The Limits of Monetary Economics

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Is the medium-of-exchange role of money relevant for Monetary Economics?
Current wisdom: Monetary Economics without $M$

Medium-of-exchange considerations are irrelevant for monetary transmission in modern high-velocity credit economies.
Current wisdom: Monetary Economics without $M$

*Medium-of-exchange considerations are irrelevant for monetary transmission in modern high-velocity credit economies.*

Based on two results:

1. Monetary equilibrium is continuous under a certain “cashless limit”
2. Real balances play small quantitative role in calibrations
Current wisdom: Monetary Economics without $M$

Medium-of-exchange considerations are irrelevant for monetary transmission in modern high-velocity credit economies.

Based on two results:

1. Monetary equilibrium is continuous under a certain “cashless limit”
2. Real balances play small quantitative role in calibrations

Both rely on a class of reduced-form monetary models (CIA/MIU)
What we do

1. Develop a model
   - explicit about the roles of money and credit in exchange
   - can exhibit any transaction velocity of money
   - allows for market power in credit intermediation

2. Study monetary policy as velocity becomes very high
Main finding

Medium-of-exchange considerations are resilient and significant.
Main findings

Given financial frictions (credit intermediaries with *some* market power):

1. As the cash-and-credit economy converges to a pure-credit economy, the monetary equilibrium does not converge to the equilibrium of the economy without money.
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1. As the cash-and-credit economy converges to a pure-credit economy, the monetary equilibrium does not converge to the equilibrium of the economy without money.

2. Effects of monetary policy remain large even as aggregate real money balances vanish along the cashless limit.
Main findings

Given financial frictions (credit intermediaries with *some* market power):

1. As the cash-and-credit economy converges to a pure-credit economy, the monetary equilibrium does not converge to the equilibrium of the economy without money.

2. Effects of monetary policy remain large even as aggregate real money balances vanish along the cashless limit.

⇒ Cashless economies are not good approximations to monetary economies—even high-velocity economies.
Money affects prices in transactions that do not involve money.
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- The *option* to engage in monetary trade disciplines the market power of financial intermediaries.

- *Off-equilibrium* small volume of monetary trades feeds back into the prices negotiated in all pure-credit nonmonetary transactions.
Related work

**Leverage and asset prices**
Kiyotaki and Moore (1997, 2005)

**Strategic bargaining advantage from holding money**

**Trade option as disciplining device for market power and mark-ups**
Bhagwati (1965), Markusen (1981), Baumol (1982), Holmes et al. (2014)

**Value of resale options**
Harrison and Kreps (1978)

**Monetary policy in cashless and near-cashless economies**
Environment

- **Time.** Discrete, infinite horizon, two subperiods per period

- **Population.** \([0, 1]\) investors, \([0, 1]\) brokers

- **Commodities.** Two divisible, nonstorable consumption goods:
  - **dividend good**
  - **general good**
Preferences

Brokers: $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (c_t - h_t)$

Investors: $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\varepsilon_t y_t + c_t - h_t)$

$\varepsilon_t$: valuation shock, i.i.d. over time, cdf $G(\cdot)$ on $[\varepsilon_L, \varepsilon_H]$
Endowments and production technology

First subperiod

- $A^s$ productive units (trees)
- Each unit yields $y_t$ dividend goods *at the end of the first subperiod*

Second subperiod

- Linear technology to transform effort into general goods
Financial assets

Money

- \( A_{t+1}^m = \mu A_t^m, \mu \in \mathbb{R}_{++} \) (implemented with lump-sum taxes)

Equity

- \( A^s \) equity shares

Bond

- issued by investors in first subperiod of \( t \), repaid next subperiod
- 1 bond = claim to 1 unit of general good
- no commitment; if issuer defaults, bond holder appropriates fraction \( \lambda \) of the issuer’s equity shares
Market structure | first subperiod: OTC trade

Two contemporaneous competitive markets
- bond market (bonds and money)
- equity market (equity and money)

