



## **Deconstructing Delays in Sovereign Debt Restructuring**

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### ABSTRACT

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Negotiations to restructure sovereign debt are time consuming, taking almost a decade on average to resolve. In this paper, we analyze a class of widely used complete information models of delays in sovereign debt restructuring and show that, despite superficial similarities, there are major differences across models in the driving force for equilibrium delay, the circumstances in which delay occurs, and the efficiency of the debt restructuring process. We focus on three key assumptions. First, if delay has a permanent effect on economic activity in the defaulting country, equilibrium delay often occurs; this delay can sometimes be socially efficient. Second, prohibiting debt issuance as part of a settlement makes delay less likely to occur in equilibrium. Third, when debt issuance is not fully state contingent, delay can arise because of the risk that the sovereign will default on any debt issued as part of the settlement.

Keyword: Sovereign debt, sovereign default, bargaining, delay.

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# 1. Introduction

Sovereign countries occasionally default on their debts. When they do, their debts to creditors have to be renegotiated. This renegotiation process often takes a great deal of time to resolve; numerous authors, surveyed in Tomz and Wright (2013), find that the average time from an initial default to a final negotiated settlement is around eight years. Since the widely held belief is that these delays are socially costly, policy makers have been actively discussing proposals for speeding up the sovereign debt restructuring process (see, for example, International Monetary Fund 2013).

In order to evaluate alternative proposals for sovereign debt restructuring reform, it is necessary to take a stand on the source of these delays in bargaining. One class of theories that have proven promising in the context of quantitative theoretical models of sovereign debt and default postulates that bargaining between the sovereign and a representative creditor occurs under complete information. In these models, delay occurs in equilibrium when the creditor and debtor jointly find it optimal to delay reaching an agreement because they expect to obtain higher payments in the future. Thus, in these models there is a sense in which delay is *privately optimal* for the sovereign debtor and its creditor. But if so, it is not clear whether delay allows any room for policy intervention that improves social welfare. Moreover, the models in the literature differ along a number of subtle dimensions, which inhibits understanding of both the causes of delay and the effect these differences have on the social efficiency of the negotiated debt restructuring outcomes.

Toward a clearer understanding of both the causes of delays in bargaining and their possible social inefficiency, in this paper we present a framework within which variants of a number of the alternative models in this literature can be nested and studied. We then proceed by simplifying the environment to highlight the key assumptions that drive results in different models. Using these simplified environments, we illustrate how, for different parameter values and different assumptions on the bargaining process, models in the framework can produce equilibrium delay that is either socially efficient or socially inefficient, all the while being privately efficient from the narrow perspective of the bargaining parties themselves. We also illustrate how different assumptions on the nature of bargaining, and in particular on the

objects over which the parties bargain as well as the debtor's ability to commit to honoring a bargain, can result in bargaining outcomes that differ significantly in terms of both payoffs and efficiency outcomes.

In all of the examples below, an important assumption will be that output in the bargaining period is low in the sense that it is costly for the debtor to transfer large amounts of output to its creditors. However, this alone is not enough to generate delay. In addition, some combination of other assumptions must be present. We emphasize three additional key modeling assumptions. The first assumption concerns the extent to which delay in bargaining has a permanent effect on economic activity in the defaulting country. Under the common assumption that delay has only an immediate effect on economic activity, delay is less likely to result in equilibrium. Departing from this assumption by, for example, assuming that delay results in permanently higher or lower levels of economic activity can lead to delay under quite general assumptions. In the case in which future economic activity is increased by delay, we show that there is also an intuitive sense in which delay can be *socially efficient* as well as privately efficient.

In the rest of the paper, we focus on cases in which being in default has only a contemporaneous impact on economic activity. In such models, delay is *socially inefficient* in the intuitive sense that it both reduces economic activity and hinders the exploitation of gains from intertemporal trade. Even so, delays in bargaining can still be privately efficient—and hence still occur in equilibrium—if the bargaining process is restricted in ways that limit the ability of the debtor to transfer resources to creditors in the future. Specifically, the second key assumption that we identify concerns whether the debtor is allowed to issue new debt as part of a debt restructuring. When debt issuance at the time of bargaining is prohibited, so that the debtor can only make a transfer out of current resources, delay can occur in equilibrium when the amount of current resources available for transfer is expected to rise in the future. However, when debt issuance is allowed, equilibrium delay does not occur even when future resources are expected to increase as the debtor simply issues more debt as part of a settlement without delay.

The third key assumption concerns whether the debtor is able to issue *state-contingent* debt as part of the settlement. If fully state-contingent debt is prohibited, then a creditor

may prefer to delay a settlement in order to wait for enough uncertainty to be resolved so that it can negotiate a fully state-contingent transfer in the future, rather than reaching agreement early on a settlement that includes debt with a repayment amount that does not vary sufficiently with future states. In such cases, it is the risk that the debt issued as part of a settlement will be defaulted upon that itself produces delay. To see this, consider the case in which debt is state-noncontingent, and suppose that the creditor and debtor were to agree on an early settlement. In such a case, the parties must choose between issuing a small amount of state-noncontingent debt that is likely to be repaid in all states in the future (and hence is highly valuable to creditors), or issuing a larger amount of state-noncontingent debt that is less likely to be repaid in full and hence has less value (per unit of debt) to creditors. In this sense, the risk of default on the state-noncontingent debt serves to reduce the value of immediate settlement. Delay, on the other hand, allows for the debtor and creditor to negotiate a fully state-contingent payment in the future, which can yield a higher expected return to creditors.

Our paper is connected to the literature as follows. Like all of the papers we review, our framework owes a debt to the seminal contribution of Merlo and Wilson (1995), who show how equilibrium delay may arise in a stylized bargaining framework in which agents have linear preferences over shares of an exogenous “pie” that is to be distributed among them. In their framework, delay is both socially (and privately) efficient whenever the pie is expected to grow fast enough over time. In contrast to their stylized framework, and in order to make contact with the sovereign debt literature, we consider an environment in which the pie to be distributed is determined endogenously from the potential for intertemporal trade between a risk-neutral set of creditors and a risk-averse debtor country that receives a flow payoff while bargaining continues without agreement.

The first paper to mention the possibility that the Merlo and Wilson (1995) model might be used to explain delays in sovereign debt restructuring was by Merlo and Wilson themselves (Merlo and Wilson 1998). In a series of papers, Marcus Miller and his coauthors began the task of interpreting sovereign debt restructuring through the lens of this model. Miller and García-Fronti (2005a) take the exact Merlo-Wilson (1998) model and calibrate parameter values to match the Argentine restructuring. This argument is further developed

in Dhillon et al. (2006). In both papers, the size of the pie is calibrated to match public statements by the parties on their bargaining positions as to the size of the recovery rate on the defaulted debt. Importantly, their analysis assumes that an early settlement locks in a permanently lower level of output, whereas delay generates a permanently higher level of output. That is, delay arises because of the assumption that delay has a permanent effect on economic activity (the first assumption we emphasized above and that we analyze in Section 3). The same basic arguments underlie Miller and García-Fronti (2005b). In all of these papers, the bargaining environment remains somewhat abstract, with a pie to be bargained over that evolves exogenously.

In contrast with these earlier papers, Benjamin and Wright (2008) and Bi (2008) were the first to endogenize the size of the pie and the nature of its variation by placing the bargaining framework within the benchmark model of sovereign borrowing. Although similar in scope, the papers differ in a great many respects. Most germane for the current purposes are the fact that Bi (2008) assumes that the parties bargain over a transfer of current resources and do not issue new debt as part of settlement (the second assumption that we emphasize above and which we examine in Section 4), and that bargaining power is constant through time (which we examine in Section 5). The assumption that the parties bargain only over a current transfer of resources is also made by Yue (2010), albeit in the context of a static Nash bargaining model. Our paper is most closely related to that of Benjamin and Wright (2008); in addition to providing a series of examples to illustrate the mechanism at work in their general framework, the current paper also expands on their analysis by emphasizing the importance of allowing borrowing and by illustrating the effect of allowing delays to have permanent effects on output.

Other approaches to modeling delays in sovereign debt restructuring also exist. One possibility is that delay serves to signal private information (see the work surveyed by Ausubel, Cramton, and Deneckere 2002). Applications of this approach to sovereign debt restructuring include Ghosal and Miller (2006) and Bai and Zhang (2012). Another is that delay is caused by collective action problems among creditors, which has been studied by Pitchford and Wright (2008, 2012, 2017) among others. All of these theories are complementary to the one studied in this paper.

