Furthermore, as TFP drops, while the real wage only slowly adjusts, the labor input further declines. This results in a substantially more severe recession. As in the previous examples, the real interest rate becomes negative and the nominal interest rate is at the zero lower bound for various periods.

The discussion in last two examples suggests that the initial deflation can be very costly in terms of output. An obvious question, then, is what can monetary policy do, if anything, to stabilize the price level and output? We consider those cases next.

4.3 Inflation Targeting

We now consider the case of a government that implements an inflation target, π^* , which for simplicity we assume constant. As long as the zero bound does not bind, so $r_t > -\frac{\pi^*}{1+\pi^*}$, inflation is determined by standard monetary policy.³¹ However, if given the target, the natural interest rate is inconsistent with the zero bound, the government needs to increase real money balances M_{t+1}/p_t and/or government bonds B_{t+1} in order to satisfy the excess demand for real assets. In order to do so, it will also need to implement a particular scheme of taxes and transfers. As we will show, because Ricardian equivalence does not hold, the way in which the taxes and transfers are executed matters a lot.

It is important to emphasize that once the economy is at the liquidity trap, real money and bonds are perfect substitutes and the only thing that matters is the sum of the two, so there is an indeterminacy in the composition of total outside liquidity. We will further discuss the subtle distinction between monetary and debt policy at the zero bound, but in order to focus on the effects of policy (monetary and/or fiscal) and without loss of generality, we assume that the government sets the quantity of money to be equal to the money required by individuals to finance their purchases of cash goods in every period, m_{t+1}^T , given by equation (12). Then, public debt accommodates the excess demand for bonds in periods during which the real interest rate equals the constant return of money $r_{t+1} = -\pi/(1 + \pi)$,

³¹Notice that because of lack of Ricardian equivalence, the tax policy associated with the monetary injections will have real effects.

$$B_{t+1} = \begin{cases} B_t & \text{if } r_{t+1} > -\frac{\pi}{1+\pi} \\ \int_0^{\hat{z}_{t+1}} \Phi_{t+1}(dz) - (\lambda_{t+1} - 1) \int_{\hat{z}_{t+1}}^{\infty} \Phi_{t+1}(dz) & \text{if } r_{t+1} = -\frac{\pi}{1+\pi} \end{cases}. \quad (32)$$

Obviously, lump-sum taxes (subsidies) must be adjusted accordingly to satisfy the government budget constraint in (8).

These conditions fully determine the evolution of the money supply, government bonds, and the aggregate level of taxes (transfers), but they leave unspecified how taxes (transfers) are distributed across entrepreneurs and workers. We consider two simple cases: first, we present results for the case in which taxes (transfers) are purely lump sum, that is, $T_t(z) = T_t^W = T_t$ for all t, z .³² We refer to this as the “lump-sum” case. The second case that we consider is one in which taxes are purely lump sum for all periods. But in periods when the government increases the supply of bonds, we assume that the proceeds from the sale of bonds, net of interest payments and the adjustment of the supply of real balances, are transferred only to the entrepreneurs according to $T_t(z) = T_t$, and $T_t^W = 0$ for all t, z . The second case captures a scenario in which the government responds to a credit crunch by bailing out productive entrepreneurs and bondholders. We refer to this as the “bailout” case.³³

The results for the case in which the government implements a constant inflation of 2%, a value in line with the price stability mandates of major central banks, are depicted in Figure 5. The solid line corresponds to the case of pure lump-sum taxes (transfer), and the dashed line shows the results for the case in which the government rebates the proceeds of the sale of bonds only to entrepreneurs. For comparison, the dotted line shows again the results for the real benchmark.

To avoid the deflation induced by the excess demand of mediums to serve as a “store of value,” the government must increase the supply of government bonds plus or money (center right panel). Furthermore, the increase in the supply of government bonds induces a further increase in the demand of these bonds by unconstrained entrepreneurs, since these agents save in anticipation of the higher taxes that will be

³²In this section, we need to specify the relative number of workers and entrepreneurs in the economy. We assume that workers are 25% of the population, $L/(1+L) = 1/4$. We choose a low share of workers, who in our model choose to be against their borrowing constraint in a steady state, to limit the non-Ricardian elements in the model.

³³The transfer to bondholders is consistent with the evidence presented by Veronesi and Zingales (2010) for the bailout of the financial sector in 2008.

