Micro Risks and Pareto Improving Policies*

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Abstract

We provide sufficient conditions for the feasibility of a Pareto-improving fiscal policy when the risk-free interest rate on government bonds is below the growth rate ($r < g$) or there is a markup between price and marginal cost. We do so in the class of incomplete markets models pioneered by Bewley-Huggett-Aiyagari, but we allow for an arbitrary amount of ex ante heterogeneity in terms of preferences and income risk. We consider both the case of dynamic inefficiency as well as the more plausible case of dynamic efficiency. The key condition is that seigniorage revenue raised by government bonds exceeds the increase in the interest rate times the initial capital stock. The Pareto improving fiscal policies weakly expand every agent's budget set at every point in time. The policies improve risk sharing and potentially guide the economy to a more efficient level of capital. We establish that debt and investment associated with Pareto-improving policies may be complements along the transition, rather than the traditional substitutes.

1 Introduction

In this paper we provide sufficient conditions for a Pareto improvement when the risk-free interest rate on government bonds is below the growth rate ($r < g$) or the economy is subject to markups. We do so in the class of incomplete markets models pioneered by Bewley-Huggett-Aiyagari, but we allow for an arbitrary amount of ex ante heterogeneity in terms of preferences and income risk. We consider both the case of dynamic inefficiency as well as the more plausible case of dynamic efficiency. The paper augments the classic dynamic inefficiency condition of Samuelson (1958) and Diamond (1965) by allowing for rich heterogeneity along dimensions other

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than age, including idiosyncratic income risk, and considering scenarios in which the economy is dynamically efficient. By looking at Pareto improvements rather than maximizing a utilitarian social welfare function, the paper also complements the important work of Aiyagari and McGrattan (1998) and Dávila, Hong, Krusell and Ríos-Rull (2012). It is the absence of cross-agent utility comparisons, or even within-person but cross-time tradeoffs in our implementation, that distinguishes our Pareto metric from the more common utilitarian welfare measures.[1]

We analyze fiscal policies where the instruments available to the government are public debt, linear taxes/subsidies, and lump-sum transfers. We study policies that leave all after-tax factor prices (and pure profits, if there are any) weakly greater at all dates, and where there are no lump-sum taxes. By weakly expanding the budget set of all agents at all dates, these policies necessarily generate a Pareto improvement. We study when these Pareto-improving fiscal policies are feasible, and consider versions that lead to both crowding out and crowding in of capital.

We establish sufficient conditions for the existence of these feasible Pareto-improving policies that involve only the knowledge of the aggregate savings schedule (that is, total private savings as a function of interest rates and government transfers) and technology (including the size of a markup, if any). In particular, the short- and long-run elasticities of aggregate household savings play a crucial role in determining feasibility. We emphasize that the relevant elasticities are the ones governing the *aggregate* savings schedule (and technology). One advantage of this "macro" or "sufficient statistic" approach is that conditional on these aggregate elasticities, the nature and extent of the underlying heterogeneity across individuals is not relevant for assessing feasibility, ensuring these policies are *robust Pareto improvements* in the sense that the planner needs no information on micro preferences or idiosyncratic risks, conditional on knowing the aggregate savings elasticity.[2]

We conduct the analysis in an environment that builds closely on the canonical model of Aiyagari (1994), where precautionary savings motives reduce the equilibrium level of interest rates. The main restriction we impose on preferences is zero wealth effect on labor supply, as in the well known "GHH" preferences of Greenwood, Hercowitz and Huffman (1988), and omit aggregate risk considerations.[3]

To understand the role of the aggregate savings schedule, consider an economy without pro-

[1]Our focus on Pareto-improving policies rather than policies that maximize a utilitarian metric has an antecedent in Werning (2007), who explores Pareto-efficient tax policies in a Mirrelesian environment. See as well Hosseini and Shourideh (2019).

[2]There are of course disadvantages to this approach. For example, our approach rules out the use of lump-sum taxes (even if available) and as a result, the policy cannot exploit the link between private borrowing constraints and government liquidity identified by Woodford (1990) and Aiyagari and McGrattan (1998). Those policies however would require information on the underlying heterogeneities, frictions and inter-temporal tradeoffs of agents in addition to knowledge about the aggregate savings behavior.

[3]We also omit idiosyncratic return risk, as in the model of Angeletos (2007).

ductivity or population growth and at a laissez-faire stationary equilibrium (with zero government debt, zero taxes, zero transfers) such that $r < 0$. It is natural to conjecture that a policy that increases government debt by some strictly positive amount could be helpful, as the interest rate is low. Issuing government bonds, however, may lead to an increase in interest rates that crowds out capital. Simply issuing debt, therefore, may eventually reduce output, wages, and profits, hurting households that rely on these sources of income. The government, however, has additional policy instruments that could be used to offset these declines. We discuss three salient cases, a policy that keeps capital constant, a policy that crowds out capital and subsidizes wages and profits, and a policy that crowds in capital while taxing wages and profits.

Consider first a government subsidy on the rental rate of capital that ensures investment remains unchanged, despite the increase in the interest rate on government bonds. This *constant-K* policy guarantees that capital, output, wages, and profits are all the same as in the laissez-faire equilibrium. To achieve a Pareto improvement that is robust to arbitrary idiosyncratic heterogeneity, the government cannot resort to taxes on wages or profits or use lump-sum taxation. If the government can finance the capital subsidy with just the revenue it receives from bond issuances and even lump-sum transfer any additional surplus, then this policy makes *every* households weakly better off: the return to wealth has increased, after-tax wages and profits have remained constant, and the government is providing a weakly positive lump-sum transfer at all dates.[4] Those agents with positive assets will be strictly better off,[5] and the policy generates a Pareto improvement.

Interestingly, this potential Pareto improvement is achieved without changes in aggregate consumption or output at any date, as capital and labor remain at their laissez-faire levels. Moreover, every household sees their budget set weakly expand at every date and idiosyncratic state, and hence every household perceives that they could increase consumption. In equilibrium, however, the higher interest rate induces some (high-income) households to postpone consumption, allowing others (low-income) to increase theirs, improving risk sharing, despite the absence of a progressive tax and transfer scheme.

The key question is then whether this constant-K Pareto improvement is feasible. We derive a simple sufficient condition. Letting B_t denote the outstanding government debt at the start of period t, r_t the interest rate paid on B_t to households, and K_0 the initial (laissez-faire) capital stock, the policy is feasible if:

$$B_{t+1} - (1 + r_t)B_t \geq (r_t - r_0)K_0$$

[4] Contrast this with the utilitarian metric of Dávila et al. (2012), which requires that a change in *relative* factor prices improved the lot of the poorest households relative to the richest.

[5] Here, we implicitly assume that the household borrowing limit is zero. In the text we show how to relax this assumption.

at all dates. The left-hand side is the revenue generated by the government at date t from the issuance of new bonds. The right-hand side represents the fiscal cost of the subsidy to capital: the increase in the interest rate, $r_t - r_0$, is the subsidy rate required to keep K constant, and K_0 is the tax base.[6] The left-hand side captures how much debt the government is asking households to absorb, while the right-hand side reflects the increase in interest rates necessary to implement it in equilibrium. The key consideration is therefore whether households are willing to increase wealth without a large increase in interest rates; that is, how large and how elastic is the aggregate demand for savings.

The role of $r < 0$ in the constant-K policy becomes clear in the steady state. The left-hand side of the inequality is positive only if the steady-state interest is below zero (or below the rate of exogenous growth g in the general case) . This means the government earns sufficient "seigniorage" from its portfolio of bonds to finance the capital subsidy. We discuss below alternative Pareto improvements that do not require $r < g$.

This sufficient condition for the feasibility of the constant-K policy holds whether the economy is dynamically efficient or inefficient, whether –and to what extent– firms have market power, and the nature of idiosyncratic heterogeneity (conditional on the aggregate savings schedule). This "triple robustness" makes the condition applicable to a wide variety of environments. However, there may be alternative policies, tailored to whether the economy is dynamically efficient or not, as well as to whether markups are large or small.

If the economy is dynamically inefficient, reducing the capital stock can increase the aggregate income available for consumption today and in the future, as in Diamond (1965). For exposition, consider a policy in which capital is neither taxed nor subsidized after debt is increased. As we discussed above, the increase in r induced by government debt issuance results in a decline in capital. Rather than subsidizing the rental rate to maintain the level of capital as in the constant-K policy case, suppose instead that the government subsidizes wages and profits to maintain after-tax wages and profits at the level of the original laissez-faire equilibrium. We show that the fiscal cost of the labor and profit subsidies are bounded above by $(r_t - r_0) \times K_0$, the same as the cost of capital subsidies in the constant-K policy. The difference here is that government debt displaces capital in the wealth of private households. All else equal, a given amount of debt therefore produces a smaller increase in the interest rate, reducing the cost of the policy. We take this insight a step further. Specifically, if the government taxes capital in order to achieve dynamic efficiency, we show that the revenue raised is *always* sufficient to cover the subsidy to wages and profits. This means a Pareto improvement is always feasible in a dynamically inefficient economy, extending Diamond's insight to the Aiyagari framework.

[6] The right hand side should be $(r_t - r_0) \times (K_0 - \underline{a})$, where $\underline{a} \leq 0$ is the loosest private household borrowing constraint. To keep the notation streamlined in the introduction, we omit this term.

If the economy is dynamically efficient, crowding out of capital is counter-productive. Indeed, given that product market markups depress capital in the laissez-faire equilibrium, it is useful to consider the feasibility of Pareto-improving fiscal policies that "crowd in" capital towards a more efficient level. This third type of fiscal policy combines a subsidy to capital with a tax on wages and profits so as to keep after-tax wages and profits at their laissez-faire levels (despite the increase in capital and output). This is a feasible Pareto-improving policy if the tax revenue plus the revenue from bond issuances is sufficient to cover the cost of the capital subsidy at all dates. Whether or not such a policy is feasible again depends on the elasticity of aggregate savings to the interest rates. But in this case, the size of the markup distortion (capturing the potential efficiency gain in output) also matters. Moreover, the additional output and associated government revenue from wage and profit taxes augment the seigniorage received from government bonds; in fact, we show it is no longer necessary that $r < g$ in order for the Pareto improvement to be fiscally feasible. In this sense, markups provide an avenue to Pareto-improving fiscal policies, even though profits remains unchanged from their laissez-faire level.[7]

In the policy that crowds in capital, a role for government debt emerges that is complementary to investment. The aggregate savings elasticity will in general be lower in the short-run than in the long-run, and hence the interest rate may overshoot its long-run level. This disproportionately raises the fiscal costs of the policy in the short-run, prior to achieving full crowding in. Issuing government debt along the transition smooths the financing of these costs, delaying the burden until interest rates are lower and capital is higher. In contrast to the traditional crowding out argument, the use of debt facilitates the transition to a higher level of capital, making government debt and capital investment *complements* rather than substitutes.

In all cases we analyze, the Pareto improving fiscal policies weakly expand every agent's budget set at every point in time. This avoids the trade off between young and old in the classic over-lapping generations settings. It also avoids the trade off between the poor and rich that is the focus of the utilitarian metric common in the Bewley-Huggett-Aiyagari literature. The latter can be motivated by an ex ante "behind the veil of ignorance" rationale, or, given ex ante homogeneity, by a "renewal" argument based on the fact that all agents eventually transit through all states (see Aiyagari and McGrattan (1998) for a related discussion). Given that income and wealth differences persist across generations (Chetty, Hendren, Kline and Saez (2014)), that some agents have limited access to financial markets (Braxton, Herkenhoff and Phillips (2020)), and agents may value inter-temporal tradeoffs differently (Krusell and Smith (1998)), working through budget sets is a robust exercise. That said, the Pareto criterion is a high threshold, and as such, it

[7] In fact, we provide sufficient conditions for the feasibility of such policies when the aggregate savings schedule is equivalent to that of a representative household, and hence the long-run interest rate is determined by the discount factor of the stand-in household.

should not be viewed as a necessary condition for policy intervention. But certainly, the availability of a Pareto improvement provides a sufficient condition for a government response. It is in this spirit we provide sufficient conditions for such a scenario.

As in any model in which seigniorage (or a liquidity premium) plays an important fiscal role, the willingness of private households to hold additional government bonds without a large increase in the interest rate is key. The empirical literature on whether and to what extent government borrowing increases the interest rate is challenged by identification concerns and has produced results with no clear consensus.[8] Theory provides some guidance, which we discuss below, but one must keep in mind that in heterogeneous agent models aggregate elasticities have a complex relationship with individual preference parameters and the processes of idiosyncratic shocks. We therefore use simulations to assess the magnitude of the aggregate savings elasticity and the feasibility of Pareto-improving fiscal policies.

Our simulation exercise assumes Epstein and Zin (1989) preferences, and is calibrated using the income process of Krueger, Mitman and Perri (2016) and the historical data on $r - g$ in the U.S. We find scope for Pareto-improving policies for a wide range of debt policies and for policies with and without capital crowding in. Our baseline experiment considers a Pareto-improving constant-K fiscal policy that starts at the laissez-faire equilibrium and slowly increases debt to 60% of output, the average observed in the US data over the last half-century. Welfare gains arise because fiscal policy improves risk sharing. These gains are larger early in the transition reflecting the larger government transfers financed by debt issuance and the fact that interest rates overshoot during the transition increasing the returns of rich households.

The second fiscal policy plan we consider consists of the same debt path as the baseline, but with capital increasing towards the golden rule. We find that this fiscal plan is also a feasible Pareto improvement and generates even larger welfare gains to all households because here policy not only helps with risk sharing but also with efficient supply expansions. Debt is an essential part of these fiscal policies, as it provides the seigniorage revenue that is used for transfers to households and subsidies required for the capital expansion. We do find, however, that seigniorage revenue from bonds has limits and features a Laffer Curve: more debt increases interest rates and therefore the relative cost for servicing the debt. In our calibration, the upper bound on debt for Pareto improving fiscal policies is about twice the level of output.

This paper is part of a fast-growing recent literature exploring fiscal policy in environments with persistently low risk-free interest rates. Mehrotra and Sergeyev (2020) use a sample of advance economies to document that $r-g$ is often negative and develop a model to study its implica-

[8] See, for example, the survey papers by Bernheim (1987a) and Seater (1993) that examine the empirical evidence and seem to draw opposing conclusions on Ricardian equivalence.

tions for debt sustainability.[9] Blanchard (2019)'s presidential address to the American Economics Association gave a major stimulus to the question of debt sustainability under low interest rates. Other recent papers are Bassetto and Cui (2018), Reis (2020), Brunnermeier, Merkel and Sannikov (2020), Ball and Mankiw (2021), and Barro (2020). Several of these papers focus on aggregate risk and build on Bohn (1995). Our paper incorporates features of this previous work, such as borrowing constraints and the potential role of markups in opening a wedge between the interest rate and the marginal product of capital. However, our focus is on designing Pareto improving policies in the presence of individual heterogeneity and incomplete markets, as in the Bewely-Huggett-Aiyagari tradition, and the role played by $r < g$.

Our work also contributes to the literature studying the effects of fiscal policies in models with heterogeneous agents. Heathcote (2005) shows the failure of Ricardian equivalence in this class of models, as temporary tax cuts financed with public debt tend to increase consumption and output because they give households that are at the borrowing constraint extra resources that are spent. Heathcote, Storesletten and Violante (2017) study optimal labor tax progressivity in this environment and illustrate sharply the tradeoff between insurance and incentives motives in response to the tax system. Dyrda and Pedroni (2020) study the optimal tax system in a quantitative version of the idiosyncratic risk model. Krueger, Ludwig and Villalvazo (2021) consider an overlapping generations model in which agents face idiosyncratic risk in the final period of life. They evaluate the tradeoffs, for general Pareto weights on different generations, of a tax on capital that reduces income risk but potentially exacerbates inter-generational inequality. Also recently, Bhandari, Evans, Golosov and Sargent (2020) explore optimal fiscal and monetary policy within the context of the heterogeneous agent model with nominal rigidities and aggregate shocks.[10] All of these papers focus on a utilitarian welfare criteria, and do not analyze the implications of $r < g$. In contemporaneous work, Kocherlakota (2021) studies the role of public debt bubbles for government deficits and expected consumption in models of heterogeneous agents that face tail risks, but abstracts from Pareto improvements and dynamic efficient environments.

The paper proceeds as follows: Section 2 lays out the environment; Section 3 provides the sufficient conditions for a Pareto improving fiscal policy; Section 3.6 discusses the aggregate interest-elasticity of household savings; Section 4 provides numerical examples; and Section 5 concludes.

[9] See also Mauro and Zhou (2021) and Jordà, Knoll, Kuvshinov, Schularick and Taylor (2019).

[10] See also Le Grand and Ragot (2020). Other recent papers that have studied the implications of transfers and government debt in heterogeneous agent models with price rigidities are Oh and Reis (2012) and Hagedorn, Manovskii and Mitman (2019).

2 Environment

The model hews closely to the canonical environment of Aiyagari (1994). In many ways, however, our environment is more general. We allow for permanent differences in the income process or preferences across households. The framework also allows for product market markups, driving a wedge between the marginal product of capital and the return on risk-free bonds. However, we do impose one assumption on preferences; namely, there is no wealth effect on labor supply which greatly simplifies tracing the impact of a change in interest rates on labor supply. In particular, the wage is a sufficient statistic for pinning down aggregate labor, regardless of other factor prices.[11]

We suppress exogenous growth in the text, but show how the model extends in the usual straight-forward way (given homothetic preferences) to growth in Appendix C. As a rule of thumb, the key condition $r < 0$ for an interest rate r is replaced with the corresponding $r < g$ where g denotes the constant exogenous growth rate of labor-augmenting productivity.

