Classic Policy Benchmarks for Economies with Substantial Inequality

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Abstract

We study a simple benchmark macroeconomic model with substantial heterogeneity among households, enough to cause the Gini coefficients for income, financial wealth, and consumption to approach those in the U.S. data in the model equilibrium. The model includes aggregate shocks as well as both permanent and temporary idiosyncratic risk. We introduce four policymaker types into this economy representing: (1) monetary policy, (2) fiscal policy, (3) labor market policy, and (4) education policy. We show that these four policymaker types, acting in concert, can achieve a first-best allocation of resources in this setting. We argue that the roles of these policymaker types are "classic" and match up well with observed policymaker roles in OECD countries. The education policymaker has a substantial impact on inequality. We hope this economy can provide a benchmark from which other aspects of the interaction between inequality and policy can be studied.

Keywords: Optimal monetary and fiscal policy, life cycle economies, heterogeneous households, credit market friction, nominal GDP targeting, non-state contingent nominal contracting, inequality, Gini coefficients.

JEL codes: E4, E5.

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1 Introduction

1.1 Motivation

There has been increasing interest in heterogeneous-agent DSGE models with monetary policy in recent years. The models hold the promise of producing substantial degrees of inequality in income, financial wealth, and consumption—enough to approach measured Gini coefficients for large modern economies. What is the role of monetary policy in such a model? In this paper, we consider a benchmark economy in this class. We show that the role of monetary policy may be largely unchanged, provided other types of macroeconomic and microeconomic policymakers are conducting their policies in an appropriate manner. Our core result represents an idealized case which we think could be perturbed in interesting ways in future research.

1.2 What we do

We construct a heterogeneous-agent economy with explicit life cycle considerations as well as intra-cohort heterogeneity. The model features three types of aggregate shocks: (1) total factor productivity, (2) labor supply, and (3) aggregate demand. We include both both permanent and temporary idiosyncratic risk at the household level. We follow Bullard and DiCecio (2021) and put the model in the context of a simple and symmetric structure that puts a premium on interpretation of results. We include four policymaking authorities representing: (1) monetary policy, (2) fiscal policy, (3) labor market policy, and (4) education policy.

1.3 Main findings

In our main finding, we describe a competitive equilibrium in which the four policymakers act in concert to attain a first-best allocation of resources defined according to a social welfare theorem. We also argue at the end of the paper that the model equilibrium can in principle be calibrated to match arbitrarily large Gini coefficients for income, financial wealth, or consumption.

We describe the policymaker roles that produce the first-best allocation as "classic" because they generally match up well with a long tradition in macroeconomic research and actual policymaking: (1) The monetary authority reacts to shocks each period in order to achieve the Wicksellian natural real rate of interest for the economy; (2) The fiscal authority raises revenue via a non-state contingent linear labor income tax on all households; (3) The labor market authority runs an unemployment insurance program; and (4) The education authority minimizes the variance of beginning-of-economic-life human capital endowments. *Hence, the main result is that classic policy prescriptions can achieve the first-best allocation of resources in this benchmark heterogeneous-agent economy.* This result may be helpful in understanding more complicated economies that deviate from this benchmark.

Some aspects of these findings may be somewhat surprising. The proposed classic policies appear broadly similar to actual policies in place in many OECD economies: (1) The monetary authority meets often and reacts to current developments; (2) Simple linear labor income taxes set for the long run by the fiscal authority can be used without distorting the labor supply; (3) OECD countries have programs designed to insure against shocks to household income; (4) OECD countries have significant investments in pre-age-20 educational systems designed to prepare people for effective participation in the economy during adulthood.

Another somewhat surprising finding is that the best policy combination drives the consumption Gini toward zero but leaves income and financial wealth Ginis substantially positive. This suggests that some, perhaps a substantial amount, of observed income and financial wealth inequality is due to life-cycle effects alone.

By design, the model has a paper-and-pencil solution despite the three aggregate shocks and the idiosyncratic risk.

1.4 Some recent literature

Galí (2018) surveys the recent literature on heterogeneous agent macroeconomics and monetary policy. Kaplan, Moll and Violante (2018) have been leaders in the burgeoning heterogeneous agent New Keynesian ("HANK") literature. The literature generally follows the Aiyagari-Bewley-Huggett tradition in emphasizing idiosyncratic household income risk which is uninsurable along with borrowing constraints. The current paper has simple forms of permanent (beginning of economic life) and temporary idiosyncratic risk but allows insurance against the temporary component, and places no borrowing constraints on households.

Bhandari, Evans, Golosov and Sargent (2021) consider a Ramsey problem in a HANK model. They find it difficult to map Ramsey-optimal fiscal-monetary policy into the types of policies typically observed in OECD economies. The current paper provides a contrasting result.

Bullard and DiCecio (2021) contains an analysis similar to the one in the current paper. Relative to that analysis the present paper has additional aggregate shocks, more idiosyncratic risk, some taxes, and policymakers other than the monetary authority. Bullard and Singh (2020) discuss nominal GDP targeting and heterogeneous labor supply in a simpler version of the model presented here. The present paper abstracts from money demand and the effective lower bound. For an analysis of these issues in a related framework, see Azariadis, Bullard, Singh and Suda (2019).

Optimal monetary policy in the present paper is a version of NGDP targeting, a topic that has a long history in monetary economics. The version used here is based on work by Koenig (2013) and Sheedy (2014). See also the discussions of the Sheedy (2014) paper by Bullard (2014) and Werning (2014).

The present paper places emphasis on idiosyncratic risk at the beginning of economic life, viewed here as approximately age 20. Huggett, Ventura and Yaron (2011) argue that a majority of lifetime earnings variance can be explained by individual characteristics at age 23. We take this finding as supporting our stylized representation of unmodeled pre-age 20 human capital accumulation as an exogenous random process that, once it occurs, has permanent effects on individual lifetime earnings.

The model also features an aggregate demand shock that affects output. Stochastic demand $D(t)$ enters the preferences of households, making them uncertain about the extent to which they may wish to consume in the future. We couple this shock with a simple version of variable labor utilization in the production technology. We casually motivate this as the "restaurant" model: Labor and fixed capital are set for the evening, but the labor has to work more intensively if a lot of customers arrive, and less intensively if fewer customers arrive. Output is higher in periods of high demand, and lower otherwise. The total labor input is hours multiplied by labor intensity, which is simply the state of aggregate demand. Roughly speaking, this causes "demand shocks to look like TFP shocks." For discussion of related but much more sophisticated versions of this idea, see for instance Bai, Rios-Rull, and Storesletten (2019) as well as Huo and Rios-Rull (2020).