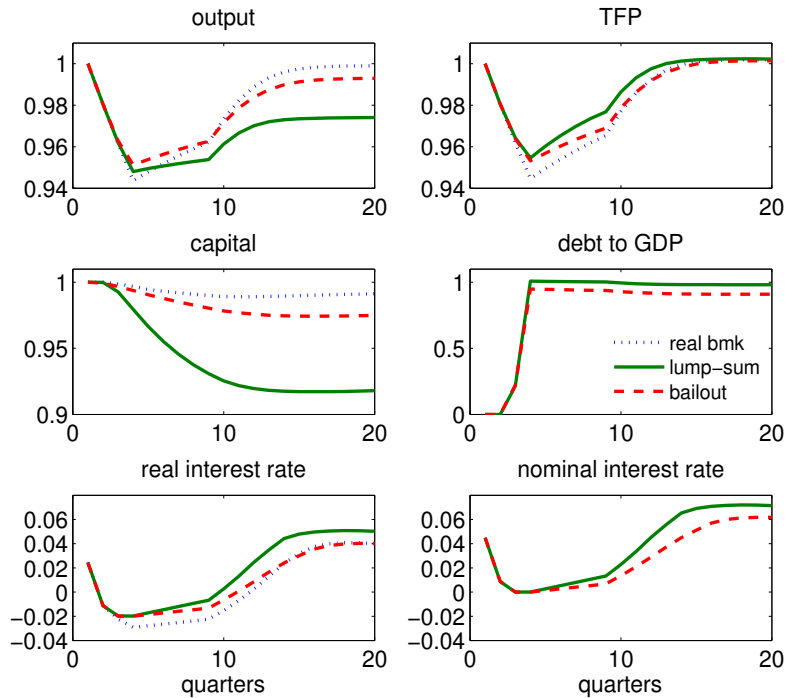


Figure 5: Aggregate Implications of a Credit Crunch with an Inflation Target: Alternative Subsidy Schemes, Inflation Target $\pi = 0.02$

raised to pay the interest on this debt.

As the top left panel of Figure 5 shows, with this policy the government accomplishes a slightly less pronounced recession at the cost of significantly more protracted recovery. The milder recession is explained by the smaller drop in TFP. When the government maintains the inflation low, the real interest rate is constrained to be higher, and therefore there is less reallocation of capital toward less productive, and previously inactive, entrepreneurs. The counterpart of the milder drop in TFP is a collapse in investment, leading to a substantial and persistent decline in the stock of capital.

In our framework, Ricardian equivalence does not hold, so increases in government debt crowd out private investment. This is true for both ways of designing the tax and transfer scheme. Note, however, that the liquidity injections are of very similar magnitudes, which implies that the steady state values of capital (represented as the points on the right-hand-side vertical axes) in the bailout and the lump-sum case are very similar, though different from the benchmark, due to the positive value of

bonds in steady state.³⁴ However, the magnitude of the crowding out is much higher when the government uses pure lump-sum taxes (solid line). In this case, part of the transfers go to workers, who in equilibrium have a large marginal propensity to consume, since they will be against their borrowing constraint in finite time.³⁵ In this case, aggregate consumption increases and investment decreases relative to the bailout case. We present the lump-sum case in spite of this counterfactual feature, just to illustrate that the magnitude of the crowding out depends not only on the total debt issued but also on the way in which taxes and transfers are distributed across the population.

In comparison to the lump-sum case, the recovery is faster when the government rebates the proceeds from the increase in the debt solely to entrepreneurs (dashed line). Nevertheless, the drop in investment is still more pronounced than in the real benchmark.³⁶

Can the government mitigate the consequences of a credit crunch by choosing alternative inflation targets? In particular, is it desirable that the government chooses a sufficiently high inflation target in order to avoid the zero lower bound? We explore this question in Figure 6. There we present the evolution of four economies differing in the level of the inflation target, $\pi = 0, 0.01, 0.02,$ and 0.03 . In all these cases, we assume that the government rebates the proceeds from the increase in the debt solely to entrepreneurs (bailout case).

The two main features of the previous example are reinforced for the economies with a lower inflation target. The lower the inflation target is, the less pronounced the recession in the short run is. To implement that lower target, the government will need a larger increase in the supply of bonds. The larger increase in the government debt will imply a larger crowding out of investment. Therefore, the recovery is slower. On the contrary, for a sufficiently large inflation target, $\pi = 0.03$ in our example, the government closely reproduces the equilibrium in the real benchmark economy. In following this inflation target, the economy is at the liquidity trap for very few periods,

³⁴The analysis of the nonzero supply of bonds in the steady state is available from the authors upon request.

³⁵In a steady state, the interest rate is strictly lower than the rate of time preferences, $(1+r_\infty)\beta < 1$. Therefore, workers, who earn a flow of labor income each period, will choose to be against their borrowing constraint in finite time.

³⁶See Appendix B for analytical comparative static results of the effect of changes in government debt in the neighborhood of $B_1 = 0$ for the simple example introduced in Section 3.3. There we consider the case in which all taxes and transfers are made to entrepreneurs, and the polar case in which workers are the only agents that are taxed and receive transfers.

