

Federal Reserve Bank of Minneapolis
Research Department

The Pass-Through of Sovereign Risk*

Luigi Bocola

Working Paper 722

April 2015

ABSTRACT

This paper examines the macroeconomic implications of sovereign credit risk in a business cycle model where banks are exposed to domestic government debt. The *news* of a future sovereign default hampers financial intermediation. First, it tightens the funding constraints of banks, reducing their available resources to finance firms (liquidity channel). Second, it generates a precautionary motive for banks to deleverage (risk channel). I estimate the model using Italian data, finding that i) sovereign credit risk was recessionary and that ii) the risk channel was sizable. I then use the model to evaluate the effects of subsidized long term loans to banks, calibrated to the ECB's longer-term refinancing operations. The presence of strong precautionary motives at the time of policy enactment implies that bank lending to firms is not very sensitive to these credit market interventions.

Keywords: Sovereign debt crises; Financial constraints; Risk; Credit policies

JEL classification: E32, E44, G01, G21

*Bocola: Northwestern University and Federal Reserve Bank of Minneapolis. First version: August 2013. This version: April 2015. I am indebted to Frank Schorfheide, Jesús Fernández-Villaverde, Iourii Manovskii and Dirk Krueger for their guidance and advice at each stage of this project. The paper is based on Chapter 1 of my PhD dissertation at the University of Pennsylvania. I am grateful for comments and suggestions from Monika Piazzesi, and two anonymous referees. I would also like to thank Manuel Amador, Javier Bianchi, V.V. Chari, Alessandro Dovis, Mark Gertler, Ellen McGrattan, Daniel Neuhann, Martin Schneider (discussant), Motohiro Yogo and participants at numerous seminars and conferences. I thank the Becker-Friedman Institute and the Macroeconomic Modeling and Systemic Risk Research Initiative for financial support. The views expressed herein are those of the author and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1 Introduction

At the end of 2009, domestic government debt holdings by banks in European peripheral countries - Greece, Italy, Portugal and Spain - were equivalent to 93% of banks' total equity. At the same time, domestic financial intermediaries in these economies were providing roughly two-thirds of the external financing of local firms. It is therefore not surprising that many empirical studies have documented a severe disruption of financial intermediation and a substantial increase in the borrowing costs of firms during the sovereign debt crisis.¹ One proposed explanation of these findings is that the exposure to distressed government bonds hurts the ability of banks to raise funds in financial markets, leading to a pass-through of their increased financing costs into the lending rates paid by firms (Committee on the Global Financial System, 2011). This view was at the core of policy discussions in Europe and was a motive for major interventions by the European Central Bank (ECB).

I argue, however, that this view is incomplete. A sovereign default can in fact be the trigger of a severe recession and have adverse effects on the performance of firms. Consequently, as an economy approaches a sovereign default, banks may start perceiving firms as more risky, and they may demand higher returns when lending to them as a fair compensation for holding this additional risk. If this mechanism is quantitatively important, policies that address the heightened liquidity problems of banks but do not reduce the increased riskiness of firms may prove ineffective in encouraging bank lending.

I analyze this mechanism in a quantitative model with financial intermediation and sovereign default risk. In the model, the *news* that the government may default in the future has adverse effects on the funding ability of banks (*liquidity channel*), and it raises the risks associated with lending to the productive sector (*risk channel*). I structurally estimate the model on Italian data and find that the risk channel is quantitatively important: it explains up to 45% of the impact of the sovereign debt crisis on firms' borrowing costs. I then use the estimated model to assess the consequences of credit market interventions adopted by the ECB in mitigating the implications of increased sovereign default risk.

My framework builds on a business cycle model with financial intermediation, in the tradition of Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). In the model, banks collect savings from households and use these funds, along with their own wealth (net worth), to buy long-term government bonds and to lend to firms. This intermediation

¹See, for example, the evidence in Acharya, Eisert, Eufinger and Hirsch (2014b) using data on syndicated loans originated by European banks, and the evidence in Bottero, Lenzu and Mezzanotti (2015) and Bofondi, Carpinelli and Sette (2013) using the Italian Credit Register.

is important because firms need external financing to buy capital goods. The model has three main ingredients. First, an agency problem between households and banks generates a constraint on the borrowing ability of the latter. This constraint on bank leverage binds only occasionally, typically when bank net worth is low. Second, financial intermediation is risky: bank net worth varies over time mainly because banks finance long-term risky assets with short-term riskless debt. Third, the probability that the government defaults on its debt in the future is time varying and follows a reduced-form rule.

To understand the mechanisms of the model, consider a scenario in which agents' expectations of a future sovereign default rise. The prospect of losses on government bonds depresses their market value today. Because banks are exposed to these assets, their net worth declines and their ability to borrow is impaired. As a result, bank lending to the productive sector drops, leading to a contraction of capital accumulation. This is a conventional liquidity channel in the literature.

In addition, the expectation of a future government default has adverse effects on the willingness of intermediaries to hold financial claims of the firms, even when banks are *currently* not constrained in their borrowing ability. Indeed, the news that the government may default raises the chance of large balance sheet losses and tight funding constraints for the banks in the future. This has two effects. First, in anticipation of hitting their funding constraints, banks act more cautiously and demand higher compensation for holding risky assets. Second, they attach a higher probability of incurring losses to their holdings of firms' claims: these assets, in fact, will lose value if the default occurs next period because they will be subject to fire sales by constrained intermediaries. Effectively, the banks now perceive private sector claims as more risky. The combination of these two effects generates a precautionary motive for banks to deleverage and to reduce their holdings of firms' claims. I refer to this second mechanism as the risk channel.

I measure the quantitative importance of liquidity and risk by estimating the structural model with Italian data. The major challenge is to separate these two propagation mechanisms because they have qualitatively similar implications for indicators of financial stress commonly used in the literature (e.g., credit spreads). I demonstrate, however, that the Lagrange multiplier on the funding constraints of banks is a function of observable variables. I construct a time series for the Lagrange multiplier and use it in estimation, along with indicators of real economic activity, to measure the cyclical behavior of the financial frictions in the model. In addition, I use credit default swap spreads on Italian government bonds and detailed data on holdings of domestic government debt by Italian banks to measure the time varying nature of sovereign risk and the exposure of banks to this risk. The estimated model has good in-sample fit and its asset pricing implications

are broadly consistent with indicators of liquidity and risk premia obtained from the cross section of Italian stock returns.

I then use the estimated model for two applications. First, I quantify the effects of the sovereign debt crisis on the financing premia of firms and on output, and I assess the relative importance of the two propagation mechanisms. I estimate that the sovereign debt crisis in Italy was responsible for a rise in the financing premia of firms that reached 80 basis points in 2011:Q4, with the risk channel explaining up to 45% of the overall effects. This increase in the financing costs of firms was associated with a decline in real economic activity: at peak (2011:Q4), the sovereign crisis was responsible for a decline in output of 1.2% in annualized terms.

Second, I evaluate the effects of a major credit market intervention adopted by the ECB in the first quarter of 2012, the longer term refinancing operations (LTROs). I model the policy as a subsidized long-term loan offered to banks and study its effects conditioning on the state of the Italian economy in 2011:Q4. I find that the average effects of LTROs on credit to firms and output are fairly small. This is because precautionary motives were sizable when the policy was enacted. Banks thus have little incentive to increase their exposure to firms, and they mainly use LTROs to cheaply substitute liabilities they have with the private sector.

This research is related to several strands of the literature. Many empirical studies have documented a strong link between sovereign and private sector interest rates, both in emerging economies and more recently in southern European countries. Several authors have recognized the importance of this relationship in different settings: see [Neumeyer and Perri \(2005\)](#) and [Uribe and Yue \(2006\)](#) in the context of business cycles in emerging markets, and [Corsetti, Kuester, Meier and Müller \(2013\)](#) for the implications of the sovereign risk pass-through for fiscal multipliers. However, in these and related papers, the reasons underlying the connection between sovereign spreads and private sector interest rates are left unmodeled. Part of the contribution of this paper is to microfound this relation in a fully specified dynamic equilibrium model.

In doing so, my paper relates to recent studies that analyze the links between sovereign defaults and the domestic banking sector. Motivated by robust empirical evidence, [Gennaioli, Martin and Rossi \(2014\)](#), [Sosa Padilla \(2013\)](#) and [Perez \(2015\)](#) study the effects of sovereign defaults on domestic banks, and the impact that the associated output losses have on the government's incentives to default.² My research is complementary to theirs:

²[Kumhof and Tanner \(2005\)](#) and [Gennaioli, Martin and Rossi \(2013\)](#) document that banks are highly exposed to domestic government debt in a large set of countries. [Borensztein and Panizza \(2009\)](#) show that sovereign defaults typically occur in proximity of banking crises.

I take sovereign default risk as given, but I explicitly model the behavior of private credit markets when sovereign risk increases. Differently from the above papers, the mere *anticipation* of a future sovereign default is recessionary, a feature that is essential for the present analysis because an actual default has not been observed over the sample. Anticipation effects have been studied in related environments by [Acharya, Drechsler and Schnabl \(2014a\)](#), [Broner, Erce, Martin and Ventura \(2014\)](#), [Fahri and Tirole \(2014\)](#) and [Cooper and Nikolov \(2013\)](#). These papers analyze in various forms the feedback loops arising between sovereigns and banks, a feature that is absent in my analysis. However, they abstract from the effects that sovereign risk has on the perceived riskiness of firms, which is the focus of my research.

This paper is also related to recent work that studies the macroeconomic implications of shocks to the balance sheet of financial intermediaries. The modeling builds on the framework developed in [Gertler and Kiyotaki \(2010\)](#) and [Gertler and Karadi \(2011\)](#), where the limited enforcement of debt contracts leads to constraints on intermediaries' leverage. In contrast to these papers, the analysis here emphasizes the precautionary motives associated with the possibility that these constraints will bind in the future, the risk channel. Technically, I am able to capture these effects because I solve the model numerically using global methods.³ These nonlinearities have been studied in different environments by [Brunnermeier and Sannikov \(2014\)](#), [He and Krishnamurthy \(2012\)](#), [Bianchi \(2013\)](#), [Bianchi and Mendoza \(2012\)](#), and [Maggiori \(2013\)](#) among others. A distinctive feature of my work is the emphasis on the quantification of these effects. By focusing on a specific historical event, the sovereign debt crisis in the euro area, I am able to measure more directly the balance sheet risk faced by intermediaries, and their associated precautionary motives. The analysis documents that the risk channel was quantitatively sizable during this episode. Moreover, it shows that the measurement of these effects provides policymakers with useful information to understand the impact of credit market interventions.⁴

Methodologically, I draw from the literature on the Bayesian estimation of dynamic equilibrium economies ([Del Negro and Schorfheide, 2011](#)), more specifically of models where nonlinearities feature prominently ([Fernández-Villaverde and Rubio-Ramírez, 2007](#)). To my knowledge, this is the first paper that estimates a model with occasionally binding financial constraints using global methods and nonlinear filters. However, other papers use related techniques in different applications (see [Gust, Lopez-Salido and Smith, 2013](#); [Bi and Traum, 2012](#)).

³In contemporaneous work, [Prestipino \(2013\)](#) studies optimal policy in a [Gertler and Karadi \(2011\)](#) economy solved numerically using global methods.

⁴There are a number of papers that study the effects of credit market interventions in related environments. See [Bianchi and Bigio \(2014\)](#) and reference therein.

Layout. The paper is organized as follows. Section 2 presents the model, and Section 3 describes the economic mechanisms at work. Section 4 estimates the structural model and analyzes its fit. Section 5 studies the propagation of a sovereign risk shock and presents the results of the first quantitative experiment. Section 6 discusses the macroeconomic implications of LTROs. Section 7 concludes.

2 The Model

I consider a standard growth model enriched with a financial sector as in Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). I introduce in this setting long term government bonds to which financial intermediaries are exposed. These securities pay in every state of nature unless the government defaults. The probability of a government default varies over time according to a reduced-form stochastic process.

The economy is populated by households, final good producers, capital good producers, and a government. Each household is composed of two types of members: *workers* and *bankers*. Workers supply labor to final good firms. Bankers borrow from capital markets in order to invest in government bonds and in claims of the firms. Firms rent labor from workers and buy capital from capital good producers in order to produce a homogeneous good. Their capital expenses are financed by the bankers. The government issues bonds and taxes households in order to finance government spending and he can default on its debt. The actions of the government are determined via fiscal rules.

In the remainder of this section, I describe the agents' decision problems, define the equilibrium, and sketch the algorithm used for the numerical solution of the model. I denote by \mathbf{S} the vector that collects the current value for the state variables, to be defined explicitly later on. The vector \mathbf{S}' denotes the value of the state vector next period.

2.1 Agents and their decision problems

2.1.1 Households

A household is composed of a fraction f of workers and a fraction $1 - f$ of bankers, with perfect consumption insurance among them. I denote by $\Pi(\mathbf{S})$ the net profits that the household receives from holdings of economic activities, $\tau(\mathbf{S})$ the lump-sum taxes from the government, and $W(\mathbf{S})$ the wage that workers receive from supplying labor to final good firms. The household values consumption c and dislikes labor l according to the flow utility $u(c, l)$ and discounts the future at the rate β . He makes contingent plans for

consumption, labor supply, and savings in order to maximize lifetime utility. Savings are managed by financial intermediaries that are run by bankers that belong to other households, and they earn a risk-free return $R(\mathbf{S})$. Taking prices as given, a household solves a canonical problem:

$$\begin{aligned} v^h(b; \mathbf{S}) &= \max_{b', c \geq 0, l \in [0, 1]} \left\{ u(c, l) + \beta \mathbb{E}_{\mathbf{S}}[v^h(b'; \mathbf{S}')] \right\}, \\ \text{s. t.} \\ c + \frac{1}{R(\mathbf{S})} b' &\leq W(\mathbf{S})l + \Pi(\mathbf{S}) + b - \tau(\mathbf{S}), \\ \mathbf{S}' &= \Gamma(\mathbf{S}), \end{aligned}$$

where $\Gamma(\cdot)$ describes the law of motion for the aggregate state variables. For future reference, I denote by $\Lambda(\mathbf{S}', \mathbf{S}) = \beta \frac{u_c(c', l')}{u_c(c, l)}$ the marginal rate of substitution for the household. In the empirical analysis, the flow utility will be $u(c, l) = \frac{[c - \frac{\lambda}{1+\nu} l^{1+\nu}]^{1-\sigma}}{1-\sigma}$.

2.1.2 Bankers

A banker uses his accumulated net worth, n , and households' savings, b' , to buy government bonds and claims on the firms. I denote by a_B and Q_B the quantity and price of government bonds acquired by the banker, and by $R_B(\mathbf{S}', \mathbf{S})$ the realized returns on these assets next period. The banker buys equity of the firms a_K at price Q_K , and he obtains the return $R_K(\mathbf{S}', \mathbf{S})$ next period.

The timing of events within a period is as follows. After observing the realization of \mathbf{S} , the banker collects the returns from government bonds, firms' equity, and he pays back his creditors. Next, a banker learns whether he exits or not. If he exits, he gives the accumulated net worth as a dividend to his household. If he does not exit, he issues new debt and buys new assets. Exit is stochastic, occurring with fixed probability $1 - \psi$.⁵

Taking prices as given, a banker chooses $\{a_B, a_K, b'\}$ to maximize the present discounted value of dividends paid to his household. I follow [Gertler and Karadi \(2011\)](#) and introduce an agency problem between bankers and their creditors. After making the portfolio choice, the banker can divert a fraction λ of the total assets and transfer these resources to his household. The costs of this action are that the creditors can force the banker into bankruptcy and recover the remaining fraction $(1 - \lambda)$ of the assets. The decision problem

⁵A banker that exits is replaced by a worker from his household. This new banker receives from the household an endowment of wealth to start the business. This endowment is equal to a fraction ω of the assets that bankers have intermediated in the previous period evaluated at current prices. The banker that exits becomes a worker, so that the relative proportion of types within the household is constant over time.

is then

$$\begin{aligned}
v^b(n; \mathbf{S}) &= \max_{a_B, a_K, b'} \mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S}) \left[(1 - \psi)n' + \psi v^b(n'; \mathbf{S}') \right] \right\}, \\
&\text{s. t.} \\
\sum_{j=\{B, K\}} Q_j(\mathbf{S}) a_j &\leq n + \frac{b'}{R(\mathbf{S})}, \\
\lambda \left[\sum_{j=\{B, K\}} Q_j(\mathbf{S}) a_j \right] &\leq v^b(n; \mathbf{S}), \\
n' &= \sum_{j=\{B, K\}} R_j(\mathbf{S}', \mathbf{S}) Q_j(\mathbf{S}) a_j - b', \\
\mathbf{S}' &= \Gamma(\mathbf{S}).
\end{aligned}$$

The first line in the feasible set states that the total assets acquired by a banker cannot exceed its liabilities. The second line is the incentive constraint: the value for the banker of defaulting on his creditors cannot exceed the value of running the business. Finally, net worth next period equals the difference between the returns on assets acquired today and the payments promised to households. Proposition 1 further characterizes this decision problem.