2.1 Households

Each household, from a measure-one continuum and indexed by $i \in [0, 1]$, draws an idiosyncratic labor productivity $z_t^i \geq 0$ at time t. We do not impose that households face the same stochastic process for idiosyncratic risk. That is, some households may face a permanently lower level of productivity or additional risk. We impose a cross-sectional independence restriction below that rules out aggregate productivity risk.

If the household provides $n_t^i \geq 0$ units of labor, it receives $w_t z_t^i n_t^i$ in labor earnings, w_t is the equilibrium wage rate per efficiency unit of labor. Without loss of generality, we assume labor taxes are paid by the firm.

A household may also receive profit income. We model this as payment to entrepreneurial talent, which, like labor productivity, is an endowment that may follow a stochastic process. Let π_t^i denote household i's return to entrepreneurial talent. Define aggregate household profit income as $\Pi_t = \int \pi_t^i di$, and household $i's$ share as $\theta_t^i \equiv \pi_t^i / \Pi_t$. Household i faces a potentially stochastic process for θ_t^i that determines its share of aggregate profits, with the restriction that $\theta_t^i \in [0, 1]$ and $\int \theta_t^i di = 1$ for all t.

At the start of period t, the household has a_t^i units of financial assets, which receive a risk-free return $(1 + r_t)$ in period t. Letting T_t denote lump sum transfers from the government, which is uniform across i, the household's budget constraint is:

$$c_t^i + a_{t+1}^i \leq w_t z_t^i n_t^i + \theta_t^i \Pi_t + (1 + r_t) a_t^i + T_t,$$

[11]Nevertheless, as we will see below, the Frisch elasticity of labor supply is not important for the analysis (beyond determining the initial equilibrium allocation), as the policies that we explore maintain a constant after-tax wage.

where c_t^i is consumption in period t.

Households are subject to a (potentially idiosyncratic) borrowing constraint $a_t^i \geq \underline{a}^i$ for all t. The fact that some households may have a tighter constraint than others captures the possibility that access to financial markets may be heterogeneous. Let $\underline{a} \equiv \inf_i \underline{a}^i$ denote the loosest borrowing constraint faced by households.[12]

The main restriction on preferences is the absence of a wealth effect on labor supply, as in the well known "GHH" preferences of Greenwood et al. (1988). In particular, let $x^i(c,n) \equiv c^i - v^i(n)$ for some convex function v^i. We write preferences recursively as $V_t^i = \phi^i(x_t^i, h_t^i(V_{t+1}^i))$, where V_t^i is household i's value and h_t^i represents a certainty equivalent operator over idiosyncratic shocks $\{z_{t+1}, \theta_{t+1}\}$, conditional on z_t, θ_t and the household's stochastic process for its shocks. This notation nests both standard "CRRA" utility as well as the recursive utility of Kreps and Porteus (1978) and Epstein and Zin (1989). We incorporate the latter to explore the different roles of risk aversion and inter-temporal elasticity of substitution in the feasibility of a Pareto improvement.

The idiosyncratic state variables for an individual household are $s \equiv (a, z, \theta)$, and the aggregate states are the (perfect foresight) sequences for factor prices $\{r_t, w_t\}$, aggregate profits $\{\Pi_t\}$, and transfers $\{T_t\}$. The household's problem can be written as follows

$$V_t^i(a,z,\theta) = \max_{a' \geq \underline{a}^i, n \in [0, \bar{n}^i], c \geq 0} \phi^i(x^i(c,n), h_t^i(V_{t+1}^i(a', z', \theta'))) \tag{1}$$

$$\text{subject to: } c + a' \leq w_t z n + \theta \Pi_t + (1 + r_t) a + T_t.$$

Note that as preferences can vary across households, we can accommodate distinct labor supply elasticities, as well as hand-to-mouth households.[13] In particular, the framework nests the classic Aiyagari (1994) with inelastic labor supply.[14]

Assuming an interior labor supply decision, household i's first-order condition with respect to labor is:

$$v_i'(n_t^i) = w_t z_t^i.$$

This implies a policy function $n_{i,t}^*(z)$, where the subscript t captures the equilibrium wage at period t.

[12]We assume below that the borrowing constraint is always above the natural borrowing limit. See Bhandari, Evans, Golosov and Sargent (2017) and Heathcote (2005) for a discussion on the role of such ad-hoc limits in breaking Ricardian equivalence.

[13]To see the later, consider the case of an aggregator $\phi^i(x,v) = h^i(x)$ for some household i. This corresponds to a household that does not value future consumption (it has a discount factor equal to 0). As a result, this household does not save and consumes its entire disposable income every period.

[14]This can be achieved by setting $v^i = 0$. In this case, the labor supply decision is not interior and the corresponding first order condition below does not hold.

Similarly, we let $a^*_{i,t}(a, z, \theta)$ and $c^*_{i,t}(a, z, \theta)$ denote the optimal saving and consumption policy functions respectively. The aggregate stock of savings chosen in period t and carried into period $t+1$ is $A_{t+1} \equiv \int a^*_{i,t}(a^i_t, z^i_t, \theta^i_t)di$.

We now state our independence assumption. Let $\boldsymbol{z}_t \equiv \{z^i_t\}_{i\in[0,1]}$ denote the state vector for productivity across households at time t.[15] Let

$$N(w_t, \boldsymbol{z}_t) \equiv \int z^i_t n^*_{i,t}(z^i_t)di = \int v_i'^{-1}(w_t z^i_t)di.$$

We make the assumption that N is independent of $\boldsymbol{z}_t$. This is a generalization of the typical assumption that v is common across households and that z is i.i.d. across i and t. The current environment requires only that aggregate labor supply is independent of the distribution, which is weaker than assuming that households are ex ante identical.[16]

2.2 Firms

The representative firm has a constant-returns technology given by $F(k, l)$, where k is capital and l effective units of labor. Firms hire labor and rent capital in competitive markets at rates r^k_t and w_t, respectively. Let τ^n_t and τ^k_t denote linear taxes on factor payments for labor and capital, respectively.

Firms may have market power in the product market. For simplicity, we assume that firms charge a price that is a constant markup over marginal cost. Let $\mu \geq 1$ be the ratio of price to marginal cost. The representative firm's first-order conditions are:

$$F_k(k, l) = \mu(1 + \tau^k_t)r^k_t$$
$$F_l(k, l) = \mu(1 + \tau^n_t)w_t.$$

Firm (pre-tax) profits are given by

$$\hat{\Pi} = \left(\frac{\mu - 1}{\mu}\right) F(k, l) = F(k, l) - (1 + \tau^k_t)r^k_t k - (1 + \tau^n_t)w_t l.$$

Profits are taxed by the government at rate τ^π_t, so after-tax profits are $\Pi = (1 - \tau^\pi_t)\hat{\Pi}$. We can think of the representative firm hiring a bundle of entrepreneurial talent that is in constant aggregate supply at after-tax price Π.

[15] We use the word vector loosely, as $\{z^i_t\}$ is a continuum of random variable realizations indexed by $i \in [0, 1]$.

[16] For example, households could belong to one of J types, each with non-trivial measure. Then within a type we can assume that the law of large numbers holds, and the aggregate is simply a weighted average across types.

2.3 Financial Intermediaries

We assume that the capital is owned by financial intermediaries.[17] Such intermediaries are competitive and borrow from the households at rate r_t, and, in turn, rent capital to firms at r_t^k and invest in government bonds at rate r_t^b. Capital depreciates at rate δ. Competition in the intermediary market ensures the following equilibrium condition at all t:

$$r_t = r_t^b = r_t^k - \delta.$$

Given the first equality, we drop the distinction between r and r^b in what follows. Note that there is also no maturity mismatch on the intermediaries' balance sheet.

2.4 Government

The government's policy consists of a sequence of taxes $\{\tau_t^n, \tau_t^k, \tau_t^\pi\}$, as well as a sequence of one-period debt issuances, $\{B_t\}$. The lump-sum transfers T_t are such that the sequential budget constraint holds at all periods:

$$T_t = \tau_t^n w_t N_t + \tau_t^k r_t^k K_t + \tau_t^\pi \hat{\Pi}_t + B_{t+1} - (1 + r_t)B_t$$

2.5 Market Clearing

Given r_t^k, w_t, and taxes, let K_t and L_t solve the representative firm's first-order conditions. Market clearing in the financial market requires $A_t = K_t + B_t$. Market clearing in the labor market requires $L_t = N_t$, where we recall that N_t is aggregate efficiency units of labor supplied by households. Finally, goods market clearing requires $C_t \equiv \int c_{i,t}^* di = F(K_t, N_t) - K_{t+1} + (1 - \delta)K_t$. By Walras law, one of the market clearing conditions is redundant given that the government's and households' budget constraints are satisfied.

Definition 1 (Equilibrium Definition)**.** *Given an initial distribution of household assets and idiosyncratic shocks $\{a_0^i, z_0^i, \theta_0^i\}_{i\in[0,1]}$ and a fiscal policy $\{B_t, \tau_t^n, \tau_t^k, \tau_t^\pi\}_{t\geq 0}$ with initial debt B_0, an equilibrium is a sequence of quantities $\{A_t, K_t, N_t, L_t\}_{t\geq 0}$, prices $\{r_t, r_t^k, w_t\}_{t\geq 0}$, and transfers $\{T_t\}_{t\geq 0}$ such that A_t and N_t are consistent with household optimization given prices and transfers, K_t and L_t are consistent with firm optimization given prices and taxes, T_t is the lump sum transfer necessary to satisfy the sequential government budget constraint, $r_t^k = r_t + \delta$, and financial, labor, and good markets clear.*

[17] As usual, this is not crucial. We could have equivalently assumed that the capital is owned directly by firms, which finance capital purchases with risk-free bonds issued to households.

We define a *stationary* equilibrium to be an equilibrium in which all sequences are constant over time.[18] We say an economy is "dynamically inefficient" if the marginal product of capital is below the rate of depreciation, $F_K(K, N) < \delta$, and "dynamically efficient" otherwise. For a given N, the "golden rule" level of capital, K^* is such that $F_K(K^*, N) = \delta$, and hence an economy is dynamically inefficient if $K > K^*$.

3 Pareto Improvements to a *Laissez-Faire* Economy

We begin with a stationary equilibrium without a government by considering the case with $\tau_t^j = T_t = B_t = 0$ for all t. This laissez-faire economy will be the benchmark from which we will search for Pareto improving policies and discuss the special role of $r < 0$.[19] Let (w_0, r_0, Π_0) denote the wage, interest rate, and aggregate profits in the initial stationary equilibrium of the laissez-faire economy, and let $(N_0, K_0 = A_0)$ denote the associated aggregate labor supply and capital stock. Unless otherwise stated, we will assume $r_0 < 0$ in what follows.

3.1 Pareto-Improving Fiscal Policies

The thought experiment we consider is a government that, starting from this environment, unexpectedly announces a new fiscal policy. Normalize $t = 0$ to be the last period in which taxes and maturing debt are zero.

In period $t = 0$, the government announces a sequence of debt issuances, taxes, and transfers $\{B_{t+1}, \tau_t^n, \tau_t^k, \tau_t^\pi, T_t\}_{t\geq 0}$. We assume that factor taxes are zero in the initial period: $\tau_0^k = \tau_0^n = \tau_0^\pi = 0$. As K_0 is inherited from the laissez-faire equilibrium, w_0 and Π_0 remain unchanged in this initial period. Other than the announcement, the only action of the government in period zero is the issuance of new bonds B_1 due next period, the proceeds of which are lump-sum rebated to households $T_0 = B_1$. Subsequent to the announcement, there is perfect foresight.

The fiscal policy will potentially involve a new sequence of transfers and factor prices going forward. We *focus on policies that keep the wage stable at w_0 and aggregate after-tax profits at* Π_0. This ensures that no agent experiences a decline in labor or profit income at each t and idiosyncratic state (z_t^i, θ_t^i).

In period zero, each household re-optimizes its consumption-saving policy to incorporate a new sequence of interest rates and transfers, $\{r_t, T_t\}_{t\geq 0}$, with r_0 given, as well as the original

[18] In the analysis that follows, we will assume that such an stationary equilibrium exists. Note that this may require additional assumptions on the stochastic processes for labor productivity and the profit share as well as on their initial cross-sectional distribution. See Açıkgöz (2018), Light (2018), and Achdou, Han, Lasry, Lions and Moll (2021) for results on existence and uniqueness of stationary equilibria in Bewley-Huggett-Aiyagari models.

[19] It is straightforward to include initial debt financed by linear taxes. For example, see footnote 22.

(w_0, Π_0). Starting from the laissez-faire stationary equilibrium in period 0, let $\mathcal{A}_{t+1}(\{r_\tau, T_\tau\}_{\tau\geq 0})$ denote the aggregate household saving in period t generated by the households' new policies.

Asset market clearing imposes a restriction on the possible combinations of transfers, capital, debt issuances, and interest rates. We formalize this restriction in the following definition:[20]

Definition 2. *A sequence $\{r_t, T_t, B_t, K_t\}_{t=0}^{\infty}$ constitutes an* "admissible sequence" *if for all $t \geq 1$:*

$$\mathcal{A}_t(\{r_\tau, T_\tau\}_{\tau=0}^{\infty}) = B_t + K_t,$$

$\{r_0, K_0\}$ represent the initial laissez-faire stationary equilibrium outcomes, $B_0 = 0$, and there exists a $\overline{B} < \infty$ such that $B_t \leq \overline{B}$ for all $t \geq 1$.

It is useful to clarify what is and is not imposed by admissibility. It imposes households' optimality over consumption-savings decisions given the sequence $\{r_t, T_t\}$ and a fixed wage, w_0, as well as asset market clearing. Given that the wage is constant and preferences are GHH, labor market clearing is satisfied for an aggregate labor supply equal to N_0.[21] Admissibility does not impose goods market clearing and the government budget constraint. By Walras Law, either one of these is sufficient to establish an allocation is achievable in equilibrium. The following result gives properties of admissible sequences that are sufficient for the feasibility of Pareto-improving fiscal policies relative to laissez-faire:[22]

Proposition 1. *If there is an admissible sequence $\{r_t, T_t, B_t, K_t\}_{t=0}^{\infty}$ such that for all $t \geq 0$:*

(i) $r_t \geq r_0$;

(ii) $T_t \geq -(r_t - r_0)\underline{a}$;

(iii) and

$$B_{t+1} - (1 + r_t)B_t - T_t \geq F(K_0, N_0) - F(K_t, N_0) - (r_0 + \delta)K_0 + (r_t + \delta)K_t, \tag{2}$$

[20]Note that the definition imposes that the debt sequence has a finite upper bound, $\overline{B}$. This is to rule out Ponzi schemes by the government.

[21]Our function $\mathcal{A}$ is closely related to the C function of Wolf (2021). Both map sequences of policy variables and equilibrium prices into a path of aggregate household spending (in Wolf's case) or saving (in the present case), starting from an initial distribution of idiosyncratic states.

[22] If there was an initial stock of debt B_0, then condition (2) in the proposition becomes:

$$B_{t+1} - (1 + r_t)B_t - T_t \geq F(K_0, N_0) - F(K_t, N_t) - (r_0 + \delta)K_0 + (r_t + \delta)K_t - r_0 B_0.$$

Note that which specific taxes were used to finance B_0 in the initial equilibrium is not relevant for this condition; only the level of debt and initial interest rate matters.

with either (i) or (ii) strict for at least one $t \geq 0$, then there exists a feasible fiscal policy that implements a Pareto improvement.

This proposition delivers a set of conditions that are sufficient for the existence of a Pareto improvement. It indicates that household heterogeneity is important only to the extent that it shapes the aggregate savings schedule, $\mathcal{A}$, which is what matters for the admissibility of the sequence. [23]

We establish the result in three steps, providing some expository remarks as we proceed. We first describe the set of tax instruments used by the government, then compute the tax revenue, and finally establish the result.

For step one, in order for the government to keep the households' after-tax wage constant, it must tax or subsidize the firm's labor input such that:

$$\frac{F_N(K_t, N_0)}{(1+\tau_t^n)\mu} = w_0.$$

Due to the GHH preferences, keeping after-tax wages constant at w_0 ensures the aggregate labor supply remains at its initial level N_0.[24] Moreover, if the issuance of debt crowds out capital ($K_t < K_0$), this involves a labor subsidy $\tau_t^n < 0$, given that $F_K(K_0, N_0) = \mu w_0$.

Similarly, the government taxes/subsidizes profits so that:

$$\Pi_t = (1-\tau_t^\pi)\hat{\Pi}_t = (1-\tau_t^\pi)(\mu-1)F(K_t, N_0)/\mu = \Pi_0$$

Finally, the government must ensure that the representative firm's choice of capital is consistent with the risk-free interest rate:

$$F_K(K_t, N_0) = (1+\tau_t^k)\mu r_t^k = (1+\tau_t^k)\mu(r_t+\delta).$$

[23] As noted, Walras Law implies an alternative representation of condition (2) in the proposition that verifies goods market clearing. Let C_t be aggregate consumption:

$$\begin{aligned} C_t &\equiv w_0 N_0 + \Pi_0 + (1+r_t)A_t - A_{t+1} + T_t \\ &= F(K_0, N_0) - (r_0+\delta)K_0 + (1+r_t)A_t - A_{t+1} + T_t. \end{aligned}$$

Then, for an admissible sequence, condition (2) is equivalent to the aggregate resource constraint:

$$C_t + K_{t+1} \leq F(K_t, N_0) + (1-\delta)K_t.$$

[24] This is the major simplification introduced by GHH. We do not need to keep track of the aggregate labor supply, or more importantly, to check that it is consistent with the aggregation of households' optimality conditions for labor and savings.