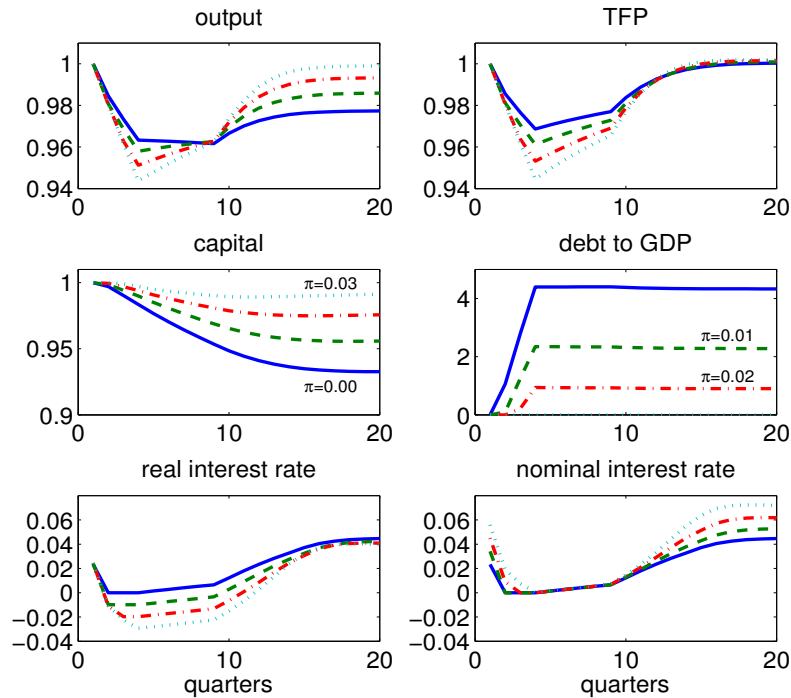


Figure 6: Aggregate Implication of a Credit Crunch with an Inflation Target: Alternative Inflation Targets, Bailout Case

and clearly, for a slightly higher target, the economy will never hit the zero bound. At this point, the allocation is very similar to the real benchmark with a deeper recession but a fast recovery.

The case of a government implementing a low inflation rate seems an attractive approach to interpreting the Great Recession in the United States. Following the 2008 crisis, the economy has been at the zero bound for several quarters, while the Fed has substantially increased its balance sheet. The Fed policy has been directed explicitly to provide the US economy with safe zero nominal interest rate moneylike assets, while inflation has been under total control. In the same direction, the federal government has substantially increased its indebtedness (supply of real assets). All of these features are reproduced by this example. Moreover, the presumption is that these policies avoided a more severe recession, although the recovery is seen as unusually slow—again, a feature of the aggregate economy in this example.

4.3.1 Monetary or Fiscal Policy?

At the zero bound, real money and bonds are perfect substitutes. Thus, standard open market operations in which the central bank exchanges money for short-term bonds have no impact on the economy. What is needed is an effective increase in the supply of government liabilities, which at the zero bound can be money or bonds. How can these policies be executed? Clearly, one way to do it is through bonds, taxes, and transfers. But another way is through a process described long ago: helicopter drops, whereby increases of money are directly transferred to agents. Sure enough, to satisfy the government budget constraint, these helicopter drops need to be compensated with future “vacuums” (negative helicopter drops).

Although the distinction between a central bank or the Treasury making direct transfers to agents may be of varying relevance in different countries because of alternative legal constraints, there is little conceptual difference in the theory. To fully control inflation during a severe credit crunch, the sum of real money plus bonds must go up at the zero bound. Otherwise, there will be an initial deflation, followed by an inflation rate that will be determined by the negative of the real interest rate. If these policies are understood as being outside the realm of central banks, then central banks should not be given tight inflation target mandates: inflation is out of their control during a liquidity trap.

5 Distribution of Welfare Impacts

In the previous section, we focused on the impact of policies on aggregate outcomes and factor prices. The aggregate figures suggest a relatively simple trade-off at the aggregate level. These dynamics, though, hide very disparate effects of a credit crunch and alternative policies among different agents. Although workers are hurt by the drop in wages, the profitability of active entrepreneurs and their welfare can increase as a result of lower factor prices. Similarly, unproductive entrepreneurs are bondholders in equilibrium, and therefore their welfare depends on the behavior of the real interest rate.

Figure 7 presents the impact of a credit crunch on the welfare of workers and on entrepreneurs of different ability under alternative inflation targets for the bailout case. We measure the welfare impact of a credit crunch in terms of the fraction of consumption that an individual is willing to permanently forgo in order to experience

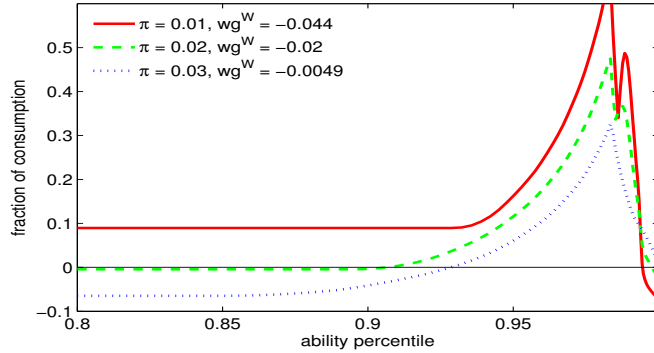


Figure 7: Distribution of Welfare Gains

a credit crunch.³⁷ If positive (negative), we refer to this measure as the welfare gains (losses) from a credit crunch and alternative policy responses.