Proposition 1. *A solution to the banker's dynamic program is*

$$v^b(n; \mathbf{S}) = \alpha(\mathbf{S})n.$$

The marginal value of wealth, $\alpha(\mathbf{S})$, solves

$$\alpha(\mathbf{S}) = \frac{\mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi) + \psi \alpha(\mathbf{S}')] R(\mathbf{S}) \}}{1 - \mu(\mathbf{S})}, \quad (1)$$

and the Lagrange multiplier on the incentive constraint satisfies

$$\mu(\mathbf{S}) = \max \left\{ 1 - \left[\frac{\mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi) + \psi \alpha(\mathbf{S}')] R(\mathbf{S}) \} N}{\lambda [Q_K(\mathbf{S}) A_K + Q_B(\mathbf{S}) A_B]} \right], 0 \right\}, \quad (2)$$

where A_B and A_K are, respectively, aggregate bankers' holdings of government bonds and firms' assets, and N is the aggregate bankers' net worth at the beginning of the period (after the realization of \mathbf{S}).

Proof. See Appendix A. □

This proposition helps us to understand the role of bank net worth in the model. Be-

cause of the linearity of the value function, we can write the incentive constraint as

$$\frac{\sum_{j=\{B,K\}} Q_j(\mathbf{S}) a_j}{n} \leq \frac{\alpha(\mathbf{S})}{\lambda}, \quad (3)$$

implying that the leverage of a banker cannot exceed the threshold $\frac{\alpha(\mathbf{S})}{\lambda}$. Bank net worth is thus a key variable regulating financial intermediation: when net worth is low, the constraint in equation (3) is more likely to bind. When this happens, the banker obtains fewer resources from households, and, as a result, he reduces his demand for government and firms' claims.

Proposition 1 also implies that heterogeneity in bankers' net worth and in their asset holdings does not affect aggregate dynamics. Hence, aggregate net worth will be a sufficient statistic for the state of the financial sector, a feature that makes the numerical analysis of the model tractable.

2.1.3 Producers

There are two types of firms: capital good producers and final good producers. The capital good producers build new capital goods using the technology $\Phi\left(\frac{i}{K}\right)K$, where K is the aggregate capital stock and i the input used in production. Taking prices as given, their decision problem is

$$\max_{i \geq 0} \left[Q_i(\mathbf{S}) \Phi\left(\frac{i}{K}\right) K - i \right].$$

Anticipating the market clearing condition, the price of new capital goods is $Q_i(\mathbf{S}) = \left[\Phi'\left(\frac{I(\mathbf{S})}{K}\right) \right]^{-1}$, where $I(\mathbf{S})$ is equilibrium aggregate investment. In the empirical analysis, I will set $\Phi(x) = a_1 x^{1-\xi} + a_2$, where $\xi \in [0, 1]$ parametrizes the elasticity of Tobin's q with respect to the investment-capital ratio. If $\xi > 0$, the price of capital moves over time, and it will be low in periods of low aggregate investment.

Final good firms produce a final output using a constant returns to scale technology, $y = k^\alpha (e^z l)^{1-\alpha}$, where k is the stock of capital goods, l stands for labor services, and z is a neutral technology shock that follows an AR(1) process in growth rates:

$$\Delta z' = (1 - \rho_z)\gamma + \rho_z \Delta z + \sigma_z \varepsilon'_z, \quad \varepsilon'_z \sim \mathcal{N}(0, 1). \quad (4)$$

Labor is rented in competitive markets at $W(\mathbf{S})$. Profit maximization implies that equi-

librium wages and profits per unit of capital are

$$W(\mathbf{S}) = (1 - \alpha) \frac{Y(\mathbf{S})}{L(\mathbf{S})}, \quad Z(\mathbf{S}) = \alpha \frac{Y(\mathbf{S})}{K}, \quad (5)$$

where $Y(\mathbf{S})$ and $L(\mathbf{S})$ are equilibrium aggregate output and labor.

Capital goods depreciate every period at the rate δ . Firms need external financing to purchase new capital goods. At the beginning of the period, they issue claims a_K to bankers at a price of $Q_K(\mathbf{S})$.⁶ In exchange, the firms pledge the realized return on a unit of the capital stock next period to the banker:

$$R_K(\mathbf{S}', \mathbf{S}) = \frac{(1 - \delta)Q_K(\mathbf{S}') + Z(\mathbf{S}')}{Q_K(\mathbf{S})}. \quad (6)$$

The realized returns $R_K(\mathbf{S}', \mathbf{S})$ move over time because of two factors: variation in firms' profits and variation in the market value of firms' claims.

2.1.4 The government

In every period, the government engages in public spending. Public spending as a fraction of output follows

$$\log(g)' = (1 - \rho_g) \log(g^*) + \rho_g \log(g) + \sigma_g \varepsilon_g', \quad \varepsilon_g' \sim \mathcal{N}(0, 1). \quad (7)$$

The government finances its expenditures by issuing long-term bonds to bankers and by levying lump-sum taxes on households. Long-term debt is introduced as in [Chatterjee and Eyigungor \(2012\)](#). In every period, a fraction π of bonds matures, and the government pays back the principal to investors. For the fraction $(1 - \pi)$ that does not mature, the government pays the coupon ι , and investors retain the right to the principal in the future. The government can default on its debt in every period by writing off a fraction $D \in [0, 1]$ of its outstanding obligations. Denoting by $Q_B(\mathbf{S})$ the pricing function for government securities, tomorrow's realized returns on a dollar invested in government bonds are

$$R_B(\mathbf{S}', \mathbf{S}) = [1 - d'D] \left[\frac{\pi + (1 - \pi) [\iota + Q_B(\mathbf{S}')] }{Q_B(\mathbf{S})} \right], \quad (8)$$

where d' is an indicator variable equal to 1 if the government defaults next period. The

⁶No arbitrage implies that the price of a unit of new capital equals in equilibrium the price of a claim issued by firms, $Q_i(\mathbf{S}) = Q_K(\mathbf{S})$.

realized returns $R_B(\mathbf{S}', \mathbf{S})$ vary over time because of two sources. First, when the government defaults, it imposes the "haircut" D on bondholders. Second, $R_B(\mathbf{S}', \mathbf{S})$ is sensitive to variation in the price of government securities: for instance, a decline in $Q_B(\mathbf{S}')$ lowers the resale value of government bonds and generates capital losses for bondholders. Note that this second effect is present even when the government does not default ($d' = 0$), and to the extent that government debt is not entirely short term ($\pi < 1$).

Denoting by B' the stock of public debt, the budget constraint of the government is

$$Q_B(\mathbf{S}) \left[\underbrace{B' - (1 - \pi)B[1 - dD]}_{\text{Newly issued bonds}} \right] = \underbrace{[\pi + (1 - \pi)\iota] B[1 - dD]}_{\text{Payments of principals and coupons}} + \underbrace{gY(\mathbf{S}) - \tau(\mathbf{S})}_{\text{Primary deficit}}, \quad (9)$$

with taxes following the fiscal rule

$$\frac{\tau(\mathbf{S})}{Y(\mathbf{S})} = t^* + \gamma_\tau \frac{B}{Y(\mathbf{S})}.$$

In order to close the model, we need to specify how sovereign risk evolves over time. I assume that in every period the economy is hit by an i.i.d. shock ε_d that follows a standard logistic distribution. The default process d' follows

$$d' = \begin{cases} 1 & \text{if } \varepsilon'_d - \Psi(\mathbf{S}; \theta_2) \geq 0 \\ 0 & \text{otherwise,} \end{cases} \quad (10)$$

where θ_2 is a vector of parameters. We can thus write the conditional probability of a sovereign default tomorrow as a logistic function of $\Psi(\mathbf{S}; \theta_2)$,

$$p^d(\mathbf{S}) \equiv \text{Prob}(d' = 1 | \mathbf{S}) = \frac{\exp\{\Psi(\mathbf{S}; \theta_2)\}}{1 + \exp\{\Psi(\mathbf{S}; \theta_2)\}}. \quad (11)$$

By appropriately choosing the function $\Psi(\mathbf{S}; \theta_2)$, one can flexibly incorporate key drivers of sovereign credit risk into the analysis. For example, $\Psi(\mathbf{S}; \theta_2)$ could depend on B or on z_t , typical determinants of sovereign risk identified in the literature ([Arellano, 2008](#)).

In the empirical application I will consider a simple specification $\Psi(\mathbf{S}; \theta_2) = s$, with s being an AR(1) process

$$s' = (1 - \rho_s) \log(s^*) + \rho_s s + \sigma_s \varepsilon'_s, \quad \varepsilon'_s \sim \mathcal{N}(0, 1). \quad (12)$$

This choice is motivated by two main considerations. First, there is substantial empirical

evidence that a large share of the variation in Italian sovereign spreads during the European crisis was driven by factors orthogonal to domestic fundamentals (Bahaj, 2013). These findings are consistent with the view, shared by many economists and policymakers, that self-fulfilling beliefs and contagion through common creditors were key drivers of sovereign risk during the European crisis. The s -shock is intended to capture these considerations. Second, this formulation allows us to clearly isolate the economic mechanisms underlying the propagation of sovereign risk without relying on reduced-form channels built in the function $\Psi(\cdot)$.

Importantly, as I will discuss in Section 4, the exogeneity of sovereign credit risk will not be used as a restriction when estimating the structural model. Thus, the identification of key model parameters will not rely on this assumption.

2.2 Equilibrium

Let $\mathbf{S} = [K, B, P, \Delta z, g, s, d]$ be the state vector, with P being the gross payments that bankers return to households at the beginning of the period.⁷ A *recursive competitive equilibrium* for this economy is given by value functions for households and bankers $\{v^b, v^h\}$, policy functions for households $\{c, l, b'\}$, and policy functions for bankers $\{a_K, a_B, b'\}$ such that, given prices $\{W, Q_K, Q_B, R\}$, **(i)** bankers' and households' policies and value functions solve their decision problems; **(ii)** the government budget constraint is satisfied; **(iii)** the market for firms' claims clears, $\int a_K(n_i; \mathbf{S}) di = K'(\mathbf{S})$; **(iv)** the market for government bonds clears, $\int a_B(n_i; \mathbf{S}) di = B'(\mathbf{S})$; **(v)** the market for households' savings clears, $\int b'(n_i; \mathbf{S}) di = b'(\mathbf{S})$; **(vi)** the goods market clears, $Y(\mathbf{S})(1 - g) = C(\mathbf{S}) + I(\mathbf{S})$; **(vii)** $\Gamma(\cdot)$ is consistent with agents' optimization and the government fiscal rules.

2.3 Numerical solution

The solution to the competitive equilibrium is obtained using global numerical methods. Here I briefly sketch the main steps of the algorithm, relegating to the online Appendix the details of the implementation and a discussion of its accuracy. Because the non-stationary technology process induces a stochastic trend in several endogenous variables, it is convenient to express the model in terms of detrended variables. For a given variable y , I define its detrended version as $\tilde{y} = \frac{y}{z}$.⁸ Let $X(\mathbf{S}) = \{\tilde{C}(\mathbf{S}), R(\mathbf{S}), \alpha(\mathbf{S}), Q_B(\mathbf{S})\}$ be

⁷As explained in the online Appendix, P will be, along with the other state and control variables, a sufficient statistic for beginning-of-period aggregate net worth N . The online Appendix is available at <https://sites.google.com/site/luigibocola/research>.

⁸The endogenous state variables of the model are detrended using the level of technology last period.

the control variables of the model, with x being an element of X . I approximate each decision rule $x(\mathbf{S})$ using piecewise smooth functions that are summarized by two sets of coefficients, $\{\gamma_{d=0}^x, \gamma_{d=1}^x\}$,

$$x(d, \tilde{\mathbf{S}}) = (1 - d)\gamma_0^{x'}\mathbf{T}(\tilde{\mathbf{S}}) + d\gamma_1^{x'}\mathbf{T}(\tilde{\mathbf{S}}),$$

where $\tilde{\mathbf{S}} = [\tilde{K}, \tilde{B}, \tilde{P}, \Delta z, g, s]$ is the vector of state variables that excludes d , and $\mathbf{T}(\cdot)$ is a vector collecting Chebyshev's polynomials. Thus, the numerical solution is indexed by the coefficients $\{\gamma_{d=0}^x, \gamma_{d=1}^x\}_x$. These coefficients are chosen so that the equilibrium conditions of the model, defined in the online Appendix, are satisfied at a set of collocation points $(d^i, \tilde{\mathbf{S}}^i) \in \{0, 1\} \times \tilde{\mathcal{S}}$. The collocation points $\tilde{\mathcal{S}}$ and the set of polynomials $\mathbf{T}(\cdot)$ are chosen using the approach of Smolyak (Krueger and Kubler, 2004). The use of a sparse grid allows one to soften the curse of dimensionality that arises when solving the model globally.

3 The Bankers' Euler Equations

Before turning to the empirical analysis, a useful step is to understand how sovereign risk affects the funding costs of firms and real economic activity in the model. It is convenient to organize the discussion around the Euler equation that governs capital accumulation in the economy.

Given the linearity of the value function of bankers, we can write their optimization problem as

$$\max_{\{a_B, a_K\}} \mathbb{E}_{\mathbf{S}} \left\{ \hat{\Lambda}(\mathbf{S}', \mathbf{S}) \left[\sum_{j=\{B, K\}} [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})] Q_j(\mathbf{S}) a_j + R(\mathbf{S}) n \right] \right\},$$

subject to the constraint that their demand for assets does not exceed a multiple of their net worth $\lambda [Q_B(\mathbf{S}) a_B + Q_K(\mathbf{S}) a_K] \leq \alpha(\mathbf{S}) n$. In the above notation, $\hat{\Lambda}(\mathbf{S}', \mathbf{S})$ is the effective discount factor of the bankers, and it is related to the marginal rate of substitution of the household as follows:

$$\hat{\Lambda}(\mathbf{S}', \mathbf{S}) = \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi) + \psi \alpha(\mathbf{S}')]. \quad (13)$$

Taking first-order conditions for the above program, we can write the Euler equation

governing the demand for firms' claims as follows:

$$\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S})R_K(\mathbf{S}', \mathbf{S})] = \mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S})R(\mathbf{S})] + \lambda\mu(\mathbf{S}). \quad (14)$$

At optimum, the demand for firms' assets is such that their risk-adjusted return equals the shadow value of funds for the bankers. As a benchmark, note that when $\mu(\mathbf{S}) = 0$ for all \mathbf{S} , equation (14) collapses to the Euler equation of the corresponding frictionless model. Indeed, the bankers will always have enough funds to arbitrage away differences between $\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S})R_K(\mathbf{S}', \mathbf{S})]$ and $\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S})R(\mathbf{S})]$ when their funding constraints are not binding. Moreover, they will discount payoffs using the marginal rate of substitution of the household because $\alpha(\mathbf{S}) = 1$ for all \mathbf{S} in this scenario.

In general, though, variation in the net worth of bankers implies that their funding constraints will bind in some states of the world, and this will affect the behavior of excess returns. In order to understand that, we can rearrange equation (14) as follows:

$$\mathbb{E}_{\mathbf{S}}[R_K(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})] = \underbrace{\frac{\lambda\mu(\mathbf{S})}{\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S})]}}_{\text{Liquidity Premium}} + \underbrace{\text{cov}_{\mathbf{S}} \left[-\frac{\hat{\Lambda}(\mathbf{S}', \mathbf{S})}{\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S})]}, R_K(\mathbf{S}', \mathbf{S}) \right]}_{\text{Risk Premium}}. \quad (15)$$

Expected excess returns arise for two distinct reasons. First, high excess returns on private sector claims may indicate the inability of bankers to exploit profitable arbitrage opportunities because of binding funding constraints. I refer to this component, represented by the first term on the right-hand side of equation (15), as a *liquidity premium*. Second, high excess returns may simply reflect fair compensation that bankers demand for holding risk, that is a premium for assets whose payouts covary negatively with the stochastic discount factor. This is a canonical *risk premium*. Importantly, the stochastic discount factor used by the bankers differs from that of the corresponding frictionless model: it is not only a function of the marginal rate of substitution of households, but also a function of the marginal value of wealth for the banker. This aspect allows the model to generate quantitatively meaningful variation in risk premia.⁹

When the funding constraints bind, bankers shed some of their assets in order to meet their leverage requirements: this implies that the demand for assets falls, leading to an increase in the liquidity premium. Even when the constraints are currently not binding, the expectations that they may bind in the future can affect excess returns by changing

⁹Note that $\hat{\Lambda}(\mathbf{S}', \mathbf{S})$ is an example of a leverage-based pricing kernel. Indeed, we can see from equation (3) that the leverage of bankers is proportional to $\alpha(\mathbf{S})$ when $\mu(\mathbf{S}) > 0$. [Adrian, Etula and Muir \(2014\)](#) provide empirical evidence in support of leverage-based pricing kernels for the U.S. economy, and [He and Krishnamurthy \(2013\)](#) discuss their asset-pricing implications in endowment economies.

the risk premium component (Aiyagari and Gertler, 1999). This happens mainly because $\alpha(\mathbf{S}')$ and $R_K(\mathbf{S}', \mathbf{S})$ tend to move in opposite direction when the funding constraints bind. In order to understand this property, consider a state $\bar{\mathbf{S}}'$ where the leverage constraint binds. By equation (1), $\alpha(\bar{\mathbf{S}}')$ is high in this state: intuitively, a dollar is more valuable to a banker when he is facing a tight funding constraint. In addition, $R_K(\bar{\mathbf{S}}', \mathbf{S})$ is likely to be low because the weak demand for firms' assets by constrained bankers puts downward pressure on the price of capital, leading to low ex post returns (see equation (6)). Taken together, these two facts imply that $\bar{\mathbf{S}}'$ is a state in which claims on the productive sector pay out little precisely when bankers are in most need of wealth. Hence, any shock that raises the likelihood of $\bar{\mathbf{S}}'$ tends to increase the risk premium demanded by the bankers.