Brokers
- always access bond market

Investors
- prob. $\alpha$: access equity market only
- prob. $\alpha_c \equiv 1 - \alpha$: access equity market and contact a bond broker

Bilateral terms of trade between investor and broker
- Nash bargaining (investor bargaining power $\theta$)
Market structure | second subperiod: centralized trade

- Walrasian trade between all brokers and investors
- equity, general good, money
Timeline and market structure
Investor trade in the OTC market | monetary economy

- Investor without credit access:
  - Sell equity
  - Hold cash
- Investor with credit access:
  - Sell equity
  - Long bond
  - Spend cash
- Buy equity
- Short bond
- Spend cash
Investor trade in the OTC market | nonmonetary economy
OTC trade | valued money | with credit access

\[
\max_{(\bar{a}^m, \bar{a}^s, \bar{a}^b, k) \in \mathbb{R}_+^2 \times \mathbb{R} \times \mathbb{R}_+} \Gamma_t(a, \varepsilon)^\theta k^{1-\theta}
\]

\[
\bar{a}^m + p_t \bar{a}^s + q_t \bar{a}^b \leq a^m + p_t a^s
\]

\[
-\lambda \phi^s_t \bar{a}^s \leq \bar{a}^b
\]

\[
0 \leq \Gamma_t(a, \varepsilon)
\]

\[
\Gamma_t(a, \varepsilon) \equiv \varepsilon y_t \bar{a}^s + W_t(\bar{a}^m, \bar{a}^s, \bar{a}^b, k) - [\varepsilon y_t \hat{a}^s_t(a, \varepsilon) + W_t(\hat{a}^m_t(a, \varepsilon), \hat{a}^s_t(a, \varepsilon), 0, 0)]
\]
Euler equation
Euler equation | equity | monetary equilibrium

\[ \phi_t^s = \beta \mathbb{E}_t \left\{ \bar{c}y_{t+1} + \phi_{t+1}^s \right\} \]

\[ + \alpha_c \theta \frac{\lambda \phi_{t+1}^s}{\phi_{t+1}^s - \lambda \phi_{t+1}^s} \int_{\epsilon_{t+1}}^{\epsilon_{t+1}^H} (\epsilon y_{t+1} + \phi_{t+1}^s - \bar{\phi}_{t+1}^s) \, dG(\epsilon) \]

\[ + \alpha_c \theta \int_{\epsilon_L}^{\epsilon_{t+1}^*} \left[ \bar{\phi}_{t+1}^s - (\epsilon y_{t+1} + \phi_{t+1}^s) \right] \, dG(\epsilon) \]
Euler equation | equity | monetary equilibrium

\[ \phi^s_t = \beta \mathbb{E}_t \left\{ \bar{\varepsilon}y_{t+1} + \phi^s_{t+1} \right\} \\
+ \alpha c \theta \frac{\lambda \phi^s_{t+1}}{\phi^s_{t+1} - \lambda \phi^s_{t+1}} \int_{\varepsilon^{**}_{t+1}}^{\varepsilon_H} \left( \varepsilon y_{t+1} + \phi^s_{t+1} - \bar{\phi}^s_{t+1} \right) dG (\varepsilon) \\
+ \alpha c \theta \int_{\varepsilon_L}^{\varepsilon^{**}_{t+1}} \left[ \bar{\phi}^s_{t+1} - (\varepsilon y_{t+1} + \phi^s_{t+1}) \right] dG (\varepsilon) \\
+ \left[ \alpha + \alpha c (1 - \theta) \right] \int_{\varepsilon_L}^{\varepsilon^*_t+1} \left[ p_{t+1} \phi^m_{t+1} - (\varepsilon y_{t+1} + \phi^s_{t+1}) \right] dG (\varepsilon) \]
Nonmonetary equilibrium
Proposition

There exists a unique RNE:

\[
\varphi^n = \bar{\varepsilon} + \alpha_c \theta \left[ \int_{\varepsilon_L}^{\varepsilon^n} (\varepsilon^n - \varepsilon) \, dG(\varepsilon) + \frac{\lambda}{1 - \lambda} \int_{\varepsilon^n}^{\varepsilon_H} (\varepsilon - \varepsilon^n) \, dG(\varepsilon) \right]
\]

