# Domestic Policies and Sovereign Default

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## Abstract

This paper incorporates fiscal and monetary policies into a model of sovereign default. In addition to the standard present-bias vs default-risk tradeoff faced by governments when choosing debt, distortionary policy instruments introduce an intertemporal tradeoff, which may mitigate or exacerbate the incentives to accumulate debt. Taxation, the money growth rate and currency depreciation all increase with the level of debt. The model reproduces standard business cycle statistics, the response of spreads, inflation and growth to terms-of-trade shocks, and the cyclical properties of fiscal and monetary policies in emerging markets. A counterfactual exercise for Argentina in 2005-2017 suggests that government expansion accounted for the rise in taxes, inflation and currency depreciation and kept output growth low, countering the benign effects of favorable terms of trade.

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# 1  Introduction

A now large literature, spanned by the work of Eaton and Gersovitz (1981), Aguiar and Gopinath (2006) and Arellano (2008), explains recurrent sovereign debt crises by contending that emerging countries underinsure against negative shocks by overborrowing during booms. It is also widely understood, though mostly ignored by this literature, that domestic policy frameworks affect the resilience of economies to shocks—e.g., see Caballero (2003). Some, notably Kehoe, Nicolini, and Sargent (2020) for the case of Latin America, go even further, conjecturing that government policies are the root of economic crises.

We study domestic fiscal and monetary policies in emerging countries: how they interact with the availability of external credit and the possibility of default; and how they react to external factors. To this effect, we extend the standard sovereign default model to include distortionary taxes, fiat money and an equilibrium nominal exchange rate. As in the standard model, debt accumulation is promoted by a present-bias (relative impatience) and hindered by default risk. When fiscal and monetary policy instruments are distortionary, we find that the government actively relies on seigniorage to finance expenditure and faces an additional intertemporal tradeoff, that may mitigate or exacerbate the incentives to accumulate debt. We also show that distortions increase with debt holdings, as the higher probability of default implies a higher service cost; thus, the money growth rate, the tax rate and the exchange rate all increase with debt. However, expected inflation is non-monotone in debt due to how debt affects the exchange rate and, in turn, the price level.

In a quantitative version of the model, calibrated to match long-term averages of emerging Latin American countries, we find that sovereign debt spreads, inflation and output growth react in an empirically plausible way to shocks to the terms of trade. The model also reproduces standard business cycle statistics as well as the cyclical properties of fiscal and monetary policies.

We also study a version of our model in which the government has access to lump-sum taxes. In this case, implemented monetary policy is no longer distortionary and the equilibrium allocation coincides with the one that would obtain from a real economy. Debt accumulation results from the standard trade-off between present-bias and default-risk. The version with lump-sum taxes is thus a variant of a standard sovereign default model, in the tradition of Eaton and Gersovitz (1981). Importantly, we show that this *EG version* of our model cannot match the levels of nominal variables, such as inflation and currency depreciation, as well as the empirical correlation of output with debt spreads, exports and the trade balance.

Finally, we contribute to the classical debate on whether external factors or domestic policies explain the poor economic performance of emerging countries. To this effect, we study the experience of Argentina in 2005-2017, following the default of 2001. Two driving forces, favorable terms of trade and an increase in government expenditure, go a long way in explaining key macroeconomic variables during that period. Our exercise suggests that government expansion accounted for the rise in taxes, inflation and currency depreciation and kept output growth low, countering the benign effects of favorable terms of trade. We also find that following a strict monetary policy, though potentially successful in containing inflation and currency depreciation, would have been detrimental without addressing the rise in spending.

Our framework consists of a tradable-nontradable (TNT) small open economy (as in Uribe and Schmitt-Grohé, 2017, §8), extended to include production, money and sovereign default. Firms produce both non-tradable goods and exported goods; agents consume non-tradable goods and imported goods. Consumers need money to finance their purchases of non-tradable goods, which gives rise to a demand for fiat money.[1] The government provides a valued public good and makes transfers to individuals; expenditures are financed with labor taxes, money creation and external debt. Government debt is issued in foreign currency to foreign risk-neutral investors. In the event of default, the government enjoys a haircut on its external liabilities but suffers temporary exclusion from financial markets and a productivity loss. We further assume that the government's inability to commit extends to all future policy actions.

We derive necessary first-order conditions to characterize government policy.[2] We show that the decision of how much debt to issue depends on three channels. The first involves distortion-smoothing: debt allows the government to trade-off intertemporally how severely the balance of payments restricts its policy. If the domestic economy is present biased (i.e., impatient relative to the rest of the world), then this factor provides incentives to accumulate debt. The second channel reflects the negative impact of more debt, which leads to a higher default premium and, thus, mitigates the desire to accumulate debt. The third channel arises because fiscal and monetary policies are distortionary. Higher debt tomorrow leads to larger future distortions when repaying and a larger default probability, both of which affect the demand for money today and, hence, the government's current budget constraint. This factor may be positive or negative, depending on agents' preferences, and thus may reinforce or mitigate the incentives to issue debt.

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[1] Though we model domestic government liabilities as fiat money, one could also interpret them more generally to include debt issued domestically in local currency as long as this debt is liquid to some extent.

[2] Arellano, Mateos-Planas, and Ríos-Rull (2019) also derive the Generalized Euler Equation to solve and characterize a sovereign default model.

We conduct several exercises to evaluate the model's ability to capture critical mechanisms observed in emerging markets. First, model-generated data are compared with data since 1980 for seven Latin American economies. In the presence of expected fluctuations in the terms of trade, the model replicates standard business cycle statistics and, more importantly, the cyclical properties of fiscal and monetary policies. Next, impulse responses to terms-of-trade shocks are estimated in the data and the model. The dynamics of the EMBI spread, inflation and real GDP all have the same signs and similar magnitudes in the model and the data. Then, we show that in the EG version of our model, when the government has access to lump-sum taxes, the model is no longer able to replicate the empirical correlation of output with spreads, exports and the trade balance. We therefore conclude that the presence of distortionary taxation is necessary to explain the average level of nominal variables, such as inflation and currency depreciation, as well as the cyclical regularities of real and external variables. Finally, feeding the model with the actual evolution of the terms of trade and government expenditure to replicate Argentina's experience during 2005-2017, the model broadly replicates the dynamics of key macroeconomic variables. We then use this simulation to run the policy counterfactuals explained above.

*Literature review*

The literature on sovereign default has evolved from the framework developed by Eaton and Gersovitz (1981) to quantitative models that account for stylized facts about business cycles in emerging countries (Aguiar and Gopinath, 2006; Arellano, 2008). Although recent models have added realistic features, such as long-term debt (Hatchondo and Martinez, 2008; Hatchondo, Martinez, and Sosa-Padilla, 2016) and sovereign-debt restructuring (Dvorkin, Sánchez, Sapriza, and Yurdagul, 2021; Yue, 2010), there are few papers concerned with the role of fiscal policy and almost no work on monetary policy.

In terms of fiscal policy analysis, Cuadra, Sánchez, and Sapriza (2010) show how a desirable counter-cyclical fiscal policy is reversed by including debt with a risk of default. The critical difference is that we introduce money, which significantly extends the scope of the analysis and complicates the environment by adding an intertemporal optimization problem for households, which the government needs to take into account when formulating policy. Bianchi, Ottonello, and Presno (2019) also argue that pro-cyclical fiscal policy is a property of countries with a high risk of default. They show that this is true even in a model with nominal rigidities and significant Keynesian stabilization gains. More recently, Anzoategui (2019) studies the effect of alternative fiscal rules.

Concerning monetary policy, there is recent work studying the currency composition of debt

and inflation (Ottonello and Perez, 2019; Sunder-Plassmann, 2018). These papers show that debt denominated in local currency raises incentives to dilute debt repayment through inflation. Closer to our work, Arellano, Bai, and Mihalache (2020) analyze the interaction of sovereign default risk with a monetary policy rule in a cashless economy. They argue that the model rationalizes the positive co-movements of sovereign spreads with domestic nominal rates and inflation in Brazil. Their work complements ours since they study the case in which central bankers in emerging markets can commit to a Taylor rule. In contrast, we assume that both fiscal and monetary policies are discretionary and useful to finance government spending. We also study the case when the government can commit to a monetary policy rule.

Another important aspect of our model is the role of nominal exchange rates. In this regard, our work connects with Na, Schmitt-Grohé, Uribe, and Yue (2018), which point to the link between devaluations and default. In a model with downward nominal wage rigidity, they show that an optimal exchange rate devaluation occurs in periods of default, lowering the real value of wages to reduce unemployment. Their paper and ours both show how to recover a "real" economy as in Eaton and Gersovitz (1981). In Na, Schmitt-Grohé, Uribe, and Yue (2018), the key is an optimal devaluation to undo the wage rigidity, while in our model, it is the availability of unconstrained lump-sum taxation.

Finally, our paper is also related to work in closed economies, e.g., Díaz-Giménez, Giovannetti, Marimón, and Teles (2008) and Martin (2009, 2011), among others. These papers study government policy without commitment in monetary economies. Unlike our work, they do not consider the role of sovereign default risk. However, these papers share important similarities in terms of the intertemporal tradeoffs faced by the government.

The paper is structured as follows. Section 2 describes the environment and characterizes the monetary equilibrium. Section 3 formulates the problem of the government, characterizes policies and derives the theoretical results. Section 4 studies the main quantitative properties of our model by focusing on an economy without anticipated aggregates shocks. Section 5 studies an economy with expected shocks to the terms of trade and evaluates the empirical plausibility of the model by comparing model-simulated data with actual data. Section 6 studies the case of Argentina in 2005-2017, to understand how external factor and domestic policies shape economic outcomes. Section 7 concludes.

## 2　Model

### 2.1　Environment

We study a small open economy populated by a large number of identical infinitely-lived agents with measure 1. Time is discrete. Throughout the paper, we make use of recursive notation, denoting next-period variables with a prime.

*Preferences, endowments and technology*

There are three private goods and one public good in the economy. First, there is a non-tradable good that is consumed and produced domestically, their quantities being denoted $c^N$ and $y^N$, respectively. Second, there is tradable imported good that is consumed domestically but not produced. Let $c^T$ denote the consumption of this imported good. Third, there is a tradable exported good that is not consumed domestically and is only produced to be exported. Let $y^T$ denote the production of this exported good. Finally, the government can transform non-tradable output $y^N$ one-to-one into a public good, $g$.

The representative household is endowed with one unit of time each period, which can be either consumed as leisure, $\ell$, or supplied in the labor market, $h$. Thus, $\ell + h = 1$.

Preferences are represented by a time-separable, expected discounted utility. Let the period utility be given by

$$u(c^N, c^T) + v(\ell) + \vartheta(g),$$

where $u$, $v$ and $\vartheta$ are strictly increasing, strictly concave, $C^2$ and satisfy standard boundary conditions. Let $\beta \in (0, 1)$ denote the discount factor. In what follows, $u_j$ denotes the partial derivative of $u$ with respect to the consumption good $c^j$, with $j = \{N, T\}$, and $v_\ell$ denotes the derivative of $v$ with respect to $\ell = 1 - h$. We assume that cross derivatives are zero; i.e., $u_{NT} = u_{TN} = 0$.

There is an aggregate production technology that transforms hours worked, $h$, into non-tradable output, $y^N$, and exported goods, $y^T$. This technology is represented by a cost function $F : \mathbb{R}^2_+ \to \mathbb{R}_+$, which is strictly increasing, strictly convex and homogeneous of degree 1. Given $h$, feasible levels of $(y^N, y^T)$ must satisfy

$$F(y^N, y^T) - h \leq 0, \tag{1}$$

where $F_j$ is the partial derivative of $F$ with respect to $y^j$, $j = \{N, T\}$.

Agents can exchange both tradable and non-tradable goods, as well as domestic currency (fiat money), while trading of other financial assets will be restricted to the government. Let $M^d$ denote individual money holdings. Prices are denominated in domestic currency (i.e., pesos) and given by $P^X$, $P^M$ and $P^N$ for exports, imports and non-tradable goods, respectively. Let $W$ denote the nominal wage in units of domestic currency.

The nominal exchange rate $E$ is defined as the units of domestic currency necessary to purchase one unit of foreign currency (i.e., pesos per dollar). We assume that the law of one price holds for tradable goods and so $P^X = Ep^T$ and $P^M = E$, where $p^T$ is the international price of exported goods and the international price of imported goods is assumed to be constant and normalized to 1. Thus, $p^T$ also stands for the terms of trade.

In order to study a stationary environment, we normalize nominal variables by the stock of the money supply, $M$. Let $\mu$ denote the growth rate of the money supply and $M' = (1 + \mu)M$ denote its law of motion. We define the corresponding normalized variables as $p^N = P^N/M$, $w = W/M$, $e = E/M$ and $m = M^d/M$.

To motivate a role for fiat money, we assume that households face a cash-in-advance constraint when purchasing non-tradable goods:

$$p^N c^N \leq m. \tag{2}$$

That is, (normalized) expenditure on non-tradable goods, $p^N c^N$, cannot exceed (normalized) money balances available at the beginning of the period, $m$.

*Government and the balance of payments*

The government provides a public good, $g$, which is transformed one-to-one from non-tradable output. It may also make lump-sum transfers to households. Let $\gamma$ be the real value (in units of the non-tradable good) of government transfers. We assume that transfers are exogenous, non-negative and represent a non-discretionary redistributive policy. To finance its expenditure, the government may tax labor income $wh$ at rate $\tau$, increase the money supply at rate $\mu$, and issue debt in international credit markets. Debt takes the form of one-period discount bonds that pay one unit of foreign currency and trade at the price $q$, also denominated in foreign currency. Let $B$ denote the value of maturing debt and $qB'$ the funds collected from issuing new debt $B'$, both expressed in foreign currency units.

We consolidate the fiscal and monetary authority and write the government budget con-

straint in (normalized) units of domestic currency as follows[3]:

$$p^N(g + \gamma) + eB \leq \tau wh + \mu + eqB'. \tag{3}$$

The balance of payments, expressed in units of foreign currency, implies

$$p^T y^T - c^T = B - qB', \tag{4}$$

where the left-hand side of (4) is the trade balance, while the right-hand side is the change in the country's net asset position plus implicit debt interest payments.

Combining (3) and (4) we can express the government budget constraint as the relationship between the external sector (the trade balance) and the public sector (the primary surplus plus seigniorage):

$$\tau wh - p^N(g + \gamma) + \mu - e(p^T y^T - c^T) \geq 0. \tag{5}$$

## 2.2 The problem of the representative firm

Local firms produce non-tradable and tradable goods by hiring labor according to the technology represented by $F$. Constant returns to scale imply that we can assume that the industry behaves as if there were a representative firm that solves the static problem

$$\max_{y^N, y^T, h} \ \{p^N y^N + ep^T y^T - wh\}$$

subject to (1). The necessary and sufficient first-order conditions imply expressions for the wage and exchange rate as functions of $(y^N, y^T, p^N, p^T)$ as follows:

$$w = \frac{p^N}{F_N}, \tag{6}$$

$$e = \frac{p^N}{p^T} \frac{F_T}{F_N}. \tag{7}$$

## 2.3 The problem of the representative household

The endogenous state of the economy consists of the amount of maturing foreign debt, $B$, and an indicator function $\mathcal{I}$, which specifies whether the government is in default ($\mathcal{I} = D$) or

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[3]As we argue in Section 5.5, when lump-sum taxes are available, as in the standard Eaton-Gersovitz model, the government sets distortionary taxes equal to zero and follows the Friedman rule. In this case, the model becomes Ricardian: the government budget constraint solves for lump-sum taxes and places no further restrictions on government policy, i.e., is not a constraint in the government's problem.

not ($\mathcal{I} = P$). As we shall explain below, the default state may last several periods while the country is excluded from international credit markets. Agents know the government's default state before making any decisions at the beginning of every period. The exogenous state of the economy is summarized by $s$ and known at the beginning of each period. The state $s$ may include any variable that evolves stochastically over time, e.g., the terms of trade, $p^T$. The set of all possible realizations for the stochastic state is $S$. Note that we are allowing for the possibility that state variables may depend on the default state.

Agents know the laws of motion of all aggregate state variables. All prices and government policies are perceived by agents as being functions of the aggregate state. This dependence is omitted to simplify notation. The period budget constraint of the household is

$$p^N c^N + e c^T + m'(1 + \mu) \leq (1 - \tau) wh + m + p^N \gamma, \tag{8}$$

where, as mentioned above, $p^N$, $w$ $e$, $m$ are all normalized by the aggregate money supply at the beginning of the period. As also mentioned above, trading in the non-tradable goods market is subject to the cash-in-advance constraint, (2).

The individual state variable is the household's (normalized) money balances at the beginning of the period, $m$. Let $V(m, B, \mathcal{I}, s)$ denote the agent's value function as a function of individual and aggregate state variables. Let $\mathbb{E}[V(m', B', \mathcal{I}', s') | B, \mathcal{I}, s]$ be the conditional expected value of the agent's value function in the next period, given current aggregate state $(B, \mathcal{I}, s)$.

The problem of the representative household is

$$V(m, B, \mathcal{I}, s) = \max_{(c^N, c^T, m', h)} u(c^N, c^T) + v(1 - h) + \vartheta(g) + \beta \mathbb{E}\left[V(m', B', \mathcal{I}', s') | B, \mathcal{I}, s\right]$$

subject to (2) and (8). As derived in Appendix A.1, the solution to this problem is characterized by

$$\frac{(1 - \tau) w u_T}{e} = v_\ell, \tag{9}$$

$$\frac{(1 + \mu) u_T}{e} = \beta \mathbb{E}\left[\frac{u'_N}{p^{N\prime}} \Big| B, \mathcal{I}, s\right] \tag{10}$$

plus constraints (2) and (8).

Conditions (9) and (10) show how policies distort households' choices. The tax rate introduces an intra-temporal wedge between the marginal utilities of consumption of tradable goods

and leisure, while the money growth rate introduces an inter-temporal wedge, as it distorts the substitution between current consumption of tradable goods and future consumption of non-tradable goods.

## 2.4 Monetary equilibrium

Since all agents are identical, $c^N$, $c^T$ and $h$ should be interpreted as referring to aggregate quantities from now on.

The resource constraint in the non-tradable sector is

$$c^N + g = y^N. \tag{11}$$

From (1) and (11), labor is a function of non-tradable (private) consumption, public expenditures and the production of tradables; i.e.,

$$h = F(c^N + g, y^T). \tag{12}$$

All agents enter the period with the same money balances, $m$. Market clearing implies that $m = m' = 1$. The Lagrange multiplier associated with the cash-in-advance constraint must be non-negative, which implies the following equilibrium condition:

$$u_N - \frac{u_T F_N p^T}{F_T} \geq 0. \tag{13}$$

If (13) is positive, then the cash-in-advance constraint (2) binds; if is equal to zero, then the cash-in-advance constraint is slack. Condition (13) reflects an inefficiency wedge, as the marginal rate of substitution between non-tradable and tradable goods is larger than the corresponding marginal rate of transformation (i.e., efficiency would dictate that agents consume relatively more non-tradable goods).

Without loss of generality, the cash-in-advance constraint (2) is satisfied with equality.[4] Then, in equilibrium

$$p^N = \frac{1}{c^N}. \tag{14}$$

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[4]If the cash-in-advance constraint is slack, then the price level $p^N$ is, in general, indeterminate. A standard assumption is to take the limiting case, when the constraint is satisfied with equality.

The equilibrium wage can be derived by combining (6) and (14):

$$w = \frac{1}{c^N F_N}. \tag{15}$$

Similarly, the equilibrium exchange rate follows from (7) and (14):

$$e = \frac{1}{c^N p^T} \frac{F_T}{F_N}. \tag{16}$$

# 3 Government policy

## 3.1 Government budget constraint in a monetary equilibrium

Below, we formulate the problem of the government following the *primal approach*. That is, we solve for allocation and debt choices that are implementable in a monetary equilibrium. In order to proceed, we need to use the equilibrium conditions derived above, to replace prices $(p^N, w, e)$ and policies $(\mu, \tau)$ in the government budget constraint (5).