The dotted line shows the welfare gains for entrepreneurs from a policy that implements a 3% inflation rate as a function of the percentile of their ability distribution. This case is very similar to the real benchmark, since the zero bound is binding only in a couple of quarters. Unproductive entrepreneurs are clearly hurt by a credit crunch, since the return on the bonds they hold becomes negative for over 10 quarters and only gradually returns to the original steady state. Their losses amount to over 5% of permanent consumption. On the contrary, entrepreneurs who become active as the credit crunch lowers factor prices, and who increase their profitability, benefit the most. The same effect increases welfare for previously active entrepreneurs, but they are hurt by the tightening of collateral constraints, which limit their ability to leverage their high productivity. The welfare losses for workers are shown by the legend of each curve. Clearly, workers are hurt by experiencing a credit crunch, since the wages drop for a number of periods. The credit crunch amounts to a permanent drop of over half a percentage point in their consumption, $wg^W = -0.005$.

The other two curves show the welfare consequences of lower inflation targets. The lower the inflation target, the higher the real interest rate, both during the credit crunch and in the new steady state.³⁸ Unproductive entrepreneurs benefit from the

³⁷For entrepreneurs, we consider the welfare of individuals that at the time of the shock have wealth equal to the average wealth of its type. For workers, their welfare is calculated assuming, as is true in the steady state of the model, that they own no wealth when the credit crunch is announced.

³⁸Given the debt policy equation (32), the government debt in the new steady state will be higher the lower the inflation target is. In the model, a higher level of government debt implies a lower level

highest interest rate. Similarly, productive entrepreneurs benefit from the lowest wages associated with the lowest capital during the transition and in the new steady state.³⁹ Although individual entrepreneurs do not internalize it, collectively they benefit from the lower wages associated with a lower aggregate stock of capital. The lower the inflation target, the lower the capital stock and the lower the wages, so the welfare of workers goes down when the target goes down.

6 Conclusions

A contraction in credit due to a tightening of collateral constraints leads to a recession and a drop in the return of safe assets. In a monetary economy, the nominal return on safe assets cannot be negative, so the negative of the rate of inflation is a lower bound on its real return. We showed that if the contraction in credit is large enough, then this constraint becomes binding and the economy enters a liquidity trap. In this case, a deflation occurs if policy is passive. This deflation may interact with collateral constraints, creating debt deflation and worsening the recession if debt obligations are in nominal terms. In addition, it creates a large drop in employment if wages are sticky.

We characterize a policy that avoids that costly deflation. That policy resembles the one followed by the Federal Reserve as a reaction to the 2008 crisis and is in line with Friedman and Schwartz's explanation of the severity of the Great Depression.

The policies that avoid the deflation involve a large increase in money or bonds, which are perfect substitutes at the zero bound. These policies do stabilize prices and output. There is a side effect of these policies, though: they generate slow recovery. We argue that many of the features of the model capture the characteristics of the last financial crisis that hit the United States starting in 2008 and the one that hit Japan in the early 1990s.

The interpretation of the crisis provided by the model in this paper is in contrast to the dominant view in most central banks and is supported by a literature that

of capital in the new steady state.

³⁹The nonmonotonic nature of the welfare effects is related to the heterogeneous impact due to the changing nature of the occupational choice of agents during the transition. For example, the entrepreneur that benefits the most is the most productive inactive entrepreneur in the steady state. As the real rate goes down, that agent becomes an entrepreneur and starts borrowing to profit from the difference between his productivity and the now low interest rate and also from the lower equilibrium wage. On the other hand, the most productive entrepreneur also benefits from the low input prices, but is hurt by the reduction in ability to borrow. Thus, although she gets a higher margin per unit of capital, she can only manage a lower amount of capital.

emphasizes price frictions. According to that literature, it is unambiguously optimal to maintain the economy at the zero bound even after the shock that drove real interest rates to negative values reverts. The model of this paper implies that avoiding the zero bound or not implies nontrivial trade-offs: ameliorating the drop in output at the cost of a slower recovery. The policy trade-offs are even more subtle when the heterogeneous effects across agents are taken into account.

Our model rationalizes the notion that the inflation determination mechanisms differ substantially when the policy authority decides to be at the zero bound. Away from the zero bound, it depends on standard monetary mechanisms. But at the zero bound, it is total outside liabilities that matter: inflation can be controlled only by managing the real interest rate so it does not become too negative.