\(\varepsilon^n \in [\varepsilon_L, \varepsilon_H]\) is the unique solution to

\[
G(\varepsilon^n) = \lambda
\]
Monetary equilibrium
Monetary equilibrium | existence

\[ \varepsilon_L < \varepsilon^* < \varepsilon^{**} = \varepsilon^n < \varepsilon_H \]

\[ \hat{I}(\lambda) \]

\[ \bar{I}(\lambda) \]
Proposition

If \( \bar{\lambda} < \lambda < \bar{\lambda} \), there exists a unique RME:

\[
\varphi = \varphi^n + [\alpha + \alpha_c (1 - \theta)] \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) dG(\varepsilon)
\]

\( \varepsilon^{**} = \varepsilon^n \) and \( \varepsilon^* \in (\varepsilon_L, \varepsilon^n) \) is the unique solution to

\[
\frac{[\alpha + \alpha_c (1 - \theta)] \int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*) dG(\varepsilon) + \alpha_c \theta [\varepsilon^n - \varepsilon^* + \frac{1}{1 - \lambda} \int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^n) dG(\varepsilon)]}{\varphi^n + [\alpha + \alpha_c (1 - \theta)] \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) dG(\varepsilon)} = \lambda
\]
Cashless limit
Limit as fraction of cash-only trades goes to zero

- $\alpha = 1 - \alpha_c \in [0, 1]$: prob. of not accessing credit

- As $\alpha \to 0$, the equity price in the RNE converges to:

$$\lim_{\alpha \to 0} \varphi^n = \bar{\varepsilon} + \theta \left[ \int_{\varepsilon_L}^{\varepsilon_H} (\varepsilon^n - \varepsilon) dG(\varepsilon) + \frac{\lambda}{1 - \lambda} \int_{\varepsilon_H}^{\varepsilon_H} (\varepsilon - \varepsilon^n) dG(\varepsilon) \right]$$
What happens to monetary equilibrium as $\alpha \to 0$?

$$\lim_{\alpha \to 0} Z = \lim_{\alpha \to 0} \frac{1}{V} = 0$$

$$\lim_{\alpha \to 0} Z = \lim_{\alpha \to 0} \frac{1}{V} > 0$$
Proposition (high policy rate)

If $\hat{\zeta} < \iota < \bar{\zeta}$ (positive inside rate), then as $\alpha \to 0$,

$$Z \to 0$$

$$\mathcal{V} \to \infty$$

$$\varphi \to \lim_{\alpha \to 0} \varphi^n + (1 - \theta) \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) \, dG(\varepsilon)$$

where $\varepsilon^* \in (\varepsilon_L, \varepsilon^n)$ is the unique solution to

$$\frac{(1-\theta) \int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^*) \, dG(\varepsilon) + \theta \left[ \varepsilon^n - \varepsilon^* + \frac{1}{1-\lambda} \int_{\varepsilon^*}^{\varepsilon_H} (\varepsilon - \varepsilon^n) \, dG(\varepsilon) \right]}{\bar{\varepsilon} + (1-\theta) \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) \, dG(\varepsilon) + \theta \left[ \int_{\varepsilon_L}^{\varepsilon^n} (\varepsilon^n - \varepsilon) \, dG(\varepsilon) + \frac{\lambda}{1-\lambda} \int_{\varepsilon^n}^{\varepsilon_H} (\varepsilon - \varepsilon^n) \, dG(\varepsilon) \right]} = l$$
Proposition (high policy rate)