To obtain an expression for the tax rate, combine (9), (15) and (16):

$$\tau = 1 - \frac{v_\ell}{u_T} \frac{F_T}{p^T}. \tag{17}$$

Similarly, the money growth rate can be written by combining (10), (14) and (16):

$$\mu = \frac{\beta \mathbb{E}\left[u_N' c^{N\prime} | B, \mathcal{I}, s\right]}{u_T c^N p^T (F_N/F_T)} - 1. \tag{18}$$

Using (14)–(18) we obtain the government budget constraint in a monetary equilibrium:

$$u_T[c^T - \gamma p^T (F_N/F_T)] - v_\ell F(c^N + g, y^T) + \beta \mathbb{E}\left[u_N' c^{N\prime} | B, \mathcal{I}, s\right] \geq 0, \tag{19}$$

which depends on $(c^N, c^T, y^T, g, c^{N\prime}, c^{T\prime})$.

## 3.2 Repayment and default

Suppose the government is currently not excluded from international credit markets. At the beginning of any such period, the government decides between repaying (P) and defaulting (D)

on its debt. If it decides to default, then debt is set to zero. Define

$$\hat{\mathcal{V}}(B, s, \varepsilon^P, \varepsilon^D) = \max\{V^P(B, s) + \varepsilon^P, V^D(s) + \varepsilon^D\},$$

where $V^P(B, s)$ and $V^D(s)$ denote the values of repayment and default, respectively, which are defined in detail below. Notice that the decision to repay or default is also influenced by random additive shocks to utility. Next, we explain how these shocks are used to obtain some useful expressions under particular assumptions on their distribution.

Assume that $\varepsilon^P$ and $\varepsilon^D$ are independently, identically distributed extreme value (Gumbel or type I extreme value) shocks. The difference between these two shocks will affect the default decision. The assumptions on the distribution of the shocks imply that this difference has mean zero and is distributed logistic; i.e., $\varepsilon = \varepsilon^P - \varepsilon^D$ follows

$$F(\varepsilon) = \frac{\exp[\varepsilon/\kappa]}{1 + \exp[\varepsilon/\kappa]},$$

where $\kappa > 0$ is the scale parameter of the distribution, which will be useful to control the variance of the $\varepsilon$ shocks.

Let $\mathcal{P}(B, s)$ be the probability of repayment for any given $(B, s)$, which can be expressed as

$$\mathcal{P}(B, s) = \Pr(V^P(B, s) - V^D(s) \geq -\varepsilon).$$

This probability has a simple expression given the assumptions on $\varepsilon^i$. Following McFadden (1974), this integral results in a closed-form expression:

$$\mathcal{P}(B, s) = \frac{\exp[V^P(B, s)/\kappa]}{\exp[V^P(B, s)/\kappa] + \exp[V^D(s)/\kappa]}, \tag{20}$$

which, in turn, implies

$$\frac{\partial \mathcal{P}(B, s)}{\partial B} = \frac{\partial V^P(B, s)}{\partial B} \frac{\mathcal{P}(B, s)(1 - \mathcal{P}(B, s))}{\kappa}. \tag{21}$$

Next, we can derive a closed-form expression for the expectation of the value function with respect to the utility shocks:

$$\mathcal{V}(B, s) = E_\varepsilon[\hat{\mathcal{V}}(B, s, \varepsilon^P, \varepsilon^D)] = \kappa \ln \left\{ \exp[V^P(B, s)/\kappa] + \exp[V^D(s)/\kappa] \right\}.$$

Using this expression, we can easily see that

$$\frac{\partial \mathcal{V}(B,s)}{\partial B} = \mathcal{P}(B,s)\frac{\partial V^P(B,s)}{\partial B}. \tag{22}$$

Expressions (21) and (22) will be useful to characterize the choice of debt as long as $V^P$ is differentiable.

## 3.3  Problem of the government

Every period, the government first decides on whether to repay or default on its debt. After that, it implements policy for the period, taking into account the response of private domestic agents and international lenders and the government policies it expects to be implemented in the future. A period policy consists of choices on the amount of future debt, the money growth rate, the tax rate and government expenditure.

If the government decided to default, then the country is excluded from international credit markets. It regains access at the beginning of the period with probability $\delta$; hence, $1/\delta$ is the expected duration of exclusion. The country reenters credit markets with a renegotiated level of debt $B^d \geq 0$, which is exogenous. This assumption implies that debt haircuts are increasing in the level of defaulted debt. While in default, $\mathcal{I} = D$, the country experiences a productivity penalty, $\Omega(s)$, which generally depends on the exogenous state of the economy.

If the government is currently repaying, the probability that it will remain in repayment status tomorrow is given by $\mathcal{P}(B',s')$ for any given $(B',s')$, as derived above. On the other hand, if the government is currently in default, the probability that it will transition to repayment status tomorrow is given by $\delta\mathcal{P}(B^d,s')$ for any given $s'$. Recall that to compute these probabilities we need to know the value functions $V^P(B,s)$ and $V^D(s)$, which we will derive below.

In equilibrium, zero expected profits by risk-neutral international lenders implies that

$$Q(B',s)B' = \frac{\mathbb{E}\left[\mathcal{P}(B',s')|s\right]B' + \mathbb{E}\left[(1-\mathcal{P}(B',s'))Q^d(s')|s\right]B^d}{1+r}, \tag{23}$$

where

$$Q^d(s') \equiv \delta Q(B^d,s') + \frac{(1-\delta)\mathbb{E}[Q^d(s'')|s']}{1+r}$$

for all $s'$. The first term in (23) reflects the expected debt repayment, while the second term reflects the expected debt recovery. $Q^d(s')$ stands for the price an investor would pay to earn $B^d$

in the period the defaulting country reenters international credit markets. Note that, although $Q^d(s')$ is an endogenous object, as it depends on $Q(B^d, s')$, the government takes it as given. Using (23) and the expression for $Q^d(s')$ we obtain the following recursion:

$$Q^d(s) = \frac{\delta \mathbb{E}\left[\mathcal{P}(B^d, s')|s\right] + \mathbb{E}\left[(1 - \delta \mathcal{P}(B^d, s'))Q^d(s')|s\right]}{1 + r} \tag{24}$$

for all $s$.

Taking the derivative of (23) with respect to $B'$ and combining with (21) implies

$$\frac{\partial[Q(B', s)B']}{\partial B'} = \mathbb{E}\left\{ \frac{\mathcal{P}(B', s')}{1 + r}\left[1 + \frac{\partial V^P(B', s')}{\partial B'}\frac{(1 - \mathcal{P}(B', s'))(B' - Q^d(s')B^d)}{\kappa}\right]\bigg|s\right\}. \tag{25}$$

As explained above, we follow the primal approach to formulate the government's problem. Hence, we use equilibrium conditions to express domestic prices, the money growth rate, and the tax rate as functions of current and future allocations. Every period, the government then chooses a debt level (when repaying) and domestic policies that implement the allocation $(c^N, c^T, y^T, g)$. These choices need to satisfy the balance of payment, (4), the government budget constraint, (19), and the non-negativity constraint, (13).

When the government is in the repayment state, $\mathcal{I} = P$, its policies are a function of the state $(B, s)$; let the relevant policy functions be denoted by $\{\mathcal{B}, \mathcal{C}^N, \mathcal{C}^T, \mathcal{Y}^T, \mathcal{G}\}$. While in the default state, $\mathcal{I} = D$, its policies are a function of the state $s$; let the relevant policy functions be denoted by $\{\bar{\mathcal{C}}^N, \bar{\mathcal{C}}^T, \bar{\mathcal{Y}}^T, \bar{\mathcal{G}}\}$.

Given these policy functions, we can define the value functions $V^P(B, s)$, $V^D(s)$ and $\mathcal{V}(B, s)$ as follows:

$$
\begin{aligned}
V^P(B, s) &= u(\mathcal{C}^N(B, s), \mathcal{C}^T(B, s)) + v(1 - F(\mathcal{C}^N(B, s) + \mathcal{G}(B, s), \mathcal{Y}^T(B, s))) + \vartheta(\mathcal{G}(B, s)) \\
&\quad + \beta \mathbb{E}[\mathcal{V}(\mathcal{B}(B, s), s')|s], \\
V^D(s) &= u(\bar{\mathcal{C}}^N(s), \bar{\mathcal{C}}^T(s)) + v(1 - F(\bar{\mathcal{C}}^N(s) + \bar{\mathcal{G}}(s), \bar{\mathcal{Y}}^T(s))) + \vartheta(\bar{\mathcal{G}}(s)) \\
&\quad + \beta \mathbb{E}[\delta \mathcal{V}(B^d, s') + (1 - \delta)V^D(s')|s], \\
\mathcal{V}(B, s) &= \kappa \ln\left\{\exp[V^P(B, s)/\kappa] + \exp[V^D(s)/\kappa]\right\},
\end{aligned}
$$

for all $(B, s)$. Using (20) and (23) these functions imply expressions for $\mathcal{P}(B, s)$ and $Q(B, s)$.

### 3.3.1 Repayment

The problem of the government in the repayment state is

$$\max_{(B',c^N,c^T,y^T,g)} u(c^N, c^T) + v(1 - F(c^N + g, y^T)) + \vartheta(g) + \beta\mathbb{E}[\mathcal{V}(B', s')|s] \qquad \text{(PP)}$$

subject to

$$p^T y^T - c^T + Q(B', s)B' - B = 0, \qquad (26)$$

$$u_T c^T - \gamma u_T p^T(F_N/F_T) - v_\ell F(c^N + g, y^T) + \beta\mathbb{E}\left[u'_N c^{N'}|P, s\right] = 0, \qquad (27)$$

$$u_N - u_T p^T(F_N/F_T) \geq 0. \qquad (28)$$

The constraints in the government's problem correspond to the balance of payment, (4), the government budget constraint, (19), and the non-negativity constraint, (13). Note that the expectation term in the government budget constraint is conditioned on the current state being repayment ($\mathcal{I} = P$); hence, the relevant transition probabilities are $\mathcal{P}(B', s')$ for repayment and $1 - \mathcal{P}(B', s')$ for default, for all $(B', s')$.

### 3.3.2 Default

The problem of the government in the default state is

$$\max_{(c^N,c^T,y^T,g)} u(c^N, c^T) + v(1 - F(c^N + g, y^T)) + \vartheta(g) + \beta\mathbb{E}[\delta\mathcal{V}(B^d, s') + (1 - \delta)V^D(s')|s] \quad \text{(DP)}$$

subject to

$$p^T y^T - c^T = 0, \qquad (29)$$

$$u_T c^T - \gamma u_T p^T(F_N/F_T) - v_\ell F(c^N + g, y^T) + \beta\mathbb{E}[u'_N c^{N'}|D, s] = 0, \qquad (30)$$

$$u_N - u_T p^T(F_N/F_T) \geq 0. \qquad (31)$$

and where total factor productivity, embedded in the cost function $F(y^N, y^T)$, is reduced by a default penalty, $\Omega(s)$.

In this case, note that the balance of payments is simply the trade balance, as the government is excluded from international credit markets. The expectation term in the government budget constraint is conditioned on the current state being default ($\mathcal{I} = D$); hence, the relevant transition probabilities are $\delta\mathcal{P}(B^d, s')$ for repayment and $1 - \delta\mathcal{P}(B^d, s')$ for default, for all $s'$.

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## 3.4 Domestic policy

We begin by characterizing domestic policy, which involves choices for the allocations $(c^N, c^T, y^T, g)$, given state $(B, s)$, repayment status $\mathcal{I} = \{P, D\}$ and a debt policy $\mathcal{B}(B)$. We make the following assumptions to simplify exposition and derive some theoretical results.

To simplify the exposition, we assume that $\gamma = 0$ in this section. In Appendix A.2 we allow $\gamma \geq 0$ and derive the conditions we use in our quantitative evaluation of the model.

Let $\Gamma(c^N, c^T, y^T, g; s) \equiv u_T p^T (F_N/F_T)$, which is an expression that shows up in the government budget and non-negativity constraints. Note that $\Gamma_T = d\Gamma/dc^T = \Gamma \times (u_{TT}/u_T) < 0$, while the convexity of $F$ implies that $\Gamma_N = \Gamma_g = \Gamma \times (F_{NN}/F_N - F_{NT}/F_T) > 0$ and $\Gamma_y = \Gamma \times (F_{NT}/F_N - F_{TT}/F_T) < 0$. Also define $\Phi \equiv v_\ell - v_{\ell\ell} F(c^N + g, y^T) > 0$.

We first consider the government's problem when repaying, (PP)–(28). Let $\xi$, $\lambda$ and $\zeta$ be the Lagrange multipliers associated with the constraints (26), (27) and (28), respectively. The necessary first-order conditions with respect to $(c^N, c^T, y^T, g)$ are

$$u_N - v_\ell F_N - \lambda F_N \Phi + \zeta(u_{NN} - \Gamma_N) = 0, \tag{32}$$

$$u_T - \xi + \lambda(u_T + u_{TT}c^T) - \zeta\Gamma_T = 0, \tag{33}$$

$$-v_\ell F_T + \xi p^T - \lambda F_T \Phi - \zeta\Gamma_y = 0, \tag{34}$$

$$-v_\ell F_N + \vartheta_g - \lambda F_N \Phi - \zeta\Gamma_g = 0. \tag{35}$$

Suppose that the non-negativity constraint (28) does not bind; i.e., $\zeta = 0$. Given $\Gamma_N = \Gamma_g$, (32) and (35) imply

$$u_N = \vartheta_g. \tag{36}$$

Using (32) and (34) to solve for the Lagrange multipliers we obtain

$$\lambda = \frac{u_N - v_\ell F_N}{F_N \Phi},$$

$$\xi = \frac{u_N F_T}{p^T F_N},$$

and so (33) implies

$$F_T \Phi(u_N - \Gamma) = p^T(u_N - v_\ell F_N)(u_T + u_{TT}c^T), \tag{37}$$

where we used the definition of $\Gamma$ to simplify the expression. We now verify under which conditions $\zeta = 0$ holds.

**Proposition 1.** *Assume that $\gamma = 0$. Then, $\zeta = 0$, and so the non-negativity constraint (28)*

*does not bind if and only if* $u_T + u_{TT}c^T \geq 0$.

*Proof.* See Appendix A.3. □

Whether (28) binds or not depends on preferences. If $u_T + u_{TT}c^T > 0$, then (28) is satisfied with strict inequality; when $u_T + u_{TT}c^T = 0$, then (28) is satisfied with strict equality but still slack. In both of these cases, $\zeta = 0$. In contrast, when $u_T + u_{TT}c^T < 0$, then (28) binds and so, $\zeta > 0$.

On the other hand, if transfers are positive (see Appendix A.2), the analysis is more involved. Under certain assumptions it is possible to show that the non-negativity constraint is slack when $u_T + u_{TT}c^T \geq 0$; but it may be slack when $u_T + u_{TT}c^T < 0$ as well. Our numerical exercises show that $\zeta = 0$ when $u_T + u_{TT}c^T \geq 0$, or $u_T + u_{TT}c^T < 0$ and $\gamma$ is sufficiently large. The intuition for this result is that transfers increase the distortions that need to be financed and, hence, provide more incentives for the government to rely on seigniorage.

To sum up, when $u_T + u_{TT}c^T \geq 0$, the solution to $(c^N, c^T, y^T, g)$ when repaying must satisfy (26), (27), (36) and (37). The first-order conditions to the government's problem in default, (DP)–(31), are functionally identical to the ones derived under repayment. Hence, the solution for allocations when in default must satisfy (29), (30), (36) and (37).

## 3.5 Debt policy

We now characterize debt choice in the event the government decides to repay its inherited debt. The necessary first-order condition of problem (PP) with respect to $B'$ is

$$\beta \frac{\partial \mathbb{E}[\mathcal{V}(B', s')|s]}{\partial B'} + \xi \frac{[\partial Q(B', s)B']}{\partial B'} + \lambda \beta \frac{\partial \mathbb{E}\left[u'_N c^{N\prime}|P, s\right]}{\partial B'} = 0. \tag{38}$$

As reflected by the three terms in (38), debt choice affects the continuation value for the government and how tightly the balance of payment and government budget constraints bind. We can further characterize this equation using some of the expressions derived above. The envelope condition of problem (PP) implies $\frac{\partial V^P(B,s)}{\partial B} = -\xi$. Hence, using (21)–(25), we obtain

$$\frac{\partial \mathbb{E}[\mathcal{V}(B', s')|s]}{\partial B'} = -\mathbb{E}[\mathcal{P}(B', s')\xi'|s]$$

$$\frac{\partial Q(B', s)B'}{\partial B'} = \mathbb{E}\left\{\frac{\mathcal{P}(B', s')}{1+r}\left[1 - \frac{(1 - \mathcal{P}(B', s'))(B' - Q^d(s')B^d)\xi'}{\kappa}\right]\Big| s\right\}.$$

The last term in (38) requires a bit more work. Given that $\mathcal{P}(B', s')$ is the probability of

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transitioning from $\mathcal{I} = P$ to $\mathcal{I}' = P$ for all $(B', s')$, we can write

$$\mathbb{E}\left[u'_N c^{N\prime}|P, s\right] = \mathbb{E}\left[\mathcal{P}(B', s')u'_N \mathcal{C}^{N\prime} + (1 - \mathcal{P}(B', s'))\bar{u}'_N \bar{\mathcal{C}}^{N\prime}|s\right],$$

where $u'_N \mathcal{C}^{N\prime}$ corresponds to the repayment state tomorrow, $\mathcal{I}' = P$, and $\bar{u}'_N \bar{\mathcal{C}}^{N\prime}$ corresponds to the default state tomorrow, $\mathcal{I}' = D$. Note that the expectation on the right-hand side (only conditional on $s$) is taken with respect to $s'$. We can take the derivative of the expression above with respect to $B'$ to obtain

$$\frac{\partial \mathbb{E}\left[u'_N c^{N\prime}|P, s\right]}{\partial B'} = \mathbb{E}\left[\mathcal{P}(B', s')(u'_N + u'_{NN}\mathcal{C}^{N\prime})\mathcal{C}^{N\prime}_B + (u'_N \mathcal{C}^{N\prime} - \bar{u}'_N \bar{\mathcal{C}}^{N\prime})\mathcal{P}'_B|s\right],$$

where $\mathcal{C}^{N\prime}_B$ and $\mathcal{P}'_B$ denote the derivatives of $\mathcal{C}^N(B', s')$ and $\mathcal{P}(B', s')$ with respect to $B'$. Recall that, when in default, allocations are not a function of $B$, i.e., $\bar{\mathcal{C}}^N(s)$ and so $\bar{\mathcal{C}}^{N\prime}_B = 0$. From (21), we have an analytical expression for $\mathcal{P}'_B$ and from the envelope condition, $\frac{\partial V^P(B,s)}{\partial B} = -\xi$. Thus, we obtain

$$\frac{\partial \mathbb{E}\left[u'_N c^{N\prime}|P, s\right]}{\partial B'} = \mathbb{E}\left\{\mathcal{P}(B', s')\left[(u'_N + u'_{NN}\mathcal{C}^{N\prime})\mathcal{C}^{N\prime}_B - \frac{(u'_N \mathcal{C}^{N\prime} - \bar{u}'_N \bar{\mathcal{C}}^{N\prime})(1 - \mathcal{P}(B', s'))\xi'}{\kappa}\right]\Big|s\right\}.$$

We can now write the equation characterizing debt choice as

$$\mathbb{E}\left\{\mathcal{P}(B', s')\left[\left(\frac{\xi}{1+r} - \beta\xi'\right) - \frac{\xi(1 - \mathcal{P}(B', s'))(B' - Q^d(s')B^d)\xi'}{\kappa(1+r)}\right]\Big|s\right\}$$
$$+ \lambda\beta\mathbb{E}\left\{\mathcal{P}(B', s')\left[(u'_N + u'_{NN}\mathcal{C}^{N\prime})\mathcal{C}^{N\prime}_B - \frac{(u'_N \mathcal{C}^{N\prime} - \bar{u}'_N \bar{\mathcal{C}}^{N\prime})(1 - \mathcal{P}(B', s'))\xi'}{\kappa}\right]\Big|s\right\} = 0. \quad (39)$$

Note that if there are no aggregate shocks (other than the extreme value shocks, $\varepsilon$), then $\mathcal{P}(B', s')$ only depends on $B'$ and is always positive; hence, the term $\mathcal{P}(B', s')$ that multiplies all the expressions in square brackets in (39) can be eliminated.

