Strategic Interactions in U.S. Monetary and Fiscal Policies*

Xiaoshan Chen† Eric M. Leeper‡ Campbell Leith§

Abstract

We estimate a model in which fiscal and monetary policy obey the targeting rules of distinct policy authorities, with potentially different objective functions. We find: (1) Time-consistent policy fits U.S. time series at least as well as instrument-rules-based behavior; (2) American policies often do not conform to the conventional mix of conservative monetary policy and debt-stabilizing fiscal policy, although economic agents expect fiscal policy to stabilize debt eventually; (3) Even after the Volcker disinflation, policies did not achieve that conventional mix, as fiscal policy did not begin to stabilize debt until the mid 1990s; (4) The high inflation of the 1970s could have been effectively mitigated by either a switch to a fiscal targeting rule or an increase in monetary policy conservatism; (5) If fiscal behavior follows its historic norm to eventually stabilize debt, current high debt levels produce only modest inflation; if confidence in those norms erodes, high debt may deliver substantially more inflation.

Keywords: Bayesian Estimation, Monetary and Fiscal Policy Interactions, Targeting rules, Markov Switching
JEL Codes: C11, E31, E63

*August 13, 2021. Previously circulated as “U.S. Monetary and Fiscal Policy—Conflict or Cooperation?” We thank Chris Sims, Harald Uhlig, Todd Walker, Tack Yun, Tao Zha and participants at the Tsinghua-CAEPR conference on monetary and fiscal policy in Beijing, the Next Steps for the Fiscal Theory in Chicago and seminars at ESRI, Dublin and the Universities of Birbeck, Birmingham, Cardiff, Nottingham and Sheffield for helpful comments.

†University of Durham; xiaoshan.chen@durham.ac.uk.
‡University of Virginia, and NBER; eleeper@virginia.edu.
§University of Glasgow; campbell.leith@glasgow.ac.uk.
1 Introduction

A large literature analyzes shifts in monetary policy regime. One important branch assesses how much of the “Great Moderation” in output and inflation volatility was simply “good luck”—a favorable shift in shock volatilities—or “good policy”—a desirable change in monetary policy rule parameters [Sims and Zha (2006)]. Many researchers date the improvement in policy making to the Volcker disinflation in 1979 or shortly after. Very little work examines the role fiscal policy played in altering inflation trends. This neglect is surprising in light of the co-movements in inflation, real interest rates, and fiscal variables, including the government debt. The upward trend in inflation before the 1980s is associated with a downward trend in the debt-GDP ratio, while the moderation in inflation coincided with a step increase in the real interest rate and a rising debt-GDP ratio, at least until 1995 [figure 1].

Bianchi (2012) and Bianchi and Ilut (2017) are notable exceptions. They build on the policy interactions in Leeper (1991) to allow for switches in the combinations of monetary and fiscal policy rules over time. Bianchi and Ilut (2017) find that a combination of passive monetary policy and active fiscal policy produced higher inflation and lower debt during the Great Inflation from 1965 to 1982. A period of policy conflict follows with both monetary and fiscal policy following active rules. Eventually, fiscal policy turns passive to stabilize debt in the face of the Fed’s anti-inflationary actions. This conventional policy mix—active money/passive fiscal—explains the sharp decline in inflation in the 1980s. Bianchi (2012) also finds that the 1970s were a period of passive monetary and active fiscal policy, but that this did not drive the high inflation of the 1970s. The key to explaining this difference is that Bianchi and Ilut (2017) estimate a set of regime change probabilities and rule parameters which imply that fiscal policy is not expected to stabilize debt: inflation surprises do the stabilizing, as in the fiscal theory of the price level (FTPL). Bianchi’s (2012) contrasting estimates imply that ultimately economic agents expect that the government will stabilize debt, so that periods of active fiscal policy do not generate inflation as they would when that long-run belief is not in place.

This paper builds on that analysis in several ways. We consider other types of policy making in addition to simple instrument rules. Monetary policy minimizes an estimated objective function with fluctuations in the degree of inflation conservatism. This minimization, using the terminology of Svensson (2003), delivers a time-consistent specific targeting rule, as in Chen, Kirsanova, and Leith (2017). We permit fiscal policy to choose among active and passive simple instrument rules, and a time-consistent specific targeting rule, where the fiscal authority, in minimizing its estimated loss function, acts as a Stackelberg leader in a game with the monetary authority. This strategic policy specification, which resembles actual policy arrangements, implies a rich set of monetary and fiscal interactions. It also fits

---

1 Leeper (1991) characterizes monetary policy as active (AM) or passive (PM) depending on whether or not it makes the nominal interest rate react strongly to inflation. A fiscal policy that adjusts taxes to ensure fiscal sustainability is passive (PF), while failing to do so is an active policy (AF).

2 Related papers include Davig (2004) and Davig and Leeper (2006, 2011), which allow for regime switching in estimated fiscal policy. Traum and Yang (2011) and Leeper, Traum, and Walker (2017) implicitly consider switches in monetary and fiscal policy by estimating a DSGE model with fixed policy rules over sub-samples.

3 See Leeper and Leith (2017) for a discussion of the FTPL in the context of both instrument and targeting rules.
data surprisingly well, comparable to the usual instrument-rules-based menu. To solve the strategic policy game between the monetary and fiscal policy makers in the face of regime switching, the paper develops a new algorithm.

The fit to data of targeting rules introduces fresh insights into the narrative of how policies have interacted and evolved in the post-war period. Under time-consistent targeting rules, movement between regimes is more nuanced and it is rare that policy combinations conform to something akin to the theoretical active/passive pairings. We find that the Great Moderation was not associated with a decisive break from poor monetary and fiscal policy. Neither was the inflation of the 1970s driven by fiscal shocks, although a different fiscal regime could have mitigated the Great Inflation as effectively as a more conservative monetary policy. We reconcile these findings with narrative evidence on the evolution of policy making. Finally, we use stochastic simulations to examine the risks to inflation posed by current high levels of government debt. Risks can be significant, but remain modest as long as fiscal authorities adhere to the historical norm by which they eventually stabilize debt. Even a small a probability that this norm will be abandoned, though, can undermine price stability dramatically.

2 The Model

Households, a monopolistically competitive production sector, and the government populate the economy. A continuum of goods enters the households’ consumption basket. Households form external consumption habits at the level of the consumption basket as a whole, what Ravn, Schmitt-Gröhe, and Uribe (2006) call “superficial” habits. Both price and inflation inertia help to capture the hump-shaped responses of output and inflation to shocks evident in VAR-based studies, as in other empirical applications of the New Keynesian model [Smets and Wouters (2003) and Christiano, Eichenbaum, and Evans (2005)].

The government levies a tax on firms’ sales revenue, which is equivalent to a tax on all labor and profit income in this model. These revenues finance government consumption, pay for transfers to households, and service the outstanding stock of government debt. Government issues a portfolio of bonds of different maturities subject to a geometrically declining maturity structure.

2.1 Households

A continuum of households indexed by $k$ and of measure one derive utility from consumption of a composite good, $C_t^k = \left( \int_0^1 (C_{kt})^{\eta-1} \, d\bar{\nu} \right)^{\frac{\eta}{\eta-1}}$, where $\eta$ is the elasticity of substitution between the goods in this basket. Households suffer disutility from hours spent working, $N^k_t$. Habits are formed at the level of the aggregate consumption good and households fail to take account of the impact of their consumption decisions on the utility of others. To facilitate data-consistent detrending around a balanced growth path without restricting preferences to be logarithmic, we assume that consumption enters the utility function scaled by the economy-wide technology trend [Lubik and Schorfheide (2006) and An and Schorfheide

---

4For a comparison of the implications for optimal policy of alternative forms of habits see Amato and Laubach (2004) and Leith, Moldovan, and Rossi (2012).
This implies that the household’s consumption norms rise with technology and are affected by habits externalities. Households maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{(X_t^k)^{1-\sigma} (\xi_t)^{-\sigma}}{1-\sigma} - \frac{(N_t^k)^{1+\varphi} (\xi_t)^{-\sigma}}{1+\varphi} \right]$$  (1)

where $X_t^k \equiv \frac{C_t^k}{A_t} - \theta \frac{C_{t-1}}{A_{t-1}}$ is the habit-adjusted consumption aggregate, $\theta$ is the habit persistence parameter ($0 < \theta < 1$), and $C_{t-1} \equiv \int_0^1 C_{t-1}^k dk$ is the cross-sectional average of consumption. Households gain utility from consuming more than other households and are disappointed if their consumption doesn’t grow in line with technical progress. Preferences are subject to a taste shock, $\ln \xi_t = \rho \ln \xi_{t-1} + \sigma \xi \varepsilon_{t}$, $\beta$ is the discount factor ($0 < \beta < 1$), and $\sigma$ and $\varphi$ are the inverses of the intertemporal elasticities of habit-adjusted consumption and work ($\sigma, \varphi > 0; \sigma \neq 1$).

The process for technology is non-stationary

$$\ln A_t = \ln \gamma + \ln A_{t-1} + \ln q_t$$
$$\ln q_t = \rho_q \ln q_{t-1} + \sigma_q \varepsilon_{q,t}$$

Households choose the composition of the consumption basket to minimize expenditure, so demand for individual good $i$ is

$$C_{it}^k = \left( \frac{P_{it}}{P_t} \right)^{-\eta} C_t^k$$

where $P_{it}$ is the price of good $i$, and $P_t = \left( \int_0^1 (P_{it})^{1-\eta} di \right)^{1-\eta}$ is the CES aggregate price index associated with the composite good consumed by households. Aggregating across households, we obtain the overall demand for good $i$ as

$$C_{it} = \int_0^1 C_{it}^k dk = \left( \frac{P_{it}}{P_t} \right)^{-\eta} C_t$$  (2)

Households choose the habit-adjusted consumption aggregate, $X_t^k$, hours worked, $N_t^k$, and the portfolio allocation, $B_t^{S,k}$ and $B_t^{M,k}$, to maximize expected lifetime utility (1), subject to the budget constraint

$$\int_0^1 P_{it} C_{it}^k dk + P_t^S B_t^{S,k} + P_t^M B_t^{M,k} = B_{t-1}^{S,k} + (1 + \rho P_t^M) B_{t-1}^{M,k} + W_t N_t^k + \Gamma_t + P_t Z_t$$

and a no-Ponzi scheme condition. Period $t$ income includes: wage income from providing labor services to goods producing firms, $W_t N_t^k$, a lump-sum transfer from the government, $Z_t$, dividends from the monopolistically competitive firms, $\Gamma_t$, and payoffs from the portfolio of assets, $B_t^{S,k}$ and $B_t^{M,k}$. Households hold two forms of government bonds. The first is the familiar one-period debt, $B_t^S$, whose price equals the inverse of the gross nominal interest rate, $P_t^S = R_t^{-1}$. The second type of bond is actually a portfolio of many bonds, which pays a declining premium of $\rho_j^{-1}$, $j$ periods after being issued where $0 < \rho < \beta^{-1}$.
The duration of the bond is \( \frac{1}{\beta P_t} \), which means that \( \rho \) can be changed to capture alternative maturity structures of debt. With this structure we need to price only a single bond, since any existing bond issued \( j \) periods ago is worth \( \rho^{j-1} \) new bonds. When \( \rho = 1 \) these bonds become infinitely lived consols.

The first-order condition for labor is

\[
\frac{W_t}{P_tA_t} = N_t^{k\varphi} X_t^{k\sigma}
\]

Household optimization yields the optimal allocation of consumption across time, based on the pricing of one-period bonds

\[
1 = \beta E_t \left[ \left( \frac{X_{t+1}^{k+1}\xi_{t+1}}{X_t^{k+1}\xi_t} \right)^{-\sigma} \frac{A_t}{A_{t+1}} \frac{P_t}{P_{t+1}} \right] R_t
\]

where we have defined the stochastic discount factor as

\[
Q_{t,t+s} \equiv \beta \left( \frac{X_{t+s}^{k+1}\xi_{t+s}}{X_t^{k+1}\xi_t} \right)^{-\sigma} \frac{A_t}{A_{t+s}} \frac{P_t}{P_{t+s}}
\]

and the geometrically declining consols

\[
P_t^M = \beta E_t \left[ \left( \frac{X_{t+1}^{k+1}\xi_{t+1}}{X_t^{k+1}\xi_t} \right)^{-\sigma} \frac{A_t}{A_{t+1}} \frac{P_t}{P_{t+1}} (1 + \rho P_{t+1}^M) \right]
\]

When all bonds have one-period duration, \( \rho = 0 \), the price of these bonds is \( P_t^M = R_t^{-1} \). Outside of this special case, the longer term bonds introduce a term structure of interest rates.

There is an associated transversality condition. Define household financial wealth in period \( t \) as

\[
D_t^k \equiv (1 + \rho P_t^M)B_{t-1}^{M,k} + B_{t-1}^{S,k}
\]

and impose the no-arbitrage conditions to rewrite the budget constraint as

\[
\int_0^1 P_t C_{it} dt + E_t Q_{t,t+1} D_{t+1}^k = D_t^k + W_t N_t^k + \Gamma_t + P_t Z_t
\]

Household optimization implies a transversality condition that combined with the no-Ponzi condition yields

\[
\lim_{T \to \infty} E_t Q_{t,T} D_t^k = 0
\]

### 2.2 Firms

Individual goods producers are subject to the constraints of Calvo (1983) contracts. Each period a firm can reset its price with probability \( 1 - \alpha \), while it retains the previous period
price with probability $\alpha$. That previous price is indexed to the steady-state rate of inflation, following Yun (1996). When a firm can choose a new price, it can do so either to maximize the present discounted value of after-tax profits, $E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s} \Gamma_{it+s}$, or to follow a simple rule of thumb as in Galí and Gertler (1999). Profits are discounted by the $s$-step ahead stochastic discount factor $Q_{t,t+s}$ and by the probability of not being able to set prices in future periods. The firm’s revenues are taxed at rate, $\tau$, which in aggregate, is equivalent to the ratio of taxes to GDP, which can be easily mapped to the data. This greatly simplifies the complexities of the tax system, but allows us to adopt a simple measure of distortionary taxation rather than the common assumption in rule-based estimations that taxes are lump-sum [Bianchi (2012) and Bianchi and Ilut (2017)]. Forward-looking profit maximizers are constrained by the demand for their good, condition (2), and the condition that all demand must be satisfied at the chosen price. An autocorrelated shock affects the desired markup, $\ln \mu_t = \rho \ln \mu_{t-1} + \sigma \varepsilon_{\mu,t}$. Firm $i$’s optimization problem is

$$\max_{\{P_t, Y_{it}\}} E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s} \left[ \left( 1 - \tau_{t+s} \right) P_{it} \pi^s - \mu_{t+s} MC_{t+s} \right] Y_{it+s} \right]$$

subject to the demand curve

$$Y_{it+s} = \left( \frac{P_{it} \pi^s}{P_{t+s}} \right)^{-\eta} Y_{t+s}$$

Optimizing firms that are able to reset price choose $P_t^f$, whose relative price satisfies

$$\frac{P_{t}^f}{P_t} = \left( \frac{\eta}{\eta - 1} \right) \frac{E_t \sum_{s=0}^{\infty} (\alpha \beta)^s \left( X_{t+s} \xi_{t+s} \right)^{-\sigma} \mu_{t+s} mc_{t+s} \left( \frac{P_{t+s} \pi^s}{P_t} \right)^{-\eta} Y_{t+s} \Gamma_{it+s}}{E_t \sum_{s=0}^{\infty} (\alpha \beta)^s \left( X_{t+s} \xi_{t+s} \right)^{-\sigma} \left( 1 - \tau_{t+s} \right) \left( \frac{P_{t+s} \pi^s}{P_t} \right)^{-1} Y_{t+s} \Gamma_{it+s}}$$

where $mc_t = \frac{MC_t}{P_t} = \frac{W_t}{P_t Ad_t}$ is the real marginal cost, given the linear production function, $Y_t = At N_t$. Under flexible prices, $mc_t = (1 - \tau_t)^{\eta-1}$. Inflation is inertial. Some firms use rules of thumb. When those firms are permitted to post a new price, they choose $P_t^b$ to obey

$$P_t^b = P_{t-1}^* \pi_{t-1}$$

so they update their price using last period’s rate of inflation rather than steady-state inflation. $P_{t-1}^*$ denotes an index of the reset prices, defined by

$$\ln P_{t-1}^* = (1 - \zeta) \ln P_{t-1}^f + \zeta P_{t-1}^b$$

where $\zeta$ is the proportion of firms that adopt rule-of-thumb pricing. With $\alpha$ share of firms keeping last period’s price (but indexed to steady-state inflation) and $1 - \alpha$ share of firms setting a new price, the law of motion of the aggregate price index is

$$(P_t)^{1-\eta} = \alpha (P_{t-1} \pi)^{1-\eta} + (1 - \alpha) (P_s)^{1-\eta}$$

\footnote{Even before considering the nature of policy, the introduction of distortionary taxation, which affects the Phillips curve, will imply that inflation is always influenced by fiscal policy.}
The setup delivers a hybrid New Keynesian Phillips curve, as Leith and Malley (2005) detail. Combine the rule-of-thumb pricing with the optimal price setting to produce

\[ \hat{\pi}_t = \chi_f \beta E_t \hat{\pi}_{t+1} + \chi_b \hat{\pi}_{t-1} + \kappa_c \left( \hat{mC}_t + \frac{\tau}{1-\tau} \hat{\pi}_t + \hat{\mu}_t \right) \]

\( \hat{\pi}_t = \ln(P_t) - \ln(P_{t-1}) - \ln(\pi) \) is the deviation of inflation from its steady-state value, \( \hat{mC}_t + \frac{\tau}{1-\tau} \hat{\pi}_t = \ln(W_t/P_t) - \ln(A_t + \frac{\tau}{1-\tau} \hat{\pi}_t - \ln((\eta - 1)/\eta) + \ln(1 - \tau) \), are log-linearized real marginal costs adjusted for the impact of the sales revenue tax, and the reduced-form parameters are defined as \( \chi_f \equiv \frac{\alpha}{\Delta}, \chi_b \equiv \frac{\zeta}{\Delta}, \kappa_c \equiv \frac{(1-\alpha)(1-\zeta)(1-\alpha\beta)}{\Delta} \), with \( \Delta \equiv \alpha(1 + \beta\zeta) + (1-\alpha)\zeta \).

### 2.3 The Government

Government choices satisfy the flow budget identity

\[ P_t^M B_t^M = (1 + \rho P_t^M) B_{t-1}^M - P_t Y_t \tau_t + P_t G_t + P_t Z_t + P_t Y_t \xi_{tp,t} \]

We assume short bonds are in zero net supply, so \( B_t^S \equiv 0 \). \( P_t^M B_t^M \) is the market value of debt, \( P_t G_t \) and \( P_t Z_t \) are government spending and transfers and \( P_t Y_t \xi_{tp,t} \) is an i.i.d. shock to the budget identity that arises from random fluctuations in the debt maturity structure.\(^6\)

Government can use distorting taxes to service government debt and to stabilize the economy. We deliberately reduce the complexity of the tax system to a single measure of distortionary taxation. With a sufficiently wide array of fiscal instruments the policy maker could address the limited set of distortions that the model contains, in a manner actual policy maker can achieve.\(^7\) Divide through by nominal GDP, \( P_t Y_t \), to rewrite the budget identity in terms of the ratio \( b_t^M \equiv \frac{P_t^M B_t^M}{P_t Y_t} \)

\[ b_t^M = \frac{(1 + \rho P_t^M) Y_{t-1}}{P_t^M Y_t} b_{t-1}^M - \tau_t + g_t + z_t + \xi_{tp,t} \]

where \( \xi_{tp,t} = \sigma_{tp} \varepsilon_{tp,t} \) and we assume that the government spending-GDP ratio, \( g_t \), evolves according to

\[ \ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \sigma_g \varepsilon_{g,t} \]

and the transfers-GDP ratio, \( z_t \), follows a similar process

\[ \ln z_t = (1 - \rho_z) \ln z + \rho_z \ln z_{t-1} + \sigma_z \varepsilon_{z,t} \]

The fiscal shocks, \( \varepsilon_{tp,t}, \varepsilon_{g,t} \) and \( \varepsilon_{z,t} \) are all standard normally distributed.

---

\(^6\)This shock breaks a singularity that arises when all the other elements of the budget identity are observables in estimation.

\(^7\)For example, in a simple New Keynesian model optimal use of multiple tax instruments can replicate the first best allocation in the same way lump-sum taxes and a production subsidy can [Correia, Nicolini and Teles (2008)]. This would render our policy problem trivial.
2.4 The Complete Model

The complete system of non-linear equations that describe the equilibrium appear in appendix A. After log-linearizing around the deterministic steady state, the model is:

Labor Supply: \( \sigma \hat{X}_t + \varphi \hat{N}_t = \hat{w}_t \)

Euler equation: \( \hat{X}_t = E_t \hat{X}_{t+1} - \frac{1}{\sigma} \left( \hat{R}_t - E_t \hat{\pi}_{t+1} - E_t \hat{q}_{t+1} \right) - \hat{\xi}_t + E_t \hat{\xi}_{t+1} \)

Bond Prices: \( \hat{P}_t^M = \frac{\rho \beta}{\gamma \pi} E_t \hat{P}_{t+1}^M - \hat{R}_t \)

Resource Constraint: \( \hat{y}_t = \hat{N}_t = \hat{c}_t + \frac{1}{1 - g} \hat{y}_t \)

Consumption Habits: \( \hat{X}_t = (1 - \theta)^{-1} (\hat{c}_t - \theta \hat{c}_{t-1}) \)

Phillips curve: \( \hat{\pi}_t = \chi_f \beta E_t \hat{\pi}_{t+1} + \chi_h \hat{\pi}_{t-1} + \kappa_c \left( \hat{w}_t + \frac{1}{1 - g} \hat{y}_t \right) \)

Govt Budget: \( \tilde{b}_t^M = \frac{1 - \beta}{\beta} \hat{b}_{t-1}^M + \frac{b_t^M}{\beta} \left( \frac{\rho \beta}{\gamma \pi} \hat{P}_t^M - \hat{P}_{t-1}^M \right) - \hat{\tau}_t + \hat{g}_t + \hat{\tau}_t = \rho \tilde{g}_t + \sigma_{\tilde{g}_t} \epsilon_{\tilde{g}_t,t} \)

Govt Spending: \( \tilde{g}_t = \rho \tilde{g}_{t-1} + \sigma \tilde{g}_t \epsilon_{\tilde{g}_t,t} \)

Transfers: \( \tilde{z}_t = \rho \tilde{z}_{t-1} + \sigma \tilde{z}_t \epsilon_{\tilde{z}_t,t} \)

Technology: \( \hat{q}_t = \rho \hat{q}_{t-1} + \sigma \hat{q}_t \epsilon_{\hat{q}_t,t} \)

Cost-Push/Markup: \( \hat{\mu}_t = \rho \hat{\mu}_{t-1} + \sigma \hat{\mu}_t \epsilon_{\hat{\mu}_t,t} \)

Preference: \( \hat{\xi}_t = \rho \hat{\xi}_{t-1} + \sigma \hat{\xi}_t \epsilon_{\hat{\xi}_t,t} \)

To close the model we specify monetary and tax policy behavior.

3 Policy Making

Policy makers follow targeting rules obtained by minimizing an objective function. We contrast the fit to data of this description of policy to a version of the model in which policy obeys the kinds of simple instrument rules in existing literature. That rules-based benchmark appears in appendix C.

---

8The fiscal variables are normalized with respect to GDP, so \( \tilde{b}_t^M, \tilde{\tau}_t, \tilde{g}_t \), and \( \tilde{z}_t \) are defined as linear deviations from their steady states. Other variables are expressed as percentage deviations from steady state. Before linearizing, output, consumption and real wages are rendered stationary by scaling by technology, \( A_t \).
3.1 Targeting Rules

Now we describe our targeting rule specifications. Chen, Kirsanova, and Leith (2017) estimate monetary policy models of the U.S. economy to find that monetary policy is best described as a time-consistent targeting rule. The fit of that description dominates both instrument-rules-based and Ramsey monetary policy. Extending this analysis to fiscal policy raises several considerations. First, monetary and fiscal authorities are independent policy makers with potentially different policy objectives. This leads us to model strategic interactions between the two policy makers: they play a game where either authority may be the Stackelberg leader—making policy decisions anticipating the reaction of the other—or a Nash equilibrium where each policy maker takes the other’s policies as given when formulating their own plans. Beetsma and Debrun (2004) argue that fiscal leadership is the best description of the interactions between monetary and fiscal authorities because in practice the monetary authority’s response to shocks is well articulated and can be anticipated by the fiscal authorities.9 Monetary policy is more nimble, able to react swiftly to news about economic conditions, including fiscal actions. We adopt this timing assumption in what follows. But we also estimated our model under the alternative assumptions of monetary leadership and the Nash solution in appendix G. This does not materially affect the fit of the model, parameter estimates, or timing of regime switches.