In this environment, news about a future sovereign default influences private sector spreads by affecting both the liquidity and the risk premium of equation (15). Starting with the former, an increase in $p(d' = 1|\mathbf{S})$ triggers a fall in the price of government bonds and a drop in the net worth of the banks.¹⁰ When these losses are sufficiently large, the leverage constraints may bind, leading to higher liquidity premia. I refer to this mechanism as the *liquidity channel*. In addition, an increase in $p(d' = 1|\mathbf{S})$ will tend to raise risk premia. When the likelihood of a future government default increases, bankers assign a higher probability of experiencing losses in their holdings of government debt next period. This is a signal that their funding constraints may bind in the future. Therefore, they act more cautiously and demand higher risk premia for holding firms' claims in their balance sheet. This is referred to as the *risk channel*.

This variation in liquidity and risk premia induced by sovereign risk is associated with adjustments in macroeconomic quantities that are well understood in the literature. Specifically, both the liquidity and the risk channel will discourage capital accumulation in the economy, and their aggregate implications will be qualitatively similar to a shock to the marginal efficiency of investment studied in the business cycle literature. Section 5 discusses these issues in more details.

It is important to stress that, despite being highly intertwined in the model, the two mechanisms are conceptually very different. The liquidity channel indicates that financial intermediaries are not lending to firms because they are currently facing funding constraints. The risk channel, instead, indicates that financial intermediaries are not demanding firms' claims because they are afraid of hitting their funding constraints next period, and they know that these will be bad assets to hold in case this event occurs.

¹⁰Note that this effect is present only when the government debt held by bankers is long term ($\pi < 1$). When $\pi = 1$, the bankers do not make capital losses on bond holdings unless the government defaults, see equation (8).

Section 6 shows that this distinction has important implications for policy evaluation.

4 Empirical Analysis

The model is estimated using Italian quarterly data. This section proceeds in three steps. Section 4.1 describes the data used in estimation. Section 4.2 illustrates the estimation strategy. Section 4.3 presents diagnostics regarding model fit.

4.1 Data

Although the previous section has described the economic mechanisms of interest, their quantitative relevance rests on the numerical value of the model parameters, especially those governing the stochastic properties of sovereign risk, the exposure of bankers to this risk, and the macroeconomic implications of the financial frictions studied in this paper. This section describes the key time series used in estimation to inform these parameters.

I use credit default swap (CDS) spreads on Italian government securities with a one-year maturity to ensure that the time-varying nature of sovereign risk in the model is realistic. The exposure of Italian banks to this risk is measured using the 2011 stress test of the European Banking Authority (EBA). Through this source, I construct net holdings of domestic government debt, classified by their maturity, for the five largest Italian banks.¹¹ This information is matched with consolidated balance sheet data at the end of 2010, obtained from Bankscope. Thus, for each of these banks, we have a measure of their holdings of Italian government debt as a fraction of the bank's total assets.

A major challenge in the literature is to select observables that are informative for the parameters governing the financial frictions in this class of models. The approach here consists in deriving a model consistent indicator of agency costs. Specifically, Appendix B.3 shows that the Lagrange multiplier on the incentive constraints of bankers is, in equilibrium, a function of their leverage, and of the spread between the interbank rate and the risk-free rate,¹²

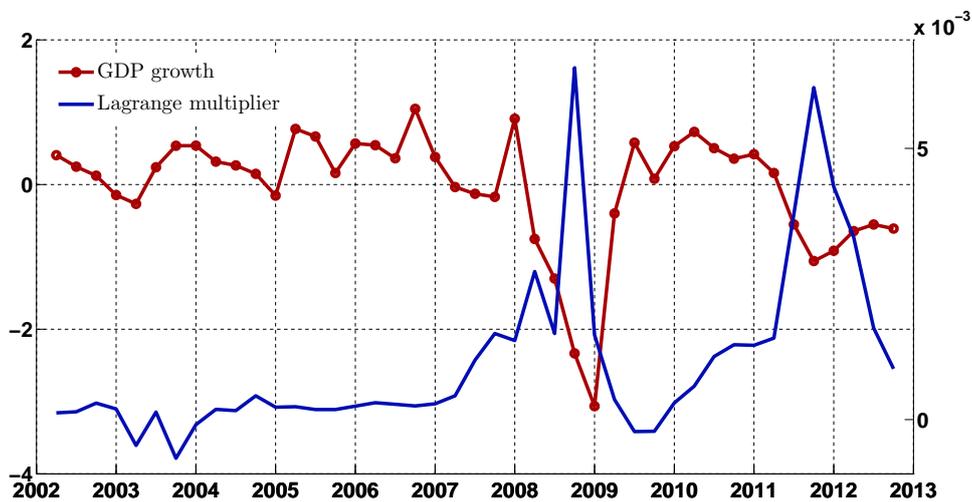
$$\mu_t = \frac{\left[\frac{R_{\text{interbank},t} - R_t}{R_t} \right] \text{lev}_t}{1 + \left[\frac{R_{\text{interbank},t} - R_t}{R_t} \right] \text{lev}_t}. \quad (16)$$

¹¹The five institutions are: Unicredit, Intesa-San Paolo, Monte dei Paschi di Siena (MPS), Banco Popolare (BPI), and Unione di Banche Italiane (UBI).

¹²An interbank market can be introduced in the model by allowing the bankers to trade a risk-free asset in zero net supply. The presence of this asset does not alter allocations. See Appendix B.3 for further details.

Therefore, equation (16) can be used to generate a time series for μ_t . I measure $R_{\text{interbank},t}$ with the prime rate on interbank unsecured loans with a week long duration for Italian banks belonging to the Euro Interbank Offered Rate (Euribor) panel. The risk-free rate R_t is matched with yields on contracts of similar duration from the interbank secured market, the Euro G.C. Repo Market (Eurepo). The leverage of financial intermediaries is calculated using the Italian flow of funds. Figure 1 reports the time series for μ_t along with real GDP growth in Italy. Two main facts stand out. First, the Lagrange multiplier is countercyclical, rising substantially in periods in which GDP growth is markedly below average. Second, it is very close to 0 for most time periods. Thus, the constraint seems to bind only occasionally in the sample.

Figure 1: GDP growth and the Lagrange multiplier: 2002:Q2-2012:Q4



Notes: Real GDP growth is the line with circles (left axis). The Lagrange multiplier on banks' leverage constraint is the solid line (right axis).

Note that the measurement of the Lagrange multiplier presents some limitations. A first concern is that spikes in the Euribor-Eurepo spread might not just reflect lack of liquidity for the banks, as it occurs in the model, but they might be an indication of heightened interbank lending risk. If that was the case, the time series for μ_t may overstate the importance of the funding constraints of bankers. However, some considerations mitigate this concern. First, the interbank rate used here relates to transactions among prime banks, typically very large and creditworthy institutions. Second, the choice of using loans with a very short duration (one week) is intended to minimize the impact of counterparty risk on the measured Lagrange multiplier. A second concern is that not all assets and liabilities in the flow of funds are reported at market value.¹³ This makes the empirical indicator

¹³When constructing the flow of funds, the Bank of Italy follows the conventions of the European System

of financial leverage not fully consistent with the one in the model, where all assets and liabilities are mark to market.

Despite these limitations, the measured Lagrange multiplier appears to be a strong signal for the funding constraints experienced by Italian banks over the sample. This can be verified from the Bank Lending Survey of the euro area. The Bank of Italy administers a quarterly survey to senior loan officers at Italian banks. The survey addresses issues such as credit standards for approving loans as well as credit terms and conditions applied to enterprises. The first question in the survey asks whether a bank's credit standards to enterprises have changed in a given quarter. Question 2 asks loan officers to rate several factors that contribute to a change in the credit standards. Among those factors, the survey considers three entries related to the "costs of funds and balance sheet constraints" for the bank: i) cost related to a bank's capital position, ii) the ability of a bank to access market financing (e.g., money or bond market financing), and iii) a bank's liquidity position. Appendix B.4 documents that the Lagrange multiplier tracks very closely the net percentage of respondents reporting a tightening of credit (R^2 of 42% in an univariate regression). Moreover, it is strongly associated to the net percentage of respondents who answered that one of the three factors described above had somewhat or considerably contributed to the change in credit standards (R^2 of, respectively, 21%, 54% and 41%). Table A-2 in Appendix B.4 further documents that the remaining "factors" that respondents can select from question 2 are fairly uncorrelated with the measured Lagrange multiplier.

In addition to the data discussed, the estimation uses additional indicators from OECD Quarterly National Accounts and other sources. See Appendix B for a detailed description of the data.

4.2 Estimation

I denote by $\theta \in \Theta$ the vector of model parameters. It is convenient to organize the discussion around the following partition, $\theta = [\theta_1, \theta_2]$

$$\theta_1 = \left[\mu^{bg}, \psi, \zeta, \sigma_z, \rho_z, \exp^{bg}, \pi, g^*, \rho_g, \sigma_g, \frac{i^{bg}}{y^{bg}}, l^{bg}, R^{bg}, \alpha, \exp^{bg}, \nu, \gamma_\tau, \text{lev}^{bg}, q_b^{bg}, \text{adj}^{bg} \right], \quad \theta_2 = [D, s^*, \rho_s, \sigma_s].$$

Conceptually, we can think of θ_1 as indexing a restricted version of the model without sovereign risk, whereas θ_2 collects the parameters determining the sovereign default pro-

of Accounts (ESA95) under which financial assets and liabilities are reported at market value. This principle, however, is not applied for activities for which there is no secondary market. Exceptions to this rule are stocks and bonds not listed on official exchanges: the market value of these assets and liabilities are imputed by looking at listed assets with similar characteristics and by applying a liquidity discount.

cess.¹⁴ Although a nonlinear analysis of the model is necessary to address the questions asked in this paper, it complicates inference substantially because repeated numerical solutions of the model are computationally very costly. I therefore estimate θ using a two-step procedure. In the first step, I infer θ_1 by estimating the model without sovereign risk on the pre-sovereign debt crisis subsample (2002:Q2-2009:Q4). This restricted version of the model has fewer state variables and is easier to analyze numerically. In the second step, and conditional on the first-step parameters, I estimate θ_2 using a retrieved time series of sovereign default probabilities.

An attractive feature of this two-step procedure is that it is more robust to misspecifications of the sovereign default process relative to a full information approach. For instance, the two-step approach does not use information from the sample correlation between GDP growth and CDS spreads in the estimation of θ . On the contrary, a full information procedure would use this moment to inform parameters value. This could be problematic, because the model interprets this moment as purely reflecting the causal effect of sovereign risk on real economic activity.

4.2.1 Estimating the model without sovereign risk

The model without sovereign risk has five state variables $\mathbf{S}_t = [\hat{K}_t, \hat{B}_t, \hat{P}_t, \Delta z_t, g_t]$. The parameters are

$$\theta_1 = \left[\underbrace{\mu^{bg}, \psi, \zeta, \sigma_z, \rho_z, \exp^{bg}, \pi, g^*, \rho_g, \sigma_g, \frac{i^{bg}}{y^{bg}}, l^{bg}, R, \alpha, \nu, \gamma_\tau, \text{lev}^{bg}, q_b^{bg}, \text{adj}^{bg}}_{\tilde{\theta}_1}, \underbrace{\phantom{\mu^{bg}, \psi, \zeta, \sigma_z, \rho_z, \exp^{bg}, \pi, g^*, \rho_g, \sigma_g, \frac{i^{bg}}{y^{bg}}, l^{bg}, R, \alpha, \nu, \gamma_\tau, \text{lev}^{bg}, q_b^{bg}, \text{adj}^{bg}}}_{\theta_1^*} \right].$$

I construct the likelihood function of the model using time series for GDP growth and the Lagrange multiplier described earlier. The cyclical behavior of the model's financial frictions is important for assessing the impact of sovereign risk on the real economy: a likelihood-based approach guarantees a high degree of consistency between the model-implied behavior for these variables and their data counterparts. However, certain parameters are likely to be only weakly affected by the information in the likelihood function, and their identification is problematic. For this reason, I determine a subset of θ_1, θ_1^* , prior to the estimation. Table 1 reports the numerical values for these parameters.

¹⁴I have reparametrized $[\lambda, \omega, \delta, \chi, \iota, \tau^*, a_1, a_2]$ with balanced growth values for, respectively, the Lagrange multiplier (μ^{bg}), the leverage ratio (lev^{bg}), the investment-output ratio ($\frac{i^{bg}}{y^{bg}}$), worked hours (l^{bg}), the price of government securities (q_b^{bg}), the ratio of government securities held by bankers to their total assets (\exp^{bg}), and the size of capital adjustment costs (adj^{bg}).

Table 1: **Parameters determined with external information**

Parameter	Description	Value
exp^{bg}	Bond holdings over total assets	0.076
π	Fraction of bond maturing	0.056
$\exp\{g^*\}$	Avg. spending-output ratio	0.210
ρ_g	Persistence, spending-output ratio	0.970
σ_g	Volatility, spending-output ratio	0.009
$\frac{i^{bg}}{y^{bg}}$	Investment-output ratio	0.200
l^{bg}	Hours worked	0.320
R^{bg}	Interest rate	1.003
α	Capital share	0.300
ν	Inverse Frisch elasticity	0.500
lev^{bg}	Leverage	5.000
γ_t	Fiscal rule	1.000
adj^{bg}	Capital adjustment costs	0.000
q_b^{bg}	Bond prices	1.000

Notes: See Appendix B for information on data sources.

I use the EBA and Bankscope data to make sure that holdings of government securities are equivalent to 7.6% of banks' total assets in a balanced growth path of the model, and that the average maturity of those bonds equals 18 months (see Table A-1 in Appendix B.2). These numbers pin down $[exp^{bg}, \pi]$. I select $[g^*, \rho_g, \sigma_g]$ from the estimation of an AR(1) on the spending-output ratio over the 1999:Q1-2009:Q4 period. The parameters $[\frac{i^{bg}}{y^{bg}}, l^{bg}, R^{bg}]$ are set to the sample average of their empirical counterparts, and α is determined using the sample average of the labor income share. The remaining parameters in θ_1^* are determined through normalizations or previous research. The parameter ν is chosen to obtain a Frisch elasticity of 2. The latter is in the high range of the estimates obtained using U.S. data (Ríos-Rull, Schorfheide, Fuentes-Albero, Kryshko and Santaaulalia-Llopis, 2012), but it is not an uncommon value in the profession for the analysis of real business cycle models. The parameter ω is chosen so that the model produces a leverage ratio of 5 in a balanced growth path, in the range of values used in the literature. I set γ_τ to 1: this choice has limited effects on aggregate dynamics, because taxes are lump-sum in the model. Following common practice in the profession, I set capital adjustment costs to zero in a balanced growth path and normalize q_b^{bg} to 1.

The remaining parameters $\tilde{\theta}_1 = [\mu^{bg}, \psi, \xi, \gamma, \rho_z, \sigma_z]$ are estimated using Bayesian meth-

ods. Letting $\mathbf{Y}_t = [\text{GDP growth}_t, \mu_t]'$, and denoting the entire sample by \mathbf{Y}^T , I first evaluate the likelihood function of the model $\mathcal{L}(\tilde{\theta}_1 | \mathbf{Y}^T)$ using the auxiliary particle filter of [Pitt and Shephard \(1999\)](#). I next characterize the posterior density of $\tilde{\theta}_1$ using the Random Walk Metropolis Hastings developed in [Schorfheide \(2000\)](#). The online Appendix provides a description of the likelihood evaluation and the estimation algorithm. The top panel of [Table 2](#) reports the prior along with posterior statistics for $\tilde{\theta}_1$.

Table 2: **Prior and posterior distribution of estimated parameters**

Parameter	Prior	Para 1	Para 2	Posterior Mean	90% Credible Set
$\mu^{bg} \times 100$	Normal	0.96	25.00	0.11	[0.01,0.20]
ψ	Normal	0.97	0.15	0.95	[0.93,0.98]
ξ	Beta	0.5	0.25	0.41	[0.22,0.58]
$\gamma \times 400$	Normal	1.25	1.00	0.70	[0.05,1.36]
ρ_z	Beta	0.35	0.25	0.54	[0.38,0.53]
$\sigma_z \times 100$	Inverse Gamma	0.75	2.00	0.81	[0.60,1.02]
s^*	Normal	-7	5	-7.27	[-9.17,-4.11]
ρ_s	Beta	0.5	0.3	0.95	[0.89,0.99]
σ_s	Inverse Gamma	0.75	4	0.60	[0.51,0.73]

Notes: Para 1 and Para 2 list the mean and standard deviation for Beta and Normal distribution; and s and v for the Inverse Gamma distribution, where $p_{IG}(\sigma|v,s) \propto \sigma^{-v-1}e^{-vs^2/2\sigma^2}$. The prior on γ is truncated at 0. The prior on μ^{bg} and ψ is truncated to the left of 0 and to the right of 1. Posterior statistics are computed using 10,000 draws from the posterior distribution of the model's parameters. The table reports equal tail probability 90% credible sets.