The Generalized Euler Equation (39) highlights three channels in the government's debt decision. The first channel, $\mathbb{E}\{\mathcal{P}(B', s')[(1+r)^{-1}\xi - \beta\xi']|s\}$, corresponds to distortion smoothing: debt allows the government to trade off intertemporally how tightly the balance of payments binds. The distortion-smoothing term has an intrinsic *present bias* since the government is relatively impatient as $\beta(1 + r) < 1$. In other words, this term would not be zero if the government kept expected distortions constant over time, i.e., set $\xi = E[\xi'|s]$. This present bias is the channel that motivates debt accumulation in the sovereign default literature.

The second channel, $-[\kappa(1+r)]^{-1}\xi\mathbb{E}\left\{\mathcal{P}(B', s')\left[(1 - \mathcal{P}(B', s'))(B' - Q^d(s')B^d)\xi'\right]|s\right\}$, cap-

tures the default premium: more debt leads to a higher probability of default and, hence, a higher interest rate. The default-premium term reflects the added financial cost due to default risk. This term, as is standard in the literature, moderates debt accumulation. $B_d > 0$ counters this effect, as lenders take into account that they partially recover their loan after a default event.

The third channel, $\lambda\beta\mathbb{E}\{\mathcal{P}(B',s')[(u'_N+u'_{NN}\mathcal{C}^{N\prime})\mathcal{C}_B^{N\prime}-(u'_N\mathcal{C}^{N\prime}-\bar{u}'_N\bar{\mathcal{C}}^{N\prime})(1-\mathcal{P}(B',s'))(\xi'/\kappa)]|s\}$, reflects an intertemporal tradeoff due to the fact that fiscal and monetary policies are distortionary. Higher debt leads to a change in policies tomorrow, which affects the demand for money today and, hence, how tightly the current government budget constraint binds.

The second and third channels described above reflect time-consistency problems in the government's problem due, respectively, to the possibility of default and the use of distortionary domestic policy instruments. In both cases, the government tomorrow does not internalize the effects of its actions on the tradeoffs faced by the government today—*bygones are bygones*. The default channel is central to standard sovereign default models, but the distortionary-policies channel is absent. Below, we analyze this latter channel in more detail.

### 3.5.1 Intertemporal tradeoff due to distortionary policies

There are two parts in the distortionary-policies term in (39), and we will analyze each in turn.

We follow Martin (2011) to interpret the expression $\mathcal{P}(B',s')(u'_N + u'_{NN}\mathcal{C}^{N\prime})\mathcal{C}_B^{N\prime}$. The envelope condition from the household's problem implies $V_m = u_N/p^N$ (see derivation in Appendix A.1). When using equilibrium condition (14) we obtain $V_m = u_N\mathcal{C}^N$, which states the equilibrium value of entering the period with an additional unit of domestic currency. We can further establish how this value changes with debt: $dV_m/dB = (u_N + u_{NN}\mathcal{C}^N)\mathcal{C}_B^N$. Hence, first part of the distortionary-policies term in (39) reflects how a change in debt directly affects the demand for money and, therefore, how tightly the government budget constraint binds. Note that this first part of the channel is multiplied by $\mathcal{P}(B',s')$, implying that it only operates within repayment states today and tomorrow (recall that $\bar{\mathcal{C}}_B^N = 0$).

The level of debt affects the level of implemented distortions since the policy instruments available to the government are distortionary; these expected distortions, in turn, affect the demand for money. Two opposing forces determine how higher debt and distortions affect the demand for money in equilibrium: a substitution effect and an income effect. The substitution effect dictates that larger distortions should lead to lower consumption and lower demand for money to finance non-tradable goods. The income effect induces households to want to mitigate

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the drop in consumption due to larger distortions; hence, it increases the demand for money. Which effect dominates depends on the curvature of $u$, more specifically, the sign of $u_N + u_{NN}c^N$. If this term is positive (negative), the substitution (income) effect dominates.

The second part of the term is the expression $(u'_N \mathcal{C}^{N\prime} - \bar{u}'_N \bar{\mathcal{C}}^{N\prime})\mathcal{P}'_B$, where $\mathcal{P}'_B = -\mathcal{P}(B', s')(1 - \mathcal{P}(B', s'))(\xi'/\kappa) < 0$. As explained above, $V_m = u_N c^N$; hence, this term reflects the impact of debt choice on the current money demand, through the change in the repayment probability. As the government issues more debt, it lowers the probability of repayment, $\mathcal{P}'_B < 0$. This matters to domestic households since the value of an extra unit of money depends on whether the government repays or defaults, as policies (and distortions) are different in each case. Again, the sign of $u'_N \mathcal{C}^{N\prime} - \bar{u}'_N \bar{\mathcal{C}}^{N\prime}$ depends on the curvature of $u$. Assuming that $\mathcal{C}^{N\prime} > \bar{\mathcal{C}}^{N\prime}$ (an assumption we verify numerically), the difference is positive (negative) if the substitution (income) effect dominates.

So, how does the distortionary-policies channel operate? First, issuing more debt today alters future fiscal and monetary policies in the repayment state, as well as the probability of repayment. Second, since domestic policy instruments are distortionary, anticipated changes in these future policies alter the marginal value of money tomorrow and, hence, households' current money-holding decisions. Third, the change in future repayment probability also alters the expected marginal value of money tomorrow, as policies differ if the government repays or defaults. Fourth, these changes in the current demand for money affect the real value of domestic government liabilities and hence, the government's budget constraint. Importantly, this effect is not internalized by the government tomorrow, which results in a time-consistency problem, as the government values current debt issuance differently today and tomorrow.

The distortionary-policies channel alters how the other two components in (39) are traded off when the government decides how much debt to issue. The effect may be positive, zero, or negative, depending on the assumptions on preferences, thus altering the standard tradeoff in sovereign debt choice. For example, if the utility is logarithmic, then $u_N + u_{NN}\mathcal{C}^N = u'_N \mathcal{C}^{N\prime} - \bar{u}'_N \bar{\mathcal{C}}^{N\prime} = 0$, and so there is no time-consistency problem due to the interplay between debt policy and the demand for money. In this case, the government would trade off its present bias with the default risk premium; i.e., the desire to accumulate debt is moderated by the extra financial cost of supporting it due to the higher default probability. Suppose instead that $u_N + u_{NN}\mathcal{C}^N < 0$, which also implies $u'_N \mathcal{C}^{N\prime} - \bar{u}'_N \bar{\mathcal{C}}^{N\prime} < 0$, as argued above. Given that $\mathcal{C}^N_B < 0$ (since higher debt implies larger distortions and, hence, lower consumption) and $\mathcal{P}^N_B < 0$ (as shown above), the distortionary-policies term would now be positive, countering the default

premium term and reinforcing the present bias term. That is, when the income effect dominates the substitution effect in the preference for the non-tradable good (and the demand for money), the distortionary-policies channel provides additional incentives to accumulate debt. In the opposite case, $u_N + u_{NN}\mathcal{C}^N > 0$, which implies $u'_N \mathcal{C}^{N\prime} - \bar{u}'_N \bar{\mathcal{C}}^{N\prime} > 0$, the distortionary-policies channel mitigates the incentive to accumulate debt.

## 3.6 Comparison to real models of sovereign default

Here, we show that our setting encompasses the celebrated Eaton and Gersovitz (1981) economy. Suppose that lump sum, unconstrained taxes $\mathcal{T}$ are available. In such a case, the government budget constraint (3) becomes $p^N(g + \gamma) + eB \leq \mathcal{T} + \tau wh + eqB'$. Proceeding as we did to derive (19), lump sum taxes can be written in terms of allocations as follows:

$$u_T[c^T - \gamma p^T (F_N/F_T)] - v_\ell F(c^N + g, y^T) + \beta\mathbb{E}\left[u'_N c^{N\prime}|\mathcal{I}, s\right] + \mathcal{T} \geq 0 \tag{40}$$

for $\mathcal{I} = \{P, D\}$. The problem of the government in the repayment state is (PP) subject to (26), (28) and (40) for $\mathcal{I} = P$. Similarly, when in default, the problem of the government is (DP) subject to (29), (31) and (40) for $\mathcal{I} = D$.

Consider now the following centralized version of the government's problem, where the only constraint is the balance of payments. In the repayment state, the problem of the government is

$$\max_{(B', c^N, c^T, y^T, g)} u(c^N, c^T) + v(1 - F(c^N + g, y^T)) + \vartheta(g) + \beta\mathbb{E}[\mathcal{V}(B', s')|s] \tag{PPEP}$$

subject to $p^T y^T - c^T + Q(B', s)B' - B = 0$. The problem of the government in the default state is defined similarly. The solution to problem (PPEP), denoted $(\hat{B}', \hat{c}^N, \hat{c}^T, \hat{y}^T, \hat{g})$, will be referred to as the *EG real allocation*. The associated lump-sum taxes necessary to finance this allocation are given by

$$\hat{\mathcal{T}} = -\hat{u}_T \hat{c}^T + \gamma \hat{u}_T p^T (\hat{F}_N/\hat{F}_T) + v_\ell F(\hat{c}^N + \hat{g}, \hat{y}^T) - \beta\mathbb{E}\left[\hat{u}'_N \hat{c}^{N\prime}|P, s\right] \tag{41}$$

We now establish the following equivalence result between the two problems.

**Proposition 2.** *Given lump sum, unconstrained taxes $\hat{\mathcal{T}}$ given by (41), the EG real allocation $(\hat{B}', \hat{c}^N, \hat{c}^T, \hat{y}^T, \hat{g})$ solves the problem* (PP), *with the corresponding value of default, $V^D(s)$, that solves* (DP), *and the constraints* (28), (31) *and* (40), *for $\mathcal{I} = \{P, D\}$, are slack.*

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*Proof.* See Appendix A.3. □

This result implies that when lump-sum taxes are available, the EG real allocation can be decentralized as a competitive equilibrium as follows. The government finds it optimal to (i) set the distortionary tax rate $\tau$ equal to zero and (ii) conduct monetary policy $\mu$ so that the cash-in-advance constraint in the household's problem does not bind, i.e., implement the Friedman rule given by

$$\hat{\mu} = \frac{\beta \mathbb{E}\left[\hat{u}'_N \hat{c}^{N\prime}|B, \mathcal{I}, s\right]}{\hat{u}_N \hat{c}^N} - 1. \tag{42}$$

Lump-sum taxes adjust so that under these policies the government budget constraint is satisfied with no need for distortions. Hence, in this version of the model, the government budget constraint is no longer a restriction in the government's problem. In effect, the policy regime becomes Ricardian.

We can write the analog of the Generalized Euler Equation (39) for the case with unconstrained lump-sum taxes as follows:

$$\mathbb{E}\left\{\mathcal{P}(B', s')\left[\frac{u_T}{1+r} - \beta u'_T - \frac{u_T B'(1 - \mathcal{P}(B', s'))u'_T}{\kappa(1+r)}\bigg|s\right]\right\} = 0. \tag{43}$$

In this case, the multiplier of the balance of payment constraint, $\xi$, is equal to $u_T$.[5] We can see that, with lump-sum taxes, the government trades off distortion-smoothing (with present bias) and the default premium. The intertemporal tradeoff due to distortionary policies is absent since the government budget constraint is automatically satisfied with lump-sum taxes and, thus, is no longer a binding restriction to policy implementation.

We now argue that the EG real allocation cannot be decentralized in the absence of lump-sum taxes, i.e., cannot be implemented if only distortionary policy instruments are available. To show this result, we focus on the case when $\hat{\mu} \leq 0$, which is isomorphic to requiring a non-negative implicit real interest rate on a domestic bond denominated in non-tradable goods.

**Proposition 3.** *Suppose that $\hat{\mu}$, given by (42), is non-positive. If lump-sum, unconstrained taxes are not available, then there are no feasible monetary and fiscal policies that decentralize the EG real allocation.*

*Proof.* See Appendix A.3. □

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[5]Note that with distortionary taxes, $\xi$ is not equal to $u_T$ in general; from (33), this would require either the government budget constraint to be slack, i.e., $\lambda = 0$, or $\lambda(u_T + u_{TT}c^T) - (\gamma\lambda + \zeta)\Gamma_T = 0$. This last case obtains when $u_T + u_{TT}c^T = \gamma = 0$, while it cannot happen when $u_T + u_{TT}c_T > 0$ and may be possible when $u_T + u_{TT}c^T < 0$.

To grasp the idea behind this result, observe that in order to implement the EG real allocation, the government budget constraint must be $\hat{e}[\hat{Q}(B', s)B' - B] = \hat{p}^N(\hat{g} + \gamma) - \hat{\mu}$. Since $\hat{g} + \gamma \geq 0$ and implementing the EG real allocation requires $\hat{\mu} \leq 0$, the left-hand side of this expression is strictly positive; hence, the government would need to run a Ponzi scheme, with $B' > B$. This would lead to $\hat{Q}(B', s) = 0$ in finite time and, thus, a contradiction.

# 4   An economy with unanticipated aggregate shocks

In this section, we study the behavior of key variables in a version of our economy with no expected aggregate shocks other than the extreme value shocks, $\varepsilon$. We start by calibrating and analyzing an economy with no shocks, i.e., when $s = \bar{s} \in S$ in all periods. Then, we study the effects of unanticipated shocks followed by perfect foresight dynamics. Most of the insights derived from this analysis will carry over to an economy with expected shocks, such as the one we study Section 5.

## 4.1   Functional forms

The utility functions for consumption and leisure are, respectively,

$$
\begin{aligned}
u(c^N, c^T) &= \alpha^N \frac{\left(c^N\right)^{1-\sigma^N}}{1 - \sigma^N} + \alpha^T \frac{\left(c^T\right)^{1-\sigma^T}}{1 - \sigma^T}, \\
v(\ell) &= \alpha^H \frac{\ell^{1-\varphi}}{1 - \varphi}.
\end{aligned}
$$

We let $\sigma^N = \sigma^T = \sigma$, which implies that $1/\sigma$ represents both the intratemporal elasticity of substitution between $c^N$ and $c^T$ and the intertemporal elasticity of substitution.

The utility function for the public good is

$$
\vartheta(g) = \alpha^G \ln g,
$$

which is a standard representation in the optimal taxation literature and close to empirical estimates.[6]

---

[6]A more general representation with constant relative risk aversion, $\alpha^G(g^{1-\nu} - 1)/(1 - \nu)$, converges to log utility as $\nu$ approaches 1. Nieh and Ho (2006) estimate values of $\nu$ around 0.8. Azzimonti et al. (2016), among others, use log utility for the public good. See also the discussion in Debortoli and Nunes (2013).

The function describing the labor requirement for production is

$$F(y^N, y^T) = \frac{\left[(y^N)^\rho + (y^T)^\rho\right]^{1/\rho}}{A},$$

where $A$ is a measure of labor productivity and $\rho$ determines how costly it is to change the composition of $y^N$ and $y^T$ that is produced, in terms of labor units.

Finally, we assume that the economy experiences a drop in productivity when the government is in default. Following the work of Arellano (2008), we allow this penalty to vary with the state of the economy. Productivity, while in the default state, takes the following form:

$$A^{def} = A \times [1 - \Omega(s)] \tag{44}$$

with

$$\Omega(s) = \max\left\{\omega_1 + \sum_{i=1}^{N} \omega_{2,i} \frac{(s_i - \bar{s}_i)}{\bar{s}_i}, 0\right\}, \tag{45}$$

where $\omega_1 > 0$ is the intercept, $N$ is the number of shocks and the slope parameter vector $\omega_2$ makes the default cost a function of the vector of shocks, $s$. The value of the shock $\bar{s}_i$ represents the steady-state value of the shock $s_i$. Note that when the economy has no shocks ($s = \bar{s}$), as the economy we study in this section, the cost of default is determined only by $\omega_1$.

## 4.2 Calibration

Table 1 shows the values of the parameters set externally. The risk-free interest rate is set at an annual 3%, in line with the average real interest rate of the world since 1985 in King and Low (2014). We calibrate the value of $\varphi$ to 1.50 so that the Frisch elasticity is one-half on average.[7] Considering the duration of a default episode from Das et al. (2012) and the length of exclusion after restructuring from Cruces and Trebesch (2013), we choose an expected period of exclusion after a default of 6 years, which implies $\delta = 1/6$.[8]

We set $\sigma_N = \sigma_T = 0.5$. As shown in the previous section, $\sigma_T < 1$ is a sufficient condition for the non-negativity constraint in the government's problem to be satisfied with strict inequality (it is also necessary when transfers $\gamma$ are zero). In addition, $\sigma_N < 1$ implies the distortionary-policies channel has a negative sign, mitigating the incentives to accumulate debt. This choice implies that imported goods are gross substitutes for non-tradable goods, as in the estimates of

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[7]We can calibrate this parameter externally because we target the value of $h$.

[8]Here, the duration of exclusion is exogenous. The country reenters with no debt after exclusion. See Dvorkin et al. (2021) for a model of endogenous restructuring.

Table 1: Exogenous parameters

| Parameter | Description | Value | Basis |
|-----------|-------------|-------|-------|
| $r$ | risk-free rate | 0.03 | long-run average |
| $\varphi$ | curvature of leisure | 1.50 | Frisch elasticity |
| $\delta$ | reentry probability | 0.17 | exclusion duration |
| $\alpha^T$ | preference share for $c^T$ | 1.00 | normalization |
| $\sigma^N$ | curvature of $c^N$ | 0.50 | see appendix B.3 |
| $\sigma^T$ | curvature of $c^T$ | 0.50 | see appendix B.3 |
| $\rho$ | elasticity of substitution between $y^N$ and $y^T$ | 1.50 | see appendix B.2 |
| $p^T$ | terms of trade | 1.00 | normalization |

Ostry and Reinhart (1992).[9]

The value of $\rho$, which determines the elasticity of substitution between $y^N$ and $y^T$ in the cost function, is set to 1.5. A number larger than 1 guarantees that $F$ is convex and, thus, ensures that the production possibilities frontier is concave.

In Appendix B.3 we support our choices of $\sigma$ and $\rho$ by studying the reaction of macroeconomic variables to shocks to the terms of trade. Though we find that many results are robust to the choice of parameter values, the reaction of spreads, inflation, output and exports favor our benchmark calibration.

Finally, $p^T$, which denotes the terms of trade, is set to 1. Our calibration strategy, described below, is such that all external variables are pinned down by other parameters. As such, any value of $p^T$ delivers the same observables, except for the exchange rate, which we do not target. In Section 5 we allow the terms of trade to evolve stochastically and calibrate the stochastic process for $\ln p_t^T$ to match the data.

The remaining parameters are calibrated *jointly* to match a set of long-run averages. We use data collected by the World Bank for Argentina, Brazil, Chile, Colombia, Mexico, Peru and Uruguay for the period 1991–2018 because of data availability. A significant fraction of the parameters can be calibrated in this simplified setup without shocks, reducing the time it takes to calibrate the model and allowing the model to fit the targets exactly. We will mention the critical parameter that reproduces each moment as we explain the choice of targets for exposition. Table 6 in Appendix B.2 shows the marginal reaction of moments to parameters.[10]

We set $A$ so that the steady state real GDP is equal to 1, making some statistics easier to read. The value of the discount factor $\beta$ helps the model produce an annual 3.8% inflation rate.