The total government revenue (before transfers) of this tax policy is given by:

$$\begin{aligned}\text{Revenue} &= \tau_t^n w_0 N_0 + \tau_t^k r_t^k K_t + \tau_t^\pi \hat{\Pi}_t \\ &= (1+\tau_t^n) w_0 N_0 + (1+\tau_t^k) r_t^k K_t - (1-\tau_t^\pi)\hat{\Pi}_t - w_0 N_0 - r_t^k K_t + \hat{\Pi}_t \\ &= \frac{F_N(K_t, N_0)N_0 + F_K(K_t, N_0)K_t}{\mu} - \Pi_0 - w_0 N_0 - r_t^k K_t + \frac{(\mu-1)F(K_t, N_0)}{\mu} \\ &= F(K_t, N_0) - \Pi_0 - w_0 N_0 - r_t^k K_t,\end{aligned}$$

where the third line uses: $(1-\tau_t^\pi)\hat{\Pi}_t = \Pi_0$; the firm's first-order condition for labor and capital; and $\hat{\Pi}_t = (\mu-1)F/\mu$. The last line follows from Euler's theorem. We can then use $r^k = r + \delta$ and $\Pi_0 = F(K_0, N_0) - r_0^k K_0 - w_0 N_0$ to obtain:

$$\text{Revenue} = F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0. \tag{3}$$

A convenient feature of this result is that the costs of a policy are pinned down by the aggregate capital stock and the interest rate. No additional information is needed, despite the potentially complicated nature of policies necessary to keep all factor prices and profits weakly increasing.[25]

Note that if $K_t < K_0$, equation (3) implies that revenue is necessarily negative. To see this, strict concavity of F implies $F(K_0, N_0) - F(K_t, N_0) > F_K(K_0, N_0)(K_0 - K_t)$. From the firm's first-order condition in the laissez-faire equilibrium, we have $F_K(K_0, N_0) = \mu(r_0 + \delta) \geq r_0 + \delta$, and hence (as $K_t < K_0$), $F(K_0, N_0) - F(K_t, N_0) > (r_0 + \delta)(K_0 - K_t)$. This implies that the value in (3) is strictly less than $(r_0 - r_t)K_t \leq 0$.

Equation (2) follows from equation (3) and the government's budget constraint. The right-hand side is the negative of equation (3). Bringing that to the other side, we have that bond issuances plus tax revenues minus lump-sum transfers must be non-negative. The inequality reflects that we allow the government to dispose of any surplus.

Finally, we verify that the new equilibrium is a Pareto improvement. By construction, wage and profit income for households remain the same as in laissez-faire in every t and idiosyncratic state. The fact that the return to financial wealth weakly increases makes every saver at time t better off. However, those with negative positions (debt) are worse off. The fact that $T_t \geq -(r_t - r_0)\underline{a}$ ensures that lump-sum transfers are large enough to make debtors weakly better off, and strictly if $a_t^i > \underline{a}$. From every household's perspective, resources are weakly greater at every

[25] Part of this tractability rests on the representative firm assumption. This allows us to track how the marginal product of labor changes in response to a policy using just the knowledge of aggregates. If there were a distribution of firms with heterogeneous capital-labor ratios, we would need to track the entire distribution's response to policy in order to compute the labor subsidy necessary to keep wages constant.

t and at every idiosyncratic state, and strictly greater for at least one household as there exists a t such that $r_t > r_0$ or $T_t > 0$. This establishes that the fiscal policy results in a Pareto improvement and concludes the proof of the proposition.

The following result provides a simpler sufficient condition for equation (2) to hold:

Claim 1. *If $F(K_t, N_0) - F(K_0, N_0) \geq r_t^k (K_t - K_0)$ and transfers are minimized $T_t = -(r_t - r_0)\underline{a}$, then a sufficient condition for (2) to hold is:*

$$B_{t+1} - (1 + r_t)B_t \geq (r_t - r_0)(K_0 - \underline{a}), \tag{4}$$

for all $t \geq 1$.

Proof. The premise and equation (3) implies that we have a lower bound on tax revenue:

$$\begin{aligned} \text{Revenue} &= F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0 \\ &\geq (r_t + \delta)(K_t - K_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0 \\ &= -(r_t - r_0)K_0. \end{aligned}$$

Substituting into (2) and setting $T_t = -(r_t - r_0)\underline{a}$ yields equation (4). □

The condition $F(K_t, N_0) - F(K_0, N_0) \geq r_t^k (K_t - K_0)$ holds immediately in two useful benchmarks. One is zero crowding out of capital, so that $K_t = K_0$. The second is when $F_K(K_t, N_0) = r_t^k$, that is, capital is undistorted relative to the risk-free rate faced by households, and the inequality holds by concavity of F in K.

To provide intuition for Proposition 1 and Claim 1, and to more fully characterize the nature of the policies, we consider three alternative policies in turn. The first is one in which capital is held at the laissez-faire level; the second crowds out capital; and the third crowds in capital by subsidizing investment.

3.2 The Constant-K Policy

The constant-K policy holds $K_t = K_0$ despite the increase in the interest rate due to debt issuance. As noted above, this policy satisfies the premise of Claim 1. The advantage of such a policy is that the sufficient condition for a feasible Pareto improvement stated in Claim 1 holds whether the economy is fully competitive, $\mu = 1$, or has markups, $\mu > 1$, and whether the economy is dynamically efficient or inefficient. In this sense, the sufficient condition is robust to the nature of idiosyncratic uncertainty and preferences conditional on aggregate savings, to whether the economy is operating competitively, and to whether capital is above or below the golden rule.

Specifically, suppose the government issues additional bonds. At a given level of capital, in order to induce households to hold more assets, the equilibrium interest rate must increase $r_t \geq r_0$, assuming that aggregate savings is increasing in r.[26] All else equal, this would raise the rental rate of capital and the representative firm would demand less capital. To avoid crowding out capital, suppose the government subsidizes the return from renting capital. Recall that r_0^k is the rental rate in the laissez-faire equilibrium, with $r_0 = r_0^k - \delta$. Let r_t be the net interest rate on government bonds in the new equilibrium at time t. Let $\tau_t^k < 0$ be a subsidy to capital such that firms pay $r_0^k = (1 + \tau_t^k)(r_t + \delta)$, and households receive $r_t \geq r_0$. As firms are paying the same after-tax rental rate, then $K_t = K_0$, and hence profits, wages, and total output remain unchanged. From equation (3), the cost of this policy is $-(r_t - r_0)K_0$.

To gain some intuition, let ss denote the new stationary equilibrium. In the limit as $t \longrightarrow \infty$, the feasibility condition in (4) becomes:

$$-r_{ss}B_{ss} \geq (r_{ss} - r_0)(K_0 - \underline{a}). \tag{5}$$

The "seigniorage" revenue ($r < 0$) from bonds must be large enough to subsidize capital as well as the compensation to borrowers.

Figure 1 depicts the tradeoff in the canonical capital market equilibrium diagram from Aiyagari (1994). The underlying calibration is provided in Section 4, but the qualitative features are fairly general. At each interest rate on the vertical axis r, the associated laissez-faire rental rate of capital is $r^k = r + \delta$. Holding labor supply constant, $N = N_0$, the downward sloping red line traces out a laissez-faire capital demand equation from the firm's first-order condition $F_K(K, N_0) = \mu r^k$.

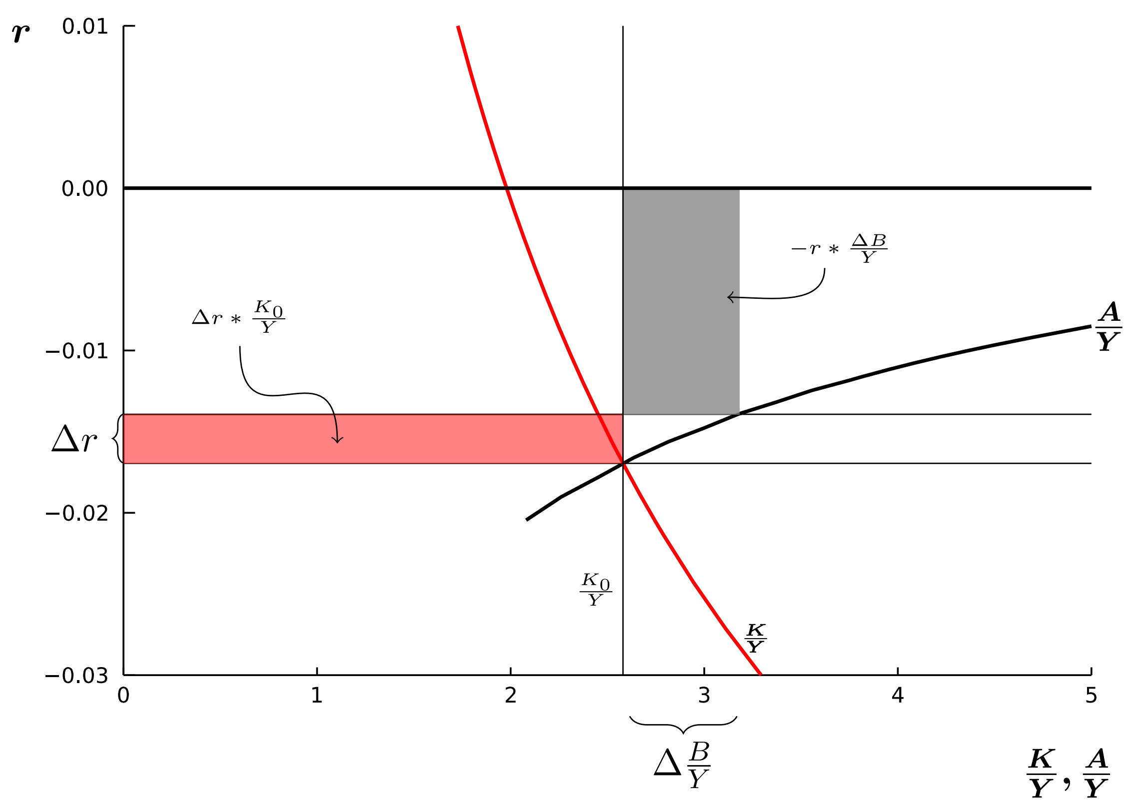
Similarly, at each candidate r, A denotes the aggregate steady-state saving of households when the wage is fixed at w_0.[27] These two curves intersect at the laisssez-faire equilibrium interest rate r_0. Note that in this parameterization, $r_0 < 0$, which is the case of interest. The quantities reflected on the horizontal axis are normalized by $Y_0 = F(K_0, N_0)$.

The fiscal policy subsidizes the rental of capital such that firms are willing to rent K_0 at all r. The width of the gray rectangle is $\Delta B/Y_0 = \frac{A_{ss}-K_0}{Y_0}$ and its height is the interest rate at the new equilibrium, hence its area is $-r_{ss}\Delta B/Y_0$. The red rectangle has height $r_{ss} - r_0$, where r_0 is the interest rate in the laissez-faire equilibrium. Its width is K_0/Y_0, where K_0 is the capital stock in the laissez-faire equilibrium. The area of this rectangle is $(r_{ss} - r_0)K_0/Y_0$. In this example, $\underline{a} = 0$. From equation (5), if the area of the gray rectangle exceeds that of the red, then *a Pareto improvement is feasible* at the steady state.

[26] More precisely, the interest rate must increase at some point along the path and converge to $\lim_{t\to\infty} r_t > r_0$. To rule out alternative cases which trade-off lower interest rates along the transition against higher long-run rates, we include $r_t \geq r_0$ as a condition for all t in Proposition 1.

[27] At each point on the household saving line, there is an associated lump-sum transfer that satisfies the government's budget constraint. At each r and implied $B = S - K$, the upward sloping line solves the household's problem for the associated transfer.

Figure 1: Net Resource Cost with Constant K



Note: This figure is a graphical depiction of the fiscal tradeoff from equation (5). All elements are normalized by the laissez-faire stationary equilibrium output $Y = Y_0$. The downward sloping line K/Y represents firm's demand for capital ($r = F_K/\mu - \delta$) and the upward sloping line A/Y depicts aggregate household saving associated with the interest rate r and the laissez-faire wage as well as the transfers generated by any fiscal surplus. The intersection is the initial laissez-faire stationary equilibrium. Fiscal costs are represented by $\Delta r * K_0/Y$, the area shaded in red, and seigniorage revenue by $-r * \Delta B/Y$, the area shaded in gray. In this example, policy holds capital at the initial laissez-faire capital stock.

The diagram reveals the key considerations in whether inequality (5) is likely to be satisfied. First, the level of the interest rate matters. That is, households must be willing to hold the economy's wealth at a low interest rate, reflecting a significant demand for precautionary savings.[28] Intuitively, and as we shall see in detail in the calibration of Section 4, this will be the case if households face significant idiosyncratic risk and are patient and risk averse. The large demand for a safe store of value provides a source of seigniorage for the government.

Second, consumers must be willing to hold new debt without a sharp increase in the interest rate. That is, the elasticity of aggregate savings to r must be sufficiently large. The intuition is that the return to saving (Δr) cannot increase significantly in response to the issuance of ΔB, as the increase in the return to capital is the amount of subsidy necessary to keep capital constant. The elasticity of the interest rate to government debt is a primary concern when discussing the "crowding out" of capital. Here, it is determining the amount of fiscal resources that must be dedicated to capital subsidies.

It is useful to pause and note some intriguing features of this Pareto improvement. Aggregate output, consumption, and investment are all held fixed at the laissez-faire level, as $K_t = K_0$. Yet every household faces a weakly bigger budget set, and strictly bigger if $a_t^i > \underline{a}$. While any household could strictly increase their consumption at all times, not all choose to do so in equilibrium, ensuring the aggregate resource condition remains unchanged. However, those with high endowment states are willing to postpone consumption due to the high return on saving. The seigniorage revenue collected from these savers is then lump-sum rebated to all households, allowing those with lower endowments to increase their consumption. This improved risk-sharing is the source of the Pareto improvement in the constant-K policy.

While this is similar to other fiscal schemes, such as a pay-as-you-go social security system, this transfer is done without taxing anyone. The sole source of revenue in the constant-K policy is the negative return on government bonds, and thus has a clear antecedent in Samuelson (1958), in which money is a substitute for "social coercion." As in a pure monetary model, the steady state seigniorage revenue depends on the infinite horizon. In contrast to Samuelson, the introduction of money or bonds is not enough for a Pareto improvement, as factor prices will change. Hence, the additional need for capital subsidies. Moreover, we shall see in Section 3.4 that a mark-up, which did not play any direct role in the above discussion, provides an alternative source of Pareto-improving fiscal policies independently of $r < 0$.

[28] For this policy, it is necessary that $r_{ss} < 0$, else (5) cannot hold.

3.3 Dynamic Inefficiency: Capital Crowding Out

If the economy is dynamically inefficient, there is social benefit from allowing the increase in interest rate due to debt issuance to crowd out capital. Consider first the case where firms are competitive, $\mu = 1$, and profits are zero. In this case, $r_0 < 0$ implies dynamic inefficiency. Suppose the government issues additional bonds. The higher r induces additional savings and crowds out capital, which is beneficial at the margin given the dynamic inefficiency, as in Diamond (1965). To keep after-tax wages from falling it is necessary that $\tau_t^n < 0$. This labor subsidy represents the primary fiscal cost of the policy.

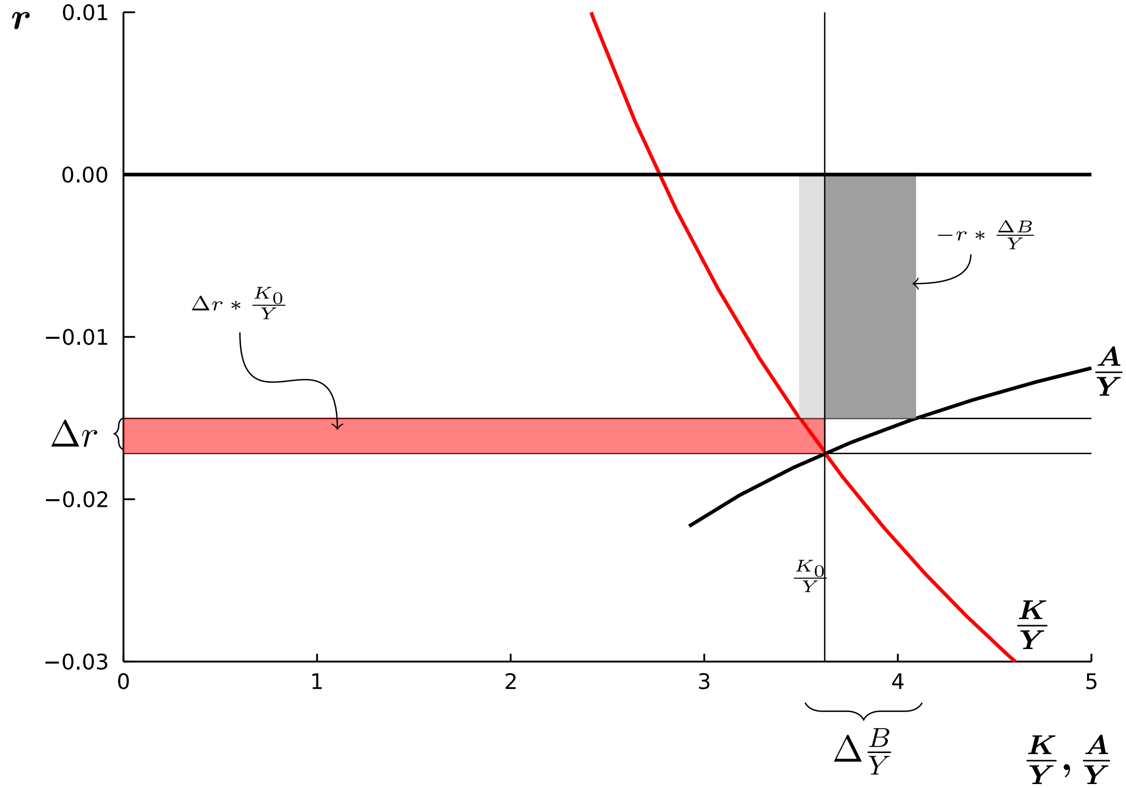
Suppose the policy sets $\tau^k = 0$, so that capital is undistorted relative to the risk-free rate. We consider a broader set of policies after providing some intuition for this case. From Claim 1, the cost of the labor subsidy is bounded by $(r_t^k - r_0^k)K_0 = (r_t - r_0)K_0$, which is the change in payments to capital. If this is large, then the share of revenue paid by firms to labor falls significantly and the subsidy to labor must be large ($\tau^n << 0$). Another interpretation is obtained by letting $w = \phi(r)$ denote the factor price frontier in the competitive laissez-faire equilibrium (so that w and r correspond to the associated factor marginal products). We have $dw/dr = -K$, and so the change in wages is approximately $\Delta w \approx -K_0 \Delta r$. This is the amount the government must make up through subsidy. Note that in the constant-K policy, $\Delta r K_0$ pinned down the cost of capital subsidy; here, the same term characterizes the cost of the labor subsidy.

