Discussion of *Imperfect Risk-Sharing and the Business Cycle*

by David Berger, Luigi Bocola and Alessandro Dovis

Dirk Krueger

University of Pennsylvania, CEPR, and NBER

August 2019
The Paper: Big Picture

- Question: Does household inequality matter for business cycles?
  - More Precisely (in this paper): Does imperfect consumption risk sharing amplify business cycle volatility?

- Why could imperfect risk sharing matter for business cycles?
  - Activates precautionary saving behavior.
  - Precautionary saving varies over the cycle (e.g. higher unemployment fears in recessions).
  - Changes aggregate consumption demand (and in NK models, output) dynamics over the cycle. How much?

- This paper: uses theory (RA representation), measurement (CEX micro data), counterfactual experiment to give answer: 20%
The Paper in a Nutshell

- Continuum of households. Time discount factor $\beta$ and
  \[ \frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\psi}}{1+\psi} \]

- Labor endowment $e(v_t)$ subject to idiosyncratic shocks (history $v^t$)
- Aggregate production subject to TFP shock $z_t$ (history $z^t$)
  \[ C(z^t) = z_t L(z^t) \]

- Financial markets: At least a one period bond $b$, potentially many
  other assets (possibly subject to trading) frictions.
- NK nominal rigidities. Largely abstracted from in the discussion.
The Paper: Three Key Contributions

• Theory: Take an equilibrium of economy with micro heterogeneity 
  \((c(z^t, v^t), l(z^t, v^t))\). Then associated \((C(z^t), L(z^t))\) form equilibrium 
  of economy with preference shocks, i.e. satisfy

\[
  z_t = \frac{\omega(z^t) L(z^t) \psi}{C(z^t) \sigma}
\]

\[
  \frac{1}{R(z^t)} = \beta \max_{v^t} \sum_{z_{t+1}} \pi(z^{t+1} | z^t) \beta(z^{t+1}, v^t) \left( \frac{C(z^{t+1})}{C(z^t)} \right)^{-\sigma}
\]

where the preference shocks (not really) satisfy

\[
  \beta(z^{t+1}, v^t) = \sum_{v^{t+1}} \pi(v^{t+1} | v^t, z^{t+1}) \left( \frac{c(z^{t+1}, v^{t+1})/C(z^{t+1})}{c(z^t, v^t)/C(z^t)} \right)^{-\sigma}
\]

\[
  \omega(z^t) = \left[ \sum_{v^t} \pi(v^t | z^t) \left( \frac{c(z^t, v^t)}{C(z^t)} \right)^{-\frac{\sigma}{\psi}} e(v^t)^{\frac{1+\psi}{\psi}} \right]^{-\psi}
\]

Content of micro heterogeneity is summarized in \(\beta(z^{t+1}, v^t), \omega(z^t)\)
The Paper: Three Key Contributions

- **Empirical:** Estimating the preference shock process
  - The $\beta(z^{t+1}, v^t), \omega(z^t)$ are highly model-dependent and model-endogenous! Progress?
  - Alternative: estimate them directly from micro data (CEX). Need data on household consumption shares $\frac{c(z^t, v^t)}{C(z^t)}$
  - Theory: stochastic process for $\frac{c(z^t, v^t)}{C(z^t)}$ key for impact of micro heterogeneity on business cycles. Thus only interested in micro models that get this process right empirically anyway.
  - Cf. sufficient statistics approach in fiscal policy (Chetty, Saez)

- **Quantitative:** compute contribution of imperfect risk sharing to business cycle fluctuations.
  - Feed $\{\beta(z^{t+1}, v^t), \omega(z^t)\}$ process into model with representative household and measure fluctuations. Do the same in model with perfect risk sharing ($\beta(z^{t+1}, v^t) \equiv 1$).
  - Key quantitative finding: 20% of Great Recession accounted for by imperfect risk sharing.
Preference Shocks and Fluctuations: A Simple Example