The prior on the technological process is centered using presample evidence and I center ξ to 0.5, a conventional value in the literature. I center the prior on μ^{bg} and ψ to their calibrated value in [Gertler and Karadi \(2011\)](#). Regarding posterior estimates, note that agency costs in the model are fairly small: the Lagrange multiplier is estimated to be 0.001 on average, a value that is roughly 10 times smaller than in the [Gertler and Karadi \(2011\)](#) calibration. This is not surprising given that the time series for μ_t is close to 0 for most of the sample. Capital adjustment costs and the technological process are in the range of what is typically obtained in the literature when using U.S. data.

4.2.2 Estimating sovereign risk

I now turn to the estimation of $\theta_2 = [D, s^*, \rho_s, \sigma_s]$. The empirical strategy consists of i) constructing a time series for the probabilities of a sovereign default and ii) using this time series, along with equations (11) and (12), to estimate θ_2 .

The first task is accomplished by exploiting the pricing equation of the model. To do so, it is convenient to introduce the following variable:

$$\hat{p}(\mathbf{S}'|\mathbf{S}) = \frac{R_{\text{interbank}}(\mathbf{S})p(\mathbf{S}'|\mathbf{S})\hat{\Lambda}(\mathbf{S}',\mathbf{S})}{\alpha(\mathbf{S})[1 - \mu(\mathbf{S})] + \lambda\mu(\mathbf{S})}, \quad (17)$$

with $p(\cdot)$ being the conditional density function of the state vector. Note that $\hat{p}(\cdot)$ is a probability measure, since it is nonnegative and integrates to 1.¹⁵ I then price a CDS on a short term zero coupon bond issued by the government.¹⁶ Using the banker's Euler equation and the definition of $\hat{p}(\cdot)$, we can express the spread on this contract as

$$CDS_t = \frac{\hat{p}_t^d D}{R_{\text{interbank},t}}, \quad (18)$$

where \hat{p}_t^d stands for the conditional probability of a sovereign default under the $\hat{p}(\cdot)$ measure. Given this relation and the definition of $\hat{p}(\cdot)$ in equation (17), we can write the actual probability of a sovereign default p_t^d as follows:

$$p_t^d = \frac{CDS_t}{D} \times \frac{\lambda[\text{lev}_t(1 - \mu_t) + \mu_t]}{\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]}. \quad (19)$$

Note that the right hand side of the above equation is known up to D and $\mathbb{E}_t[\hat{\Lambda}_{t+1}|d_{t+1} = 1]$. I set the haircut D to 0.55, a value that is consistent with the Greek debt restructuring of 2012 (Zettelmeyer, Trebesch and Gulati, 2013). For the second term, I indirectly use the model's restrictions to approximate this object using an empirical counterpart to the model's stochastic discount factor. The online Appendix describes this step in details.

After obtaining a time series for p_t^d , we can estimate the logistic model defined by equations (11) and (12). The bottom panel of Table 2 reports prior and posterior statistics for $[s^*, \rho_s, \sigma_s]$.

4.3 Model fit

I now present an analysis of model fit. First, I verify whether the model performs well in the sample by checking whether it reproduces key moments of the time series used in estimation. The second experiment checks whether the model's predictions for liquidity

¹⁵To see this last property, note that the return on a risk-free security traded by bankers can be written as $R_{\text{interbank}}(\mathbf{S}) = \frac{\alpha(\mathbf{S})[1 - \mu(\mathbf{S})] + \lambda\mu(\mathbf{S})}{\mathbb{E}_S[\hat{\Lambda}(\mathbf{S}',\mathbf{S})]}$ using equation (14) and equation (1).

¹⁶The contract specifies that, in the event of a government default at $t + 1$, the seller of the CDS pays the principal to the buyer in exchange of $(1 - D)$.

and risk premia are consistent with comparable measures obtained from the cross section of Italian stock returns.

4.3.1 Lagrange multiplier, output growth and sovereign default probabilities

The analysis of the in-sample fit is conducted through posterior predictive checks.¹⁷ I summarize the behavior of the data by computing a set of sample statistics for the Lagrange multiplier, output growth, and sovereign default probabilities. Then, I compute the model implied posterior distributions for the same statistics via simulation techniques. The check consists of comparing the statistics computed from actual data with their model-implied distribution.

We can start by examining the performance of the model regarding GDP growth and the Lagrange multiplier. Their joint behavior in the data is summarized using mean, standard deviation, first-order autocorrelation, skewness, kurtosis, and their correlation. These sample statistics are collected in \mathcal{S} . The model-implied densities for these sample statistics, $p(\mathcal{S}|\mathbf{Y}^T)$, are computed via simulations.¹⁸ Figure 2 reports the 5th and 95th percentile of the model-implied density (the box) along with its median (the bar) and the sample counterpart (the dot).

The model generates trajectories for the Lagrange multiplier and GDP growth whose moments are in line with those observed in the data. The main discrepancy in the data lies in the excess kurtosis for the GDP growth trajectory: the model has difficulty replicating the depth of the 2009:Q1 recession. Note that the model captures part of the left skewness of GDP growth observed in the data. This nonlinearity is the result of two properties of the estimated model: the amplification of the leverage constraint and the fact that it binds in recessions. See [Guerrieri and Iacoviello \(2013\)](#) for a discussion of macroeconomic asymmetries generated by models with occasionally binding financial constraints.

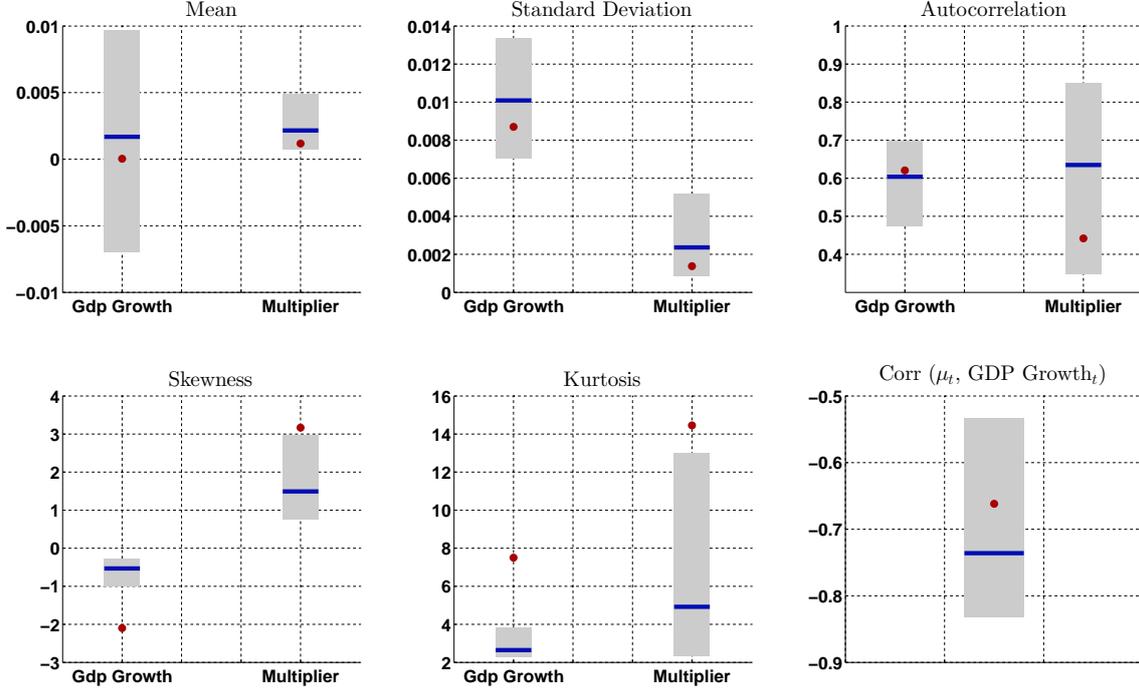
Table 3 reports the posterior predictive checks for p_t^d . As we can see, the logistic model used in this paper captures key features of the empirical distribution of sovereign default probabilities.

Overall, the results in this section suggest that: i) the cyclical behavior of the leverage constraint in the estimated model is empirically reasonable, and ii) agents in the model have beliefs about the time-varying nature of sovereign credit risk that closely track what

¹⁷See [Aruoba, Bocola and Schorfheide \(2013\)](#) for a discussion of the use of posterior predictive checks for the evaluation of dynamic stochastic general equilibrium models.

¹⁸Let $\{\theta_n\}_{n \in N}$ be draws from $p(\theta|\mathbf{Y}^T)$. For each θ_n , I solve the model, simulate trajectories of length T for the endogenous variables, and compute the sample statistics on the simulated path, $\mathcal{S}(\theta_n)$. The density $p(\mathcal{S}|\mathbf{Y}^T)$ is then approximated from $\{\mathcal{S}(\theta_n)\}$. In this experiment, I set $N = 100$ and $T = 100$.

Figure 2: Posterior predictive checks: Lagrange multiplier and output growth



Notes: Dots correspond to the value of the statistic computed from actual data. Solid horizontal lines indicate medians of posterior predictive distribution for the sample statistic, and the boxes indicate the equal tail 90% credible set associated with the posterior predictive distribution.

was observed in the data.

4.3.2 Evidence from the cross section of stock returns

I now evaluate the asset-pricing implications of the model. As discussed in Appendix C, the Euler equations of bankers define the following asset pricing model

$$\mathbb{E}[R_{j,t+1}^e] = b_0 + b_1\beta_j, \quad \beta_j = \frac{\text{cov}\left(-\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}, R_{j,t+1}\right)}{\text{var}\left(\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}\right)}, \quad (20)$$

where $\hat{\Lambda}_{t,t+1}$ is defined as

$$\hat{\Lambda}_{t,t+1} = \beta \exp\{-\Delta c_{t+1}\}[(1 - \psi) + \psi\lambda \text{lev}_{t+1}], \quad (21)$$

and $R_{j,t}$ and $R_{j,t}^e$ are, respectively, realized returns and realized excess returns on asset j at time t . In the above equations, $b_0 = \mathbb{E}\left[\frac{\lambda\mu_t}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}\right]$ is the mean of the liquidity premium while $b_1 = \text{var}\left(\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}\right)$ can be thought as the price for holding risk. These moments are

Table 3: **Posterior predictive checks: sovereign default probabilities**

Statistic	Data	Posterior Median	90% Credible Set
Median	0.02	0.07	[0.00,2.69]
Mean	0.23	0.18	[0.01,8.95]
Standard Deviation	0.39	0.28	[0.01,13.59]
Autocorrelation	0.87	0.80	[0.62,0.92]
Skewness	2.05	1.88	[1.20,4.61]
Kurtosis	6.30	9.56	[3.57,29.01]

Notes: Based on 1,000 draws from the posterior distribution of $[s^*, \rho_s, \sigma_s]$. For each draw, I simulate the $\{s_t\}$ process for 100 periods. Statistics are computed on each of these 1,000 samples. The table reports the posterior median and equal tail probability 90% credible set for the posterior predictive distributions.

taken with respect to the ergodic distribution of the model state variables.

The logic of the exercise is as follows. Assuming that equation (20) holds for stocks traded in the Italian exchange, we can estimate b_0 and b_1 using a *two-pass* procedure. We can then ask whether the asset-pricing implications of the model are consistent, qualitatively and quantitatively, with these indicators of liquidity and risk premia. The estimation of equation (20) uses quarterly returns data from Compustat Global along with a time series of the stochastic discount factor in equation (21). Appendix C describes in details the data and the estimation procedure. Moreover, it confronts this specification with three benchmarks: the capital asset pricing model (CAPM), the three-factor model of Fama and French (1993), and a consumption-based version of $\hat{\Lambda}_{t,t+1}$ obtained by setting ψ to 0 in equation (21). In this section, I briefly report the main results.

The first result is that $\hat{\Lambda}_{t,t+1}$ outperforms these benchmark models in pricing the cross section of Italian stock returns. A successful model predicts that the pricing errors, the residuals of the cross sectional regression, are close to zero. Table A-3 in Appendix C shows that the benchmark specification achieves a better fit with respect to that of the CAPM and of the three-factor model of Fama and French (1993). Moreover, the analysis shows that the leverage component of $\hat{\Lambda}_{t,t+1}$ is key for pricing the cross section of returns, and this mirrors the mechanism in the structural model. These results are consistent with the work of Adrian et al. (2014), who find that a single factor based on the leverage of brokers and dealers prices the cross section of stock returns in the U.S. economy very well.

Second, the estimates of equation (20) are consistent with small average liquidity premia. Although imprecise, the point estimate for b_0 implies an average liquidity premium

of 11 basis points in annualized terms, remarkably close to the sample average of the interbank spread (8 basis points) used in the calculation of μ_t . This provides additional support for the measurement of the Lagrange multiplier proposed in this paper, and it stands in contrast to calibrations of the [Gertler and Karadi \(2011\)](#) model encountered in the literature, which typically imply substantially larger liquidity premia.

Third, and consistent with the structural model, stocks that pay out little when $\hat{\Lambda}_{t,t+1}$ is high earn higher excess returns relative to stocks that are less correlated to $\hat{\Lambda}_{t,t+1}$. In terms of magnitudes, the estimated b_1 implies an average annual excess return of 14% for an asset with $\beta_j = 1$. [Appendix C.3](#) shows that the structural model can generate values for $\text{var}_t \left(\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]} \right)$ that are consistent with these estimates.

5 The Propagation of Sovereign Risk

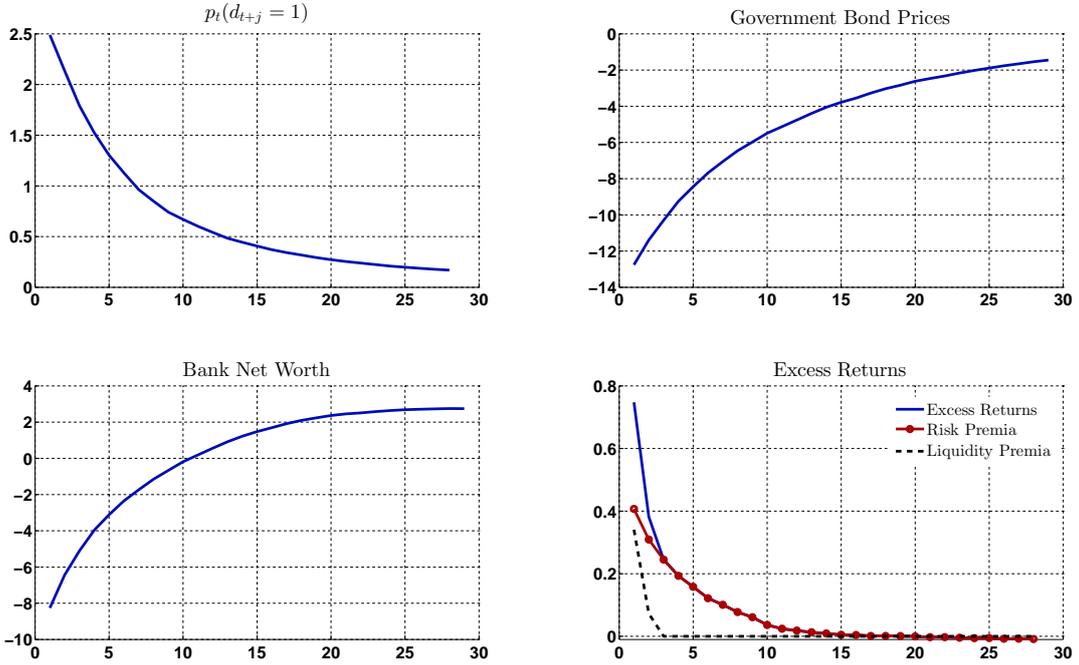
We now turn to the key experiment of this paper and study the macroeconomic effects of an increase in sovereign risk. As a preliminary step, [Section 5.1](#) uses impulse response analysis to explain the propagation mechanisms of an s -shock. [Section 5.2](#) proposes a counterfactual to measure the effect of the Italian sovereign debt crisis of 2010-2011 on the financing premia of firms and real economic activity. [Section 5.3](#) discusses in more detail the behavior of consumption conditional on an s -shock.

5.1 Impulse response functions

We can start by considering the reaction of liquidity and risk premia to a sovereign risk shock. [Figure 3](#) plots the response of key financial variables to an increase in s of 3.6 standard deviations. The model parameters are fixed at their posterior mean $\bar{\theta}$ in these experiments.

The shock increases the probability of a government default next quarter from 0.07% to 2.5%. Following this impulse, the price of government bonds declines because of the risk of experiencing losses on these assets in the future. This decline in the price of government securities feeds back into the balance sheet of bankers, generating a fall in their net worth. In this example, these losses are large enough to trigger the funding constraints of bankers. As a result, the shadow cost of funds for intermediaries increases, and these higher financing costs are passed on to the nonfinancial firms. In [Figure 3](#) we can observe that this increase in sovereign risk implies an increase of 35 basis points in the liquidity premium of equation [\(15\)](#).