Figure 2 replicates Figure 1 for the calibration of Section 4 in which $\mu = 1$. As before, seigniorage revenue is the area of the gray rectangle, and the fiscal cost of wage subsidies is the red rectangle. With capital crowding out, the width of the gray rectangle is $\Delta B/Y_0$ is greater than the increase in aggregate household wealth. That is $K_{ss} < K_0$, and $\Delta B = \Delta A - \Delta K > \Delta A$. In Figure 2 the part of seigniorage generated by displacing capital is to the left of the vertical line K_0/Y and is shaded a lighter color.

The fact that crowding out of capital raises seigniorage without necessitating that households hold more wealth opens the door to a broader set of fiscal policies that are feasible. Namely, those that increase the crowding out of capital to make "room" for additional debt issuance at a given interest rate. This outcome can be implemented by a positive τ^k that depresses K_t. While the additional seigniorage and revenue from $\tau^k > 0$ relax the fiscal constraints (and may bring the economy closer to the golden rule), the lower capital stock requires a larger wage subsidy. There is thus a limit to the feasibility of additional crowding out.

It is straightforward to show, however, that if the economy is dynamically inefficient, the government can always generate additional resources by crowding out capital to the golden rule level K^* (that is, the value that solves $F_K(K^*, N_0) = \delta$):

Figure 2: Fiscal Tradeoff in the Steady State: Crowding Out



Note: This figure is a graphical depiction of the fiscal tradeoff from equation (5). All elements are normalized by the laissez-faire stationary equilibrium output $Y = Y_0$. The downward sloping line K/Y represents firm's demand for capital ($r = F_K - \delta$) and the upward sloping line A/Y depicts aggregate household saving associated with the interest rate r and the laissez-faire wage as well as the transfers generated by any fiscal surplus. The intersection is the initial laissez-faire stationary equilibrium. Fiscal costs are represented by $\Delta r * K_0/Y$, the area shaded in red, and seigniorage revenue by $-r * \Delta B/Y$, the area shaded in gray. The part of seigniorage generated by displacing capital is to the left of the vertical line K_0/Y and is shaded a lighter color. The remaining seigniorage represents additional household saving.

Claim 2. *Consider the case where the laissez-faire stationary economy is dynamically inefficient (that is, $K_0 > K^*$). Then the sequence $\{r_t, T_t, B_t, K_t\}$ with $r_t = r_0$, $T_t = 0$, $K_t = K^*$ and $B_t = K_0 - K^* > 0$ for all $t \geq 1$ and $T_0 = 0$ is admissible. In addition, such sequence satisfies the conditions (i), (ii) and (iii) of Proposition 1. And in particular, the inequality (2) is strict for all $t \geq 0$; that is, the government raises strictly positive revenue in all periods without affecting any household's utility.*

Proof. That the sequence is admissible follows directly from the fact that $\{r_t, T_t\}$ remain as in the stationary laissez-faire equilibrium, and thus $A_t = K_0$ for all $t \geq 0$; $B_t + K_t = A_t = K_0$; and B_t is bounded.

Conditions (i) and (ii) in Proposition 1 are satisfied. From the fiscal resource condition (2), we have for $t = 0$ the government raises $B \equiv K_0 - K^*$ in additional resources by issuing bonds. As K_0 is fixed, there are no fiscal costs in the initial period. For $t > 0$ the fiscal resource condition (2) is:

$$\begin{aligned} & B - (1 + r_0)B + F(K^*, N_0) - F(K_0, N_0) - (r_0 + \delta)K^* + (r_0 + \delta)K_0 \\ & = F(K^*, N_0) - F(K_0, N_0) - \delta K^* - r_0(K^* + B) + \delta K_0 + r_0 K_0 \\ & = F(K^*, N_0) - \delta K^* - (F(K_0, N_0) - \delta K_0), \end{aligned}$$

where we use $K^* + B = K_0$ for the last equality. By definition of the golden rule capital stock, K^* maximizes $F(K, N) - \delta K$. Hence, the policy generates strictly positive resources for all $t \geq 0$. Such policy does not affect any household's utility as all prices and incomes remain unchanged. □

In terms of Figure 2, distorting capital at a given r increases the width of the gray rectangle without increasing the cost. The claim states the government can do this at the initial r_0, generating zero net costs. The fact that this policy generates a surplus suggests that there is scope to raise welfare by providing a public good, as long as the public good does not significantly alter households' savings or labor-supply choices. The result extends the classic Samuelson-Diamond result to the Aiyagari framework. As we already saw in the constant-K case, dynamic inefficiency is not a necessary condition for Pareto-improving fiscal policies. However, the next result states that it is necessary for the competitive case, at least for the class of policies considered by Proposition 1:

Claim 3. *Consider $\mu = 1$. If $r_0 > 0$, then there is no admissible sequence that satisfies the conditions of Proposition 1.*

Proof. Condition (iii) in Proposition 1 requires

$$B_{t+1} - (1 + r_t)B_t + F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0 - T_t \geq 0.$$

If $\mu = 1$, we have $F_K(K_0, N_0) = (r_0 + \delta)$. Concavity of F implies

$$F(K_0, N_0) - F(K_t, N_0) \geq (r_0 + \delta)(K_0 - K_t).$$

Thus a necessary condition for condition (iii) is:

$$B_{t+1} - (1 + r_t)B_t + (r_0 - r_t)K_t - T_t \geq 0.$$

Re-arranging:

$$B_{t+1} - B_t \geq r_t B_t + (r_t - r_0)K_t + T_t.$$

If $r_t > r_0$ or $T_t > 0$ for some t, the right-hand side is strictly positive at t even if $B_t = 0$, implying a strictly positive increase in debt. After this period, the right-hand side is always strictly positive as $r_t \geq r_0 > 0$ and $T_t \geq 0$, generating an explosive path of debt, violating the upper bound on debt required for an admissible sequence. □

So far in this subsection, we have considered crowding out capital in an economy without markups. Consider the case of a small markup, where by small we mean that the economy remains dynamically inefficient when $r_0 < 0$.[29] We shall consider the alternative case in the next subsection. Again, we start with the policy of $\tau^k = 0$, so that $F_K = \mu(r_t + \delta)$. The complication relative to the competitive case is that with a markup, a Pareto improvement requires not only weak increases in factor prices but also a weak increase in profits. That is, $\tau_t^\pi \leq 0$ so that $(1-\tau_t^\pi)\hat{\Pi}_t = \Pi_0$.

Using the fact that $\hat{\Pi}_t = \Pi_0/(1 - \tau_t^\pi)$ and $\tau^k = 0$, equation (3) implies:

$$\tau_t^n w_0 N_0 + \frac{\tau_t^\pi}{1 - \tau_t^\pi}\Pi_0 = F(K_t, N_0) - F(K_0, N_0) - r_t^k K_t + r_0^k K_0,$$

Concavity implies $F(K_0, N_0) \leq F(K_t, N_0) + F_K(K_t, N_0)(K_0 - K_t)$. Using the fact that $F_K(K_t, N_0) = \mu r_t^k$, we have:

$$\tau_t^n w_0 N_0 + \frac{\tau_t^\pi}{1 - \tau_t^\pi}\Pi_0 \geq (\mu - 1)r_t^k(K_t - K_0) - (r_t - r_0)K_0.$$

The first term on the right-hand side is due to the markup and adds to the fiscal burden as $K_t < K_0$. The fall in K leads to a fall in profits, which must be offset by subsidies in order to ensure a Pareto improvement. Here, it makes the improvement harder to achieve if capital is elastic.

The counter-part to condition (4) is now

$$B_{t+1} - (1 + r_t)B_t \geq -(\mu - 1)r_t^k(K_t - K_0) + (r_t - r_0)(K_0 - \underline{a}).$$

For the steady-state comparison of Figure 2, we can think of losing part of the gray rectangle over the ΔK part of the horizontal axis if $\mu > 1$. The markup implies some of the gray area is used to compensate the decline in profits.

To summarize, if the economy with markups is also dynamic inefficient, then, as in the competitive case, crowding out capital leads to a Pareto improvement.[30] We turn our attention next to the case where the economy is dynamically efficient.

[29] That is, $\mu < \delta/(r_0 + \delta)$.

[30] Note that claim 2 required only dynamic inefficiency, not $\mu = 1$.

3.4 Capital Crowding In

We now consider a policy in which the government subsidizes investment in order to "crowd in" capital. The policy is relevant for an economy in which the markup is large. In particular, suppose the economy is dynamically efficient, despite the fact that $r_0 < 0$. As dynamic efficiency requires $F_K = \mu(r_0 + \delta) > \delta$, the markup is bounded below by $\delta/(r_0 + \delta) > 1$.

The constant-K analysis of Section 3.2 applies to this environment, as that policy is robust to the level of the markup. However, given that the markup depresses the level of capital in the laissez-faire equilibrium relative to the first-best, it may be feasible to crowd in capital. In particular, rather than using government debt to increase the supply of financial assets, the government may also find it beneficial to induce additional investment. It can do so by subsidizing capital to the point that $(1 + \tau^k)(r_t + \delta) < r_0 + \delta$. The additional capital raises pre-tax profits and wages. This possibility begs the question of whether the government can tax these latter factors, keeping the after-tax prices at the laissez-faire levels, and generate sufficient revenue to subsidize capital.

Note that the fiscal policies we allow for do not tax away pure profits completely, as we need to ensure that households earning profits in the laissez-faire economy continue to do so – and to the same extent – in the economy with fiscal policy. Nor can the subsidy to investment be paid for with lump-sum taxation, as a Pareto improvement requires $T_t \geq 0$. Thus, our policies are distinct from more familiar fiscal policies that directly respond to markup distortions by redistributing rents or that implement the competitive allocation via taxes and subsidies.

As a start, suppose we take the extreme of zero revenue from seigniorage, so $B_t = 0$ for all t. Instead, the government implements policy to increase the capital stock, $K_t > K_0$. For simplicity of exposition, let $\underline{a} = 0$ and $T_t = 0$, so that any excess revenue is discarded.

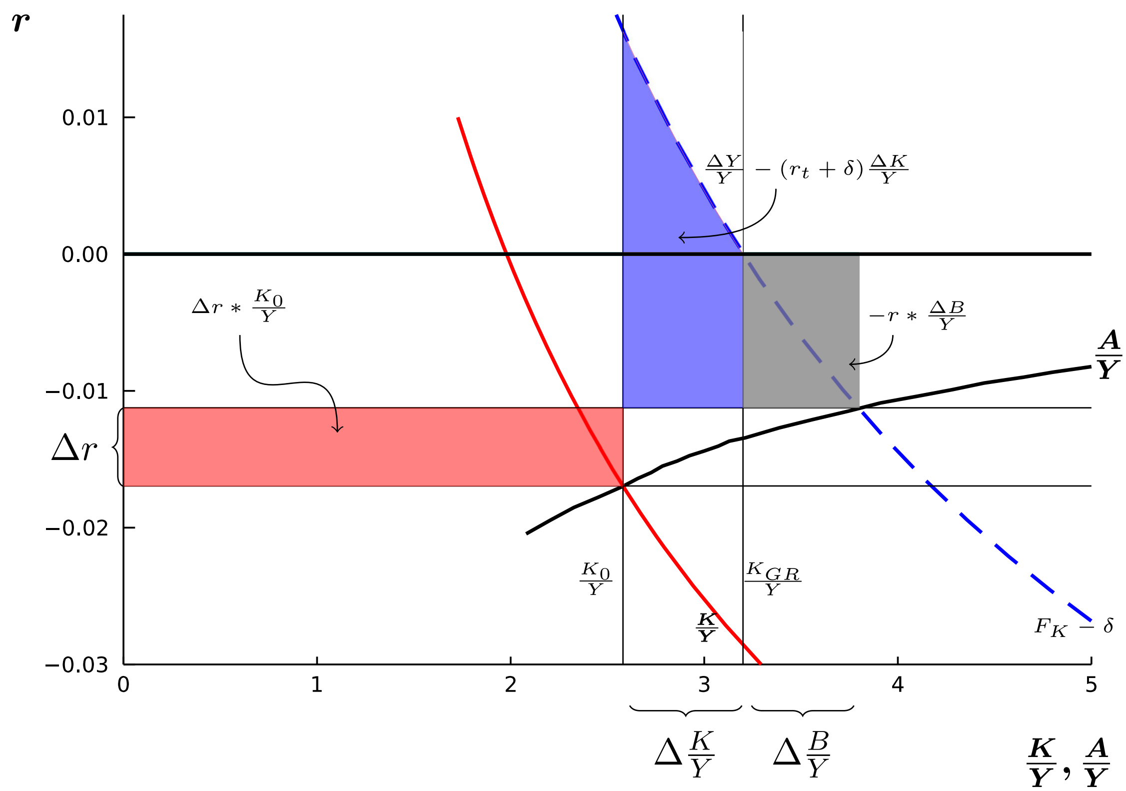
For this crowding in policy to be an equilibrium, households must be willing to hold additional wealth. That is, it must be part of an admissible sequence $\{r_t, T_t = 0, B_t = 0, K_t\}_{t\geq 1}$. Recall that admissible sequences are generated by the household's problem, and reflect the mapping from sequences $\{r_t, T_t\}$ to aggregate wealth, A_t, given (w_0, Π_0). The experiment sets $A_t = K_t = A_0 + \Delta K_t$ for $t \geq 1$, where $\Delta K_t \equiv K_t - K_0$.

Associated with the admissible sequence are taxes such that $F_K(K_t, N_0) = \mu(1 + \tau_t^k)(r_t + \delta)$, $F_N(K_t, N_0) = (1 + \tau_t^n)w_0$, and $(1 - \tau_t^\pi)F(K_t, N_0) = F(K_0, N_0)$, with $K_t = K_0 + \Delta K$. From the proof of Proposition 1, tax revenues are:

$$F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0.$$

Given that $B_t = T_t = 0$ in this experiment, feasibility requires this expression must be weakly greater than zero. A positive markup implies that $F_K(K_0, N_0) > (r_0 + \delta)K_0$, so for small ΔK, resources are increasing in K_t *for a given interest rate*. For a small increase $\Delta K \approx 0$, revenues are

Figure 3: Net Resource Cost with Crowding In



Note: This figure reproduces the asset demand and supply curves from Figure 1 for the case in which policy *crowds in* capital. The dashed downward sloping blue line represents the marginal product of capital minus depreciation. The capital demand curve K/Y is distorted relative to this benchmark by the markup. In this example, policy crowds in capital to the “golden rule” level K_{GR}. The gain from crowding in of capital is the area between $F_K - \delta$ and the new steady state r.

approximately

$$F_K(K_0, N_0)\Delta K_t - (r_0 + \delta)\Delta K_t - \Delta r_t K_0,$$

where $\Delta r_t \equiv r_t - r_0$ and we set second-order terms to zero.[31] The representative firm's first order condition in the laissez-faire equilibrium implies $F_K(K_0, N_0) = \mu(r_0 + \delta)$. Thus, to a first order and assuming $\Delta r_t > 0$, a sufficient condition for revenue to be weakly positive is

$$\mu - 1 \geq \xi_0, \tag{6}$$

where $\xi_0 \equiv \frac{\Delta r_t}{\Delta K_t}\left(\frac{K_0}{r_0+\delta}\right)$ is the inverse elasticity of aggregate wealth to the interest rate.

The left-hand side of (6) is the markup, and the right-hand side is the elasticity of the interest rate to aggregate wealth, using the fact that $K_t = A_t$ in this experiment. This condition is easier to satisfy if the distortion of capital due to market power is large and if the elasticity of wealth to the interest rate ($1/\xi_0$) is large. For such a combination, to a first order, crowding in of capital is a feasible Pareto improvement.

Crowding in of capital allows the economy to (eventually) produce more efficiently, but foregoes the revenue generated by issuing bonds along the path. In the initial period, issuing bonds generates more resources to the government than directing additional savings to investment, but in the steady state, the policy of crowding in capital generates greater fiscal resources.

Of course, policy does not have to be only debt or only crowding-in of capital. In fact, there is a theoretical case for a combination of both. As noted, equation (6) is easier to satisfy the greater the elasticity of private saving to the interest rate. In many settings, the long-run elasticity is greater than short-run elasticity, a property Paul Samuelson linked to the LeChatelier principle original developed in chemistry. This property holds in the quantitative Aiyagari model explored in Section 4. In the current context of inducing greater investment, this potentially implies an over-shooting of the interest rate along the transition. A higher interest rate raises the fiscal cost of capital subsidies. If the short-run elasticity is too small to satisfy (6), the government can issue debt along the transition to finance the associated crowding-in fiscal policy. In the new stationary equilibrium, the additional resources can then be used to service (if $r_\infty > 0$) or pay down the debt. Thus, along the transition, debt and additional capital become *complements* rather than substitutes in engineering a Pareto improvement. We give an example of such a hybrid policy in Section 4.