Labor supply—hours worked—in the model depends on the agent's position in the life cycle and the agent's relative productivity at that point in the life cycle, but not on the real wage. Given these labor supply findings, an appropriately specified cross-section regression or panel regression on changes in labor supply with respect to changes in real compensation using data from the model equilibrium would yield coefficients of zero, broadly consistent with the literature in this area. For a discussion of recent research in this area, see for instance Chang, Kim, Kwon, and Rogerson (2020). One interpretation of this result is that macroeconomic policies in OECD countries are broadly similar to the ones in this model and may therefore be pushing these regression coefficients toward zero.

Doepke and Schneider (2006) study redistribution across households in response to inflation shocks. They argue that the effects of such shocks in the historical data are quantitatively large because so many assets are held and traded in nominal terms. We think this result helps motivate our assumption concerning nominal contracting. From the perspective of the current model, the Doepke and Schneider (2006) result might be interpreted as evidence that actual monetary policy was not optimal over this period.

2 Environment

2.1 Demographics, symmetry, and notation

We work with a class of models that can become quite complicated very quickly. To control the heterogeneity we wish to discuss, we make an important set of symmetry assumptions. The upshot of the symmetry assumptions is that, under certain conditions, they will allow us to guess and verify the general equilibrium solution of the model, which is essentially that the real interest rate will be equal to the stochastic rate of output growth each period. For more discussion of these assumptions and their role, we refer the interested reader to Bullard and DiCecio (2021). Relative to that paper, we have multiple aggregate shocks here instead of just one, as well as taxes and other features.

Each period, a new continuum of households enters the economy, makes economic decisions over the next $T+1 = 241$ periods ("quarterly"), and then exits the economy. To fix ideas, a quarterly cohort in the U.S. would be comprised of, very approximately, one million people. We are using a continuum to represent this.

The symmetry assumptions are:[1] (1) All households have log preferences defined over consumption and leisure; (2) All households have a discount factor of unity; (3) All households are endowed with a personal productivity profile at the beginning of their life cycle which begins low, rises to a peak exactly in the middle of the life cycle, and then declines in a symmetric fashion; (4) The aggregate production technology is linear in the aggregate effective labor input. We allow variable utilization of the labor input.

We generally use the notation that small letters represent agent-level quantities and large letters represent aggregate quantities, and also that subscripts represent the date of entry into the economy and dates in parentheses represent the real time of the model. We typically express variables in real terms throughout this paper. In particular, real consumption is denoted c and the real wage is denoted w. However, there are also nominal quantities in the model due to the non-state contingent nominal contracting (NSCNC) assumption. Accordingly, we denote net asset holding a understanding that it is expressed in nominal terms. The aggregate price level is denoted by P.

2.2 Preferences

For an agent i entering the economy at date t preferences are given by

$$v_{t,i} = \sum_{s=0}^{T} \eta \ln \tilde{c}_{t,i}\left(t+s\right) + \left(1-\eta\right) \ln \ell_{t,i}\left(t+s\right). \tag{1}$$

where $\tilde{c}_{t,i}\left(t+s\right) \equiv c_{t,i}\left(t+s\right) D\left(t+s\right)$ and the state of aggregate demand D is given by

$$D\left(t+1\right) = \delta\left(t,t+1\right) D\left(t\right) \tag{2}$$

where δ is stochastic and discussed below. The preferences for other households are analogous and the state of demand is experienced by all households at each date. Following Bai, Ríos-Rull and Storesletten (2017), we will allow the state of aggregate demand to influence productive activity in the economy.

[1] For a broader discussion of the symmetry assumptions, see Bullard and DiCecio (2021).

2.3 Production technology

The linear production technology transforms labor hours into aggregate real output $Y(t)$ according to

$$Y(t) = Q(t) D(t) N(t) L(t) \tag{3}$$

where $Q(t)$ is the level of aggregate total factor productivity, $N(t)$ is the index of the size of the labor force, $D(t)$ is the state of aggregate demand, and $L(t)$ is aggregate labor hours supplied when the size of the labor force is fixed (which corresponds to a normalization $N(t) = 1$). We sometimes call this "core labor hours." It is a feature of the equilibrium we study that $L(t)$ will be a constant $\bar{L}(1-\bar{u})$ for all t, where $\bar{u}$ is the unemployment rate. This production technology therefore says simply that output can increase over time either because total factor productivity improves, the size of the labor force expands, or the intensity of hours worked increases in order to meet relatively high demand according to the "restaurant model" described earlier.

Total hours worked is given by $N(t) L(t)$, and $N(t)$ follows a stochastic process as described below, and this means that increases or decreases in the size of the labor force apply to all cohorts simultaneously in order to maintain the symmetry assumptions of the model.[2] The state of demand dictates how intensively the labor hours will be used at each date, with an average value of unity. The total labor input at date t is $N(t) D(t) L(t)$. We assume that

$$\begin{aligned} Q(t) &= \lambda(t-1,t) Q(t-1) & (4) \\ N(t) &= \nu(t-1,t) N(t-1) & (5) \\ D(t) &= \delta(t-1,t) D(t-1) & (6) \end{aligned}$$

where $\lambda(t-1,t)$ is the gross growth rate of productivity between date $t-1$ and date t, $\nu(t-1,t)$ is the gross growth rate of the labor force between date $t-1$ and date t, and $\delta(t-1,t)$ is the gross growth rate of the state of aggregate demand between date $t-1$ and date t.[3] We specify the means of λ and ν as values greater than one, and the mean of δ equal to one.[4]

If we denote the fraction of time spent working by household i of cohort $t-s$ as $[1-\ell_{t-s,i}(t)] \in (0,1)$, core labor hours $L(t)$ can be written as

$$L(t) = (1-\bar{u}) \int \{e_{0,i}[1-\ell_{t,i}(t)] + e_{1,i}[1-\ell_{t-1,i}(t)] + \cdots + e_{T,i}[1-\ell_{t-T,i}(t)]\} dF_{\xi}. \tag{7}$$

[2] We think of changes in the index $N(t)$ as a form of symmetric immigration, where the same number of new households enter or exit the model at each age node carrying the assets that they would have had if they had been in the economy their entire life.

[3] A wide range of stochastic processes for λ, ν and δ would work, as long as they imply Q, N, and $D > 0$ $\forall t$.

[4] We ignore issues related to the effective lower bound in this paper. The zero lower bound or effective lower bound would be encountered with a sufficiently negative set of shocks combined with enough serial correlation to cause the expected rate of nominal GDP growth (=nominal consumption growth) to be negative. See Azariadis et al. (2019) for a discussion of this issue.