Figure 3: Impulse response functions to an s -shock: financial variables



Notes: Impulse response functions are computed via simulations initialized at the ergodic mean of the state vector. The conditional probability of a default, $p_t(d_{t+j} = 1)$, is reported in percentages. $Q_{B,j}$ and N_j (linearly detrended) are expressed as percentage deviations from their ergodic mean value. Returns are reported in annualized basis points.

From the figure, we can also see that the risk premium component increases by 40 basis points following the impulse to s . As explained in Section 3, this increase in risk premia emerges because bankers now attach a higher likelihood to a state of the world (a sovereign default) in which claims of the productive sector pay out little precisely when bankers are in most need of wealth.

In order to understand the mechanisms that make the s -shock a priced factor for firms' claims, we can further decompose the risk premium on these assets as follows:

$$\text{cov}_t \left[-\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}, R_{K,t+1} \right] \approx \sigma_t \left[\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]} \right] \times \sigma_t[R_{K,t+1}], \quad (22)$$

where the first component, the conditional standard deviation of the (normalized) pricing kernel, can be interpreted as the *price* of risk, and the second component, the standard deviation of firms' payouts, can be seen as an indicator of the *quantity* of risk.¹⁹

Table 4 reports these components under two different conditioning sets: one in which the probability of a sovereign default next quarter is 0.07% (*Low* s_t) and one in which this

¹⁹The right-hand side of equation (22) is a good approximation to risk premia because $\rho_t \left[-\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}, R_{K,t+1} \right]$ is close to 1 and does not vary much over the state space. See Table 4.

probability is 2.5% (*High* s_t). As documented in Figure 3, risk premia on firms' claims increase by roughly 40 basis points when default risk changes from 0.07% to 2.5%. This change in risk premia is due to an increase both in the price and in the quantity of risk: the former increases by a factor of four, and the conditional volatility of firms' payouts more than doubles.

Table 4: **Decomposing risk premia**

	Risk Premia	$\sigma_t \left[\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]} \right]$	$\sigma_t[R_{K,t+1}]$	$\rho_t \left[-\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}, R_{K,t+1} \right]$
Low s_t	0.049	0.024	0.006	0.85
High s_t	0.452	0.106	0.012	0.89

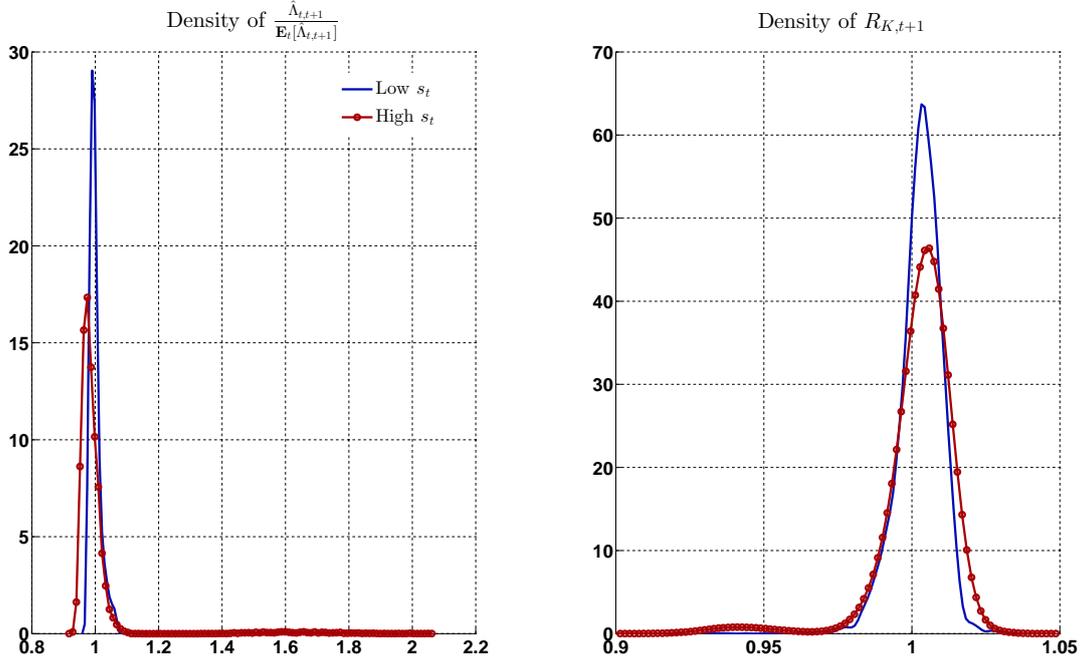
Notes: The conditional moments are computed by simulations. For the first row, simulate $M = 10,000$ realizations for $\{\hat{\Lambda}\}$ and $\{R_K\}$. Each simulation is initialized at the ergodic mean of the state vector and has length 2. The moments are computed using $\{\hat{\Lambda}_2^m\}_{m=1}^M$ and $\{R_{K,2}^m\}_{m=1}^M$. The second row repeats the same procedure, but simulations are initialized as follows: i) s -shock is set so that $p_t^d = 0.025$, and ii) the other state variables are set at their ergodic mean. Risk premia are in annualized basis points.

Figure 4 helps us understand this result. There, I plot the density of $\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}$ and $R_{K,t+1}$ under these two conditioning sets. We can observe two main facts from the figure. First, the increase in the price of risk occurs mainly because the right tail of the $\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}$ distribution becomes fatter. After the s -shock, agents place more probability on a default of the government next period. In this state of the world, the funding constraints of bankers are very tight because of the large losses on government bond holdings. The anticipation of this event makes bankers effectively act as if they were more averse to risk, and it leads them to charge higher prices for holding risky assets. Second, the increase in the conditional volatility of firms' payouts occurs because the left tail of the $R_{K,t+1}$ distribution becomes fatter. A government default in the model leads to tight funding constraints for bankers. Fire-sales of firms' assets by constrained intermediaries put downward pressure on $Q_{K,t+1}$ because of capital adjustment costs. This leads to low ex post returns. When s increases, bankers attach a higher probability to this event, and they therefore perceive future returns on firms' claims as more volatile. In other words, the sovereign default acts as a major economic disaster, and an increase in its likelihood increases the perceived riskiness of the productive sector.²⁰

Figure 5 displays the behavior of macroeconomic aggregates. We can observe that

²⁰Note that the sources of disaster risk in the model are shocks to the valuation of unproductive assets (government securities) that propagates to the productive ones via the balance sheet of financial intermediaries. In [Gourio \(2012\)](#) and related papers, the sources of disaster risk are shocks to production efficiency.

Figure 4: Price and quantity of risk



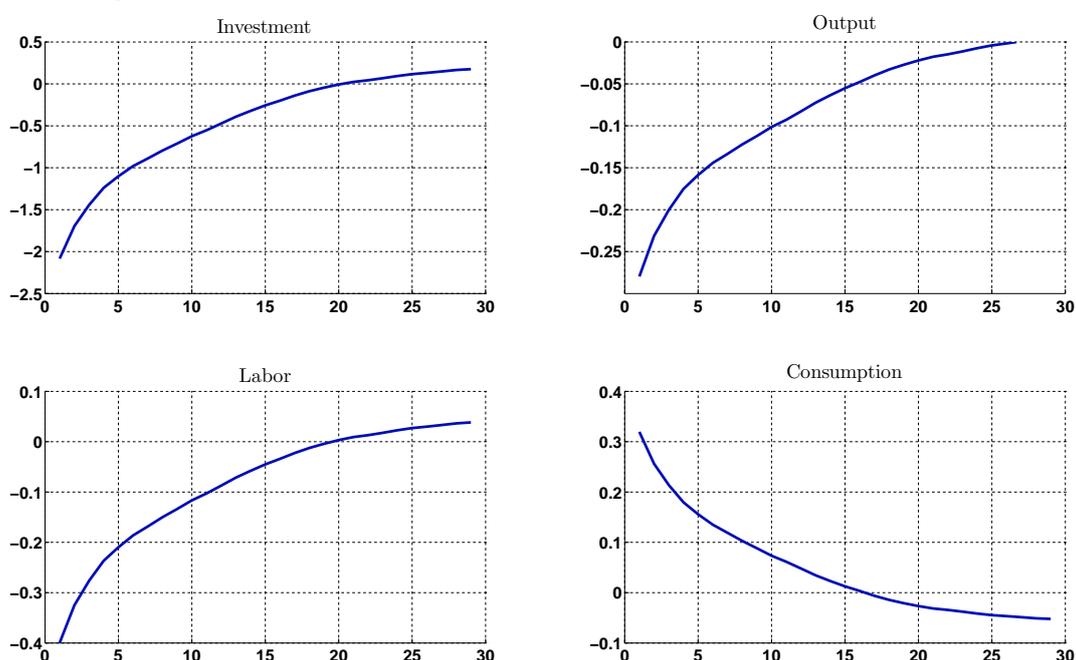
Notes: The left panel reports the density of the normalized pricing kernel for different conditioning sets. The solid line is constructed by applying a kernel density smoother to $\{\hat{\Lambda}_2^m\}_{m=1}^M$ simulated conditioning on the ergodic mean as described in the note to Table 4, and normalized by their average. The line with circles repeats the same procedure, this time generating the simulations as in the “High s_t ” scenario of Table 4. The right panel reports the same information for $R_{K,t+1}$.

aggregate investment drops by 2% on impact. The fall in aggregate investment is related to the behavior of liquidity and risk premia discussed above. On the one hand, the tightening of the funding constraints leads bankers to shed firms’ assets. On the other hand, the more cautious attitude of intermediaries and the increase in the perceived riskiness of the firms generate a precautionary motive for the bankers to deleverage and to reduce their exposure to the productive sector. Both of these factors contribute to a reduction in the demand for capital goods. Hours worked and output also decline, respectively, by 0.4% and 0.25%. Therefore, a mere increase in the probability of a sovereign default depresses real economic activity. Note that the decline of output on impact is entirely driven by the behavior of hours, because capital is predetermined and technology is fixed in this experiment. This drop in hours is motivated by intertemporal labor supply motives: allocating savings to banks is less attractive for households because of agency costs, and this depresses their incentives to work. For this same reason, consumption increases on impact. Therefore, while being recessionary, the s -shock generates imperfect comovement between consumption on the one side, and investment, output, and hours on the other side. As I explain in Section 5.3, the behavior of consumption is not a robust feature of

the environment, and it can be changed by considering sensible extensions of the model.

Note that the speed of recovery of macroeconomic variables is time-varying. In the first few periods after the sovereign risk shock, the leverage constraint binds. The recovery in bank net worth loosens these funding constraints and it stimulates capital accumulation. From period 4 onward, the constraints are not binding anymore and the recovery in net worth has limited positive spillovers: in period 11, net worth is already above trend (see Figure 3) while real economic activity is still depressed partly because of the precautionary motives associated to the risk channel.

Figure 5: Impulse response functions to an s -shock: quantities



Notes: See note to Figure 3. Variables are linearly detrended and expressed as percentage deviations from their ergodic mean value.

5.2 Sovereign risk, firms' financing premia, and real economic activity

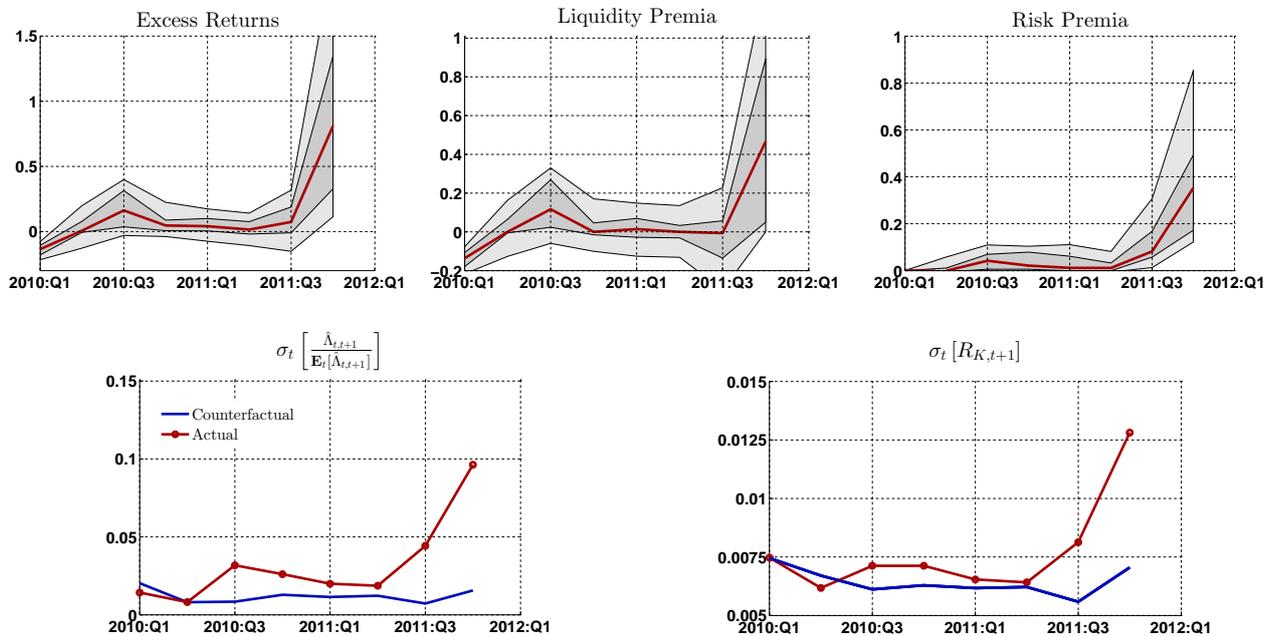
We can now use the model to measure the effects of the sovereign debt crisis on the financing premia of Italian firms and on real economic activity, and to quantify the relative importance of liquidity and risk premia in driving this propagation. This can be done through a counterfactual experiment. Specifically, we can proceed in three steps:

1. Estimate the historical path for the structural shocks. Technically, this is done by applying the particle filter to the observable variables $\mathbf{Y}_t = [\text{GDP growth}_t, \mu_t, p_t^d]$.

From this first step, we obtain the density $p(\varepsilon_t | \mathbf{Y}^t, \theta^i)$ for the structural shocks $\varepsilon_t = [\varepsilon_{z,t}, \varepsilon_{g,t}, \varepsilon_{s,t}]'$ over the period 2010:Q1-2011:Q4.

2. Generate a counterfactual “no sovereign crisis” scenario for the endogenous variables of the model by setting the probability of an Italian default at its ergodic mean throughout the sample. That is, we can feed into the model the historical realizations of the shocks estimated in step 1, with the exception that $\varepsilon_{s,t}$ is set to 0 $\forall t$.
3. We can then isolate the effect of heightened sovereign risk on a variable x by taking the difference between its *actual* and *counterfactual* trajectory.²¹

Figure 6: Sovereign risk and excess returns



Notes: The statistics in the top panels are generated as follows. Draw $N = 10$ values of θ from its posterior distribution. For each θ^i , draw $M = 10$ trajectories from $p(\mathbf{S}_t | \mathbf{Y}^t, \bar{\theta})$ for the time period 2010:Q1-2011:Q4. For each trajectory, compute the path for the variables of interest. Next, generate a counterfactual path for the state vector and for the variables of interest by setting $\varepsilon_{s,t} = 0 \forall t$. The solid lines in the figures are the pointwise averages of the difference between these two paths over the M trajectories. The dark and light shaded areas represent, respectively, a 60% and 90% equal tail probability credible sets for the variables of interest. The statistics in the bottom panel are constructed as explained in Table 4, and they represent averages of the actual and counterfactual paths over the $N \times M$ realizations.

The top panel of Figure 6 reports the estimated impact of the sovereign debt crisis of the 2010:Q1-2011:Q4 period on excess returns and on its decomposition into liquidity and risk premia. The model predicts that the sharp increase in the probability of an Italian default in the last two quarters of 2011 led to an increase in excess returns of 81

²¹The actual trajectory for variable x is obtained by fitting the model with the time path for the structural shocks estimated in step 1.

basis points at peak. The figure also shows that the risk channel was quantitatively an important force in the propagation of sovereign credit risk in Italy: at the end of 2011:Q4, risk premia accounted for 45% of the pass-through of sovereign risk on the financing rates of firms. The bottom panel of the figure decomposes further the behavior of risk premia into price and quantity of risk, defined in equation (22). We can verify that both indicators have been substantially affected by the sovereign debt crisis: the price of risk increases by roughly 4 times, and the conditional volatility of firms' payouts almost doubles. This last finding suggests that the transmission of risk from the sovereign to the private sector is quantitatively an important force in the model.

We can also explore the effect of the sovereign debt crisis on output. Table 5 reports the difference between actual and counterfactual trajectories in the last two quarters of 2011. The model predicts that output would have been, respectively, 0.63% and 1.18% higher in annualized terms in absence of the observed tensions in sovereign markets. Table 5 further computes the predicted effect of the sovereign crisis for 2012. That is, I generate forecasts for linearly detrended output conditioning on two different scenarios in 2011:Q4: the posterior mean of the estimated state vector, and its counterfactual mean generated under the "no sovereign crisis" scenario. We can verify that the model estimates output to be persistently below trend following the sharp rise in the probability of an Italian default in 2011.