The rest of this paper is organized as follows. Section 2 outlines the general environment, which is later specialized to emphasize different aspects of bargaining, and describes different notions of efficiency in bargaining. Section 3 examines the delays that arise when delay is assumed to have permanent effects on output. This section also discusses the notion of socially efficient and inefficient delays, while the rest of the paper examines cases in which delay is socially inefficient although privately efficient. Section 4 examines the role of the assumption that the agents bargain only over current transfers (as in Yue 2010 and Bi 2008) as opposed to bargaining over debt issuance (as in Benjamin and Wright 2008). Section 5 then examines the role of uncertainty and incompleteness in debt markets in producing delay when agents can bargain over debt issues, while Section 6 concludes.

## 2. A General Environment

In this section, we outline some features of our environment that are common across all of the examples we consider below. This approach also serves to set notation and focus attention on the key modeling assumptions that drive the differing results below. Specifically, consider the problem of a sovereign country that begins in default on a debt  $b_{-1}$ . Time evolves discretely, is indexed by  $t$ , and goes on forever so that  $t = 0, 1, 2, \dots$ . The infinite horizon assumption is common in the literature. However, it will be convenient to work with environments that have an effective finite horizon, and below we will make assumptions to ensure that play effectively concludes after some terminal period  $T$ . We assume that the initial level of defaulted debt  $b_{-1}$  is sufficiently large that it will never be repaid and hence does not constrain negotiations over its restructuring.<sup>1</sup>

The debtor country has nonlinear preferences over state-contingent sequences of consumption  $c_t$  ordered by

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t) \right],$$

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<sup>1</sup>The assumption that the debt is large rules out cases in which the creditor can extract a large repayment on a small level of debt and hence serves to bound recovery rates above at 100%. Benjamin and Wright (2008) consider a model in which the sovereign has the “outside option” to always repay the debt in full, which plays a similar role. Other authors (such as Bi 2008) assume that agents bargain over the proportion by which the face value of the debt is reduced in default, which is directly bounded above by one.

where the expectation operator conditions over information available at the end of time zero or, equivalently, at the beginning of time one. We restrict consumption to be non-negative, which serves to place bounds on the amount of resources that can be transferred to the creditor in any period, and assume that  $u$  is strictly increasing, strictly concave, and unbounded below; these assumptions are satisfied by the constant relative risk aversion class of period-utility functions with a coefficient weakly greater than one. Note that the assumption that preferences are non-linear and that there is a flow payoff to the debtor country while bargaining continues are departures from the Merlo and Wilson (1995) framework.

The debt is owed to a set of risk-neutral creditors who are assumed to be able to perfectly coordinate and bargain as one entity with the debtor country; this is a point of departure from the models of delays in sovereign debt restructuring negotiations that focus on coordination problems among creditors, such as Pitchford and Wright (2008, 2012, 2017). The representative creditor values state-contingent sequences of transfers  $tr_t$  from (and to, if negative) the country according to

$$E_0 \sum_{t=0}^{\infty} q^t tr_t.$$

We assume that  $\beta = q$  so that there are no gains from trade from tilting consumption. Hence, if there were no frictions from default and bargaining over a settlement, the country would enjoy a constant state-noncontingent level of consumption. This is a departure from the sovereign debt literature that typically assumes that the country is more impatient than the creditor, or  $\beta < q$ , in order to generate long-run borrowing in equilibrium (e.g. Arellano 2008, Aguiar and Gopinath 2006, and Tomz and Wright 2007). In our framework, an assumption of a rising output profile (on average) over time plays a similar role in ensuring that our debtor country has a desire to borrow.

As emphasized in the introduction, one of the key assumptions driving the results in the literature is the extent to which economic activity in the debtor country  $y_t$  is assumed to vary with past default and repayment decisions (and hence with equilibrium delay). Another key assumption concerns the degree of state contingency in debt contracts, and hence also the amount of uncertainty in the environment. We adopt notation that is flexible enough to



capture a range of possibilities. Specifically, for each  $t$ , we denote the state of the economy by  $\theta_t = (z_t, s_t)$ . Here,  $z_t$  is an exogenous state taking on a finite number of values that determines the amount of output available to the country  $y_t$  as well as the conduct of bargaining to be described below. This exogenous state is assumed to be Markov with the probability of observing state  $z_t$  at time  $t$  given state  $z_{t-1}$  in the previous period, given by  $\pi(z_t|z_{t-1})$ . The term  $s_t$  takes on values in  $\{n, d\}$  and serves to index whether the country *enters the period* in default  $d$  or not  $n$ . The initial state  $\theta_0 = (z_0, s_0)$  is given, with our assumption that the debtor begins in default pinning down  $s_0 = d$  and the value  $z_0$  parameterizing the initial expectation operator. A history of the economy up to and including the beginning of time  $t$  is therefore denoted by

$$\theta^t = (z^t, s^t) = (\theta_0, \theta_1, \dots, \theta_t) = ((z_0, s_0), (z_1, s_1), \dots, (z_t, s_t)).$$

The state for the debtor country at the start of a period is given by the state of the economy plus the level of outstanding debt  $b_{t-1}$  determined in the previous period.

In general, we can allow for the level of economic activity in the defaulting economy to depend on the entire history of the economy, plus the default decision in the current period, or  $y_t = y(\theta^t, s_{t+1})$ . Note that  $s_{t+1}$  is the default status at the start of period  $t+1$  and hence reflects the default or settlement decision made in period  $t$ . For simplicity, we will assume that output depends on the exogenous state only through its contemporaneous realization  $z_t$ , so that  $y_t = y(\theta^t, s_{t+1}) = y(z_t, s_{t+1})$ . This specification is still rich enough to capture the possibility that default and delay in bargaining in the past result in higher output in the future; this might be thought of capturing (in a reduced-form way) the possibility that protracted debt negotiations allow the country time to undertake productivity-enhancing reforms. Alternatively, it is also general enough to capture the possibility that protracted negotiations have progressively higher economic costs. We consider both cases in Section 3. The quantitative sovereign default literature, almost without exception, focuses on the much simpler case in which output depends only on whether the country is in default during the period, so that  $y(\theta^t, s_{t+1}) = y(z_t, s_{t+1})$ . In this case, an output penalty in default that is

paid only if the country defaults in the period is represented by

$$y(z_t, d) < y(z_t, n).$$

Note that transfers to creditors are simply the difference between the debtor country's output and consumption.

As noted above, it will simplify matters substantially to be able to truncate all interesting economic decisions after a finite amount of time  $T$ . We do this by assuming an initial value  $z_0$  and a transition matrix  $\pi$  such that, after  $T$  periods, the economy converges to an absorbing state  $z^*$  where

$$y(z^*, s^{t+1}) = y(z^*, s^t) = y^*,$$

for all  $t \geq T - 1$  and all  $s^{t+1}$ . As the state is absorbing, and as  $\beta = q$ , there are no further gains from intertemporal trade beyond period  $T$ . Moreover, as there is also no default penalty (output levels are the same regardless of the past history of default encoded in  $s^{t+1}$ ), a debtor country will always default (there is no cost in terms of output or lost capital market access), while a country in default will never settle. (there is nothing to be gained in terms of additional output or capital market access). This approach allows us to treat this infinite horizon bargaining and borrowing model as if it has a finite horizon. In the cases examined below,  $T$  will equal 2, except in the last case when we consider fluctuations in bargaining power.

The remaining details of the model concern the nature of bargaining when a country is in default and the nature of borrowing when a country has settled its default and is able to borrow again. As the precise details here will differ in each of the cases examined below, we elaborate on them in greater detail as we go along. However, certain aspects remain constant. When it comes to bargaining, we assume that the creditor makes a take-it-or-leave-it offer to the country in the first period. After the first period, the probability that the creditor makes the offer is governed by the exogenous state  $z_t$ . That is, we have a stochastic alternating offer bargaining model, although with the exception of the final example, we will assume that this model takes on a trivial form (expectations about who makes offers in the future determine

bargaining power today, and the last example will consider fluctuations in bargaining power).