If $\zeta < \iota < \tilde{\zeta}$ (positive inside rate), then as $\alpha \to 0$,

$$
Z \to 0 \quad \forall \to \infty
$$

$$
\varphi \to \lim_{\alpha \to 0} \varphi^n + (1 - \theta) \int_{\varepsilon_L}^{\varepsilon_*} (\varepsilon^* - \varepsilon) dG(\varepsilon)
$$

where $\varepsilon^* \in (\varepsilon_L, \varepsilon^n)$ is the unique solution to

$$
\frac{(1 - \theta) \int_{\varepsilon_*}^{\varepsilon^*} (\varepsilon - \varepsilon^*) dG(\varepsilon) + \theta \left[ \varepsilon^n - \varepsilon^* + \frac{1}{1 - \lambda} \int_{\varepsilon}^{\varepsilon^*} (\varepsilon - \varepsilon^n) dG(\varepsilon) \right]}{\varepsilon + (1 - \theta) \int_{\varepsilon}^{\varepsilon_L} (\varepsilon - \varepsilon) dG(\varepsilon) + \theta \left[ \int_{\varepsilon}^{\varepsilon^n} (\varepsilon^n - \varepsilon) dG(\varepsilon) + \frac{\lambda}{1 - \lambda} \int_{\varepsilon}^{\varepsilon^n} (\varepsilon - \varepsilon^n) dG(\varepsilon) \right]} = \lambda
$$
Limit as fraction of cash-only trades goes to zero

**Proposition (high policy rate)**

If $\hat{\zeta} < \iota < \bar{\zeta}$ (*positive inside rate*), then as $\alpha \to 0$,

$$Z \to 0$$
$$\mathcal{V} \to \infty$$

$$\varphi \to \lim_{\alpha \to 0} \varphi^n + (1 - \theta) \int_{\epsilon^*}^{\epsilon_L} (\epsilon^* - \epsilon) \, dG(\epsilon)$$

where $\epsilon^* \in (\epsilon_L, \epsilon^n)$ is the unique solution to

$$\frac{(1 - \theta) \int_{\epsilon^*}^{\epsilon_H} (\epsilon - \epsilon^*) \, dG(\epsilon) + \theta \left[ \epsilon^n - \epsilon^* + \frac{1}{1 - \lambda} \int_{\epsilon^*}^{\epsilon_L} (\epsilon - \epsilon^n) \, dG(\epsilon) \right]}{\bar{\epsilon} + (1 - \theta) \int_{\epsilon^*}^{\epsilon_L} (\epsilon - \epsilon^*) \, dG(\epsilon) + \theta \left[ \int_{\epsilon_L}^{\epsilon^*} (\epsilon^n - \epsilon) \, dG(\epsilon) + \frac{\lambda}{1 - \lambda} \int_{\epsilon^n}^{\epsilon_H} (\epsilon - \epsilon^n) \, dG(\epsilon) \right]} = l$$
Intuition
Why the discontinuity as $\alpha \to 0$?

\[
\lim_{\alpha \to 0} \frac{Z}{\varphi} = \lim_{\alpha \to 0} \frac{1}{\mathcal{V}} = 0 < \lim_{\alpha \to 0} (\varphi - \varphi^n)
\]
Why the discontinuity as $\alpha \to 0$?

$$\lim_{\alpha \to 0} \frac{Z}{\varphi} = \lim_{\alpha \to 0} \frac{1}{Y} = 0 < \lim_{\alpha \to 0} (\varphi - \varphi^n) = (1 - \theta) \int_{\epsilon_L}^{\epsilon^*} (\epsilon^* - \epsilon) dG(\epsilon)$$
Why the discontinuity as $\alpha \to 0$?

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\lim_{\alpha \to 0} \frac{Z}{\varphi} = \lim_{\alpha \to 0} \frac{1}{V} = 0 < \lim_{\alpha \to 0} (\varphi - \varphi^n) = (1 - \theta) \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) \, dG(\varepsilon)
\]

\[
\lim_{\alpha \to 0} \frac{Z/\varphi}{\alpha} = \lim_{\alpha \to 0} \frac{G(\varepsilon^*)}{1 - G(\varepsilon^*) \alpha} = \lim_{\alpha \to 0} G(\varepsilon^*) > 0
\]
Why the discontinuity as $\alpha \rightarrow 0$?