A Additional Proofs

Proof of Lemma 1: Use the solutions for the real interest rate and capital (25) and (24) to write

$$1 + i_t = \left[\alpha \theta_{ss} \left(\frac{1 + \theta_{ss}}{2} K_t \right)^{\alpha-1} + (1 - \delta) \right] \frac{K_t}{K_{t+1}} = \frac{\left[\alpha \theta_{ss} \left(\frac{1 + \theta_{ss}}{2} \right)^{\alpha-1} K_t^\alpha + (1 - \delta) K_t \right]}{\beta \left[\alpha \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha K_t^\alpha + (1 - \delta) K_t \right]}.$$

Assume now, toward a contradiction, that

$$1 + i_t = \frac{\left[\alpha \theta_{ss} \left(\frac{1 + \theta_{ss}}{2} \right)^{\alpha-1} K_t^\alpha + (1 - \delta) K_t \right]}{\beta \left[\alpha \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha K_t^\alpha + (1 - \delta) K_t \right]} < 1.$$

Then,

$$\alpha \theta_{ss} \left(\frac{1 + \theta_{ss}}{2} \right)^{\alpha-1} K_t^\alpha + (1 - \delta) K_t < \beta \left[\alpha \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha K_t^\alpha + (1 - \delta) K_t \right],$$

which can be written as

$$\alpha \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha K_t^\alpha \left(\frac{2\theta_{ss}}{1 + \theta_{ss}} - \beta \right) + (1 - \delta) K_t (1 - \beta) < 0.$$

Assumption (30) implies that the first term on the left-hand side is positive. As δ and $\beta \in (0, 1)$, this is a contradiction. \square

Proof of Lemma 2: Assume, toward a contradiction, that $i_1 > 0$. Then

$$\bar{M} = (1 - \beta)K_1p_0$$

so

$$\frac{p_1}{p_0} = \frac{K_1}{K_2},$$

and the solution for the nominal interest rate is given by

$$1 + i_1 = (1 + r_1) \frac{p_1}{p_0} = \left[\frac{2\theta_l}{(1 + \theta_l)} \left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) + (1 - \delta) \right] \frac{K_1}{K_2},$$

but

$$\frac{K_1}{K_2} = \frac{\alpha \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha K_{ss}^\alpha + (1 - \delta) K_{ss}}{\alpha \left(\frac{1 + \theta_l}{2} \right)^\alpha K_{ss}^\alpha + (1 - \delta) K_{ss}} = \frac{\alpha \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha + (1 - \delta) K_{ss}^{1-\alpha}}{\alpha \left(\frac{1 + \theta_l}{2} \right)^\alpha + (1 - \delta) K_{ss}^{1-\alpha}}.$$

Replacing the solution for K_{ss} , we obtain

$$\frac{K_1}{K_2} = \frac{\alpha \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha + (1 - \delta) \frac{\alpha}{\frac{1}{\beta} - 1 + \delta} \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha}{\alpha \left(\frac{1 + \theta_l}{2} \right)^\alpha + (1 - \delta) \frac{\alpha}{\frac{1}{\beta} - 1 + \delta} \left(\frac{1 + \theta_{ss}}{2} \right)^\alpha} = \frac{1/\beta}{\left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) + (1 - \delta)}.$$

Then

$$1 + i_1 = \frac{1}{\beta} \frac{\frac{2\theta_l}{(1 + \theta_l)} \left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) + (1 - \delta)}{\left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) + (1 - \delta)}. \quad (33)$$

Thus, for the lower bound on the nominal interest rate to be binding, we need

$$\frac{1}{\beta} \frac{\frac{2\theta_l}{(1 + \theta_l)} \left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) + (1 - \delta)}{\left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) + (1 - \delta)} < 1,$$

which implies that

$$\frac{2\theta_l}{(1 + \theta_l)} \left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) + (1 - \delta) < \beta \left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) + \beta(1 - \delta)$$

or

$$(1 - \delta)(1 - \beta) < \left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) \left[\beta - \frac{2\theta_l}{(1 + \theta_l)} \right].$$

We now briefly characterize the function

$$f(\theta_l) = \left(\frac{1 + \theta_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) \left[\beta - \frac{2\theta_l}{(1 + \theta_l)} \right].$$

Equation (30) implies that $f(\theta_{ss}) = 0$. On the other hand,

$$f(0) = \left(\frac{1}{1 + \theta_{ss}} \right)^\alpha \frac{(\rho + \delta)}{1 + \rho}.$$