When it comes to borrowing, we consider borrowing that is state-noncontingent except for the possibility of default. Default is deterred both by exclusion from financial markets (a country that defaults spends a period in autarky before it can bargain with creditors over a settlement and the right to borrow again) and by reductions in output while in default. In Sections 3 and 4, we will focus on cases in which the environment is deterministic so that default does not occur in equilibrium, before returning to cases with uncertainty and possible equilibrium default in Section 5.

### **3. Persistent Effects of Delay on Output**

We begin by focusing attention on assumptions about the extent to which past default and delays in bargaining influence future economic activity in the debtor country. We abstract from the other assumptions of note by assuming that the environment is deterministic and by allowing the debtor to issue debt as part of a settlement. In the first subsection, we present the computation of the equilibrium bargain in detail; in succeeding subsections, we proceed with greater brevity.

#### **A. Delay Increases Output after Future Settlement**

The easiest way to produce delays in bargaining is to make these delays attractive to society as a whole; this was the point of Merlo and Wilson (1995). This process is not straightforward in the context of a sovereign debt restructuring model that satisfies the properties of the above framework, since delay in settling a debt means that there is delay before trade can occur; hence, potential gains from trade between the sovereign and its creditors resulting from the sovereign's desire to smooth its consumption over time are inevitably lost. Furthermore, a common assumption is that output is also reduced during a default, which results in further social losses. However, if the modeler is prepared to assume that longer delays lead directly to higher output, then delay can become attractive for society. This assumption might not be unreasonable if we think that longer delays result in crises that lead to structural economic reforms that increase future output.

To see this, consider the following example, which specializes the general environment above in several ways. We make three assumptions for simplicity. First, we assume that there is no uncertainty and that the transition matrix takes a form that reaches the absorbing state by  $T = 2$ . This will be the case if

$$\pi(z^*|z_1) = \pi(z_1|z_0) = 1,$$

where the subscript is doing double duty indexing both states and dates. Second, we assume that the creditor makes a take-it-or-leave-it offer in every period. Third, we assume that the output cost of default in period 1 is a simple proportional reduction  $\gamma$  in output, so that

$$\begin{aligned} y(z_1, (d, d, d)) &= (1 - \gamma) y(z_1, (d, d, n)), \\ y(z_1, (d, n, d)) &= (1 - \gamma) y(z_1, (d, n, n)). \end{aligned}$$

The key assumption for this example is that, following a delay in period 0, output is higher in period 1 than it would be if the sovereign and creditor had immediately settled in period 0. This would appear to capture the ideas of Miller and García-Fronti (2005a,b) and Dhillon et al. (2006), who present very stylized models close to those originally proposed by Merlo and Wilson (1995), and in which an early settlement locks in a permanently lower level of output, whereas delay generates a permanently higher level of output. Moreover, in order to ensure that the debtor has a strong desire to borrow, we assume that the endowment process is increasing over time. Specifically, we assume

$$y(z_0, (d, n)) < y(z_1, (d, n, n)) < y(z_1, (d, d, n)) < y^*.$$

In words, if the debtor does not settle in period 0 but does settle in period 1, it receives  $y(z_1, (d, d, n))$ , which is larger than what it would have received had it settled in period 0, borrowed and repaid that debt in period 1, or  $y(z_1, (d, n, n))$ . Both of these output levels in period 1 are higher than output in period 0.

To see the effect of these assumptions, we solve the model working backward from the (effectively) terminal period 2. As is hopefully clear from the assumptions and discussion

above, a country in period 2 has nothing to gain from participating in international financial markets; it will simply consume its endowment. Moreover, as there are no penalties for default, a country in default will never settle with its creditors, while a country with debt will never repay. Hence, any debts  $b$  issued in period 1 that come due in period 2 will have a price of zero in period 1, or  $p_1(b) = 0$ . The value to being a country in period 2 is therefore given by

$$V(z^*, (d, s_1, s_2)) = \frac{u(y^*)}{1 - \beta} \equiv V^*,$$

regardless of default and settlement behavior in previous periods 0 and 1, and regardless of the decision to default or settle in period 2.

Working backward to period 1, there are two possibilities. One is that the country did not settle its default on debt with creditors in period 0. In this case, the problem of the creditor (which always makes the offer in this example) is to offer a settlement comprising a current transfer from the debtor  $\tau_1$  and a new debt issue  $b_1$  that maximizes its own payoff subject to the offer that delivers the debtor at least as much utility had it rejected the offer. Formally, this offer solves

$$\max_{\tau_1, b_1} \tau_1 + b_1 \times p_1(b_1),$$

subject to

$$u(y(z_1, (d, d, n)) - \tau_1) + \beta V^* \geq u(y(z_1, (d, d, d))) + \beta V^*.$$

As  $p_1(b) = 0$  for all  $b$ , the choice of debt is irrelevant, and we set it to zero. The settlement in period 1 thus takes the form of a transfer of current resources of size

$$\tau_1^* = y(z_1, (d, d, n)) - y(z_1, (d, d, d)) = \gamma y(z_1, (d, d, n))$$

under our assumptions. The value to the country of entering period 1 without a settlement

is therefore

$$V(z_1, (d, d)) = u(y(z_1, (d, d, d))) + \frac{\beta}{1-\beta} u(y^*).$$

The second possibility is that the debtor country settled its default in period 0 and issued debt of  $b_0$  as part of the settlement. The problem of the debtor country (the creditor has no decision to make as the debtor country is not in default) is whether to repay its debt. If it does not repay, it receives the default level of output and moves on to period 2 for a value of

$$u(y(z_1, d, n, d)) + \beta V^*.$$

If it does repay, it receives the higher level of output and can, in principle, borrow again. However, given our assumptions on period 2, creditors correctly anticipate that the country would never repay any new debt and will not buy any debt the country chose to offer. Hence, we set the debt issuance decision to zero, yielding the debtor country a payoff from the repayment of debt  $b$  of

$$u(y(z_1, d, n, n) - b) + \beta V^*.$$

Clearly, the debtor country will repay if and only if

$$b \leq y(z_1, (d, n, n)) - y(z_1, (d, n, d)) = \gamma y(z_1, (d, n, n)) \equiv b^*.$$

Knowing this, creditors in period 0 will buy debt up to but not exceeding  $b^*$ , generating a unit price for debt of  $p_0(b) = \beta$  for debt less than this level, and zero otherwise. The value to the debtor of ending up in period 1 with an amount of debt  $b$  is therefore

$$V(z_1, (d, n); b) = \begin{cases} u(y(z_1, d, n, n) - b) + \frac{\beta}{1-\beta} u(y^*) & b \leq b^* \\ u(y(z_1, d, n, d)) + \frac{\beta}{1-\beta} u(y^*) & b \geq b^*. \end{cases}$$

Finally, working backward to period 0, to find out whether the creditor wants to make

an offer that is accepted, we compute the best possible acceptable offer to the debtor. In this case, the problem of a creditor is to propose a settlement consisting of a current transfer of resources  $\tau_0$  and a new debt issue  $b_0$  to maximize the value of its payoff subject to the offer being accepted. Formally, this solves

$$\max_{\tau_0, b_0} \tau_0 + b_0 \times p_0(b_0),$$

subject to

$$u(y(z_0, (d, n)) - \tau_0) + \beta V(z_1, (d, n); b) \geq u(y(z_0, (d, d))) + \beta V(z_1, (d, d)).$$

Under our assumption that the period 0 endowment is low relative to the period 1 level, the optimal acceptable settlement sets  $b_0^* = b^*$  while  $\tau_0^*$  solves

$$u(y(z_0, (d, n)) - \tau_0^*) + \beta u(y(z_1, d, n, d)) \geq u(y(z_0, (d, d))) + \beta u(y(z_1, (d, d, d))).$$

When does delay occur? The creditor will make a settlement offer that the debtor will accept, rather than delay until period 1, if

$$\tau_0^* + \beta b^* = \tau_0 + \beta \gamma y(z_1, (d, n, n)) \geq \beta \tau_1^* = \beta \gamma y(z_1, (d, d, n)).$$

This result is generally ambiguous. On the one hand, the creditor extracts more resources in period 1 from a delayed settlement—due to the expanded economy—than it can extract in repayment of debts issued in period 0 ( $\tau_1^* > b^*$ ). On the other hand, a delayed settlement means that the creditor passes on extracting the payment  $\tau_0^*$  in period 0. However, if the period 0 endowment is close to zero, the default cost to output in period 0 is small, and/or the debtor country's utility function displays a lot of curvature in the neighborhood of the period 0 endowment,  $\tau_0^*$  will be small and the creditor will prefer to delay settlement.