Table 5: **Output losses of the sovereign crisis**

	2011:Q3	2011:Q4	2012:Q1	2012:Q2	2012:Q3	2012:Q4
Mean	-0.63	-1.18	-1.36	-1.24	-1.14	-0.98
	[-0.89,-0.34]	[-1.59,-0.80]	[-1.51,-0.98]	[-1.40,-0.79]	[-1.32,-0.62]	[-1.21,-0.52]

Notes: The first two columns report the posterior mean for the differences between the actual and counterfactual level of output, as described in the note to Figure 6. The remaining columns report the differences in the point forecasts for output conditioning on the posterior mean of the state vector in 2011:Q4 under the actual and counterfactual scenario. Equal tail probability 60% credible sets in parenthesis.

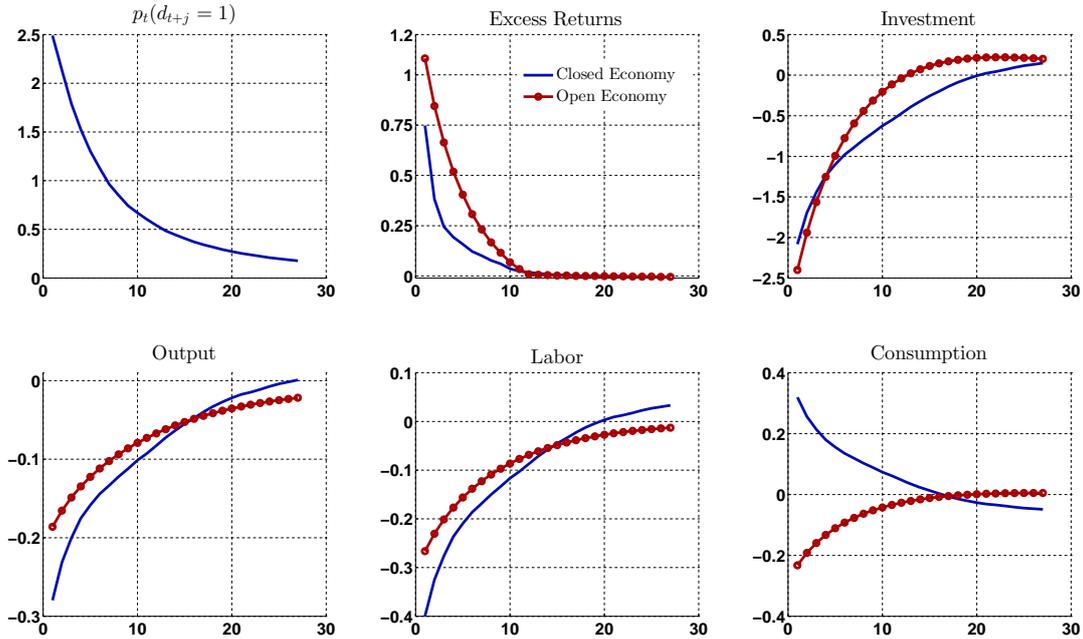
5.3 The response of consumption to an s -shock

As shown in Figure 5, when sovereign risk increases, output, investment, and hours fall on impact while consumption rises. This "comovement problem" arises frequently in neoclassical settings when considering the response to nontechnology shocks (Barro and King, 1984). Holding technology fixed, the only way that the model can generate a contemporaneous drop in output is through a reduction in hours' worked. For this to happen, labor

supply needs to decline because K is predetermined and z is held fixed. The comovement problem, then, arises because an increase in consumption is necessary to generate a decline in labor supply under the households' preferences used in this paper.

This feature of the model can be changed with slight modifications to the baseline environment. Specifically, assume that bankers can now borrow from the international capital market at the fixed interest rate R^* . Because of a no-arbitrage condition, the returns on households' savings will also earn the risk-free return R^* in equilibrium. In addition, consider two ingredients that are routinely used in the analysis of small open economy models: i) preferences that mute the wealth effect on labor supply and ii) working capital requirements for the firms. As in Neumeyer and Perri (2005), firms need to borrow a fraction ϕ of the wage bill before production takes place. These loans are obtained from the bankers at the beginning of the period, and they pay the gross return $R_{W,t} = R^* + \lambda \frac{\mu_t}{\mathbb{E}_t[\bar{\lambda}_{t,t+1}]}$ at the end of the period.

Figure 7: Impulse response functions to an s -shock: open vs. closed economy



Notes: See note to Figure 3.

Figure 7 overlays the responses to a sovereign risk shock for the small open economy version of the model (lines with circles) and for the benchmark specification (solid lines).²² First, note that in the open economy model consumption declines conditional on an in-

²²The parametrization of the open economy model is the same as the benchmark. However, because the preferences described above are not consistent with balanced growth, technology is assumed to follow a simple AR(1) process. The parameter ϕ is set to 1 for illustrative purposes.

crease in s . This happens because the negative wealth effect induced by a tightening of bankers' leverage constraint is not counteracted by the reduction in the risk-free rate, since R^* is now fixed. Second, the implications for investment, output, hours, and excess returns are qualitatively and quantitatively comparable to those obtained in the closed economy model. However, the underlying forces that generate the decline in hours and output on impact differ. In the open economy model, the decline in hours worked occurs because the tightening of bankers' funding constraints leads to an increase in $R_{W,t}$, and these higher financing costs for working capital translate into a lower demand for labor by the firm.

6 Refinancing Operations

In the previous section, we used the structural model to measure the effects of the sovereign debt crisis on the Italian economy and to quantify the relative importance of liquidity and risk in driving this propagation. This section shows that this decomposition provides useful insights into understanding the effects of credit market interventions in this class of models.

This point is illustrated by simulating the effects of the *Longer Term Refinancing Operations* (LTROs) implemented by the ECB in December 2011 and February 2012. At the height of the euro area sovereign debt crisis, the ECB launched two unconventional LTROs with the aim of loosening funding pressures on the banking sectors of distressed economies. Relative to canonical open market operations in the euro area,²³ these interventions featured a long maturity (36 months), a fixed interest rate, and special rules for the collateral that could be used by banks. Moreover, the two LTROs were the largest refinancing operations in the history of the ECB, as more than 1 trillion euros were lent to banks through these interventions. I model LTROs as a nonstationary version of the discount window lending considered in [Gertler and Kiyotaki \(2010\)](#). The government gives bankers the option at $t = 1$ to borrow resources up to a threshold \bar{m} . The loans have a fixed interest rate R_m , and bankers repay the principal and all accrued interest at a future date T .²⁴ The parameters of this policy are calibrated to the actual intervention of the ECB

²³Open market operations in the euro area are conducted through refinancing operations. These are similar to repurchase agreements: banks put up acceptable collateral with the ECB and receive cash loans. Prior to 2008, there were two major types of refinancing operations: main refinancing operations (loans with a weekly maturity) and LTROs, with a three-month maturity.

²⁴The government finances this intervention via lump-sum taxes and has perfect monitoring of banks. This last assumption implies that these liabilities do not count for the leverage constraints of bankers and that these government loans do not perfectly crowd out households' deposits. See [Gertler and Kiyotaki](#)

by setting $R_m = 1.00$, T to 12 quarters, and $\bar{m} = 0.1\hat{Y}^{bg}$.

We can evaluate the effects of the policy on output and the private sector spreads by simulating the model forward with and without the intervention, and by subsequently taking the difference between these two paths.²⁵ Because the model is nonlinear, the effects of the policy depend on the state variables at which these simulations are initialized. Therefore, an integral part of the evaluation of LTROs is to specify initial conditions. In order to make the exercise more realistic, I choose initial conditions so that the economy is in a state of financial recession. I do so by simulating a long trajectory from the model and by selecting state variables $\{\mathbf{S}_i^*\}$ at which output growth is 2.5 standard deviations below average and excess returns are 2.5 standard deviations above average. For each \mathbf{S}_i^* , I calculate the impact effect of the LTROs on output, excess returns, and their decomposition into liquidity and risk premia. Moreover, I repeat these simulations by drawing initial conditions from $p(\mathbf{S}_{2011:Q4} | \mathbf{Y}^{2011:Q4}, \bar{\theta})$, the empirical distribution of the state variables for the Italian economy in 2011:Q4.

In order to interpret the results, it is convenient to define the following indicator:

$$\delta_i = \frac{\text{cov}_{\mathbf{S}_i^*} \left[-\frac{\hat{\Lambda}(\mathbf{S}', \mathbf{S}_i^*)}{\mathbb{E}_{\mathbf{S}_i^*}[\hat{\Lambda}(\mathbf{S}', \mathbf{S}_i^*)]}, R_K(\mathbf{S}', \mathbf{S}_i^*) \right]}{\mathbb{E}_{\mathbf{S}_i^*}[R_K(\mathbf{S}', \mathbf{S}_i^*) - R(\mathbf{S}_i^*)]}.$$

This variable $\delta_i \in [0, 1]$ can be seen as an indicator of the importance of the precautionary motives described in this paper at a given state \mathbf{S}_i^* . When $\delta_i = 0$, expected excess returns purely reflect binding funding constraints for the banks, whereas $\delta_i = 1$ means that the private sector spreads reflect compensation for holding risk. Figure 8 reports the results. Each circle represents the impact effect of LTROs on an outcome variable at a given state \mathbf{S}_i , and the associated δ_i . The filled red circles report these indicators when the states are drawn from the financial recession simulations while the empty blue ones report the results when the simulations are initialized from $p(\mathbf{S}_{2011:Q4} | \mathbf{Y}^{2011:Q4}, \bar{\theta})$.

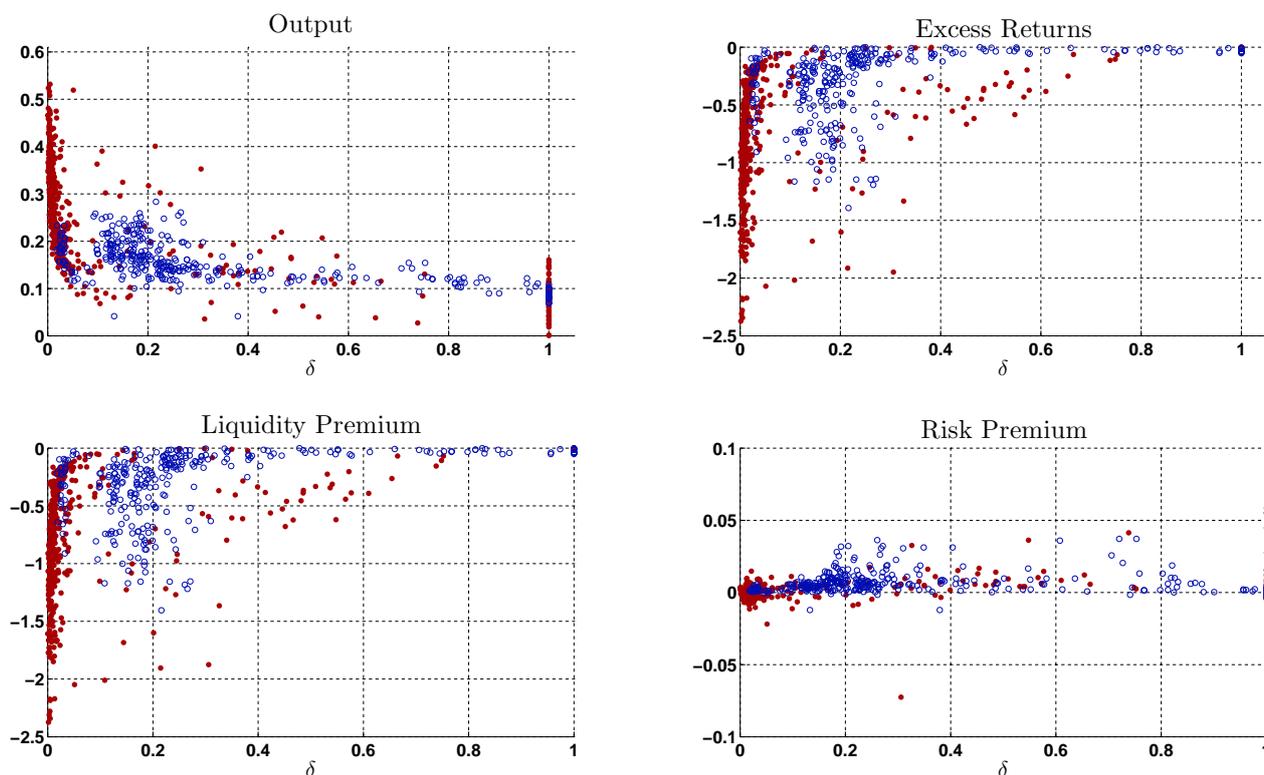
On average, LTROs lead to an increase in output and a decline in the excess returns that bankers demand for holding firms' assets. However, the figure shows a significant amount of state dependence in the macroeconomic implications of the policy. LTROs are particularly effective in stimulating real economic activity in regions of the state space in which liquidity premia are sizable ($\delta_i \approx 0$). Liquidity constraints, by definition, prevent bankers from undertaking profitable investments: LTROs relax these constraints and stimulate lending to the productive sector. The same intervention, however, has substantially

(2010) for a discussion of this point.

²⁵See the online Appendix for additional details.

weaker effects when implemented in regions of the space space in which risk premia are sizable ($\delta_i \approx 1$). High risk premia are an indication that bankers are not particularly willing to hold firms' claims in their balance sheets. Therefore, refinancing operations tend to have limited effects on real economic activity because bankers use these funds to substitute part of their liabilities with households rather than lending to the productive sector.

Figure 8: The macroeconomic effects of refinancing operations



Notes: For each \mathbf{S}_i , compute by simulations the impact effect of LTROs on variable x , defined as $\mathbb{E}_{\mathbf{S}_i}[x_1^{ltro} - x_1^{no\ ltro}]$, and compute variable δ . Each circle in the figure represents a combination of these two indicators at a particular \mathbf{S}_i . The filled circles report the results when \mathbf{S}_i is drawn from the financial recession set, while empty circles report results when simulations are initialized from $p(\mathbf{S}_{2011:Q4} | \mathbf{Y}^{2011:Q4}, \bar{\theta})$. Returns are in annualized percentages.

The structural model allows one to measure the relative importance of these mechanisms, a decomposition that is important to evaluate these interventions. This can be seen by comparing the effects of LTROs under the two conditioning sets. In a typical financial recession, one would predict refinancing operations to increase output by 0.35% on impact, and to reduce interest rate spreads by 0.72%. When conditioning on the state of the Italian economy in 2011:Q4, instead, these numbers are 0.15% and 0.25% respectively. Therefore, the model predicts LTROs to be quite ineffective in stimulating bank lending at the height of the debt crisis. This depends on the fact that precautionary motives,

measured by the relative importance of risk premia, were sizable at the time.

7 Conclusion

This paper proposed a quantitative model for studying the transmission of sovereign risk on the borrowing costs of firms and on real economic activity. The mere prospect of a future default of the government hampers financial intermediation. On the one hand, banks that are exposed to risky government bonds are less able to borrow from capital markets: this hampers their ability to finance firms (*liquidity channel*). On the other hand, the default of the government may itself be a source of risk for the firms: this reduces the willingness of financial intermediaries to hold claims of the private sector (*risk channel*). The estimation of the model shows that both mechanisms were important during the sovereign debt crisis in Italy. Moreover, the analysis suggests that the provision of liquidity to the banking sector by the ECB had limited effects on bank lending to firms. Indeed, the model detects strong precautionary motives for financial intermediaries at the height of the debt crisis, which indicates that intermediaries were reluctant to hold claims of the the productive sector.

Abstracting from the current application, the analysis underscores the importance of measuring the sources that drive the movements in asset prices. Understanding whether firms' financing premia during crises arise because of lack of liquidity in financial markets or because of fair compensation for increased risk is key information for policymakers. Incorporating the nonlinearities emphasized in this paper in larger-scale models used for policy evaluation is technically challenging. Moreover, given their relevance for policy, there is a need for developing tools for their empirical validation in the data. These areas represent exciting opportunities for future work.

References

- Acharya, Viral V., Itamar Drechsler, and Philipp Schnabl, "A Pyrrhic Victory? Bank Bailouts and Sovereign Credit Risk," *Journal of Finance*, 2014, 69 (6), 2689–2739.
- , Tim Eisert, Christian Eufinger, and Christian Hirsch, "Real Effects of the Sovereign Debt Crisis in Europe: Evidence from Syndicated Loans," 2014. manuscript, New York University Stern School of Business.
- Adrian, Tobias, Erkki Etula, and Tyler Muir, "Financial Intermediaries and the Cross-Section of Asset Returns," *Journal of Finance*, 2014, 69 (6), 2557–2596.