- $t = 0, 1$
- No initial heterogeneity, no risk in $t = 0$ (i.e. $z_0 = e_0 = 1$).
- $t = 1$: we have $z \in \{z_b, z_g\}$ with $\pi = 0.5$. Also $v \in \{u, m\}$ with $\pi = 0.5$ and
  
  $$e(z, v) = \begin{cases} 
  1 - \varepsilon(z) & \text{if } v = v_u \\
  1 + \varepsilon(z) & \text{if } v = v_m 
  \end{cases}$$
- $\sigma = 1$ (log-utility) and $\psi = 1$ (quadratic cost of labor).
- Three economies
  1. Representative agent (RA) economy: $\varepsilon(z) \equiv 0$.
  2. Complete markets economy (CM): $\varepsilon(z) > 0$, but Arrow securities that pay contingent on $v$ realizations. Perfect risk sharing.
  3. Incomplete markets economy (SIM). Only risk free bond in zero net supply (imperfect risk sharing).
**Preference Shocks and Equilibrium Allocations**

### Table: Preference Shocks and Equilibrium Allocations

<table>
<thead>
<tr>
<th>Statistic</th>
<th>RA</th>
<th>CM</th>
<th>SIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega(z) )</td>
<td>1</td>
<td>( \frac{2}{(1-\varepsilon(z))^2+(1+\varepsilon(z))^2} &lt; 1 )</td>
<td>1</td>
</tr>
<tr>
<td>( \beta(z) )</td>
<td>1</td>
<td>1</td>
<td>( \frac{1}{1-\sigma^2_\varepsilon(z)} &gt; 1 )</td>
</tr>
<tr>
<td>( L_0 = C_0 )</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( L_1(z) )</td>
<td>1</td>
<td>( \omega(z)^{-\frac{1}{2}} &gt; 1 )</td>
<td>1</td>
</tr>
<tr>
<td>( C_1(z), Y_1(z) )</td>
<td>( z )</td>
<td>( z\omega(z)^{-\frac{1}{2}} &gt; z )</td>
<td>( z )</td>
</tr>
<tr>
<td>( \frac{Y_1(z_b)}{Y_1(z_g)} )</td>
<td>( \frac{z_b}{z_g} )</td>
<td>( \left( \frac{\omega(z_g)}{\omega(z_b)} \right)^{0.5} \frac{z_b}{z_g} )</td>
<td>( \frac{z_b}{z_g} )</td>
</tr>
<tr>
<td>( q = \frac{1}{R} )</td>
<td>( \frac{\beta}{2} \sum z \frac{1}{z} )</td>
<td>( \frac{\beta}{2} \sum z \frac{1}{z\omega(z)^{-\frac{1}{2}}} &lt; q^{RA} )</td>
<td>( \frac{\beta}{2} \sum z \frac{1}{z(1-\sigma^2_\varepsilon(z))} &gt; q^{RA} )</td>
</tr>
</tbody>
</table>

- Labor supply reacts to idiosyncratic productivity shocks. CM v.s SIM? Wealth effect \( (\sigma = 1) \).
- Full consumption insurance: \( \beta^{CM}(z) = 1 \). Imperfect insurance in SIM represented as patience: \( \beta^{SIM}(z) > 1 \). Drives down \( R^{SIM} \).
- Business cycles: More volatile in CM than in RA if \( \sigma^2_\varepsilon(z_b) > \sigma^2_\varepsilon(z_g) \).
- No difference in SIM vs. RA? No link in SIM from \( \beta^{CM}(z) \) to \( Y \) since no capital \( R \) adjusts flexibly to \( \beta^{CM}(z) \). NK elements break this.

Dirk Krueger (Penn,NBER,CEPR)  Imperfect Risk Sharing and Cycles  August 2019  7 / 11
Labor supply reacts to idiosyncratic productivity shocks. CM v.s SIM? Wealth effect (\(\sigma = 1\)). \(L_1^{CM}(z) > 1\) needs \(\omega_1^{CM}(z) < 1\).
Preference Shocks and Equilibrium Allocations