The marginal product of core labor is

$$w(t) = Q(t)\, N(t)\, D(t), \tag{8}$$

and the associated marginal product of an extra hour of labor is $Q(t)\, D(t)$. We conclude that

$$w(t) = \lambda(t-1,t)\, \nu(t-1,t)\, \delta(t-1,t)\, w(t-1). \tag{9}$$

Given that $L(t)$ will turn out to be a constant, aggregate real output evolves simply as

$$Y(t) = \lambda(t-1,t)\, \nu(t-1,t)\, \delta(t-1,t)\, Y(t-1) \tag{10}$$

in the equilibrium under optimal macroeconomic policy. We will show below that the gross real interest rate equals the stochastic gross real output growth rate, that is, $\lambda(t-1,t)\, \nu(t-1,t)\, \delta(t-1,t)$ period by period in the equilibrium we study.

2.4 Life-cycle productivity

Each household i is randomly and permanently assigned a personal productivity profile $e_i = \{e_{0,i}, e_{1,i}, ..., e_{240,i}\}$ when entering the model. This profile dictates how much productivity $e_{s,i}$ each agent will have to sell in an economywide competitive labor market at each date of their life cycle s.

In this paper we assume that profiles are symmetric—they begin low, rise and peak exactly in the middle of life, then decline symmetrically back to the low level. Profiles are restricted to be consistent with interior solutions to all household problems—households will choose to work low hours but not zero.

To make this assignment for each household, we begin with a baseline productivity profile which is denoted $e = \{e_0, e_1, ..., e_{240}\}$. Each household then draws a scaling factor ξ from a lognormal distribution as they enter the model, that is, $\ln(\xi) \sim \mathcal{N}(\mu, \sigma^2)$.[5] The product of their scaling factor and the baseline productivity profile permanently determines their life-cycle productivity, that is, ξe. Accordingly, there will be arbitrarily rich and arbitrarily poor households in the economy.

The rationale for a permanent life-cycle productivity assignment is simple: The assignment of productivity profiles is a stand-in for the human capital development (not modeled here) that takes place before age 20 in actual economies, including schooling, parenting and any job experience. Huggett, Ventura and Yaron (2011) argue that differences in initial conditions (human capital at age 23) are more important than subsequent shocks in explaining lifetime income.

Figure 1 shows a baseline productivity profile that meets the conditions we have outlined. We will use this baseline profile to illustrate key aspects of the model. This particular baseline profile has productivity in the middle portion of the life cycle 50%

[5]For illustrative purposes, ξ is drawn from a uniform distribution $U[a,b]$ in the figures shown later in the paper.

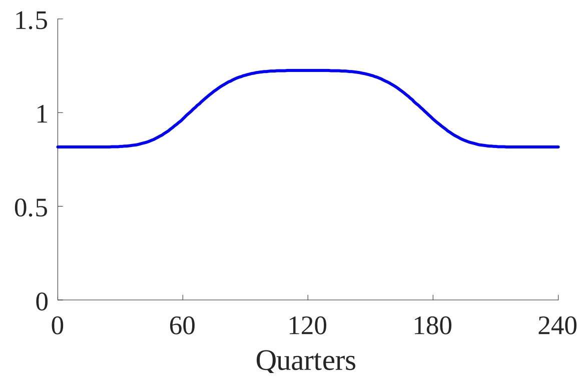

Figure 1: A baseline personal productivity endowment profile. The profile is symmetric and peaks in the middle period of the life cycle at a level about 50% greater than at the beginning or end.

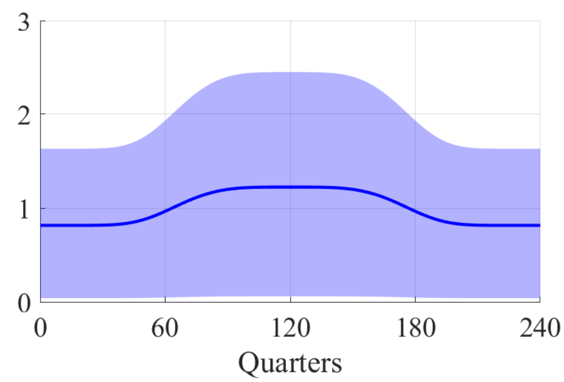

Figure 2: The mass of endowment profiles with the scaling factor drawn from a uniform distribution $U\,[0.05, 1.95]$. Drawing from a lognormal distribution is harder to visualize, but such a distribution would include arbitrarily rich and arbitrarily poor households. The endowment Gini is about 35%.

higher than the productivity nearer the beginning or end of the life cycle. Figure 2 shows the mass of productivity in the economy using a uniform distribution for ξ (instead of a lognormal) to ease the visual presentation.

2.5 Additional idiosyncratic risk

Households can earn income in a competitive economywide labor market by supplying hours along with the productivity they have available at that date. Agents also face idiosyncratic shocks to employment opportunities during their lifetime $s = 0, 1, ..., 240$, which we will call "unemployment shocks," given by

$$u_{s,i} = \begin{cases} 1 & \Pr\left(u_{s,i} = 1\right) = q \\ 0 & \Pr\left(u_{s,i} = 0\right) = 1 - q \end{cases} \tag{11}$$

The agent reports to work at the beginning of each period, but in some periods the job opportunity is closed and we say $u_{s,i} = 0$. This situation is publicly observable, and the agent can earn no income via market-based employment. We think of these idiosyncratic shocks as being *i.i.d.* and uncorrelated with the aggregate shocks. Because of the continuum of agents in each cohort, this process will lead to a constant aggregate unemployment rate of $\bar{u} = 1 - q$ in the equilibrium we study.

2.6 Household credit

The overlapping-generations structure creates a large private credit market essential to good macroeconomic performance. Households relatively early in the life cycle will have less labor income and will want to pull consumption forward in the life cycle, while other households nearer middle age will have more labor income which they will wish to lend to the younger households as a means of moving consumption later in the life cycle. Given this situation, the key asset is therefore *privately issued* household debt. To fix ideas (and this is not explicitly modeled), we like to think of the privately-issued debt in the model as "mortgage-backed securities." U.S. household debt in the second quarter of 2021 was about \$15 trillion, which was equal to about two-thirds of GDP. Relatively young households use mortgage debt to pull consumption forward in the life cycle.

There is an important friction in the credit market which motivates the role for monetary policy in the model. Households borrow in nominal terms at an agreed nominal interest rate determined in a competitive market, promising to pay off in the following period, also in nominal terms, and this contract does not depend on the state of the economy or other shocks. This friction is non-state contingent nominal contracting (NSCNC). There are two aspects to this assumption: (1) The non-state contingent aspect means that real resources are misallocated via this friction; and (2) The nominal aspect means that the monetary authority may be able to fix the distortion to the equilibrium through appropriate monetary policy. Indeed, the monetary policy described below effectively converts the non-state contingent nominal contracts between agents into real, state contingent contracts which are optimal under the homothetic preferences used here.