Second, while Chen, Kirsanova, and Leith (2017) find strong evidence that monetary policy has been conducted with reference to an objective function, albeit with switches in the degree of conservatism within that objective over time, it is not obvious that fiscal policy has been similarly optimizing.10 This leads us to posit that monetary policy follows a targeting rule—with changes in degree of conservatism—while fiscal policy switches between instrument-rules and time-consistent targeting rules, as fit to data dictates. We compare this description of policy with simple instrument rules in Section 4.4.1 below.

An obvious approach to defining policy objectives would be to use the micro-founded welfare function based on the utility of the households that populate the economy.11 But estimation with micro-founded weights is problematic. Because the micro-founded weights are functions of structural parameters, they place very tight cross-equation restrictions on the model, which are likely to deteriorate fit to data. With standard estimates of the degree of price stickiness, for example, the micro-founded weight attached to inflation can be over 100 times that attached to output [Woodford (2003, chapter 6)]. Targeting rules based on such a strong anti-inflation objective would be wildly inconsistent with observed inflation volatility. Instead, we follow Chen, Kirsanova, and Leith (2017) and adopt a form of the objective function for each policy maker which is consistent with the representative agents’ utility, but we freely estimate the weights within that objective function. Using the terminology of Svensson (2003) this objective function constitutes a general targeting rule, which then

---

9 Fiscal leadership is not fiscal dominance and does not imply that the fiscal authority forces the central bank to accommodate its actions. Leadership means that the central bank takes fiscal policy as given and it has a well-known reaction to the state of the economy, which the fiscal authority takes into account when setting policy. For example, the fiscal authority might anticipate that the central bank will act to stabilize inflation in the face of a fiscal stimulus.

10 Or if it has involved a formal optimization, this may reflect political objectives/frictions as in, Song, Storesletten, and Zilibotti (2012) rather than those contained in a conventional general targeting rule.

11 See appendix B for the micro-founded welfare function.
implies a specific targeting rule after optimization subject to the constraints implied by the
decentralized equilibrium and the nature of the strategic interactions with the other policy
maker. The objective function for the monetary authority is

$$\Gamma_0^M = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \omega_1 \left( \tilde{X}_t + \tilde{\xi}_t \right)^2 + \omega_2 \left( \tilde{y}_t - \frac{\sigma}{\varphi} \tilde{\xi}_t \right)^2 \\
+ \omega_3 \left( \tilde{\pi}_t - \tilde{\pi}_{t-1} \right)^2 + \omega_M^S \tilde{\pi}_t^2 + \omega_R (\Delta \tilde{R}_t)^2 \right\} \quad (3)$$

Under the monetary policy specification, we consider potential switches in the weight at-
tached to inflation stabilization, $\omega_{\pi,S_t}^M$. That normalized weight can switch between $\omega_{\pi,S_t=1}^M = 1$
in the More-Conservative (MC) regime and $0 < \omega_{\pi,S_t=2}^M < 1$ in the Less-Conservative (LC)
regime. The monetary authority also values smooth interest rates.

Fiscal policy minimizes

$$\Gamma_0^F = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \omega_1 \left( \tilde{X}_t + \tilde{\xi}_t \right)^2 + \omega_2 \left( \tilde{y}_t - \frac{\sigma}{\varphi} \tilde{\xi}_t \right)^2 \\
+ \omega_3 \left( \tilde{\pi}_t - \tilde{\pi}_{t-1} \right)^2 + \omega_F^M \tilde{\pi}_t^2 + \omega_R (\Delta \tilde{R}_t)^2 \right\} \quad (4)$$

The objective of the fiscal authority can differ from that of the monetary authority only
in the weight attached to inflation, $\omega_F^M$, the presence of a tax rate-smoothing term, and
the absence of interest-rate smoothing. In essence, the two policy makers share the same
conception of social welfare, but the government may appoint a monetary authority with an
aversion to inflation that differs from that of society, to reflect Rogoff’s (1985) arguments.

Habits externalities introduce the preference shock, $\tilde{\xi}_t$, into the objective functions.
Habits confront policy makers with a trade off. When $\xi_t$ is high, utility of consumption
and disutility of work are low. Policy makers will want to induce more labor, but any higher
consumption from that labor produces a lower utility gain.

### 3.2 Instrument Rules

The Fiscal Targeting Rule (TF) regime corresponds to $s_t = 1$. But fiscal behavior need not
optimize at all times. When fiscal policy is not following a targeting rule—when it is not
minimizing (4)—it obeys the tax instrument rule

$$\tilde{\tau}_t = \rho_{\tau,si} \tilde{\tau}_{t-1} + \left( 1 - \rho_{\tau,si} \right) \left( \delta_{\tau,si} \tilde{b}_{t-1}^M + \delta_y \tilde{y}_t \right) + \sigma_{\tau} \varepsilon_{\tau,t} \quad (5)$$

The coefficient on debt, $\delta_{\tau,si}$, and the persistence of the tax rate, $\rho_{\tau,si}$, are subject to regime
switching with $s_t = 2$ the Passive Fiscal (PF) regime and $s_t = 3$ the Active Fiscal (AF)
regime. The value of the coefficient on debt determines fiscal regime, with $\delta_{\tau,si=2} > \frac{1}{\beta} - 1$
in the PF regime and $\delta_{\tau,si=3} = 0$ in the AF regime. These simple instrument rules are intended
to capture fiscal behavior when policy is not obviously geared towards attaining conventional
macroeconomic policy objectives, perhaps due to political considerations, but where we can
still classify policy as being consistent with debt stabilization, or not.

Transition matrices for monetary and fiscal policy regimes are

$$\Phi = \begin{bmatrix} \phi_{11} & 1 - \phi_{22} \\ 1 - \phi_{11} & \phi_{22} \end{bmatrix}, \quad \Psi = \begin{bmatrix} \psi_{11} & 1 - \psi_{22} - \psi_{23} & \psi_{31} \\ \psi_{12} & \psi_{22} & 1 - \psi_{31} - \psi_{33} \\ 1 - \psi_{11} - \psi_{12} & \psi_{23} & \psi_{33} \end{bmatrix}$$
where $\phi_{ii} = \Pr[S_t = i|S_{t-1} = i]$ and $\psi_{ii} = \Pr[s_t = i|s_{t-1} = i]$.

We also permit fundamental shock volatilities to change, a feature of existing explanations of the Great Moderation. Failure to do so can bias the identification of shifts in policy [Sims and Zha (2006)]. Standard deviations of shocks to technology ($\sigma_{q,k}$), preferences ($\sigma_{\xi,k}$), and cost-push ($\sigma_{\mu,k}$) may switch independently, with $k_t = 1$ the low volatility regime and $k_t = 2$ the high volatility regime. The transition matrix for the shock volatilities is

$$H = \begin{bmatrix} h_{11} & 1 - h_{22} \\ 1 - h_{11} & h_{22} \end{bmatrix}$$

where $h_{ii} = \Pr[k_t = i|k_{t-1} = i]$.

To solve the targeting rule problem, we develop a new algorithm with two policy makers under different structures of strategic interaction: when one policy maker can act as a Stackelberg leader in the policy game and when policy makers move simultaneously as part of a Nash equilibrium. Our algorithm incorporates potential changes in policy makers’ preferences over time (see appendices D and E).

## 4 Estimation

The empirical analysis uses seven U.S. time series on real output growth ($\Delta GDP_t$), annualized domestic inflation ($INF_t$), the federal funds rate ($FFR_t$), the annualized debt-GDP ratio ($B_t/GDP_t$), government spending ratio ($G_t/GDP_t$), transfers ratio ($Z_t/GDP_t$) and federal tax revenue ratio ($T_t/GDP_t$) from 1955Q1 to 2008Q3. All data are seasonally adjusted and at quarterly frequencies. Output growth is the log difference of real GDP, multiplied by 100. Inflation is the log difference of the GDP deflator, scaled by 400. The four fiscal variables—debt, government spending, transfers and taxes—are normalized with respect to GDP and multiplied by 100. Appendix F describes the dataset in detail.

The data are linked to the law of motion of states through the measurement equation

$$\begin{bmatrix} \Delta GDP_t \\ INF_t \\ FFR_t \\ G_t/GDP_t \\ T_t/GDP_t \\ Z_t/GDP_t \\ B_t/GDP_t \end{bmatrix} = \begin{bmatrix} \gamma^Q + \Delta\tilde{y}_t + \tilde{q}_t \\ \pi^A + 4\tilde{\pi}_t \\ r^A + \pi^A + 4\gamma^Q + 4\tilde{R}_t \\ 100g + \tilde{g}_t \\ 100\tau + \tilde{\tau}_t \\ 100z + \tilde{z}_t \\ \frac{100}{4}b^M + \frac{17}{4}b^M \end{bmatrix}$$

where parameters, $\gamma^Q$, $\pi^A$, $r^A$, $g$, $\tau$, $z$ and $b^M$ represent the steady-state values of output growth, inflation, real interest rates the government spending-GDP ratio, transfers-GDP ratio, the tax rate, and debt-GDP on a quarterly basis.

Steady-state values of fiscal variables and output growth are fixed at their means over the sample period. The government spending-GDP ratio ($g$) is 8%, transfers ($z$) is 9.19%, the federal tax revenues to GDP ratio ($\tau$) is 17.5%, the federal debt to annualized output ratio ($b^M$) is 31%, and quarterly output growth ($\gamma^Q$) is 0.46%. The steady-state real interest rate

12 The joint transition matrix governing the monetary-fiscal-shock regime is $\Phi \otimes \Psi \otimes H$, to yield 12 regimes under time-consistent targeting rules.
(r^A) is 1.8% and the inflation target (\pi^A) is 2%. The average real interest rate, r^A, is linked to the discount factor, \beta = \left(1 + r^A/400\right)^{-1}. Average maturity of outstanding government debt is 5 years [Leeper and Zhou (2021, table 2)]. The inverse of Frisch elasticity of labor supply, \varphi, is set to 2.\textsuperscript{13}

We approximate the likelihood function using Kim’s (1994) filter, and then combine it with the prior distribution to obtain the posterior distribution. A random walk Metropolis-Hastings algorithm generates four chains of 540,000 draws each, after discarding the first 240,000 draws, and saving 1 in every 100 draws. Brooks-Gelman-Rubin potential reduction scale factors, reported in appendix H, are all below the 1.1 upper bound for convergence.

4.1 Prior Distributions

Table 1 reports the priors of the targeting rule model, which consists of priors that are common to the instrument-rules-based estimation in appendix C, as well as those for parameters specific to the targeting rules, such as the weights on the objective function. Priors for most of the parameters are relatively loose and broadly consistent with the literature that estimates New Keynesian models. We choose the normal distribution for the inverse of the intertemporal elasticity of substitution, \sigma, with a prior mean of 2.5. Habits formation, indexation, and the AR(1) parameters of the technology, cost-push, taste, transfers, government spending shocks follow a beta distribution with a mean of 0.5 and a standard deviation of 0.15. The Calvo parameter for the probability of no price change, \alpha, is set so that the average length of the contract is around one year with a fairly tight prior around that value. A looser prior on this parameter tends to yield implausibly high estimates of the degree of price stickiness.

The parameters specific to targeting rules include the relative weights attached to the output (\omega_1 and \omega_2), changes in inflation (\omega_3), and interest rate smoothing (\omega_R) in the monetary policy objective function. We normalize to 1 the weight on inflation stabilization in the MC regime, \omega_{\pi,S_t=1}. The micro-founded objective function implies that the relative weights on other objectives should be very small. Small values for the remaining freely estimated weights are consistent with the Fed’s anti-inflation stance.\textsuperscript{14} We assume a fairly loose beta distribution with a mean of 0.5 for those weights. In the LC regime, \omega_{\pi,S_t=2} also obeys a beta distribution with mean 0.5. \Phi is the 2 \times 2 transition matrix for monetary policy where a beta distribution is used for its diagonal elements, \phi_{ii}, with a prior mean of 0.95 and a standard deviation of 0.05. This prior implies that the average duration for each monetary regime is about 20 quarters, and values can vary between 6.6 and 100 quarters within the 90% confidence interval.

Unlike monetary policy, the fiscal policy maker may not always minimize its loss function. Fiscal behavior may switch among two tax instrument rules and a time-consistent targeting rule. Priors over the passive and active fiscal rules are set to be broadly consistent with the

\textsuperscript{13}It can be difficult to estimate the inverse of Frisch elasticity without using labor market data. The value \varphi = 2 is consistent with the estimate of Smets and Wouters (2007). This value is in line with microeconomic estimates using household level data as in MaCurdy (1981).

\textsuperscript{14}This is also in line with empirical findings of Favero and Rovelli (2003) and Ozlale (2003) who also estimated policy objective functions for the Federal Reserve.
literature that estimates fiscal rules [Bianchi and Ilut (2017) and Leeper, Traum, and Walker (2017)]. In the TF regime, fiscal objectives parallel monetary objectives, but allow the fiscal authority’s weight on inflation stabilization, $\omega_B$, to differ from the monetary authority’s. We do not presume that the fiscal authority will be either more or less inflation-conservative than the central bank, so $\omega_B$ follows a gamma distribution with prior mean of 1 and values below 1 receive around 57% of the a priori probability. We also replace interest rate smoothing with a tax rate smoothing term, $\omega_r$, to reflect the possibility that the fiscal authority wants to avoid large variations in tax rates. The prior distribution over $\omega_r$ is beta. With a total of three fiscal regimes, the elements estimated in the $3 \times 3$ fiscal transition matrix, $\Psi$, follow a dirichlet distribution. Election cycles may give fiscal regimes shorter duration than monetary regimes. This is reflected in the prior distribution of diagonal elements, $\psi_{ii}$, in $\Psi$ that corresponds to an average duration of 10 quarters for each fiscal regime with values ranging between 5 and 25 quarters in the 90% confidence bands.

Finally, we allow high- and low-volatility states for technology, preference and cost-push shocks. Priors on the standard deviations of shocks are symmetric across regimes and are quite loose. $h_{ii}$ are diagonal elements on the $2 \times 2$ transition matrix for shock volatilities that follow a beta distribution with prior belief that each shock regime lasts for 10 quarters.

We consciously specify priors for the transition probabilities that favor neither one policy permutation over another, nor the nature of transitions between regimes. This contrasts to Bianchi (2012) and Bianchi and Ilut (2017) who only consider three possible policy permutations, omitting the pairing of PM/PF. Those papers also restrict the movement between policy regimes and limited how long the AM/AF regime may last.

4.2 Posterior Estimates

Table 1 presents posterior parameter estimates. These include when the monetary authority implements a targeting rule, taking fiscal policies as given, while the monetary authority’s objective function may switch in its inflation aversion over time—between More or Less Conservative. The fiscal authority acts as a Stackelberg leader in the game with the monetary authority, so the fiscal authority conducts policy anticipating the response of the Fed. Fiscal policy may switch between this leadership role (TF) and simple passive or active fiscal rules, labeled PF and AF. Joint monetary-fiscal behavior produces six regimes: MC/TF, MC/PF, MC/AF, LC/TF, LC/PF and LC/AF.

Monetary policy always follows a time-consistent targeting rule. It attaches the weight $\omega_{\pi,S_t} = 0.61$ to inflation stabilization in the LC regime (relative to 1 in the MC regime). Data are highly informative about the fiscal authority’s aversion to inflation. The posterior estimate under TF is $\omega_B = 0.32$. Fiscal authorities are substantially less averse to inflation than is the central bank, even when monetary policy is Less Conservative. These estimates are consistent with Rogoff’s (1985) idea that the government should appoint a conservative central banker with a stronger dislike of inflation than the government. The optimal degree of inflation conservatism for a delegated central bank is 1.4, well above the normalized weight of 1 under the MC regime. Additional gains from conservatism, however, come from reducing inflation volatility below levels observed in data.

Estimates of the deep model parameters are similar to those under rules-based policy—see appendix C—with a modest rise in the intertemporal elasticity of substitution, $\sigma$, to 3.2,
indexation, $\zeta$, to 0.37, and the degree of habits, $\theta$, to 0.81. The other significant difference is that the estimated degree of persistence of the cost-push shock process, $\rho$, rises from 0.21 to 0.93 as we move from the rules-based estimation to the targeting rule estimation, while the variance of i.i.d. innovations to the cost-push shock fall dramatically. The combined effect of these differences is that the standard deviation of the cost-push shock process is actually lower under the targeting rule estimation.\(^{15}\) Although cost-push shocks generate a meaningful trade off for policy makers by raising inflation and reducing output, they do not rise to implausible levels in explaining the data when policy minimizes a loss function. Appendix I reports results from the Komunjer and Ng (2011) identification test, along with plots of the prior and posterior densities.

### 4.3 Model Comparison

Does modeling strategic interactions between policy makers in the form of targeting rules deliver a reasonable statistical fit to data? Table 2 reports the log marginal likelihood values for models with instrument rules and strategic targeting rule policies to provide a basis for comparison. We compute Geweke’s (1999) modified harmonic mean estimator and the statistic that Sims, Waggoner, and Zha (2008) propose to draw similar conclusions. The latter method is designed for models with time-varying parameters, where the posterior density may be non-Gaussian. The two models fit data equally well.

We also present the marginal likelihood associated with an intermediate case in which we allow monetary policy to be time-consistent with switches in the degree of conservatism, while fiscal policy switches between active and passive rules, without the possibility of the fiscal authority following a targeting rule.\(^{16}\) The targeting rule model’s fit is also comparable to the intermediate model's: episodes of fiscal Stackelberg leadership can help explain the data, even when those episodes occur relatively infrequently. Fiscal leadership is consistent with specific policy episodes. Fiscal leadership also affects fit because of the impact it has on other policy regimes through expectations. We discuss this issue below.

Model comparisons lead to a key finding that speaks to the bulk of the literature that estimates policy rules. Targeting rules fit data at least as well as instrument rules or a combination of monetary targeting rules policy and fiscal instrument rules. This is a surprising outcome in light of the additional restrictions that this form of policy imposes.

### 4.4 Regime Switching

We model monetary policy as fluctuating between the more (MC) and less (LC) conservative targeting rules. Fiscal policy can move among a targeting rule (TF), a passive instrument rule (PF), and an active instrument rule (AF). Figure 2 reports probabilities of each policy/volatility regime over the sample and table 3 details the long-run probabilities of being in each policy regime. Before connecting these estimated policy shifts to narrative descriptions

\(^{15}\) The unconditional standard deviation of the cost-push shock process under the rules-based estimation is 4.9% (13%) in the low (high) volatility regimes, but is only 1.5% (4.2%) under the targeting rule estimation. This compares to an unconditional standard deviation of the cost-push process in Smets and Wouters (2007) of 14.7%.

\(^{16}\) Parameter estimates of this intermediate model are available upon request.
of the evolution of monetary and fiscal policies, we compare the estimated model’s behavior with targeting rules to conventional pairings of active/passive rules.

4.4.1 UNDERSTANDING POLICY BEHAVIOR Figure 3 plots the response to a 10% fiscal transfers shock under the three descriptions of fiscal policy we use—passive rule, active rule, and time-consistent targeting rule. These are paired with either the more or less conservative monetary targeting rule or an active/passive Taylor rule from the instrument-rule estimation in Appendix C. Responses in the figure come from turning off the probability of switching to an alternative policy regime. Making regimes permanent highlights the basic properties of the different descriptions of policy.

In column one, fiscal policy passively adjusts the tax rate to ensure fiscal solvency. Debt-GDP ratio rises to finance higher transfers, but higher debt is gradually unwound by a sustained increase in taxation. When paired with an active monetary policy rule, this conventional policy stabilizes debt with minimal impact on inflation (solid line). Under a monetary policy targeting rule inflation rises; it rises more when monetary policy is less conservative (dash-dotted vs. dotted lines). A targeting rule enhances the inflationary impact due to the debt-stabilization bias that Leeper and Leith (2017) and Leeper, Leith, and Liu (2021) discuss. This bias reflects the policy makers’ desire to return debt to steady-state, which would not be the case if they were pure tax smoothers acting under commitment. The debt stabilization bias is driven by the fact that higher debt creates an inflationary bias problem as the monetary authority is tempted to raise inflation to reduce the real value of government debt. Returning debt to steady-state mitigates the associated inflationary bias. This mechanism, linking debt and inflation, is absent in instrument rule-based descriptions of policy.

In column 2 fiscal policy is active, failing to adjust taxes to stabilize debt. This fiscal behavior requires inflation surprises to revalue debt, as in the FTPL. With the estimated passive monetary policy rule, fiscal expansion produces an initial burst of inflation (solid line). When we assume a targeting rule for monetary policy, the path for inflation is largely the same regardless of how conservative the policy maker is (dash-dotted and dotted lines). This is because the magnitude of the required inflation surprise is determined by the size of the fiscal shock. It is important to stress just how large the inflationary impact of the fiscal shock is when there is no prospect of the fiscal authority acting to stabilize debt.

The third column of figure 3 reports impacts of higher transfers under a time-consistent fiscal targeting rule. As in the first column, inflation rises modestly, particularly when the central bank is conservative. Fiscal leadership combined with a conservative central bank allows the policy makers to resist the debt stabilization bias and to pursue a near tax-smoothing policy without generating significant inflation. A less conservative central bank tolerates higher inflation, which prompts the fiscal authority to stabilize debt more aggressively to remove the inflationary bias problem that elevated debt generates.

In summary, the implications of the monetary policy targeting rule depend on the fiscal regime with which it is paired. When fiscal policy is active, the monetary authority has to generate the inflation surprises necessary to stabilize debt, regardless of the authority’s

\[17\] The prolonged increase in inflation is because the passive monetary policy is both inertial and close to satisfying the Taylor principle. A more passive rule would avoid the sustained increase in inflation beyond the maturity of the debt stock [Leeper and Leith (2017)].
inflation aversion. This is akin to the PM/AF regime in Leeper (1991). But when fiscal policy is either passive or following a targeting rule, the same monetary targeting rules produce more modest inflation because fiscal policy carries most of the burden of stabilizing debt. The size of the burden depends on the central bank’s inflation aversion. This has similarities to the AM/PF regime, although the debt stabilization bias creates a link between debt and inflation that would not, otherwise, be present.

It is tempting to infer from the results that episodes of high inflation, like the 1970s, likely stem from the absence of debt-stabilizing fiscal policy. This is not necessarily the case. Figure 4 plots responses to the same transfers shock under the same policy permutations, except that the estimated probabilities of switching to other policy regimes are reinstated. Agents use those probabilities to form expectations about future policies. We find that results under the first column are similar to those in figure 3. But column 2 no longer exhibits a large burst of inflation to stabilize debt. Instead, both debt and inflation trend upwards when the central bank implements a targeting rule, particularly when the monetary authority is less conservative. Differences between the two figures stem from what Leeper and Zha (2003) call “expectations formation effects.” As shown in table 3, estimates imply that the fiscal authority will eventually stabilize debt by reverting to a passive fiscal rule with the ergodic probability 0.62.\(^{18}\) Until it does so, debt rises to generate a modest increase in inflation due to the debt stabilization bias. By breaking the association between high inflation and active fiscal policy, under our estimates the Volcker disinflation does not require a prompt switch to a passive fiscal rule to explain why both the level and volatility of inflation fell.

In the third column of figure 4 the nature of the cross-regime expectation effects is different. If regimes were permanent, the fiscal authority would allow debt to rise for a sustained period. With switching in place, the fiscal authority anticipates that policy will revert to the passive rule and that this will involve an increase in tax rates to unwind any increase in debt that the transfers shock produced. Given the forward-looking nature of the Phillips curve, the anticipated rise in distortionary taxation fuels current inflation. Fiscal policy cuts taxes today to mitigate the rise in inflation, especially when the central bank is not strongly inflation averse. This tax cut means that fiscal policy is not stabilizing debt while this regime is in place. But when the regime switches to passive fiscal behavior and tax increases come, those increases will be greater. This leads to a further cut in taxes and a spiral of rising debt and inflation. Although the fiscal targeting regime cannot last forever, the behavior is consistent with observed data, particularly during periods when the shocks imply a decline in debt, alongside a gradual decrease in inflation.

In our setting the level of debt and the fiscal consequences of shocks will always impact on inflation. The magnitude of that impact depends crucially on agents’ long term expectations regarding the nature of debt stabilization. If agents expect fiscal policy will eventually switch to stabilize debt, the inflation impacts can be modest, even if the prevailing fiscal regime is not stabilizing. In contrast, when agents do not expect fiscal policy to ultimately stabilize government debt, as in Bianchi and Ilut (2017), low and stable inflation requires passive fiscal behavior. By using a model with lump-sum taxes and simple policy rules, Bianchi and Ilut omit two mechanisms that link debt and inflation in our setup: (i) the debt-stabilization

\(^{18}\)In combination with the other estimated policy parameters this is sufficient to ensure the model does not exhibit the kinds of equilibria associated with the FTPL even during episodes where fiscal policy is active.
bias, which connects a rising inflation bias to rising debt, and (ii) with passive fiscal policy, rising debt raises expected distorting taxes, which fuel current inflation. These mechanisms give fiscal policy a central role in our interpretations of data, even though the move to more conservative monetary policy is also important, as standard monetary interpretations of the Volcker era assert.