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[9]However, the estimates in Ostry and Reinhart (1992) are in the range of 1.22-1.27 and our calibration implies an elasticity equal to 2. In Appendix B.3, we study how our results change when setting $\sigma^N = \sigma^T = 1.5$, which implies that the goods are complements with an elasticity of 0.66.

[10]The corresponding expressions for each target are presented in Appendix B.1.

Table 2: Calibration parameters and targets

| Parameter | Value | Statistic | Target/Non-stochastic Model |
|---|---|---|---|
| $A$ | 1.4575 | Real GDP | 1.000 |
| $\beta$ | 0.8675 | Inflation, % | 3.800 |
| $\gamma$ | 0.1082 | Transfers/GDP | 0.117 |
| $\alpha^N$ | 2.6888 | Exports/GDP | 0.209 |
| $\alpha^H$ | 0.9265 | Employment/Population | 0.587 |
| $\alpha^G$ | 0.4240 | Gov. Consumption/GDP | 0.133 |
| $B^d$ | 0.1854 | Debt/GDP | 0.185 |
| $\omega_1$ | 0.0228 | Haircut, Share of Debt | 0.305 |
| $\kappa$ | 0.0235 | Default, % | 0.700 |

The parameter $\gamma$ matches the ratio of transfers to GDP, which in the data average 11.7%. The value of $\alpha^H$ allows the model to hit the long-run average for the employment-to-population ratio, 0.59. The weight in the utility of the government consumption good, $\alpha^G$, delivers government consumption over GDP of 13.3%. The parameter $\alpha^N$ takes a value that allows the model to reproduce the ratio of exports to GDP, which is 21% in the data.

The values of $\omega_1$ and $B^d$ determine the costs and benefits of default, respectively. Thus, they are used to reproduce the implied haircut obtained by the country in default, which is 30.5% (Dvorkin, Sánchez, Sapriza, and Yurdagul, 2021), and the external debt-to-GDP ratio, which is 18.5% in our data set.

The scale parameter in the distribution of taste shocks, $\kappa$, determines the risk of sovereign default in the steady state and is calibrated to reproduce a default rate of 0.7% annual. We choose a default rate target that is lower than the more typical 2% since in this version of the model, default only occurs due to the extreme value "non-fundamental" shocks.[11]

## 4.3 Equilibrium policy

Figure 1 shows equilibrium policies in the repayment and default states as functions of beginning-of-period debt. These policy functions are computed using the benchmark calibration from Table 2.[12]

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[11]See the numbers calculated by Tomz and Wright (2013) for different sets of countries. Alternatively, we could target the average EMBI for these countries. Matching the average for this period, about 300 basis points, would require a higher value of $\kappa$ and a smaller value of $\beta$. That calibration, which we also experimented with, yields similar results.

[12]The equilibrium is solved globally using the equations derived above. The algorithm uses 100 equally spaced gridpoints between 0 and 1 for debt. Note that using as few as 10 gridpoints results in visually indistinguishable (interpolated) equilibrium functions. Figure 1 shows debt up to 0.5 since the probability of repayment is essentially zero for higher values. We use cubic splines to interpolate the evaluation of future policies and their derivatives. As a reference of precision, the sum of squared residuals for the Generalized Euler Equation is $7e^{-21}$. See also

Figure 1: Equilibrium policies as functions of debt
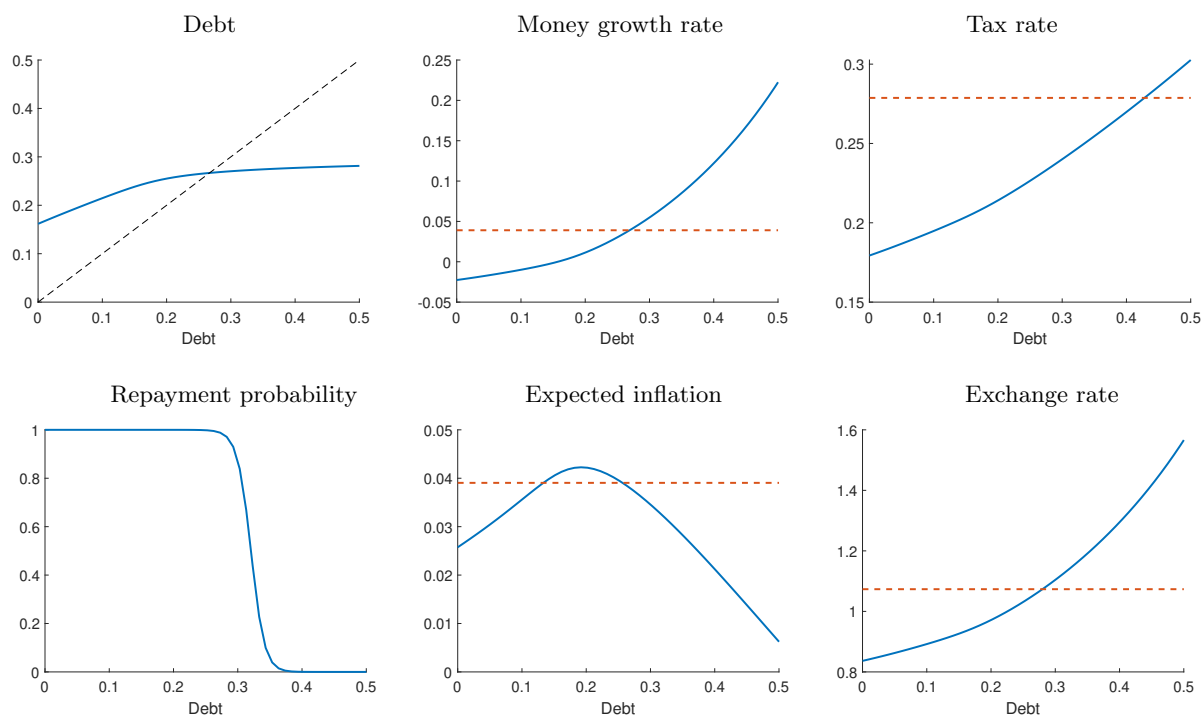


Note: Policy functions in repay are solid lines, while in default are the dashed ones.

As we can see from Figure 1, conditional on repayment, debt choice, the money growth rate, the tax rate and the exchange rate are all increasing in debt. In contrast, the repayment probability is decreasing in debt as the incentives to default become larger. Higher debt implies a higher interest rate due to a lower repayment probability and so requires larger distortions to finance it. Hence, domestic policy involves faster money printing and higher taxation as debt grows.

Policy remains constant while the country is in default. The difference between the policy functions in default and repayment is informative. For example, if the country starts with $b = 0.4$ (roughly 50% above the steady state) and the government decides to repay its debt, which is extremely unlikely, the money growth rate is about 12%; in contrast, if it decides to default, the money growth rate declines to 4%.

Conditional on repaying today, expected inflation between today and tomorrow (expected, since there are two possible states tomorrow, repayment or default) is non-monotonic in debt. It is increasing for low values of debt and decreasing for higher values of debt. Note that inflation, measured as the increase in the price level of consumption goods, has two components: non-tradable and imported. The expected inflation in nontradable goods increases with debt

because the money growth rate increases with debt. However, expected inflation in imported goods is decreasing in debt for large debt values. This pattern occurs because conditional on repayment, an increasing proportion of debt must be repaid, resulting in exchange rate appreciation. Combining these two forces results in the non-monotonicity of overall expected inflation as a function of debt.[13]

In terms of allocations, Figure 10 in Appendix B.3 shows that in the benchmark economy, $c^N$, $c^T$, $g$, $y^N$ and $h$ are decreasing in debt, while $y^T$ is increasing in debt. Again this follows from higher debt implying larger domestic policy distortions (higher $\mu$ and $\tau$) and a higher exchange rate (which discourages imports and promotes exports).

## 4.4 Dynamics after an unexpected shock

We now study dynamics to further illustrate the model's inner working and understand the impact of different shocks. We consider the effect of an unexpected shock at $t = 0$; the shock is then assumed to gradually die out following a known process that ends at date $t = T$.[14] Hence, once the process for a particular parameter is determined, the model is solved backwards from $t = T$ to $t = 0$, assuming that at $t = T + 1$ the economy returns to normal. We then simulate the path of the economy starting from its steady state at $t = -1$ and assuming that the extreme value shocks are small enough to not trigger default. For a given parameter $x$, we assume an after-shock value, $x_0$, and then a process $x_t = \bar{x} + \rho_x(x_{t-1} - \bar{x})$, where $\bar{x}$ corresponds to its pre-shock (steady state) value (see Table 2).

We consider adverse shocks to the (log of) terms of trade, $\ln p^T$; productivity, $A$; and government transfers, $\gamma$. The sizes of the shocks were chosen such that all of them imply a similar impact on inflation.[15] For the persistence, we set $\rho_p = 0.8803$, as estimated in Appendix C.2, and $\rho_A = \rho_\gamma = 0.9$ so that all shocks have similar duration. In all cases, we assume $\omega_{2,x} = 0$ so that the simulations do not include the added impact of changing the cost of default due to an adverse shock. Figure 2 shows the simulated path for selected variables in response to the three alternative shocks described above.

All shocks considered imply similar dynamics: real output falls, while debt over GDP, inflation, depreciation, spreads and the primary deficit increase.[16]

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[13]A similar result is obtained if we instead define inflation as the increase in the GDP deflator.

[14]In later sections, we analyze the economy assuming these shocks are fully anticipated.

[15]We set $\ln p_0 = -0.0756$, $A_0 = \bar{A} \times 0.99$ and $\gamma_0 = \bar{\gamma} \times 1.1$.

[16]We define spreads as the yield of the bond maturing this period, given the level of debt, $B$, and the realization of shocks, $s$, but before the realization of the extreme value shocks $\varepsilon$, and minus the yield of a risk-free bond. That is, $\text{Spread}(B, s) = \left( \mathcal{P}(B, s) + (1 - \mathcal{P}(B, s)) \frac{B^d}{B} Q^d(s) \right)^{-1} - 1$.

Figure 2: Dynamics following unexpected adverse shocks
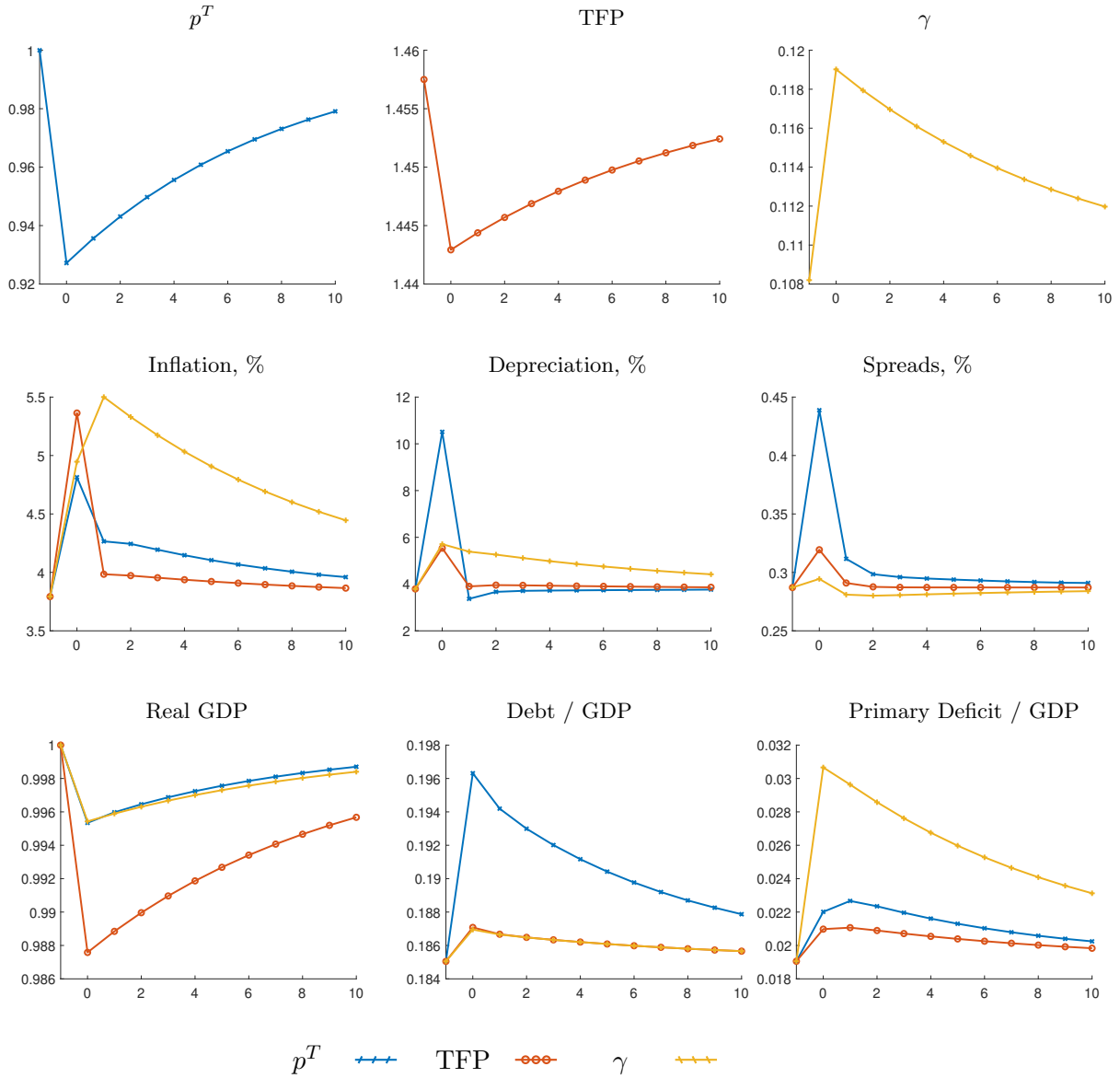
It is worth noting that, though debt over GDP increases, debt in dollars actually decreases very slightly in response to these shocks. If we allowed for $\omega_2 \neq 0$ (as we do in Section 5), then there would be an additional reason to lower the debt in response to adverse shocks, as the incentives to default and, hence, the spreads would increase further.[17]

We can see that shocks to the terms of trade have a significant impact on currency depreciation, spreads, and debt over GDP (and this last one mainly through the effects of the depreciation). Shocks to productivity affect mainly real GDP. Shocks to transfers impact government policy: inflation, currency depreciation and the primary deficit all increase persistently.

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[17]For this mechanism to work we would need $\omega_{2,p} > 0$, $\omega_{2,A} > 0$ and $\omega_{2,\gamma} < 0$.

# 5 An economy with terms-of-trade shocks

This section analyzes a version of our economy when the terms of trade, $p^T$, evolve stochastically over time. We start from the benchmark calibration of the previous section and modify it so that the model with anticipated shocks predicts a higher default rate and fits the targeted levels of debt and haircuts. Then, we show how model-simulated data compare with the actual data from the seven countries used in the calibration.

## 5.1 Recalibration

The calibration of the model with terms-of-trade shocks uses the same parameters as in Table 2 except for two. First, the value of debt after default, $B^d$, now takes the value 0.149 so that the model with shocks reproduces the same targeted median haircuts. Second, the value of the cost of default, $\omega_1$, takes the value 0.059 to match the debt level.

We also need to calibrate $\omega_2$, which from (45) determines how the cost of default changes as the economy deteriorates. This parameter played no role in the non-stochastic economy but is critical for determining the economy's default rate with aggregate shocks. We set $\omega_2 = 0.656$ so that the model replicates a default rate of 2%. Figure 3 displays the total cost of default as a function of the terms of trade, $p^T$. The total cost of default in the steady state, which is given exclusively by $\omega_1$, is close to 6%. With the choice of $\omega_2$, this cost decreases to zero when the terms of trade are slightly less than 10% below the steady state and increases to 20% when $p^T$ is 20% above the steady state.

Figure 3: Cost of default in terms of reduction in TFP
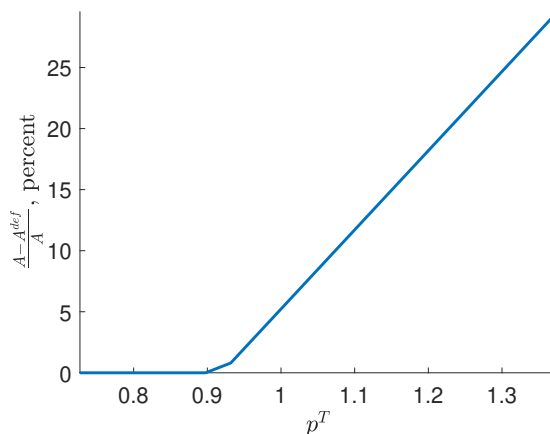


We assume the terms of trade follow the process $\ln(p^T_{t+1}) = \rho_p \ln(p^T_t) + \varepsilon_{t+1}$, where $\varepsilon \sim N(0, \sigma^2_p)$. The estimated values are $\rho_p = 0.8803$ and $\sigma_p = 0.0756$.[18]

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[18]In Appendix C.2, we show that the estimations of this process for each of the seven countries we use for our

Since our model adds several new features to the standard sovereign default model, we evaluate how well the model reproduces relevant properties of the data. To do this, we simulate data from the model and compute some key moments that are presented together with their data counterpart.

## 5.2 Business cycle statistics

A standard practice is to compare key macroeconomic statistics in the model and the data. To do this comparison, we use time series for the seven countries in our sample. We detrend GDP and consumption using a band-pass filter as in Christiano and Fitzgerald (2003) to separate a time series into trend and cyclical components.[19] Table 3 shows that fluctuations in output, exports and the trade balance are smaller in the model than in the data. Underpredicting the volatility of these variables is to be expected given that the model only has one shock driving all fluctuations.[20] In the empirical literature, the percent of fluctuations in output accounted for by terms-of-trade shocks varies between about 10% to 40% depending mostly on the country and period considered (Drechsel and Tenreyro, 2017; Schmitt-Grohé and Uribe, 2018). Our model with only terms-of-trade shocks generates 17% of the variance of GDP in the data.

Table 3: Business Cycles and Policy Statistics

| Moment | Model | Data |
|---|---|---|
| Std. Dev. (y) | 0.038 | 0.016 |
| Std. Dev. (trade balance/Y) | 0.035 | 0.017 |
| Std. Dev. (c) / Std. Dev. (y) | 1.193 | 2.478 |
| Std. Dev. (spreads) | 3.923 | 3.303 |
| Std. Dev. (exports/Y) | 0.052 | 0.021 |
| Std. Dev. (depreciation) | 0.196 | 0.102 |
| Correlation(trade balance/Y,y) | -0.357 | -0.177 |
| Correlation(c,y) | 0.846 | 0.589 |
| Correlation(spreads,y) | -0.362 | -0.073 |
| Correlation(exports/Y,y) | -0.178 | -0.141 |
| Correlation(depreciation,y) | -0.252 | -0.226 |
| Correlation(depreciation,spreads) | 0.431 | 0.205 |

Note: Data for Argentina, Brazil, Chile, Colombia, Mexico, Peru and Uruguay from 1980 to 2018. Y refers to nominal GDP; c and y are the cyclical components of real consumption and real GDP per capita, respectively. See appendix C.1 for details.

calibration and for a series for commodity prices yield similar results.

[19]We set the minimum and maximum periods of oscillation of cyclical component at 2 and 16 years, respectively.

[20]In the model the only source of volatility is terms-of-trade shocks. These shocks have a larger effect on consumption than in output because they induce the sovereign to reduce the external debt position. An economy with other sources of fluctuation, such as TFP shocks, will likely have a relatively larger volatility in output than in consumption, bringing the model closer to the data (see Figure 2).

Though not targeted in the calibration, the model reproduces the correlations between output and other relevant variables we observe in the data. For example, the model replicates two salient properties of emerging markets, namely, that the ratio of trade balance-to-GDP and bond spreads are counter-cyclical. The model also reproduces the negative correlation between the ratio of exports-to-GDP and economic activity. This occurs because, both in the model and the data, downturns are associated with currency depreciations.