In summary, delay will occur in equilibrium if output in period 0 is sufficiently low (so that  $\tau_0^*$  is small) and delay sufficiently expands the economy (so that  $b_0$  is small relative to  $\tau_1^*$ ).

## B. Output Losses in Default Grow with Delay

In the previous example, delay occurred because the creditor could extract more resources bargaining tomorrow than it could enforce in a debt repayment. This was because delay expanded the debtor country's economy, yielding more resources for the creditor to extract. Similar logic applies to the case in which the cost of remaining in default to the creditor increases with delay. In this case, even though *the size of the economy in the event of a settlement is the same*, the creditor can extract more resources by delaying than it can by enforcing a debt repayment because *delay reduces the size of the economy when the country is in default*.

To see this, suppose now that the output of the debtor country when not in default is the same regardless of the past history of delay, so that

$$y(z_1, (d, d, n)) = y(z_1, (d, n, n)),$$

but assume that output in default is lower if there was a delay in settlement so that

$$y(z_1, (d, d, d)) < y(z_1, (d, n, d)).$$

The analysis of this case proceeds in much the same way as for the previous example, except that under our new assumptions, we have

$$b^* = y(z_1, (d, n, n)) - y(z_1, (d, n, d)) < \tau_1^* = y(z_1, (d, d, n)) - y(z_1, (d, d, d)).$$

Although the formula differs, once again, the creditor can extract a greater transfer of resources from the debtor following delay than it can enforce in a debt repayment. The formula for the optimal  $\tau_0^*$  and the comparison of creditor payoffs from immediate settlement versus delay are analogous. As before, if the period 0 endowment is high, or utility is highly curved in the neighborhood of this endowment so that it is very costly for the country to make a payment of current resources, the creditor will once again prefer to delay settlement until period 1.



### C. The Social and Private Optimality of Delay

In both of the above examples, and indeed in all of the examples in this paper, agreement on a bargain to restructure a debt in a default is voluntary. Likewise, agreement to delay restructuring a debt is voluntary. Hence, we will refer to all equilibrium debt restructuring agreements negotiated in these models as privately efficient, and any resulting delay in equilibrium as *privately optimal delay*. An important question is whether these agreements can ever be described as socially efficient and hence as producing socially optimal delay. If so, there would appear to be no scope for beneficial intervention by another government or supranational institution.

The above examples provide a useful window on this question. Start with the example in Section 3.B. In that case, delay always results in lower levels of output for the country. Moreover, delay also resulted in potential gains from trade from allowing borrowing in period 0 to be wasted. In an intuitive sense, therefore, delay in this example is socially inefficient: there is *socially suboptimal delay*. More precisely, if a social planner intervened to force the parties to agree to a settlement in period 0, there would be potential for a Pareto improvement; with more output in the first period— $y(z_0, (d, n))$  is greater than  $y(z_0, (d, d))$ —and the same output in all other periods, there is potential for both creditor and debtor country consumption to be weakly increased in all periods. Whether an actual Pareto improvement can be achieved depends on whether the social planner has the power to coerce appropriate transfers from the debtor to the creditor. After all, the creditor prefers delay because it can extract more resources from the debtor following delayed settlement than it can extract in the form of a debt repayment from an early settlement. Whether there is room for a supranational institution like the International Monetary Fund (IMF) to improve social welfare requires taking a stand on what it is feasible for the IMF to achieve from its lending programs as constrained under its articles of agreement.

The social optimality of delay in the example in Section 3.A is generally ambiguous because in that case, delay increases equilibrium output. If a social planner were to force the creditor and debtor to settle in period 0, they would be forgoing the extra output in period 1. Indeed, it is possible to envisage a situation in which parameter values were such that

the gains from trade in period 0 were small and were dominated by the additional increase in output produced by delay so that we have *socially optimal delay* in addition to privately optimal delay. In this case, even if the social planner was able to coerce arbitrary transfers from the debtor to the creditor, there would be no potential for a Pareto improvement if the social planner were to force an immediate settlement. In this case, it is also clear that there is no potential for a supranational institution like the IMF to increase welfare in a Pareto sense by intervening to reduce or eliminate delay.

In the rest of this paper, we focus attention on cases in which delay has no persistent effect on output. In the examples that follow, delay will be socially suboptimal in the same intuitive sense introduced above. Specifically, we say that there is *socially suboptimal delay* if a settlement without delay would be *potentially Pareto improving* in the sense that a social planner with the power to coerce arbitrary transfers between the debtor and creditor could generate a strict Pareto improvement. We focus first in Section 4 on assumptions about debt issuance as part of debt restructuring negotiations in an environment with no uncertainty. In Section 5, we then introduce uncertainty and the possibility of default on debt issued as part of a restructuring.

## 4. Debt Issuance as Part of a Default Settlement

In what follows, we focus on cases in which delay has no persistent effect on output. Specifically, we assume that

$$y_t = y(z_t, s_{t+1}),$$

so that it depends on the exogenous state and the decision to default, repay, or settle in period  $t$  (which is encoded in  $s_{t+1}$ ). Nonetheless, we show that delay can arise in equilibrium if the ability of the debtor and creditor to bargain over future payments is limited in some way relative to their ability to bargain over current payments. In this section, we focus on the nature of bargaining, and in particular on whether the sovereign and its creditors can issue new debt as part of a settlement, as a determinant of delay. In practice, almost all debt settlements are accompanied by an issue of new debt. Nonetheless, a number of models

of delays in bargaining over sovereign debt restructuring explicitly exclude the parties from bargaining over debt issuance. We show that this assumption plays a significant part in generating delay in these models.

### A. Default Settlement without Debt Issuance

As noted above, a number of papers in the literature assume that the sovereign and its creditors bargain over a settlement that consists of only a transfer of current resources. This is a feature of the bargaining model of Yue (2010) and of the variant of the Merlo and Wilson framework studied by Bi (2008). In particular, Bi assumes that a transfer equal to some fraction of the face value of the defaulted debt is paid and that the sovereign then enters the next period with zero debt outstanding. In this subsection, we restrict attention to a deterministic environment and show that when debt issuance as part of a settlement is prohibited, delay occurs as long as output is increasing enough over time. In the next subsection, we show that delay disappears once borrowing is allowed. We then reintroduce uncertainty and default risk in Section 5 and show that we can observe delay even when state-noncontingent debt issuance is allowed.

Specifically, consider a version of the framework exactly as in Section 3 above but without the dependence of output on past defaults and delay. For simplicity, we will assume that default costs are a constant proportion  $\gamma$  of output in *both* periods 0 and 1, so that

$$y(z_t, d) = (1 - \gamma)y(z_t, n).$$

We maintain the assumption that output is increasing over time, regardless of delay, so that

$$y(z_0, s) < y(z_1, s) < y^*.$$

The departure of this section from what went before is that we will assume that the creditor, when making offers to the debtor, is restricted to only offer a transfer of current resources. Moreover, following Bi (2008), the debtor is prohibited from issuing new debt until it enters the next period in good standing with its creditors. This is important: if the

country were able to borrow in the same period, the creditor would take this into account when making a settlement offer, with the result being identical to that from bargaining over debt and resource transfers simultaneously.

To see how prohibiting debt issuance affects the results, note that the analysis proceeds similarly to that described in Section 3.A above. As the country will not repay any debts contracted in period 1 for repayment in period 2, creditors would not choose to make settlement offers including debt in period 1 even if they were allowed to do so. The first departure from the analysis of the previous section concerns the problem of a country that settles its default in period 0. Because no debt issuance is allowed as part of the settlement, the country enters period 1 without debt and hence has no debt to default upon. Moreover, because creditors understand that they will not honor debts contracted in period 1 for repayment in period 2, they cannot issue debt. Such a country simply consumes its endowment and receives

$$V(z_1, n) = u(y(z_1, n)) + \frac{\beta}{1 - \beta} u(y^*).$$

The second difference compared to Section 3, and the main difference from our point of view, concerns the analysis of bargaining in period 0. Now, the best acceptable offer that the creditor could make in period 0 maximizes  $\tau_0$  subject to

$$u(y(z_0, n) - \tau_0) + \beta V(z_1, (d, n)) \geq u(y(z_0, d)) + \beta V(z_1, (d, d)),$$

so that, after substituting for these values,  $\tau_0^*$  solves

$$u(y(z_0, n) - \tau_0^*) + \beta u(y(z_1, n)) = u(y(z_0, d)) + \beta u(y(z_1, d)).$$

The creditor must then compare the level of  $\tau_0^*$  to that of

$$\beta \tau_1^* = \beta \gamma y(z_1, n),$$

as before.