As $\delta > \rho$,

$$\frac{\delta}{1 - \delta} > (1 - \beta) = \frac{\rho}{1 + \rho}$$

so

$$\theta_{ss} < 1 < \frac{\delta}{1 - \delta} \frac{1 + \rho}{\rho}$$

and

$$1 + \theta_{ss} < \frac{\delta + \rho}{(1 - \delta)\rho}.$$

Thus,

$$(1 + \theta_{ss})^\alpha < 1 + \theta_{ss} < \frac{\delta + \rho}{(1 - \delta)\rho}$$

and

$$(1 + \theta_{ss})^\alpha (1 - \delta)\rho < \delta + \rho$$

or

$$\frac{\rho}{1 + \rho} (1 - \delta) = (1 - \beta)(1 - \delta) < \left(\frac{1}{1 + \theta_{ss}} \right)^\alpha \frac{\delta + \rho}{1 + \rho}.$$

Thus, by the intermediate value theorem, there exists a $\tilde{\theta}_l \in (0, \theta_{ss})$ such that

$$(1 - \delta)(1 - \beta) = \left(\frac{1 + \tilde{\theta}_l}{1 + \theta_{ss}} \right)^\alpha (\rho + \delta) \left[\beta - \frac{2\tilde{\theta}_l}{(1 + \tilde{\theta}_l)} \right].$$

Since $f(\theta_l)$ is decreasing, the zero bound will bind for all $\theta_l \in (0, \tilde{\theta}_l]$. \square

Proof of Lemma 3: The ratio of the price level at $t = 0$ to the price level in the steady state p_{ss} is given by

$$\begin{aligned} \frac{p_0}{p_{ss}} &= (1 + r_1) \frac{p_1}{p_{ss}} \\ &= \frac{1}{\beta} \frac{\frac{2\theta_l}{(1+\theta_l)} \left(\frac{1+\theta_l}{1+\theta_{ss}} \right)^\alpha (\rho + \delta) + (1 - \delta)}{\left(\frac{1+\theta_l}{1+\theta_{ss}} \right)^\alpha (\rho + \delta) + (1 - \delta)}, \end{aligned}$$

which is equal to the right-hand side of (33) and, therefore, it is strictly less than one provided $\theta_l \in (0, \tilde{\theta}_l)$. \square

B The Effect of Public Debt Around $B = 0$

In this appendix we characterize the effect of public debt on GDP for two limiting cases. First, we consider the example presented in Section 3.3.2, where only entrepreneurs pay taxes and receive subsidies associated with the temporary one-period increase in government debt. For this case, we show that GDP tends to be an increasing function of the level of public debt in the neighborhood of $B = 0$. Second, we consider the polar case in which only workers pay taxes and receive subsidies associated with the temporary one-period increase in government debt. In this case, we show that GDP is a decreasing function of the level of public debt in the neighborhood of $B = 0$. These examples illustrate that the net effect of government debt on aggregate output depends on the particular implementation of the debt policy and on the relative size of workers and entrepreneurs in the population.

B.1 Taxing/Subsidizing Only Entrepreneurs

Differentiating (26) around $B_1 = 0$,

$$\left. \frac{\partial K_1}{\partial B_1} \right|_{B_1=0} = -(1-\beta) \left[1 - \int \frac{(1+r_{ss})}{R_{ss}(z)} dz \right]. \quad (34)$$

Similarly, differentiating (22) around $B_1 = 0$,

$$\left. \frac{\partial Z_1}{\partial B_1} \right|_{B_1=0} = \alpha Z_{ss} K_{ss}^{-1} \frac{1-\theta}{1+\theta}. \quad (35)$$

Thus, the net effect on GDP around $B_1 = 0$ is as follows:

$$\begin{aligned} \left. \frac{\partial Y_1}{\partial B_1} \right|_{B_1=0} &= \alpha Z_{ss} K_{ss}^{\alpha-1} \frac{1-\theta}{1+\theta} - \alpha Z_{ss} K_{ss}^{\alpha-1} (1-\beta) \left[1 - \int \frac{(1+r_{ss})}{R_{ss}(z)} dz \right] \\ &= \alpha Z_{ss} K_{ss}^{\alpha-1} \left[\frac{1-\theta}{1+\theta} - (1-\beta) \left[1 - \int \frac{(1+r_{ss})}{R_{ss}(z)} dz \right] \right]. \end{aligned}$$

Finally, using the expressions for $R_1(z)$ and solving the integral, we have

$$\left. \frac{\partial Y_1}{\partial B_1} \right|_{B_1=0} = \alpha Z_{ss} K_{ss}^{\alpha-1} (1-\theta) \left[\frac{1}{1+\theta} - (1-\beta) \left[1 - \frac{1+r_{ss}}{r_{ss}+\delta} \theta \log \left(\frac{r_{ss}+\delta}{1+r_{ss}} \frac{1}{\theta} + 1 \right) \right] \right],$$

where around $B_1 = 0$ the real interest rate $r_{ss} = (\rho + \delta)2\theta/(1 + \theta) - \delta$. It is straightforward to show that this expression is positive for β close to 1 or θ close to 0.