A graphical depiction of the steady state after such a hybrid policy is given in Figure 3. The novelty is the area between the curve labelled $F_K - \delta$ and the new long-run interest. In particular,

[31] Note that we impose that Δr_t is of the same order as ΔK_t, which implicitly assumes a continuous mapping from the sequence of interest rates to the sequence of aggregate wealth.

recall that fiscal revenue before transfers in the new steady state is given by

$$\begin{aligned}
&F(K_\infty, N_0) - F(K_0, N_0) - (r_\infty + \delta)K_\infty + (r_0 + \delta)K_0 \\
&= \int_{K_0}^{K_\infty} (F_K - \delta - r_\infty) dK - \Delta r \times K_0.
\end{aligned}$$

In Figure 3, the shaded area under the marginal product curve and above the new steady-state r represents the first term, while the rectangle $\Delta r \times K_0$ represents the final term. The additional shaded rectangle labelled $-r\Delta B$ represents seigniorage from debt issuance. Again, this depiction refers to the stationary equilibrium and ignores the transition. Nevertheless, it graphically highlights the fiscal resources gained by crowding in capital when $F_K - \delta > r$ due to a markup distortion.

The additional revenue (eventually) raised from crowding in of capital reduces, and potentially eliminates, the need for seigniorage in the long-run. Thus, markups provide an independent source of Pareto improvements above and beyond $r < 0$. We can make this point in stark terms by considering a complete markets economy in which the long-run interest rate is pinned down by $1/\beta - 1 > 0$. We turn our attention to this case next.

3.5 The Complete Markets Benchmark

The complete-markets, representative agent benchmark is a useful environment to shed light on three key facets of the above analysis. The first is the role of the elasticity of aggregate savings. In the complete markets case, the long-run elasticity is infinite, and transition dynamics from the household side are pinned down by the consumer's Euler equation. The second facet is the role of debt along the transition. This is not independent of the first, as the finite short-run elasticity of savings implies the interest rate overshoots its long-run level, and the government can use debt along the transition to smooth the costs associated with the temporarily high interest rate. The third facet is that the markup on its own, independent of risk sharing considerations or $r < 0$, opens the door to Pareto-improving fiscal policy, despite the fact that after-tax profits remain bounded below by the laissez-faire equilibrium.

Note that the representative agent benchmark is nested in our notation. The technology side is the same as in the neoclassical growth model, but augmented with a constant markup. Given the Ricardian structure of the neoclassical model, we can set transfers to zero without loss of generality.[32] Thus, the household budget constraint and national income accounting imply "admissible

[32] In particular, for a fixed sequence of distortionary taxes, there exist many alternative paths for debt and lump-sum taxes/transfers that deliver the same allocation. To see this, given that in the laissez-faire, $B_t = 0$ and the representative agent must hold the aggregate capital stock, we can ignore the role of $\underline{a}$ in condition (ii) of Proposition 1, and focus on $T_t \geq 0$. Now, suppose that $\{r_t, T_t, B_t, K_t\}$ is an admissible sequence that satisfies condition (iii) of

sequences" satisfying the same resource conditions:

$$\begin{aligned} C_t &= F(K_t, N_0) + (1-\delta)K_t - K_{t+1} \\ &= w_0 N_0 + \Pi_0 + (1+r_t)A_t - A_{t+1}, \end{aligned}$$

where $A_t = K_t + B_t$. Household saving behavior is characterized by the representative agent's Euler equation

$$u_c(C_t, N_0) = \beta(1+r_t)u_c(C_{t+1}, N_0),$$

where we assume the case of time-separable preferences. This condition is the restriction imposed by complete markets. Thus, admissible sequences are those that satisfy the resource condition, the Euler equation, and the upper bound on government debt $\overline{B}$.

To make the analysis as transparent as possible, we consider a very simple policy. At time 0, starting from the laissez-faire steady state, the government induces a small, permanent increase in the capital stock, $K_t = K_1 > K_0$. We also assume the government does not issue government bonds in period zero, $B_1 = 0$, implying $A_1 = K_1 > K_0 = A_0$. For this to be consistent with household optimization, $r_1 > 1/\beta - 1 = r_0$. We assume for $t \geq 2$, the economy is in steady state. Thus, for $t \neq 1$, $r_t = 1/\beta - 1 = r_0$.

Working backwards from $t = 2$, the fact that $r_2 = 1/\beta - 1$ implies that $C_1 = C_2$ due to the Euler equation. The household's budget constraint implies:

$$\begin{aligned} C_1 &= w_0 N_0 + \Pi_0 + (1+r_1)A_1 - A_2 \\ C_2 &= w_0 N_0 + \Pi_0 + r_0 A_2, \end{aligned}$$

where the second line uses the fact we are in steady state for $t \geq 2$. These expressions plus the fact that $C_1 = C_2$ and $r_1 > r_0$ implies that $A_2 > A_1$. As $K_1 = K_2$, we have $B_2 = A_2 - K_1 = A_2 - A_0 > 0$. Thus, the government issues debt in period $t = 1$, and then rolls it over indefinitely.

To see the role of debt from the household's perspective, note that the interest rate is falling between periods $t = 1$ and $t = 2$. Consumption smoothing induces the household to save some of the temporarily high capital income, and the government accommodates this by issuing debt. Note that the government is issuing debt at r_0, taking advantage of the infinite elasticity of savings in the steady state.

Proposition 1 with $r_t \geq r_0 = 1/\beta - 1 > 0$. Let us recursively define $\Delta_{t+1} \equiv (1+r_t)\Delta_t + T_t$, starting with $\Delta_0 = 0$. The fact that $T_t \geq 0$ implies that $\Delta_t \geq 0$. Then the sequence $\{r_t, \hat{T}_t, \hat{B}_t, K_t\}$ with $\hat{T}_t = 0$ for all $t \geq 0$ and $\hat{B}_t = B_t - \Delta_t \leq B_t$ is also admissible. In addition, $B_{t+1} - (1+r_t)B_t - T_t = \hat{B}_{t+1} - (1+r_t)\hat{B}_t$, and thus the sequence $\{r_t, \hat{T}_t, \hat{B}_t, K_t\}$ also satisfies condition (iii) of Proposition 1.

From the household's budget constraints, plus $A_2 = B_2 + A_1 = B_2 + K_1$, we can solve out

$$B_2 = \left(\frac{r_1 - r_0}{1 + r_0}\right) K_1.$$

Thus, the larger is r_1, the more the government issues debt.

The size of r_1 depends on the amount of investment being induced, $K_1 - K_0$, as well as the curvature of the utility function. Let $r_1 + \delta = \boldsymbol{R}(K_1)$ define the mapping from K_1 to $r_1 + \delta$, conditional on preferences.[33] Note that $\boldsymbol{R}(K_0) = r_0 + \delta$. The short-run response of the interest rate to the government policy is given by $\boldsymbol{R}'(K)$. Let $\xi_0^{CM} \equiv (\boldsymbol{R}'(K)K)/\boldsymbol{R}(K)$ evaluated at $K = K_0$ represent the (short-run) elasticity of the interest rate to aggregate wealth in this complete markets case.

We can now state:

Claim 4. *A Pareto improvement is feasible if*

$$\mu - 1 \geq \frac{r_0}{1 + r_0} \xi_0^{CM}$$

Proof. Conditions (ii) and (iii) in Proposition 1 require in the representative agent complete markets set up that for all $t \geq 0$

$$F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0 + B_{t+1} - (1 + r_t)B_t \geq 0.$$

For period $t = 0$, this holds with $B_1 = 0$. For an arbitrary $K_t = K_1 = K$, let $H_1(K)$ denote the left-hand side of this expression evaluated at $t = 1$:

$$\begin{aligned} H_1(K) &\equiv F(K, N_0) - F(K_0, N_0) - (r_1 + \delta)K + (r_0 + \delta)K_0 + B_2 \\ &= F(K, N_0) - F(K_0, N_0) - \boldsymbol{R}(K)K + \boldsymbol{R}(K_0)K_0 + \left(\frac{\boldsymbol{R}(K) - \boldsymbol{R}(K_0)}{1 + r_0}\right) K. \end{aligned}$$

Thus, $H_1(K) \geq 0$ implies that the government can finance K in period 1 without negative transfers. Note that $H_1(K_0) = 0$, and

$$\begin{aligned} H_1'(K_0) &= F_K(K_0, N_0) - \boldsymbol{R}'(K_0)K_0 - \boldsymbol{R}(K_0) + (1 + r_0)^{-1}\boldsymbol{R}'(K_0)K_0 \\ &= (\mu - 1)\boldsymbol{R}(K_0) - \frac{r_0}{1 + r_0}\boldsymbol{R}'(K_0)K_0 \\ &= \boldsymbol{R}(K_0)\left(\mu - 1 - \frac{r_0}{1 + r_0}\frac{1}{\xi_0^{CM}}\right), \end{aligned}$$

where the second line uses $F_K(K_0, N_0) = \mu(r_0 + \delta) = \mu\boldsymbol{R}(K_0)$. Thus the condition of the proposition establishes that $H_1(K) \geq 0$ for $t = 1$ and small increases in $K > K_0$.

[33] $\boldsymbol{R}$ is defined by the Euler equation and budget constraint in periods 0. In particular, $C_0 = w_0N_0 + \Pi_0 + (1+r_0)K_0 - K_1$. $C_1 = C_2 = w_0N_0 + \Pi_0 + r_0(K_1 + B_2))$, where B_2 is defined in the text as a function of r_1 and K_1. Then $r_1 = \boldsymbol{R}(K_1)$ solves $u_c(C_0, N_0) = \beta(1 + r_1)u_c(C_1, N_0)$.

For $t \geq 2$, we have $r_t = r_0$ and $B_t = B_2 = (r_1 - r_0)K/(1 + r_0)$. Hence, we can define

$$H_t(K) \equiv F(K, N_0) - F(K_0, N_0) - \boldsymbol{R}(K_0)(K - K_0) - r_0 \left(\frac{\boldsymbol{R}(K) - \boldsymbol{R}(K_0)}{1 + r_0} \right) K.$$

Again, this is zero when $K = K_0$, and

$$H_t'(K_0) = (\mu - 1)\boldsymbol{R}(K_0) - \frac{r_0}{1 + r_0} \boldsymbol{R}'(K_0)K_0 = H_1'(K_0).$$

Hence, the condition that $H_1'(K_0) \geq 0$ is the same for arbitrary $t \geq 1$. □

This condition contains many of the characteristics of the incomplete markets case. In particular, a large markup creates room for a Pareto improvement. In addition, the smaller the (short-run) increase in interest rate, the more easily the condition is satisfied. Note that the markup term is scaled by $r_0/(1 + r_0)$, which reflects the advantage offered to the government by an infinite long-run elasticity. That is, in both the complete and incomplete markets cases, issuing debt along the path helps mitigate the cost of a smaller short-run elasticity of savings. However, the ability to issue debt in the transition at r_0 reflects the infinitely elastic supply of savings in the long run at $1/\beta$, a feature of the complete markets example but one that does not hold in general for incomplete markets.

3.6 The Elasticity of Aggregate Savings

The robust conclusion from the above is that a Pareto improvement is facilitated by a very elastic aggregate savings function with respect to the interest rate. Feasibility turns on this key statistic. Unfortunately, there is little clear cut empirical or theoretical guidance on the magnitude of this elasticity.

Testing the sensitivity of interest rates to changes in government debt or deficits was an active area of empirical research in the 1980s and 1990s.[34] Perhaps surprisingly, there are a number of empirical studies that conclude the Ricardian equivalence benchmark, in the spirit of Barro (1974), of no change in the interest rate is a reasonable description of the data. Nevertheless, there are other empirical estimates that conclude otherwise, and our reading of this literature is that there is no clear consensus.

In the Bewley-Huggett-Aiyagari literature there are a few theoretical results. For example, for the case of CRRA utility, Benhabib, Bisin and Zhu (2015) show that as $a \longrightarrow \infty$, the household saving function's sensitivity to the risk-free interest rate is increasing in the inter-temporal elasticity of substitution (IES). A similar result is proved by Achdou et al. (2021). Thus the derivative with respect to r is governed by the IES, with a larger IES indicating a more elastic response, at least

[34] See the surveys and associated references of Barth, Iden and Russek (1984); Bernheim (1987b); Barro (1989); Elmendorf and Gregory Mankiw (1999); Gale and Orszag (2003); and Engen and Hubbard (2005).

for the very wealthy. At the other end of the asset domain, Achdou et al. (2021) shows that, for those at the lowest income realization and approaching the borrowing constraint, the sensitivity of savings to r also depends positively on the IES.

These results pertain to individual savings behavior at the extremes of the asset distribution. To explore the elasticity of *aggregate* saving to the interest rate in a stationary equilibrium and during the transition, and how this elasticity varies with preference parameters, we turn to the a calibrated version of the model in the next section. The quantitative model can also speak to whether the sufficient conditions for a Pareto improving policy are satisfied in a plausible calibration. This is the topic of Section 4.

4 Simulations

In this section we present simulation results for various policy experiments. Our benchmark focuses on the dynamically efficient economy with markups. In this setting, we explore constant-K policies as well as policies with capital crowding in. For contrast, we also briefly present a competitive economy that is dynamically inefficient with capital crowding out.

The simulated economies allow us to assess the scope for Pareto-improving fiscal policies in a calibrated quantitative model as well as compute transition dynamics and welfare implications. The quantitative experiments will also underscore how government debt is used in implementing Pareto-improving policies.

4.1 Parameter Settings

The utility function we consider for households is of the Epstein-Zin form

$$V_{it} = \left\{(1-\beta)x_{it}^{1-\zeta} + \beta\left(\mathbb{E}_z V_{it+1}^{1-\gamma}\right)^{\frac{1-\zeta}{1-\gamma}}\right\}^{\frac{1}{1-\zeta}}$$

where β is the discount factor, $1/\zeta$ is the elasticity of inter-temporal substitution, γ is the risk aversion coefficient, and x is the composite of consumption and labor

$$x_{it} = c_{it} - n_{it}^{1/\nu}.$$

The parameter ν controls the Frisch elasticity of the labor supply. We set some of the preference parameters to conventional values in the literature and others as part of the calibration. The elasticities of substitution and of labor supply are set to the common parameters values of 1 and 0.2, respectively. The discount factor and coefficient of risk aversion are set as part of the calibration exercise described below. We set the borrowing constraint to zero for all households.

An important part of the parametrization is the stochastic structure for idiosyncratic shocks. We adopt the structure and estimates from Krueger et al. (2016) that use micro data on after tax labor earnings from the PSID. Idiosyncratic productivity shocks z_{it} contain a persistent and transitory component and their process is as follows

$$\begin{aligned} \log z_{it} &= \tilde{z}_{it} + \varepsilon_{it} \\ \tilde{z}_{it} &= \rho^z \tilde{z}_{it-1} + \eta_{it} \end{aligned}$$

with persistence ρ^z and innovations of the persistent and transitory shocks (ε, η), with associated variances given by $(\sigma^2_\varepsilon, \sigma^2_\eta)$. We set the three parameters controlling this process $(\rho^z, \sigma^2_\varepsilon, \sigma^2_\eta)$ to .9695, .0384, and .0522 respectively to reflect the estimated earnings risk in Krueger et al. (2016) for employed individuals. We discretize this process into 10 points based on Tauchen (1986).

We take a parsimonious approach to allocating profits to households. In particular, we assume a distinct class of entrepreneurs that are endowed with managerial talent and consume profit distributions in a hand-to-mouth manner. While stark, this approach offers a number of advantages. First, it approximates that a significant share of entrepreneurial rents accrue to a small share of the population. Second, under this assumption, they do not affect factor prices, and so we can solve the economy without taking a stand on the idiosyncratic details of the entrepreneurial class. Finally, and related to the previous point, the analysis is invariant to the extent to which profits are offset by fixed costs versus representing pure rents.

The technology specification is Cobb-Douglas, $F(K, N) = K^\alpha N^{1-\alpha}$. We use standard values for the coefficient α and for the depreciation rate of capital δ. The values are $\alpha = 0.3$ and $\delta = 0.1$. The markup parameter μ is set to 1.4, which is within the estimates in Basu (2019).[35]

We calibrate the discount factor and the coefficient of relative risk aversion by targeting a capital-output ratio of 2.5, based on Aiyagari and McGrattan (1998) and Krueger et al. (2016), and an interest rate of -1.4%, which is the difference between the average one-year treasury rates and average nominal GDP growth in the United States since 1962.[36] While our focus is on Pareto-improving policies relative to a laissez-faire benchmark, the empirical moments are generated from an economy with government debt. Hence, we simulate a stationary economy with a debt-to-output ratio of 60%, which is the average value in the US since 1966, and choose preference parameters to match moments from this economy to the data. The resulting values are $\{\beta = 0.993, \theta = 5.5\}$.

[35] As noted above, some of this markup may represent fixed costs. The aggregate markup may also reflect smaller markups at different stages of production in a vertical supply chain, as in Ball and Mankiw (2021). In fact, 1.4 is close to the number they use in their numerical exercises.

[36] This estimate is consistent with the ones in Blanchard (2019) and Mehrotra and Sergeyev (2020).