Table 3 shows a positive correlation between the currency depreciation rate and the spreads in sovereign debt. This result connects our work with the work of Na, Schmitt-Grohé, Uribe, and Yue (2018), although through a different mechanism. They emphasize that devaluations are optimal during a debt crisis because they allow for a decline in real wages when nominal wages are rigid. In our model, a negative shock to the terms of trade implies both a rise in spreads and the rate of currency depreciation. Note, however, that the correlation between currency depreciation and spreads is not unique or entirely attributable to the shock to the terms of trade. As previously shown in Figure 2, other shocks would explain this fact as well. The additional mechanism at work here is the endogenous loosening of monetary policy as a response to adverse shocks.

Overall, our findings highlight that (i) the model captures fundamental macroeconomic mechanisms well and (ii) terms-of-trade shocks are an important source of fluctuations for emerging markets.

## 5.3 Fluctuations in fiscal and monetary policies

This subsection asks two related questions: How do monetary and fiscal policies vary over the business cycles when the government cannot commit to future policies? and Does the model capture the cyclicality of policy in emerging markets?

First, we study the cyclicality of monetary policy. Following Figure 2 in Vegh and Vuletin (2015), we compute the correlations between the cyclical components of the inflation tax and real GDP. They define the inflation tax as inflation/(1+inflation). We detrend the series using Christiano and Fitzgerald (2003), as in the previous subsection. This allows us to compare the model and the average of the data for the seven countries in our sample. The results, presented in Table 4, show that in recessions, when real output falls, the inflation tax increases both in the model and in the data. In our model, this occurs because as the terms of term deteriorate, output declines, the exchange rate depreciates, risk of default increases, and the government increases the money growth rate. Importantly, recall from Figure 2 in the previous section that

in our model a similar pattern would emerge with recessions induced by TFP or government expenditure shocks.

Table 4: Policy over the business cycle

| Moment | Model | Data |
|---|---|---|
| Corr. (inflation tax, y) | -0.5261 | -0.2143 |
| Corr. (real expenditure, y) | 0.4666 | 0.2600 |
| Corr. (personal income tax rate, y) | -0.1223 | -0.1710 |

Note: Data are the average of Argentina, Brazil, Chile, Colombia, Mexico, Peru and Uruguay. The variable y is the cyclical component of real GDP per capita.

Second, we analyze the cyclicality of fiscal policy. In this case, we consider two variables that are government choices in our model: expenditures and labor taxes. Following Figure 1 in Vegh and Vuletin (2015), we proceed as with the inflation tax and compute the correlation between the cyclical components of real government expenditure and real GDP, both in the model and the data. Similarly, we study the behavior of taxation by looking at the cyclical components of the personal income tax rate and real GDP. The results, presented in Table 4, show that government expenditure decreases and taxes increase during bad times—the government follows "austerity policies" during recessions. This policy, which is optimal in our model because they government must often repay debt in "bad times," is typical of emerging markets. Fiscal policy's procyclicality stands in stark contrast with the behavior in developed economies, where fiscal policy is generally countercyclical (Frankel et al., 2013). For example, in Japan, the correlation between the cyclical component of output and real government expenditure was −0.2 during the same period.

The fact that the model reproduces these correlations is a valuable validation of the most significant contribution of our model: incorporating domestic policies in a model of sovereign default.

## 5.4 Response to terms-of-trade shocks

This section analyzes how spreads, inflation and real GDP respond to shocks to the terms of trade, $p^T$. Importantly, none of these responses were targeted in our calibration of the model. The estimation in the data uses local projections, following Jordà (2005). To compare these with the model, we perform the same estimation on simulated time series. All the responses, which are plotted in Figure 4, correspond to a 10% decline in the terms of trade.

Comparing the response of the spread to a terms-of-trade shock is useful to evaluate how we

Figure 4: Response of variables to a 10% decline in $p^T$



Note: See Appendix C.3 for more information on the regressions.

model the government's incentives to default on sovereign debt—in particular, our assumptions for the functional form of the cost of default. The first panel in Figure 4 shows that, in the data, a 10% decline in the terms of trade increases the EMBI spread by about 50 basis points, an effect that persists over the next year and then declines to zero.[21] The dashed red line corresponds to the same estimation using model-simulated data. The response of the model is larger in the first period but lower in the second period. Notably, the overall response has the same sign and a similar magnitude.

Next, we compare inflation's reaction to a shock to the terms of trade (middle panel in Figure 4), which is useful to test our assumptions for the inclusion of money in the model. The results suggest that a 10% fall in the terms of trade in the data implies an increase in inflation of slightly less than 2 percentage points.[22] As in the case of the EMBI spread, the model's reaction is initially more considerable than in the data, 3 percentage points in this case, and also less persistent.

The last panel of Figure 4 displays the impact of a terms-of-trade shock on real GDP. Among other things, this comparison helps validate our choices for the curvature of the utility function $u(c^N, c^T)$ and the elasticity of substitution between $y^T$ and $y^N$ in the production cost function. In the data, the effect is significantly different from zero but has large standard errors. On average, real GDP growth falls by close to 1 percentage point the year of a 10% fall in the terms of trade. Replicating such an effect in the model is challenging. Kehoe and Ruhl (2008)

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[21]As a comparison, Drechsel and Tenreyro (2017) finds that "a 10 percent deviation of commodity prices from their long-run mean can move Argentina's real interest spread by almost 2 percentage points" (i.e., 200 basis points). Our estimates are more conservative because we include more countries with fewer debt crises than Argentina.

[22]We truncated inflation at 100% annual. Otherwise, hyperinflation episodes in Argentina and Brazil dominate the value of any statistic.

show that in a multi-sector model, the first-order effect of changes in the terms of trade on real GDP is zero. Our model's structure allows for a novel mechanism, as policy distortions need to increase to repay the sovereign debt when the terms of trade deteriorate. This mechanism generates a 0.5 percentage points decline the year of the shock, which is smaller than the point estimate in the data but within 2 standard errors.[23]

## 5.5   The role of distortionary domestic policies

In this section, we argue that adding distortionary policies improves the ability of the standard sovereign default model to account for important properties of emerging markets.

The EG version of our model with lump-sum taxes differs from standard models, such as Arellano (2008), in several key aspects. First, our model allows for production, while standard models typically assume output is an endowment, though there are some exceptions (e.g., Cuadra, Sánchez, and Sapriza, 2010 and Mendoza and Yue, 2012). We do this since there is no role for distortionary taxation in an endowment economy. Second, we allow for tradable and non-tradable goods. Although some papers in the literature allow for tradable and non-tradable goods, they assume there is a stochastic endowment of tradable goods (see Bianchi, Ottonello, and Presno, 2019 and Bianchi and Sosa-Padilla, 2020). Third, our model incorporates money and monetary policy. As shown in Proposition 2, when lump-sum taxes are available, the cash-in advance constraint is slack and monetary policy is not distortionary.

Table 5 compares our benchmark with two versions of the model with lump-sum taxes. In the first version, we keep the benchmark parameters; in the second version, we recalibrate the economy to match the same moments as in our benchmark model, except for inflation which it cannot match by construction—see Proposition 2. Since we lose a moment, we leave $\beta$, which we use primarily to target inflation, as in the benchmark economy. Given that economies with lump-sum taxes cannot match the observed rates of inflation and currency depreciation, we focus our comparison in Table 5 on real and external variables.

Following a negative shock to the terms of trade, the exchange rate depreciates and this leads to a decline in non-tradable output and an increase in tradable output. However, in the benchmark economy, the decline in non-tradable output is deeper, while the increase in tradable output is milder. As a result, output declines in the benchmark economy while it increases when lump-sum taxes are available. What explains this difference in outcomes? With lower terms of

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[23]In the literature, there exist other channels to generate a larger effect of terms-of-trade shocks on output. See Kohn et al. (forthcoming).

Table 5: The Role of Distortionary Policies

| | Data | Benchmark | EG version | |
| | | | Non-recalibrated | Recalibrated |
|---|---|---|---|---|
| Std. Dev. (y) | 0.038 | 0.016 | 0.018 | 0.016 |
| Std. Dev. (trade balance/Y) | 0.035 | 0.017 | 0.016 | 0.018 |
| Std. Dev. (c) / Std. Dev. (y) | 1.193 | 2.478 | 1.866 | 2.096 |
| Std. Dev. (spreads) | 3.923 | 3.303 | 3.415 | 2.996 |
| Std. Dev. (exports/Y) | 0.052 | 0.021 | 0.022 | 0.023 |
| Correlation(trade balance/Y, y) | -0.357 | -0.177 | 0.249 | 0.293 |
| Correlation(c,y) | 0.846 | 0.589 | 0.288 | 0.196 |
| Correlation(spreads,y) | -0.362 | -0.073 | 0.059 | 0.079 |
| Correlation(exports/Y,y) | -0.178 | -0.141 | 0.124 | 0.148 |

trade, the ability of the government to repay its debt decreases, which implies a rise in debt spreads. This prompts a reduction in debt which needs to be financed. If the government has access to lump-sum taxes, then the rise in taxation implies a decrease in household income, which in turn, decreases consumption and leisure. As a consequence, employment and thus, output, increases. When the government does not have access to lump-sum taxes, as in our benchmark economy, the increase in taxation discourages employment and explains the contraction in output. Since debt spreads, exports and the trade balance move in the same direction in both models, the correlation of these variables with output goes in opposite directions.

Regardless of which of the two calibrations of the EG version of our model is considered, our results and analysis strongly suggest that ruling out unconstrained lump-sum taxation and allowing for distortionary taxation improves the correlation of output with critical variables, such as debt spreads, exports-to-GDP and the trade balance. Table 8 in the appendix shows that this result also holds in two economies recalibrated after setting $\rho = 2$ and, alternatively, $\sigma = 1.5$.

# 6 How external factors and domestic policy shape economic outcomes: the case of Argentina, 2005-17

One of the classic debates in explaining outcomes in emerging countries concerns the role of external factors and government policy. In Latin America, there is a long tradition connecting economic performance with the evolution of commodity prices. In recent work, Drechsel and Tenreyro (2017) and Fernández et al. (2018) find that the contribution of commodity price shocks to output fluctuations is close to 40%. Poor economic performance is also attributed to

government policy, e.g., high fiscal deficits and inflation. Recently, Kehoe et al. (2020) take this view by stating, "our fundamental hypothesis is that, despite their different manifestations, all economic crises in Latin America have been the result of poorly designed or poorly implemented macro-fiscal policies."

This section sheds light on this debate using a calibration of external factors and domestic policies for Argentina, from 2005 to 2017. During this period, Argentina fits the profile of a commodity-exporting emerging economy with rising fiscal deficit and inflation. Argentina experienced favorable terms of trade (good luck) due to a global boom in commodity prices. At the same time, government spending grew considerably and inflation rose significantly. Our exercise consists of simulating the Argentine experience and running counterfactuals to understand the role of external factors (favorable terms of trade) and domestic policy (increased government expenditure coupled with accommodative monetary policy).

## 6.1 Calibrating the driving forces: Argentina 2005–2017

After the default of 2001, Argentina had only a limited number of debt issuances until 2016. To capture this period, we assume that between 2005 and 2015, Argentina was excluded from international credit markets, but it was no longer suffering a TFP penalty for being in the default state. The assumption of exclusion is evaluated below in a counterfactual exercise. The assumption of no TFP cost is motivated by two facts: (i) the period of analysis starts towards the end of the expected exclusion period and (ii) there was no output boom when Argentina fully reentered credit markets in 2016.[24]

Except for the changes outlined above, we use the calibration from Section 5, which, recall, targets the average of several Latin American countries for 1991-2017. We use this economy as a starting point and subject it to the changes in terms of trade and government spending experienced by Argentina from 2005 onward.

The terms of trade are expected to evolve as described in the previous section and the simulation follows a particular realization path, as observed in the data. The initial value for the terms of trade are set at $p^T = 0.913$, which, in the data, matches the value in 2005 relative to the average between 1991 and 2017.[25] The increase in government expenditure is modeled as a series of unexpected, permanent shocks. Since the actual levels in Argentina do not

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[24]Note that we maintain the assumption that the country would experience a TFP penalty if it were to default in the future.

[25]We also computed the exercises for different initial values of $p^T$. Our results do not change fundamentally if we instead started at the average value for the terms of trade.

Figure 5: Exogenous changes
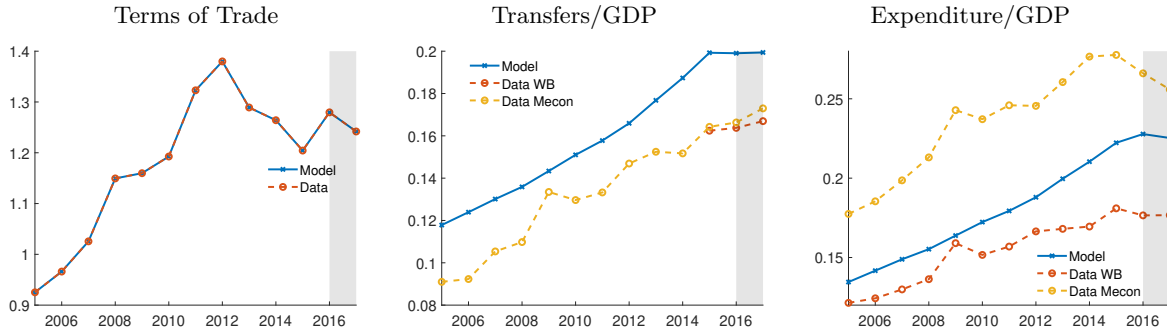


Note: The shaded areas correspond to the period in which Argentina had access to credit markets. "WB" is the World Bank, and "Mecon" is the Ministry of Economics in Argentina. For Terms of Trade we report the data of the World Bank. More details on the sources are presented in Appendix C.1.

exactly match our calibration, we instead target the changes in transfers and total expenditure. Specifically, we model a series of unexpected and permanent increases in two model parameters. First, we increase the transfer $\gamma$ to match the increase in transfers to GDP in the data (middle panel). Second, we raise the multiplicative parameter in the government good's utility function, $\alpha_G$, to match the increase in total government expenditure to GDP (right panel).[26]

Figure 5 shows the evolution of the exogenous driving forces of the Argentine experience. The blue solid lines show the outcomes from the model simulation, while the other lines represent data according to different sources, as appropriate. The left panel shows the evolution of the terms of trade, the other two panels show the increase in transfers and total expenditure, both in terms of GDP.

We compare the evolution of key macro variables in the model to those in the data to show that the simulation captures the Argentine experience reasonably well. Figure 6 presents the evolution of six macroeconomic variables. The top panel shows that the tax revenue-to-GDP ratio (which, recall, equals the tax rate in the model) and inflation increased significantly to finance government expansion while debt grew once the country reentered credit markets. The bottom left panel shows that the currency depreciation rate also increased during this period.[27] The other two bottom panels show that the paths for real GDP and GDP measured in dollars match the data well. Overall, these findings suggest that although other things occurred in Argentina during this period, the driving forces we model capture a significant share of the macroeconomic performance.

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[26]Specifically, we assume that between 2005 and 2015, $\gamma$ and $\alpha_g$ grow at 5.14% and 6.40% per year, respectively.

[27]During part of this period, Argentina had multiple nominal exchange rates. The figure shows the devaluation in the official exchange rate (retrieved from the World Bank database) and in the *black market* rate (as reported in recent updates of the data set constructed in Ferreres, 2005).

Figure 6: Comparison of Key Macro Variables with Argentinean Data
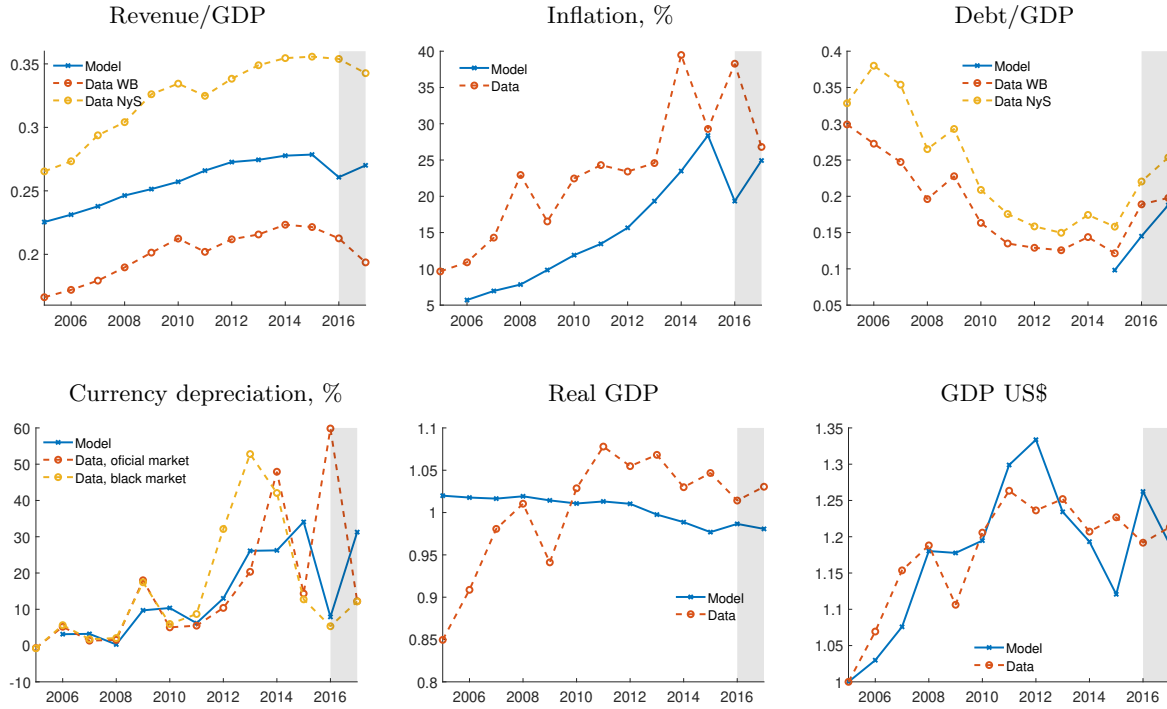


Note: The shaded areas correspond to the period in which Argentina had access to credit markets. "WB" is the World Bank, and "NyS" corresponds to recent updates of the data set constructed by Ferreres (2005). Data for inflation is from "NyS". For depreciation, the official exchange rate is retrieved from the World Bank and in the *black market* as reported in "NyS". Real GDP is from the WB, and GDP in US$ is from "NyS". More details on the sources are presented in Appendix C.1.

## 6.2 The role of external factors and domestic policy

Having shown that the model fits the evolution of macroeconomic variables in Argentina reasonably well, we proceed to perform some counterfactual exercises. We start by redoing our simulation assuming only one of the driving forces is present at a time. Figure 7 shows the evolution of several macroeconomic variables under alternative scenarios: only shocks to $p^T$, only fiscal expansion and the benchmark with all shocks.

The improvement in the terms of trade does not contribute to the increase in government expenditure, though it has a clear effect on revenue. Similarly, more favorable terms of trade do not explain the increase in inflation and depreciation, but they do contribute to short-term fluctuations in these variables. We can also see that the rise in the terms of trade have a positive effect on real GDP, although it is not very significant.

The expansion of government explains the rise in inflation and currency depreciation and, in part, the increase in tax revenue. The permanent nature of these changes implies that the impact on debt is minor. Importantly, the increase in distortions associated with larger expenditure proves a significant drag to the real economy, pushing real GDP down.