Households that are entering the economy at date t are assumed to hold no net nominal assets, which we refer to simply as "net assets." Households that entered into the economy in previous periods will generally have a non-zero net asset position at date t. We will denote this by $a_{t-s,i}(t)$ for $s = 1, ..., T$ and $i \in (0, \infty)$, which indicates the net asset holdings carried into the current period from date $t-1$ by each member of each cohort that entered the economy at the various dates $t-s$. There will therefore be a net asset distribution in the economy that we will have to track as part of the equilibrium, the hallmark of this literature since Krusell and Smith (1998). However, in this paper it will be easy to track this distribution because all net asset positions will be linear in the real wage.

2.7 Timing protocol

A timing protocol dictates the sequence of events within a period. We assume that nature moves first and chooses a continuum of draws defining the heterogeneous productivity profiles for the entering cohort and values for the stochastic processes implying values for the productivity growth rate $\lambda(t-1,t)$, the labor force growth rate $\nu(t-1,t)$, and the growth in the state of demand $\delta(t-1,t)$, and hence a value for today's real wage $w(t)$. The monetary policymaker moves next and chooses a value for the price level $P(t)$, as described below. Households then take $w(t)$ and $P(t)$ as known, supply labor—potentially encountering an unemployment shock—and make decisions about today's consumption and net assets to be carried into the next period. The nominal contracting procedure described in the next subsection determines the nominal interest rate, which we denote by $R^n(t,t+1)$.

2.8 Nominal interest rate contracts

All households meet in a competitive market for nominal loans. Households contract by fixing the nominal interest rate on consumption loans one period in advance. From the cohort t life cycle household Euler equation, the non-state contingent gross nominal interest rate in effect from period t to period $t+1$, denoted $R^n(t,t+1)$, is given by[6]

$$R^n(t,t+1)^{-1} = E_t\left[\frac{\tilde{c}_{t,i}(t)}{\tilde{c}_{t,i}(t+1)}\frac{P(t)}{P(t+1)}\right]. \tag{12}$$

We call this the contracted gross nominal interest rate, or simply the "contract rate." Recall that $\tilde{c}_{s,i}(t) = c_{s,i}(t)D(t)$, indicating that the expected change in aggregate demand plays a role in the determination of the nominal interest rate. The E_t operator indicates that households must use information available as of the end of period t and before the realization of aggregate shocks. This expression is the same for all life cycle households $i \in (0,\infty)$ as well as for households that entered the economy at earlier dates in the equilibria we study. We can interpret this equation as indicating that the contract nominal interest rate will be equal to the expected rate of nominal GDP growth each period.

2.9 Additional assumptions

There are no borrowing constraints in this version of the model, but interested readers may wish to consult Azariadis et al. (2019) for a version with some households excluded from credit markets. There is stochastic labor force (i.e., population) growth in this version. There is no default—all debts are repaid according to contractual arrangements. Prices are flexible. There is no capital in this version. We assume all policies we describe below are set credibly for all time $t \in (-\infty,+\infty)$.

[6] See Chari and Kehoe (1999) for more details.

3 Four policy authorities

To fix ideas, we assume there are four policymaking authorities in this economy. These authorities have clear assignments and powers as delineated below.

3.1 The monetary authority

The monetary policymaker uses the ability to observe aggregate shocks at the beginning of the period and its control over today's price level to meet a targeting criterion $\forall t$ given by[7]

$$P(t) = \frac{R^n(t-1,t)}{\lambda(t-1,t)\,\nu(t-1,t)\,\delta(t-1,t)} P(t-1). \tag{13}$$

The term $R^n(t-1,t)$ is the contract nominal interest rate effective between date $t-1$ and date t, which is the expected rate of nominal GDP growth as described above. The term $\lambda(t-1,t)\,\nu(t-1,t)\,\delta(t-1,t)$ is the realized rate of real output growth between date $t-1$ and date t. A hallmark of the recent literature on nominal GDP targeting is the idea of "countercyclical price level movements" which is what this rule delivers.[8]

3.2 The fiscal authority

The role of the fiscal authority is to raise an exogenously-assigned fixed fraction of real output as government revenue. The fiscal authority is allowed to use linear labor income taxation, denoted by tax rate τ^f levied on gross labor income for the period, in order to accomplish this task. We have not explored other types of taxation in the current version of the paper. The government revenue raised simply leaves the economy, and we have not explored other options on this dimension of the model in this version.

3.3 The labor authority

The labor authority provides unemployment insurance for unemployed households in the economy. The labor authority is allowed to raise revenue for this purpose through a linear labor income tax, denoted τ^u, levied on gross labor income for the period. At the beginning of each period, the labor authority observes the households who have received the unemployment shock. These households report to the unemployment office and remain there for the length of time they would otherwise be working. At the end of that length of time, the labor authority pays out the amount of income

[7]For a discussion of an optimal targeting criterion for monetary policy, see Giannonni and Woodford (2004).

[8]We assume the inflation target is exogeneously given and for convenience we set the net target rate equal to zero.

that the household would have otherwise earned during that period. In this way, the household receives the same amount of income and enjoys the same level of leisure that they would have without the unemployment shock. They pay taxes τ^f and τ^u on the gross unemployment insurance payout.

3.4 The education authority

The education authority has the power to influence the dispersion of beginning-of-economic-life human capital shocks ξ which we are assuming are lognormally distributed. We will denote the standard deviation of the distribution by σ. We think of the stochastic nature of this process as representing randomness in the development of human capital that occurs before agents enter the model and begin economic life. Some agents receive a lot of human captial, while others receive very little, as represented by the scaling factor draw, and this has permanent effects on the agent's ability to earn income over the life cycle. By reducing the value of σ, the education authority can smooth out this random process of human capital assignment, in effect insuring households against the permanent shock to human capital. In the limit, the education authority would use this power to drive $\sigma \to 0$, meaning that all households would receive the same hump-shaped life cycle productivity profile. In the theorem below, we assume that $\sigma \to 0$ is in fact the policy of the education authority.

One might think that the $\sigma \to 0$ policy is somewhat extreme. There may be an underlying talent distribution in the economy, so that different agents would have different life cycle productivity profiles even in the case where the education authority does all that is possible to equalize human capital across agents. In this interpretation, there would be some minimum level beyond which σ cannot be reduced. We denote this minimum level by $\sigma_{\min}$. For the theorem, we use $\sigma_{\min} = 0$, but in the illustrative examples, we also allow $\sigma_{\min} > 0$.