4.4.2 Monetary Policy Regimes

Looking at monetary policy alone, periods of the LC regime capture all those identified as passive in the rules-based estimation [appendix C]. But there are other periods in which monetary policy remains less conservative. Figure 2 shows that the late 1950s gave way to fluctuations in conservatism throughout the first half of the 1960s. Debate surrounds the anti-inflation stance of monetary policy in the 1950s: Romer and Romer (2002a) argue that policy makers appeared to recognize the need to fight inflation with monetary tightening, while Friedman’s (1960) concern was that the policy of targeting free reserves implied a less-conservative regime. Our switches in monetary policy regime in the late 1950s and early 1960s mirror this debate: relatively benign macroeconomic outcomes can be described as a mixture of more or less conservative monetary policy in this period.

By the mid 1960s, Romer and Romer (2002b) find that monetary policy makers believed that, although buoyant output drove higher inflation, inflation itself would soon stabilize without requiring a significant recession. This is consistent with the switch to the less conservative regime that we see in the mid 1960s.

The Romers suggest that policy makers internalized the Friedman-Phelps accelerationist Phillips curve in the 1970s, but with an initially overoptimistic assessment of the natural rate of unemployment. That optimism morphed into a pessimistic view of the output costs of fighting inflation. This explains the loss of inflation conservatism throughout the 1970s.

The Volcker disinflation did not really take hold until 1982 [Chen, Kirsanova, and Leith (2017)]. The switch to high conservatism in 1982 occurred once monetary policy makers acknowledged the costs of inflation [Romer and Romer (2002b)]. That switch also corresponds with Volcker’s assessment of when his deflation had finally become credible. Finally, the temporary loss of conservatism in 1987 reflects the operation of the “Greenspan put,” as monetary policy responded to the Black Monday stock market crash of that year [Bornstein and Lorenzoni (2018)].

Our estimates of the movements between periods of more- or less-conservative monetary policy display some subtlety in dating the loss of conservatism in the 1960s/70s, but are broadly in line with other monetary-policy-only analyses of the Great Moderation using either targeting rules [Chen, Kirsanova, and Leith (2017)] or active/passive instrument rules [Sims and Zha (2006)]. We do not deviate far from the standard narrative in this respect, although observed outcomes depend crucially on the associated fiscal regime as we now document.

4.4.3 Fiscal Policy Regimes

Romer and Romer (2009, 2010) extensively analyze post-war tax changes. They distinguish among tax policies designed to reduce the budget deficit,
attempts to affect aggregate demand, actions intended to pay for specific spending initiatives, and tax reforms aimed at enhancing long-run growth.

Throughout the 1950s and 1960s fiscal authorities ran either fiscal surpluses or small deficits, so the debt-GDP ratio gradually declined [figure 1]. In the brief period in the 1950s, which our estimates identify as the application of the fiscal targeting rule, Romer and Romer (2010) do not find any significant tax changes other than as a response to changes in spending. The relative stability of taxes, falling debt levels, and low, but slightly falling inflation observed in this period are all consistent with the targeting fiscal rule. In the next decade, there are some limited tax measures designed to match additional spending commitments like the expansion of highways and social security. The slower pace of debt reduction and rising inflation suggest that policy is no longer following a targeting rule, switching to passive.

By the end of 1960s, the debt-GDP ratio has fallen below the implicit steady state and the Romers do not find instances of tax cuts designed to return debt back to steady state. Tax cuts at the time aimed to boost aggregate demand and reduce unemployment. Those cuts were relatively small and were unable to overcome the fiscal drag generated by high inflation and a progressive tax system with non-indexed tax brackets. The upward trend in the tax burden, at a time of high inflation and low debt, explains why the estimates find that fiscal policy is predominantly active in the 1970s. Instances of non-active fiscal policy in this period are associated with the more sizeable tax cuts. The Nixon administration’s tax reforms of 1970 appear as a passive policy, which then turned to a targeting rule as fiscal policy was further loosened before the 1972 election. Policy was optimizing in the sense that reducing tax revenues as a share of GDP reduced the inflationary impact of distortionary taxation at a time when inflation was rising sharply, but debt levels were low. Ford’s tax rebate in 1975 appears as a fleetingly passive fiscal policy when the debt-GDP ratio had fallen below its steady-state value.

The relatively low debt-GDP ratio in the 1970s and the fact that fiscal policy is expected to turn passive in the long-run mean that the high inflation of that period cannot be attributed to the Fed generating inflation to reduce the real value of government debt. Nevertheless, we shall show below that a different fiscal regime could have offset the inflation of that era just as effectively as a switch to a more conservative monetary policy. In this sense, the inflationary outcomes of the 1970s are as much a fiscal as monetary phenomenon.

The reason fiscal policy is identified as active in the 1970s differs from the reason in the 1980s to the mid-1990s. The former was a decade when fiscal authorities failed to cut taxes despite debt falling below steady state; in the latter period government did not generate sufficient tax revenues to prevent debt from rising rapidly. President Reagan introduced measures to mitigate the increase in the deficit in 1982 and enhance the sustainability of Social Security in 1983. But these were dominated by the tax cuts contained in the earlier Economic Recovery Tax Act of 1981, which were phased in over three steps between 1982 and 1984. The Reagan administration also significantly reduced the progressivity of the tax system by eliminating tax brackets and indexing remaining brackets to inflation. The tax burden fell significantly and the debt-GDP ratio rose. There was no attempt to reduce the deficit under President George H. W. Bush either, until he broke his “no new taxes” pledge.

in budget negotiations with Congress in 1991. Dominance of large exogenous tax cuts over deficit targeting in the 1980s is consistent with active fiscal policy, but is hard to reconcile with explanations of the Great Moderation which rely on a near simultaneous shift to a passive fiscal policy.

Only with the Omnibus Budget Reconciliation Act of 1993 under President Clinton does fiscal policy emerge from the active regime to enter a sustained period of targeting or passive policy regimes. As in the 1950s, which our estimates label as a targeting regime, the second half of the 1990s is also marked by low and gradually falling inflation and debt. Although our fiscal targeting rule is destabilizing if not expected to be permanent, in periods of favorable fiscal shocks these features are identified by our model as constituting fiscal policy under a targeting rule. Targeting fiscal behavior gives fiscal policy a prominent role in producing the observed low rates of inflation. Instrument-rules-based studies credit monetary policy fully with delivering those favorable inflation outcomes. In those studies, fiscal policy passively adjusts (lump-sum) taxes to stabilize debt, but plays no role in determining inflation.

Active fiscal behavior re-emerges around President G. W. Bush’s cuts taxes in 2001 and 2003, partly to promote long-term growth and partly to offset the macroeconomic shock associated with the 9/11 terrorist attacks. The ultimate switch to passive policy after 2005 is not obviously due to any observed discrete policy changes, but likely reflects the increase in revenues generated by the booming economy leading up to the financial crisis that began in 2007.

In their dating of fiscal regimes, our estimation differs most clearly from the narrative in Bianchi and Ilut (2017). We do not find that debt levels or fiscal shocks drove the inflation of the 1970s, nor that fiscal policy switched decisively to a passive regime in the early 1980s. Instead, our estimates suggest that the fiscal policies of Reagan and the first George Bush did not avert the rising debt levels seen in this period. We obtain different inferences because our specification permits modest inflation to coexist temporarily without tax backing for government debt. These outcomes can coexist because economic agents anticipate that debt will be stabilized through fiscal policy eventually. Bianchi and Ilut’s (2017) setup implies the opposite belief, under which the fiscal repercussions of the shocks of the 1980s would generate too much inflation, relative to the data, if fiscal policy were to remain active in that period.

4.5 Welfare Gaps

To gain further insight into which features of the data drive the identification of the various policy regimes, we examine the welfare-relevant “gaps” policy makers aim to close. We consider four gaps: inflation, output, taxes, and debt, where inflation and debt gaps measure the deviation of the variable from its steady state or target value. The output gap, $\hat{y}_t - \hat{\gamma}^*_t$, computes the deviation of output from the level of output that would be chosen by the social planner, $\hat{\gamma}^*_t$ [appendix J]. This gap reflects the extent to which the policy maker is unable to achieve the desired level of output due to nominal inertia, the habits externality, fiscal constraints, and time-consistency problems. It measures the trade offs between inflation and the real economy embedded in the estimated objective function, but reduces those to a single measure. The tax gap, $\tau_t - \tau^*_t$, is the difference between the actual tax rate, $\tau_t$, and the rate that a policy maker could choose to eliminate cost-push shocks, $\tau^*_t = -(1 - \tau)\hat{\mu}_t$. 

18
This reflects the fact that distortionary taxation acts like a cost-push shock in the Phillips curve, so that tax cuts can offset realized cost-push shocks driven by variations in the desired markup. Inflation and tax gaps are often, to some extent, mirror images of each other, as both are influenced by the estimated cost-push shocks.

The top two panels of figure 5 plot the inflation and output gaps alongside the probability that monetary policy is in the LC regime. Less-conservative monetary policy arises when for a given output gap, inflation is unusually high. Although there is a sizeable negative output gap in the early 1970s, this was not as large relative to the levels of excess inflation found during the Volcker disinflation. This is why the Volcker period shows up as a switch to more conservative monetary policy. Similarly, a more conservative policy maker would not have permitted the modest rise in inflation that was associated with the loosening of monetary policy after the stock market crash of 1987.

The bottom two panels of figure 5 plot the tax and debt gaps, alongside the probabilities of being in the TF and PF fiscal regimes. Realizations of the targeting rule fiscal regime in the 1950s and in 1995 correspond to periods when the tax, output, and inflation gaps are modest, with debt returning to steady state and inflation falling slowly. Passive fiscal policy is associated with debt-stabilizing movements in taxation predominantly in the 1960s. Exit from the passive fiscal regime in the late 1960s corresponds to a period of rising tax gap that was not consistent with the negative debt gap in the 1970s; these gaps are then reversed from the 1980s to the mid 1990s. Seen in this way, the prolonged periods of active fiscal behavior—throughout the 1970s and then the 1980s until 1995—are due to tax policies that fail to stabilize debt in both directions.

We now turn to re-examine the role fiscal policy played in the inflation of the 1970s, before considering the inflationary risks posed by the currently high levels of debt seen in the US.

5 Avoiding the Great Inflation with Fiscal Leadership

Because our estimates find no decisive shift in fiscal behavior to support Volcker’s monetary policy, it is tempting to conclude that the disinflation was largely a monetary phenomenon. Does that mean the inflation of the 1970s could have been avoided had Paul Volcker been appointed earlier? Or that fiscal policy played no part in the inflation of the 1970s? Figure 6 plots the rate of inflation observed in the 1970s alongside counterfactual outcomes had the shocks been the same, but the policy regime differed. The first comparison is what would have happened had the Fed been more conservative throughout the sample, even although fiscal policy remained active (but with the expectation that, ultimately, policy would have switched to other regimes in line with estimated transition probabilities). Here we see a sizeable drop in inflation in the 1970s had the Fed been more conservative, falling from an average of 6.4% to 4.6%.

But it is possible to explore how much fiscal policy could have reduced the 1970s inflation. Had monetary policy remained less conservative throughout the 1970s, but the fiscal authorities had adopted a targeting rule then, even though the policy is not expected to be permanent, inflation would have fallen even further to 4.2%. Since the fiscal targeting rule uses distortionary taxation to offset cost-push shocks, which were prevalent in the period, this can improve inflation outcomes more than the adoption of a more conservative
monetary policy. Still better inflation outcomes arise by combining a conservative central bank with a targeting rule fiscal authority: inflation would have averaged 3.55% (or 3.35% if the policy were considered permanent). Although the Volcker disinflation was achieved without contemporaneous fiscal support, similar or better inflation performance could have been achieved by the fiscal authority adopting a targeting rule, even if that policy was not expected to last.

6 High Debt and Inflation Risks

Two powerful global shocks in quick succession—the financial crisis of 2008 and the Covid-19 pandemic of 2020—dramatically raised government debt levels. Do elevated debt levels increase inflation risks? We use the estimated model to assess these risks.

Imagine that the American economy has emerged from the pandemic recession to return to steady state except for the debt-GDP ratio. That ratio stands at 82.6%, compared to the calibrated steady state value of 31%. We conduct 100,000 stochastic simulations of the model, allowing policy regimes to evolve randomly, but shutting down the other economic shocks.

We consider two scenarios for how monetary and fiscal policies evolve from the high-debt initial condition: (1) policies follow historic norms; (2) with small probability, historic norms are overthrown and policy enters an absorbing active fiscal state. In both scenarios, monetary policy fluctuates between MC and LC regimes, obeying estimated transition probabilities.

6.1 Maintain Policy Norms

To maintain historic norms, policy behavior evolves according to the estimated transition probabilities that table 1 reports. We randomly select the initial policy regime using the ergodic distribution in table 3. Figure 7 plots the median—black solid line—and shaded fan chart percentiles for debt and inflation over 400 periods. There is a significant, but not overwhelming, increase in inflation which mirrors the projected paths of government debt. High initial debt levels worsen the inflationary bias problem that stems from the policy makers’ incentives to induce inflation surprises that reduce the real value of debt. The median path quickly rises to 5%, which corresponds to the rate of CPI inflation in the US in May 2021. Inflation rises further in the short term as the inertial inflation process evolves and debt levels rise further under many scenarios. Debt-GDP overshoots steady state along the median path because the fiscal policy makers’ objective function penalizes rapid adjustments in tax rates. This penalty extends fiscal consolidation over many decades.

This simulation assumes that in the long run debt returns to its postwar mean. Because stabilization occurs only gradually, inflation remains away from its long-run target throughout that process. In this scenario, very long-term inflation expectations are anchored firmly on target inflation. But expected inflation, as measured by the median of realizations, can deviate significantly and persistently from target.

---

21 As of April 2021, the market value of debt held by the public was 82.6%, according to the Dallas Fed, https://www.dallasfed.org/research/econdata/govdebt.

6.2 Erosion of Policy Norms

One can imagine many ways in which policy norms could change, with each possibility generating different inflation implications from high debt. We consider a minimal deviation from norms to underscore how sensitive model predictions are to seemingly minor changes in beliefs about policy behavior. A critical feature of beliefs based on historic norms is that in the long run fiscal policy adjusts tax rates to stabilize debt. We perturb the norm by introducing a small probability of transitioning from the temporary active fiscal regime to an absorbing state in which fiscal policy does not adjust taxes to stabilize debt.

With the additional permanent active fiscal regime, transition probabilities are given by

$$
\Psi = \begin{bmatrix}
\psi_{11} & 1 - \psi_{22} - \psi_{23} & \psi_{31} & 0 \\
\psi_{12} & \psi_{22} & 1 - \psi_{31} - \psi_{33} & 0 \\
1 - \psi_{11} - \psi_{12} & \psi_{23} & \psi_{33} - q & 0 \\
0 & 0 & q & 1
\end{bmatrix}
$$

where the $\psi_{ij}$’s are estimated values reported in table 1 and $q$ is the probability of entering the permanent active fiscal regime.

We repeat the exercise in section 6.1 with this modified transition matrix. When $q$ is small—we use $q = .001$ and $q = .005$—remaining probabilities in $\Psi$ are only little affected, but with large impacts on the inflationary potential of high debt.

In the top panel of figure 8, with probability $q = 0.001$ the economy will never leave the active fiscal regime once it enters. Even this small risk that policy makers will abandon the norm that eventually fiscal policy stabilizes debt raises median inflation by one percentage point in the short-to-medium runs. Other simulated inflation paths display similar upward shifts with the best short-term inflation outcomes now over 6%. The lower panels of figure 8 increase the transition probability to $q = 0.005$. Now inflation rises dramatically: in initial periods, all simulated paths lie above 10%; for the first 50 periods, all inflation realizations exceed 5%.

Two effects drive the worsening inflation outcomes: the occurrence of entering the absorbing state and the expectations formation effects that the risk of doing so generates. If fiscal policy turns permanently active when debt is above steady state, inflation jumps to return debt to steady state, as column 2 of figure 3 depicts. Higher levels of debt when fiscal policy turns permanently active amplify the jump in inflation.\textsuperscript{23} As in column 2 of figure 3 the inflation surprise lasts only as long as the maturity structure of the outstanding debt stock, so debt is quickly stabilized. Effects of surprise inflation on debt explain the kinks in the median path for debt around its steady-state; debt return to steady state 20 periods after the economy enters the permanently active regime.

Even if the economy does not enter the permanent active fiscal regime during a given simulation, the risk of doing so creates expectational spillover effects. Expectational effects arise from the anticipation of a jump in inflation, should the permanent active fiscal regime occur in the future. Higher expected inflation shifts the Phillips curve to raise current inflation. These effects augment the inflationary bias associated with a given level of debt.

\textsuperscript{23}By symmetry, if debt is below steady state when the absorbing fiscal regime is realized, there is a deflationary jump, explaining the risk of deflation in the lower panel of figure 8.
which was already present, to exacerbate the debt stabilization bias. The higher inflation in figure 8 is a mixture of higher inflation from transitioning to the permanent active fiscal regime and the worsening of the debt stabilization bias. Even if the economy doesn’t enter the permanently active regime in the near future, a small likelihood of doing so can dramatically increase inflation outcomes as long as debt remains high by historical standards.

Maintaining the norm that fiscal authorities will eventually, as they have in the past, take the actions necessary to stabilize debt is essential to avoid a large increase in inflation. Tightness of the inflation distribution around the median underscores that the nature of the regime at any point in time matters far less than beliefs about the nature of debt stabilization in the long-run. The fact that we see inflationary pressures rising in current data, but not dramatically, suggests that belief in stabilizing fiscal policy remains.

7 Conclusions

There has been much debate on the extent to which the Great Moderation was due to good luck or good (monetary) policy. There has been less emphasis on the role that fiscal policy plays in the improved economic outcomes. Work that examines this issue reaches contradictory conclusions: Bianchi (2012) finds that fiscal policy did not begin to stabilize debt until the early 1990s, although economic agents did expect that the fiscal authorities would eventually act to stabilize debt; Bianchi and Ilut (2017) find the opposite—fiscal policy turned passive in the early 1980s and this switch was crucial to enabling the active monetary policy to reduce inflation. We generalize these results by considering a richer description of policy involving a mixture of instrument and targeting rules, with potential shifts in the conservatism of the central bank, the introduction of distortionary taxation, and by broadening the nature of the transitions between monetary and fiscal policy regimes.

In this environment, inflationary outcomes are always the joint outcome of both monetary and fiscal policy, offering fresh interpretations of monetary and fiscal policy interactions. We do not find that the inflation of the 1970s was driven by either the level of debt or the fiscal consequences of shocks. The narrative that the switch in monetary policy at the time of the Volcker disinflation was associated with a similar switch in fiscal policy making from a regime where the fiscal authorities did not act to stabilize debt to one where they did, does not fit time series data. Instead, we find that the Volcker disinflation occurred around 1982, but fiscal policy didn’t abandon its active policy until 1995; even then this policy was subject to further revisions. There are numerous switches between the various permutations of policies, with a passive fiscal policy still not clearly supporting the post-Volcker monetary conservatism observed in the data.

Although the Great Moderation was largely driven by a shift in monetary policy, counterfactuals suggest that adopting a fiscal targeting rule could have reduced the 1970s inflation just as dramatically. The key to finding that the Volcker disinflation did not require an immediate fiscal response is that economic agents anticipated that fiscal authorities would eventually act to stabilize debt. Stochastic simulations show that if that implicit promise to maintain historic fiscal norms were ever in doubt, elevated debt-GDP from the Covid-19 pandemic could raise inflation dramatically. If the norms are expected to be maintained, higher debt should drive a more modest rise in inflation.
Figure 1: United States Data.
Figure 2: Markov Switching Probabilities: Policy and Volatility Switches under Strategic Policy
Figure 3: Impulse Response to a 10 percent Transfers Shock Under Different Policy Permutations. All plots assume that there is no expectation of policy switching. Column 1 combines a passive fiscal rule with a more- or less-conservative monetary targeting rule or an active monetary rule estimated in appendix C. Column 2 combines the active fiscal rule with a more- or less-conservative monetary targeting rule or the passive monetary rule estimated in appendix C. Column 3 combines the fiscal targeting rule with a more- or less-conservative monetary targeting rule.
Figure 4: Impulse Response to a 10 percent Transfers Shock Under Different Policy Permutations (No Credibility). All plots assume that there is the expectation of switching to alternative policy regimes in line with the estimates. Column 1 combines a passive fiscal rule with a more- or less-conservative monetary targeting rule or an active monetary rule estimated in appendix C. Column 2 combines the active fiscal rule with a more- or less-conservative monetary targeting rule or the passive monetary rule estimated in appendix C. Column 3 combines the fiscal targeting rule with a more- or less-conservative monetary targeting rule.
Figure 5: Output, Inflation, Tax, Debt and Policy Regimes. The output gap measures the difference between output and what would be chosen by a social planner given the estimated objective function as a percentage, as Appendix J describes. Inflation and debt gaps measure the deviation from steady-state and the tax gap is the difference between the percentage tax rate and the tax rate that would perfectly offset the inflationary impact of cost push shocks. All gaps are measured on the left scale and the probability of policy regimes on the right scale.
Figure 6: Actual and Counterfactual U.S. Inflation. Counterfactuals condition on remaining in the specified policy regime, but equilibrium embeds estimated beliefs that regime may change. MC is time-consistent more-conservative monetary policy; TF is targeting-rule fiscal behavior.
Figure 7: Model simulated 100,000 times for 400 periods. Economy in steady-state initially, except the debt-GDP ratio is 82.6%. Initial policy regime drawn randomly. Policy regimes can switch in line with estimated transition probabilities.
Figure 8: Model simulated 100,000 times for 400 periods. Economy in steady-state initially, except the debt-GDP ratio is 82.6%. Initial policy regime drawn randomly. Policy regimes can switch in line with estimated transition probabilities, adjusted to include a risk, $q$, of the active fiscal regime becoming permanent.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Posterior</th>
<th>Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Targeting policy parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\omega_1$, $\hat{X}_t - \hat{\xi}_t$,</td>
<td>0.221</td>
<td>0.208</td>
</tr>
<tr>
<td>$\omega_2$, $\hat{y}_t - \frac{\sigma}{\varphi} \hat{\xi}_t$,</td>
<td>0.256</td>
<td>0.247</td>
</tr>
<tr>
<td>$\omega_3$, change in inflation</td>
<td>0.422</td>
<td>0.420</td>
</tr>
<tr>
<td>$\omega_{M,S_t=1}$, inflation</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\omega_{M,S_t=2}$, inflation</td>
<td>0.611</td>
<td>0.601</td>
</tr>
<tr>
<td>$\omega_{R}$, change in interest</td>
<td>0.739</td>
<td>0.724</td>
</tr>
<tr>
<td>$\omega_{F,S_t=1}$, inflation</td>
<td>0.298</td>
<td>0.316</td>
</tr>
<tr>
<td>$\omega_{T,s_t=1}$, change in tax</td>
<td>0.699</td>
<td>0.659</td>
</tr>
<tr>
<td>$\rho_{r,s_t=2}$, lagged tax rate</td>
<td>0.964</td>
<td>0.950</td>
</tr>
<tr>
<td>$\rho_{r,s_t=3}$, lagged tax rate</td>
<td>0.932</td>
<td>0.935</td>
</tr>
<tr>
<td>$\delta_{T,s_t=2}$, tax resp. to debt</td>
<td>0.045</td>
<td>0.050</td>
</tr>
<tr>
<td>$\delta_{T,s_t=3}$, tax resp. to debt</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>$\delta_{y_t}$, tax resp. to output</td>
<td>0.001</td>
<td>0.032</td>
</tr>
</tbody>
</table>

| **Deep parameters** | | |
| $\sigma$, Inverse of intertemp | 3.102 | 3.208 | [2.759,3.631] | N | 2.50 | [2.09,2.91] |
| $\alpha$, Calvo | 0.780 | 0.774 | [0.751,0.797] | B | 0.75 | [0.71,0.78] |
| $\zeta$, inflation inertia | 0.353 | 0.366 | [0.277,0.458] | B | 0.50 | [0.33,0.66] |
| $\theta$, habit persistence | 0.802 | 0.810 | [0.736,0.885] | B | 0.50 | [0.33,0.66] |
| $\varphi$, Inverse of Frisch | 2.00 | 2.00 | - | - | 2.00 | Fixed |

| **Serial correlation of shocks** | | |
| $\rho_{\xi_t}$, taste | 0.938 | 0.942 | [0.931,0.953] | B | 0.50 | [0.25,0.75] |
| $\rho_{\mu_t}$, cost-push | 0.938 | 0.931 | [0.912,0.949] | B | 0.50 | [0.25,0.75] |
| $\rho_{\eta_t}$, productivity | 0.274 | 0.280 | [0.211,0.350] | B | 0.50 | [0.25,0.75] |
| $\rho_{z_t}$, transfers | 0.968 | 0.971 | [0.960,0.982] | B | 0.50 | [0.25,0.75] |
| $\rho_{g_t}$, government | 0.986 | 0.984 | [0.978,0.989] | B | 0.50 | [0.25,0.75] |

Table 1: Targeting Rules. Under targeting rules, we have six policy permutations: MC/TF, MC/PF, MC/AF, LC/TF, LC/PF, LC/AF. For monetary policy switches, $S_t = 1$ is the MC regime and $S_t = 2$ is the LC regime. For fiscal policy, the TF policy regime corresponds to $s_t = 1$, while the PF and AF regimes correspond to $s_t = 2$ and $s_t = 3$, respectively. Weights $\omega_1, \omega_2, \omega_3$ are constant across monetary and fiscal policy regimes.
### Table 1: Targeting Rules (continued). For volatility, $k_t = 1$ is the low volatility regime and $k_t = 2$ is the high volatility regime.
Table 2: Model Comparison. The intermediate model treats monetary policy as time-consistent targeting rule with changes in the degree of inflation conservatism, while fiscal policy switches between the PF and AF regimes. The targeting rule model adds to the intermediate model the possibility that fiscal policy may switch to an additional TF regime.