<table>
<thead>
<tr>
<th>Statistic</th>
<th>RA</th>
<th>CM</th>
<th>SIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega(z)$</td>
<td>1</td>
<td>$\frac{2}{(1-\varepsilon(z))^2+(1+\varepsilon(z))^2} &lt; 1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$\beta(z)$</td>
<td>1</td>
<td>$\frac{1}{1-\sigma_\varepsilon^2(z)} &gt; 1$</td>
<td></td>
</tr>
<tr>
<td>$L_0 = C_0$</td>
<td>1</td>
<td>$\omega(z)^{-\frac{1}{2}} &gt; 1$</td>
<td></td>
</tr>
<tr>
<td>$L_1(z)$</td>
<td>1</td>
<td>$z\omega(z)^{-\frac{1}{2}} &gt; z$</td>
<td></td>
</tr>
<tr>
<td>$C_1(z), Y_1(z)$</td>
<td>$z$</td>
<td></td>
<td>$z$</td>
</tr>
<tr>
<td>$\frac{Y_1(z_b)}{Y_1(z_g)}$</td>
<td>$\frac{z_b}{z_g}$</td>
<td>$\left(\frac{\omega(z_g)}{\omega(z_b)}\right)^{0.5} \frac{z_b}{z_g}$</td>
<td>$\frac{z_b}{z_g}$</td>
</tr>
<tr>
<td>$q = \frac{1}{R}$</td>
<td>$\frac{\beta}{2} \sum z \frac{1}{z}$</td>
<td>$\frac{\beta}{2} \sum z \frac{1}{z\omega(z)^{-\frac{1}{2}} &lt; q^{RA}}$</td>
<td>$\frac{\beta}{2} \sum z \frac{1}{z(1-\sigma_\varepsilon^2(z))} &gt; q^{RA}$</td>
</tr>
</tbody>
</table>

- Labor supply reacts to idiosyncratic productivity shocks. CM v.s SIM? Wealth effect ($\sigma = 1$). $L_1^{CM}(z) > 1$ needs $\omega_1^{CM}(z) < 1$.
- Full consumption insurance: $\beta^{CM}(z) = 1$. Imperfect insurance in SIM represented as patience: $\beta^{SIM}(z) > 1$. Drives down $R^{SIM}$.
### Preference Shocks and Equilibrium Allocations

<table>
<thead>
<tr>
<th>Statistic</th>
<th>RA</th>
<th>CM</th>
<th>SIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega(z)$</td>
<td>$\frac{2}{(1-\varepsilon(z))^2+(1+\varepsilon(z))^2} &lt; 1$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$\beta(z)$</td>
<td>$1$</td>
<td>$1$</td>
<td>$\frac{1}{1-\sigma^2_\varepsilon(z)} &gt; 1$</td>
</tr>
<tr>
<td>$L_0 = C_0$</td>
<td>$1$</td>
<td>$1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$L_1(z)$</td>
<td>$1$</td>
<td>$\omega(z)^{-\frac{1}{2}} &gt; 1$</td>
<td>$1$</td>
</tr>
<tr>
<td>$C_1(z), Y_1(z)$</td>
<td>$z$</td>
<td>$z\omega(z)^{-\frac{1}{2}} &gt; z$</td>
<td>$z$</td>
</tr>
<tr>
<td>$\frac{Y_1(z_b)}{Y_1(z_g)}$</td>
<td>$\frac{z_b}{z_g}$</td>
<td>$\left(\frac{\omega(z_g)}{\omega(z_b)}\right)^{0.5}$</td>
<td>$\frac{z_b}{z_g}$</td>
</tr>
<tr>
<td>$q = \frac{1}{R}$</td>
<td>$\frac{\beta}{2} \sum z \frac{1}{z}$</td>
<td>$\frac{\beta}{2} \sum z \frac{1}{z\omega(z)^{-\frac{1}{2}}} &lt; q^{RA}$</td>
<td>$\frac{\beta}{2} \sum z \frac{1}{z(1-\sigma^2_\varepsilon(z))} &gt; q^{RA}$</td>
</tr>
</tbody>
</table>