B.2 Taxing/Subsidizing Only Workers

In this case,

$$\left. \frac{\partial K_1}{\partial B_1} \right|_{B_1=0} = -1, \quad (36)$$

and the effect on TFP is also given by (35). Thus,

$$\left. \frac{\partial Y_1}{\partial B_1} \right|_{B_1=0} = -\alpha Z_{ss} K_{ss}^{\alpha-1} \frac{2\theta}{1+\theta} < 0.$$

C Environment with Sticky Wages

In this appendix we describe the extension with rigid wages that is solved in Section 4.2.2. In order to allow for sticky wages, we now consider the case in which workers are grouped into households with a continuum of members indexed by $h \in [0, 1]$, each supplying a differentiated labor input l_{ht} . Each member is endowed with a unit of time. Preferences of the household are described by

$$\sum_{t=0}^{\infty} \beta^t [\zeta \nu \log c_{1t}^W + \zeta (1 - \nu) \log c_{2t}^W + (1 - \zeta) \log (N_t)],$$

where leisure is

$$N_t = 1 - \int_0^1 l_{ht} dh. \quad (37)$$

The differentiated labor varieties aggregate up to the labor input L_t , used in production by individual entrepreneurs, according to the Dixit-Stiglitz aggregator

$$L_t = \left[\int_0^1 l_{ht}^{\frac{\eta-1}{\eta}} dh \right]^{\frac{\eta}{\eta-1}}, \eta > 1. \quad (38)$$

Each member of the household, which supplies a differentiated labor variety, behaves under monopolistic competition. They set wages as in Calvo (1983), with the probability of being able to revise the wage $1 - \alpha^w$. This lottery is also *i.i.d.* across workers and over time. The workers that are not able to set wages in period 0 all share the same wage w_{-1} . Other prices are taken as given.

There is a representative firm that produces homogeneous labor to be used in production by the entrepreneurs using the production function (38). The representative firm minimizes $\int_0^1 w_{ht} l_{ht} dh$, where w_{ht} is the wage of the h -labor, for a given aggregate L_t , subject to (38). The demand for l_{ht} is

$$l_{ht} = \left(\frac{w_{ht}}{w_t} \right)^{-\eta} L_t, \quad (39)$$

where W_t is the aggregate wage level, given by

$$w_t = \left[\int_0^1 w_{ht}^{1-\eta} dh \right]^{\frac{1}{1-\eta}}. \quad (40)$$

It follows that $\int_0^1 w_{ht} n_{ht} dh = w_t L_t$. In order to simplify the analysis, we also assume that workers are hand to mouth. In this case, the representative worker maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t [\zeta \nu \log c_{1t}^W + \zeta (1 - \nu) \log c_{2t}^W + (1 - \zeta) \log (N_t)]$$

subject to

$$c_{1t}^W + c_{2t}^W + \frac{m_{t+1}^W}{p_t} = \frac{1}{p_t} \int_0^1 w_{ht} l_{ht} dh + \frac{m_t^W}{p_t} - T_t^W,$$

$$l_{ht} = \left(\frac{w_{ht}}{w_t} \right)^{-\eta} L_t,$$

and

$$c_{1t}^W \leq \frac{m_t^W}{p_t}.$$

Note that although consumption and total labor will not be stochastic, each particular w_{ht} will be a random variable. From the first-order conditions of representative workers, we obtain

$$w_{ht} = \tilde{w}_t = \frac{\eta}{\eta - 1} \sum_{j=0}^{\infty} \xi_{t+j} \frac{1 - \zeta}{\zeta(1 - \nu)} \frac{p_{t+j} c_{2t+j}^W}{N_{t+j}},$$

where

$$\xi_{t+j} = \frac{(\beta \alpha^w)^j \frac{\zeta(1-\nu)}{c_{2t+j}^W} \frac{1}{p_{t+j}} w_{t+j}^\eta L_{t+j}}{\sum_{j=0}^{\infty} (\alpha^w \beta)^j \frac{\zeta(1-\nu)}{c_{2t+j}^W} \frac{1}{p_{t+j}} w_{t+j}^\eta L_{t+j}}$$

and

$$\sum_{j=0}^{\infty} \xi_{t+j} = 1.$$

The evolution of the cost of a composite unit of labor is

$$w_t = \left[(1 - \alpha^w) \tilde{w}_t^{1-\theta^w} + \alpha^w w_{t-1}^{1-\theta^w} \right]^{\frac{1}{1-\theta^w}},$$

and

$$L_t = \left[\alpha^w \left(\frac{w_{t-1}}{w_t} \right)^{-\theta^w} + (1 - \alpha^w) \left(\frac{\tilde{w}_t}{w_t} \right)^{-\theta^w} \right]^{-1} (1 - N_t)$$

solves for the aggregate composite labor input given aggregate leisure.

To implement this extension, we follow Correia et al. (2013) and calibrate $\zeta = 0.3$, $\eta = 3$, and $\alpha^w = 0.85$. To simplify the calculations, we consider the cashless limit. The other parameter values are set as in the other numerical examples.