<table>
<thead>
<tr>
<th>Model</th>
<th>Log Marginal Data Density</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Geweke</td>
</tr>
<tr>
<td>Targeting Rules</td>
<td>−1410.254</td>
</tr>
<tr>
<td>Intermediate Model</td>
<td>−1416.304</td>
</tr>
<tr>
<td>Rules-Based Policy</td>
<td>−1418.116</td>
</tr>
</tbody>
</table>

Table 3: Long-run Regime Probabilities. The table reflects the ergodic probabilities of being in each permutation of monetary and fiscal policy regime given the estimated transition probabilities in table 1.

<table>
<thead>
<tr>
<th>Regimes</th>
<th>Less Conservative</th>
<th>More Conservative</th>
<th>Sum Columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Active Fiscal</td>
<td>0.11</td>
<td>0.13</td>
<td>0.25</td>
</tr>
<tr>
<td>Passive Fiscal</td>
<td>0.29</td>
<td>0.33</td>
<td>0.62</td>
</tr>
<tr>
<td>Fiscal Targeting</td>
<td>0.06</td>
<td>0.07</td>
<td>0.13</td>
</tr>
<tr>
<td>Sum Rows</td>
<td>0.46</td>
<td>0.54</td>
<td>1.00</td>
</tr>
</tbody>
</table>
REFERENCES


APPENDICES FOR STRATEGIC INTERACTIONS IN U.S.
POLICIES FOR ONLINE PUBLICATION*

Xiaoshan Chen†  Eric M. Leeper‡  Campbell Leith§

CONTENTS

A System of Non-Linear Equations 1

B Derivation of Objective Functional Form 2
  B.1 The Social Planner’s Problem 2
  B.2 Quadratic Representation of Social Welfare 3

C Rules-Based Estimation 6
  C.1 Posterior Estimates: Rules-Based Policy 9
  C.2 Regime Switching Rules-Based Policy 10

D Leadership Equilibria Under Discretion 13
  D.1 Policy of the Follower 15
  D.2 Policy of the Leader 17

E Nash Equilibrium under Discretion 20
  E.1 Policy Maker A 21
  E.2 Policy Maker B 22

F Data Appendix 25

G Alternative Leadership Regimes 26

H Convergence 32

I Model Identification 33

J Alternative Social Planner’s Allocation 36

*August 13, 2021.
†University of Durham; xiaoshan.chen@durham.ac.uk.
‡University of Virginia, and NBER; eleeper@virginia.edu
§University of Glasgow; campbell.leith@glasgow.ac.uk.
A System of Non-Linear Equations

\[ N_t^{k\phi} X_t^{k\sigma} = \frac{W_t}{A_t P_t} \equiv w_t \]

\[ 1 = \beta E_t \left[ \left( \frac{X_t^{k\xi_{t+1}}}{X_t^{k\xi_t}} \right)^{-\sigma} \frac{A_t}{A_{t+1}} \frac{P_t}{P_{t+1}} \right] R_t \]

\[ P_t^M = \beta E_t \left[ \left( \frac{X_t^{k\xi_{t+1}}}{X_t^{k\xi_t}} \right)^{-\sigma} \frac{A_t}{A_{t+1}} \frac{P_t}{P_{t+1}} (1 + \rho P_t^M) \right] \]

\[ N_t = \left( \frac{Y_t}{A_t} \right) \int_0^1 \left( \frac{P(t)}{P_t} \right)^{-\eta} dt \]

\[ P_t Y_t = P_t C_t + P_t G_t \]

\[ \frac{P_t^f}{P_t} = \left( \frac{\eta}{\eta - 1} \right) \frac{E_t \sum_{s=0}^{\infty} (\alpha \beta)^s (X_{t+s}^{k\xi_{t+s}})^{-\sigma} \mu_{t+s} mc_{t+s} \left( \frac{P_{t+s}^{\pi-s}}{P_t} \right)^{\eta} Y_{t+s}^{\pi+s}}{E_t \sum_{s=0}^{\infty} (\alpha \beta)^s (X_{t+s}^{k\xi_{t+s}})^{-\sigma} (1 - \tau_{t+s}) \left( \frac{P_{t+s}^{\pi-s}}{P_t} \right)^{\eta-1} Y_{t+s}^{\pi+s}} \]

\[ mc_t = \frac{W_t}{A_t P_t} \]

\[ P_t^b = P_{t-1}^{\pi-t-1} \]

\[ \ln P_{t-1}^* = (1 - \zeta) \ln P_{t-1}^f + \zeta P_{t-1}^b \]

\[ (P_t)^{1-\eta} = \alpha (P_{t-1}^{\pi})^{1-\eta} + (1 - \alpha) (P_t)^{1-\eta} \]

\[ b_t^M = \frac{(1 + \rho P_t^M) Y_{t-1}^s b_{t-1}^M}{\pi_t Y_t} - \tau_t + g_t + z_t + \xi_{t+p,t} \]

\[ \ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \sigma_g \varepsilon_{g,t} \]

\[ \ln z_t = (1 - \rho_z) \ln z + \rho_z \ln z_{t-1} + \sigma_z \varepsilon_{z,t} \]

\[ \ln A_t = \ln \gamma + \ln A_{t-1} + \ln q_t \]

\[ \ln q_t = \rho_q \ln q_{t-1} + \sigma_q \varepsilon_{q,t} \]

\[ \ln \mu_t = \rho_\mu \ln \mu_{t-1} + \sigma_\mu \varepsilon_{\mu,t} \]

\[ \ln \xi_t = \rho_\xi \ln \xi_{t-1} + \sigma_\xi \varepsilon_{\xi,t} \]

The equation describing the evolution of price dispersion, \( \int_0^1 \left( \frac{P(t)}{P_t} \right)^{-\eta} dt \) is not needed to tie down the equilibrium upon log-linearization.

In order to render this model stationary we need to scale certain variables by the non-stationary level of technology, \( A_t \) such that \( k_t = K_t/A_t \) where \( K_t = \{ Y_t, C_t, W_t/P_t \} \). Fiscal variables (i.e. \( P_t^M B_t^M/P_t, G_t \) and \( Z_t \)) are normalized with respect to \( Y_t \). All other real variables are naturally stationary. Applying this scaling, the steady-state equilibrium conditions reduce to:
\[ N^\phi X^\sigma = w \]
\[ 1 = \beta \left( R \pi^{-1} \right) / \gamma = \beta r / \gamma \]
\[ p^M = \frac{\beta}{\gamma \pi - \beta \rho} \]
\[ y = N \]
\[ y = \frac{c}{1 - g} \]
\[ X = c (1 - \theta) \]
\[ mc = w \]
\[ \frac{\eta}{\eta - 1} = \frac{1 - \tau}{mc} \]
\[ b^M = \left( \frac{\beta}{1 - \beta} \right) s \]

To determine the steady state value of labor, we substitute for \( X \) in terms of \( y \) and then, using the aggregate production function, we obtain the following expression,

\[ y^{\sigma + \psi} [(1 - g) (1 - \theta)]^\sigma = \frac{\eta - 1}{\eta} (1 - \tau), \tag{A.1} \]

where \( g \) is the steady state share of government spending in output. We shall contrast this with the labor allocation/output that would be chosen by a social planner to obtain a measure of the steady-state distortion inherent in this economy which features distortionary taxation, monopolistic competition and the habits externality.

**B Derivation of Objective Functional Form**

**B.1 The Social Planner’s Problem**

In order to assess the scale of the steady-state inefficiencies caused by the monopolistic competition, tax and habits externalities it is helpful to contrast the decentralized equilibrium with that which would be attained under the social planner’s allocation. The social planner ignores the nominal inertia and all other inefficiencies and chooses real allocations that maximize the representative consumer’s utility subject to the aggregate resource constraint, the aggregate production function, and the law of motion for habits-adjusted consumption:

\[
\max_{\{x_t, c_t^*, g_t^*\}} E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{X_t^{1-\sigma} \xi_t^{-\sigma}}{1-\sigma} + \chi \frac{(G_t^*/A_t)^{1-\sigma} (\xi_t)^{-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi} \xi_t^{-\sigma}}{1+\phi} \right)
\]

s.t. \( Y_t^* = C_t^* + G_t^* \)
\( Y_t^* = A_t N_t^* \)
\( X_t^* = C_t^*/A_t - \theta C_{t-1}^*/A_{t-1} \)

The optimal choice implies the following relationship between the marginal rate of substitution between labor and habit-adjusted consumption and the intertemporal marginal rate
of substitution in habit-adjusted consumption
\[
\frac{(N_t^*)^\sigma}{(X_t^*)^{-\sigma}} = \left[1 - \theta \beta E_t \left(\frac{X_{t+1}^* \xi_{t+1}}{X_t^* \xi_t}\right)^{-\sigma}\right].
\]
The steady state equivalent of this expression can be written as,
\[
(N^*)^{\sigma+\sigma} \left[(1 - \frac{G^*}{Y^*}) (1 - \frac{1}{Y^*})\right]^{\sigma} = (1 - \theta \beta).
\]
where the optimal share of government consumption in output is given by,
\[
\frac{G_t^*}{Y_t^*} = \chi^\frac{1}{\sigma} \left(\frac{Y_t^*}{A_t}\right)^{-\sigma + \frac{\sigma}{\sigma}}
\]
In steady state these can be combined to give the optimal share of government consumption in output,
\[
\frac{G^*}{Y^*} = (1 + (1 - \theta)^{-1}\chi^{-\frac{1}{\sigma}}(1 - \theta \beta)^{\frac{1}{\sigma}})^{-1}
\]
which can then used to get the steady state level of output under the social planner’s allocation. We shall assume that the share of government spending in GDP in the data matches this, such that the data is calibrating the value of \(\chi\). Doing so facilitates the construction of a quadratic objective function.

If we contrast this with the allocation achieved in the steady-state of our decentralized equilibrium (A.1), assuming that the steady state share of government consumption to GDP is the same, we can see that the two will be identical whenever the following relationship between the markup, the tax rate and the degree of habits holds,
\[
\frac{\eta}{\eta - 1} = \frac{1 - \tau}{1 - \theta \beta}
\]
Notice that in the absence of habits this condition could only be supported by a negative tax rate. However, for the data given level of taxation and the estimated degree of habits this condition will define our steady-state markup, enabling us to adopt an efficient steady-state and thereby avoiding a steady-state inflationary bias problem when describing optimal policy.

**B.2 Quadratic Representation of Social Welfare**

Individual utility in period \(t\) is
\[
\frac{X_t^{1-\sigma} \xi_t^{-\sigma}}{1 - \sigma} + \chi \frac{(G_t/A_t)^{1-\sigma} \xi_t^{-\sigma}}{1 - \sigma} - \frac{N_t^{1+\varphi} \xi_t^{-\sigma}}{1 + \varphi}
\]
where \(X_t = C_t - \theta C_{t-1}\) is the habit-adjusted aggregate consumption. Before considering the elements of the utility function, we need to note the following general result relating to second order approximations
\[
\frac{Y_t - Y}{Y_t} = \hat{Y}_t + \frac{1}{2} \hat{Y}_t^2 + O[2]
\]
3
where $\hat{Y}_t = \ln \left( \frac{Y_t}{\hat{Y}} \right)$ and $O[2]$ represents terms that are of order higher than 2 in the bound on the amplitude of the relevant shocks. This will be used in various places in the derivation of welfare. Now consider the second order approximation to the first term,

$$\frac{X_t^{1-\sigma} \xi_t^{1-\sigma}}{1 - \sigma} = X^{1-\sigma} \left( \frac{X_t - X}{X} \right) - \frac{\sigma}{2} X^{1-\sigma} \left( \frac{X_t - X}{X} \right)^2 - \sigma X^{1-\sigma} \left( \frac{X_t - X}{X} \right) (\xi_t - 1) + \text{tip} + O[2]$$

where $\text{tip}$ represents ‘terms independent of policy’. Using the results above this can be rewritten in terms of hatted variables

$$\frac{X_t^{1-\sigma} \xi_t^{1-\sigma}}{1 - \sigma} = X^{1-\sigma} \left\{ \hat{X}_t + \frac{1}{2} (1 - \sigma) \hat{X}_t^2 - \sigma \hat{X}_t \hat{\xi}_t \right\} + \text{tip} + O[2].$$

In pure consumption terms, the value of $X_t$ can be approximated to second order by:

$$\hat{X}_t = \frac{1}{1 - \theta} \left( \hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) - \frac{\theta}{1 - \theta} \left( \hat{c}_{t-1} + \frac{1}{2} \hat{c}_{t-1}^2 \right) - \frac{1}{2} \hat{X}_t^2 + O[2]$$

and to a first order,

$$\hat{X}_t = \frac{1}{1 - \theta} \hat{c}_t - \frac{\theta}{1 - \theta} \hat{c}_{t-1} + O[1]$$

which implies

$$\hat{X}_t^2 = \frac{1}{(1 - \theta)^2} \left( \hat{c}_t - \theta \hat{c}_{t-1} \right)^2 + O[2]$$

Therefore,

$$\frac{X_t^{1-\sigma} \xi_t^{1-\sigma}}{1 - \sigma} = X^{1-\sigma} \left\{ \frac{1}{1 - \theta} \left( \hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) - \frac{\theta}{1 - \theta} \left( \hat{c}_{t-1} + \frac{1}{2} \hat{c}_{t-1}^2 \right) + \frac{1}{2} (-\sigma) \hat{X}_t^2 - \sigma \hat{X}_t \hat{\xi}_t \right\} + \text{tip} + O[2]$$

Summing over the future,

$$\sum_{t=0}^{\infty} \beta^t \frac{X_t^{1-\sigma} \xi_t^{1-\sigma}}{1 - \sigma} = X^{1-\sigma} \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1 - \theta \beta}{1 - \theta} \left( \hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) - \frac{1}{2} \sigma \hat{X}_t^2 - \sigma \hat{X}_t \hat{\xi}_t \right\} + \text{tip} + O[2].$$

Similarly for the term in government spending,

$$\chi \frac{g_t^{1-\sigma} \xi_t^{1-\sigma}}{1 - \sigma} = \chi \left\{ \frac{1 - \theta \beta}{1 - \theta} \left( \hat{g}_t + \frac{1}{2} \hat{g}_t^2 \right) - \frac{1}{2} \sigma \hat{g}_t^2 - \sigma \hat{g}_t \hat{\xi}_t \right\} + \text{tip} + O[2]$$

While the term in labour supply can be written as

$$\frac{N_t^{1+\phi} \xi_t^{1-\sigma}}{1 + \phi} = \bar{N}^{1+\phi} \left\{ \hat{N}_t + \frac{1}{2} (1 + \varphi) \hat{N}_t^2 - \sigma \hat{N}_t \hat{\xi}_t \right\} + \text{tip} + O[2]$$

Now we need to relate the labour input to output and a measure of price dispersion. Aggregating the individual firms’ demand for labour yields,

$$N_t = \left( \frac{Y_t}{A_t} \right) \int_0^1 \left( \frac{P(i)_t}{P_t} \right)^{-\eta} di$$
It can be shown (see Woodford (2003, Chapter 6)) that

\[ \tilde{N}_t = \hat{y}_t + \ln[\int_0^1 \left( \frac{P(i)}{F_t} \right)^{-\eta} \, di] \]

\[ = \hat{y}_t + \frac{\eta}{2} \text{var}_t \{p(i)_t\} + O[2] \]

which implies

\[ \tilde{N}_t^2 = \hat{y}_t^2 \]

so we can write

\[ \frac{N_{t+1}^{1+\varphi}}{1+\varphi} = N^{1+\varphi} \{ \hat{y}_t + \frac{1}{2} (1+\varphi) \hat{y}_t^2 - \sigma \hat{y}_t \hat{\xi}_t + \frac{\eta}{2} \text{var}_t \{p_t(i)\} \} + \text{tip} + O[2] \]

Welfare is then given by

\[ \Gamma_0 = \mathbf{X}^{1-\sigma} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1-\theta}{1-\theta} \left( \hat{c}_t + \frac{1}{2} \hat{g}_t^2 \right) - \frac{1}{2} \sigma \hat{X}_t^2 - \sigma \hat{X}_t \hat{\xi}_t \right\} \]

\[ + \chi \hat{g}^{1-\sigma} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \hat{g}_t + \frac{1}{2} (1-\sigma) \hat{g}_t^2 - \sigma \hat{g}_t \hat{\xi}_t \right\} \]

\[ - N^{1+\varphi} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \hat{y}_t + \frac{1}{2} (1+\varphi) \hat{y}_t^2 - \sigma \hat{y}_t \hat{\xi}_t + \frac{\eta}{2} \text{var}_t \{p_t(i)\} \right\} \]

\[ + \text{tip} + O[2] \]

From the steady-state of our model, and its comparison with the social planner’s allocation we know that \( \mathbf{X}^{1-\sigma} (1-\theta \beta) = (1-\theta) \frac{\gamma}{\varphi} N^{1+\varphi} \). Similarly, assuming the same share of government spending in GDP across the social planner’s and decentralized equilibrium, we also know that, \( \chi \hat{g}^{1-\sigma} = \frac{\gamma}{\varphi} N^{1+\varphi} \). Using the fact that,

\[ \frac{c}{y} \hat{c}_t = \hat{y}_t - (1-\frac{c}{y}) \hat{g}_t - \frac{1}{2} \frac{c}{y} \hat{c}_t^2 - \frac{1}{2} (1-\frac{c}{y}) \hat{g}_t^2 + \frac{1}{2} \hat{g}_t^2 + O[2] \]

we can collect the levels terms and write the sum of discounted utilities as:

\[ \Gamma_0 = -\frac{1}{2} N^{1+\varphi} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\sigma(1-\theta)}{1-\theta \beta} \left( \hat{X}_t + \hat{\xi}_t \right)^2 + \frac{\sigma \varphi}{2} \left( \hat{g}_t + \hat{\xi}_t \right)^2 \right\} + \text{tip} + O[2] \]

Using the result from Eser, Leith, and Wren-Lewis (2009) that

\[ \sum_{t=0}^{\infty} \beta^t \text{var}_t \{p_t(i)\} = \frac{\alpha}{(1-\beta \alpha)(1-\alpha)} \sum_{t=0}^{\infty} \beta^t \left[ \xi_t^2 + \zeta \alpha^{-1} (\hat{\xi}_t - \hat{\xi}_{t-1})^2 \right] + O[2]. \]

we can write the discounted sum of utility as,

\[ \Gamma_0 = -\frac{1}{2} N^{1+\varphi} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\sigma(1-\theta)}{1-\theta \beta} \left( \hat{X}_t + \hat{\xi}_t \right)^2 + (\varphi) \left( \hat{y}_t - \frac{\varphi}{\varphi} \hat{\xi}_t \right)^2 \right\} + \text{tip} + O[2] \]
where we have put the terms in public consumption into tip since they are treated as an exogenous process and therefore independent of policy.

After normalising the coefficient on inflation to one, we can write the microfounded objective function as,

$$\Gamma_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \Phi_1 \left( \hat{X}_t + \hat{\xi}_t \right)^2 + \Phi_2 \left( \hat{y}_t - \frac{\bar{\pi}_t}{\bar{\pi}_t} \right)^2 + \zeta \alpha - 1 \left( 1 - \zeta \right) \left( \hat{\pi}_t - \hat{\pi}_{t-1} \right)^2 + \hat{\pi}_t^2 \right\}$$

where the weights on the two real terms are functions of model structural parameters, where

$$\Phi_1 = \frac{\sigma(1-\theta)(1-\beta_0)(1-\alpha)\xi}{\alpha y}$$
$$\Phi_2 = \frac{\varphi(1-\beta_0)(1-\alpha)}{\alpha y}$$

C Rules-Based Estimation

In this section we undertake an estimation of our model when describing policy using simple rules. This serves to create a set of benchmark results which we can contrast with our estimates which allow for strategic interactions between monetary and fiscal policy. In doing so it is important to note that while we extend the analysis of Bianchi (2012) and Bianchi and Ilut (2017) in some ways, this does not overturn their key results. Bianchi and Ilut argue that restricting the number and transition pattern of regimes is data-preferred largely as a result of the fact that the PM/AF and PM/PF regimes are very similar in terms of their dynamic responses to shocks. This is no longer the case when taxes are assumed to be distortionary where the inflationary impact of variation in taxes becomes a key ingredient in identifying policy regimes. Nevertheless, this results in a similar narrative in terms of the evolution of monetary and fiscal policy to the existing literature - fiscal policy turns active in the late 1960s and monetary policy turns active shortly afterwards, only regaining its activism following the Volcker disinflation in 1982. However, under our Rules-Based estimation the transition to a complementary passive fiscal regime was, unlike Bianchi and Ilut, not decisively achieved in 1982, and really only emerged a decade later in 1992.

When considering policy described by simple rules, we assume fiscal policy follows a simple tax rule,

$$\bar{\tau}_t = \rho_{\tau,s_t} \bar{\tau}_{t-1} + (1 - \rho_{\tau,s_t}) \left( \delta_{\tau,s_t} \hat{b}_{t-1}^M + \delta_y \hat{y}_t \right) + \sigma_{\tau} \varepsilon_{\tau,t}$$

where we assume the coefficient on debt, $\delta_{\tau,s_t}$, and the persistence of the tax rate, $\rho_{\tau,s_t}$ are subject to regime switching with $s_t = 1$ indicating the Passive Fiscal (PF) regime and $s_t = 2$ being the Active Fiscal (AF) regime. The fiscal policy regimes are determined by the value of coefficient on debt with $\delta_{\tau,s_t=1} > \frac{1}{\beta} - 1$ in the PF regime and $\delta_{\tau,s_t=2} = 0$ in the AF regime.

When U.S. monetary policy is described as a generalized Taylor rule, we specify this rule following An and Schorfheide (2007),

$$\hat{R}_t = \rho_{R,s_t} \hat{R}_{t-1} + (1 - \rho_{R,s_t}) \left[ \psi_{1,s_t} \hat{\pi}_t + \psi_{2,s_t} \left( \Delta \hat{y}_t + \hat{q}_t \right) \right] + \sigma_R \varepsilon_{R,t}$$

where the Fed adjusts interest rates in response to movements in inflation and deviations of output growth from trend. We allow the rule parameters $(\rho_{R,s_t}, \psi_{1,s_t}, \psi_{2,s_t})$ to switch between active and passive policy regimes. The Active Monetary (AM) policy regime corresponds to
$S_t = 1$, while the Passive Monetary (PM) policy regime corresponds to $S_t = 2$. The labeling implies that $\psi_{1,S_t=1} > 1$ and $0 < \psi_{1,S_t=2} < 1$.