- Labor supply reacts to idiosyncratic productivity shocks. CM v.s SIM? Wealth effect ($\sigma = 1$). $L_1^{CM}(z) > 1$ needs $\omega_1^{CM}(z) < 1$.
- Full consumption insurance: $\beta^{CM}(z) = 1$. Imperfect insurance in SIM represented as patience: $\beta^{SIM}(z) > 1$. Drives down $R^{SIM}$.
- Business cycles: More volatile in CM than in RA if $\sigma^2_\varepsilon(z_b) > \sigma^2_\varepsilon(z_g)$.
### Preference Shocks and Equilibrium Allocations

<table>
<thead>
<tr>
<th>Statistic</th>
<th>RA</th>
<th>CM</th>
<th>SIM</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega(z)$</td>
<td>1</td>
<td>$\frac{2}{(1-\varepsilon(z))^2+(1+\varepsilon(z))^2} &lt; 1$</td>
<td>1</td>
</tr>
<tr>
<td>$\beta(z)$</td>
<td>1</td>
<td>$\frac{1}{1-\sigma_\varepsilon^2(z)} &gt; 1$</td>
<td></td>
</tr>
<tr>
<td>$L_0 = C_0$</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>$L_1(z)$</td>
<td>1</td>
<td>$\omega(z)^{-\frac{1}{2}} &gt; 1$</td>
<td></td>
</tr>
<tr>
<td>$C_1(z), Y_1(z)$</td>
<td>$z$</td>
<td>$z\omega(z)^{-\frac{1}{2}} &gt; z$</td>
<td>$z$</td>
</tr>
<tr>
<td>$\frac{Y_1(z_b)}{Y_1(z_g)}$</td>
<td>$\frac{z_b}{z_g}$</td>
<td>$\left(\frac{\omega(z_g)}{\omega(z_b)}\right)^{0.5} \frac{z_b}{z_g}$</td>
<td>$\frac{z_b}{z_g}$</td>
</tr>
<tr>
<td>$q = \frac{1}{R}$</td>
<td>$\frac{\beta}{2} \sum_z \frac{1}{z}$</td>
<td>$\frac{\beta}{2} \sum_z \frac{1}{z\omega(z)^{-\frac{1}{2}}} &lt; q^{RA}$</td>
<td>$\frac{\beta}{2} \sum_z \frac{1}{z(1-\sigma_\varepsilon^2(z))} &gt; q^{RA}$</td>
</tr>
</tbody>
</table>

- Labor supply reacts to idiosyncratic productivity shocks. CM v.s SIM? Wealth effect ($\sigma = 1$). $L_1^{CM}(z) > 1$ needs $\omega_1^{CM}(z) < 1$.
- Full consumption insurance: $\beta^{CM}(z) = 1$. Imperfect insurance in SIM represented as patience: $\beta^{SIM}(z) > 1$. Drives down $R^{SIM}$.
- Business cycles: More volatile in CM than in RA if $\sigma_\varepsilon^2(z_b) > \sigma_\varepsilon^2(z_g)$
- No difference in SIM vs. RA? No link in SIM from $\beta(z)$ to $Y$ since no capital $R$ adjusts flexibly to $\beta(z)$. NK elements break this.
Birds Eye Comment (1): How Robust is the Result

• Applies to large class of HANK models.

• Applies to models with capital accumulation (under certain assumptions).

• Can handle fairly general asset market structure.

• Likely does not generalize to discount factor or asset return heterogeneity. Important because literature has used these to get wealth heterogeneity right.

• Also needs interiority of labor supply. Rules out extensive margin, unemployment.
Birds Eye Comment (2): How Useful is the Result

- **Big Positives**
  - Powerful tool to measure answer to an important specific quantitative question.
  - Useful diagnostic tool: what aspects of (models of) household heterogeneity really matter for amplification of business cycles?

- **Limitations**
  - Theoretical result is not a substitute for actually solving the heterogeneous agent model unless model $\beta(z), \omega(z)$ fits data perfectly.
  - Cannot be used for counterfactual policy analysis (stimulus, anyone?) since the preference shocks $\{\beta(z_{t+1}, v^t), \omega(z^t)\}$ are not invariant to policy.
Conclusions

- Great paper!
  - A powerful general theoretical representation result.
  - Careful measurement using micro data.
  - Uses the theory and measurement to give quantitative answer to important question of policy relevance.
THANK YOU