References

- BERNANKE, B. AND M. GERTLER (1989): “Agency Costs, Net Worth, and Business Fluctuations,” *American Economic Review*, 79, 14–31.
- BERNANKE, B. S., M. GERTLER, AND S. GILCHRIST (1999): “The financial accelerator in a quantitative business cycle framework,” in *Handbook of Macroeconomics*, ed. by J. B. Taylor and M. Woodford, Elsevier, vol. 1 of *Handbook of Macroeconomics*, chap. 21, 1341–1393.
- BRUNNERMEIER, M. K. AND Y. SANNIKOV (2013): “The I Theory of Money,” Manuscript, Princeton University.
- BUERA, F. J., R. FATTAL-JAEF, AND Y. SHIN (2014): “Anatomy of a Credit Crunch: From Capital to Labor Markets,” Working Paper 19997, National Bureau of Economic Research.
- BUERA, F. J. AND B. MOLL (2012): “Aggregate Implications of a Credit Crunch,” Working Paper 17775, National Bureau of Economic Research.
- CALVO, G. A. (1983): “Staggered Prices in a Utility-Maximizing Framework,” *Journal of Monetary Economics*, 12, 383–398.
- CHRISTIANO, L., M. EICHENBAUM, AND S. REBELO (2011): “When Is the Government Spending Multiplier Large?” *Journal of Political Economy*, 119, 78–121.
- COOLEY, T., R. MARIMON, AND V. QUADRINI (2004): “Aggregate Consequences of Limited Contract Enforceability,” *Journal of Political Economy*, 112, 817–847.
- CORREIA, I., E. FARHI, J. P. NICOLINI, AND P. TELES (2013): “Unconventional Fiscal Policy at the Zero Bound,” *American Economic Review*, 103, 1172–1211.
- CURDIA, V. AND G. EGGERTSSON (2009): “What Caused the Great Depression?” Manuscript, Federal Reserve Bank of New York.
- CURDIA, V. AND M. WOODFORD (2010): “Credit Spreads and Monetary Policy,” *Journal of Money, Credit and Banking*, 42, 3–35.
- DIAZ-ALEJANDRO, C. (1985): “Good-bye Financial Repression, Hello Financial Crash,” *Journal of Development Economics*, 19, 1–24.
- DRAUTZBURG, T. AND H. UHLIG (2011): “Fiscal Stimulus and Distortionary Taxation,” Working Paper 17111, National Bureau of Economic Research.

- EGGERTSSON, G. B. AND M. WOODFORD (2003): “The Zero Bound on Interest Rates and Optimal Monetary Policy,” *Brookings Papers on Economic Activity*, 34, 139–235.
- FISHER, I. (1933): “The Debt-Deflation Theory of Great Depressions,” *Econometrica*, 1, 337–357.
- FRIEDMAN, M. AND A. SCHWARTZ (1963): *A Monetary History of the United States, 1867-1960*, Princeton, NJ: Princeton University Press.
- GALÍ, J., F. SMETS, AND R. WOUTERS (2011): “Unemployment in an Estimated New Keynesian Model,” Working Paper 17084, National Bureau of Economic Research.
- GUERRIERI, V. AND G. LORENZONI (2011): “Credit Crises, Precautionary Savings, and the Liquidity Trap,” Working Paper 17583, National Bureau of Economic Research.
- JERMANN, U. AND V. QUADRINI (2012): “Macroeconomic Effects of Financial Shocks,” *American Economic Review*, 102, 238–71.
- KIYOTAKI, N. (1998): “Credit and Business Cycles,” *Japanese Economic Review*, 49, 18–35.
- KIYOTAKI, N. AND J. MOORE (1997): “Credit Cycles,” *Journal of Political Economy*, 105, 211–48.
- (2012): “Liquidity, Business Cycles, and Monetary Policy,” Working Paper 17934, National Bureau of Economic Research.
- KRUGMAN, P. R. (1998): “It’s Baaack: Japan’s Slump and the Return of the Liquidity Trap,” *Brookings Papers on Economic Activity*, 29, 137–206.
- LUCAS, R. E. AND J. P. NICOLINI (2013): “On the Stability of Money Demand,” Manuscript, Research Department, Federal Reserve Bank of Minneapolis.
- MOLL, B. (forthcoming): “Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?” *American Economic Review*.
- SVENSSON, L. (1985): “Money and Asset Prices in a Cash-in-Advance Economy,” *Journal of Political Economy*, 93, 919–944.

VERONESI, P. AND L. ZINGALES (2010): “Paulson’s Gift,” *Journal of Financial Economics*, 97, 339–368.

WERNING, I. (2011): “Managing a Liquidity Trap: Monetary and Fiscal Policy,” Working Paper 17344, National Bureau of Economic Research.