Figure 7: Evolution of Key Macro Variables in Alternative Scenarios

All shocks ✶✶✶   Only $p^T$ shocks ⊶⊶⊶   Only fiscal expansion ●—●—●
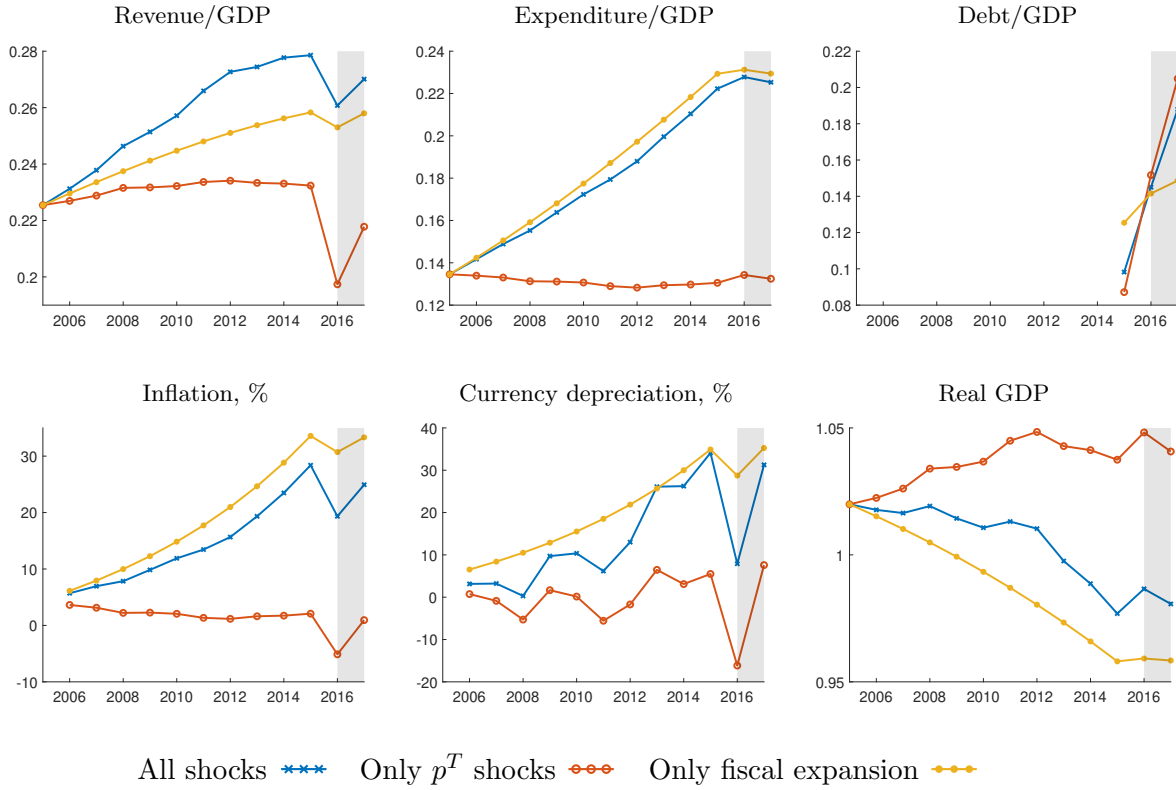
Our simulations suggest that, had there not been a dramatic expansion of the government, Argentina would have been more able to reap the benefits of favorable terms of trade: inflation and currency depreciation would have been low and output high. Note, however, that the rapid expansion of foreign currency debt after the country regained access to credit markets appears entirely attributable to international circumstances rather than the profligacy of its government.

## 6.3   The role of monetary policy

A natural follow-up question is, what would have happened had Argentina adopted a more disciplined monetary policy? Figure 8 shows the effects of assuming a constant money growth rate throughout this period, a type of policy that was eventually adopted in late 2018 to curb inflation.[28]   Relative to our benchmark, this monetary policy would have kept inflation low and stable (except for the year in which the country reentered international credit markets). In addition, currency depreciation would have mostly been impacted by the variation in the terms of trade. However, implementing a constant money growth rate would have required a significant increase in taxation to finance the expansion of government. This suboptimal choice

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[28]In our simulation, we set the money growth rate equal to the average rate in the benchmark economy, $\mu = 0.0418$.

Figure 8: Evolution of Key Macro Variables with Constant $\mu$ and Reentry in 2005

Benchmark ✕✕✕    Constant $\mu$ ○○○    Reentry in 2005 •••
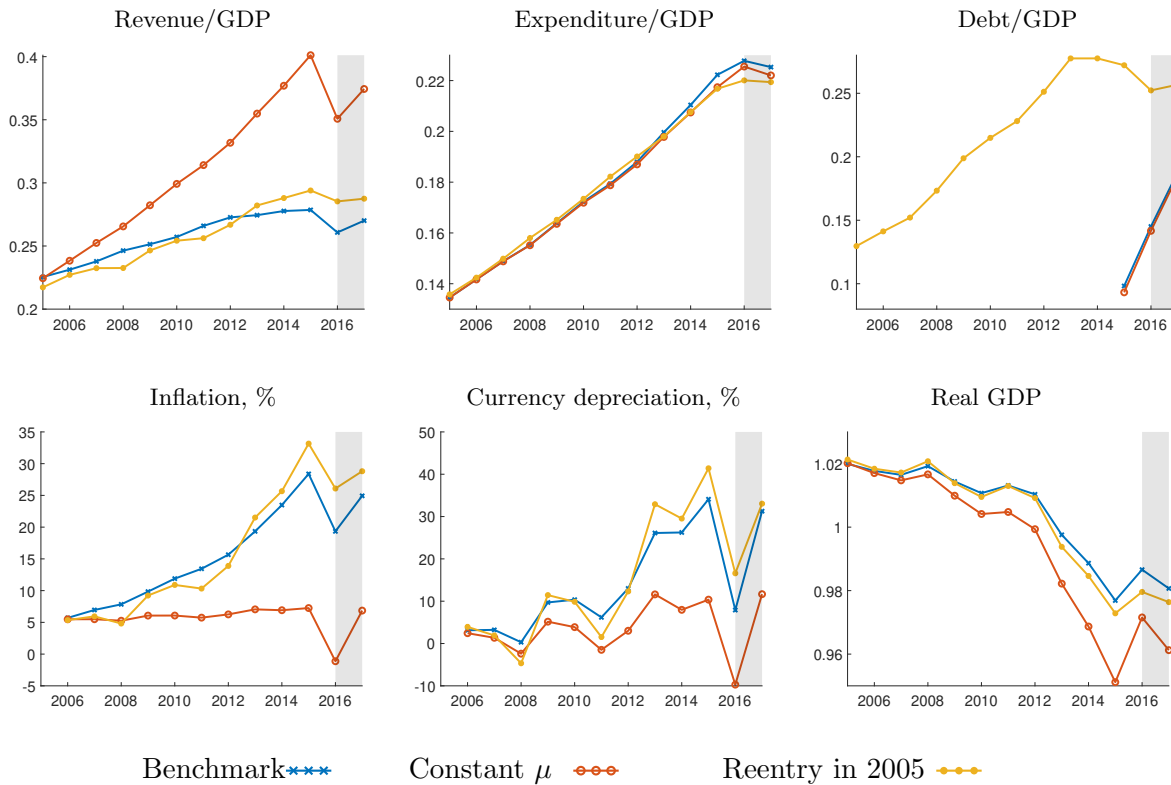
of distortions would have resulted in a lower real GDP.

To better understand the counter-cyclical role of policy in offsetting terms-of-trade shocks, we now focus on 2013, when the terms of trade fell by 6.6% after reaching their peak in 2012. Relative to the benchmark, the effect of this event when adopting a constant monetary policy is smaller on inflation (0.8 percentage points vs 3.7 percentage points) and currency depreciation (8.6 percentage points vs 13.1 percentage points). However, taxes must increase more (2.3 percentage points vs 0.2 percentage points), so the decline in real GDP is more significant (−1.7% vs −1.3%).

These results provide a cautionary tale for policy recommendation: adhering to a conservative monetary policy in the face of an expanding government may succeed in keeping inflation under control, but without addressing the underlying issue, it may also lead to a deeper economic problem. In addition, a strict monetary policy hinders the government's ability to handle external shocks. In our counterfactual exercise, we allowed revenue to adjust as necessary, so the cost is borne by the real economy in the form of lower real output. One could also imagine a scenario in which taxes cannot be raised sufficiently due to political considerations. If the central bank were not to relent under these circumstances, this scenario would likely lead to a

debt crisis.

## 6.4  The role of exclusion from international credit markets

From 2005 to 2015, we assumed the country was excluded from international credit markets since, in reality, access to external credit was minimal during this period. We now study the role of this exclusion by simulating an economy that can issue external debt throughout the entire period. The yellow lines in Figure 8 follow the economy with access to international credit markets, as highlighted by the evolution of debt in the right top panel.

Except for debt issuance, the differences with the benchmark economy are relatively small. Having access to external credit allows the government to implement lower distortions (taxes and inflation) early on, but at the cost of increasing them in later years as debt builds up and its service costs increase. At the end of the period of analysis, real GDP is slightly lower with access to credit than in the benchmark economy.

Again, we find it instructive to focus on the reaction of policy and real variables in 2013, when the terms of trade dropped sharply after reaching their peak. *A priori*, one would think that having access to external debt would enable the government to better smooth the shock. However, for the case when the government has access to international credit markets as of 2005, the country reaches 2012 with a high debt-to-GDP ratio—recall that, in the benchmark, the government is still not allowed to issue external debt. Relative to the benchmark simulation, when the government has access to international credit, there is a larger reaction of inflation (7.6 percentage points vs 3.7 percentage points) and currency depreciation (20.6 percentage points vs 13.1 percentage points), which lead to a larger contraction in real GDP ($-1.5\%$ vs $-1.3\%$). Evidently, the presence of high debt, and therefore, high policy distortions, limits the capacity of the country to absorb the fall in the terms of trade and implies a more extreme response.

## 7  Concluding remarks

Emerging economies experience recurrent debt crises, in part, due to their tendency to over-borrow during good times. Their fragility to adverse shocks is likely also a consequence of inadequate economic policy frameworks; for example, a lack of fiscal discipline and an excessive reliance on seigniorage and currency depreciation.

Our paper connects domestic policies to sovereign default and economic outcomes. We modeled fiscal and monetary policies as being inherently distortionary and assumed the government

lacks commitment to both external credit repayment and the conduct of its domestic policies. Our framework led to new insights on the tradeoffs faced by governments when deciding their level of indebtedness, the probability of repayment and the determination of domestic policies. We then showed that the model is able to reproduce standard business cycle statistics and the dynamic responses of policies and macroeconomic aggregate to terms-of-trade shocks in emerging markets.

We used the experience of Argentina in 2005-2017 to study the role of external shocks and domestic policies in shaping economic outcomes. During that period, Argentina experienced exceptionally favorable terms of trade, but also embarked on a significant expansion of its government. The lesson we draw is that the rise in spending explained the increase in taxation, inflation and currency depreciation and low output growth.

In follow-up research, we use the setup developed here to analyze the fiscal and monetary policy response to the COVID-19 pandemic in emerging markets. We model the pandemic as a "perfect storm" combining four unexpected shocks and decompose their individual contribution to policy and economic outcomes.

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# Appendix

# A  Theory

## A.1  Derivations

In order to characterize the solution to the household's problem, let $\chi$ and $\psi$ denote the Lagrange multipliers associated with constraints (8) and (2), respectively. The necessary first-order conditions with respect to $(c^N, c^T, h, m')$ for an interior solution are

$$u_N - p^N(\chi + \psi) = 0, \tag{46}$$

$$u_T - e\chi = 0, \tag{47}$$

$$-v_\ell + \chi(1 - \tau)w = 0, \tag{48}$$

$$\beta\mathbb{E}\left[V'_m | \mathcal{I}, s\right] - \chi(1 + \mu) = 0, \tag{49}$$

where $V_m$ denotes the partial derivative of $V$ with respect to the individual state variable, $m$. The corresponding envelope condition implies that $V_m = \chi + \psi$. From (46) and (47) we can solve for the Lagrange multipliers,

$$\chi = \frac{u_T}{e}, \tag{50}$$

$$\psi = \frac{u_N}{p^N} - \frac{u_T}{e}. \tag{51}$$

Replacing these expressions in (48) and (49) yields (9) and (10). Using (7) to replace $e$ in (51) and imposing $\psi \geq 0$ yields (13).

*The government budget constraint*

Take the government budget constraint (5), multiply both sides by $F_N c^N$ and use (12), (14), (15) and (16) to obtain

$$\tau F(y^N, y^T) + F_N(\mu c^N - g - \gamma) - (F_T/p^T)(p^T y^T - c^T) \geq 0.$$

Next, replace the tax rate, $\tau$, using (17) and the money growth rate, $\mu$, using (18) to obtain the government budget constraint in a competitive equilibrium,

$$[1 - (F_T/p^T)(v_\ell/u_T)]F(c^N + g, y^T) - F_N(c^N + g + \gamma) + \beta(F_T/p^T)\mathbb{E}\left[u'_N c^{N\prime} | \mathcal{I}, s\right]/u_T - (F_T/p^T)(p^T y^T - c^T) \geq 0.$$

Since $F(y^N, y^T) = F_N y^N + F_T y^T = F_N(c^N + g) + F_T y^T$ we obtain

$$(F_T/p^T) \left\{ c^T - (v_\ell/u_T)F(c^N + g, y^T) + \beta \mathbb{E}\left[u'_N c^{N'}|\mathcal{I}, s\right]/u_T \right\} - \gamma F_N \geq 0,$$

which after multiplying both sides by $u_T(p^T/F_T)$ implies (19).

## A.2  Domestic policy with transfers

We now relax the assumption of $\gamma = 0$ and allow transfers to be positive; i.e., $\gamma \geq 0$. The necessary first-order conditions with respect to $(c^N, c^T, y^T, g)$ are

$$u_N - v_\ell F_N - \lambda(F_N \Phi + \gamma \Gamma_N) + \zeta(u_{NN} - \Gamma_N) = 0, \tag{52}$$

$$u_T - \xi + \lambda(u_T + u_{TT}c^T - \gamma \Gamma_T) - \zeta \Gamma_T = 0, \tag{53}$$

$$-v_\ell F_T + \xi p^T - \lambda(F_T \Phi + \gamma \Gamma_y) - \zeta \Gamma_y = 0, \tag{54}$$

$$-v_\ell F_N + \vartheta_g - \lambda(F_N \Phi + \gamma \Gamma_g) - \zeta \Gamma_g = 0. \tag{55}$$

Suppose that the non-negativity constraint (28) does not bind; i.e., $\zeta = 0$. Given $\Gamma_N = \Gamma_g$, (52) and (55) imply

$$u_N = \vartheta_g. \tag{56}$$

Using (52) and (54) to solve for the Lagrange multipliers we obtain

$$\lambda = \frac{u_N - v_\ell F_N}{F_N \Phi + \gamma \Gamma_N},$$

$$\xi = \frac{u_N F_T \Phi + \gamma[v_\ell F_T \, \Gamma_N + (u_N - v_\ell F_N)\Gamma_y]}{p^T(F_N \Phi + \gamma \Gamma_N)},$$

and so (53) implies

$$F_T \Phi(u_N - \Gamma) - \gamma \left[(u_T p^T - v_\ell F_T)\Gamma_N - (u_N - v_\ell F_N)\Gamma_y\right] = p^T(u_N - v_\ell F_N)(u_T + u_{TT}c^T - \gamma \Gamma_T), \tag{57}$$

where we used the definition of $\Gamma$ to simplify the expression.

## A.3  Proofs

*Proof of Proposition 1.* Note that (28) implies $u_N - \Gamma \geq 0$. Since $\Phi > 0$, the left-hand side of (37) is non-negative. Next, $\lambda > 0$ implies $u_N - v_\ell F_N > 0$. Hence, $u_N - \Gamma > 0$ if and only if $u_T + u_{TT}c^T > 0$, while $u_N - \Gamma = 0$ if and only if $u_T + u_{TT}c^T = 0$. If preferences are such

that $u_T + u_{TT}c^T < 0$, then (37) cannot be satisfied—a contradiction. In this case, $\zeta > 0$ and therefore, (28) binds. $\qquad\square$

*Proof of Proposition 2.* Consider the EG real allocation $(\hat{B}', \hat{c}^N, \hat{c}^T, \hat{y}^T, \hat{g})$ that solves the problem (PPEP) where lump-sum, unconstrained taxes $\mathcal{T}$ make the government budget constraint becomes (41). In order to prove this result, we first solve the problem (PPEP) and then we construct the monetary policy and taxes $(\hat{\mu}, \hat{\tau})$ as well as the prices $(\hat{p}^N, \hat{e}, \hat{w})$ that support this allocation as an equilibrium in our setting.

The necessary first-order conditions characterizing the EG real allocation are

$$\hat{u}_N = \hat{v}_\ell \hat{F}_N, \tag{58}$$

$$\hat{v}_\ell \hat{F}_T = p^T \hat{u}_T, \tag{59}$$

$$\hat{\vartheta}_g = \hat{v}_\ell \hat{F}_N, \tag{60}$$

which imply $\frac{\hat{u}_N \hat{F}_T}{p^T} = \hat{u}_T \hat{F}_N$; i.e., the non-negative constraint (28), which we ignore to derive the EG real allocation, is satisfied with equality. The balance of payment implies

$$p^T \hat{y}^T - \hat{c}^T + \hat{Q}(B', s)\hat{B}' - \hat{B} = 0. \tag{61}$$

We now construct the policies and prices that support the EG real allocation, we have that the price of non-tradable goods and wages are determined by

$$\hat{p}^N = \frac{1}{\hat{c}^N},$$

$$\hat{w} = \frac{\hat{p}^N}{\hat{F}_N},$$

while the exchange rate is determined by

$$\hat{e} = \frac{\hat{p}^N}{p^T} \frac{\hat{F}_T}{\hat{F}_N},$$

The monetary policy has to be tailored so that

$$\hat{\mu} = \frac{\beta \mathbb{E}\left[\hat{u}'_N \hat{c}^{N\prime} | B, \mathcal{I}, s\right]}{\hat{u}_T \hat{c}^N p^T (\hat{F}_N / \hat{F}_T)} - 1,$$

as it has to decentralize money holdings such that $m' = m = 1$. Since $\frac{\hat{u}_N \hat{F}_T}{p^T} = \hat{u}_T \hat{F}_N$, we obtain

$$\hat{\mu} = \frac{\beta \mathbb{E}\left[\hat{u}'_N \hat{c}^{N'}|B, \mathcal{I}, s\right]}{\hat{u}_N \hat{c}^N} - 1. \tag{62}$$

On the other hand, taxes are given by

$$\hat{\tau} = 1 - \frac{\hat{v}_\ell}{\hat{u}_T} \frac{\hat{F}_T}{p^T} = 0.$$

Finally, lump-sum transfers are designed to make the budget constraint of the government (41) hold so that

$$\hat{\mathcal{T}} = \hat{p}^N(\hat{g} + \gamma) - \hat{\mu} + \hat{e}(p^T \hat{y}^T - \hat{c}^T).$$

$\square$

*Proof of Proposition 3.* As shown in Proposition 2, the real EG allocation implies zero labor taxes when monetary policy is given by (42). Thus, combining the balance of payments with the government budget constraint when lump-sum taxes are not available implies

$$\hat{e}[\hat{Q}(\hat{B}', s)\hat{B}' - B] = \hat{p}^N(\hat{g} + \gamma) - \hat{\mu}, \tag{63}$$

i.e., a non-linear first-order difference equation in domestic debt, $B$.

First, consider a steady state in an environment with no aggregate shocks $s$. Since $\hat{g} + \gamma \geq 0$ and $\hat{\mu} \leq 0$ (see (62) above), imply $\hat{e} > 0$ and $\hat{Q}(\hat{B}', s) < 1$, it follows that any steady state would require $B < 0$; i.e., the government must accumulate a sufficiently large amount of assets to finance its expenditures. This asset position would never be reached, as (63) implies that the amount of $B$ is strictly increasing and positive when the initial stock of debt is positive.

