Discussion of *Imperfect Risk-Sharing and the Business Cycle*

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The Paper: Big Picture

- Question: Does household inequality matter for business cycles ?
  - More Precisely (in this paper): Does imperfect consumption risk sharing amplify business cycle volatility ?
- Why could imperfect risk sharing matter for business cycles?
  - Activates precautionary saving behavior.
  - Precautionary saving varies over the cycle (e.g. higher unemployment fears in recessions).
  - Changes aggregate consumption demand (and in NK models, output) dynamics over the cycle. How much?
- This paper: uses theory (RA representation), measurement (CEX micro data), counterfactual experiment to give answer: 20%

The Paper in a Nutshell

- Continuum of households. Time discount factor β and

$$\frac{c^{1-\sigma}}{1-\sigma} - \frac{l^{1+\psi}}{1+\psi}$$

- Labor endowment $e(v_t)$ subject to idiosyncratic shocks (history v^t)
- Aggregate production subject to TFP shock z_t (history z^t)

$$C(z^t) = z_t L(z^t)$$

- Financial markets: At least a one period bond b, potentially many other assets (possibly subject to trading) frictions.
- NK nominal rigidities. Largely abstracted from in the discussion.

The Paper: Three Key Contributions

- Theory: Take an equilibrium of economy with micro heterogeneity $(c(z^t, v^t), l(z^t, v^t))$. Then associated $(C(z^t), L(z^t))$ form equilibrium of economy with preference shocks, i.e. satisfy

$$z_t = \frac{\omega(z^t) L(z^t)^{\psi}}{C(z^t)^{\sigma}}$$

$$\frac{1}{R(z^t)} = \beta \max_{v^t} \sum_{z_{t+1}} \pi(z^{t+1}|z^t) \beta(z^{t+1}, v^t) \left(\frac{C(z^{t+1})}{C(z^t)} \right)^{-\sigma}$$

where the preference shocks (not really) satisfy

$$\beta(z^{t+1}, v^t) = \sum_{v^{t+1}} \pi(v^{t+1}|v^t, z^{t+1}) \left(\frac{c(z^{t+1}, v^{t+1})/C(z^{t+1})}{c(z^t, v^t)/C(z^t)} \right)^{-\sigma}$$

$$\omega(z^t) = \left[\sum_{v^t} \pi(v^t|z^t) \left(\frac{c(z^t, v^t)}{C(z^t)} \right)^{-\frac{\sigma}{\psi}} e(v_t)^{\frac{1+\psi}{\psi}} \right]^{-\psi}$$

Content of micro heterogeneity is summarized in $\beta(z^{t+1}, v^t), \omega(z^t)$

The Paper: Three Key Contributions

- Empirical: Estimating the preference shock process
  - The $\beta(z^{t+1}, v^t), \omega(z^t)$ are highly model-dependent and model-endogenous! Progress?
  - Alternative: estimate them directly from micro data (CEX). Need data on household consumption shares $\frac{c(z^t,v^t)}{C(z^t)}$
  - Theory: stochastic process for $\frac{c(z^t,v^t)}{C(z^t)}$ key for impact of micro heterogeneity on business cycles. Thus only interested in micro models that get this process right empirically anyway.
  - Cf. sufficient statistics approach in fiscal policy (Chetty, Saez)
- Quantitative: compute contribution of imperfect risk sharing to business cycle fluctuations.
  - Feed $\{\beta(z^{t+1}, v^t), \omega(z^t)\}$ process into model with representative household and measure fluctuations. Do the same in model with perfect risk sharing ($\beta(z^{t+1}, v^t) \equiv 1$).
  - Key quantitative finding: 20% of Great Recession accounted for by imperfect risk sharing.