- Aiyagari, Rao S. and Mark Gertler**, ““Overreaction” of Asset Prices in General Equilibrium,” *Review of Economic Dynamics*, 1999, 2 (1), 3–35.
- Arellano, Cristina**, “Default Risk and Income Fluctuations in Emerging Economies,” *American Economic Review*, 2008, 98 (3), 690–712.
- Aruoba, Boragan S., Luigi Bocola, and Frank Schorfheide**, “Assessing DSGE Model Non-linearities,” 2013. NBER Working Paper No. 19693.
- Bahaj, Saleem**, “Systemic Sovereign Risk: Macroeconomic Implications in the Euro Area,” 2013. Manuscript, University of Cambridge.
- Barro, Robert J. and Robert G. King**, “Time-Separable Preferences and Intertemporal-Substitution Models of Business Cycles,” *Quarterly Journal of Economics*, 1984, 99 (4), 817–839.
- Bi, Huixin and Nora Traum**, “Estimating Sovereign Default Risk,” *American Economic Review*, 2012, 102 (3), 161–166.
- Bianchi, Javier**, “Efficient Bailouts?,” 2013. manuscript, Federal Reserve Bank of Minneapolis.
- **and Enrique G. Mendoza**, “Overborrowing, Financial Crises and “Macroprudential Policies”,” 2012. Manuscript, University of Pennsylvania.
- **and Saki Bigio**, “Banks, Liquidity Management and Monetary Policy,” 2014. Federal Reserve Bank of Minneapolis Staff Report 503.
- Bofondi, Marcello, Luisa Carpinelli, and Enrico Sette**, “Credit Supply during a Sovereign Debt crisis,” 2013. Temi di Discussione N° 909, Bank of Italy.
- Borensztein, Eduardo and Ugo Panizza**, “The Costs of Sovereign Default,” *IMF Staff Papers*, 2009, 9 (56), 683–741.
- Bottero, Margherita, Simone Lenzu, and Filippo Mezzanotti**, “The Lending Channel at the Onset of the Sovereign Crisis,” 2015. Manuscript, Harvard Business School.
- Broner, Fernando, Aitor Erce, Alberto Martin, and Jaume Ventura**, “Sovereign Debt Markets in Turbulent Times: Creditor Discrimination and Crowding-Out Effects,” *Journal of Monetary Economics*, 2014, 61, 114–142.
- Brunnermeier, Markus K. and Yuliy Sannikov**, “A Macroeconomic Model with a Financial Sector,” *American Economic Review*, 2014, 104 (2), 379–421.
- Chatterjee, Satyajit and Burcu Eyigungor**, “Maturity, Indebtedness, and Default Risk,” *American Economic Review*, 2012, 102 (6), 2674–2699.

- Committee on the Global Financial System**, “The Impact of Sovereign Credit Risk on Bank Funding Conditions,” 2011. CGFS Papers No. 43, Bank for International Settlements.
- Cooper, Russell and Kalin Nikolov**, “Government Debt and Banking Fragility: The Spreading of Strategic Uncertainty,” 2013. NBER Working Paper No. 19278.
- Corsetti, Giancarlo, Keith Kuester, André Meier, and Gernot J. Müller**, “Sovereign Risk, Fiscal Policy and Macroeconomic Stability,” *Economic Journal*, 2013, 123 (566), 99–132.
- Del Negro, Marco and Frank Schorfheide**, “Bayesian Macroeconometrics,” in J. Geweke, G. Koop, and H. Van Dijk, eds., *The Oxford Handbook of Bayesian Econometrics*, Oxford University Press, 2011.
- Fahri, Emmanuel and Jean Tirole**, “Deadly Embrace: Sovereign and Financial Balance Sheets Doom Loops,” 2014. manuscript, Harvard University.
- Fama, Eugene F. and Kenneth R. French**, “Common Risk Factors on the Returns of Stocks and Bonds,” *Journal of Financial Economics*, 1993, 33 (1), 3–56.
- Fernández-Villaverde, Jesús and Juan F. Rubio-Ramírez**, “Estimating Macroeconomic Models: A Likelihood Approach,” *Review of Economic Studies*, 2007, 74 (4), 1059–1087.
- Gennaioli, Nicola, Alberto Martin, and Stefano Rossi**, “Banks, Government Bonds, and Default: What Do the Data Say?,” 2013. Manuscript, London School of Economics.
- , —, and —, “Sovereign Default, Domestic Banks and Financial Institutions,” *Journal of Finance*, 2014, 69 (2), 819–866.
- Gertler, Mark and Nobuhiro Kiyotaki**, “Financial Intermediation and Credit Policy in Business Cycle Analysis,” in B. Friedman and M. Woodford, eds., *Handbook of Monetary Economics*, 3A, Elsevier, 2010.
- and **Peter Karadi**, “A Model of Unconventional Monetary Policy,” *Journal of Monetary Economics*, 2011, 58 (1), 17–34.
- Gourio, François**, “Disaster Risk and Business Cycles,” *American Economic Review*, 2012, 102 (6), 2734–2766.
- Guerrieri, Luca and Matteo Iacoviello**, “Collateral Constraints and Macroeconomic Asymmetries,” 2013. International Finance Discussion Papers No. 1082, Board of Governors of the Federal Reserve System.
- Gust, Christopher, David Lopez-Salido, and Matthew E. Smith**, “The Empirical Implications of the Interest-Rate Lower Bound,” 2013. Manuscript, Federal Reserve Board.
- He, Zhiguo and Arvind Krishnamurthy**, “A Macroeconomic Framework for Quantifying Systemic Risk,” 2012. Manuscript, Kellogg School of Management.

- and —, “Intermediary Asset Pricing,” *American Economic Review*, 2013, 103 (2), 732–770.
- Krueger, Dirk and Felix Kubler**, “Computing Equilibrium in OLG Models with Stochastic Production,” *Journal of Economic Dynamics and Control*, 2004, 28 (7), 1411–1436.
- Kumhof, Michael and Evan Tanner**, “Government Debt: A Key Role in Financial Intermediation,” 2005. IMF Working Paper WP/05/57.
- Lewellen, Jonathan, Stefan Nagel, and Jay Shanken**, “A Skeptical Appraisal of Asset Pricing Tests,” *Journal of Financial Economics*, 2010, 96 (2), 175–194.
- Maggiore, Matteo**, “Financial Intermediation, International Risk Sharing and Reserve Currencies,” 2013. Manuscript, Harvard University.
- Neumeyer, Pablo A. and Fabrizio Perri**, “Business Cycles in Emerging Economies: The Role of Interest Rates,” *Journal of Monetary Economics*, 2005, 52 (2), 345–380.
- Perez, Diego J.**, “Sovereign Debt, Domestic Banks and the Provision of Public Liquidity,” 2015. manuscript, Stanford University.
- Pitt, Michael K. and Neil Shephard**, “Filtering via Simulation: Auxiliary Particle Filters,” *Journal of the American Statistical Association*, 1999, 94 (446), 590–599.
- Prestipino, Andrea**, “Financial Crises and Policy,” 2013. manuscript, Board of Governors of the Federal Reserve System.
- Ríos-Rull, José-Víctor, Frank Schorfheide, Cristina Fuentes-Albero, Maxym Kryshko, and Raul Santaaulalia-Llopis**, “Methods versus Substance: Measuring the Effects of Technology Shocks,” *Journal of Monetary Economics*, 2012, 59 (8), 826–846.
- Schorfheide, Frank**, “Loss Function-Based Evaluation of DSGE Models,” *Journal of Applied Econometrics*, 2000, 15 (6), 645–670.
- Shanken, Jay**, “On the Estimation of Beta-Pricing Models,” *Review of Financial Studies*, 1992, 5 (1), 1–55.
- Sosa Padilla, César**, “Sovereign Defaults and Banking Crises,” 2013. Manuscript, McMaster University.
- Uribe, Martin and Vivian Z. Yue**, “Country Spreads and Emerging Countries: Who Drives Whom?,” *Journal of International Economics*, 2006, 69 (1), 6–36.
- Zettelmeyer, Jeromin, Christoph Trebesch, and Mitu Gulati**, “The Greek Debt Restructuring: An Autopsy,” 2013. CESifo Working Paper No. 4333.

APPENDIX

A Proof of Proposition 1

Rewrite the decision problem of a banker by eliminating the demand for households' savings:

$$\begin{aligned}
 v^b(n; \mathbf{S}) &= \max_{a_B, a_K} \mathbb{E}_{\mathbf{S}} \left\{ \Lambda(\mathbf{S}', \mathbf{S}) \left[(1 - \psi)n' + \psi v^b(n'; \mathbf{S}') \right] \right\}, \\
 \text{s. t.} \\
 n' &= \sum_{j=\{B, K\}} [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})] Q_j(\mathbf{S}) a_j + R(\mathbf{S})n, \\
 \lambda \left[\sum_{j=\{B, K\}} Q_j(\mathbf{S}) a_j \right] &\leq v^b(n; \mathbf{S}), \\
 \mathbf{S}' &= \mathbf{\Gamma}(\mathbf{S}).
 \end{aligned}$$

Guess that the value function is $v^b(n, \mathbf{S}) = \alpha(\mathbf{S})n$, and define $\hat{\Lambda}(\mathbf{S}', \mathbf{S})$ as

$$\hat{\Lambda}(\mathbf{S}', \mathbf{S}) = \Lambda(\mathbf{S}', \mathbf{S})[(1 - \psi) + \psi\alpha(\mathbf{S}')]. \quad (23)$$

Necessary and sufficient conditions for an optimum are

$$\mathbb{E}_{\mathbf{S}} \left\{ \hat{\Lambda}(\mathbf{S}', \mathbf{S}) [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})] \right\} = \lambda \mu(\mathbf{S}), \quad (24)$$

$$\mu(\mathbf{S}) \left(\alpha(\mathbf{S})n - \lambda \sum_{j=\{B, K\}} Q_j(\mathbf{S}) a_j \right) = 0. \quad (25)$$

Substituting the guess in the dynamic program and using the law of motion for n' , we can rewrite the decision problem as

$$v^b(n, \mathbf{S}) = \max_{a_B, a_K} \left\{ \sum_{j=\{B, K\}} \mathbb{E}_{\mathbf{S}} \left\{ \hat{\Lambda}(\mathbf{S}', \mathbf{S}) [R_j(\mathbf{S}', \mathbf{S}) - R(\mathbf{S})] \right\} Q_j(\mathbf{S}) a_j \right\} + \mathbb{E}_{\mathbf{S}} \left\{ \hat{\Lambda}(\mathbf{S}', \mathbf{S}) \right\} R(\mathbf{S})n,$$

subject to the constraint

$$\lambda \left[\sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j \right] \leq \alpha(\mathbf{S})n.$$

Note that the first term on the right-hand side of the objective function equals $\mu(\mathbf{S})\alpha(\mathbf{S})n$.²⁶ Thus, the value function under the guess takes the following form:

$$\alpha(\mathbf{S})n = \mu(\mathbf{S})\alpha(\mathbf{S})n + \mathbb{E}_{\mathbf{S}} \{ \hat{\Lambda}(\mathbf{S}', \mathbf{S}) \} R(\mathbf{S})n.$$

Solving for $\alpha(\mathbf{S})$ and using the definition of $\hat{\Lambda}(\mathbf{S}', \mathbf{S})$ in equation (23), we obtain

$$\alpha(\mathbf{S}) = \frac{\mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi) + \psi\alpha(\mathbf{S}')] R(\mathbf{S}) \}}{1 - \mu(\mathbf{S})},$$

which is the expression for the marginal value of wealth in equation (1). The guess is verified if $\mu(\mathbf{S}) < 1$. From equation (25) we obtain:

$$\mu(\mathbf{S}) = \max \left\{ 1 - \frac{\mathbb{E}_{\mathbf{S}} \{ \Lambda(\mathbf{S}', \mathbf{S}) [(1 - \psi) + \psi\alpha(\mathbf{S}')] R(\mathbf{S}) \} n}{\lambda [Q_K(\mathbf{S})a_K + Q_B(\mathbf{S})a_B]}, 0 \right\} < 1.$$

Finally, note that financial leverage is the same across bankers when $\mu(\mathbf{S}) > 0$. Therefore, $\frac{n}{\lambda \sum_{j=\{B,K\}} Q_j a_j}$ equals $\frac{N}{\lambda [\sum_{j=\{B,K\}} Q_j A_j]}$ when the constraint binds.

B Data Appendix

B.1 Credit default swap (CDS) spread

Daily CDS spreads are on one year Italian government securities (RED code: 4AB951), and they are obtained from Markit. The restructuring clause of the contract is CR (complete restructuring). The spread is denominated in basis points and paid quarterly. The data are accessed from Wharton Research Data Services.

²⁶Indeed, when $\mu(\mathbf{S}) = 0$, this term equals zero because risk-adjusted returns on assets demanded by the banker equals $\mathbb{E}_{\mathbf{S}} [\hat{\Lambda}(\mathbf{S}', \mathbf{S}) R(\mathbf{S})]$ by equation (24). When $\mu(\mathbf{S}) > 0$, this term can be written as

$$\mu(\mathbf{S})\lambda \sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j.$$

Using the condition in (25), we can then express $\mu(\mathbf{S})\lambda \sum_{j=\{B,K\}} Q_j(\mathbf{S})a_j$ as $\mu(\mathbf{S})\alpha(\mathbf{S})n$.

B.2 Banks' exposure to the Italian government

The European Banking Authority (EBA) published information on holdings of government debt by European banks participating in the 2011 stress test. Five Italian banks were in this pool: Unicredit, Intesa-San Paolo, Monte dei Paschi di Siena (MPS), Banco Popolare (BPI), and Unione di Banche Italiane (UBI). Results of the stress test for each of these five banks are available at <http://www.eba.europa.eu/risk-analysis-and-data/eu-wide-stress-testing/2011/results>. The exposure of each bank to the Italian central and local government is measured as net direct positions (bonds, loans and advances) plus direct sovereign exposures in derivative contracts. The data are available by the maturity of the financial instrument, and reflect positions as of December 31, 2010. This information is reported in Table A-1. The table reports also total financial assets for these institutions as of 31st of December 2010. This latter information is obtained using consolidated banking data from Bankscope, accessed from Wharton Research Data Services.

Table A-1: Exposure to domestic sovereign by major Italian banks: end of 2010

	3Mo	1Yr	2Yr	3Yr	5Yr	10Yr	15Yr	Tot.	Tot. Assets
Intesa	17.18	9.31	2.46	4.87	7.71	6.42	10.12	58.08	658.76
Unicredit	17.78	9.85	2.78	6.12	4.24	5.90	1.44	48.11	929.49
MPS	5.61	4.96	3.92	3.57	1.35	3.71	8.91	32.03	240.70
BPI	3.90	1.65	1.15	3.64	0.78	0.39	0.25	11.76	134.17
UBI	1.27	3.56	0.22	0.30	0.54	2.47	1.76	10.11	129.80
Total	45.75	29.32	10.53	18.5	14.61	18.89	22.48	160.01	2092.99

Notes: Data are reported in billions of euros.

B.3 Lagrange multiplier

As a preliminary step, I introduce an interbank market in the model. Bankers can trade between themselves, after observing the realization of \mathbf{S} , a claim that offers one unit of the numeraire next period in exchange for $Q_{\text{interbank}}(\mathbf{S})$ units of the numeraire today. I

let $a_{\text{interbank}}(n_i, \mathbf{S})$ be the position of a banker with net worth n_i in this market. Market clearing implies $\int a_{\text{interbank}}(n_i, \mathbf{S}) di = 0$. Letting $R_{\text{interbank}}(\mathbf{S})$ be the yield on this asset, we can write the bankers' Euler equation as²⁷

$$R_{\text{interbank}}(\mathbf{S}) = R(\mathbf{S}) + \frac{\lambda \mu(\mathbf{S})}{\mathbb{E}_{\mathbf{S}}[\hat{\Lambda}(\mathbf{S}', \mathbf{S})]}. \quad (26)$$

I now show that the Lagrange multiplier can be expressed as a function of the interbank spread $[R_{\text{interbank}}(\mathbf{S}) - R(\mathbf{S})]$ and of the leverage of bankers.

Proposition 2. *In equilibrium, the Lagrange multiplier on the incentive constraint of bankers is a function of financial leverage and of the spread between the interbank rate and the risk-free rate:*

$$\mu_t = \frac{\left[\frac{R_{\text{interbank},t} - R_t}{R_t} \right] lev_t}{1 + \left[\frac{R_{\text{interbank},t} - R_t}{R_t} \right] lev_t}. \quad (27)$$

Proof. From equation (1), we can write $\mathbb{E}_t[\hat{\Lambda}_{t,t+1}] = \frac{\alpha_t(1-\mu_t)}{R_t}$. Hence, we can write equation (26) as

$$\left[\frac{R_{\text{interbank},t} - R_t}{R_t} \right] = \frac{\lambda}{\alpha_t} \frac{\mu_t}{1 - \mu_t}.$$

Equation (27) follows from the fact that $\frac{\alpha_t}{\lambda}$ equals financial leverage when $\mu_t > 0$. \square

The data used to construct the time series for the Lagrange multiplier are the following:

Interbank spread: The prime interbank rate is the Euribor rate on loans with a duration of one week. From 2004 onward, this is computed as the average rate for Italian banks on the Euribor panel. Prior to 2004, there is no information on individual banks, and $R_{\text{interbank},t}$ is measured as the average rate across all the banks in the panel. For the computation of the Lagrange multiplier, R_t is the prime rate for interbank secured transactions (Eurepo) on loans with a duration of one week. The rates are monthly (2002:M4-2012:M12), and they can be downloaded at <http://www.euribor-ebf.eu/euribor-org/euribor-rates.html>. Quarterly rates are obtained by compounding the monthly information.

Financial leverage: I use the Italian quarterly flow of funds (*Conti Finanziari*) to obtain a time series for total financial assets and total equity of the financial sector. First, I match

²⁷Note that because bankers are indifferent between buying and selling these claims, the introduction of this market does not alter the dynamics of the model.

banks in the model with monetary and financial institutions (MFIs).²⁸ Second, I construct a time series for bank equity as the difference between total financial assets and total debt liabilities. This latter is defined as total liabilities minus shares and other equities (liabilities) and mutual fund shares (liabilities). Financial leverage is the ratio between total assets and equity. Data can be downloaded at <http://bip.bancaditalia.it/4972unix/>.