We treat the economy with debt as being generated from a Pareto-improving fiscal policy starting from the laissez-faire economy. That is, in simulating the economy with government debt during moment matching, we assume that tax policy is such that after-tax wages and capital are the same as in the laissez-faire economy,. We refer to this constant-K policy as our *baseline fiscal policy.*

Table 1: Baseline Constant-K Policy and Laissez-Faire Economies

	Data	Constant-K Policy	Laissez-Faire
Aggregates			
Public Debt (% output)	60	60	0
Interest Rates(%)	-1.4	-1.4	-1.7
Capital (rel. output)	2.5	2.6	2.6
Wealth Distribution			
Q1 Wealth Share	-1	1	1
Q2 Wealth Share	1	4	4
Q3 Wealth Share	4	11	10
Q4 Wealth Share	13	24	23
Q5 Wealth Share	83	61	63

Table 1 presents some moments in the steady states of the economy with baseline constant-K fiscal policy and the laissez-faire economy. The level of public debt, interest rates, and capital in the economy with the baseline fiscal policy matches the data moments by construction.[37] The table shows that an increase in debt to output of 60% increases interest rates by 0.3%. We also present some moments on the wealth distribution in the steady states, namely the wealth share of each asset quintile, and compare it with data as reported in Krueger et al. (2016). Our model economies generate skewed distributions of wealth, with most of the wealth being held by the top quintile of the distribution, although not quite as skewed as the data. Also in our model economies, a small fraction of agents are at their borrowing constraint at any period, about 2%.

4.2 Pareto Improvements in Dynamically Efficient Economies

We now describe the transition as the government implements its fiscal policy and assess the scope for Pareto improvements. The economy starts in the laissez-faire steady state and transitions to the steady state with fiscal policy. We perform two policy experiments, one in which capital is held constant, the constant-K policy, and one in which capital eventually reaches the

[37] The economy is dynamically efficient also by construction. To see this, $F_K = \alpha Y/K = 0.3/2.5 = 0.12$, which is greater than the depreciation rate of 0.10.

golden rule level, the crowding in policy. Both of these experiment are undertaken in the dynamically efficient economy. We also perform sensitivity over preference parameters, and postpone discussion of an economy without markups to the final section.

The two experiments confirm that Pareto-improving fiscal policies are feasible in a calibrated model. In particular, the calibrated aggregate savings schedule is sufficiently elastic to allow the government to increase both debt and capital without resorting to lump-sum taxation.

4.2.1 Baseline Constant-K Policy

We start with a policy plan that transitions from a laissez-faire stationary equilibrium to a new steady state with the baseline fiscal policy. In particular, the government takes the economy from zero debt to a level of 60% of output, while keeping after-tax wages and profits constant. Our posited path of public debt is depicted at the top left panel of Figure 4; debt increases monotonically until it reaches its steady state level of 60% of output. Also by construction in the constant-K policy, capital is held fixed at the laissez-faire level, as depicted in the top middle panel of Figure 4. Given the policy of constant capital and wages, output and consumption (reported in the lower middle panel) are not changing. This will be different in the subsequent experiment with capital crowding in.

Given this path of debt and capital, we solve for the equilibrium interest rates path r_t and associated government transfers T_t. The computational algorithm and other details are reported in Appendix A.

The top right panel of Figure 4 plots the path for government transfers and seigniorage revenue from debt issuance $B_{t+1} - (1 + r_t)B_t$, both relative to output. Transfers are larger on impact, about 5% of output, remain positive throughout the transition, and settle to a small positive level in the steady state, of about 0.1% of output. The difference between transfers and seigniorage revenue is equal to the tax revenues, which is negative due to the capital subsidies.

The bottom left panel in Figure 4 plots the path for the interest rate. Interest rates rise with public debt to induce households to hold a greater stock of aggregate wealth. Note that interest rates overshoot during the transition, which is the Le Chatelier principle at work; namely, the short-run elasticity of assets to interest rates is lower than its long-run level. The sharp spike in interest rates makes the policy fiscally expensive, as exposited in Section 3. However, the cost is more than offset by the funds raised directly by debt issuance as seen by the elevated transfers early in the transition.

The bottom right panel plots the dispersion of household consumption relative to the laissez-faire dispersion. Consumption dispersion decreases by about 10% upon the introduction of the fiscal policy plan, as households with low assets and low productivity benefit from government transfers that support their consumption. As transfers fall over time, consumption dispersion increases, but remains about 2% below the one in the laissez-faire economy. The smaller long-run consumption dispersion reflects that households on average hold a greater stock of precautionary savings given the elevated interest rate.

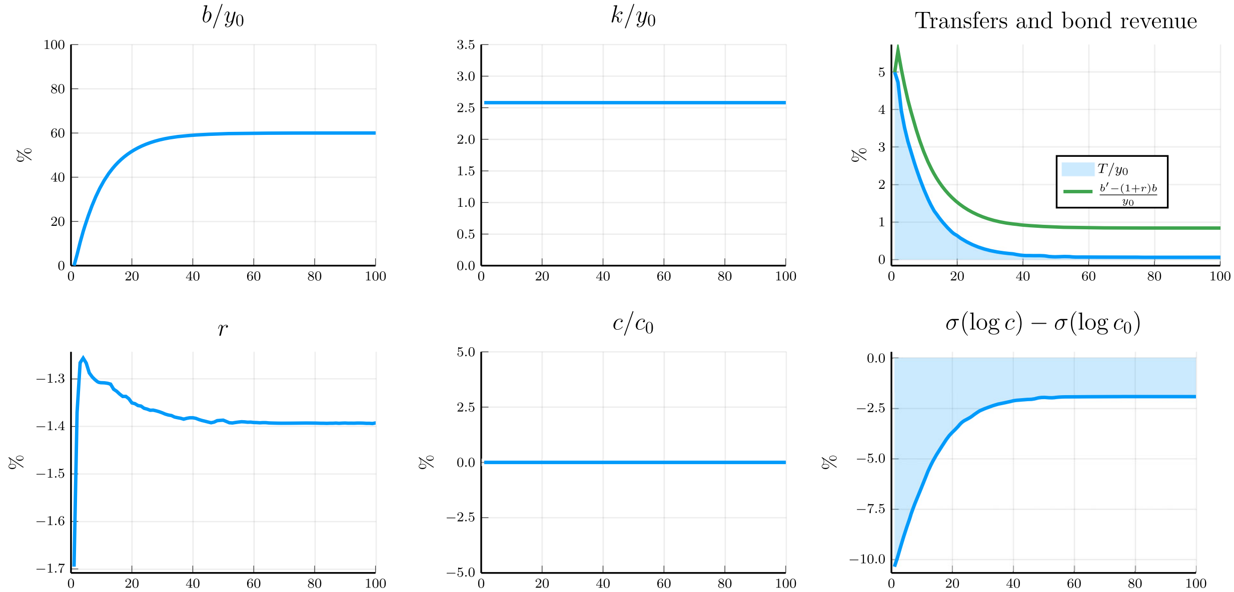


Figure 4: Constant-K Policy Transition

The transition paths of positive transfers and higher interest rates imply that our baseline constant-K fiscal policy is Pareto improving. We now evaluate the magnitude of the welfare gains. Table 2 Column 1 reports welfare for various households upon the announcement of the policy. Welfare is measured in consumption equivalence units relative to the laissez-faire economy. Across the distribution of households for assets and productivity (a, z), the economy with fiscal policy delivers higher welfare for every household. The table reports five measures of welfare gains: the mean gain, the minimum gain, and the mean gains for the bottom ten percent of the asset distribution, the 40-60th percentiles of the asset distribution, and the top 10 percent of the asset distribution. The mean welfare gains are computed by integrating over idiosyncratic states, conditional on belonging to the respective asset bin, weighted by the invariant distribution of the laissez-faire economy.

The mean welfare gain is 2.62% and the minimum gain is 2.09%. Looking across the wealth distribution, welfare gains are greatest for the poorest households. While all households receive the same transfer, the poorer households benefit relatively more in percentage terms. However, gains are not monotonic in wealth. The top decile of asset holders experience a greater welfare gain than those in the middle of the asset distribution. This reflects the fact that the benefits of a higher interest rate are increasing in wealth. At some point in the distribution, this effect dominates the uniform transfer, generating a non-monotonicity in percentage welfare gain as a function of initial wealth.

We can also compute welfare gains comparing the new steady state to the laissez-faire steady state, ignoring the transition. Welfare gains are more modest in the new steady state relative to the gains enjoyed at $t = 0$ due to the declining path of transfers. Nevertheless, all households are better off in the new steady state, with an average welfare gain of 1.8%.

Table 2: Changes in Welfare

	Constant-K Policy	Crowding-In Policy
Welfare Gains at Announcement (%)		
Overall Mean	2.62	5.16
Overall Minimum	2.09	4.48
Poor ($\leq$ 10 pct)	3.57	5.19
Middle Wealth (40-60 pct)	2.30	4.95
Rich (>90 pct)	2.82	7.12

The preceding established that a Pareto-improving fiscal policy targeting a long-run debt-to-output level of 60% is feasible. This result reflects that a debt level of this magnitude can be absorbed by households with only a modest increase in interest rates. This raises the question of whether even higher levels of debt are feasible while still ensuring all agents are weakly better off. To answer this question, we revisit the logic of Figure 1. In particular, long-run seigniorage is given by $-rB$, while the costs are captured by $\Delta r \times K_0$. In Figure 5 we plot these two components for stationary equilibria with different levels of debt to output for the constant-K policy. At each debt level, tax policy is set to deliver laissez-faire wages and profits.

Up until debt levels of roughly twice the level of output, seigniorage exceeds fiscal costs, implying positive lump-sum transfers to households. Beyond this level of debt, the increase in interest rate makes weakly positive transfers infeasible. Note that these two curves intersect while seigniorage is still increasing in debt. Eventually, r becomes close enough to zero that seigniorage begins to decline in debt. The peak of this Laffer curve occurs at debt levels roughly four times output. Feasible Pareto-improving levels of debt, however, are much lower than this peak.[38]

While Figure 5 only establishes that the policy is feasible in the new steady state, the analysis of transition dynamics in the baseline case above suggests that feasibility in the steady state is the critical metric. Along the transition, the government is a net issuer of bonds. As long as the revenue from the net issuances dominates any overshooting of the interest rate, feasibility rests

[38] Bassetto and Sargent (2020) argue in an OLG framework that the peak of the debt Laffer curve may occur while r is strictly below the growth rate, making $r < g$ an unreliable guide for expanding government funds via debt issuance.

on long-run considerations.

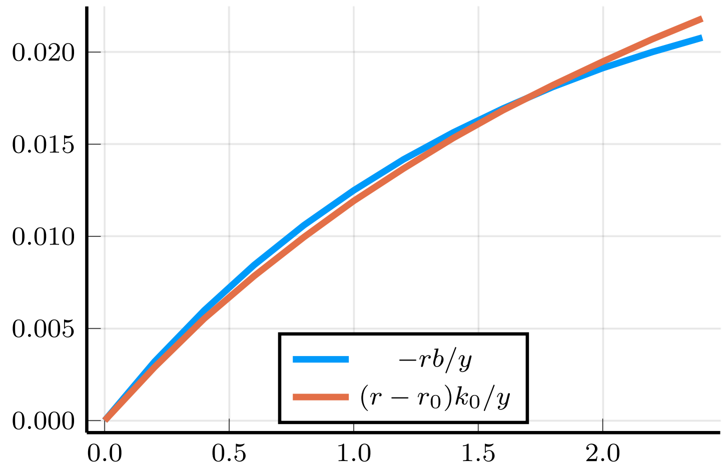


Figure 5: Steady State Seigniorage and Tax Revenue across Debt

4.2.2 The Role of Preferences

The analysis of Section 3 revealed that a key consideration in the feasibility of Pareto-improving policies is the elasticity of the aggregate savings function with respect to the interest rate. The short-run and long-run elasticities, in turn, depend on household preferences and the extent of exposure to idiosyncratic risk in a non-trivial manner. To assess this sensitivity quantitatively, we perform comparative static exercises to our baseline constant-K policy experiment by varying household preferences. We vary the inter-temporal elasticity of substitution (IES), the coefficient of risk aversion, and the discount factor (β), and trace out the path of interest rates and transfers in response to an increase in government debt. We present the case for alternative IES parameters, and defer the sensitivity to risk aversion and β to the appendix. In all experiments, the increase in government debt is as in the baseline reaching 60 percent of initial aggregate output, keeping in mind that the laissez-faire capital stock varies across experiments.

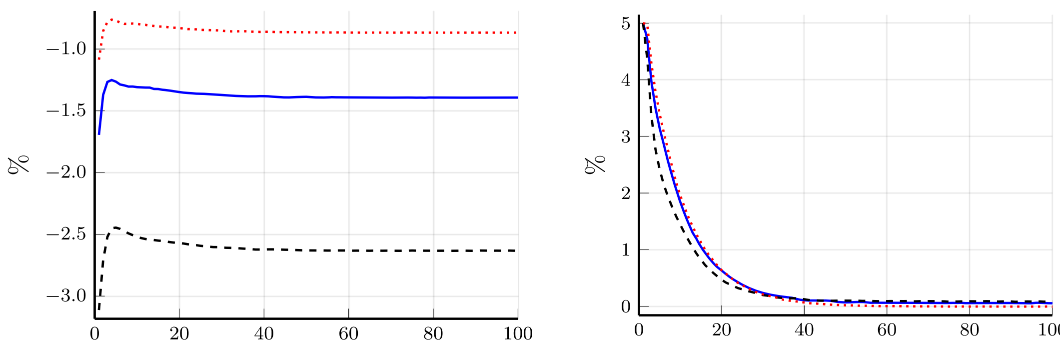
In Figure 6 we plot the path of interest rates (panel (a)) and transfers (panel (b)) for different values of the IES; namely 0.5, the benchmark 1.0, and 1.5, holding all other parameters as in the benchmark, including risk aversion. The pattern confirms the conjecture that a higher IES requires a smaller increase in interest rates to absorb the government debt, both in the short-run and long-run.

The value of the IES, however, also affects the initial level of the interest rate. In particular, a low elasticity implies lower level of interest rates throughout the transition, which also matters for the feasibility of Pareto improving policies. As seen in the path for transfers in Figure 6, it is the *more* elastic preferences that eventually require negative transfers (albeit very small, on the order of 10^{-4} of initial output). With higher elasticity, while the small increase in interest rates requires a small subsidy to capital, the fact that rates are close to zero implies lower seigniorage revenue, and the latter slightly dominates the former and impedes a Pareto improvement.[39]

Appendix Figures A.2 and A.3 contain the same simulated time series for alternative coefficients of relative risk aversion (CRRA) and time discount factors β. Intuitively, a higher CRRA implies a lower laissez-faire interest rate as well as a less elastic aggregate savings function. Nevertheless, the former dominates, implying feasibility for a CRRA of 10.0, but not for a CRRA of 2.0. Similarly, a lower discount factor (more impatience), implies a higher initial interest rate, leaving less fiscal space for Pareto-improving policies.

Figure 6: Alternative Inter-temporal Elasticities

(a) Path of r_t (b) Path of T_t



Note: This figure displays the path of interest rates (Panel a) and transfers as a percentage of laissez-faire output (Panel b) associated with the baseline fiscal policy under alternative preference parameterizations. In both panels, the solid line is the benchmark IES=1.0; the dotted red line is IES=1.5; and the dashed black line is IES=0.5.

4.2.3 Crowding In Policy

We now consider a fiscal policy plan that engineers an increase in capital that reaches the golden rule level in the new steady state. In particular, with this crowding in policy, capital relative to output increases from 2.5 to 3.0.[40] We assume that the government also pursues the same path of debt issuance as in the previous experiment. Capital subsidies are set to ensure firms rent the targeted level of capital given the prevailing interest rate, and labor and profit taxes are set such that after-tax wages and profits remain constant. We find that transfers are positive throughout, and hence the fiscal plan is a feasible Pareto improvement.

[39] The fact that the laissez-faire interest rate varies with the IES while holding risk aversion constant stems from the fact that precautionary savings depend on more than the extent of risk aversion. Kimball and Weil (1992) show that with Kreps-Porteus preferences, the strength of the precautionary savings motive is determined by attitudes toward both risk and inter-temporal substitution.

[40] Recall that $F_K = \alpha Y/K$, and $\delta = 0.10$, and hence given $\alpha = 0.3$ the golden rule is achieved at $K/Y = 3.0$.

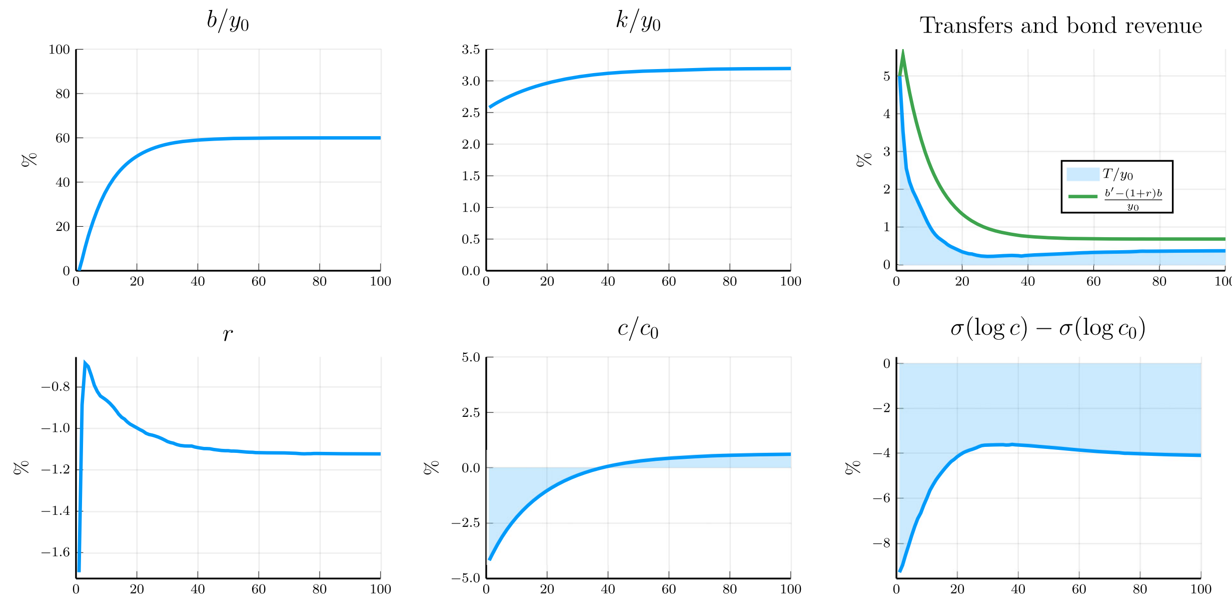


Figure 7: Crowding In Policy Transition

Figure 7 plots the variables of interest during this transition. The layout of the figure is the same as Figure 4. Along the path, we normalize quantities by the initial laissez-faire income, keeping in mind that contemporaneous income is increasing with capital.