By considering both fiscal and monetary policy changes, we can distinguish four policy regimes under Rules-Based policy. They are AM/PF, AM/AF, PM/PF and PM/AF. Leeper (1991) shows that, in the absence of regime switching, the existence of a unique solution to the model depends on the nature of the assumed policy regime. A unique solution can be found under both the AM/PF and PM/AF regimes, what Leeper and Leith (2017) refer to as the M and F-regimes, respectively. In the former monetary policy actively targets inflation and fiscal policy adjusts taxes to stabilize debt, while under the latter combination the fiscal authority does not adjust taxes to stabilize debt and the monetary authority does not actively target inflation in order to facilitate the stabilization of debt. In contrast, no stationary solution and multiple equilibria are obtained under the AM/AF and PM/PF regimes, respectively. However, when regime switching is considered, the existence and uniqueness of a solution also depends on the transition probabilities of the potential regime changes as economic agents anticipate the transition to different policy regimes. Specifically, we allow monetary and fiscal policy rule parameters to switch independently of each other. The transition matrices for monetary policy and fiscal policy are as follows

$$ P = \begin{bmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{bmatrix}, \quad Q = \begin{bmatrix} q_{11} & 1 - q_{22} \\ 1 - q_{11} & q_{22} \end{bmatrix}, $$

where $p_{ii} = \Pr[S_t = i|S_{t-1} = i]$ and $q_{ii} = \Pr[s_t = i|s_{t-1} = i]$. In addition, we also account for a possible shift in fundamental shock volatilities which has been used as a potential explanation of the Great Moderation. Failure to do so could potentially bias the identification of shifts in policy (see Sims and Zha (2006)). Therefore, we allow for independent regime switching in the standard deviations of technology ($\sigma_{q,k_t}$), preference ($\sigma_{\xi,k_t}$) and cost-push ($\sigma_{\mu,k_t}$) shocks, with $k_t = 1$ being in the low volatility regime and $k_t = 2$ in the high volatility regime. The transition matrix for the shock volatilities is as follows

$$ H = \begin{bmatrix} h_{11} & 1 - h_{22} \\ 1 - h_{11} & h_{22} \end{bmatrix}, $$

where $h_{ii} = \Pr[k_t = i|k_{t-1} = i]$.$^1$

We adopt the solution algorithm proposed by Farmer, Waggoner, and Zha (2011) to solve the model with Markov-switching in policy rule parameters. Since this algorithm implies that economic agents anticipate the Markov switching between different policy rules, there will be spillovers across policy regimes which will turn out to be crucial in determining the relative performance of alternative policies.

Table C.1 presents the priors and posterior estimates for the Rules-Based policy. For the interest rate rule parameters, we set symmetric priors for the parameter of the lagged interest rate and the parameter of output growth, whereas asymmetric and truncated priors are used for the parameter of inflation to ensure that $\psi_{1,S_t=1} > 1$ in the AM regime and $0 < \psi_{1,S_t=2} < 1$ in the PM regime. Similarly, for the tax rule, a symmetric prior is used for the parameter of lagged tax rate, while the parameter of debt is restricted to be zero in the AM/PF

$^1$The joint transition matrix governing the monetary-fiscal-shock regime is then $P = P \otimes Q \otimes H$. In total, there are eight regimes in the Rules-Based model.
regime and positive in the PF regime. Overall, the priors of the policy rule parameters imply four distinct fiscal and monetary policy regimes: AM/PF, AM/AF, PM/PF and PM/AF. In addition, variances of shocks are chosen to be highly dispersed inverted Gamma distributions to generate realistic volatilities for the endogenous variables.

Table C.1: Rules-Based Policy. Under the Rules-Based policy, we have four alternative policy permutations: AM/PF, AM/AF, PM/PF and PM/AF. For monetary policy switches, $S_t = 1$ is the AM regime and $S_t = 2$ is the PM regime. For fiscal policy switches, $s_t = 1$ is the PF regime and $s_t = 2$ is the AF regime. $\delta_y$ is assumed to be time-invariant across regimes.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mode</th>
<th>Mean</th>
<th>5%</th>
<th>95%</th>
<th>Type</th>
<th>Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>AM/AF</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{R,S_t=1}$, lagged interest rate</td>
<td>0.860</td>
<td>0.880</td>
<td>0.833</td>
<td>0.906</td>
<td>B</td>
<td>0.50</td>
<td>0.15</td>
</tr>
<tr>
<td>$\psi_{1,S_t=1}$, interest rate resp. to inflation</td>
<td>1.454</td>
<td>1.485</td>
<td>1.289</td>
<td>1.688</td>
<td>G</td>
<td>2.00</td>
<td>0.50</td>
</tr>
<tr>
<td>$\psi_{2,S_t=1}$, interest rate resp. to output</td>
<td>0.686</td>
<td>0.695</td>
<td>0.483</td>
<td>0.926</td>
<td>G</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>$\rho_{s,S_t=2}$, lagged tax rate</td>
<td>0.763</td>
<td>0.762</td>
<td>0.610</td>
<td>0.846</td>
<td>B</td>
<td>0.70</td>
<td>0.15</td>
</tr>
<tr>
<td>$\delta_{r,s=2}$, tax rate resp. to debt</td>
<td>0.000</td>
<td>0.078</td>
<td>0.000</td>
<td>0.177</td>
<td>G</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$\delta_y$, tax rate resp. to output</td>
<td>0.000</td>
<td>0.078</td>
<td>0.000</td>
<td>0.177</td>
<td>G</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>PM/PF</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{R,S_t=2}$, lagged interest rate</td>
<td>0.869</td>
<td>0.856</td>
<td>0.819</td>
<td>0.896</td>
<td>B</td>
<td>0.50</td>
<td>0.15</td>
</tr>
<tr>
<td>$\psi_{1,S_t=2}$, interest rate resp. to inflation</td>
<td>0.982</td>
<td>0.904</td>
<td>0.810</td>
<td>0.990</td>
<td>G</td>
<td>0.80</td>
<td>0.15</td>
</tr>
<tr>
<td>$\psi_{2,S_t=2}$, interest rate resp. to output</td>
<td>0.581</td>
<td>0.583</td>
<td>0.288</td>
<td>0.938</td>
<td>G</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>$\rho_{s,S_t=1}$, lagged tax rate</td>
<td>0.437</td>
<td>0.466</td>
<td>0.308</td>
<td>0.623</td>
<td>B</td>
<td>0.70</td>
<td>0.15</td>
</tr>
<tr>
<td>$\delta_{r,s=1}$, tax rate resp. to debt</td>
<td>0.000</td>
<td>0.078</td>
<td>0.000</td>
<td>0.177</td>
<td>G</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$\delta_y$, tax rate resp. to output</td>
<td>0.000</td>
<td>0.078</td>
<td>0.000</td>
<td>0.177</td>
<td>G</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>PM/AF</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{R,S_t=2}$, lagged interest rate</td>
<td>0.869</td>
<td>0.856</td>
<td>0.819</td>
<td>0.896</td>
<td>B</td>
<td>0.50</td>
<td>0.15</td>
</tr>
<tr>
<td>$\psi_{1,S_t=2}$, interest rate resp. to inflation</td>
<td>0.982</td>
<td>0.904</td>
<td>0.810</td>
<td>0.990</td>
<td>G</td>
<td>0.80</td>
<td>0.15</td>
</tr>
<tr>
<td>$\psi_{2,S_t=2}$, interest rate resp. to output</td>
<td>0.581</td>
<td>0.583</td>
<td>0.288</td>
<td>0.938</td>
<td>G</td>
<td>0.50</td>
<td>0.25</td>
</tr>
<tr>
<td>$\rho_{s,S_t=2}$, lagged tax rate</td>
<td>0.763</td>
<td>0.725</td>
<td>0.610</td>
<td>0.846</td>
<td>B</td>
<td>0.70</td>
<td>0.15</td>
</tr>
<tr>
<td>$\delta_{r,s=2}$, tax rate resp. to debt</td>
<td>0.000</td>
<td>0.078</td>
<td>0.000</td>
<td>0.177</td>
<td>G</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td>$\delta_y$, tax rate resp. to output</td>
<td>0.000</td>
<td>0.078</td>
<td>0.000</td>
<td>0.177</td>
<td>G</td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>
### Appendix C: Strategic Interactions U.S. Policies

#### Table C.1: Rules-Based Policy

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Posterior</th>
<th>Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mode</td>
<td>Mean</td>
</tr>
<tr>
<td><strong>Deep parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$, Inv. of intertemp. elas. of subst.</td>
<td>2.500</td>
<td>2.509</td>
</tr>
<tr>
<td>$\alpha$, Calvo parameter</td>
<td>0.798</td>
<td>0.800</td>
</tr>
<tr>
<td>$\zeta$, inflation inertia</td>
<td>0.387</td>
<td>0.339</td>
</tr>
<tr>
<td>$\theta$, habit persistence</td>
<td>0.464</td>
<td>0.524</td>
</tr>
<tr>
<td>$\varphi$, Inverse of Frisch elasticity</td>
<td>2.00 2.00</td>
<td>-</td>
</tr>
<tr>
<td><strong>Serial correlation of exogenous processes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_z$, AR coeff., taste shock</td>
<td>0.893</td>
<td>0.886</td>
</tr>
<tr>
<td>$\rho_\mu$, AR coeff., cost-push shock</td>
<td>0.153</td>
<td>0.209</td>
</tr>
<tr>
<td>$\rho_q$, AR coeff., productivity shock</td>
<td>0.427</td>
<td>0.406</td>
</tr>
<tr>
<td>$\rho_z$, AR coeff., transfers</td>
<td>0.977</td>
<td>0.976</td>
</tr>
<tr>
<td>$\rho_g$, AR coeff., government spending</td>
<td>0.981</td>
<td>0.980</td>
</tr>
<tr>
<td><strong>Standard deviations of exogenous processes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\xi,k_t=1}$, taste shock</td>
<td>0.555</td>
<td>0.532</td>
</tr>
<tr>
<td>$\sigma_{\xi,k_t=2}$, taste shock</td>
<td>1.235</td>
<td>1.215</td>
</tr>
<tr>
<td>$\sigma_{\mu,k_t=1}$, cost-push shock</td>
<td>4.845</td>
<td>4.051</td>
</tr>
<tr>
<td>$\sigma_{\mu,k_t=2}$, cost-push shock</td>
<td>12.734</td>
<td>11.772</td>
</tr>
<tr>
<td>$\sigma_{q,k_t=1}$, productivity shock</td>
<td>0.510</td>
<td>0.572</td>
</tr>
<tr>
<td>$\sigma_{q,k_t=2}$, productivity shock</td>
<td>1.111</td>
<td>1.275</td>
</tr>
<tr>
<td>$\sigma_{tp}$, term premium shock</td>
<td>3.258</td>
<td>3.293</td>
</tr>
<tr>
<td>$\sigma_g$, government spending shock</td>
<td>0.246</td>
<td>0.249</td>
</tr>
<tr>
<td>$\sigma_z$, transfers shock</td>
<td>0.300</td>
<td>0.303</td>
</tr>
<tr>
<td>$\sigma_r$, tax rate shock</td>
<td>0.359</td>
<td>0.361</td>
</tr>
<tr>
<td>$\sigma_{R}$, interest rate shock</td>
<td>0.205</td>
<td>0.211</td>
</tr>
</tbody>
</table>

**Transition probabilities**

<table>
<thead>
<tr>
<th></th>
<th>Mode</th>
<th>Mean</th>
<th>5%</th>
<th>95%</th>
<th>Type</th>
<th>Mean</th>
<th>Std Dev</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{11}$, monetary policy: remaining active</td>
<td>0.972</td>
<td>0.971</td>
<td>0.955</td>
<td>0.989</td>
<td>B</td>
<td>0.95</td>
<td>0.02</td>
</tr>
<tr>
<td>$p_{22}$, monetary policy: remaining passive</td>
<td>0.933</td>
<td>0.915</td>
<td>0.877</td>
<td>0.956</td>
<td>B</td>
<td>0.95</td>
<td>0.02</td>
</tr>
<tr>
<td>$q_{11}$, fiscal policy: remaining passive</td>
<td>0.955</td>
<td>0.952</td>
<td>0.929</td>
<td>0.978</td>
<td>B</td>
<td>0.95</td>
<td>0.02</td>
</tr>
<tr>
<td>$q_{22}$, fiscal policy: remaining active</td>
<td>0.935</td>
<td>0.918</td>
<td>0.882</td>
<td>0.954</td>
<td>B</td>
<td>0.95</td>
<td>0.02</td>
</tr>
<tr>
<td>$h_{11}$, volatility: remaining with low volatility</td>
<td>0.958</td>
<td>0.951</td>
<td>0.926</td>
<td>0.977</td>
<td>B</td>
<td>0.95</td>
<td>0.02</td>
</tr>
<tr>
<td>$h_{22}$, volatility: remaining with high volatility</td>
<td>0.910</td>
<td>0.905</td>
<td>0.875</td>
<td>0.935</td>
<td>B</td>
<td>0.95</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table C.1: Rules-Based Policy (continued). For volatility, $k_t = 1$ is the low volatility regime and $k_t = 2$ is the high volatility regime.

### C.1 Posterior Estimates: Rules-Based Policy

The posterior parameter estimates of the Rules-Based policy are reported in Table C.1. Our estimates of the structural parameters are broadly in line with other studies: an intertemporal elasticity of substitution, $\sigma = 2.5$; a measure of price stickiness, $\alpha = 0.8$, implying that price...
contracts typically last for just over one year; a degree of price indexation, $\zeta = 0.34$, and a significant estimate of the degree of habits, $\theta = 0.52$.

Under the Rules-Based policy, we have four alternative policy permutations: AM/PF, AM/AF, PM/PF and PM/AF. In order to allow for maximum flexibility in describing the policy regimes, we initially allowed for variations in rule parameters across the four policy regimes. Therefore, for example, the active monetary policy rule parameters in the AM/PF regime can differ from those in the AM/AF regime. Indeed, we find significant variations in the AM and PF regimes depending on which policy they are combined with. However, the PM and AF regimes appeared to be similar regardless of which policy they were paired with. Therefore, we restrict the PM and AF to be the same across their respective paired regimes. The resultant policy regimes imply that the passive monetary policy is inertial, $\rho_{R,S_t=2} = 0.86$, and only falling slightly short of the Taylor principle, $\psi_{1,S_t=2} = 0.9$, with a significant coefficient on output, $\psi_{2,S_t=2} = 0.58$. While an active monetary policy paired with a passive fiscal policy (AM/PF) is both inertial, $\rho_{R,S_t=1} = 0.88$, and very aggressive in targeting inflation, $\psi_{1,S_t=1} = 2.9$, with a relatively strong response to output, $\psi_{2,S_t=1} = 0.67$. When fiscal policy is active, then an associated active monetary policy is far less aggressive as interest rate inertia falls, $\rho_{R,S_t=1} = 0.61$, along with the response to inflation, $\psi_{1,S_t=1} = 1.48$, while the response to output increases, $\psi_{2,S_t=1} = 0.70$. Since the AM/AF regime is inherently unstable, it would appear that the conflict between the monetary and fiscal authority results in a moderation in the conservatism of monetary policy even while that policy remains active. Similarly, the passive fiscal policy is far more inertial, $\rho_{r,S_t=1} = 0.96$, and less responsive to debt, $\delta_{r,S_t=1} = 0.04$, when it is paired with an active monetary policy (AM/PF) than when the passive fiscal policy is paired with a passive monetary policy (PM/PF) where tax rate inertia falls, $\rho_{r,S_t=1} = 0.46$, and the response to debt rises, $\delta_{r,S_t=1} = 0.08$. These kinds of differences in estimation across regimes could reflect the nature of the interaction between monetary and fiscal policy. In the case of the AM/AF regime the policy is unstable and only rendered determinate because of spillovers from other policy permutations, so that the moderation in monetary policy would serve to mitigate the unstable debt dynamics caused by rising debt service costs under the active policy policy. Similarly, a passive fiscal policy which raises distortionary taxes to stabilize debt is likely to fuel inflation and lead to rising debt service costs when monetary policy is active. This is less of a danger when monetary policy is passive, so that fiscal policy can be relatively more aggressive in responding to debt in the latter case. These results suggest that the stance of one (or both) policy maker(s) is dependent on the policies of the other. This can be analyzed more formally by considering targeting rules where one policy maker takes into account the actions of the other.

C.2 Regime Switching Rules-Based Policy

Figure C.1 details the movements across fiscal and monetary policy regimes when the policy is described by Rules-Based policy. The first panel describes the probability of being in the passive fiscal policy regime, the second the active fiscal policy regime, and the third panel gives the probability of being in the passive monetary policy regime (with its complement being the active monetary regime). Taking these together, we observe that the conventional policy assignment (i.e. AM/PF) prevails right up until the late 1960s in contrast to the findings in Bianchi (2012) or Bianchi and Ilut (2017) who suggest that policy had already
deviated from the textbook assignment by then. Fiscal policy then turns active in 1969, and monetary policy turns passive shortly afterwards. There is a brief attempt at disinflation in 1973, but we essentially stay in the PM regime until Volcker. Afterwards monetary policy stays active, and there are brief flirtations with passive fiscal policy around 1975 and 1981-1982, although none stick until 1992. Therefore, the AM/PF regime did not re-emerge until 1992. This result is consistent with Bianchi (2012) but different with Bianchi and Ilut (2017). Finally, we find a brief relaxation of monetary policy in the aftermath of the bursting of the dot com bubble around 2001, while fiscal policy remains passive.

Our estimates suggest that regimes that are determinate because of the expectations of returning to either the AM/PF or PM/AF regime actually describe observed policy configurations for much of our sample period. The AM/PF and PM/AF regimes are estimated to be in place for 60% and 12% of the sample period, respectively, while the PM/PF regime appears to be the least frequently observed regime which is only present for 2% of the time. This is consistent with Bianchi and Ilut (2017) in that the PM/PF regime does not appear to be a significant regime. The remaining 26% of the sample period is described by the AM/AF regime, which is inherently unstable in the absence of expectations that we would return to either the AM/PF or PM/AF regimes.

In short, the Rules-Based estimation is consistent with a narrative where fiscal policy ceases to act to stabilize debt in the 1970s, with monetary policy turning passive shortly afterwards. Monetary policy then actively targets inflation following the appointment of Paul Volcker, but fiscal policy does not decisively turn passive in support of that policy until the early 1990s. That the Rules-Based estimation would identify this pattern of regime change can easily be seen in the broad trends in inflation, interest rates and debt contained in Figure 1 in the paper. The PM/AF regime of the 1970s is associated with high inflation, the AM/AF regime of the 1980s with the tightening of monetary policy, falling inflation and rising debt, the AM/PF regime of the 1990s with the ongoing stabilization in inflation and the debt to GDP ratio. We shall see in the main text that the estimation based on strategic policy allows for a more nuanced description of the evolution of policy regimes.
Figure C.1: Markov Switching Probabilities: Policy and Volatility Switches under Rules-Based Policy
### D Leadership Equilibria Under Discretion

This section demonstrates how to solve non-cooperative dynamic games in the Markov jump-linear quadratic systems. Consider an economy with two policy makers: a leader ($L$) and a follower ($F$).

\[
X_{t+1} = A_{11k_{t+1}}X_t + A_{12k_{t+1}}x_t + B_{11k_{t+1}}u_t^L + B_{12k_{t+1}}u_t^F + C_{k_{t+1}}\varepsilon_{t+1},
\]

(D.1)

\[
E_tH_{k_{t+1}}x_{t+1} = A_{21j_t}X_t + A_{22j_t}x_t + B_{21j_t}u_t^L + B_{22j_t}u_t^F.
\]

(D.2)

where $X_t$ is a $n_1 \times 1$ vector of predetermined variables; $x_t$ is a $n_2 \times 1$ vector of forward-looking variables; $u_t^L$ and $u_t^F$ are the control variables, and $\varepsilon_t$ contains a vector of zero mean i.i.d. shocks. Without loss of generality, the shocks are normalized so that the covariance matrix of $\varepsilon_t$ is an identity matrix, and the covariance matrix of the shocks to $X_{t+1}$ is $C'_{k_{t+1}}C_{k_{t+1}}$.

The matrices $A_{11k_{t+1}}$, $A_{12k_{t+1}}$, $H_{k_{t+1}}$, $B_{11k_{t+1}}$, $B_{12k_{t+1}}$, $A_{21j_t}$, $A_{22j_t}$, $B_{21j_t}$, and $B_{22j_t}$ can each take $n$ different values, corresponding to the $n$ modes $k_{t+1} = 1, 2, ..., n$ in period $t + 1$, and $j_t = 1, 2, ..., n$ in period $t$. The modes follow a Markov process with constant transition probabilities:

\[
P_{jk} = Pr\{k_{t+1} = k|j_t = j\}, \quad j, k = 1, 2, ..., n
\]

Let $P$ denote the $n \times n$ transition matrix $[P_{jk}]$ and the $1 \times n$ vector $p \equiv (p_1, ..., p_n)$ denote the probability distribution of the modes in period $t$,

\[
p_{t+1} = p_tP.
\]

Finally, the $1 \times n$ vector $\overline{p}$ denotes the unique stationary distribution of the modes,

\[
\overline{p} = \overline{p}P.
\]

We assume that the intertemporal loss functions of the two policy makers are defined by the quadratic loss function

\[
\mathbb{E}_t \sum_{\tau=0}^{\infty} \frac{1}{2} \beta^\tau L_{jt+\tau}^u,
\]

where $L_{jt}^u$ is the period loss with $u = F$ for the follower and $u = L$ for the leader, respectively. The period loss, $L_{jt}^u$, can take different value corresponding to the $n$ modes in period $t$. The period loss satisfies

\[
L_{jt}^u = Y_{jt}^uA_{jt}^uY_{jt}^u,
\]

where $A_{jt}^u$ is a symmetric and positive semi-definite weight matrix. $Y_{jt}^u$ are $n_y$ vectors of target variables for the follower and leader.

\[
Y_{jt}^u = D_{jt}\begin{bmatrix} X_t \\ x_t \\ u_t^L \\ u_t^F \\ u_t \end{bmatrix},
\]
It follows that the period loss function can be rewritten as

\[
L^u_{jt} = \begin{bmatrix} X_t \\ x_t \\ u^L_t \\ u^F_t \end{bmatrix} \begin{bmatrix} X_t \\ x_t \\ u^L_t \\ u^F_t \end{bmatrix}',
\]

(D.3)

where \(W^u_{jt} = D^u A^u_j D^u\) is symmetric and positive semidefinite, and

\[
W^u_{jt} = \begin{bmatrix} Q^u_{11jt} & Q^u_{12jt} & P^u_{11jt} & P^u_{12jt} \\ Q^u_{21jt} & Q^u_{22jt} & P^u_{21jt} & P^u_{22jt} \\ P^u_{11jt} & P^u_{21jt} & R^u_{11jt} & R^u_{12jt} \\ P^u_{12jt} & P^u_{22jt} & R^u_{12jt} & R^u_{22jt} \end{bmatrix}
\]

is partitioned with \(X_t, x_t, u^L_t\) and \(u^F_t\).

The follower and leader decide their policy \(u^F_t\) and \(u^L_t\) in period \(t\) to minimize their intertemporal loss functions defined in (D.3) under discretion subject to (D.1), (D.2), \(X_t\) and \(j_t\) given. The follower also observes the current decision \(u^L_t\) of the leader. Furthermore, two policy makers anticipate that they will reoptimize in period \(t+1\). Reoptimization will result in the two instruments and the forward-looking variables in period \(t+1\) being functions of the predetermined variables and the mode in period \(t+1\) according to

\[
u^L_{t+1} = -F^L_{k_t+1} X_{t+1},
\]

(D.4)

\[
u^F_{t+1} = -G^F_{k_t+1} X_{t+1} - D^F_{k_t+1} u^L_{t+1},
\]

(D.5)

\[
x_{t+1} = -N_{k_t+1} X_{t+1},
\]

(D.6)

where \(k_t+1 = 1, \ldots, n\) are the \(n\) modes at period \(t+1\). The dynamics of the predetermined variables will follow

\[
X_{t+1} = M_{j_t k_t+1} X_t + C_{k_t+1} \varepsilon_{t+1},
\]

where

\[
M_{j_t k_t+1} = A_{11k_t+1} - A_{12k_t+1} N_{j_t} - B_{11k_t+1} F^L_{j_t} - B_{12k_t+1} G^F_{j_t} + B_{12k_t+1} D^F_{j_t} F^L_{j_t},
\]

First, by (D.6) and (D.1) we have,

\[
E^t H_{k_t+1} x_{t+1} = -E^t H_{k_t+1} N_{k_t+1} X_{t+1} = -E^t H_{k_t+1} N_{k_t+1} \left( A_{11k_t+1} X_t + A_{12k_t+1} x_t + B_{11k_t+1} u^L_t + B_{12k_t+1} u^F_t \right)
\]

where \(E^t H_{k_t+1} N_{k_t+1} = \sum_{k=1}^n P_{j_t k_t+1} H_{k_t+1} N_{k_t+1}\), conditional on \(j_t = 1, 2, \ldots n\) at the period. Combining this with (D.2) gives

\[
-E^t H_{k_t+1} N_{k_t+1} \left( A_{11k_t+1} X_t + A_{12k_t+1} x_t + B_{11k_t+1} u^L_t + B_{12k_t+1} u^F_t \right) = A_{21j_t} X_t + A_{22j_t} x_t + B_{21j_t} u^L_t + B_{22j_t} u^F_t.
\]

Solving for \(x_t\) we obtain
\[ x_t = -J_{jt}X_t - K^L_{jt}u^L_t - K^F_{jt}u^F_t, \] (D.7)

where

\[
J_{jt} = \left( A_{22jt} + \sum_{k=1}^{n} P_{jk^{k+1}t} H_{k^{k+1}t} N_{k^{k+1}t} A_{12k^{k+1}t} \right)^{-1} \left( A_{21jt} + \sum_{k=1}^{n} P_{jk^{k+1}t} H_{k^{k+1}t} N_{k^{k+1}t} A_{11k^{k+1}t} \right),
\]

\[
K^L_{jt} = \left( A_{22jt} + \sum_{k=1}^{n} P_{jk^{k+1}t} H_{k^{k+1}t} N_{k^{k+1}t} A_{12k^{k+1}t} \right)^{-1} \left( B_{21jt} + \sum_{k=1}^{n} P_{jk^{k+1}t} H_{k^{k+1}t} N_{k^{k+1}t} B_{11k^{k+1}t} \right),
\]

\[
K^F_{jt} = \left( A_{22jt} + \sum_{k=1}^{n} P_{jk^{k+1}t} H_{k^{k+1}t} N_{k^{k+1}t} A_{12k^{k+1}t} \right)^{-1} \left( B_{22jt} + \sum_{k=1}^{n} P_{jk^{k+1}t} H_{k^{k+1}t} N_{k^{k+1}t} B_{12k^{k+1}t} \right).
\]

We assume that \( A_{22jt} + \sum_{k=1}^{n} P_{jk^{k+1}t} H_{k^{k+1}t} N_{k^{k+1}t} A_{12k^{k+1}t} \) is invertible.