B.4 Bank Lending Survey

The indicators of credit tightening and bank funding constraints are constructed from responses to the Bank Lending Survey of the euro area by loan officers at Italian banks. Specifically, I use the responses to question 1 and question 2 in the survey. Data are quarterly (2003:Q1-2012:Q4) and they can be downloaded at <https://www.ecb.europa.eu/stats/money/surveys/lend/html/index.en.html>.

Question 1: "Over the past three months, how have your bank's credit standards as applied to the approval of loans or credit lines to enterprises changed?" Respondents have five options: "Tightened considerably" (1), "Tightened somewhat" (2), "Remained basically unchanged" (3), "Eased somewhat" (4) and "Eased considerably" (5).

The indicator of credit tightening is the percentage of respondents that indicated either (1) or (2) less the percentage of respondents that indicated either (4) or (5). A value of 100% means that all the respondents in the survey indicated that credit standards have tightened considerably or somewhat in the preceding quarter. The indicator is de-meant and standardized.

Question 2: "Over the past three months, how have the following factors affected your bank's credit standards as applied to the approval of loans or credit lines to enterprises changed (as described in question 1)? Please, rate the contribution of the following factors to the tightening or easing of credit standards." Respondents have nine options, divided into three categories. Category A ("Cost of funds and balance sheet constraints"): "cost related to your bank's capital position" (1), "Your bank's ability to access market financing (e.g., money or bond market financing, incl. true-sale securitization)" (2), "Your bank's liquidity position" (3). Category B ("Pressure from competition"): "competition from other banks" (4), "competition from non-banks" (5), "competition from market financing" (6). Category C ("Perception of risk"): "expectations regarding general economic activity" (7), "industry or firm-specific outlook" (8), "risk on the collateral demanded" (9).

²⁸This category includes commercial banks, money market funds, and the domestic central bank. I use balance sheet information for the Bank of Italy to exclude the latter from this pool.

The indicators of banks' funding constraints are the percentage of respondents who replied that factors (1), (2), or (3) contributed "considerably" or "somewhat" to a tightening of credit less the percentage of respondents who indicated that these same factors contributed "considerably" or "somewhat" to an easing of credit. In the same vein, I obtain indicators for the remaining six factors. These variables de-meanded and standardized.

Table A-2 reports the results of a regression of each of these indicators on the measured Lagrange multiplier.

Table A-2: **Bank Lending Survey and the Lagrange multiplier**

Dependent variable	$\frac{\mu_t - \bar{\mu}_t}{\sigma(\mu_t)}$	R^2
Credit standards	0.65 (5.43)	0.42
Capital position	0.45 (3.44)	0.21
Ability to access to market finance	0.73 (2.63)	0.54
Liquidity position	0.64 (2.12)	0.41
Competition form banks	0.25 (2.49)	0.06
Competition from non-banks	-0.14 (-0.62)	0.02
Competition from market	-0.26 (-0.91)	0.07
Expectations of economic activity	0.33 (1.92)	0.11
Industry or firm-specific outlook	0.13 (0.90)	0.01
Risk on collateral demanded	0.19 (1.14)	0.02

Notes: Each row reports the results of an univariate regression of the indicators constructed from the Bank Lending Survey on the (standardized) Lagrange multiplier. Intercepts are not reported. Robust t-statistics in parenthesis.

B.5 Other time series

Real gross domestic product: OECD Quarterly National Accounts, millions of national currency, volume estimates (reference year 2010), 1980:Q1-2012:Q4.

Real private final consumption expenditures: OECD Quarterly National Accounts, millions of national currency, volume estimates (reference year 2010), 1980:Q1-2012:Q4.

Real general government final consumption expenditures: OECD Quarterly National Accounts, millions of national currency, volume estimates (reference year 2010), 1980:Q1-2012:Q4.

Real gross capital formation: OECD Quarterly National Accounts, millions of euros, volume estimates (reference year 2010), 1980:Q1-2012:Q4.

Labor income share: EU KLEMS,²⁹ ratio of total labor compensation and value added, 1970-2007 (annual). Data can be downloaded at <http://www.euklems.net/>.

Worked hours: EU KLEMS, average numbers of hours worked per year by person engaged, 1970-2007 (annual). The series is scaled by $(24 - 8) \times 7 \times 52$. Data can be downloaded at <http://www.euklems.net/>.

C Evidence from the Cross Section of Italian Stock Returns

From equation (15), we can write expected excess returns on any asset j held by bankers as

$$\mathbb{E}[R_{j,t+1}^e] = b_0 + b_1\beta_j, \quad (28)$$

where $b_0 = \mathbb{E}\left[\frac{\lambda\mu_t}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}\right]$ is the mean of the liquidity premium, common to all assets,

$$\beta_j = \frac{\text{cov}\left(-\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}, R_{j,t+1}\right)}{\text{var}\left(\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]}\right)} \quad (29)$$

²⁹EU KLEMS is an industry-level, and productivity research project funded by the European Commission. The acronym stands for EU-level analysis of capital (K), labor (L), energy (E), materials (M), and service inputs (S).

is the *beta* of security j , an indicator of the risk of the asset, and $b_1 = \text{var} \left(\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]} \right)$ can be thought as the price for holding risk. These moments are taken with respect to the ergodic distribution of the model state variables.

This section explains how (b_0, b_1) are estimated using information from the cross section of stock returns and compares these estimates with corresponding indicators of liquidity and risk premia obtained from the structural model. Section C.1 discusses the data, Section C.2 describes the procedure, and Section C.3 presents the results. The online Appendix reports some sensitivity analysis.

C.1 Data

Data on Stocks: End-of-month returns for stocks traded on the Italian exchange are obtained from Compustat Global Security Daily. I select stocks whose country of incorporation is Italy. For each of these stocks, I obtain the following information: global company key, company legal name, SIC code (*SIC*), shares outstanding (*CSHOC*), trading volume (*CSHTRD*), closing price (*PRCCD*), daily total return factor (*TFRD*), adjustment factor (*AJEXDI*), end-of-month indicator. The returns for security j in month t are defined as

$$R_{j,t} = \log \left(\frac{\frac{PRCCD_{j,t}}{AJEXDI_{j,t}} TFRD_{j,t}}{\frac{PRCCD_{j,t-1}}{AJEXDI_{j,t-1}} TFRD_{j,t-1}} \right).$$

I construct $R_{j,t}$ for each stock in the sample for the period 1999:M1-2011:M12, and I drop securities that have missing observations over the sample period. This leaves 116 stocks. I further restrict the sample by discarding securities whose average trading volume falls in the bottom 10th percentile. As is customary in the literature, I aggregate these stocks into a smaller number of portfolios. I consider two procedures:

- *Industry and Size.* Stocks are partitioned by industry into *Manufacturing* ($SIC \leq 30$), *Services* ($SIC \in [40, 60)$ and > 70), and *Financial* ($SIC \in [60, 70)$). Within each group, stocks are sorted by quintiles of the size distribution, obtaining 15 portfolios. Returns on the portfolios are value weighted.
- *Beta.* First, the beta of each stock is estimated via a time series regression (see below for details). Second, stocks are partitioned by the deciles of the estimated distribution of the betas, obtaining 10 portfolios. Returns on the portfolios are value weighted.

Quarterly returns are obtained by compounding the monthly series.

Risk Free Rate: The risk-free rate is matched with the yields on zero-coupon German government securities with a residual maturity of 6 months (Nelson-Siegel-Svensson method). The data are obtained from the Deutsche Bundesbank at http://www.bundesbank.de/Navigation/EN/Statistics/Time_series_databases/Macro_economic_time_series/its_list_node.html?listId=www_s140_it03a

Stochastic Discount Factor: I construct a quarterly time series (1999:Q1-2011:Q4) for $\hat{\Lambda}_{t,t+1}$ using equation (21) and the growth rate of real private final consumption expenditures (Δc_{t+1}), the financial leverage indicator (lev_{t+1}), and the posterior mean of (β, ψ, λ) .

C.2 Two-Pass Estimation

The estimation of (b_0, b_1) is accomplished using a two-pass procedure. First, the beta of each portfolio is estimated via time series regressions. Specifically, I consider a first-order log-linear approximation to the stochastic discount factor

$$\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]} \approx 1 + \log(\hat{\Lambda}_{t,t+1}) - \mathbb{E}_t[\log(\hat{\Lambda}_{t,t+1})] = 1 + e_{\hat{\Lambda},t+1},$$

and I obtain the innovations $\{e_{\hat{\Lambda},t+1}\}$ by fitting an AR(1) process to $\log(\hat{\Lambda}_{t,t+1})$. The beta defined in equation (28) is estimated as the slope coefficient of a linear projection of $R_{j,t+1}$ on $e_{\hat{\Lambda},t+1}$ for portfolio j .

Second, after averaging $\{R_{j,t}^e\}$ over time, I estimate (b_0, b_1) via a cross-sectional regression of average excess returns on their respective betas. In the benchmark specification, I pool the two sets of portfolios to obtain 25 observations in this cross-sectional regression. Moreover, I restrict the estimation to the 1999:Q1-2007:Q4 period in order to minimize the impact of the extremely low excess returns observed during the crisis on the sample averages. The online Appendix check the sensitivity of the results to these choices.

C.3 Results

The first column of Table A-3 reports OLS estimates for the cross-sectional regression (28). The remaining columns report estimates for alternative specifications of equation (28) that differ in the "factors" used to calculate the betas of the portfolios. Columns 2 and 3 estimate two benchmarks in the empirical asset-pricing literature, the capital asset pricing

model (CAPM) and the three-factor model of Fama and French (1993).³⁰ In column 4, the betas are calculated with respect to a consumption-based version of $\hat{\Lambda}_{t,t+1}$ obtained by setting ψ to 0 in equation (21).³¹ The bottom panel of the table reports diagnostics for the cross-sectional regressions: the adjusted R^2 , confidence interval for this statistic, the mean absolute pricing error (in annualized percentages) and a test statistic for the null hypothesis that the pricing errors are jointly zero with its associated p -value. Test statistics and p -values are computed using the method of Shanken (1992) that corrects for the uncertainty associated with the estimation of the betas in the first step of the procedure. Confidence intervals on the adjusted R^2 are computed by bootstrap, following Lewellen, Nagel and Shanken (2010).

Regarding the estimates, note that the fit of the model outperforms that of the CAPM and of the three-factor model of Fama and French (1993). For example, the first specification achieves an R^2 of 46%, while these two benchmark asset pricing models achieve, respectively, 20% and 26%. Moreover, a comparison of the first and the fourth column reveals that the leverage component in $\hat{\Lambda}_{t,t+1}$ is important for pricing the cross-section of stock returns: the consumption based version of $\hat{\Lambda}_{t,t+1}$ achieves an R^2 of only 15%.

The point estimate of b_0 suggests small liquidity premia on average, 11 basis points in annualized terms. However, this coefficient is not significantly different from zero. The point estimate of b_1 implies an annualized premium of roughly 14% for an asset with a $\beta_j = 1$. It is interesting to compare the estimated b_1 with comparable indicators in the structural model. Table A-4 reports $\text{var}_t \left(\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]} \right)$ under different conditioning sets, along with other statistics that are relevant to understand its behavior. Specifically, the table reports the expected value of bankers' marginal value of wealth conditional on the constraints binding next period, and the conditional standard deviation of the two components of the stochastic discount factor: the marginal value of wealth of the banker and aggregate consumption growth.

The first column of Table A-4 reports these statistics when the state vector is at the ergodic mean. We can verify that the price of risk is small in this region of the state space. This happens because of two reasons. First, at the ergodic mean bankers have a buffer stock of wealth that insures them against regular fluctuations in the price of capital. Second, the probability of a government default next period is small ($p^d = 0.001$). Hence, balance sheet risk for financial intermediaries is negligible, and the model does

³⁰The "factors" for the CAPM and the three-factor model of Fama and French are downloaded from Stefano Marmi data library at http://homepage.sns.it/marmi/Data_Library.html.

³¹For these three specifications, the second set of portfolios is generated by sorting stocks according to their betas with the factors considered. For the three-factor Fama and French specification, the stocks are sorted with respect to their beta with the market return.

Table A-3: OLS estimates of equation (28)

	Benchmark	CAPM	Three-factor FF	No Leverage
Intercept	0.11 (0.75)	10.68 (0.25)	6.38 (0.87)	3.81 (0.37)
$\beta^{\hat{\Lambda}}$	14.11 (3.78)			
β^{MKT}		-9.29 (5.70)	-5.91 (5.71)	
β^{SMB}			1.67 (3.61)	
β^{HML}			7.05 (3.85)	
$\beta^{\hat{\Lambda}, \psi=0}$				-0.60 (0.27)
R^2	0.46	0.20	0.26	0.15
L.B. R^2	0.31	0.02	0.02	0.02
MAPE	4.65	4.55	3.60	4.84
$T^2(\chi^2)$	17.16	22.32	21.75	21.01
p-value	0.84	0.56	0.47	0.64

Notes: The dependent variable is the sample mean of annualized excess returns for the 25 portfolios. The first column estimates equation (28). The remaining columns estimate, respectively, the CAPM, the three-factor model of Fama and French, and a specification of equation (28) where ψ is set to 0 in equation (21). The bottom panel reports test diagnostics, including R^2 , the lower bound on an 80% confidence interval for the sample R^2 , the mean absolute pricing errors (MAPE), and a χ^2 statistic that tests whether the pricing errors are jointly zero. Standard errors and p-values are computed using the method of Shanken (1992).

not generate a particularly volatile stochastic discount factor.

The model, however, can generate substantially larger values for $\text{var}_t \left(\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]} \right)$ in regions of the state space where the risk faced by intermediaries is not trivial. The second column of Table A-4 reports the same statistics when the likelihood of a sovereign default next period is 5%, holding fixed the other state variables at their ergodic mean values. Now bankers anticipate that they may face severe losses next period on their government bond holdings, and these losses may tighten their funding constraints. Moreover, the net worth losses for the bankers are large in those states of the world, implying a substantial increase in their marginal value of wealth: from the second row of Table A-4 we can verify that the mean of α_{t+1} conditional on the constraints binding next period increases from 1.04 to 1.31. This explains why the conditional volatility of α_{t+1} is high in this region of

Table A-4: Price of risk in the model

	Ergodic mean	$p_t^d = 0.05$	$p_t^d = 0.05$ and low N_t	Filtered states
$400 \times \text{var}_t \left(\frac{\hat{\Lambda}_{t,t+1}}{\mathbb{E}_t[\hat{\Lambda}_{t,t+1}]} \right)$	0.23	4.67	22.06	3.98
$\mathbb{E}_t[\alpha_{t+1} \mu_{t+1} = 1]$	1.04	1.31	1.33	1.13
$100 \times \sigma_t(\alpha_{t+1})$	2.65	12.44	33.19	11.55
$100 \times \sigma_t(\Delta c_{t+1})$	0.46	0.61	0.63	0.48

Notes: The conditional moments are computed by simulation. For the first column, simulate $M = 1,500$ realizations for $\{\hat{\Lambda}\}$, $\{\alpha\}$, $\{\mu\}$, $\{\Delta c\}$. Each simulation is initialized at the ergodic mean of the state vector and has length 2. The moments are computed using the second period of these simulations. The remaining columns repeat this procedure, but simulations are initialized at a different point of the state space. The second column initializes the simulations as follows: i) s -shock is set so that $p_t^d = 0.05$, and ii) the other state variables are set at their ergodic mean. The third column initializes the simulations as in the second column, with the exception that P is set so that net worth is 3 standard deviations below average. The fourth column initializes the simulations at the mean of the state vector under $p(\mathbf{S}_t | \mathbf{Y}^{2011:Q4}, \bar{\theta})$, computed with the particle filter.

the state space: bankers now place more mass on the right tail of the α_{t+1} distribution, and this implies that its conditional distribution spreads out. The higher volatility of α_{t+1} translates into a higher conditional volatility for the stochastic discount factor: the price of risk is now roughly 5% in annualized terms. Interestingly, the last two rows of the table clarifies that the asset-pricing properties of the model are mostly driven by α_{t+1} rather than by the behavior of consumption growth. This feature of the model is consistent with the result, discussed earlier, that the leverage component of $\hat{\Lambda}_{t,t+1}$ is essential for pricing the cross section of Italian stock returns. Note that the balance sheet risk implied by the prospect of a sovereign default depends on the other state variables on the model. For instance, column 3 shows that the price of risk reaches 22.06% once we condition on $p_t^d = 0.05$ and on very low initial values of bankers net worth (3 standard deviations below average). These examples indicate that the model is capable of generating values for the price of risk in the range of those estimated using cross-sectional information on stock returns.

The last column of Table A-4 reports these calculations for a region of the state space that is empirically relevant for the Italian economy during our sample. Specifically, I use the particle filter to estimate the state of the Italian economy in 2011:Q4, and I compute the above statistics conditioning on these states.³² In this region of the state space, the model produces a price of risk of 3.98%, roughly one third of the estimated value in Table A-3.

³²See Section 5.2 for details on the filtering procedure.