The first two panels of the figure's top row present the posited paths of debt and capital. The top right panel illustrates that government transfers are positive throughout the transition. These transfers fall in the middle of the transition and increase towards the end of the transition. Transfers increase towards the end because interest rates are declining and capital is increasing, easing the fiscal burden.

As in the constant-K policy experiment, seigniorage revenue from borrowing falls during the transition but settles at a lower level, due to the higher interest rates. As seen in the bottom left panel of the figure, interest rates rise more with a fiscal policy that crowds in capital because households need to be induced to hold the additional capital as well as debt.

The bottom middle panel shows that aggregate consumption falls early in transition, as the economy increases investment in new capital, and settles above the laissez-faire level in the new steady state with higher capital. The dispersion of household consumption, however, remains uniformly below the level in the laissez-faire economy throughout the transition. As seen in the bottom right panel, the standard deviation drops about 9%, and increases to about 4% lower. Thus, fiscal policy improves risk sharing because of larger transfers as well as a larger stock of household wealth, which earns a higher rate of return.

The second column of Table 2 reports the welfare gains for this experiment. Welfare increases for all households both upon the fiscal policy announcement and also in the new steady state.

The mean welfare gain is 5.16% and the minimum gain is 4.48%. In this case, fiscal policies benefit the rich households more than poor households, with gains upon impact of 7.12% and 5.19%, respectively. Nevertheless, as before, households in the middle of the wealth distribution gain the least in percentage terms. The gains in this policy experiment are much larger than for the baseline policy, because they not only reflect better risk sharing but also a higher level of capital and consumption in the long-run.

The crowding in experiment assumed that the government issued debt in the same manner as it did in our baseline policy. In the analysis of Section 3, we argued that debt issuance may be useful along the transition to a higher capital stock, if the short-run elasticity of household savings is significantly lower than the long-run elasticity. This configuration made debt issuance a complement to capital accumulation. We can explore this property in greater depth using the quantitative model.

Specifically, we study an alternative fiscal policy that implements the same path of capital as in our crowding in experiment, but with zero debt issuance. The transition dynamics for this case are presented in Figure 8.

The top right panel of the figure shows that without public debt, the government needs to lump-sum tax households early in the transition. The large increase in the interest rate necessary to induce households to hold more wealth (the bottom left panel) implies large fiscal costs from capital rental subsidies. In the transition with debt, the government could use debt issuance to smooth this burden. Without debt, the government must lump-sum tax early in the transition, which implies some households may be strictly worse off. These losses are also reflected by the higher standard deviation of consumption early in the transition, which is plotted in the lower right panel. Increasing capital towards a more efficient level without debt is not a feasible Pareto improving policy because it requires lump-sum taxes during the transition, despite delivering positive transfers in the new steady state.

This experiment suggests that public debt is an important tool in Pareto-improving capital expansions. In this sense, government debt and capital expansions can be complements rather than the traditional substitutes, providing a contrast with Diamond (1965).

4.3 Pareto Improvements in Dynamic Inefficient Economies

We now discuss the effects of fiscal policies in the competitive economy that is dynamically inefficient. The fiscal policy we analyze here is one that crowds out capital by letting capital adjust with interest rates in an undistorted way. The policy instruments consist of a time path for debt, labor subsidies, and the resulting transfers for households. Recall that we design labor subsidies to guarantee that the wage households receive with fiscal policy equals that in the laissez-faire

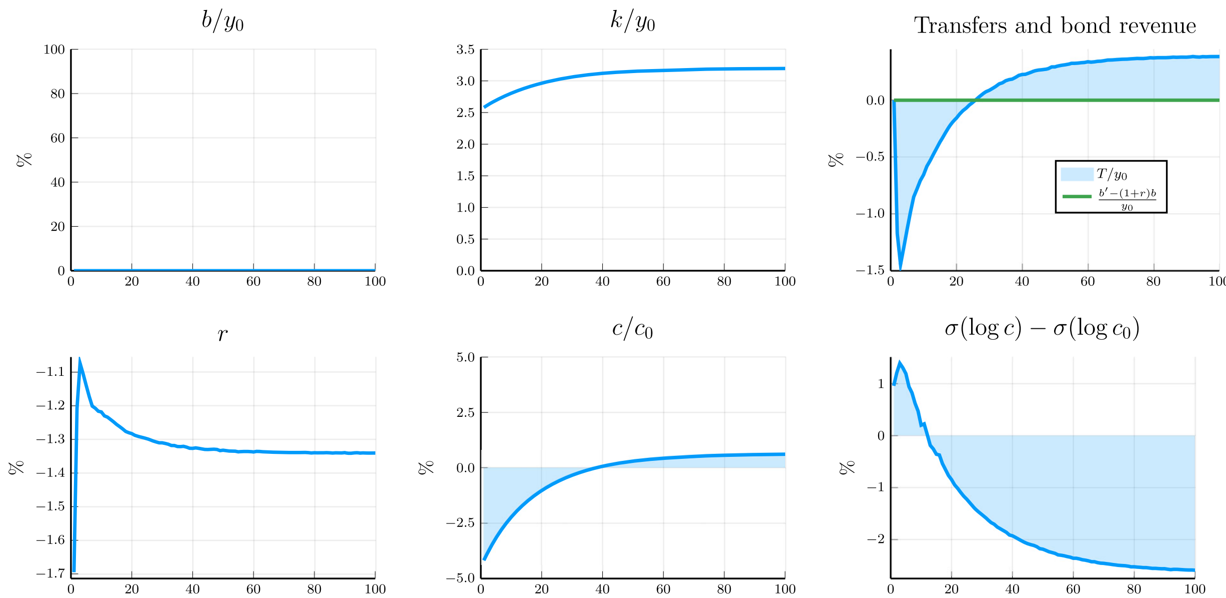


Figure 8: Fiscal Policy Transition with Crowding In and No Debt

economy. The path for government debt is identical to the one considered in the baseline fiscal policy experiment. Finally, the parameters of this economy are the same as those we calibrated for the baseline policy, with the exception that markups are zero.

We find that this fiscal policy plan in the competitive economy is also Pareto improving. As in the baseline economy, fiscal policy in this case improves risk sharing because it delivers positive transfers and higher returns on households' wealth. An additional force in the dynamically inefficient economy is that government debt is also useful because it crowds out unproductive capital.

As shown in Appendix B, the transition dynamics share many features of the baseline economy: the standard deviation of consumption falls about 8% on impact and settles at about 1% below the laissez-faire, transfers are higher early in the transition than later in the transition, and interest rates overshoot early in the transition. Capital to output in the laissez-faire economy is larger in the absence of markups. Moreover, capital falls with fiscal policy because the increase in interest rates crowds out capital. The decline in investment early in the transition boosts aggregate consumption, which settles at a higher level than in the laissez-faire economy because lower capital increases consumption in the dynamically inefficient economy. The fiscal policy plan here also gives rise to significant welfare gains of 3.0 % on average upon the announcement of the policy.

5 Conclusion

We provided sufficient conditions for the feasibility of Pareto-improving fiscal policies in the Bewley-Huggett-Aiyagari model when the risk-free interest rate on government bonds is below the growth rate ($r < g$) or there is a markup ($\mu > 1$). The key condition is that seigniorage revenue raised by government bonds exceeds the increase in the interest rate times the initial capital stock. As long as the aggregate household savings schedule is sufficiently elastic and/or the markup is large, such Pareto-improving policies are feasible. In this sense, we have shown that feasibility of a Pareto improvement depends on an aggregate elasticity, not on the finer details of idiosyncratic heterogeneity that underpin this elasticity. In calibrated examples using U.S. data on household heterogeneity and historical data on interest rates and growth rates, we find scope for Pareto improving policies for a wide range of debt and tax policies.

The government uses seigniorage debt revenue to provide transfers to households and to subsidize factor prices. These policies are welfare improving for all households because they improve risk sharing and can give rise to beneficial supply expansions. We find scope for Pareto improving fiscal policies with and without capital crowding in, and in both dynamically efficient and inefficient economies. We find that debt is a useful tool, specially for fiscal policies that expand capital.

Many governments around the world are rapidly expanding their public debt in the context of low interest rates. Our analysis points to a force that increases the benefits of such expansions. The analysis provided simple conditions for fiscal feasibility, complementing the typical dynamic inefficiency condition of Samuelson (1958) and Diamond (1965). We have tried for analytical clarity in an extension of the canonical Bewley environment, rather than a full fledged quantitative model for policy design in the current context. In particular, we have abstracted from aggregate risk. Integrating the possibility that interest rates rise in response to aggregate shocks would certainly increase the fiscal costs of higher debt. The benefits from increasing debt to improve risk sharing and for supply expansions would then have to be balanced against the costs of having to tax future generations to pay for the debt if interest rates rise.

References

Achdou, Yves, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll, "Income and Wealth Distribution in Macroeconomics: A Continuous-Time Approach," *Review of Economic Studies*, 2021.

Aiyagari, S. R., "Uninsured Idiosyncratic Risk and Aggregate Saving," *The Quarterly Journal of Economics*, August 1994, *109* (3), 659–684.

Aiyagari, S.Rao and Ellen R. McGrattan, "The optimum quantity of debt," *Journal of Monetary Economics*, October 1998, *42* (3), 447–469.

Angeletos, George-Marios, "Uninsured idiosyncratic investment risk and aggregate saving," *Review of Economic dynamics*, 2007, *10* (1), 1–30.

Ball, Laurence M and N Gregory Mankiw, "Market Power in Neoclassical Growth Models," National Bureau of Economic Research Working Paper 28538, 2021.

Barro, Robert, *Modern business cycle theory*, Cambridge, Mass: Harvard University Press, 1989.

_ , "r minus g," Harvard University Working Paper 2020.

Barro, Robert J, "Are government bonds net wealth?," *Journal of political economy*, 1974, *82* (6), 1095–1117.

Barth, James R., George Iden, and Frank S. Russek, "DO FEDERAL DEFICITS REALLY MATTER?," *Contemporary Economic Policy*, September 1984, *3* (1), 79–95.

Bassetto, Marco and Thomas J. Sargent, "Shotgun Wedding: Fiscal and Monetary Policy," *Annual Review of Economics*, August 2020, *12* (1), 659–690.

_ **and Wei Cui**, "The fiscal theory of the price level in a world of low interest rates," *Journal of Economic Dynamics and Control*, 2018, *89*, 5–22.

Basu, Susanto, "Are price-cost markups rising in the United States? A discussion of the evidence," *Journal of Economic Perspectives*, 2019, *33* (3), 3–22.

Benhabib, Jess, Alberto Bisin, and Shenghao Zhu, "The wealth distribution in Bewley economies with capital income risk," *Journal of Economic Theory*, September 2015, *159*, 489–515.

Bernheim, B Douglas, "Ricardian equivalence: An evaluation of theory and evidence," *NBER macroeconomics annual*, 1987, *2*, 263–304.

Bernheim, B. Douglas, "Ricardian Equivalence: An Evaluation of Theory and Evidence," in "NBER Macroeconomics Annual 1987, Volume 2," The MIT Press, 1987, pp. 263–316.

Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J. Sargent, "Public debt in economies with heterogeneous agents," *Journal of Monetary Economics*, 2017, *91*, 39–51.

— , — , — , **and Thomas Sargent**, "Inequality, Business Cycles and Monetary-Fiscal- Policy," *University of Minnesota*, 2020.

Blanchard, Olivier, "Public Debt and Low Interest Rates," *American Economic Review*, April 2019, *109* (4), 1197–1229.

Bohn, Henning, "The Sustainability of Budget Deficits in a Stochastic Economy," *Journal of Money, Credit and Banking*, February 1995, *27* (1), 257.

Braxton, J Carter, Kyle F Herkenhoff, and Gordon M Phillips, "Can the unemployed borrow? implications for public insurance," Technical Report, National Bureau of Economic Research 2020.

Brunnermeier, Markus K, Sebastian A Merkel, and Yuliy Sannikov, "Debt as safe asset: Mining the bubble," *Princeton University work in progress*, 2020.

Chetty, Raj, Nathaniel Hendren, Patrick Kline, and Emmanuel Saez, "Where is the land of opportunity? The geography of intergenerational mobility in the United States," *The Quarterly Journal of Economics*, 2014, *129* (4), 1553–1623.

Dávila, Julio, Jay H. Hong, Per Krusell, and José-Víctor Ríos-Rull, "Constrained Efficiency in the Neoclassical Growth Model With Uninsurable Idiosyncratic Shocks," *Econometrica*, 2012, *80* (6), 2431–2467.

Diamond, Peter A., "National Debt in a Neoclassical Growth Model," *The American Economic Review*, 1965, *55* (5), 1126–1150.

Dyrda, Sebastian and Marcelo Pedroni, "Optimal fiscal policy in a model with uninsurable idiosyncratic shocks," *University of Toronto*, 2020.

Elmendorf, Douglas W. and N. Gregory Mankiw, "Government debt," in J. B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, Vol. 1 of *Handbook of Macroeconomics*, Elsevier, 1999, chapter 25, pp. 1615–1669.

Engen, Eric M. and R. Glenn Hubbard, "Federal Government Debt and Interest Rates," in "NBER Macroeconomics Annual 2004, Volume 19," MIT Press, April 2005, pp. 83–160.

Epstein, Larry G. and Stanley E. Zin, "Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, July 1989, *57* (4), 937.

Gale, William G. and Peter R. Orszag, "Economic effects of sustained budget deficits," *National Tax Journal*, September 2003, *56* (3), 487–502.

Grand, François Le and Xavier Ragot, "Managing Inequality over Business Cycles: Optimal Policies with Heterogeneous Agents and Aggregate Shocks," SciencesPo Working Paper 2020.

Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman, "Investment, Capacity Utilization, and the Real Business Cycle," *The American Economic Review*, 1988, *78* (3), 402–417.

Hagedorn, Marcus, Iourii Manovskii, and Kurt Mitman, "The fiscal multiplier," National Bureau of Economic Research Working Paper 25571, 2019.

Heathcote, Jonathan, "Fiscal Policy with Heterogeneous Agents and Incomplete Markets," *The Review of Economic Studies*, 01 2005, *72* (1), 161–188.

_ , Kjetil Storesletten, and Giovanni L Violante, "Optimal tax progressivity: An analytical framework," *The Quarterly Journal of Economics*, 2017, *132* (4), 1693–1754.

Hosseini, Roozbeh and Ali Shourideh, "Retirement financing: An optimal reform approach," *Econometrica*, 2019, *87* (4), 1205–1265.

Jordà, Òscar, Katharina Knoll, Dmitry Kuvshinov, Moritz Schularick, and Alan M Taylor, "The rate of return on everything, 1870–2015," *The Quarterly Journal of Economics*, 2019, *134* (3), 1225–1298.

Kimball, Miles and Philippe Weil, "Precautionary Savings and Consumption Smoothing across Time and Possibilities," National Bureau of Economic Research Working Paper 3976 1992.

Kocherlakota, Narayana, "Public Debt Bubbles in Heterogeneous Agent Models with Tail Risk," *University of Rochester*, 2021.

Kreps, David M. and Evan L. Porteus, "Temporal Resolution of Uncertainty and Dynamic Choice Theory," *Econometrica*, January 1978, *46* (1), 185.

Krueger, Dirk, Alexander Ludwig, and Sergio Villalvazo, "Optimal Taxes on Capital in the OLG Model with Uninsurable Idiosyncratic Income Risk," Working Paper 2021.

_ , Kurt Mitman, and Fabrizio Perri, "Macroeconomics and Household Heterogeneity," in "Handbook of Macroeconomics," Elsevier, 2016, pp. 843–921.

Krusell, Per and Anthony A Smith Jr, "Income and wealth heterogeneity in the macroeconomy," *Journal of political Economy*, 1998, *106* (5), 867–896.

Light, Bar, "Uniqueness of equilibrium in a Bewley–Aiyagari model," *Economic Theory*, December 2018, *69* (2), 435–450.

Mauro, Paolo and Jing Zhou, "rg¡ 0: Can We Sleep More Soundly?," *IMF Economic Review*, 2021, *69* (1).

Mehrotra, Neil and Dmitriy Sergeyev, "Debt Sustainability in a Low Interest Rate World," Federal Reserve Bank of New York Working Paper 2020.

Oh, Hyunseung and Ricardo Reis, "Targeted transfers and the fiscal response to the great recession," *Journal of Monetary Economics*, 2012, *59*, S50–S64.

Ömer T. Açıkgöz, "On the existence and uniqueness of stationary equilibrium in Bewley economies with production," *Journal of Economic Theory*, January 2018, *173*, 18–55.

Reis, Ricardo, “The constraint on Public Debt when r<g but g<m,” London School of Economics Working Paper 2020.

Samuelson, Paul A., “An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money,” *Journal of Political Economy*, 1958, *66* (6), 467–482.