Whether there is delay will again depend on the curvature of  $u$  and the size of the endowment in period 0. Note that under our assumptions, it is straightforward to show that

$$\tau_0^* > \gamma y(z_0, n),$$

or that the transfer is at least as large as the output cost in period 0. If utility is close to linear and the period 0 endowment is large, the debtor country can be induced to make a settlement in period 0 that is considerably larger than the output cost of default in period 0. This is because a debtor country that settles in period 0 receives higher output in period 0 *and in period 1*, without the period 1 extra output reduced by the need to repay new debts contracted in period 0. If so, the transfer in period 0 will be large and delay will not occur. If, however, the period 0 endowment is small or  $u$  displays a lot of curvature in the neighborhood of  $y(z_0, d)$ , the transfer will still fall short of the value of the resources that can be extracted by bargaining in period 1, and delay will occur.

Next, we show that, even under these assumptions, allowing borrowing as part of a default settlement always eliminated delay in this environment.

## B. Default Settlement with Debt Issuance

Now consider the exact same set of assumptions except that we allow the sovereign and its creditor to bargain over both a transfer of current resources (which could be negative) as well as new debt issuance  $b_0$  in period 0. To see how this matters, note that if the country had arrived in period 1 with some level of debt negotiated as part of a settlement in period 0 given by  $b$ , then, as in Section 3.A above, it would have repaid that debt as long as

$$b \leq \gamma y(z_1, n) \equiv b^*.$$

Hence, the creditor would never agree to a settlement involving more than  $b^*$  in debt.

In this case, the optimal period 0 settlement offer solves

$$\max_{\tau_0, b_0} \tau_0 + \beta b_0,$$

subject to

$$u(y(z_0, n) - \tau) + \beta u(y(z_1, n) - b_0) + \frac{\beta^2}{1 - \beta} u(y^*) \geq u(y(z_0, d)) + \beta u(y(z_1, d)) + \frac{\beta^2}{1 - \beta} u(y^*)$$

$$b_0 \leq b^*.$$

It is straightforward to show that, under our assumption that output is increasing, the solution to this problem involves the creditor setting the largest debt issuance level  $b_0 = b^*$ , along with a transfer of  $\tau_0 = \gamma y(z_0, n)$ , for a total settlement value of

$$\gamma(y(z_0, n) + \beta y(z_1, n)),$$

which is always strictly greater than the value of delaying and making a new settlement offer tomorrow, which offers a discounted return of only  $\beta b^* = \beta \gamma y(z_1, n)$ .

The mechanics behind this result are quite instructive as to the differences between the results of this section and those of Section 3. In Section 3, because delay had persistent effects on output, the ability of a creditor to extract resources from the debtor in the future grew with delay. As a result, the creditor was able to extract more resources by delaying settlement to period 1 than it could by settling in period 0 and issuing debt that paid off in period 1. Put differently, the ability of the creditor to get the debtor to honor debts in period 1 was unaffected by delay (since debt issuance could occur only if there was no delay in settlement in period 0) while the ability to get the debtor to make a settlement in current resources in period 1 was increasing in delay. Hence, the creditor found it optimal to delay and extract current resources.

When delay does not have a persistent effect on output, the incentive to hand over resources to honor a debt is identical to the incentive to hand over resources to settle an outstanding default; both are governed by the output cost of default in that period and that period alone. Since settling in period 0 yields the creditor a payoff in current resources in period 0 and a payoff in period 1 through the repayment of debt that is exactly the same as the entire payment that the creditor could receive by waiting, the creditor always chooses to settle in period 0, and there is no delay.

To summarize, the assumption that the sovereign and its creditors cannot bargain

over new debt issuance, and the assumption that the sovereign cannot borrow itself until the next period, together produce delay in equilibrium. Allowing for either form of debt issuance eliminates delay in this example. This naturally begs the question of what can be done to restore delay as a possible equilibrium outcome. In the next section, we consider debt issuance in an uncertain environment in which the sovereign may default on the new debt issue. In such a world, although delay causes the creditor to miss out on a current payment of resources, it allows the creditor to avoid losing resources through a default in the second period. Thus, delay may once again be optimal.

## 5. Uncertainty and Default upon a Settlement

In the previous section we showed that, when output was deterministic and delay had no persistent effect on economic activity, allowing the parties to negotiate over new debt issues eliminated delay. The reason was that a settlement in period 0 allowed the creditor to extract period 0 resources *and* extract period 1 resources via the country's debt issuance. Delaying settlement to period 1 meant forgoing period 0 resources while obtaining the same amount of period 1 resources; the amount of period 1 resources was the same because the ability to extract payment in period 1 is determined by the same forces—the contemporaneous output cost of default—that also determine the ability to enforce debts coming due in period 1.

We also showed in Section 3.B that if we allow default costs to rise with the duration of default, then the creditor may have an incentive to delay settlement so as to take advantage of the greater ability to extract resources after a long default. In that case, the ability to extract second-period resources in a settlement was *different* from the ability to extract second-period resources via debt issuance because the ability to enforce debts was lower.

In this section, we explore another reason why delayed bargaining may be able to extract more resources than early bargaining: the possibility that the country will default on any debt issued as part of an early agreement. To generate default risk in equilibrium, we allow for uncertainty in either the level of output in the country *or* the level of bargaining power in negotiations. Combined with the assumption that debt is state-noncontingent but for the possibility of default, early settlement must involve either a low level of default-risk-free debt or a higher level of debt that is subject to default risk. In contrast, delayed settlement

allows the creditor to extract resources later more efficiently as it can tailor its offers to the realized state of nature. This opens up the possibility for delays in equilibrium.

### A. Uncertain Output

To begin, we consider a case in which uncertainty over period 1 output gives rise to the risk of default on any debt issued as part of a settlement in period 0. Specifically, we assume that  $z_1$  can take one of two values corresponding to either a high  $z_1^H$  or low  $z_1^L$  level of output with

$$\pi(z_1^H|z_0) = \pi \text{ and } \pi(z_1^L|z_0) = 1 - \pi.$$

As before, we set  $T = 2$  so that

$$\pi(z^*|z_1^H) = \pi(z^*|z_1^L) = 1,$$

and output is constant in all states and dates after the second period. We assume that the creditor makes the proposal in every period so that there is no uncertainty in bargaining power. We assume that the output cost of default in period 1 is in proportion  $\gamma$  to output and that output is increasing over time so that

$$y(z_0, n) < y(z_1^L, d) \text{ and } y(z_1^H, n) < y^*,$$

to ensure that the country always wants to borrow in periods 0 and 1, and so that delay in reaching agreement is socially suboptimal as the debtor is unable to exploit the potential gains from trade from borrowing.

As before, we solve for the equilibrium recursively starting in period 2, which is the same as in Section 3. The analysis of period 1 is also analogous to that above, with the exception that the precise outcomes depend upon the outcome of uncertainty. If there was no settlement agreement in period 0 and output is high—a probability  $\pi$  event—the creditor proposes  $\tau_1^*(z_1^H) = \gamma y(z_1^H, n)$ , whereas if output is low the creditor proposes  $\tau_1^*(z_1^L) = \gamma y(z_1^L, n)$ . On the other hand, if there was a settlement agreement in period 0 involving debt



issuance of  $b_0$ , this debt will be repaid as long as it is no larger than  $b^*(z_1^H) = \gamma y(z_1^H, n)$  in the high output state and no larger than  $b^*(z_1^L) = \gamma y(z_1^L, n)$  in the low output state, where under our assumptions  $b^*(z_1^L) < b^*(z_1^H)$ . Once again, and as in Section 4, under the assumption that delay has no permanent effect on output, the creditor's ability to extract repayment of a debt in period 1 is the same as its ability to extract a transfer in a settlement. Unlike in Section 4, the ability to extract resources varies by state, which cannot be exploited by the creditor if it settles in period 1 with an issuance of state-noncontingent debt; this is the source of delay in this example.