Second, substituting \( x_t \) from (D.1) using (D.7) gives

\[ X_{t+1} = \tilde{A}_{jkt+1} X_t + \tilde{B}^L_{jkt+1} u^L_t + \tilde{B}^F_{jkt+1} u^F_t + C_{k^{t+1}t+1}, \] (D.8)

where

\[
\tilde{A}_{jkt+1} = A_{11k^{k+1}t} - A_{12k^{k+1}t} J_{jt},
\]

\[
\tilde{B}^L_{jkt+1} = B_{11k^{k+1}t} - A_{12k^{k+1}t} K^L_{jt},
\]

\[
\tilde{B}^F_{jkt+1} = B_{12k^{k+1}t} - A_{12k^{k+1}t} K^F_{jt}.
\]

**D.1 Policy of the Follower**

Using (D.7) in the follower’s loss function (D.3) gives

\[
L^F_{jt} = \begin{bmatrix} X_t \\ x_t \\ u^L_t \\ u^F_t \end{bmatrix}' \begin{bmatrix} Q^F_{11jt} & Q^F_{12jt} & P^F_{11jt} & P^F_{12jt} \\ Q^F_{21jt} & Q^F_{22jt} & P^F_{21jt} & P^F_{22jt} \\ P^F_{11jt} & P^F_{12jt} & R^F_{11jt} & R^F_{12jt} \\ P^F_{21jt} & P^F_{22jt} & R^F_{21jt} & R^F_{22jt} \end{bmatrix} \begin{bmatrix} X_t \\ x_t \\ u^L_t \\ u^F_t \end{bmatrix}
\] (D.9)

where
\[ \tilde{Q}_{jt}^F = Q_{11jt}^F - Q_{12jt}^F J_{jt} - J_{jt}' Q_{21jt}^F + J_{jt}' Q_{22jt}^F J_{jt}, \]
\[ \tilde{P}_{1jt}^F = P_{11jt}^F - Q_{12jt}^F K_{jt}^L + J_{jt}' Q_{22jt}^F K_{jt}^L - J_{jt}' P_{21jt}^F, \]
\[ \tilde{P}_{2jt}^F = P_{12jt}^F - Q_{12jt}^F K_{jt}^F + J_{jt}' Q_{22jt}^F K_{jt}^F - J_{jt}' P_{21jt}^F, \]
\[ \tilde{R}_{1jt}^F = K_{jt}' Q_{22jt}^F K_{jt}^L - K_{jt}' P_{22jt}^F - P_{21jt}^F K_{jt}^L + P_{11jt}^F, \]
\[ \tilde{R}_{12jt}^F = K_{jt}' Q_{22jt}^F K_{jt}^F - K_{jt}' P_{22jt}^F - P_{21jt}^F K_{jt}^F + R_{12jt}^F, \]
\[ \tilde{R}_{22jt}^F = K_{jt}' Q_{22jt}^F K_{jt}^F - K_{jt}' P_{22jt}^F - P_{21jt}^F K_{jt}^F + R_{22jt}^F. \]

The optimal value of the problem in period \( t \) is associated with the symmetric positive semidefinite matrix \( V_{kt+1}^F \) and it satisfies the Bellman equation:
\[ X_t V_{jt}^F X_t = \min_{u_{jt}^F} \left\{ L_{jt}^F + \beta E_t \left[ X_{t+1}^F V_{kt+1}^F X_{t+1} \right] \right\} \tag{D.10} \]
subject to (D.8) and (D.9). The first-order condition with respect to \( u_{jt}^F \) is
\[ 0 = X_t \tilde{P}_{2jt}^F + u_t^L \tilde{R}_{12jt}^F + u_t^F \tilde{R}_{22jt}^F + \beta E_t X_t \tilde{A}_t^t V_{kt+1}^F \tilde{A}_t^t + \beta E_t u_t^F \tilde{B}_t^F V_{kt+1}^F \tilde{B}_t^F. \]

This leads to the optimal policy function \( u_t^F \) of the follower
\[ u_t^F = -G_{kt+1}^F X_{t+1} - D_{kt+1}^F u_{t+1}^L, \tag{D.11} \]
where
\[ G_{kt+1}^F = \left( \tilde{R}_{22jt}^F + \beta \sum_{k=1}^n P_{jkkt+1}^F \tilde{B}_j^F \tilde{A}_j^F \right)^{-1} \left( \tilde{P}_{2jt}^F + \beta \sum_{k=1}^n P_{jkkt+1}^F \tilde{B}_j^F \tilde{A}_j^F \right), \]
\[ D_{kt+1}^F = \left( \tilde{R}_{22jt}^F + \beta \sum_{k=1}^n P_{jkkt+1}^F \tilde{B}_j^F \tilde{A}_j^F \right)^{-1} \left( \tilde{R}_{12jt}^F + \beta \sum_{k=1}^n P_{jkkt+1}^F \tilde{B}_j^F \tilde{A}_j^F \right). \]

Furthermore, using (D.4) and (D.11) in (D.7) gives
\[ x_t = -N_{jt} X_t, \tag{D.12} \]
where
\[ N_{jt} = J_{jt} - K_{jt}^L F_{jt} - K_{jt}^F G_{jt}^F + K_{jt}^F D_{jt}^F F_{jt}, \]
and using (D.4) and (D.11) and (D.12) in (D.1) gives
\[ X_{t+1} = M_{jtkt+1}^t X_t + C_{kt+1} \varepsilon_{t+1}, \]
where
\[ M_{jtkt+1}^t = A_{11kt+1} - A_{12kt+1} N_{jt} - B_{11kt+1} F_{jt} - B_{12kt+1} G_{jt}^F + B_{12kt+1} D_{jt}^F F_{jt}. \]
Finally, using (D.4), (D.8), (D.9) and (D.11) in (D.10) results in

\[
V_{jt}^F \equiv \tilde{Q}_{jt}^F - \tilde{P}_{1jt}^F F_{jt}^L - F_{jt}^L \tilde{P}_{1jt}^F + F_{jt}^L \tilde{P}_{1jt}^F - \tilde{P}_{2jt}^F F_{jt}^L - \tilde{P}_{1jt}^F F_{jt}^L, \\
+ \beta \sum_{k=1}^{n} P_{jkt+1} \left( \tilde{A}_{jkt+1} - \tilde{B}_{jkt+1}^L F_{jt}^L \right)' \left( \begin{array}{cc} \tilde{A}_{jkt+1} - \tilde{B}_{jkt+1}^L F_{jt}^L \\ \tilde{B}_{jkt+1}^L F_{jt}^L \end{array} \right) V_{kt+1}^F \\
- \left[ \begin{array}{c} \tilde{P}_{jt}^F - \tilde{R}_{1jt}^F \\ \tilde{P}_{2jt}^F - \tilde{R}_{1jt}^F \\
+ \beta \sum_{k=1}^{n} P_{jkt+1} \left( \tilde{A}_{jkt+1} + \tilde{B}_{jkt+1}^L F_{jt}^L \right)' \left( \begin{array}{cc} \tilde{A}_{jkt+1} + \tilde{B}_{jkt+1}^L F_{jt}^L \\ \tilde{B}_{jkt+1}^L F_{jt}^L \end{array} \right) \end{array} \right]^{-1} \\
\left[ \begin{array}{c} \tilde{P}_{jt}^F - \tilde{R}_{1jt}^F \\ \tilde{P}_{2jt}^F - \tilde{R}_{1jt}^F \\
+ \beta \sum_{k=1}^{n} P_{jkt+1} \tilde{B}_{jkt+1}^L V_{kt+1}^F \left( \tilde{A}_{jkt+1} + \tilde{B}_{jkt+1}^L F_{jt}^L \right) \end{array} \right],
\]

\section*{D.2 Policy of the Leader}

Using (D.7) and (D.11) in the leader’s loss function (D.3) gives

\[
L_{jt}^L = \begin{bmatrix} X_t \\ x_t^L \\ u_t^L \end{bmatrix}' \begin{bmatrix} Q_{11jt}^L & Q_{12jt}^L & P_{11jt}^L & P_{12jt}^L \\ Q_{21jt}^L & Q_{22jt}^L & P_{21jt}^L & P_{22jt}^L \\ P_{11jt}^L & P_{12jt}^L & R_{11jt}^L & R_{12jt}^L \\ P_{21jt}^L & P_{22jt}^L & R_{21jt}^L & R_{22jt}^L \end{bmatrix} \begin{bmatrix} X_t \\ x_t^L \\ u_t^L \end{bmatrix},
\tag{D.13}
\]

where

\[
\tilde{Q}_{jt}^L = Q_{11jt}^L - P_{12jt}^L C_{jt}^F - G_{jt}^F P_{12jt}^L + G_{jt}^F P_{22jt}^L C_{jt}^F - Q_{12jt}^L \tilde{J}_{jt}, \\
\tilde{P}_{jt}^L = P_{11jt}^L - Q_{12jt}^L \tilde{K}_{jt} - P_{12jt}^L D_{jt}^F + \tilde{J}_{jt}^F Q_{22jt}^L \tilde{K}_{jt} - \tilde{J}_{jt}^F P_{21jt}^L, \\
\tilde{R}_{jt}^L = R_{11jt}^L + \tilde{K}_{jt}^F Q_{22jt}^L \tilde{K}_{jt} - R_{12jt}^L D_{jt}^F - D_{jt}^F R_{12jt}^L + D_{jt}^F P_{22jt}^L D_{jt}^F, \\
\tilde{R}_{jt} = \begin{bmatrix} \tilde{J}_{jt}^F P_{21jt}^L + \tilde{K}_{jt}^F P_{22jt}^L D_{jt}^F - P_{21jt}^L \tilde{K}_{jt} + D_{jt}^F P_{22jt}^L \tilde{K}_{jt} \end{bmatrix},
\]

and \( \tilde{J}_{jt} = (J_{jt} - K_{jt}^F G_{jt}^F) \) and \( \tilde{K}_{jt} = (K_{jt}^L - K_{jt}^F D_{jt}^F) \).

The value of the problem in period \( t \) is associated with the symmetric positive semidefinite matrix \( V_{kt+1}^L \) and it satisfies the Bellman equation

\[
X_t V_{jt}^L X_t = \min_{u_t^L} \left\{ L_{jt}^L + \beta E_t \left[ X_{t+1}^L V_{kt+1}^L X_{t+1} \right] \right\},
\tag{D.14}
\]

17
subject to (D.8), (D.11) and (D.13). The first-order condition with respect to $u_t^L$ is

$$0 = X_t^L \tilde{P}^L_{jt} + u_t^L \tilde{R}^L_{jt} + \beta E_t X_t' \left( \tilde{A}_{jt,k+1} - \tilde{B}^F_{jt,k+1} \tilde{G}^F_{jt} \right)' V_{k+1}^L \left( \tilde{B}^L_{jt,k+1} - \tilde{B}^F_{jt,k+1} D^F_{jt} \right)$$

$$+ \beta E_t u_t^L \left( \tilde{B}^L_{jt,k+1} - \tilde{B}^F_{jt,k+1} D^F_{jt} \right)' V_{k+1}^L \left( \tilde{B}^L_{jt,k+1} - \tilde{B}^F_{jt,k+1} D^F_{jt} \right).$$

This leads to the optimal policy function of the leader

$$u_t^L = -F_{jt}^L X_t,$$  \hspace{1cm} (D.15)

where

$$F_{jt}^L = \left[ \tilde{P}^L_{jt} + \beta \sum_{k=1}^{n} P_{jt,k+1} \left( \tilde{P}^L_{jt,k+1} - \tilde{B}^F_{jt,k+1} D^F_{jt} \right)' V_{k+1}^L \left( \tilde{B}^L_{jt,k+1} - \tilde{B}^F_{jt,k+1} D^F_{jt} \right) \right]^{-1}.$$

Furthermore, using (D.11) and (D.15) in (D.7) gives

$$x_t = -N_{jt} X_t,$$  \hspace{1cm} (D.16)

where

$$N_{jt} = J_{jt} - K_{jt} F_{jt} - K_{jt}^F G_{jt} + K_{jt}^F D_{jt} F_{jt}^L,$$

and using (D.11), (D.15) and (D.16) in (D.1) gives

$$X_{t+1} = M_{jt,k+1} X_t + C_{k+1} \epsilon_{t+1},$$

where

$$M_{jt,k+1} = A_{11k+1} - A_{12k+1} N_{jt} - B_{11k+1} F_{jt}^L - B_{12k+1} G_{jt} + B_{12k+1} D_{jt}^F F_{jt}^L.$$

Finally, using (D.8), (D.11), (D.13) and (D.15) in (D.14) results in

$$V_{jt}^L = \tilde{Q}_{jt}^L + \beta \sum_{k=1}^{n} P_{jt,k+1} \left( \tilde{A}_{jt,k+1} - \tilde{B}^F_{jt,k+1} G^F_{jt} \right)' V_{k+1}^L \left( \tilde{A}_{jt,k+1} - \tilde{B}^F_{jt,k+1} G^F_{jt} \right)$$

$$- \left[ \tilde{P}_{jt}^L + \beta \sum_{k=1}^{n} P_{jt,k+1} \left( \tilde{A}_{jt,k+1} - \tilde{B}^F_{jt,k+1} G^F_{jt} \right)' V_{k+1}^L \left( \tilde{B}^L_{jt,k+1} - \tilde{B}^F_{jt,k+1} D^F_{jt} \right) \right]^{-1}.$$

$$\tilde{R}_{jt}^L + \beta \sum_{k=1}^{n} P_{jt,k+1} \left( \tilde{B}^L_{jt,k+1} - \tilde{B}^F_{jt,k+1} D^F_{jt} \right)' V_{k+1}^L \left( \tilde{B}^L_{jt,k+1} - \tilde{B}^F_{jt,k+1} D^F_{jt} \right) \right]^{-1}.$$
To sum up, the first order conditions to the optimization problem (D.1), (D.2) and (D.3) can be written in the following form:

\[ N_{jt} = J_{jt} - K_{jt} L_{jt} - K_{jt} F_{jt} + K_{jt} D_{jt} F_{jt} \]

\[ V_{jt}^F = \tilde{Q}_{jt}^F - \tilde{F}_{1jt}^F L_{jt} - F_{jt}^L L_{jt} + F_{jt}^L R_{1jt}^F \]

\[ + \beta \sum_{k=1}^{n} P_{jk_{k+1}} \left( \tilde{A}_{jk_{k+1}} - \tilde{B}_{jk_{k+1}} L_{jt} \right) V_{k_{k+1}}^F \left( \tilde{A}_{jk_{k+1}} - \tilde{B}_{jk_{k+1}} L_{jt} \right) \]

\[ - \left[ \tilde{P}_{2jt}^F - \tilde{P}_{12jt}^F \beta \sum_{k=1}^{n} P_{jk_{k+1}} \left( \tilde{A}_{jk_{k+1}} + \tilde{B}_{jk_{k+1}} L_{jt} \right) V_{k_{k+1}}^F \tilde{P}_{jk_{k+1}}^F \right]^{-1} \]

\[ \left[ \tilde{P}_{2jt}^F - \tilde{P}_{12jt}^F + \beta \sum_{k=1}^{n} P_{jk_{k+1}} \tilde{B}_{jk_{k+1}} V_{k_{k+1}}^F \left( \tilde{A}_{jk_{k+1}} + \tilde{B}_{jk_{k+1}} L_{jt} \right) \right] \]

\[ V_{jt}^L = \tilde{Q}_{jt}^L + \beta \sum_{k=1}^{n} P_{jk_{k+1}} \left( \tilde{A}_{jk_{k+1}} - \tilde{B}_{jk_{k+1}} G_{jt}^F \right) V_{k_{k+1}}^L \left( \tilde{A}_{jk_{k+1}} - \tilde{B}_{jk_{k+1}} G_{jt}^F \right) \]

\[ - \left[ \tilde{P}_{jt}^L + \beta \sum_{k=1}^{n} P_{jk_{k+1}} \left( \tilde{A}_{jk_{k+1}} - \tilde{B}_{jk_{k+1}} D_{jt}^F \right) V_{k_{k+1}}^L \left( \tilde{B}_{jk_{k+1}} - \tilde{B}_{jk_{k+1}} G_{jt}^F \right) \right]^{-1} \]

\[ \left[ \tilde{R}_{jt}^L + \beta \sum_{k=1}^{n} P_{jk_{k+1}} \left( \tilde{B}_{jk_{k+1}} - \tilde{B}_{jk_{k+1}} G_{jt}^F \right) V_{k_{k+1}}^L \left( \tilde{A}_{jk_{k+1}} - \tilde{B}_{jk_{k+1}} G_{jt}^F \right) \right] \]

\[ F_{jt}^L = \left[ \tilde{R}_{jt}^L + \beta \sum_{k=1}^{n} P_{jk_{k+1}} \left( \tilde{B}_{jk_{k+1}} - \tilde{B}_{jk_{k+1}} D_{jt}^F \right) V_{k_{k+1}}^L \left( \tilde{B}_{jk_{k+1}} - \tilde{B}_{jk_{k+1}} D_{jt}^F \right) \right]^{-1} \]

\[ \left[ \tilde{P}_{jt}^L + \beta \sum_{k=1}^{n} P_{jk_{k+1}} \left( \tilde{B}_{jk_{k+1}} - \tilde{B}_{jk_{k+1}} D_{jt}^F \right) V_{k_{k+1}}^L \left( \tilde{A}_{jk_{k+1}} - \tilde{B}_{jk_{k+1}} G_{jt}^F \right) \right] \]

\[ G_{k_{k+1}}^F = \left( \tilde{R}_{2jt}^F + \beta \sum_{k=1}^{n} P_{jk_{k+1}} \tilde{B}_{jk_{k+1}} V_{k_{k+1}}^F \tilde{B}_{jk_{k+1}} \right) \left( \tilde{P}_{2jt}^F + \beta \sum_{k=1}^{n} P_{jk_{k+1}} \tilde{B}_{jk_{k+1}} V_{k_{k+1}}^F \tilde{A}_{jk_{k+1}} \right) \]

\[ D_{k_{k+1}}^F = \left( \tilde{R}_{1jt}^F + \beta \sum_{k=1}^{n} P_{jk_{k+1}} \tilde{B}_{jk_{k+1}} V_{k_{k+1}}^F \tilde{B}_{jk_{k+1}} \right) \left( \tilde{P}_{1jt}^F + \beta \sum_{k=1}^{n} P_{jk_{k+1}} \tilde{B}_{jk_{k+1}} V_{k_{k+1}}^F \tilde{B}_{jk_{k+1}} \right) \]
The discretion equilibrium is a fixed point \((N, V^L, V^F) \equiv \{N_{jt}, V^L_{jt}, V^F_{jt}\}_{jt=1}^n\) of the mapping and a corresponding \(\{F^L_j, C^F_j, D^F_j\}_{jt=1}^n\). The fixed point can be obtained as the limit of \((N_t, V^L_t, V^F_t)\) when \(t \to -\infty\).

**E Nash Equilibrium under Discretion**

Consider an economy with two policy makers, A and B, who decide their policy simultaneously.

\[
X_{t+1} = A_{11k_{t+1}}X_t + A_{12k_{t+1}}x_t + B_{11k_{t+1}}u^A_t + B_{12k_{t+1}}u^B_t + C_{k_{t+1}}\varepsilon_{t+1}, \quad (E.1)
\]

\[
E_tH_{k_{t+1}}X_{t+1} = A_{21j_t}X_t + A_{22j_t}x_t + B_{21j_t}u^A_t + B_{22j_t}u^B_t, \quad (E.2)
\]

where \(X_t\) is a \(n_1\) vector of predetermined variables; \(x_t\) is a \(n_2\) vector of forward-looking variables; \(u^A_t\) and \(u^B_t\) are the two policy makers’ instruments, and \(\varepsilon_t\) contains a vector of zero mean i.i.d. shocks. Without loss of generality, the shocks are normalized so that the covariance matrix of \(\varepsilon_t\) is an identity matrix, and the covariance matrix of the shocks to \(X_{t+1}\) is \(C_j^t\).

The period loss function of policy makers, A and B, is defined as in equation (D.3) with \(u = A\) and \(u = B\), respectively. Policy makers A and B simultaneously decide their policy \(u^A_t\) and \(u^B_t\) in period \(t\) to minimize their intertemporal loss functions defined in (D.3) under discretion subject to (E.1), (D.2), \(X_t\) and \(j_t\) given. Reoptimization in period \(t + 1\) result in the two instruments and the forward-looking variables being functions of the predetermined variables and the mode as follows

\[
u^A_{t+1} = -F_{k_{t+1}}^A X_{t+1}, \quad (E.3)
\]

\[
u^B_{t+1} = -F_{k_{t+1}}^B X_{t+1}, \quad (E.4)
\]

\[
x_{t+1} = -N_{k_{t+1}}X_{t+1}. \quad (E.5)
\]

Combining equations (E.1), (E.2) and (E.5), we solve for \(x_t\)

\[
x_t = -J_tX_t - K^A_{jt}u^A_t - K^B_{jt}u^B_t, \quad (E.6)
\]

where

\[
J_{jt} = \left( A_{22j_t} + \sum_{k=1}^n P_{jt{k_{t+1}}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}} \right)^{-1} \left( A_{21j_t} + \sum_{k=1}^n P_{jt{k_{t+1}}} H_{k_{t+1}} N_{k_{t+1}} A_{11k_{t+1}} \right),
\]

\[
K^A_{jt} = \left( A_{22j_t} + \sum_{k=1}^n P_{jt{k_{t+1}}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}} \right)^{-1} \left( B_{21j_t} + \sum_{k=1}^n P_{jt{k_{t+1}}} H_{k_{t+1}} N_{k_{t+1}} B_{11k_{t+1}} \right),
\]

\[
K^B_{jt} = \left( A_{22j_t} + \sum_{k=1}^n P_{jt{k_{t+1}}} H_{k_{t+1}} N_{k_{t+1}} A_{12k_{t+1}} \right)^{-1} \left( B_{22j_t} + \sum_{k=1}^n P_{jt{k_{t+1}}} H_{k_{t+1}} N_{k_{t+1}} B_{12k_{t+1}} \right).
\]
By substituting $x_t$ from (E.1) using (E.6) gives

$$X_{t+1} = \tilde{A}_{jk_{t+1}} X_t + \tilde{B}^A_{jk_{t+1}} u_t^A + \tilde{B}^B_{jk_{t+1}} u_t^B + C_{k_{t+1}} \varepsilon_{t+1}; \quad (E.7)$$

where

$$\tilde{A}_{jk_{t+1}} = A_{11k_{t+1}} - A_{12k_{t+1}} J_{jt},$$

$$\tilde{B}^A_{jk_{t+1}} = B_{11k_{t+1}} - A_{12k_{t+1}} K^A_{jt},$$

$$\tilde{B}^B_{jk_{t+1}} = B_{12k_{t+1}} - A_{12k_{t+1}} K^B_{jt}. \quad (E.8)$$

### E.1 Policy Maker A

Substitute (E.4) and (E.6) in the policy maker A’s period loss function gives

$$L^A_{jt} = \begin{bmatrix} X_t^A & x_t \end{bmatrix}' \begin{bmatrix} Q^{A}_{11j} & Q^{A}_{12j} & P^{A}_{11j} & P^{A}_{12j} \\ Q^{A}_{21j} & Q^{A}_{22j} & P^{A}_{21j} & P^{A}_{22j} \\ P^{A}_{11j} & P^{A}_{21j} & R^{A}_{21j} & R^{A}_{22j} \\ P^{A}_{12j} & P^{A}_{22j} & R^{A}_{21j} & R^{A}_{22j} \end{bmatrix} \begin{bmatrix} X_t^A \\ x_t \end{bmatrix} + \beta E_t \begin{bmatrix} X_{t+1}^A (V^A_{k_{t+1}} X_{t+1}) \end{bmatrix} \quad (E.9)$$

where

$$\tilde{Q}^A_{jt} = Q^{A}_{11j} - Q^{A}_{12j} \tilde{J}^B_{jt} - \tilde{J}^B_{jt} Q^{A}_{21j} + \tilde{J}^B_{jt} Q^{A}_{22j} \tilde{J}^B_{jt} + F^{B}_{jt} R^{A}_{22j} F^{B}_{jt} + F^{B}_{jt} P^{A}_{22j} \tilde{J}^B_{jt} + \tilde{J}^B_{jt} P^{A}_{22j} F^{B}_{jt} - P^{A}_{22j} F^{B}_{jt} F^{B}_{jt},$$

$$\tilde{P}^A_{jt} = -Q^{A}_{12j} K^A_{jt} - \tilde{J}^B_{jt} Q^{A}_{22j} K^A_{jt} + P^{A}_{11j} - \tilde{J}^B_{jt} P^{A}_{21j} + F^{B}_{jt} P^{A}_{22j} K^A_{jt} - F^{B}_{jt} P^{A}_{12j},$$

$$\tilde{R}^A_{jt} = K^A_{jt} Q^{A}_{22j} K^A_{jt} - K^A_{jt} P^{A}_{21j} - P^{A}_{21j} K^A_{jt} + R^{A}_{11j},$$

and $\tilde{J}^B_{jt} = J_{jt} - K^B_{jt} F^B_{jt}.$

The optimal value of the problem in period $t$ is associated with the symmetric positive definite matrix $V^A_{k_{t+1}}$ and it satisfies the Bellman equation:

$$X_t V^A_{jt} X_t = \min_{u^A_{jt}} \{ L^A_{jt} + \beta E_t [X'_{t+1} V^A_{k_{t+1}} X_{t+1}] \} \quad (E.9)$$

subject to (E.4), (E.6) and (E.8). The first-order condition with respect to $u^A_{jt}$ is

$$0 = X_t' \tilde{P}^A_{jt} + u^A_{jt} \tilde{R}^A_{jt} + \beta E_t X_t' \begin{bmatrix} \tilde{A}_{jk_{t+1}} - \tilde{B}^B_{jk_{t+1}} F^B_{jt} \end{bmatrix}' V^A_{k_{t+1}} \tilde{B}^B_{jk_{t+1}} + \beta E_t u^A_{jt} \tilde{B}^A_{jk_{t+1}} V^A_{k_{t+1}} \tilde{B}^A_{jk_{t+1}}.$$
where

\[
F_j^A = \left( \hat{R}_j^A + \beta \sum_{k=1}^n P_{j,k_t+1} \hat{B}_{j,k_t+1}^A V_{k_t+1}^A \hat{B}_{j,k_t+1}^A \right)^{-1} \left[ \hat{P}_j^A + \beta \sum_{k=1}^n P_{j,k_t+1} \hat{B}_{j,k_t+1}^A V_{k_t+1}^A \left( \hat{A}_{j,k_t+1} - \hat{B}_{j,k_t+1}^B F_{jt} \right) \right]
\]