Seater, John J, “Ricardian equivalence,” *Journal of economic literature*, 1993, *31* (1), 142–190.

Tauchen, George, “Finite state markov-chain approximations to univariate and vector autoregressions,” *Economics letters*, 1986, *20* (2), 177–181.

Werning, Iván, “Pareto Efficient Income Taxation,” MIT Working Paper 2007.

Wolf, Christian, “Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result,” MIT Working Paper 2021.

Woodford, Michael, “Public Debt as Private Liquidity,” *The American Economic Review*, 1990, *80* (2), 382–388.

Appendix A Computational Algorithm

This appendix describes the computational algorithm we use in solving the model. Our procedure consists of two steps. We first compute the initial and final stationary equilibria. The initial one is the laissez-faire equilibrium and the final one has fiscal policy. We then compute the transition of this economy with shooting algorithms. We describe the algorithm for the dynamically efficient economy with markups.

A.1 Stationary Equilibrium

Initial The laissez-faire initial stationary equilibrium is the standard Bewley-Hugget-Aiyagari model. We compute it with a value function iteration over a savings grid, and solve for the equilibrium wages and interest rates that clear markets. The objects we record are prices $\{w_0, r_0\}$, aggregate capital, labor, profits, and the limiting distribution of households over idiosyncratic state $\{a, z\}$, namely $\{K_0, N_0, \Pi_0, \Lambda_0(a, z)\}$. We denote the objects in the initial stationary equilibrium by 0.

Final The final stationary equilibrium is indexed by H.

1. We set a target levels for capital K_H and debt B_H.
2. Choose fiscal policies $\{\tau_H^n, \tau_H^\pi\}$ to keep wages and profits as in the initial equilibrium using

$$\begin{aligned} F_L(K_H, N_0) &= \mu(1 + \tau_H^n) w_0 \\ \Pi_0 &= (1 - \tau_H^\pi)(\mu - 1) F(K_H, N_0)/\mu \end{aligned}$$

3. Guess an equilibrium interest rate r_H.
4. We recover the initial implied τ_H^k from

$$F_K(K_H, N_0) = \mu(1 + \tau_H^k)(r_H + \delta)$$

5. Use the government budget is a constraint to recover transfers T_H given our settings and the guess for interest rates

$$T_H = F(K_H, N_0) - F(K_0, N_0) - (r_H + \delta)K_H + (r_0 + \delta)K_0 + r_H B_H$$

6. Solve households problem

$$\begin{aligned} V_H(a, z; r_H) = \max_{a' \geq \underline{a}, c, n} & \ \phi(x(c, n), EV_H(a', z'; r_H)) \\ \text{subject to: } & \ c + a' \leq w_0 z n + (1 + r_H)a + T_H \end{aligned}$$

   - Gives value and savings policy functions $V_H(a, z; r_H)$, $a'_H(a, z; r_H)$ and labor supply without wealth effects $n_H(z)$, and a household distribution $\Lambda_H(a, z; r_H)$

7. Use the asset market clearing condition and firm's optimal capital condition to obtain a new guess on interest rates $\tilde{r}_H$

$$\begin{aligned} B_H + \tilde{K}_H =& \int a'_H(a, z; r_H)\pi(z', z)d\Lambda_H(a, z; r_H) \\ F_K(\tilde{K}_H, N_0) =& \mu(1 + \tau^k_H)(\tilde{r}_H + \delta) \end{aligned}$$

8. We go back to step 3, and repeat the procedure until $\tilde{K}_H$ is close to the target capital level.

A.2 Transition

At time 0, the government announces a sequence of fiscal policies that implement a sequence of capital and debt $\{K_t, B_t\}_{t=0}^H$. We will assume that at period H the economy is in the final stationary equilibrium.

1. Choose fiscal policies $\{\tau^n_t, \tau^\pi_t\}$ to keep wages and profits as in the initial equilibrium using

$$\begin{aligned} F_L(K_t, N_0) =& \mu(1 + \tau^n_t)w_0 \\ \Pi_0 =& (1 - \tau^\pi_t)(\mu - 1)F(K_t, N_0)/\mu \end{aligned}$$

2. Guess sequence of interest rates $\{r_t\}_{t=0}^H$

   - Recover the capital taxes given our target capital sequence and interest rate guess

   $$F_K(K_t, N_0) = \mu(1 + \tau^k_t)(r_t + \delta)$$

   - Recover transfers from government budget constraint using sequence of debt

   $$T_t = F(K_t, N_0) - F(K_0, N_0) - (r_t + \delta)K_t + (r_0 + \delta)K_0 + B_{t+1} - (1 + r_t)B_t$$

3. Solve households problem backwards

   - Start with period $H - 1$ problem. Note that we have the value at period H from the stationary equilibrium

   $$\begin{aligned} V_t(a, z) = \max_{a' \geq \underline{a}, c, n} \phi(x(c, n), h(V_{t+1}(a', z'))) \\ \text{subject to: } c + a' \leq w_0 z n + (1 + r_t)a + T_t \end{aligned}$$

   - Store saving decision rules: $a'_t(a, z)$.

4. Iterate forwards, update interest rates: The resulting aggregate savings from Step 2 will not be equal to the targets.

   - Start with initial distribution $\Lambda_0(a, z)$ and apply the decision rules from above.

- Use the asset market clearing condition to obtain the resulting capital sequence $\tilde{K}_{t+1}$

$$B_{t+1} + \tilde{K}_{t+1} = \int a'_t(a,z)\pi(z',z)d\Lambda_t(a,z)$$

- Use firm's optimal capital condition to obtain a candidate new sequence of interest rates $\tilde{r}_t$

$$F_K(\tilde{K}_t, N_0) = \mu(1+\tau^k_t)(\tilde{r}_t + \delta)$$

- Update the sequence of interest rates such that $r^{new}_t = \lambda r^{old}_t + (1-\lambda)\tilde{r}_t$, for attenuation parameter $\lambda = 0.5$. If the new sequence of interest rates is close enough to the old sequence, we stop, otherwise we go back to step 2.

Appendix B Dynamic Inefficient Transition

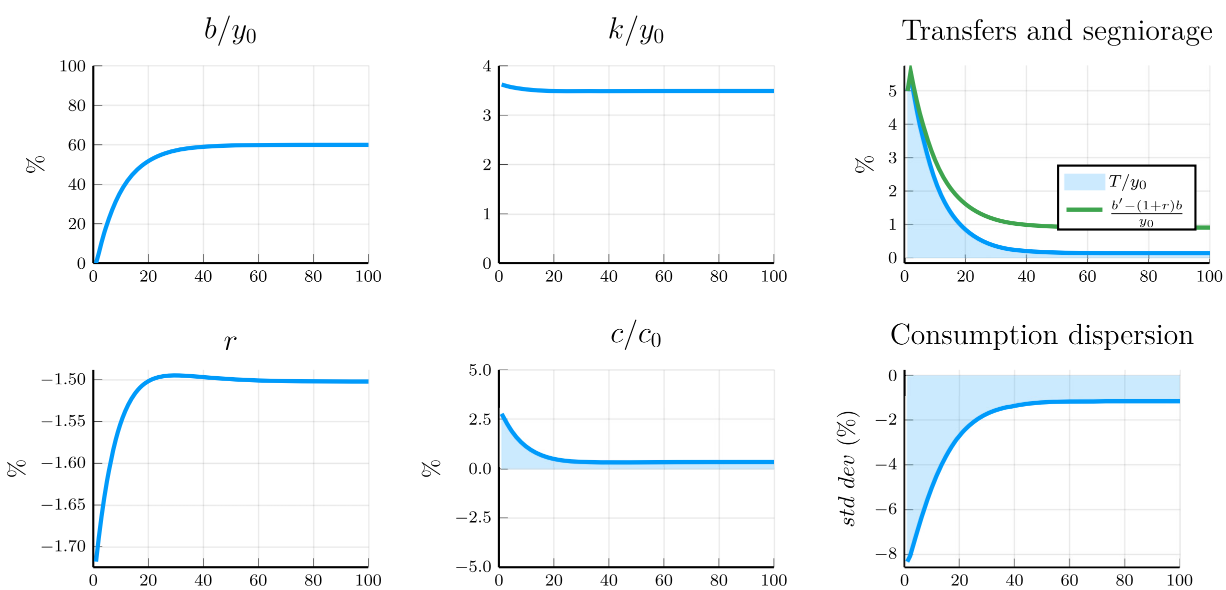


Figure A.1: Fiscal Policy Transition in Dynamic Inefficient Economy

Appendix C The Growth Economy

In this appendix, we show how the key expressions of Section 2 are modified by the presence of exogenous labor-augmenting technological growth. The derivations are standard and are included for completeness.

Assume technology is given by

$$Y_t = F(K_t, (1+g)^t L_t),$$

where $g \geq 0$ is the constant rate of growth of labor-augmenting technology. Letting a tilde denote variables divided by $(1+g)^t$, constant returns implies:

$$\tilde{Y}_t \equiv (1+g)^{-t} Y_t = F(\tilde{K}_t, L_t).$$

The representative firm's first-order conditions are (dropping t subscripts):

$$F_k(\tilde{K}, L) = \mu(1+\tau^k) r^k$$
$$F_l(\tilde{K}, L) = \mu(1+\tau^n)\tilde{w}.$$

We also have $\tilde{\Pi} = (1-\tau^\pi)(\mu-1)F(\tilde{K}, L)/\mu$.

Given the absence of a wealth effect on labor supply, we assume that the disutility of working grows at rate g as well (dropping i and t indicators):

$$x(c,n) = c - (1+g)^t v(n),$$

giving us

$$\tilde{x}(\tilde{c}, n) \equiv (1+g)^{-t} x(c,n) = \tilde{c} - v(n).$$

We also assume that the borrowing constraint is scaled by $(1+g)^t$.

We can write the household's problem as:

$$\begin{aligned} V_t(a,z,\theta) = \max_{a',n,c} \ & \phi(x(c,n), h(V_{t+1}(a',z',\theta'))) \\ \text{s.t. } & c + a' \leq w_t z n + \theta \Pi_t + (1+r_t)a + T_t \\ & a' \geq (1+g)^{t+1}\underline{a}, \end{aligned}$$

where we have altered the last constraint to account for growth and h is a certainty equivalent operator. The constraint set can be rewritten as

$$\begin{aligned} \tilde{c} + (1+g)\tilde{a}' &\leq \tilde{w}_t z n + \theta\tilde{\Pi}_t + (1+r_t)\tilde{a} + \tilde{T}_t \\ \tilde{a}' &\geq \underline{a}. \end{aligned}$$

Thus, if (c, n, a') is feasible at time t then $(\tilde{c}, n, \tilde{a}')$ satisfies the normalized constraint set, and vice versa. Assuming ϕ is constant-returns in x and h is homogeneous of degree 1, if $V_t(a,z,\theta)$ satisfies the consumer's Bellman equation, then $\tilde{V}_t(\tilde{a},z,\theta) \equiv (1+g)^{-t} V_t(a,z,\theta)$ satisfies

$$\tilde{V}_t(\tilde{a}, z, \theta) = \max_{\tilde{c},n,\tilde{a}'} \phi(\tilde{x}(\tilde{c}, n), (1+g)h(\tilde{V}_{t+1}(\tilde{a}', z', \theta'))),$$

subject to the normalized constraint set, and vice versa.[41]

Note that for an interior optimum for n, the first-order condition can be expressed:

$$v'(n) = z\tilde{w}.$$

[41]For the simulations, we use $\phi(x,h) = ((1-\beta)x^{1-\zeta} + \beta h^{1-\zeta})^{1/(1-\zeta)}$. In this case, we can define $\tilde{\beta} \equiv \beta(1+g)^{1-\zeta}$, and write $\tilde{\phi}(\tilde{x}, h) = ((1-\tilde{\beta})\chi\tilde{x}^{1-\zeta} + \tilde{\beta}h^{1-\zeta})^{1/(1-\zeta)}$, where $\chi \equiv (1-\beta)/(1-\tilde{\beta})$. This is well defined as long as $\tilde{\beta} \leq 1$. Growth can be accommodated by re-scaling the discount factor, as expected with homogeneous preferences.

Hence, labor supply is constant as long as $\tilde{w}$ remains constant.

The government's budget constraint can be rewritten in normalized form:

$$\tilde{T}_t = \tau_t^n \tilde{w}_t N_t + \tau_t^k r_t^k \tilde{K}_t + \tau_t^\pi \tilde{\Pi}_t/(1-\tau_t^\pi) + (1+g)\tilde{B}_{t+1} - (1+r_t)\tilde{B}_t.$$

Let $\tilde{X}_t \equiv \tau_t^n \tilde{w}_0 N_0 + \tau_t^k r_t^k \tilde{K}_t + \tau_t^\pi \tilde{\Pi}_0/(1-\tau_t^\pi)$ denote normalized tax revenue before transfers when keeping after tax normalized wages and profits constant. Following the same steps as the proof of Proposition 1, we have

$$\tilde{X}_t = F(\tilde{K}_t, N_0) - F(\tilde{K}_0, N_0) - (r_t+\delta)\tilde{K}_t + (r_0+\delta)\tilde{K}_0.$$

Condition (iii) of Proposition 1 (equation (2)) becomes:

$$(1+g)\tilde{B}_{t+1} - (1+r_t)\tilde{B}_t - \tilde{T}_t \geq F(\tilde{K}_0, N_0) - F(\tilde{K}_t, N_0) - (r_0+\delta)\tilde{K}_0 + (r_t+\delta)\tilde{K}_t.$$

Condition (ii) becomes $\tilde{T}_t \geq -(r_t - r_0)\underline{\tilde{a}}$, and condition (i) remains unchanged. Note that in a steady state (that is, relevant aggregates grow at rate g), Condition (iii) becomes

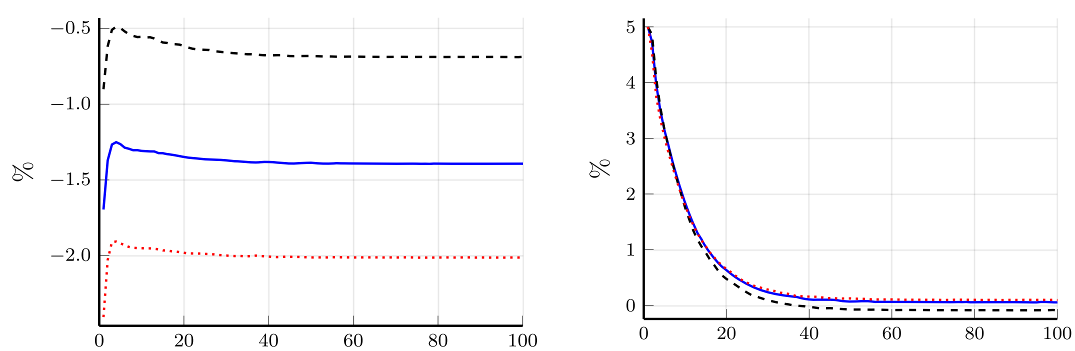
$$(g - r_{ss})\tilde{B}_{ss} - \tilde{T}_{ss} \geq F(\tilde{K}_0, N_0) - F(\tilde{K}_{ss}, N_0) - (r_0+\delta)\tilde{K}_0 + (r_t+\delta)\tilde{K}_{ss}.$$

Hence, debt increases government revenues in the steady state as long as $g > r_{ss}$. Expressions in Claims 1 and 2 are adjusted in a similar fashion to obtain normalized equivalents.

Appendix D Additional Comparative Statics with respect to Preference Parameters

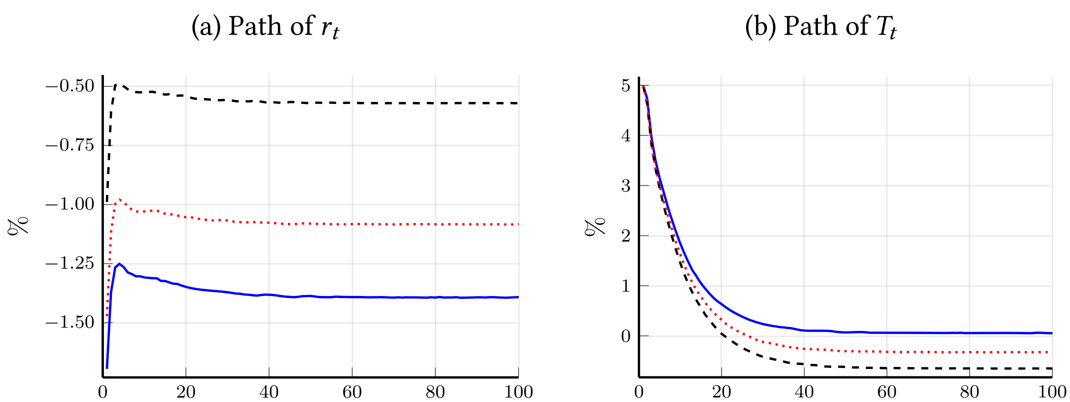
Figure A.2: Alternative Relative Risk Aversion Coefficients

(a) Path of r_t (b) Path of T_t



Note: This figure displays the path of interest rates (Panel a) and transfers as a percentage of laissez-faire output (Panel b) associated with the baseline fiscal policy under alternative preference parameterizations for the coefficient of relative risk aversion (CRRA). In both panels, the solid line is the benchmark CRRA=5.5; the dotted red line is CRRA=10.0; and the dashed black line is CRRA=2.0.

Figure A.3: Alternative Discount Factors



Note: This figure displays the path of interest rates (Panel a) and transfers as a percentage of laissez-faire output (Panel b) associated with the baseline fiscal policy under alternative preference parameterizations for time discount factors. In both panels, the solid line is the benchmark β-0.993; the dotted red line is β=0.98; and the dashed black line is β=0.97.