Preference Shocks and Fluctuations: A Simple Example

- $t = 0, 1$
- No initial heterogeneity, no risk in $t = 0$ (i.e $z_0 = e_0 = 1$).
- $t = 1$: we have $z \in \{z_b, z_g\}$ with $\pi = 0.5$. Also $v \in \{u, m\}$ with $\pi = 0.5$ and

$$e(z, v) = \begin{cases} 1 - \varepsilon(z) & \text{if } v = v_u \\ 1 + \varepsilon(z) & \text{if } v = v_m \end{cases}$$

- $\sigma = 1$ (log-utility) and $\psi = 1$ (quadratic cost of labor).
- Three economies
  1. Representative agent (RA) economy: $\varepsilon(z) \equiv 0$.
  2. Complete markets economy (CM): $\varepsilon(z) > 0$, but Arrow securities that pay contingent on v realizations. Perfect risk sharing.
  3. Incomplete markets economy (SIM). Only risk free bond in zero net supply (imperfect risk sharing).

Preference Shocks and Equilibrium Allocations

Statistic	RA	CM	SIM
$\omega(z)$	1	$\frac{2}{(1-\varepsilon(z))^2+(1+\varepsilon(z))^2} < 1$	1
$\beta(z)$	1	1	$\frac{1}{1-\sigma_\varepsilon^2(z)} > 1$
$L_0 = C_0$	1	1	1
$L_1(z)$	1	$\omega(z)^{-\frac{1}{2}} > 1$	1
$C_1(z), Y_1(z)$	z	$z\omega(z)^{-\frac{1}{2}} > z$	z
$\frac{Y_1(z_b)}{Y_1(z_g)}$	$\frac{z_b}{z_g}$	$\left(\frac{\omega(z_g)}{\omega(z_b)}\right)^{0.5} \frac{z_b}{z_g}$	$\frac{z_b}{z_g}$
$q = \frac{1}{R}$	$\frac{\beta}{2}\sum_z \frac{1}{z}$	$\frac{\beta}{2}\sum_z \frac{1}{z\omega(z)^{-\frac{1}{2}}} < q^{RA}$	$\frac{\beta}{2}\sum_z \frac{1}{z(1-\sigma_\varepsilon^2(z))} > q^{RA}$

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- Labor supply reacts to idiosyncratic productivity shocks. CM v.s SIM? Wealth effect ($\sigma = 1$). $L_1^{CM}(z) > 1$ needs $\omega_1^{CM}(z) < 1$.

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- Full consumption insurance: $\beta^{CM}(z) = 1$. Imperfect insurance in SIM represented as patience: $\beta^{SIM}(z) > 1$. Drives down R^{SIM}.

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- Business cycles: More volatile in CM than in RA if $\sigma_\varepsilon^2(z_b) > \sigma_\varepsilon^2(z_g)$
- No difference in SIM vs. RA? No link in SIM from $\beta(z)$ to Y since no capital R adjusts flexibly to $\beta(z)$. NK elements break this.

Birds Eye Comment (1): How Robust is the Result

- Applies to large class of HANK models.
- Applies to models with capital accumulation (under certain assumptions).
- Can handle fairly general asset market structure.
- Likely does not generalize to discount factor or asset return heterogeneity. Important because literature has used these to get wealth heterogeneity right.
- Also needs interiority of labor supply. Rules out extensive margin, unemployment.

Birds Eye Comment (2): How Useful is the Result

- Big Positives
  - Powerful tool to measure answer to an important specific quantitative question.
  - Useful diagnostic tool: what aspects of (models of) household heterogeneity really matter for amplification of business cycles?
- Limitations
  - Theoretical result is not a substitute for actually solving the heterogeneous agent model unless model $\beta(z), \omega(z)$ fits data perfectly.
  - Cannot be used for counterfactual policy analysis (stimulus, anyone?) since the preference shocks $\{\beta(z^{t+1}, v^t), \omega(z^t)\}$ are not invariant to policy.

Conclusions

- Great paper!
    - A powerful general theoretical representation result.
    - Careful measurement using micro data.
    - Uses the theory and measurement to give quantitative answer to important question of policy relevance.

THANK YOU