Finally, consider period 0. To make a proposal that will be accepted, the creditor has (essentially) two options: it can propose a settlement with low debt  $b_0 = b^*(z_1^L)$  which is repaid with certainty, or it can propose a settlement with high debt  $b_0 = b^*(z_1^H)$  which is repaid only in the high output state (probability  $\pi$ ) and is defaulted upon in the low output state. Associated with each of these debt offers is an offer of a transfer of current resources. If the creditor proposes the high debt level, the transfer is

$$\tau_0^*(b^*(z_1^H)) = y(z_0, n) - y(z_0, d).$$

Intuitively, by setting the high debt level, the creditor ensures that the debtor receives its default utility level in both states in period 1: if the high state eventuates, the debtor repays and is left with its default output level to consume; if the low state occurs, the debtor defaults and receives the default level of output. However, if the creditor proposes the low debt level, the optimal transfer solves

$$u(y(z_0, n) - \tau_0) + \beta\pi u(y(z_1^H, n) - b^*(z_1^L)) = u(y(z_0, d)) + \beta\pi u(y(z_1^H, d)),$$

generating

$$\tau_0^*(b^*(z_1^L)) > \tau_0^*(b^*(z_1^H)).$$

Unlike the high debt offer, if the high state eventuates, the debtor repays only a small amount, leaving the debtor with higher consumption; the creditor therefore extracts these resources

by proposing a higher transfer from the debtor in period 0.

We will observe delay in bargaining if and only if the expected settlement from delay exceeds the better of these two possible period 0 settlement offers,

$$\beta (\pi \tau_1^* (z_1^H) + (1 - \pi) \tau_1^* (z_1^L)) > \max \{ \tau_0^* (b^* (z_1^H)) + \pi \beta b^* (z_1^H), \tau_0^* (b^* (z_1^L)) + \beta b^* (z_1^L) \}.$$

In the limit, as period 0 output approaches zero, or as utility becomes more curved in the neighborhood of the period 0 output level, current period transfers are very costly and the optimal period 0 transfer levels will be low, so that this condition approaches

$$\beta \gamma \{ \pi y (z_1^H, n) + (1 - \pi) y (z_1^L, n) \} > \beta \gamma \max \{ \pi y (z_1^H, n), y (z_1^L, n) \},$$

which always holds as long as there is some uncertainty in output, or  $\pi \in (0, 1)$ . That is, the creditor always finds it optimal to wait until period 1 output is revealed before making a state-contingent offer, instead of settling in period 1 for a small amount of debt or risking default on a larger level of borrowing. Conversely, if there is no uncertainty, so that  $\pi = 1$  or  $\pi = 0$ , this strict inequality is always violated, and we are back to the case studied in Section 4.B in which we have no delay.

In this example, the ability to extract transfers in the future varied with the state because output varied with the state. Combined with the assumption that debtors cannot issue state-contingent debt as part of an early settlement, the creditor found it optimal to delay settlement until uncertainty about future output was resolved. In the next section, we show that this logic remains true even when output is constant across states of the world, as long as the ability to extract settlements can still vary as a result of differences in creditor bargaining power.

## B. Uncertain Bargaining Power

Next, we consider a case in which output is deterministic but fluctuations in bargaining power generate uncertainty, default and delays in debt restructuring. As bargaining power in an alternating offer bargaining model is almost entirely determined by the probability that an

agent proposes a settlement in the future, we must allow for nontrivial bargaining in period 2 to generate changes in bargaining power in period 1. Uncertainty about this bargaining power will then generate delay.

Specifically, consider a modified version of the previous example in which output is deterministic in period 1 and where  $T = 3$  so that the output penalty from default is positive in period 2 (but zero in periods 3 and beyond). For simplicity, we'll assume that the output cost is in proportion  $\gamma$  to output in periods 1 and 2.

Regarding bargaining power, we assume that the creditor proposes a settlement with certainty in periods 0 and 1, but that the probability of the creditor proposing a settlement in period 2 is random and becomes known in period 1. Specifically, in the high creditor bargaining power state (probability  $1 - \pi$ ), it is revealed in period 1 that the creditor proposes with certainty in period 2, whereas in the low creditor bargaining power state (probability  $\pi$ ), it is revealed that the creditor proposes with probability  $1 - \lambda$  in period 2. This uncertainty is encoded in realizations of the state in periods 1 and 2, or  $z_1$  and  $z_2$ . Note that output is assumed to be invariant across realizations of the state in these periods (output is deterministic), and hence we suppress the state superscript when referencing output.

To solve the model, we use backward induction. As before, in period 3 our assumptions ensure that no bargains are made and the country always defaults on any debt. Next consider period 2. If the debtor enters period 2 not in default with debt  $b$ , bargaining power is irrelevant (as assumed in Section 2, even if the country defaults, it spends a period in autarky, and so bargaining over a settlement does not occur until period 3 and would yield no payments or costs by assumption), and the debtor will repay if  $b \leq b_2^* = \gamma y(z_2, n)$  and receive payoff

$$V(z_2, b) = u(y(z_2, n) - \min\{b, \gamma y(z_2, n)\}) + \beta V^*.$$

Hence, in period 1, creditors will pay a unit price of  $p_1(b) = \beta$  per unit debt up to  $b_2^*$  and zero for all debt thereafter.

If the debtor enters period 2 in default and the creditor makes the proposal, which occurs with probability  $\pi + (1 - \pi)(1 - \lambda)$ , reflecting the possibility of ending up in the high creditor bargaining power state in which it always proposes, or because in the low creditor

bargaining state where it proposes with probability  $1 - \lambda$ , the creditor is able to extract  $\tau_2 = \gamma y(z_2, n)$  in settlement payments, leaving the debtor with

$$u(y(z_2, d)) + \beta V^*.$$

If, however, the debtor makes the proposal in period 2 (a probability  $(1 - \pi)\lambda$  event), the creditor receives nothing and the debtor gets

$$u(y(z_2, n)) + \beta V^*.$$

Now consider period 1. Whether the debtor enters period 1 in default or enters having already settled in period 0 and is deciding whether to default on debt issued as part of the settlement, the debtor's decisions will be affected by its expectations about bargaining in period 2. These expectations will be affected by the state of nature, which is revealed at the start of period 1. We will first consider the problem of a debtor that enters period 1 in default after having failed to settle in period 0, before turning to the problem of a debtor that entered period 1 not in default with some level of debt issued as part of the settlement. In each case, the analysis will vary according to whether bargaining power is high.

Suppose that the debtor enters period 1 in default. The state reveals bargaining power, which is determined by the probability that the creditor makes a proposal in period 2, but in period 1 the creditor always makes the proposal that consists of a transfer  $\tau_1$  and a debt issuance level  $b_1$  designed to maximize

$$\tau_1 + b_1 p_1(b_1).$$

Note that the bond price function does not depend on bargaining power; however, the constraint on the creditor will depend on bargaining power, and hence so will the equilibrium choices. In the high creditor bargaining power state  $z_1^H$  (a probability  $1 - \pi$  event), for a proposal to be acceptable to the debtor, it must satisfy

$$u(y(z_1, n) - \tau_1) + \beta V(z_2, b_1) \geq u(y(z_1, d)) + \beta u(y(z_2, d)) + \beta^2 V^*.$$

Under our assumption that output is increasing over time, the optimal acceptable proposal in this state involves the largest debt level  $b_1^*(z_1^H) = b_2^*$  and setting  $\tau_1$  so that constraint binds, or  $\tau_1^*(z_1^H) = \gamma y(z_1, n)$ . This yields the debtor its default or autarky payoff:

$$V(z_1^H, d) = u(y(z_1, d)) + \beta u(y(z_2, d)) + \beta^2 V^*.$$

Conversely, in the low bargaining power state  $z_1^L$  (a probability  $\pi$  event), for a proposal to be acceptable to the debtor, it must satisfy a tighter constraint:

$$u(y(z_1, n) - \tau_1) + \beta V(z_2, b_1) \geq u(y(z_1, d)) + \beta [\lambda u(y(z_2, n)) + (1 - \lambda) u(y(z_2, d))] + \beta^2 V^*,$$

reflecting the fact that in this state, the debtor expects to do better in the future on average. Under our assumptions on output, the solution still sets  $b_1^*(z_1^L) = b_2^*$ , whereas  $\tau_1^*(z_1^L)$  solves

$$(1) \quad u(y(z_1, n) - \tau_1^*(z_1^L)) = u(y(z_1, d)) + \beta \lambda [u(y(z_2, n)) - u(y(z_2, d))],$$

which, under our assumptions,<sup>2</sup> satisfies  $0 < \tau_1^*(z_1^L) < \gamma y(z_1, n)$ . This yields the debtor

$$V(z_1^L, d) = u(y(z_1, d)) + \beta [\lambda u(y(z_2, n)) + (1 - \lambda) u(y(z_2, d))] + \beta^2 V^*,$$

a payoff greater than its autarky value. This result is intuitive; in the low creditor bargaining power state, the payoff of the creditor is smaller, whereas that of the debtor is larger.