Furthermore, using (E.4) and (E.10) in (E.6) gives

\[
x_t = -N_{j_t} X_t,
\]

where

\[
N_{j_t} = J_t - K_{jt}^A - K_{jt}^B F_{jt}^B
\]

and using (E.4), and (E.10) and (E.11) in (20) gives

\[
X_{t+1} = M_{j_{t+1} t} X_t + C_{k_{t+1} t+1},
\]

where

\[
M_{j_{t+1} t} = A_{11k_{t+1}} - A_{12k_{t+1}} N_{j_t} - B_{11k_{t+1}} F_{jt}^A - B_{12k_{t+1}} F_{jt}^B
\]

Finally, using (E.4), (E.7), (E.8) and (E.10) in (E.9) results in

\[
V_j^A = \tilde{Q}_j^A + \beta \sum_{k=1}^n P_{j,k_t+1} \left( \tilde{A}_{j,k_t+1} - \tilde{B}_{j,k_t+1}^B F_{jt} \right)' V_{k_t+1}^A \left( \tilde{A}_{j,k_t+1} - \tilde{B}_{j,k_t+1}^B F_{jt} \right)
- \left( \tilde{P}_j^A + \beta \sum_{k=1}^n P_{j,k_t+1} \left( \tilde{A}_{j,k_t+1} - \tilde{B}_{j,k_t+1}^B F_{jt} \right)' V_{k_t+1}^A \tilde{B}_{j,k_t+1}^A \right)
\left( \tilde{R}_j^A + \beta \sum_{k=1}^n P_{j,k_t+1} \tilde{B}_{j,k_t+1}^A V_{k_t+1}^A \tilde{B}_{j,k_t+1}^A \right)^{-1}
\left[ \tilde{P}_j^A + \beta \sum_{k=1}^n P_{j,k_t+1} \tilde{B}_{j,k_t+1}^A V_{k_t+1}^A \left( \tilde{A}_{j,k_t+1} - \tilde{B}_{j,k_t+1}^B F_{jt} \right) \right]
\]

E.2 Policy Maker B

Using (E.10) and (E.6) in policy maker B’s period loss function gives

\[
L_j^B = \left[ \begin{array}{c} X_t \\ x_t \\ u_t^A \\ u_t^B \end{array} \right]' \left[ \begin{array}{cccc} Q_{11j_t}^B & Q_{12j_t}^B & P_{11j_t}^B & P_{12j_t}^B \\ Q_{21j_t}^B & Q_{22j_t}^B & P_{21j_t}^B & P_{22j_t}^B \\ P_{11j_t}^B & P_{12j_t}^B & P_{11j_t}^B & P_{12j_t}^B \\ P_{21j_t}^B & P_{22j_t}^B & P_{21j_t}^B & P_{22j_t}^B \end{array} \right] \left[ \begin{array}{c} X_t \\ x_t \\ u_t^A \\ u_t^B \end{array} \right]
= \left[ \begin{array}{c} X_t \\ x_t \\ u_t^A \\ u_t^B \end{array} \right]' \left[ \tilde{Q}_j^B \tilde{P}_j^B \tilde{R}_j^B \tilde{P}_j^B \right] \left[ \begin{array}{c} X_t \\ x_t \\ u_t^A \\ u_t^B \end{array} \right]
\]
where
\[
\begin{align*}
\tilde{Q}^B_{jt} &= Q^B_{11jt} - Q^B_{12jt} \tilde{J}^A_{jt} - \tilde{J}^A_{jt} Q^B_{21jt} + \tilde{J}^A_{jt} Q^B_{22jt} \tilde{J}^A_{jt} + F^A_{jt} R^B_{11jt} F^A_{jt} + F^B_{jt} P^B_{11jt} F^A_{jt} - F^A_{jt} P^B_{11jt}, \\
\tilde{R}^B_{jt} &= -Q^B_{12jt} K^B_{jt} + \tilde{J}^A_{jt} Q^B_{22jt} K^B_{jt} - F^A_{jt} R^B_{12jt} + P^B_{12jt} - \tilde{J}^A_{jt} P^B_{22jt} + F^A_{jt} P^B_{21jt} K^B_{jt}, \\
\tilde{F}^B_{jt} &= K^B_{jt} Q^B_{22jt} K^B_{jt} - K^B_{jt} P^B_{22jt} - P^B_{22jt} K^B_{jt} + R^B_{22jt},
\end{align*}
\]
and \( \tilde{J}^A_{jt} = (J^A_{jt} - K^A_{jt} F^A_{jt}) \).

The optimal value of the problem in period \( t \) is associated with the symmetric positive semidefinite matrix \( V^B_{kt+1} \) and it satisfies the Bellman equation:
\[
X_t V^B_{jt} X_t = \min_{u^B_{jt}} \left\{ L^B_{jt} + \beta E_t \left[ X^B_{t+1} V^B_{kt+1} X_{t+1} \right] \right\} \tag{E.13}
\]
subject to (E.10), (E.6) and (E.12). The first-order condition with respect to \( u^B_{jt} \) is
\[
0 = X_t \tilde{P}^B_{jt} + u^B_{jt} \tilde{R}^B_{jt} + \beta E_t X_t' \left( \tilde{A}_{jtkt+1} - \tilde{B}^A_{jtkt+1} F^A_{jt} \right)' V^B_{kt+1} \tilde{B}^B_{jtkt+1} + \beta E_t u^B_{jt} \tilde{B}^B_{jtkt+1} V^B_{kt+1} \tilde{B}^B_{jtkt+1} + \beta E_t u^B_{jt} \tilde{P}^B_{jtkt+1} V^B_{kt+1} \tilde{B}^B_{jtkt+1}.
\]

The optimal policy function of the follower is given by
\[
u^B_{jt} = -F^B_{jt} X_t, \tag{E.14}\]
where
\[
\begin{align*}
F^B_{jt} &= \left( \tilde{R}^B_{jt} + \beta \sum_{k=1}^n P_{jkkt+1} \tilde{B}^B_{jkkt+1} V^B_{kt+1} \tilde{B}^B_{jkkt+1} \right)^{-1} \\
&\quad \left[ \tilde{P}^B_{jt} + \beta \sum_{k=1}^n P_{jkkt+1} \tilde{B}^B_{jkkt+1} V^B_{kt+1} \left( \tilde{A}_{jkkt+1} - \tilde{B}^A_{jkkt+1} F^A_{jt} \right) \right].
\end{align*}
\]
Furthermore, using (E.10) and (E.14) in (E.6) gives
\[
x_t = -N_{jt} X_t, \tag{E.15}\]
where
\[
N_{jt} = J_t - K^A_{jt} F^A_{jt} - K^B_{jt} F^B_{jt},
\]
and using (E.10) and (E.14) and (E.15) in (20) gives
\[
X_{t+1} = M_{jkt_{t+1}} X_t + C_{kt_{t+1}} e_{t+1},
\]
where
\[
M_{jkt_{t+1}} = A_{11kt_{t+1}} - A_{12kt_{t+1}} N_{jt} - B_{11kt_{t+1}} F^A_{kt_{t+1}} - B_{12kt_{t+1}} F^B_{jt}.
\]
Finally, using (E.7), (E.10), (E.12) and (E.14) in (E.13) results in
\[ V^B_{jt} = \tilde{Q}^B_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \left( \tilde{A}_{jtk_{t+1}} - \tilde{B}^A_{jtk_{t+1}} F^A_{k_{t+1}} \right)' V^B_{k_{t+1}} \left( \tilde{A}_{jtk_{t+1}} - \tilde{B}^A_{jtk_{t+1}} F^A_{k_{t+1}} \right) \]

\[ - \left[ \tilde{P}^B_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \left( \tilde{A}_{jtk_{t+1}} - \tilde{B}^A_{jtk_{t+1}} F^A_{jt} \right)' V^B_{k_{t+1}} \tilde{B}^B_{jtk_{t+1}} \right] \]

\[ \left( \tilde{R}^B_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \tilde{B}^B_{jtk_{t+1}} V^B_{k_{t+1}} \tilde{B}^B_{jtk_{t+1}} \right)^{-1} \]

\[ \left[ \tilde{P}^B_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \tilde{B}^B_{jtk_{t+1}} V^B_{k_{t+1}} \left( \tilde{A}_{jtk_{t+1}} - \tilde{B}^A_{jtk_{t+1}} F^A_{jt} \right) \right] \]

To sum up, the first order conditions to the optimization problem can be written in the following form:

\[ N_{jt} = J_{jt} - K^A_{jt} F^A_{jt} - K^B_{jt} F^B_{jt} , \]

\[ V^A_{jt} = \tilde{Q}^A_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \left( \tilde{A}_{jtk_{t+1}} - \tilde{B}^B_{jtk_{t+1}} F^B_{jt} \right)' V^A_{k_{t+1}} \left( \tilde{A}_{jtk_{t+1}} - \tilde{B}^B_{jtk_{t+1}} F^B_{jt} \right) \]

\[ - \left( \tilde{P}^A_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \left( \tilde{A}_{jtk_{t+1}} - \tilde{B}^B_{jtk_{t+1}} F^B_{jt} \right)' V^A_{k_{t+1}} \tilde{B}^A_{jtk_{t+1}} \right) \]

\[ \left( \tilde{R}^A_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \tilde{B}^A_{jtk_{t+1}} V^A_{k_{t+1}} \tilde{B}^A_{jtk_{t+1}} \right)^{-1} \]

\[ \left[ \tilde{P}^A_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \tilde{B}^A_{jtk_{t+1}} V^A_{k_{t+1}} \left( \tilde{A}_{jtk_{t+1}} - \tilde{B}^B_{jtk_{t+1}} F^B_{jt} \right) \right] \]

\[ V^B_{jt} = \tilde{Q}^B_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \left( \tilde{A}_{jtk_{t+1}} - \tilde{B}^A_{jtk_{t+1}} F^A_{k_{t+1}} \right)' V^B_{k_{t+1}} \left( \tilde{A}_{jtk_{t+1}} - \tilde{B}^A_{jtk_{t+1}} F^A_{k_{t+1}} \right) \]

\[ - \left[ \tilde{P}^B_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \left( \tilde{A}_{jtk_{t+1}} - \tilde{B}^A_{jtk_{t+1}} F^A_{jt} \right)' V^B_{k_{t+1}} \tilde{B}^B_{jtk_{t+1}} \right] \]

\[ \left( \tilde{R}^B_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \tilde{B}^B_{jtk_{t+1}} V^B_{k_{t+1}} \tilde{B}^B_{jtk_{t+1}} \right)^{-1} \]

\[ \left[ \tilde{P}^B_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \tilde{B}^B_{jtk_{t+1}} V^B_{k_{t+1}} \left( \tilde{A}_{jtk_{t+1}} - \tilde{B}^A_{jtk_{t+1}} F^A_{jt} \right) \right] \]
\[ F^A_{jt} = \left( \tilde{R}^A_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \tilde{B}^A_{jtk_{t+1}} V^A_{k_{t+1}} \tilde{B}^A_{jtk_{t+1}} \right)^{-1} \]
\[ \tilde{P}^A_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \tilde{B}^A_{jtk_{t+1}} V^A_{k_{t+1}} \left( A_{jtk_{t+1}} - \tilde{B}^B_{jtk_{t+1}} F^B_{jt} \right) \]

\[ F^B_{jt} = \left( \tilde{R}^B_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \tilde{B}^B_{jtk_{t+1}} V^B_{k_{t+1}} \tilde{B}^B_{jtk_{t+1}} \right)^{-1} \]
\[ \tilde{P}^B_{jt} + \beta \sum_{k=1}^{n} P_{jtk_{t+1}} \tilde{B}^B_{jtk_{t+1}} V^B_{k_{t+1}} \left( A_{jtk_{t+1}} - \tilde{B}^A_{jtk_{t+1}} F^A_{jt} \right) \]

The discretion equilibrium is a fixed point \((N, V^A, V^B) \equiv \{ N_{jt}, V^A_{jt}, V^B_{jt} \}_{jt=1}^{n}\) of the mapping and a corresponding \((F^A, F^B) \equiv \{ F^A_{jt}, F^B_{jt} \}_{jt=1}^{n}\). The fixed point can be obtained as the limit of \((N_t, V^A_t, V^B_t)\) when \(t \to -\infty\).

F Data Appendix

We follow Bianchi and Ilut (2017) in constructing our fiscal variables. The data for government spending, tax revenues and transfers, are taken from National Income and Product Accounts (NIPA) Table 3.2 (Federal Government Current Receipts and Expenditures) released by the Bureau of Economics Analysis. These data series are nominal and in levels.

**Government Spending.** Government spending is defined as the sum of consumption expenditure (line 21), gross government investment (line 42), net purchases of nonproduced assets (line 44), minus consumption of fixed capital (line 45), minus wage accruals less disbursements (line 33).

**Total tax revenues.** Total tax revenues are constructed as the difference between current receipts (line 38) and current transfer receipts (line 16).

**Transfers.** Transfers is defined as current transfer payments (line 22) minus current transfer receipts (line 16) plus capital transfers payments (line 43) minus capital transfer receipts (line 39) plus subsidies (line 32).

**Federal government debt.** Federal government debt is the market value of privately held gross Federal debt, which is downloaded from Dallas Fed website.

The above three fiscal variables are normalized with respect to Nominal GDP. **Nominal GDP** is taken from NIPA Table 1.1.5 (Gross Domestic Product).

**Real GDP.** Real GDP is take download from NIPA Table 1.1.6 (Real Gross Domestic Product, Chained Dollars).

**The GDP deflator.** The GDP deflator is obtained from NIPA Table 1.1.5 (Gross Domestic Product).

**Effective Federal Funds Rate.** Effective Federal Funds Rate is taken from the St. Louis Fed website.
G Alternative Leadership Regimes

In this appendix we present the estimation results in the case of the alternative leadership regimes where (1) the monetary authority acts as a the Stackelberg leader and (2) both policy makers act simultaneously in defining a Nash equilibrium. It can be seen from these results the parameter estimates and log-likelihoods are very similar to the case of fiscal leadership considered in the paper. As a result the underlying narrative does not change if we make a different assumptions about the role of policy leadership.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Targeting rules parameters</th>
<th>Deep parameters</th>
<th>Serial correlation of shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Posterior</td>
<td>Mean [5%, 95%]</td>
<td>Mean [5%, 95%]</td>
<td>Mean [5%, 95%]</td>
</tr>
<tr>
<td>ω₁, Ẋₜ – Ẋₜ,</td>
<td>0.202 (0.226) [0.146, 0.303]</td>
<td>B 0.50 [0.33, 0.66]</td>
<td></td>
</tr>
<tr>
<td>ω₂, Ṥₜ – ẋₜ,</td>
<td>0.185 (0.267) [0.191, 0.341]</td>
<td>B 0.50 [0.33, 0.66]</td>
<td></td>
</tr>
<tr>
<td>ω₃, change in inflation</td>
<td>0.275 (0.422) [0.264, 0.587]</td>
<td>B 0.50 [0.33, 0.66]</td>
<td></td>
</tr>
<tr>
<td>ω₄⁵, inflation</td>
<td>1.00 (1.00) -</td>
<td>B 0.50 [0.33, 0.66]</td>
<td></td>
</tr>
<tr>
<td>ω₅⁵, inflation</td>
<td>0.584 (0.630) [0.516, 0.755]</td>
<td>B 0.50 [0.33, 0.66]</td>
<td></td>
</tr>
<tr>
<td>ω₆, change in interest</td>
<td>0.689 (0.719) [0.558, 0.880]</td>
<td>B 0.50 [0.25, 0.75]</td>
<td></td>
</tr>
<tr>
<td>ω₇, inflation</td>
<td>0.322 (0.291) [0.187, 0.397]</td>
<td>G 1.00 [0.30, 2.04]</td>
<td></td>
</tr>
<tr>
<td>ω₈, inflation</td>
<td>0.723 (0.632) [0.461, 0.797]</td>
<td>B 0.50 [0.25, 0.75]</td>
<td></td>
</tr>
<tr>
<td>ω₉, lagged tax rate</td>
<td>0.967 (0.904) [0.792, 0.974]</td>
<td>B 0.70 [0.42, 0.92]</td>
<td></td>
</tr>
<tr>
<td>ω₁⁰, lagged tax rate</td>
<td>0.930 (0.941) [0.916, 0.970]</td>
<td>B 0.70 [0.42, 0.92]</td>
<td></td>
</tr>
<tr>
<td>ω₁¹, tax resp. to debt</td>
<td>0.049 (0.047) [0.035, 0.060]</td>
<td>G 0.05 [0.00, 0.18]</td>
<td></td>
</tr>
<tr>
<td>ω₁², tax resp. to debt</td>
<td>0.00 (0.00) -</td>
<td>G 0.10 [0.00, 0.45]</td>
<td></td>
</tr>
<tr>
<td>ω₁³, tax resp. to output</td>
<td>0.001 (0.032) [0.000, 0.073]</td>
<td>G 0.10 [0.00, 0.45]</td>
<td></td>
</tr>
</tbody>
</table>

Table G.1: Monetary Policy Leadership

In this appendix we present the estimation results in the case of the alternative leadership regimes where (1) the monetary authority acts as a the Stackelberg leader and (2) both policy makers act simultaneously in defining a Nash equilibrium. It can be seen from these results the parameter estimates and log-likelihoods are very similar to the case of fiscal leadership considered in the paper. As a result the underlying narrative does not change if we make a different assumptions about the role of policy leadership.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Targeting rules parameters</th>
<th>Deep parameters</th>
<th>Serial correlation of shocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Posterior</td>
<td>Mean [5%, 95%]</td>
<td>Mean [5%, 95%]</td>
<td>Mean [5%, 95%]</td>
</tr>
<tr>
<td>ω₁, Ẋₜ – Ẋₜ,</td>
<td>0.202 (0.226) [0.146, 0.303]</td>
<td>B 0.50 [0.33, 0.66]</td>
<td></td>
</tr>
<tr>
<td>ω₂, Ṥₜ – ẋₜ,</td>
<td>0.185 (0.267) [0.191, 0.341]</td>
<td>B 0.50 [0.33, 0.66]</td>
<td></td>
</tr>
<tr>
<td>ω₃, change in inflation</td>
<td>0.275 (0.422) [0.264, 0.587]</td>
<td>B 0.50 [0.33, 0.66]</td>
<td></td>
</tr>
<tr>
<td>ω₄⁵, inflation</td>
<td>1.00 (1.00) -</td>
<td>B 0.50 [0.33, 0.66]</td>
<td></td>
</tr>
<tr>
<td>ω₅⁵, inflation</td>
<td>0.584 (0.630) [0.516, 0.755]</td>
<td>B 0.50 [0.33, 0.66]</td>
<td></td>
</tr>
<tr>
<td>ω₆, change in interest</td>
<td>0.689 (0.719) [0.558, 0.880]</td>
<td>B 0.50 [0.25, 0.75]</td>
<td></td>
</tr>
<tr>
<td>ω₇, inflation</td>
<td>0.322 (0.291) [0.187, 0.397]</td>
<td>G 1.00 [0.30, 2.04]</td>
<td></td>
</tr>
<tr>
<td>ω₈, inflation</td>
<td>0.723 (0.632) [0.461, 0.797]</td>
<td>B 0.50 [0.25, 0.75]</td>
<td></td>
</tr>
<tr>
<td>ω₉, lagged tax rate</td>
<td>0.967 (0.904) [0.792, 0.974]</td>
<td>B 0.70 [0.42, 0.92]</td>
<td></td>
</tr>
<tr>
<td>ω₁⁰, lagged tax rate</td>
<td>0.930 (0.941) [0.916, 0.970]</td>
<td>B 0.70 [0.42, 0.92]</td>
<td></td>
</tr>
<tr>
<td>ω₁¹, tax resp. to debt</td>
<td>0.049 (0.047) [0.035, 0.060]</td>
<td>G 0.05 [0.00, 0.18]</td>
<td></td>
</tr>
<tr>
<td>ω₁², tax resp. to debt</td>
<td>0.00 (0.00) -</td>
<td>G 0.10 [0.00, 0.45]</td>
<td></td>
</tr>
<tr>
<td>ω₁³, tax resp. to output</td>
<td>0.001 (0.032) [0.000, 0.073]</td>
<td>G 0.10 [0.00, 0.45]</td>
<td></td>
</tr>
</tbody>
</table>

Table G.1: Monetary Policy Leadership
### Table G.1: Monetary Policy Leadership (continued).