$$\lim_{\alpha \rightarrow 0} \frac{Z}{\varphi} = \lim_{\alpha \rightarrow 0} \frac{1}{\mathcal{Y}} = 0 < \lim_{\alpha \rightarrow 0} (\varphi - \varphi^n) = (1 - \theta) \int_{\epsilon_L}^{\epsilon^*} (\epsilon^* - \epsilon) \, dG(\epsilon)$$

$$\lim_{\alpha \rightarrow 0} \frac{Z}{\varphi} = \lim_{\alpha \rightarrow 0} \frac{G(\epsilon^*)}{1 - G(\epsilon^*)} \alpha = \lim_{\alpha \rightarrow 0} G(\epsilon^*) > 0$$

$$\lim_{\lambda \rightarrow \lambda} \frac{Z}{\varphi} = \lim_{\lambda \rightarrow \lambda} \frac{G(\epsilon^*)}{[1 - G(\epsilon^*)] \alpha + \alpha_c} = 0$$
Quantitative analysis
The monetary equilibrium is not continuous under the cashless limit; is the discontinuity quantitatively relevant?

Are these monetary frictions important for monetary policy transmission in modern high-velocity credit economies?

→ Study monetary transmission to asset prices (well documented empirically)
## Calibration

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{t+1} = e^{x_{t+1}} y_t$</td>
<td>$g = .04$</td>
<td>Ludvigson-Letttau (05)</td>
</tr>
<tr>
<td>$x_{t+1} \sim \mathcal{N}(g, \Sigma^2)$</td>
<td>$\Sigma = .12$</td>
<td></td>
</tr>
<tr>
<td>dividend process</td>
<td>$\delta = .075$</td>
<td>risk proxy</td>
</tr>
<tr>
<td>asset depreciation</td>
<td>$\rho^p = .0447$</td>
<td>3-M ED future (94-08)</td>
</tr>
<tr>
<td>nominal policy rate</td>
<td>$\pi - g = .0269$</td>
<td>CPI inflation (94-08)</td>
</tr>
<tr>
<td>inflation rate</td>
<td>$r = .0178$</td>
<td>$\rho^p - (\pi - g)$</td>
</tr>
<tr>
<td>real risk-free rate</td>
<td>$1 - \lambda = .25$</td>
<td>Rule 4210 (FINRA)</td>
</tr>
<tr>
<td>margin</td>
<td>$\alpha = .04$</td>
<td>$\mathcal{V} = 25$ daily (CHIPS)</td>
</tr>
<tr>
<td>fraction with no credit</td>
<td>$1 - \theta = .84$</td>
<td>2.3% margin spread</td>
</tr>
<tr>
<td>broker market power</td>
<td>$\ln \varepsilon \sim \mathcal{N}(-\frac{1}{2}\Sigma_e^2, \Sigma_e^2)$</td>
<td>2.08</td>
</tr>
</tbody>
</table>
| idiosyncratic shocks                          | $\mathcal{S} \equiv \left| \frac{d\phi_s}{d\rho^p} \right| = 11$ | $\mathcal{S}$ = 11
Quantitative exercises

- Compute asset price responses to increases in $\rho^p$ for all $(\alpha, \lambda, \theta)$

- Since response is negative, report the absolute value of the semi-elasticity of the asset price to the policy rate, i.e.,

$$S = \left| \frac{d\phi^s}{d\rho^p} \right|$$
\[ \lim_{\alpha \to 0} S \text{ as a function of } \lambda \text{ and } \theta \]
Reduced-form models of money demand
The recursive equilibrium conditions of our model can be obtained from the following representation:

$$\max_{\{c_t, h_t, a_{t+1}\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t [U(c_{1t}, c_{2t}) + c_t - h_t]$$

s.t. $c_t + \phi_s a_{t+1} + \phi_m a_{t+1} = h_t + (\bar{\epsilon} y_t + \phi_s) a_s + \phi_m a^m + T_t$

$$c_{1t} = \frac{a^m_t}{p_t} y_t$$

$$c_{2t} = a_s^t y_t$$
Reduced-form money

The recursive equilibrium conditions of our model can be obtained from the following representation:

\[
\max_{\{c_t, h_t, a_{t+1}\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t [U(c_{1t}, c_{2t}) + c_t - h_t]
\]

s.t. \( c_t + \phi^s_t a^s_{t+1} + \phi^m_t a^m_{t+1} = h_t + (\bar{\epsilon} y_t + \phi^s_t) a^s_t + \phi^m_t a^m_t + T_t \)

\[
\begin{align*}
    c_{1t} &= \frac{a^m_t}{p_t} y_t \\
    c_{2t} &= a^s_t y_t 
\end{align*}
\]

with

\[
U(c_{1t}, c_{2t}) \equiv u^z c_{1t} + u^s c_{2t}
\]
Reduced-form money

The recursive equilibrium conditions of our model can be obtained from the following representation:

$$\max_{\{c_t, h_t, a_{t+1}\}} \mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[U(c_{1t}, c_{2t}) + c_t - h_t \right]$$

s.t. $c_t + \phi_t a_{t+1}^s + \phi_t a_{t+1}^m = h_t + (\bar{\epsilon} y_t + \phi_t a_{t+1}^s) + \phi_t a_{t+1}^m + T_t$

$$c_{1t} = \frac{a_{t}^m}{p_t} y_t$$

$$c_{2t} = a_{t}^s y_t$$

with

$$U(c_{1t}, c_{2t}) \equiv u^z c_{1t} + u^s c_{2t}$$
Reduced-form money | first-order conditions

\[ \varphi = \bar{\epsilon} + u^s \]

\[ \iota \geq \frac{u^z}{\varphi}, \text{ with } " = " \text{ if } 0 < Z \]

- Since \( u^s \) and \( u^z \) are treated as "deep" parameters:
  - \( \varphi \) determined independently of \( \iota \) and money
  - \( Z > 0 \) only if \( \iota = \frac{u^z}{\varphi} \), and \( Z = 0 \) if \( \iota > \frac{u^z}{\varphi} \)

→ Monetary considerations are irrelevant
Reduced-form money | first-order conditions

\[
\varphi = \bar{\epsilon} + u^s
\]

\[
\iota \geq \frac{u^z}{\varphi}, \text{ with } " = " \text{ if } 0 < Z
\]

- But \( u^s \) and \( u^z \) are not “deep” parameters...

\[
u^s = [\alpha + \alpha_c (1 - \theta)] \int_{\varepsilon_L}^{\varepsilon^*} (\varepsilon^* - \varepsilon) \, dG (\varepsilon)
\]

\[
+ \alpha_c \theta \left[ \int_{\varepsilon_L}^{\varepsilon^{**}} (\varepsilon^{**} - \varepsilon) \, dG (\varepsilon) + \frac{\lambda}{1 - \lambda} \int_{\varepsilon^{**}}^{\varepsilon_H} (\varepsilon - \varepsilon^{**}) \, dG (\varepsilon) \right]
\]
Reduced-form money | first-order conditions

\[ \varphi = \bar{\varepsilon} + u^s \]

\[ \iota \geq \frac{u^z}{\varphi}, \text{ with } “=“ \text{ if } 0 < \bar{Z} \]

- But \( u^s \) and \( u^z \) are not “deep” parameters...
- The utility function itself changes with monetary policy

\[ u^s = u^s (\iota) \]
Reduced-form money | first-order conditions

\[ \phi = \bar{\epsilon} + u^s \]

\[ \iota \geq \frac{u^z}{\phi}, \text{ with } " = " \text{ if } 0 < Z \]

- But \( u^s \) and \( u^z \) are not “deep” parameters...
- The shape of the utility function depends on:
  - policy, credit conditions, and mark-ups in financial markets

\[ U \left( c, \frac{M}{p}; \iota, \alpha, \lambda, \theta \right) \]
Medium-of-exchange considerations are important for monetary transmission—even in near-cashless economies where credit supports a large volume of transactions with arbitrarily small real balances.
Thank you all

for Minnesota Economics