Now suppose that the debtor enters period 1 after settling in period 0 with a debt issuance of  $b_0$ . In period 1, the debtor must decide whether to default or repay this debt, and if it repays, it may borrow again. If the debtor repays and borrows again, bargaining power is irrelevant, and its problem is to choose a new debt issuance level  $b_1$  to maximize its welfare given the bond price function described above, or

$$V^R(z_1, b_0) = \max_{b_1} u(y(z_1, n) - b_0 + \beta b_1) + \delta V(z_2, b_1),$$

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<sup>2</sup>Strict positivity of  $\tau_1^*(z_1^L)$  follows from the fact that  $y(z_2, n) > y(z_1, n)$ , the assumption of a proportional default cost of output, and the strict concavity of  $u$ .

subject to

$$b_1 \leq b_2^*,$$

where we have imposed the fact that the debtor will never issue any debt beyond the level where its price falls to zero. The solution to this problem under our assumption that output is increasing is  $b_1 = b_2^*$ .

If the debtor country defaults, it knows that it will enter period 2 with defaulted debt, and hence bargaining power will matter. Given what we found above for period 2, in the high creditor bargaining power state, if the debtor defaults it expects to receive

$$u(y(z_1, d)) + \beta u(y(z_2, d)) + \beta^2 V^*,$$

and hence it defaults if and only if

$$b_0 > b_1^*(z_1^H) \equiv \gamma y(z_1, n) + \beta \gamma y(z_2, n).$$

By contrast, in the low creditor bargaining power state, if the debtor defaults it expects to receive

$$u(y(z_1, d)) + \beta \{ \lambda u(y(z_2, n)) + (1 - \lambda) u(y(z_2, d)) \} + \beta^2 V^*,$$

and hence it defaults if and only if  $b_0 > b_1^*(z_1^L)$  where  $b_1^*(z_1^L)$  solves

$$(2) \quad u(y(z_1, n) - b_1^*(z_1^L) + \beta \gamma y(z_2, n)) = u(y(z_1, d)) + \beta \lambda [u(y(z_2, n)) - u(y(z_2, d))],$$

which implies

$$b_1^*(z_1^L) < \gamma y(z_1, n) + \beta \gamma y(z_2, n) = b_1^*(z_1^H).$$

Intuitively, because the debtor expects on average to do better in the future in the low creditor bargaining power state, it cannot credibly commit to repaying as much debt in this state.

Note also that by comparing equations (1) and (2), we obtain

$$(3) \quad b_1^*(z_1^L) = \tau_1^*(z_1^L) + \beta\gamma y(z_2, n).$$

Under our maintained assumption that the level of output in period 0 is small enough so that period 0 transfers can be ignored, we have enough information to establish our result on the existence of delay.<sup>3</sup> Specifically, in the limit as these period 0 transfers vanish, we will observe delay if and only if the discounted expected payment from delay is at least as large as what the creditor can obtain from a settlement that is acceptable to the debtor today. If the creditor delays a settlement, then it expects to receive in period 1 the (undiscounted) amount

$$\begin{aligned} & \pi(\gamma y(z_1, n) + \beta\gamma y(z_2, n)) + (1 - \pi) [\tau_1^*(z_1^L) + \beta\gamma y(z_2, n)] \\ &= \pi b_1^*(z_1^H) + (1 - \pi) b_1^*(z_1^L), \end{aligned}$$

where we have used equation (3). Ignoring period 0 transfers, if the creditor instead settles immediately by issuing the default-free amount of debt  $b_1^*(z_1^L)$ , it evidently does worse as  $b_1^*(z_1^L) < b_1^*(z_1^H)$ . If instead it issues the higher amount of debt  $b_1^*(z_1^H)$ , it receives this full amount in period 1 with probability  $\pi$ , while in the probability  $1 - \pi$  event that creditor bargaining power is low, the debtor defaults and the creditor receives a settlement of expected value

$$(1 - \lambda) \beta\gamma y(z_2, n),$$

reflecting the risk that the debtor makes the proposal and the creditor receives nothing. But again, this is always worse than delay; in that case, the creditor receives a current transfer in period 1 of  $\tau_1^*(z_1^L)$  and a level of debt worth  $\beta\gamma y(z_2, n)$  that is certain to be repaid.

Note also that if there is no default risk (*either*  $p$  or  $\lambda$  are zero), then  $\tau_1^*(z_1^L) = \gamma y(z_1, n)$ ,  $b_1^*(z_1^L) = b_1^*(z_1^H)$ , and the creditor always settles in period 1, as we are back in the case of Section 4.B.

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<sup>3</sup>Values for the period 0 transfers outside of this limit are collected in the appendix.

## 6. Conclusions

Bargaining to restructure sovereign debts in default takes time; historically, roughly eight years pass on average between an initial default and a final restructuring agreement. In this paper, we have studied one class of explanations for this delay that involve bargaining in the presence of complete information between a sovereign country and a representative international creditor. In such a world, we show that delays can arise and that they can be socially inefficient when there are limits on the ability of the debtor country to share future surplus with creditors as part of a restructuring agreement. These limits sometimes arise in models, perhaps inadvertently, as a result of assumptions made by the author, such as the assumption to restrict debt issuance as part of a restructuring agreement. They may also arise because of the fundamental problem of sovereign debt: it is hard for debtors to commit to being able to repay their debts. In this sense, an inability to pay one's debts both causes the problem of default and also serves to increase the cost of default by making renegotiation more difficult.

In focusing on bargaining between a debtor and a coordinated group of creditors with complete information, we have abstracted from other potential causes of delays in bargaining. There are two obvious alternative candidates. The first follows much of the theoretical literature on delays in bargaining by emphasizing delays as a tool for signaling private information about aspects of the bargaining process, including the resources available to the debtor or the levels of patience of the bargaining parties (see the work surveyed by Ausubel, Cramton, and Deneckere 2002). Elements of these ideas have been applied in a sovereign debt context by Bai and Zhang (2012) and Ghosal and Miller (2006).

The second alternative explanation takes inspiration from some recent experiences with sovereign debt restructuring in which a small number of creditors have held up an agreement to restructure debts. Examples of models of this phenomenon are presented by Pitchford and Wright (2008, 2012, 2017). The processes examined in the present paper should be complementary to these alternative mechanisms, and a task for future research is to elucidate the relative importance of these alternative mechanisms while also studying the extent to which they complement and amplify one another.



A primary motivation for the current paper was an understanding of the extent to which bargaining that is efficient for the participants may give rise to a potential role for government intervention. As shown above, efficient private bargaining may give rise to socially inefficient outcomes when bargaining is limited by concerns such as the inability of a sovereign to commit to honoring the terms of the bargain. Whether a supranational policy maker like the IMF or a creditor country government can improve on these social outcomes in such an environment depends on whether they can act on the causes of this inefficiency. As it is likely both impossible and undesirable to impose completely binding agreements on debtor nations, is there such a role for supranationals or creditor country governments? One possibility involves the provision of collateral to back debts issued as part of a debt restructuring. For example, during the resolution of the 1980s debt crisis under the Brady Plan, some countries engaged in the subsidized purchase of U.S. Treasury securities as collateral to partially back new debts issued as part of the settlement.<sup>4</sup> This collateral plausibly enhanced the sovereign country's ability to commit to honoring the new debts (or at least increased the value of these debts) and thus may have assisted in the conclusion of restructuring negotiations. One option to speed up future restructuring negotiations would be for a supranational organization or creditor country government to provide, or subsidize the purchase of, such collateral. The extent to which this plan was important in history and could make a difference in practice is an open quantitative question.