Consider now the general case in a stochastic environment. Observe that since $\hat{p}^N(\hat{g} + \gamma) - \hat{\mu} \geq 0$, then $B > 0$ implies that $B' > 0$ as

$$\frac{\hat{B}'}{(1+r)} \geq \hat{Q}(\hat{B}', s)\hat{B}' = B + \frac{\hat{p}^N}{\hat{e}}(\hat{g} + \gamma) - \frac{\hat{\mu}}{\hat{e}}, \tag{64}$$

and so $\hat{B}' \geq (1+r)B + (1+r)\left(\frac{\hat{p}^N}{\hat{e}}(\hat{g} + \gamma) - \frac{\hat{\mu}}{\hat{e}}\right)$. Therefore, as $r > 0$, the sequence of debt for this allocation is strictly increasing and unbounded as long as $B_0 > 0$. We argue that this cannot be an equilibrium path. To see this, define $\bar{y}^T$ as the unique solution to $F(0, \bar{y}^T) = 1$, i.e., the highest level of the tradable good that can be produced as $h = 1$ and $\hat{y}^N = \hat{c}^N + \hat{g} = 0$.

Let $\bar{p^T} = \max p^T$ and conjecture that $Q(B', s) = 0$ for all $B' \geq \frac{(1+r)}{r} \bar{p}^T \bar{y}^T$ and all $s$. From the balance of payments, non-default tradable consumption can be written as

$$\hat{c}^T = \hat{Q}(\hat{B}', s)\hat{B}' + p^T \hat{y}^T - B \leq \max_{B'}\{\hat{Q}(\hat{B}', s)\hat{B}'\} + \bar{p}^T \bar{y}^T - B \leq \frac{1}{1+r} \frac{(1+r)}{r} \bar{p}^T \bar{y}^T + \bar{p}^T \bar{y}^T - B.$$
(65)

Therefore, if $B = \frac{(1+r)}{r} \bar{y}^T$, then (65) implies that non-default tradable consumption cannot be positive and leads to a contradiction, as the EG allocation is interior and consequently the outcome must be default; i.e., $Q\left(\frac{(1+r)}{r} \bar{y}^T, s\right) = 0$ for all $s$. Therefore, since $Q$ is decreasing, $Q(B', s) = 0$ for all $B' \geq \frac{(1+r)}{r} \bar{y}^T$ and all $s$ and validates the conjecture.

To conclude the proof, observe that as the sequence of debt would be strictly increasing and unbounded, it would be larger than $\frac{(1+r)}{r} \bar{y}^T$ in finite time and thus contradicts (64) since $Q(B', s) = 0$ for all $B' \geq \frac{(1+r)}{r} \bar{y}^T$ and all $s$ . $\qquad\square$

# B  Quantitative Results

## B.1  Definition of macroeconomic aggregates

- Nominal GDP (in pesos, normalized by the money stock),

$$Y_t = e_t p_t^T y_t^T + p_t^N y_t^N.$$

- GDP in foreign currency (USD),

$$Y_t^{USD} = p_t^T y_t^T + \frac{1}{e_t} p_t^N y_t^N.$$

- The GDP deflator (in pesos, normalized by the money stock)

$$P_t^y = \left(\frac{e p^T y^T}{Y}\right) e_t p_t^T + \left(\frac{p^N y^N}{Y}\right) p_t^N$$

- Real GDP,

$$Y_t^R = \frac{Y_t}{P_t^y}.$$

- Consumption expenditures (in pesos, normalized by the money stock),

$$C_t = e_t c_t^T + p_t^N c_t^N.$$

- Consumption price index (in pesos, normalized by the money stock),

$$P_t^c = \left(\frac{ec^T}{C}\right) e_t + \left(\frac{p^N c^N}{C}\right) p_t^N.$$

- Inflation, measured as the change in the consumption price index,

$$\pi_t = \frac{P_t^C}{P_{t-1}^C}(1 + \mu_{t-1}) - 1.$$

- Currency depreciation

$$\Delta_t = \frac{e_t}{e_{t-1}}(1 + \mu_{t-1}) - 1$$

Note that inflation and currency depreciation are corrected by the money growth rate, since prices are normalized by the money stock.


## B.2  Identification

To provide a heuristic proof of identification, we compute how each parameter would change if we change one target at a time by 10 percent. The results, presented in Table 6, justify the link between parameters and targets mentioned in the calibration section. The first column shows how each parameter change when we target a default rate 10 percent larger, i.e., 1.1 percent instead of 1 percent. Note that the more significant change is for $\kappa$. By increasing $\kappa$ 9.68 percent and adjusting all the parameters (except $\omega_1$) very slightly, the model can replicate all the targets perfectly. Thus, we selected $\kappa$ as the critical parameter to get the default rate.

In the second column of Table 6, we present the percent change in each parameter that would allow the model to replicate a debt to GDP ratio 10 percent larger. In addition to the change in $\kappa$, which we already show is key to replicating the default rate, the most substantial change is in $B_d$ followed by $\omega_1$. Clearly, these parameters are important to determine debt because they determine the benefits and costs of default. We pick $B_d$ for debt because its adjustment is larger and highlights $\omega_1$ for matching haircuts because it is the larger adjustment to match the haircut among the remaining parameters.

Continuing with the same logic, we connect each parameter in Table 6 with a moment.

Table 6: Percent change in each parameter when a target is increased by 10 percent

| | Default | Debt | Haircut | G | Hours | Exports | Inflation | Transfers | Real GDP |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Target increased by 10 percent | | | | |
| $\kappa$ | **9.68** | 8.99 | 6.07 | -11.79 | 57.65 | 6.39 | 7.34 | -22.34 | 4.88 |
| $B_d$ | 0.00 | **10.00** | -4.39 | 0.00 | 0.00 | -4.13 | 0.00 | 0.00 | 10.00 |
| $\omega_1$ | 3.39 | 9.33 | **5.85** | -4.95 | 22.31 | 5.95 | 2.74 | -9.61 | 0.00 |
| $\alpha_g$ | 0.02 | 0.23 | 0.01 | **11.72** | -4.22 | -7.92 | -0.27 | 3.17 | 4.88 |
| $\alpha_h$ | 0.01 | 0.07 | 0.00 | -1.80 | **-27.77** | -6.86 | 0.33 | -1.52 | 4.88 |
| $\alpha_n$ | 0.02 | 0.23 | 0.01 | 0.53 | -4.22 | **-9.32** | -0.27 | 3.17 | 0.00 |
| $\beta$ | -0.01 | -0.09 | 0.00 | -1.54 | 4.40 | 1.25 | **0.64** | -3.07 | 0.00 |
| $\gamma$ | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | -0.15 | 0.00 | **10.00** | 10.00 |
| $A$ | 0.00 | 0.00 | 0.00 | 0.00 | -9.09 | -1.03 | 0.00 | 0.00 | **10.00** |

Note: Each number represents the percentage change in the parameter when the target is increased by 10 percent.

## B.3 The choice of $\rho$ and $\sigma$

This section discusses the choice of $\sigma_N = \sigma_T = 0.5$ and $\rho = 1.5$ by comparing the results for alternative parameters. In particular, we consider $\sigma_N = \sigma_T = 1.5$ and $\rho = 2$. Recall that we set $\sigma_N = \sigma_T = 0.5$ because $\sigma_T < 1$ is sufficient for the non-negativity constraint in the government's problem to be satisfied with strict inequality (it is also necessary when transfers $\gamma$ are zero). The value of $\rho$ determines the elasticity of substitution between $y^N$ and $y^T$ in the cost function and is set to 1.5. A number larger than 1 ensures that the production possibilities frontier is concave.

To be able to perform this comparison, we re-calibrate the model without terms-of-trade shocks twice to make sure that the model with $\sigma = 1.5$ and the one with $\rho = 2$ fit the targets well. Next, Figure 9 shows how policies and allocations change with the alternative calibrations.

The economy calibrated with $\sigma = 1.5$ shows some important qualitative differences with the benchmark economy. First, the money growth rate is decreasing in debt over the relevant range, while expected inflation takes a faster dive as debt increases. Though not shown, $\mu$ becomes increasing for sufficiently high debt, but this occurs for a range of debt that makes the government likely to default. In terms of allocations, the one critical difference is that hours is increasing rather than decreasing in debt. These differences serve to identify which calibration of the model fits the data better. More on this below.

The economy calibrated with $\rho = 2$ is qualitatively similar to the benchmark economy. Differences are quantitative, though they are calibrated to match the same steady state statistics. In this sense, varying $\rho$, i.e., the elasticity of substitution between nontradable output and exports in the cost function, changes how the economy behaves outside the steady state and, thus,

Figure 9: Policies as functions of debt, alternative calibrations

Benchmark —— $\sigma = 1.5$ --- $\rho = 2$ ······
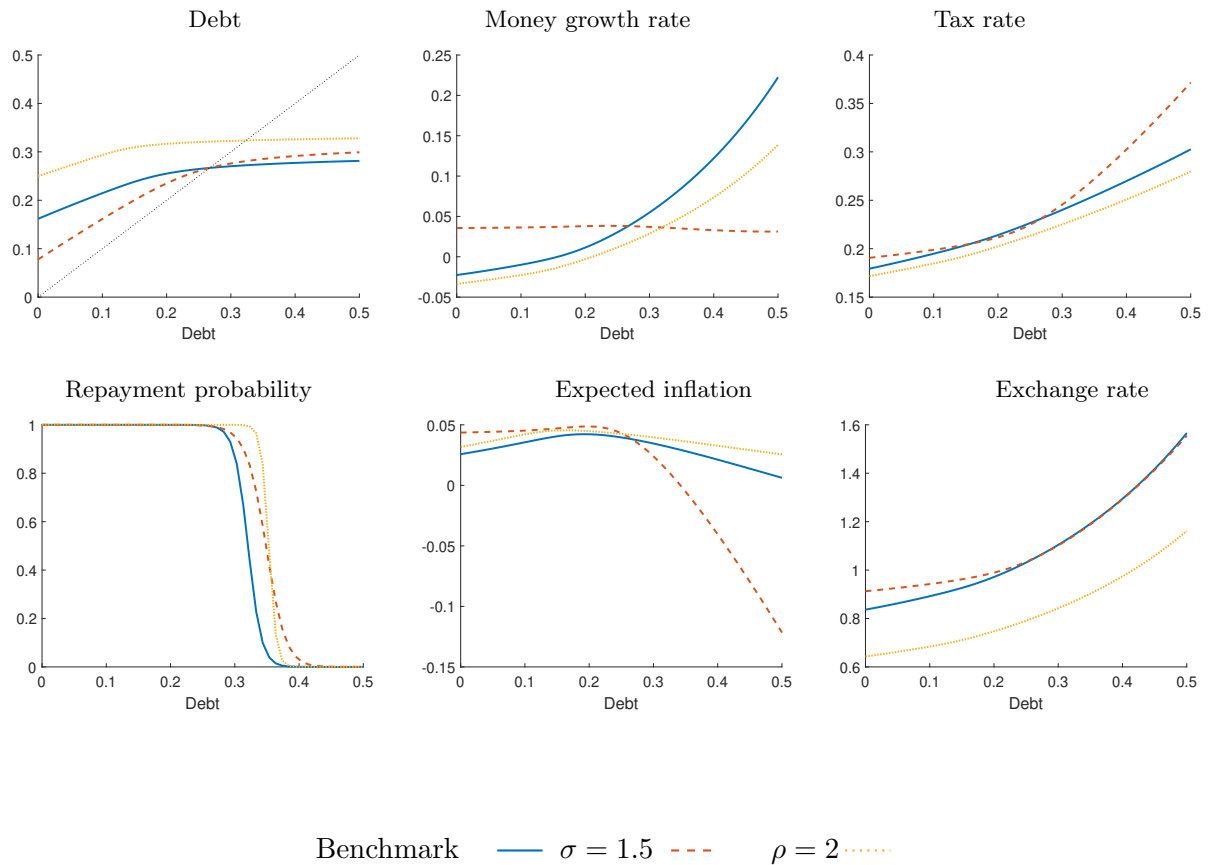
determine how it reacts to shocks.

We now study dynamics to further illustrate the model's inner working and the differences across different calibration. In Figure 11, we consider here the reaction to unexpected shocks to terms of trade. When we consider the economy calibrated with $\sigma = 1.5$ we see important differences: real GDP and exports actually increase in response to a fall in export prices. This is a critical source of identification for which calibration is preferred; as we describe in more detail below, the behavior of output and exports favors the benchmark calibration. Another interesting difference is the behavior of inflation. In the benchmark economy, inflation increases and exhibits persistence; in the economy calibrated with $\sigma = 1.5$, inflation first increases above and then falls below the steady state, thus displaying more erratic dynamics and less persistence.

The economy calibrated with $\rho = 2$ is qualitative similar to the benchmark economy. In this case, identification of the preferred value for $\rho$ is quantitative. As Figure 11 shows, the response of the spread is much more pronounced when $\rho$ is calibrated to a higher value.

We also re-calibrated the model with terms-of-trade shocks to evaluate the non-targeted statistics in Tables 3-4. We present all the moments in Table 7, where we added at the bottom

Figure 10: Equilibrium allocations when repaying as functions of debt, alternative calibrations
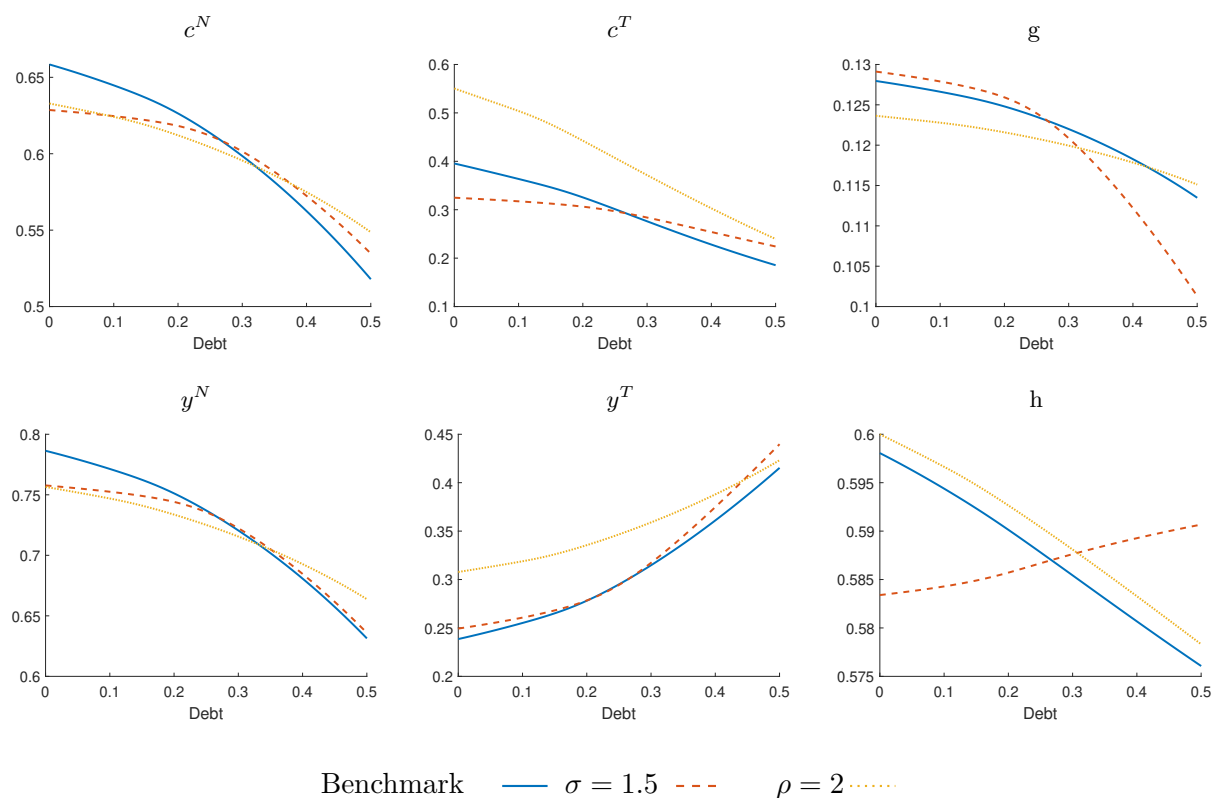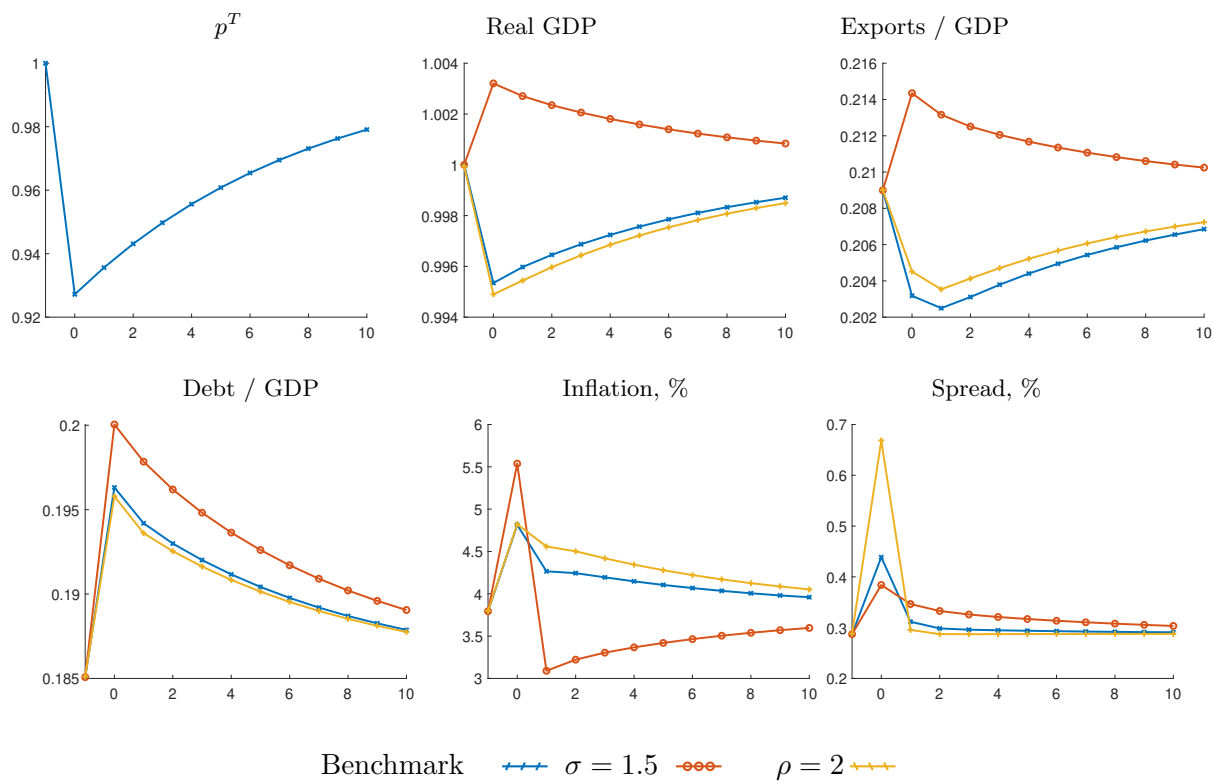


Benchmark ——— $\sigma = 1.5$ - - - - $\rho = 2$ ·········

Figure 11: Dynamics following unexpected adverse shock to $p^T$, alternative calibrations



Benchmark —+—+— $\sigma = 1.5$ —o—o—o— $\rho = 2$ —*—*—

the average absolute distance to the moments. This last statistic is revealing of how better our

preferred calibration fits these moments. This measure of the fit of non-targeted moments in twice as large for the economy with $\rho = 2$ and more than three times larger in the economy with $\sigma = 1.5$.

In particular, we find that the economy with $\sigma = 1.5$ does a worse job fitting these moments because it generates a positive correlation of trade balance/Y with output, a negative correlation of real expenditure with output, no correlation of real consumption and real output, and acyclical inflation. In the case of the economy with $\rho = 2$, the most poorer fit is due to the fact that it generates a positive correlation of exports/Y with output and very pro-cyclical tax rates.