4 Main theorem

We now state the main theorem:

Theorem 1 *Assume symmetry as defined above. Assume the following policy mix: (1) the monetary authority credibly uses the targeting criterion given above, $\forall t$. (2) The fiscal authority raises a fixed fraction of real output via the linear labor income tax τ^f levied on all labor income and unemployment benefits in the society. (3) The labor authority runs an unemployment insurance program as defined above and funds it with the linear labor income tax τ^u levied on all labor income and unemployment benefits in the society. (4) The education authority minimizes the variance of beginning-of-economic-life human capital endowments by setting $\sigma = 0$. Given this policy mix, the competitive equilibrium gross real interest rate, $R(t-1,t)$, is equal to the stochastic gross rate of real output growth given by $\lambda(t-1,t)\,\nu(t-1,t)\,\delta(t-1,t)$ $\forall t$.*

Corollary 2 *A social planner maximizing a social welfare function with equal weight on all agents for all t and discounting into the infinite past and infinite future using the real interest rate would conclude that the competitive equilibrium is a social optimum.*

Corollary 3 *For any two life cycle households i and $i\prime$ in the model at each date t that share the same life-cycle productivity profile, consumption is equalized.*

Corollary 4 *Consumption growth is equalized for all households at each date.*
Proof. *See Appendix.* ■

We provide a heuristic discussion of the model solution here, and provide details in the Appendix.

4.1 Heuristic discussion

We think in terms of a stationary equilibrium in which time extends from the infinite past to the infinite future. The policy mix as described above is maintained credibly for all time. A single household i entering the economy at an arbitrary date t has the preferences given above and faces a lifetime budget constraint containing linear labor income taxes, unemployment shocks, and nominal net assets. We can substitute the monetary policymaker price level targeting criterion and the unemployment insurance program into this lifetime budget constraint to eliminate the uncertainty faced by the household and then solve the household's problem. The solution features date t consumption and leisure choices that depend solely on information available at date t and not on any future expectations, reflecting the insurance provided against both aggregate and idiosyncratic risk. The consumption choices, as well as the net asset holding of this household, will depend linearly on the real wage. Leisure choices will not depend on the real wage. These same features apply to the choice problems of all other members of this cohort with different productivity profiles, as well as to all members of all cohorts entering the economy at earlier dates. The general equilibrium condition is that the net asset holding in the economy sums to zero. We guess and verify that a gross real interest rate equal to the real output growth rate satisfies this condition at each date t.

4.2 Additional intuition

To gain somewhat more intuition about how the solution works, we consider individual household budget constraints.

Households have a simple sequence of budget constraints which can be aggregated into a consolidated remaining lifetime budget constraint, which is standard. For the cohort entering the economy at date t, and given the unemployment insurance program, household i faces

$$\sum_{s=0}^{T}\left(\frac{P(t+s)}{P(t)}\frac{\tilde{c}_{t,i}(t+s)}{\mathcal{R}^{n}(t,t+s)}\right)$$
$$\leq \sum_{s=0}^{T}\left(\frac{P(t+s)}{P(t)}\frac{\left(1-\tau^{f}\right)(1-\tau^{u})\,e_{s,i}w(t+s)\left(1-\ell_{t,i}(t+s)\right)}{\mathcal{R}^{n}(t,t+s)}\right), \quad (14)$$

where

$$\mathcal{R}^{n}(t,t+s)=\begin{cases}\prod_{j=1}^{s}R^{n}(t+j-1,t+j) & s>0\\ 1 & s=0\end{cases}. \quad (15)$$

Households entering the economy at earlier dates have a similar constraint over their remaining lifetime but also have a net asset position that they carry into date t, denoted by $a_{t-1,i}(t)$, $a_{t-2,i}(t)$,. . . , $a_{t-T,i}(t)$.

Now let us consider just one term in this budget constraint (14), the one applicable to date $t+1$ given by

$$\cdots\frac{P(t+1)}{P(t)}\frac{\tilde{c}_{t,i}(t+1)}{R^{n}(t,t+1)}\cdots$$
$$\leq\cdots\frac{P(t+1)}{P(t)}\frac{\left(1-\tau^{f}\right)(1-\tau^{u})\,e_{1,i}\left[1-\ell_{t,i}(t+1)\right]w(t+1)}{R^{n}(t,t+1)}\cdots. \quad (16)$$

The uncertainty in this expression is coming from the future real wage $w(t+1)$ as well as future consumption $\tilde{c}_{t,i}(t+1)$. We can substitute the monetary policy targeting criterion (13) directly into this expression. Noting that $w(t+1) = \lambda(t,t+1)\,\nu(t,t+1)\,\delta(t,t+1)\,w(t)$ and that ℓ choices will depend on contemporaneous consumption choices alone, the stochastic elements, $\lambda(t,t+1)$, $\nu(t,t+1)$, and $\delta(t,t+1)$ will cancel on the right-hand side and thus future income will become deterministic from the perspective of the household. This cancellation occurs for all other terms on the right-hand side of this expression, as well as for all other similar expressions for all other agents in the economy. The solution dictates a consumption plan for the household which is to wait and see how rapidly the economy grows each period and then increase consumption by that amount. The consumption growth rate can be thought of either in terms of the bundle $\tilde{c}$ or actual consumption c.

The monetary policymaker is providing a form of insurance to households against aggregate shocks. The policymaker is credibly transforming ex ante non-state contingent nominal credit contracts into ex post state contingent real contracts. This is "equity share contracting" whereby each participant sharing an identical life cycle productivity profile receives an equal portion of that period's output; this is optimal contracting under homothetic preferences as we have here. The education authority makes sure that there is only one life cycle productivity profile. The labor market authority insures against idiosyncratic risk. The fiscal and labor authorities are able to use linear labor income taxes to fund their programs because leisure choices depend only on position in the life cycle. In particular, life cycle households will work more nearer the middle of the life cycle independently of aggregate shocks.

4.3 Intuition for leisure choices and tax policy

Given the monetary policy, the labor market authority sets a tax τ^u that is linear in labor earnings. The fiscal authority sets a tax τ^f that is also linear in labor earnings. The household i first-order condition for leisure can then be written as

$$\ell_{t,i}\left(t+s\right) = \left(1-\eta\right)\frac{\bar{e}_i}{e_{s,i}} = \left(1-\eta\right)\frac{\bar{e}}{e_s},\ \forall i, \tag{17}$$

where $\bar{e} = \frac{1}{T+1}\sum_{s=0}^{T} e_s$ and $\bar{e}_i = \frac{1}{T+1}\sum_{s=0}^{T} e_{s,i}$. The scaling factor ξ and the two taxes τ^u and τ^f are all linear in e and therefore cancel out in this expression—so taxing in this manner is nondistortionary. This result requires that all labor income is taxed at these rates for all time.

5 Characterizing the policies

In this section we first describe how the monetary and education policies work. We then characterize the equilibrium in a series of schematic graphs mostly representing the cross-sectional distribution of households at each date. In the graphs, we will show both the case where $\sigma_{\min} = 0$ and the case where $\sigma_{\min} > 0$. We can interpret this latter case as one where the education policymaker cannot drive the variance of the scaling factor all the way to zero.

5.1 Calibration

This model is too stylized to provide a complete characterization of the U.S. data, but we use a simple calibration nevertheless to illustrate key properties of the equilibrium. This calibration corresponds to the one used in Bullard and DiCecio (2021).