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Posterior</th>
<th>Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>Mean</td>
<td>5%, 95%</td>
</tr>
<tr>
<td><strong>Standard deviation of shocks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\sigma_{\xi,k_t=1, \text{taste}})</td>
<td>0.811</td>
<td>0.892</td>
</tr>
<tr>
<td>(\sigma_{\xi,k_t=2, \text{taste}})</td>
<td>2.141</td>
<td>2.368</td>
</tr>
<tr>
<td>(\sigma_{\mu,k_t=1, \text{cost-push}})</td>
<td>0.654</td>
<td>0.591</td>
</tr>
<tr>
<td>(\sigma_{\mu,k_t=2, \text{cost-push}})</td>
<td>2.208</td>
<td>1.898</td>
</tr>
<tr>
<td>(\sigma_{q,k_t=1, \text{productivity}})</td>
<td>0.689</td>
<td>0.678</td>
</tr>
<tr>
<td>(\sigma_{q,k_t=2, \text{productivity}})</td>
<td>1.236</td>
<td>1.272</td>
</tr>
<tr>
<td>(\sigma_{\tau_p, \text{term premium}})</td>
<td>2.533</td>
<td>2.581</td>
</tr>
<tr>
<td>(\sigma_{g, \text{government}})</td>
<td>0.161</td>
<td>0.163</td>
</tr>
<tr>
<td>(\sigma_{z, \text{transfer}})</td>
<td>0.303</td>
<td>0.304</td>
</tr>
<tr>
<td>(\sigma_{\tau, \text{tax rate}})</td>
<td>0.235</td>
<td>0.249</td>
</tr>
<tr>
<td><strong>Transition probabilities</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\phi_{11, \text{remaining mc}})</td>
<td>0.952</td>
<td>0.951</td>
</tr>
<tr>
<td>(\phi_{22, \text{remaining lc}})</td>
<td>0.965</td>
<td>0.939</td>
</tr>
<tr>
<td>(\psi_{11, \text{remaining targeting}})</td>
<td>0.868</td>
<td>0.878</td>
</tr>
<tr>
<td>(\psi_{12, \text{targeting to passive}})</td>
<td>0.005</td>
<td>0.007</td>
</tr>
<tr>
<td>(\psi_{22, \text{remaining passive}})</td>
<td>0.963</td>
<td>0.949</td>
</tr>
<tr>
<td>(\psi_{23, \text{passive to active}})</td>
<td>0.005</td>
<td>0.013</td>
</tr>
<tr>
<td>(\psi_{33, \text{remaining active}})</td>
<td>0.918</td>
<td>0.909</td>
</tr>
<tr>
<td>(\psi_{31, \text{active to targeting}})</td>
<td>0.002</td>
<td>0.006</td>
</tr>
<tr>
<td>(h_{11, \text{remaining lv}})</td>
<td>0.971</td>
<td>0.964</td>
</tr>
<tr>
<td>(h_{22, \text{remaining hv}})</td>
<td>0.894</td>
<td>0.890</td>
</tr>
</tbody>
</table>
### Targeting rules parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mode</th>
<th>Mean</th>
<th>5%, 95%</th>
<th>Type</th>
<th>Mean</th>
<th>5%, 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_1, \hat{X}_t - \hat{\xi}_t$</td>
<td>0.226</td>
<td>0.221</td>
<td>[0.142, 0.296]</td>
<td>B</td>
<td>0.50</td>
<td>[0.33, 0.66]</td>
</tr>
<tr>
<td>$\omega_2, \hat{y}_t - \frac{2}{\sqrt{3}} \hat{\xi}_t$</td>
<td>0.200</td>
<td>0.261</td>
<td>[0.191, 0.334]</td>
<td>B</td>
<td>0.50</td>
<td>[0.33, 0.66]</td>
</tr>
<tr>
<td>$\omega_3^M$, change in inflation</td>
<td>0.309</td>
<td>0.420</td>
<td>[0.251, 0.573]</td>
<td>B</td>
<td>0.50</td>
<td>[0.33, 0.66]</td>
</tr>
<tr>
<td>$\omega_{M,S_t=1}^F$, inflation</td>
<td>1.00</td>
<td>1.00</td>
<td>-</td>
<td>-</td>
<td>1.00</td>
<td>Fixed</td>
</tr>
<tr>
<td>$\omega_{M,S_t=2}^F$, inflation</td>
<td>0.616</td>
<td>0.621</td>
<td>[0.498, 0.742]</td>
<td>B</td>
<td>0.50</td>
<td>[0.33, 0.66]</td>
</tr>
<tr>
<td>$\omega_B^F$, change in interest</td>
<td>0.667</td>
<td>0.715</td>
<td>[0.558, 0.881]</td>
<td>B</td>
<td>0.50</td>
<td>[0.35, 0.75]</td>
</tr>
<tr>
<td>$\omega_{r,s_t=1}^F$, inflation</td>
<td>0.334</td>
<td>0.302</td>
<td>[0.187, 0.409]</td>
<td>G</td>
<td>1.00</td>
<td>[0.30, 2.04]</td>
</tr>
<tr>
<td>$\omega_{r,s_t=1}^{\tau}$, change in tax</td>
<td>0.670</td>
<td>0.622</td>
<td>[0.462, 0.796]</td>
<td>B</td>
<td>0.50</td>
<td>[0.25, 0.75]</td>
</tr>
<tr>
<td>$\rho_{\tau,s_t=2}$, lagged tax rate</td>
<td>0.967</td>
<td>0.899</td>
<td>[0.799, 0.972]</td>
<td>B</td>
<td>0.50</td>
<td>[0.42, 0.92]</td>
</tr>
<tr>
<td>$\rho_{\tau,s_t=3}$, lagged tax rate</td>
<td>0.931</td>
<td>0.943</td>
<td>[0.917, 0.968]</td>
<td>B</td>
<td>0.50</td>
<td>[0.42, 0.92]</td>
</tr>
<tr>
<td>$\rho_{\tau,s_t=2}$, tax resp. to debt</td>
<td>0.050</td>
<td>0.047</td>
<td>[0.035, 0.059]</td>
<td>G</td>
<td>0.05</td>
<td>[0.00, 0.18]</td>
</tr>
<tr>
<td>$\delta_{r,s_t=3}$, tax resp. to debt</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
<td>-</td>
<td>0.00</td>
<td>Fixed</td>
</tr>
<tr>
<td>$\delta_y$, tax resp. to output</td>
<td>0.002</td>
<td>0.031</td>
<td>[0.000, 0.071]</td>
<td>G</td>
<td>0.10</td>
<td>[0.00, 0.45]</td>
</tr>
</tbody>
</table>

### Serial correlation of shocks

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mode</th>
<th>Mean</th>
<th>5%, 95%</th>
<th>Type</th>
<th>Mean</th>
<th>5%, 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{\xi}$, taste</td>
<td>0.953</td>
<td>0.944</td>
<td>[0.933, 0.954]</td>
<td>B</td>
<td>0.50</td>
<td>[0.25, 0.75]</td>
</tr>
<tr>
<td>$\rho_{\mu}$, cost-push</td>
<td>0.930</td>
<td>0.933</td>
<td>[0.915, 0.951]</td>
<td>B</td>
<td>0.50</td>
<td>[0.25, 0.75]</td>
</tr>
<tr>
<td>$\rho_{y}$, productivity</td>
<td>0.290</td>
<td>0.274</td>
<td>[0.206, 0.345]</td>
<td>B</td>
<td>0.50</td>
<td>[0.25, 0.75]</td>
</tr>
<tr>
<td>$\rho_{z}$, transfers</td>
<td>0.968</td>
<td>0.972</td>
<td>[0.961, 0.983]</td>
<td>B</td>
<td>0.50</td>
<td>[0.25, 0.75]</td>
</tr>
<tr>
<td>$\rho_{g}$, government</td>
<td>0.988</td>
<td>0.984</td>
<td>[0.979, 0.990]</td>
<td>B</td>
<td>0.50</td>
<td>[0.25, 0.75]</td>
</tr>
</tbody>
</table>

Table G.2: The Nash Solution
### Posterior Prior

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mode</th>
<th>Mean</th>
<th>[5%, 95%]</th>
<th>Type</th>
<th>Mean</th>
<th>[5%, 95%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\xi,t=1}$, taste</td>
<td>0.911</td>
<td>0.897</td>
<td>[0.639, 1.147]</td>
<td>IG</td>
<td>0.50</td>
<td>[0.11, 1.49]</td>
</tr>
<tr>
<td>$\sigma_{\xi,t=2}$, taste</td>
<td>2.251</td>
<td>2.363</td>
<td>[1.606, 3.130]</td>
<td>IG</td>
<td>0.50</td>
<td>[0.11, 1.49]</td>
</tr>
<tr>
<td>$\sigma_{\mu,t=1}$, cost-push</td>
<td>0.653</td>
<td>0.593</td>
<td>[0.470, 0.715]</td>
<td>IG</td>
<td>0.50</td>
<td>[0.11, 1.49]</td>
</tr>
<tr>
<td>$\sigma_{\mu,t=2}$, cost-push</td>
<td>2.303</td>
<td>1.917</td>
<td>[1.391, 2.463]</td>
<td>IG</td>
<td>0.50</td>
<td>[0.11, 1.49]</td>
</tr>
<tr>
<td>$\sigma_{q,t=1}$, productivity</td>
<td>0.687</td>
<td>0.681</td>
<td>[0.604, 0.756]</td>
<td>IG</td>
<td>0.50</td>
<td>[0.11, 1.49]</td>
</tr>
<tr>
<td>$\sigma_{q,t=2}$, productivity</td>
<td>1.274</td>
<td>1.274</td>
<td>[1.052, 1.480]</td>
<td>IG</td>
<td>0.50</td>
<td>[0.11, 1.49]</td>
</tr>
<tr>
<td>$\sigma_{t_{p_t}}$, term premium</td>
<td>2.546</td>
<td>2.584</td>
<td>[2.333, 2.841]</td>
<td>IG</td>
<td>2.00</td>
<td>[0.63, 4.89]</td>
</tr>
<tr>
<td>$\sigma_{g}$, government</td>
<td>0.161</td>
<td>0.163</td>
<td>[0.150, 0.176]</td>
<td>IG</td>
<td>0.50</td>
<td>[0.11, 1.49]</td>
</tr>
<tr>
<td>$\sigma_{z}$, transfer</td>
<td>0.303</td>
<td>0.304</td>
<td>[0.280, 0.329]</td>
<td>IG</td>
<td>0.50</td>
<td>[0.11, 1.49]</td>
</tr>
<tr>
<td>$\sigma_{T_t}$, tax rate</td>
<td>0.232</td>
<td>0.248</td>
<td>[0.219, 0.277]</td>
<td>IG</td>
<td>0.50</td>
<td>[0.11, 1.49]</td>
</tr>
</tbody>
</table>

### Transition Probabilities

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Mode</th>
<th>Mean</th>
<th>[5%, 95%]</th>
<th>Type</th>
<th>Mean</th>
<th>[5%, 95%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{11}$, remaining mc</td>
<td>0.955</td>
<td>0.953</td>
<td>[0.925, 0.981]</td>
<td>B</td>
<td>0.95</td>
<td>[0.848, 0.998]</td>
</tr>
<tr>
<td>$\phi_{22}$, remaining lc</td>
<td>0.970</td>
<td>0.938</td>
<td>[0.896, 0.979]</td>
<td>B</td>
<td>0.95</td>
<td>[0.848, 0.998]</td>
</tr>
<tr>
<td>$\psi_{11}$, remaining targeting</td>
<td>0.882</td>
<td>0.876</td>
<td>[0.846, 0.904]</td>
<td>D</td>
<td>0.90</td>
<td>[0.807, 0.967]</td>
</tr>
<tr>
<td>$\psi_{12}$, targeting to passive</td>
<td>0.005</td>
<td>0.006</td>
<td>[0.000, 0.015]</td>
<td>D</td>
<td>0.05</td>
<td>[0.002, 0.151]</td>
</tr>
<tr>
<td>$\psi_{22}$, remaining passive</td>
<td>0.963</td>
<td>0.950</td>
<td>[0.926, 0.976]</td>
<td>D</td>
<td>0.90</td>
<td>[0.807, 0.967]</td>
</tr>
<tr>
<td>$\psi_{23}$, passive to active</td>
<td>0.006</td>
<td>0.013</td>
<td>[0.001, 0.025]</td>
<td>D</td>
<td>0.05</td>
<td>[0.002, 0.151]</td>
</tr>
<tr>
<td>$\psi_{33}$, remaining active</td>
<td>0.917</td>
<td>0.909</td>
<td>[0.884, 0.934]</td>
<td>D</td>
<td>0.90</td>
<td>[0.807, 0.967]</td>
</tr>
<tr>
<td>$\psi_{31}$, active to targeting</td>
<td>0.0031</td>
<td>0.006</td>
<td>[0.000, 0.011]</td>
<td>D</td>
<td>0.05</td>
<td>[0.002, 0.151]</td>
</tr>
<tr>
<td>$h_{11}$, remaining with lv</td>
<td>0.973</td>
<td>0.964</td>
<td>[0.945, 0.983]</td>
<td>B</td>
<td>0.90</td>
<td>[0.807, 0.967]</td>
</tr>
<tr>
<td>$h_{22}$, remaining with lv</td>
<td>0.892</td>
<td>0.890</td>
<td>[0.860, 0.922]</td>
<td>B</td>
<td>0.90</td>
<td>[0.807, 0.967]</td>
</tr>
</tbody>
</table>

Table G.2: The Nash Solution (continued).
## Log Marginal Data Density

<table>
<thead>
<tr>
<th>Model</th>
<th>Geweke</th>
<th>Sims, Waggoner, Zha</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monetary Leader</td>
<td>-1408.923</td>
<td>-1409.531</td>
</tr>
<tr>
<td>Nash</td>
<td>-1409.326</td>
<td>-1410.003</td>
</tr>
<tr>
<td>Fiscal Leader</td>
<td>-1410.254</td>
<td>-1410.561</td>
</tr>
<tr>
<td>Intermediate Model</td>
<td>-1416.304</td>
<td>-1416.392</td>
</tr>
<tr>
<td>Rules-Based Policy</td>
<td>-1418.116</td>
<td>-1418.541</td>
</tr>
</tbody>
</table>

Table G.3: Model Comparison
Figure G.1: Markov Switching Probabilities: Policy and Volatility Switches under Alternative Leadership Regimes. Solid lines are from monetary leadership, whereas dashed lines are from Nash solution.
## H. Convergence

A random walk Metropolis-Hastings algorithm is then used to generate four chains consisting of 540,000 draws each (with the first 240,000 draws being discarded and 1 in every 100 draws being saved). Brooks-Gelman-Rubin potential reduction scale factors (PRSF) are all below the 1.1 benchmark value used as an upper bound for convergence. FPSR values for Rules-Based Policy and Targeting rules are presented in Table H.1.

### Table H.1: Brooks-Gelman-Rubin potential reduction scale factors.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>PSRF</th>
<th>Parameters</th>
<th>PSRF</th>
<th>Parameters</th>
<th>PSRF</th>
<th>Parameters</th>
<th>PSRF</th>
<th>Parameters</th>
<th>PSRF</th>
<th>Rules-based policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>AM/PF</td>
<td></td>
<td>PM/AF</td>
<td></td>
<td>σ</td>
<td>1.00</td>
<td>σμ(κt=1)</td>
<td>1.01</td>
<td>p11</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>ρR,S1=1</td>
<td>1.00</td>
<td>ρR,S2=2</td>
<td>1.06</td>
<td>α</td>
<td>1.00</td>
<td>σξ(κt=2)</td>
<td>1.00</td>
<td>p22</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>ψ1,S1=1</td>
<td>1.00</td>
<td>ψ1,S2=2</td>
<td>1.01</td>
<td>ϑ</td>
<td>1.00</td>
<td>σμ(κt=1)</td>
<td>1.00</td>
<td>q11</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>ψ2,S1=1</td>
<td>1.00</td>
<td>ψ2,S2=2</td>
<td>1.01</td>
<td>ϑ</td>
<td>1.00</td>
<td>σμ(κt=2)</td>
<td>1.00</td>
<td>q22</td>
<td>1.01</td>
<td></td>
</tr>
<tr>
<td>ρτ,κt=1</td>
<td>1.00</td>
<td>ρτ,κ2=1</td>
<td>1.00</td>
<td>ϕ</td>
<td>fixed</td>
<td>σq(κt=1)</td>
<td>1.00</td>
<td>h11</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>δτ,κt=1</td>
<td>1.00</td>
<td>δτ,κ2=1</td>
<td>1.00</td>
<td>ρξ</td>
<td>1.01</td>
<td>σq(κt=2)</td>
<td>1.00</td>
<td>h22</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>δτ,κt=2</td>
<td>1.02</td>
<td>δτ,κ2=2</td>
<td>1.02</td>
<td>ρμ</td>
<td>1.00</td>
<td>στp</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AM/AF</td>
<td></td>
<td>PM/AF</td>
<td></td>
<td>ρκ</td>
<td>1.00</td>
<td>σκ</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρR,S1=1</td>
<td>1.00</td>
<td>ρR,S2=2</td>
<td>1.06</td>
<td>ρκ</td>
<td>1.00</td>
<td>σκ</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ψ1,S1=1</td>
<td>1.00</td>
<td>ψ1,S2=2</td>
<td>1.01</td>
<td>ρκ</td>
<td>1.00</td>
<td>σκ</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ψ2,S1=1</td>
<td>1.00</td>
<td>ψ2,S2=2</td>
<td>1.01</td>
<td>ρκ</td>
<td>1.00</td>
<td>σκ</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρτ,κt=2</td>
<td>1.02</td>
<td>ρτ,κ2=2</td>
<td>1.02</td>
<td>ρκ</td>
<td>1.00</td>
<td>σκ</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>δτ,κt=2</td>
<td>fixed</td>
<td>δτ,κ2=2</td>
<td>fixed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>δκ</td>
<td>1.02</td>
<td>δκ</td>
<td>1.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Targeting rules

<table>
<thead>
<tr>
<th>Parameters</th>
<th>PSRF</th>
<th>Parameters</th>
<th>PSRF</th>
<th>Parameters</th>
<th>PSRF</th>
<th>Parameters</th>
<th>PSRF</th>
<th>Parameters</th>
<th>PSRF</th>
<th>Rules-based policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>ω1</td>
<td>1.00</td>
<td>σ</td>
<td>1.00</td>
<td>σμ(κt=1)</td>
<td>1.01</td>
<td>ϕ11</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ω2</td>
<td>1.00</td>
<td>α</td>
<td>1.01</td>
<td>σμ(κt=2)</td>
<td>1.00</td>
<td>ϕ22</td>
<td>1.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ω3</td>
<td>1.00</td>
<td>ϑ</td>
<td>1.00</td>
<td>σq(κt=1)</td>
<td>1.00</td>
<td>ψ11</td>
<td>1.01</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ωR</td>
<td>1.00</td>
<td>ρξ</td>
<td>1.02</td>
<td>σξ(κt=1)</td>
<td>1.01</td>
<td>ψ23</td>
<td>1.02</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ωτ</td>
<td>1.00</td>
<td>ρμ</td>
<td>1.01</td>
<td>στp</td>
<td>1.01</td>
<td>ψ33</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ωκ</td>
<td>1.01</td>
<td>ρκ</td>
<td>1.00</td>
<td>σκ</td>
<td>1.00</td>
<td>ψ31</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρτ,κt=2</td>
<td>1.03</td>
<td>ρκ</td>
<td>1.02</td>
<td>σκ</td>
<td>1.00</td>
<td>h11</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρτ,κt=3</td>
<td>1.01</td>
<td>ρκ</td>
<td>1.01</td>
<td>σκ</td>
<td>1.01</td>
<td>h22</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>δτ,κt=2</td>
<td>1.02</td>
<td>δτ,κ2=2</td>
<td>1.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>δτ,κt=3</td>
<td>fixed</td>
<td>δτ,κ2=3</td>
<td>fixed</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>δκ</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table H.1: Brooks-Gelman-Rubin potential reduction scale factors.
I Model Identification

We apply the Komunjer and Ng (2011) identification test to analyze our targeting rule model. Komunjer and Ng (2011) study the local identification of a DSGE model from its linearized solution. Their test uses the restrictions implied by equivalent spectral densities to obtain rank and order conditions for identification. Minimality and left-invertibility are necessary and sufficient conditions for identification. It is important to note that the Komunjer and Ng (2011) identification test only applies to covariance stationary processes. Therefore, the parameters associated with Markov-switching shock variances cannot be incorporated into the test.

Nevertheless, it is possible to test the identification of structural parameters and the transition probabilities associated with policy changes. We can solve our model assuming that policy stays in one regime, while the private agents in the economy are aware that there are probabilities of policy switching to a different regime. In total, we have six policy regimes: MC/TF, LC/TF, MC/PF, LC/PF, MC/AF and LC/AF.

Our targeting rule model has an estimated parameter vector of dimension \( n_\theta = 35 \), seven observables and seven exogenous shocks (i.e. \( n_Y = n_\varepsilon = 7 \)). The model is square. Thus, Proposition 2-S in Komunjer and Ng (2011) is employed to assess identification. Overall, the test does not indicate that any parameters are unidentified. In Table I.1 the required rank for identification of each regime is presented, along with the Tol at which the model passes the rank requirement.\(^2\)

In addition, we plot draws from the the prior and posterior distributions of parameters for the targeting rule model in Figure I.1. Visual inspection reveals that the priors are widely dispersed around the respective means, whereas posteriors are more concentrated. In other words, the data are informative with respect to these parameters.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Tolerance</th>
<th>( \Delta^s_\lambda )</th>
<th>( \Delta^s_T )</th>
<th>( \Delta^s_U )</th>
<th>( \Delta^s )</th>
<th>Pass</th>
</tr>
</thead>
<tbody>
<tr>
<td>MC/TF</td>
<td>1.0e-03</td>
<td>35</td>
<td>144</td>
<td>49</td>
<td>228</td>
<td>YES</td>
</tr>
<tr>
<td>LC/TF</td>
<td>1.0e-03</td>
<td>35</td>
<td>144</td>
<td>49</td>
<td>228</td>
<td>YES</td>
</tr>
<tr>
<td>MC/PF</td>
<td>1.0e-04</td>
<td>35</td>
<td>100</td>
<td>49</td>
<td>184</td>
<td>YES</td>
</tr>
<tr>
<td>LC/PF</td>
<td>1.0e-04</td>
<td>35</td>
<td>100</td>
<td>49</td>
<td>184</td>
<td>YES</td>
</tr>
<tr>
<td>MC/AF</td>
<td>1.0e-04</td>
<td>35</td>
<td>100</td>
<td>49</td>
<td>184</td>
<td>YES</td>
</tr>
<tr>
<td>LC/AF</td>
<td>1.0e-04</td>
<td>35</td>
<td>100</td>
<td>49</td>
<td>184</td>
<td>YES</td>
</tr>
</tbody>
</table>

Table I.1: Komunjer and Ng (2011) Identification Test.

\(^2\)Using the same notation as in Komunjer and Ng (2011), the required rank for identification is \( rank(\Delta^s) = rank(\Delta^s_\lambda + \Delta^s_T + \Delta^s_U) = n_\theta + n_X^2 + n_\varepsilon^2 \), where \( n_\theta \) is the number of estimated parameters, \( n_X \) is the number of minimal state variables, and \( n_\varepsilon \) is the number of exogenous shocks.
Figure I.1: Prior and Posterior Distributions of Parameters. The panels depict 500 draws from prior and posterior distributions from the estimates of our targeting rule model. The draws are plotted for pairs of estimated parameters and the intersections of lines signify prior (solid) and posterior (dashed) means, respectively.
Figure I.1: Prior and Posterior Distributions of Parameters (continued). The panels depict 500 draws from prior and posterior distributions from the estimates of our targeting rule model. The draws are plotted for pairs of estimated parameters and the intersections of lines signify prior (solid) and posterior (dashed) means, respectively.
J Alternative Social Planner’s Allocation

In this section we outline the social planner’s allocation associated with our estimated model. Normally such an allocation would be obtained by maximising utility subject to resource and technology constraints as in Appendix B above. However, in order to generate insight into our policy maker’s decisions we need to consider the estimated objective function. Therefore we maximise the following objective function,

\[
L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \omega_1 \left( \hat{X}_t^* + \hat{\xi}_t \right)^2 + \omega_2 \left( \hat{y}_t^* - \frac{\sigma}{\varphi} \hat{\xi}_t \right)^2 \right\},
\]

subject to the definition of habits adjusted consumption,

\[
\hat{X}_t^* = (1 - \theta)^{-1} \left( \hat{y}_t^* - \frac{1}{1 - g} \hat{y}_t - \theta \hat{y}_{t-1}^* + \theta \frac{1}{1 - g} \hat{y}_{t-1} \right)
\]

where the star superscripts denote the fact that we are considering the social planner’s allocation. The first order condition this implies is given by,

\[
\omega_1 \left( (1 - \theta)^{-1} \left( \hat{y}_t^* - \frac{1}{1 - g} \hat{y}_t - \theta \hat{y}_{t-1}^* + \theta \frac{1}{1 - g} \hat{y}_{t-1} \right) + \hat{\xi}_t \right) (1 - \theta)^{-1} + \omega_2 \left( \hat{y}_t^* - \frac{\sigma}{\varphi} \hat{\xi}_t \right) \]

\[
= \theta \omega_1 \left( (1 - \theta)^{-1} \left( E_t \hat{y}_{t+1}^* - \frac{1}{1 - g} \rho g \hat{y}_t - \theta \hat{y}_t^* + \theta \frac{1}{1 - g} \hat{y}_t \right) + \rho \hat{\xi}_t \right) (1 - \theta)^{-1}.
\]

This describes the desired path for output that would be chosen by the social planner conditional on the exogenous path for government spending. This can be used to construct a welfare relevant output gap \( \hat{y}_t - \hat{y}_t^* \) which captures the extent to which the policy maker is unable to achieve this desired level of output due to nominal inertia, the habits externality, fiscal constraints and time-consistency problems. Effectively, it reflects the welfare trade-offs between inflation and the real economy implied by the estimated objective function, but reduces those to a single measure.

In order to identify why the estimations adopts particular regimes at particular points of time it is also helpful to get a measure of various fiscal gaps, specifically the tax and debt gaps. The tax gap is the difference between \( \tilde{\tau}_t \) and the tax rate that the social planner would choose to eliminate cost-push shocks, \( \tilde{\tau}_t^* = -(1 - \tau) \tilde{\mu}_t \), so that we have a tax gap, \( \tilde{\tau}_t - \tilde{\tau}_t^* \).
REFERENCES