## **7. Appendix: Additional Calculations from Section 5.B**

This appendix collects some calculations that are useful in establishing the results of Section 5.B, as well as some others that serve as a complete characterization of this example but were otherwise superfluous to the main result.

First, note from the arguments in the text that if the country enters period 1 with

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<sup>4</sup>The U.S. General Accounting Office calculated that the purchase by Mexico, for example, of \$30.2 billion in zero coupon bonds at face value for \$2.99 billion amounted to a (small) subsidy of approximately \$192 million.

debt  $b_0$ , its value in the high creditor bargaining power state is given by

$$V(z_1^H, b_0) = u(y(z_1, n) + \beta\gamma y(z_2, n) - \min\{b_0, \beta b_1^* + \gamma y(z_1, n)\}) + \beta u(y(z_2, d)) + \beta^2 V^*.$$

Similarly, the debtor country's value in the low creditor bargaining power state is given by

$$\begin{aligned} V(z_1^L, b_0) &= \max \left\{ u(y(z_1, n) + \beta\gamma y(z_2, n) - b_0) + \beta u(y(z_2, d)) + \beta^2 V^*, \right. \\ &\quad \left. u(y(z_1, d)) + \beta \{ \lambda u(y(z_2, n)) + (1 - \lambda) u(y(z_2, d)) \} + \beta^2 V^* \right\}. \end{aligned}$$

To compute the equilibrium period 0 transfers, note that any period 0 proposal of  $\tau_0$  and  $b_0$  that is acceptable to the debtor must satisfy

$$\begin{aligned} &u(y(z_0, n) - \tau_0) + \beta \{ \pi V(z_1^H, b_0) + (1 - \pi) V(z_1^L, b_0) \} \\ &\geq u(y(z_0, d)) + \beta \{ \pi V(z_1^H, d) + (1 - \pi) V(z_1^L, d) \}. \end{aligned}$$

Substituting for these value functions, we get

$$\begin{aligned} &u(y(z_0, n) - \tau_0) + \beta\pi (u(y(z_1, n) + \beta\gamma y(z_2, n) - \min\{b_0, \beta\gamma y(z_2, n) + \gamma y(z_1, n)\}) + \beta u(y(z_2, d)) + \\ &\quad \beta(1 - \pi) \max \{ u(y(z_1, n) + \beta\gamma y(z_2, n) - b_0) + \beta u(y(z_2, d)) + \beta^2 V^*, \\ &\quad u(y(z_1, d)) + \beta \{ \lambda u(y(z_2, n)) + (1 - \lambda) u(y(z_2, d)) \} + \beta^2 V^* \} \\ &\geq u(y(z_0, d)) + \beta\pi (u(y(z_1, d)) + \beta u(y(z_2, d)) + \beta^2 V^*) + \\ &\quad \beta(1 - \pi) (u(y(z_1, d)) + \beta [\lambda u(y(z_2, n)) + (1 - \lambda) u(y(z_2, d))]) + \beta^2 V^*. \end{aligned}$$

Rearranging yields

$$\begin{aligned} &u(y(z_0, n) - \tau_0) + \beta\pi (u(y(z_1, n) + \beta\gamma y(z_2, n) - \min\{b_0, \beta\gamma y(z_2, n) + \gamma y(z_1, n)\})) + \\ &\quad \beta(1 - \pi) \max \{ u(y(z_1, n) + \beta\gamma y(z_2, n) - b_0) + \beta u(y(z_2, d)), \\ &\quad u(y(z_1, d)) + \beta \{ \lambda u(y(z_2, n)) + (1 - \lambda) u(y(z_2, d)) \} \} \\ &\geq u(y(z_0, d)) + \beta\pi (u(y(z_1, d))) + \\ &\quad \beta(1 - \pi) (u(y(z_1, d)) + \beta [\lambda u(y(z_2, n)) + (1 - \lambda) u(y(z_2, d))]). \end{aligned}$$

From this, we can see that if the creditor proposes a settlement with  $b_0 = b_1^*(z_1^H)$ , the optimal transfer  $\tau_0^*(b_1^*(z_1^H)) = y(z_0, n) - y(z_0, d)$ . However, if the creditor proposes a settlement with  $b_0 = b_1^*(z_1^L)$ , the optimal transfer  $\tau_0^*(b_1^*(z_1^L))$  solves

$$\begin{aligned} & u(y(z_0, n) - \tau_0^*(b_1^*(z_1^L))) + \beta\pi(u(y(z_1, n) + \beta\gamma y(z_2, n) - b_1^*(z_1^L))) \\ & \geq u(y(z_0, d)) + \beta\pi(u(y(z_1, d))). \end{aligned}$$

Note also that

$$\begin{aligned} & u(y(z_0, n) - \tau_0^*(b_1^*(z_1^L))) \\ & \geq u(y(z_0, d)) - \beta^2\pi\lambda[u(y(z_2, n)) - u(y(z_2, d))]. \end{aligned}$$

## References

- [1] Aguiar, M. and G. Gopinath. (2006). “Defaultable Debt, Interest Rates and the Current Account.” *Journal of International Economics*, 69(1): 64-83.
- [2] Arellano, C. (2008). “Default Risk and Income Fluctuations in Emerging Economies.” *American Economic Review* 98(3): 690-712.
- [3] Ausubel, L. M., P. Cramton, and R. J. Deneckere. (2002). “Bargaining with Incomplete Information,” in *Handbook of Game Theory with Economic Applications*, ed. R. J. Aumann and S. Hart, 1897-1945. Amsterdam: Elsevier.
- [4] Bai, Y., and J. Zhang (2012). “Duration of Sovereign Debt Renegotiation.” *Journal of International Economics* 86(2): 252-268.
- [5] Benjamin, D. and M. L. J. Wright (2008). “Recovery Before Redemption.” Working paper, UCLA.
- [6] Bi, R. (2008). “‘Beneficial’ Delays in Debt Restructuring Negotiations.” IMF Working Paper, 08/38.
- [7] Dhillon, A., J. García-Fronti, S. Ghosal, and M. Miller (2006). “Debt Restructuring and Economic Recovery: Analysing the Argentine Swap.” *World Economy* 29(4): 377-398.

- [8] Ghosal, S. and M. Miller (2006). “Delay in Debt Swaps: Growth and Sustainability.” Working paper, University of Warwick.
- [9] International Monetary Fund (2013). “Sovereign Debt Restructuring: Recent Developments and Implications for the Fund’s Legal and Policy Framework.” Policy paper, IMF.
- [10] Merlo, A., and C. Wilson (1995). “A Stochastic Model of Sequential Bargaining with Complete Information.” *Econometrica* 63(2): 371-399.
- [11] Merlo, A., and C. Wilson (1998). “Efficient Delays in a Stochastic Model of Bargaining.” *Economic Theory* 11(1): 39-55.
- [12] Miller, M., and J. García-Fronti (2005a). “Bargaining and sustainability: a technical note on applying the Merlo-Wilson model.” Working paper, University of Warwick.
- [13] Miller, M. and J. García-Fronti (2005b). “Case study: Restructuring Argentine debt: a renegotiation game?” Working paper, Centre for the Study of Globalisation and Regionalisation, Warwick.
- [14] Pitchford, R., and M. L. J. Wright. (2008). “Restructuring the Sovereign Debt Restructuring Mechanism.” Working paper, UCLA.
- [15] Pitchford, R., and M. L. J. Wright (2012). “Holdouts in Sovereign Debt Restructuring: A Theory of Negotiation in a Weak Contractual Environment.” *Review of Economic Studies* 79(2): 812-837.
- [16] Pitchford, R., and M. L. J. Wright (2017). “Settlement Games with Rank-Order Payoffs and Applications to Sovereign Debt Restructuring.” *Economic Theory* 64(4): 847-876.
- [17] Tomz, M., and M. L. J. Wright (2007). “Do Countries Default in ‘Bad Times’?” *Journal of the European Economic Association* 5(2-3): 352-360.
- [18] Tomz, M., and M. L. J. Wright (2013). “Empirical Research on Sovereign Debt and Default.” *Annual Review of Economics* 5(1): 247-272.
- [19] Yue, V. Z. (2010). “Sovereign Default and Debt Renegotiation.” *Journal of International Economics* 80(2): 176-187.