We use the following baseline endowment profile:

$$e_s = f\left(s\right) = 2 + \exp\left[-\left(\frac{s-120}{60}\right)^4\right]. \tag{18}$$

Given the baseline income profile in (18), setting

$$\eta = \left[1 - \left(1-0.19\right)/\left(\frac{\sum_{s=0}^{T}\frac{\bar{e}}{e_s}}{T+1}\right)\right] = 0.21 \tag{19}$$

results in the fraction of time worked being 19 percent, as in the U.S. economy (see Bullard and Feigenbaum, 2007).

We assume that the endowment scaling factor is drawn from a lognormal distribution, i.e., $\ln(\xi) \sim \mathcal{N}(\mu, \sigma^2)$, and we choose the within-cohort dispersion, σ, to match exactly the consumption Gini coefficient for the U.S. $G_{C,U.S.} = 32\%$ (Heathcote et al., 2010): $\sigma = 2\,\mathrm{erf}^{-1}(0.32) = 0.5833$.[9]

Finally, we set the average growth rate of productivity, $\bar{\lambda}$, to a 1.5% annual rate and the average growth rate of the labor force, $\bar{\nu}$, at 1.5% (annual rate); the average net growth rate of the state of aggregate demand is zero.

5.2 Monetary policy

The monetary policymaker controls the price level $P(t)$ directly and follows a price level targeting criterion given by

$$P(t) = \frac{R^n(t-1,t)}{\lambda(t-1,t)\,\nu(t-1,t)\,\delta(t-1,t)} P(t-1), \tag{20}$$

where $R^n(t-1,t)$, the gross nominal interest rate, is equal to the expected gross rate of NGDP growth between dates $t-1$ and t. The targeting criterion works because it provides a form of insurance for all households against current and future aggregate shocks. The implementation via a policy rule is not unique—see Bullard and Singh (2020).

In Figure 3, we show one implementation of the price level targeting criterion in response to a negative shock to economic growth. In the upper left hand panel, the output growth rate falls on impact but subsequently recovers. The panel in the lower right shows that the inflation rate rises in equal and opposite proportion to the negative growth shock, a hallmark of nominal GDP targeting. This response can be understood as one that validates all nominal contracts made in the previous period. The upper right panel shows that the nominal interest rate does not change in the period of the shock, but subsequently falls before gradually returning to steady state. The lower left panel shows the evolution of the price level. This implementation of the price level targeting criterion has standard features in that the nominal interest rate falls following a negative shock to the economy and subsequently recovers slowly.

5.3 Education policy

"Education policy" in this model, admittedly somewhat abstract, can influence what happens to agents before they enter the model and begin making economic decisions at approximately age 20. As we have it, the education authority influences the productivity profile dispersion parameter σ. One could interpret this as an idealized

[9] The Gini coefficients discussed below are invariant to μ.

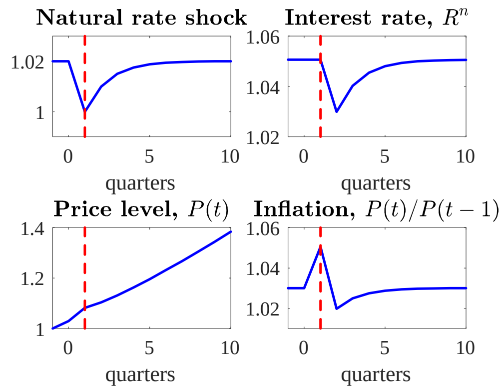

Figure 3: Monetary policy responds to a decrease in the natural rate, i.e., a decrease in λ, ν or δ, by increasing the inflation rate in the period of the shock. Subsequently, inflation converges to its long-run equilibrium value from below. The nominal interest rate drops in the period after the shock. The Phillips curve correlation is high.

insurance market that operates before households enter the economy. In the theorem, we use the limiting case in which $\sigma_{\min} = 0$, which means all households receive the same lifetime productivity profile. This would be a "perfectly equal" economy in that the talent distribution would collapse to just one life-cycle pattern. This will be represented by a blue line in the figures below.

An education policy of this type would drive the consumption Gini to zero. However, the income and wealth Gini coefficients would remain close to observed values—these are driven mostly by the life-cycle structure as we shall see in the figures below. In the figures, we use both the idealized case $\sigma_{\min} = 0$, the blue line, and the less ideal case $\sigma_{\min} > 0$, the blue mass.

5.4 Hours worked over the life cycle

Figure 4 displays hours worked over the life cycle (the blue line), leisure decisions (the green line), and average time devoted to market work across the U.S. labor force (the red line). We stress that there is no shading in this graph. This is because all households in this economy, rich and poor, work the same pattern of hours over their life cycle. Hours worked depends only on the stage of the life cycle and the relative productivity at that point in the life cycle. For all agents, the pattern of productivity over the life cycle is the same in this economy, and hence the pronounced pattern in the graph with young agents and older agents working very little, but middle aged agents working substantially more hours.

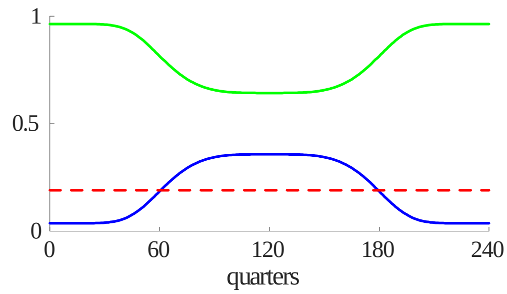

Figure 4: Cross section: Leisure decisions (green), labor supply decisions (blue) and fraction of work time in U.S. data, 19% (red). The labor/leisure choice depends on age only. High-income households plan to work the same hours as low-income households at each age. A certain percentage of the continuum of households in each cohort is unemployed but insured.

5.5 Labor income mass

Households are more productive in the middle of the life cycle than they are at the beginning or the end. Only relative productivity matters for the choices of hours worked. This means all households want to work more when they are also more productive in the middle of the life cycle. This creates substantial labor income inequality. Figure 5 shows schematically how the labor income mass expands in the middle of the life cycle. In this figure, there is a continuum of households at each vertical slice, but the dispersion in labor income is larger in the middle.

5.6 Consumption mass

The households in the model wish to take the uneven labor income they receive over their life cycle and convert it into equal consumption. The credit market in the model allows this but has a friction (NSCNC) that is fully mitigated by the monetary policymaker. This means that any two households at two different points in the life cycle, but who are using the same life cycle productivity profile, will consume the same amount. This is shown in Figure 6, where the red box indicates the observed consumption in the society at a point in time.

It is important to recall, however, that the economy is also growing at a stochastic rate. In the next period, this cross-sectional picture will look the same, but the blue and red masses will have shifted up somewhat depending on the growth that occurred during that period. This is shown in Figure 7, which gives a time series perspective on consumption evolution.

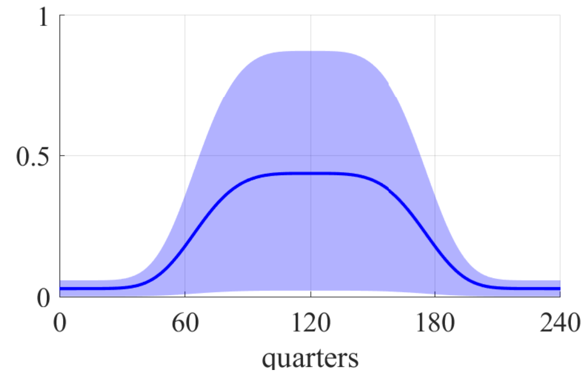

Figure 5: Cross section: Labor income profiles with unemployment insurance. Personal productivity peaks at the middle of the life cycle, and households work more at that time as well, making income even more concentrated in the peak earning years. The blue line depicts the limiting case $\sigma_{\min} = 0$.

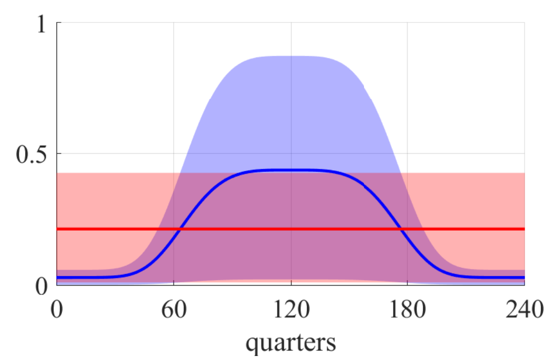

Figure 6: Cross section: Schematic consumption mass (red) and labor income mass (blue). Under optimal monetary policy, the private credit market reallocates uneven labor income into perfectly equal consumption along each productivity profile. The consumption Gini is 31.7%, similar to values calculated from U.S. data. The solid lines depicts the limiting case with $\sigma_{\min} = 0$.

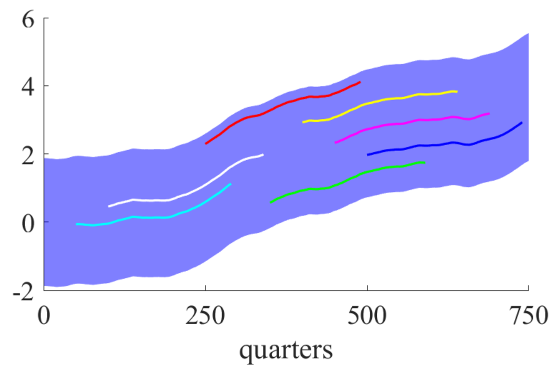

Figure 7: Time series: Evolution of the distribution of log consumption (shaded area) and examples of individual log consumption profiles (colored lines). Under optimal monetary policy, individual consumption profiles share the same stochastic trend as aggregate consumption.

5.7 Net asset holding mass

Households hold net assets at each date as schematically shown in Figure 8. Because we have a closed economy and no asset in net positive supply, this figure summarizes the borrowing and lending in the economy, and it must integrate to zero. The blue line indicates the net asset holdings under the conditions of the theorem, and the blue mass indicates the holdings when the variance of life cycle profiles is positive. The borrowers occupy the left portion of this diagram, and lenders are on the right portion. All households, rich and poor, utilize the credit market—peak indebtedness around quarter 60 in the figure involves both relatively rich households at the most negative point of the blue mass, as well as relatively poor households just below the horizontal axis (for $\sigma_{\min} > 0$). The Wicksellian natural real rate of interest, which is also the output and consumption growth rate in this model, exactly balances the credit needs in the society.

5.8 Marginal propensities to consume

The model produces variation in the marginal propensity to consume. We can write a typical agent's consumption as linear in the real wage

$$c_{t,i}\left(t+s\right)=\eta\xi\bar{e}\left(1-\tau^{f}\right)\left(1-\tau^{u}\right)w\left(t+s\right). \tag{21}$$

Labor income is also linear in the real wage

$$Y_{1;t,i}\left(t+s\right)=\xi e_{s}\left[1-\ell_{t}\left(t+s\right)\right]\left(1-\tau^{f}\right)\left(1-\tau^{u}\right)w\left(t+s\right). \tag{22}$$

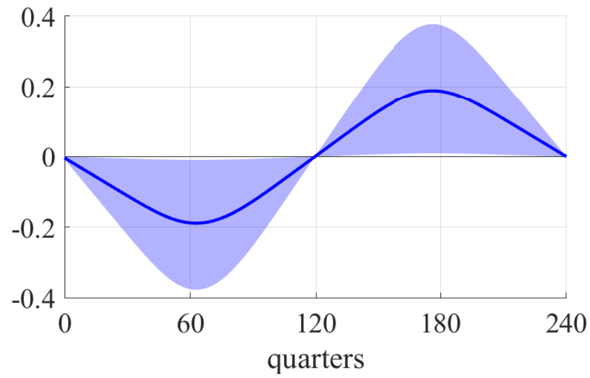

Figure 8: Cross section: Schematic net asset holding mass relative to GDP by cohort. Borrowing, the negative values to the left, peaks at stage 60 of the life cycle (age ~35), while positive assets peak at stage 180 of life (age ~65). The financial wealth Gini is 72.7%, similar to values calculated in U.S. data. The blue line depicts the limiting case with $\sigma_{\min} = 0$.

Hence, the MPC can be calculated as follows:

$$MPC = \frac{dc_{t,i}/dw}{Y_{1;t,i}/dw} = \frac{\eta \bar{e} \xi \left(1-\tau^f\right)\left(1-\tau^u\right)}{e_s \xi \left[1-(1-\eta)\frac{\bar{e}}{e_s}\right]\left(1-\tau^f\right)\left(1-\tau^u\right)} = \frac{\eta \bar{e}}{e_s - (1-\eta)\,\bar{e}}. \tag{23}$$

Accordingly, Figure 9 shows that there will be two points where the MPC is 1, one where households are generally young and indebted, and another when households are older financial wealth holders.

6 Inequality

The model has three notions of income, depending on how one wishes to treat capital income. One idea would be to focus only on pretax labor income, which we will denote by Y_1. Another is to consider pretax labor income plus non-negative capital income, which we call Y_2. The third is the non-negative component of total income, which we label as Y_3. The model Gini coefficients for these three concepts are somewhat different: $G_{Y_1} = 56.1\%$, $G_{Y_2} = 51.5\%$, $G_{Y_3} = 59.5\%$. Figures 5, 10, and 11 display the schematic differences for the three income concepts.

We wish to compare model Gini coefficients to data on inequality in the U.S. For the consumption Gini we use Heathcote, Perri and Violante (2010) who estimate $G_{C,\text{U.S.}} = 32\%$. For income we use an estimate due to the Congressional Budget Office (2016), pre-taxes/transfers, of $G_{Y,\text{U.S.}} = 51\%$. For financial wealth we use Davies, Sandström, Shorrocks and Wolff (2011): $G_{W,\text{U.S.}} = 80\%$. These are compared to the model Gini coefficients in Table 1.

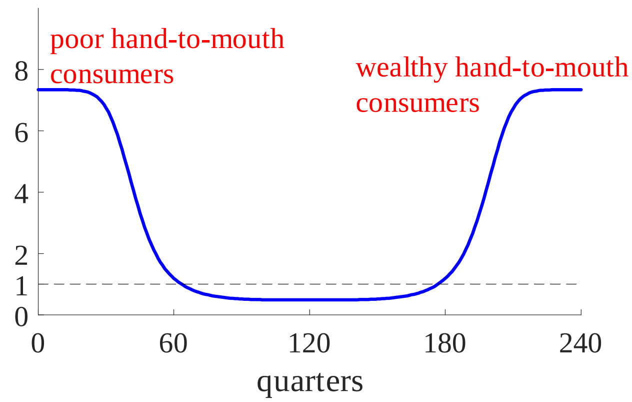

Figure 9: Cross section: Marginal propensity to consume out of labor income by cohort. Notice that the MPC does not depend on the endowment scaling factor, ξ, and hence there is no shaded region in this figure.

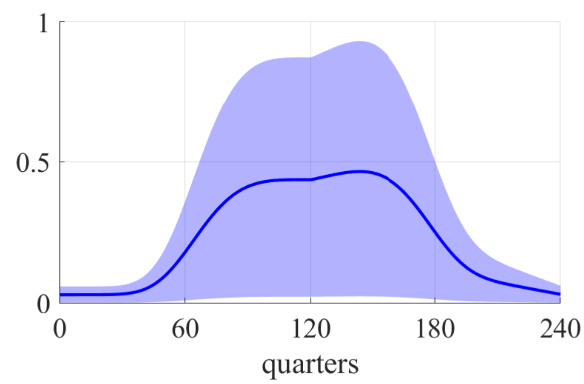

Figure 10: Cross section: Profiles of labor income and non-negative capital income. The blue line depicts the limiting case with $\sigma_{\min} = 0$.

	Wealth		**Income**		**Consumption**
	W	Y_1	Y_2	Y_3	C
U.S. data	80%		51%		32%
Lognormal	72.4%	55.7%	51.1%	59.0%	32%

Table 1: Gini coefficients in the U.S. data and in the model with uniform and lognormal productivity.

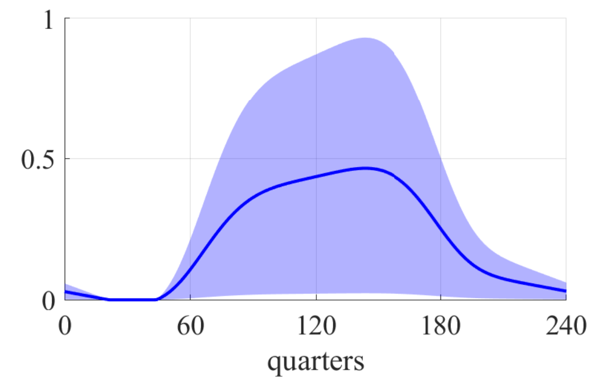

Figure 11: Cross section: Profiles of non-negative total income. The blue line depicts the limiting case with $\sigma_{\min} = 0$.

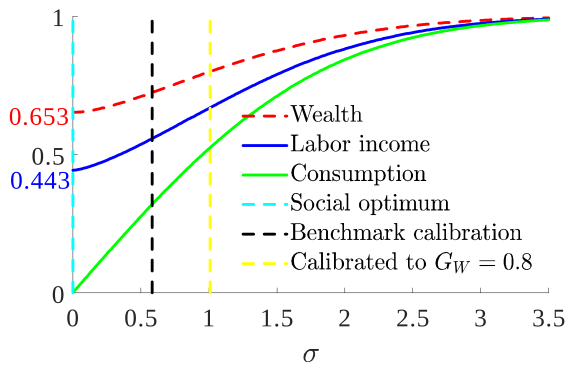

Figure 12: As the dispersion of productivity profiles, σ, increases, the Gini coefficients increase. The ordering $G_W > G_Y > G_C$ is preserved. The case where $\sigma_{\min} = 0$ has $G_C = 0$, but $G_W = 65.3\%$ and $G_Y = 44.3\%$.

Figure 12 shows the Gini coefficients of the model as a function of the intra-cohort dispersion parameter σ. The values of the Gini coefficients between zero and one are on the vertical axis, and the value of σ is on the horizontal axis. The financial wealth Gini is in red, the income Gini is in blue, and the consumption Gini is in green. The model preserves the ranking of the Gini coefficients with the financial wealth Gini as the largest, the income Gini as the second largest, and the consumption Gini as the lowest. As σ increases to the right in the figure, all three Ginis converge to one—the economy becomes more and more unequal on all dimensions. The benchmark case is indicated by the vertical black line in the figure—these are the values reported in Table 1, and the calibration in this case matches the U.S. data with respect to the consumption Gini but allows the income and financial wealth Ginis to be determined by the equilibrium.

However, the figure indicates that one could also match the financial wealth Gini of 0.80 estimated from the U.S. data—at the cost of somewhat higher Gini coefficients for income and consumption. The vertical yellow line indicates this case.

As σ moves to the left in the figure—all the way to zero—the conditions of the theorem in the paper are met, and this illustrates the social optimum. This drives the consumption Gini all the way to zero. However, even though all households have the same life cycle productivity profile in this situation, the income Gini is still 0.44 and the financial wealth Gini is still 0.65. This is indicates that an important fraction of observed inequality as measured by wealth and income Ginis is driven by life cycle factors according to the model.

7 Conclusions

We study a simple and stylized economy in which a classic combination of macroeconomic and microeconomic policies can deliver a first-best allocation of resources even in the presence of substantial inequality in income, financial wealth, and consumption. In particular, a monetary policymaker reacts to aggregate shocks each period in order to achieve the Wicksellian natural rate of interest. This in turn enables non-distortionary linear labor income taxes to fund government expenditures as well as an unemployment insurance program. An education policy can then drive the consumption Gini toward zero but would leave income and wealth Ginis at positive levels due to life cycle effects. These classic benchmarks may be useful in understanding the effects of macroeconomic policy for models in this class going forward.

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A Appendix

Incomplete. We refer the reader to the appendix in J. Bullard and R. DiCecio (2021) "Optimal Monetary Policy for the Masses" for more details.