Table 7: Business Cycles and Policy Statistics for Alternative Parameters

| Moment | Bench. | $\rho = 2$ | $\sigma = 1.5$ | Data |
|---|---|---|---|---|
| Std. Dev. (y) | 0.0182 | 0.0080 | 0.0087 | 0.0553 |
| Std. Dev. (trade balance/Y) | 0.0166 | 0.0079 | 0.0068 | 0.0351 |
| Std. Dev. (c) / Std. Dev. (y) | 2.0205 | 3.0225 | 1.6456 | 1.2455 |
| Std. Dev. (spreads) | 4.1396 | 3.7378 | 1.2486 | 3.9227 |
| Std. Dev. (exports/Y) | 0.0207 | 0.0141 | 0.0124 | 0.0515 |
| Correlation(trade balance/Y, y) | -0.0815 | -0.0616 | 0.1985 | -0.3518 |
| Correlation(c, y) | 0.6335 | 0.7755 | -0.0753 | 0.8658 |
| Correlation(spreads, y) | -0.1252 | -0.0929 | 0.3088 | -0.3873 |
| Correlation(exports/Y, y) | -0.0545 | 0.3421 | 0.3011 | -0.1808 |
| Correlation(inflation tax, y) | -0.3152 | -0.4278 | -0.0402 | -0.1986 |
| Correlation(real expenditure, y) | 0.3229 | 0.4691 | -0.1869 | 0.2557 |
| Correlation(Personal income tax, y) | 0.0208 | 0.3740 | 0.2690 | 0.0014 |
| Average absolute distance to data | 0.1810 | 0.3406 | 0.5605 | – |

Note: Data is the average of the numbers Argentina, Brazil, Chile, Colombia, Mexico, Peru and Uruguay. The variable y is the cycle of GDP. The inflation tax is defined as in Vegh and Vuletin (2015): inflation/ ( 1 + inflation).

Finally, we analyze how our conclusions about the performance of the EG version of our model shown in Table 5 of the paper change when we recalibrate the model for different values for $\rho$ and $\sigma$. The results are presented in Table 8. The main result is that our conclusion are robust to the values of $\rho$ and $\sigma$. In particular, the correlations of trade balance, exports, and debt spreads with real GDP have a positive sign, contrary to what we observe in the data.

## B.4 Policy functions in an economy with terms-of-trade shocks

The equilibrium is solved globally, using the equations derived above. The algorithm uses 21 equally spaced gridpoints for debt, between 0 and 1.5, and 21 gridpoints for $p^T$ estimated with the Tauchen method with a bandwidth of 2 (i.e., a multiple 2 of the unconditional standard

Table 8: The Role of Distortionary Policies, Alternative Parameters

| | Data | Benchmark | Recalibrated EG version $\rho = 2$ | $\sigma = 1.5$ |
|---|---|---|---|---|
| Std. Dev. (y) | 0.0379 | 0.016 | 0.0066 | 0.0076 |
| Std. Dev. (trade balance/Y) | 0.0351 | 0.017 | 0.0072 | 0.0068 |
| Std. Dev. (c) / Std. Dev. (y) | 1.1929 | 2.478 | 3.1856 | 1.403 |
| Std. Dev. (spreads) | 3.9227 | 3.303 | 2.3942 | 0.9576 |
| Std. Dev. (exports/Y) | 0.0516 | 0.021 | 0.0167 | 0.0127 |
| Correlation(trade balance/Y, y) | -0.3566 | -0.177 | 0.2492 | 0.5082 |
| Correlation(c,y) | 0.8458 | 0.589 | 0.3955 | -0.2494 |
| Correlation(spreads,y) | -0.3622 | -0.073 | 0.0344 | 0.4601 |
| Correlation(exports/Y,y) | -0.1777 | -0.141 | 0.3209 | 0.4593 |

deviation). To compute expectations we interpolate policy functions with a modified Akima piecewise cubic Hermite interpolation in a dense grid of 20001 equally spaced points in debt and 501 equally spaced points in $p^T$ for which we estimate its corresponding transition matrix with the Tauchen method.[29] We experimented we different grid sizes, different interpolations schemes (e.g., linear), and different ways of computing the expectations (e.g., computing its corresponding integral). The final choice of grid points and methods is the most efficient allocation of computing time. For example, either computing the integral, or increasing the size of the grids, deliver the same solution but requires more computing time.

Figures 12 and 13 show some policies and allocations as function of the repayment status, debt and terms of trade. For example, the top-left figure shows the policy function of debt issuances as a function current debt (in the horizontal axis) and terms of trade (ranging from blue for lower terms of trade to yellow for higher terms of trade). The dashed lines correspond to the allocations when the economy is in autarky.

# C  Data

## C.1  Data Sources

This section lists the sources for all the variables used in the main body of the paper.

Variables in Table 2:

- "Inflation" is Inflation, consumer prices (annual %) from the World Bank. Indicator Code

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[29]The interpolated value at a query point is based on a piecewise function of polynomials with degree at most three evaluated using the values of neighboring grid points in each respective dimension. The Akima formula is modified to avoid overshoots.

Figure 12: Policies as function of debt, $p^T$, and repayment status
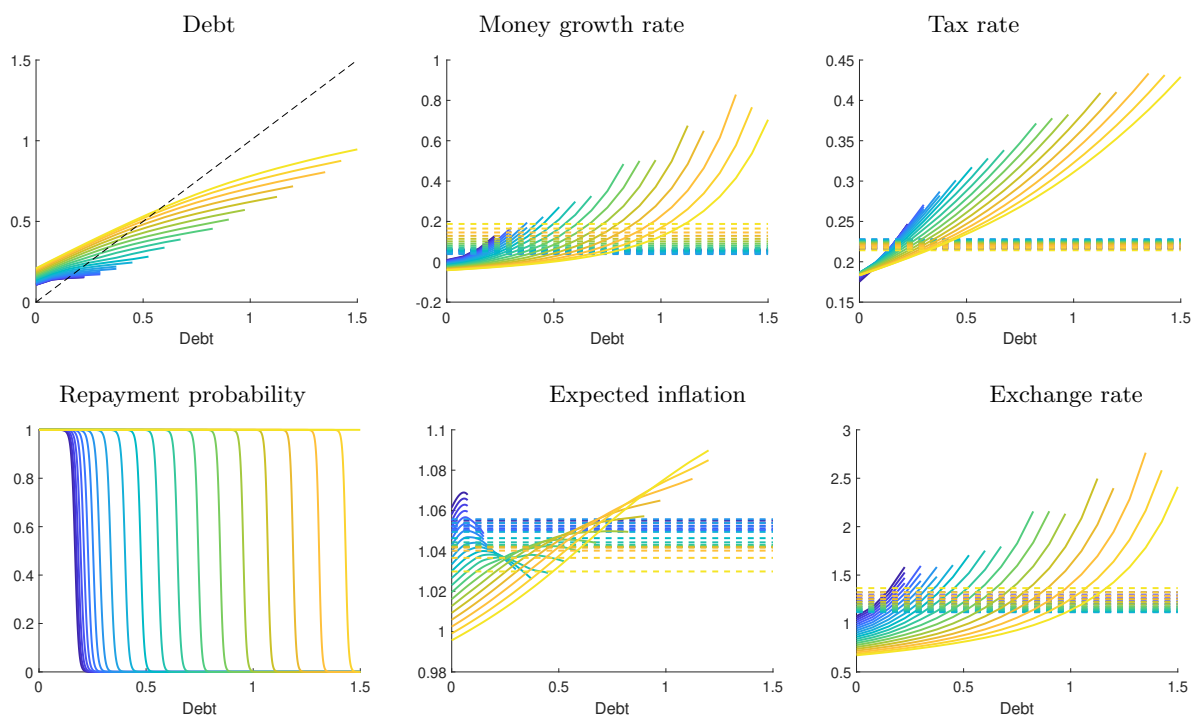


Debt

Money growth rate

Tax rate

Repayment probability

Expected inflation

Exchange rate

Note: Policy functions in repay are solid lines, while in default are the dashed ones. The different lines ranging from blue to yellow correspond to states with higher terms of trade.

Figure 13: Equilibrium allocations as function of debt, $p^T$, and repayment status



$c^N$

$c^T$

$g$

$y^N$

$y^T$

$h$
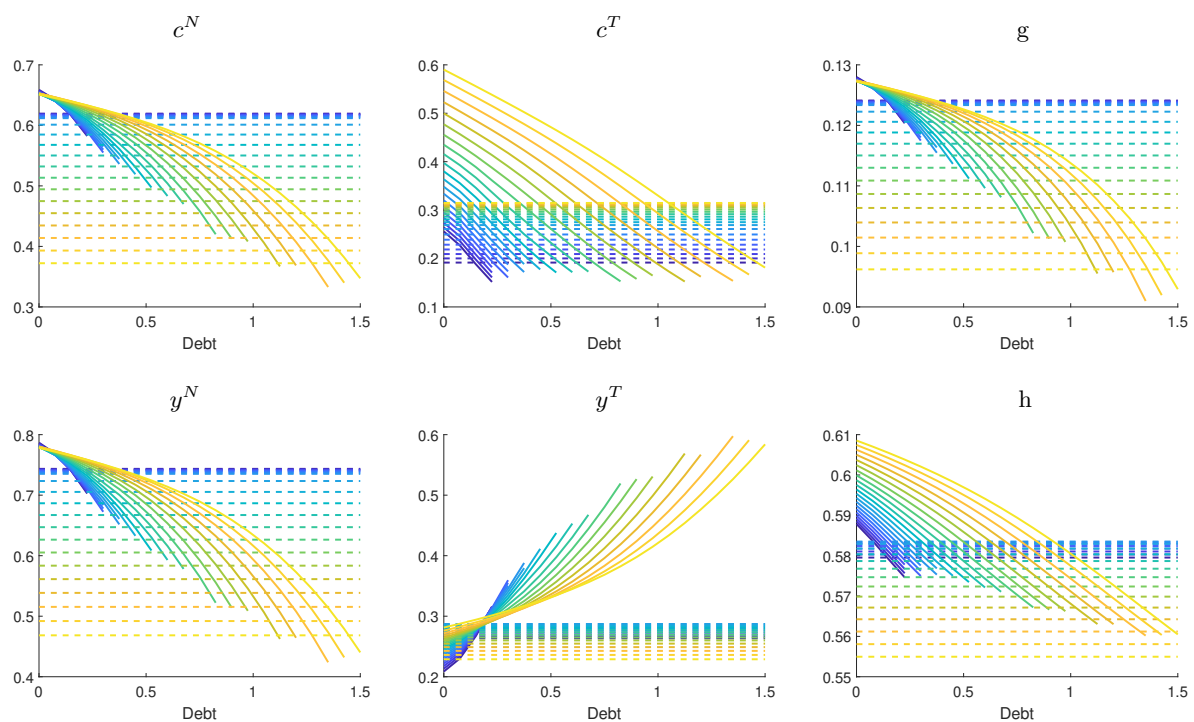
Note: Policy functions for repayment are solid lines, while in default are the dashed ones. The different lines ranging from blue to yellow correspond to states with increasingly higher terms of trade.

FP.CPI.TOTL.ZG.

- "Transfers/GDP" constructed as the product of two series from the World Bank. Subsidies and other transfers (% of expense) with indicator code GC.XPN.TRFT.ZS and Expense (% of GDP) with indicator code GC.XPN.TOTL.GD.ZS.

- "Exports/GDP" is Exports of goods and services (% of GDP) from the World Bank. Indicator code NE.EXP.GNFS.ZS.

- "Employment/Population" is Employment to population ratio, 15+, total (%) (modeled ILO estimate). Indicator code SL.EMP.TOTL.SP.ZS.

- "Gov. Consumption/GDP" is General government final consumption expenditure (% of GDP) from the World Bank. Indicator code NE.CON.GOVT.ZS.

- "Debt/GDP" is Public External Debt (%GDP) computed using the ratio of the following two variables from the World Bank. External debt stocks, public and publicly guaranteed (PPG) (DOD, current US$) with indicator code DT.DOD.DPPG.CD and GDP (current US$) with indicator code NY.GDP.MKTP.CD.

- "Haircut, Share of Debt" is the median "SZ haircut, HSZ" in Table 2 of Dvorkin et al. (2021).

- "Default rate" is obtained from Tomz and Wright (2013). They construct a database of 176 sovereign entities spanning 1820 to 2012. The frequency of default is sensitive to the sample being analyzed. They mention that their findings are "similar to the 2% default probability that is a target for many calibrated versions of the standard model," which is the number we use as well. The unconditional probability of a country with positive debt (a borrower) defaulting on debts owed to commercial creditors is 1.7% per year. Nevertheless, this probability is higher in developing countries. Note also in Figure 2 of Tomz and Wright (2013) that in a typical year, there are no defaults or there is one country in default. We considered this fact when calibrating a significantly lower default rate in the model with only $\varepsilon$ shocks.

The sources for variables used in Table 3 are:

- "Real GDP growth" is GDP per capita (constant LCU) from the World Bank. Indicator Code NY.GDP.PCAP.KN.

- "Trade balance" is Trade balance (% GDP) computed using two variables from the World Bank. Trade (% of GDP) with indicator code NE.TRD.GNFS.ZS and the variable Exports of goods and services (% of GDP) mentioned above.

- "Consumption" is Consumption per capita constructed using two World Bank Variables. Households and NPISHs Final consumption expenditure (constant LCU) with indicator code NE.CON.PRVT.KN and Total Population with indicator code SP.POP.TOTL.

- "Spreads" is the J.P. Morgan Emerging Markets Bond Spread (EMBI+) obtained from the World Bank. Indicator Indicator Id: EMBIG.

The additional sources for Table 4 is Vegh and Vuletin (2015). For real expenditure and inflation tax, the numbers in the column "Data" corresponds to the averages for the countries in our sample of the correlations reported by Vegh and Vuletin (2015). The only difference with the numbers that they report and what we do with the simulated data from the model is that the detrending method is different. We use these numbers because the series made publicly available are already detrended. For taxes, we use the file they made available, "data_AEJEP.dta", and using the variable "individual_tr," we follow the same detrending procedure to make the correlation more comparable. For this variable, Vegh and Vuletin (2015) present in Figure 11 the correlation in growth rates; i.e., the correlation between the change in the personal income tax rate and GDP growth. If we follow that procedure, our results confirm the similarity of the model and the data—we obtain $-0.2319$ in the model and $-0.1009$ in the data.

Figure 4 uses only a new variable: the terms of trade index. We use data from ECLAC - CEPALSTAT, Economic Indicators and statistics, External sector. The index is called "terms of trade and purchasing power of exports". The same data is used to estimate the autoregressive process for terms of trade in Appendix C.2.

The data sources for Figure 5 and 6 that came from the World Bank, also referred to as "WB," were already described. The other sources are:

- Transfers/GDP labeled "Mecon" is taken from the Ministry of Economics of Argentina information about "Gasto Publico Consolidado 1980-2017 por finalidad" as a share of GDP and it corresponds to the sum of "II.2.2. Obras sociales - Atención de la salud," "II.2.3. INSSJyP - Atención de la salud," "II.6. Previsión social," and "II.7. Trabajo." Data can be found in `https://www.argentina.gob.ar/economia/politicaeconomica/macroeconomica/gastopublicoconsolidado`.

- Expenditure/GDP labeled "Mecon" is taken from the Ministry of Economics of Argentina information about "Gasto Publico Consolidado 1980-2017 por finalidad" as a share of GDP and it corresponds to the "'Gasto Publico sin Servicios de la Deuda Publica (IV)" minus transfers as defined above.

- "Revenue/ GDP" labeled as "NyS" corresponds to data from the "Fundacion Norte y Sur," file C7.2, tab "Ingresos, Gastos y Resultado del Sector Público Argentino," column "Ingresos totales % PIB,". Data can be found in `https://dossiglos.fundacionnorteysur.org.ar/series`.

- "Debt/ GDP" labeled as "NyS" corresponds to data from the "Fundacion Norte y Sur," file C7.2, tab "Deuda publica nacional interna y externa consolidada," column "Deuda Publica Externa".

- "Currency Depreciation % Data, black market" corresponds to data from the "Fundacion Norte y Sur," file C8.2, tab "Tipo de Cambio," column "Dolar Libre."

## C.2    Estimation of a stochastic process for the terms of trade

We use data on terms of trade as described above and the time series of commodity prices used by Drechsel and Tenreyro (2017). Before estimating the autoregressive process, we take logs of the series and subtract the mean.

Table 9 presents the results. The time period is 1980 to 2019. The coefficients $\rho_p$ and $\sigma_p$ are both similar for all the seven countries, so we use the average in our benchmark calibration. It is reassuring that the estimation results for the commodity price index presented at the bottom of Table 9 are also quite similar to the average.

We did not de-trend the series before estimating the stochastic process so as to include long-duration cycles in the terms of trade (often referred to as "super-cycles") in our quantitative exercises and keep the model and the data more comparable. We have also estimated these stochastic processes after de-trending the time series for terms of trade. The main difference is that the resulting value of $\rho_p$ is smaller, which implies that shocks are less persistent.

To get a simple idea of the role of persistence, we present in Figure 14 the response to an unexpected shock to the terms of trade with three values of persistence: the benchmark, $\rho_p = 0.8803$, a higher value, $\rho_p = 0.98$, and a lower value $\rho_p = 0.8$. The dynamics after the shock are perfect-foresight, as in the exercises in Section 4.4. Changing $\rho_p$ mainly affects the persistence of real GDP and fiscal variables (debt over GDP and primary deficit over GDP);

Table 9: Estimation of process for $p^T$

| | $\rho_p$ | $\sigma_p$ | Number of years |
|---|---|---|---|
| Argentina | 0.9302 | 0.0608 | 40 |
| | (0.0567) | (0.0064) | |
| Brazil | 0.8742 | 0.0656 | 40 |
| | (0.0759) | (0.0072) | |
| Colombia | 0.9174 | 0.0851 | 40 |
| | (0.0588) | (0.0095) | |
| Mexico | 0.8216 | 0.0702 | 40 |
| | (0.1339) | (0.0036) | |
| Chile | 0.9154 | 0.1022 | 40 |
| | (0.0868) | (0.0106) | |
| Peru | 0.9327 | 0.0735 | 40 |
| | (0.0728) | (0.0063) | |
| Uruguay | 0.7707 | 0.0718 | 40 |
| | (0.0928) | (0.0072) | |
| Average | 0.8803 | 0.0756 | |
| Commodity price index | 0.8757 | 0.0910 | 36 |
| | (0.0965) | (0.0134) | |

Note: Standard errors in parenthesis.

it also affects spreads on impact. The reason for this last result is that a more persistent fall in terms of trade limits the capacity for debt repayment more severely. Interestingly, the persistence of inflation, currency depreciation and spreads are not altered significantly.

## C.3 Local projections

We consider three alternative left-hand-side variables: inflation, EMBI spreads, and GDP growth (i.e., $ln(GDP_t) - ln(GDP_{t-1})$). We refer to these variables as $y_t^i$, where $i$ refers to the country and $t$ to the year. The right-hand-side variable of interest is the log(terms of trade), and we refer to this variable as $lp_t^i$.

The difference of a variable $\delta$ periods ahead with the same variable one period ago is $\Delta y_{t+\delta,t-1}^i = y_{t+\delta}^i - y_{t-1}^i$. The panel regression we run to obtain the response to terms of trade shocks is

$$\Delta y_{t+\delta,t-1}^i = \alpha^\delta + \beta^\delta \Delta lp_{t,t-1}^i + \text{controls}.$$

We run this regression 12 times: for each of the three alternative left-hand-side variables and for $\delta = \{0, 1, 2, 3\}$. The controls consist of two lags of $\Delta y_{t+\delta,t-1}^i$, two lags of $\Delta lp_{t,t-1}^i$, and country fixed effects.

Figure 14: Shocks to the terms of trade: the role of persistence
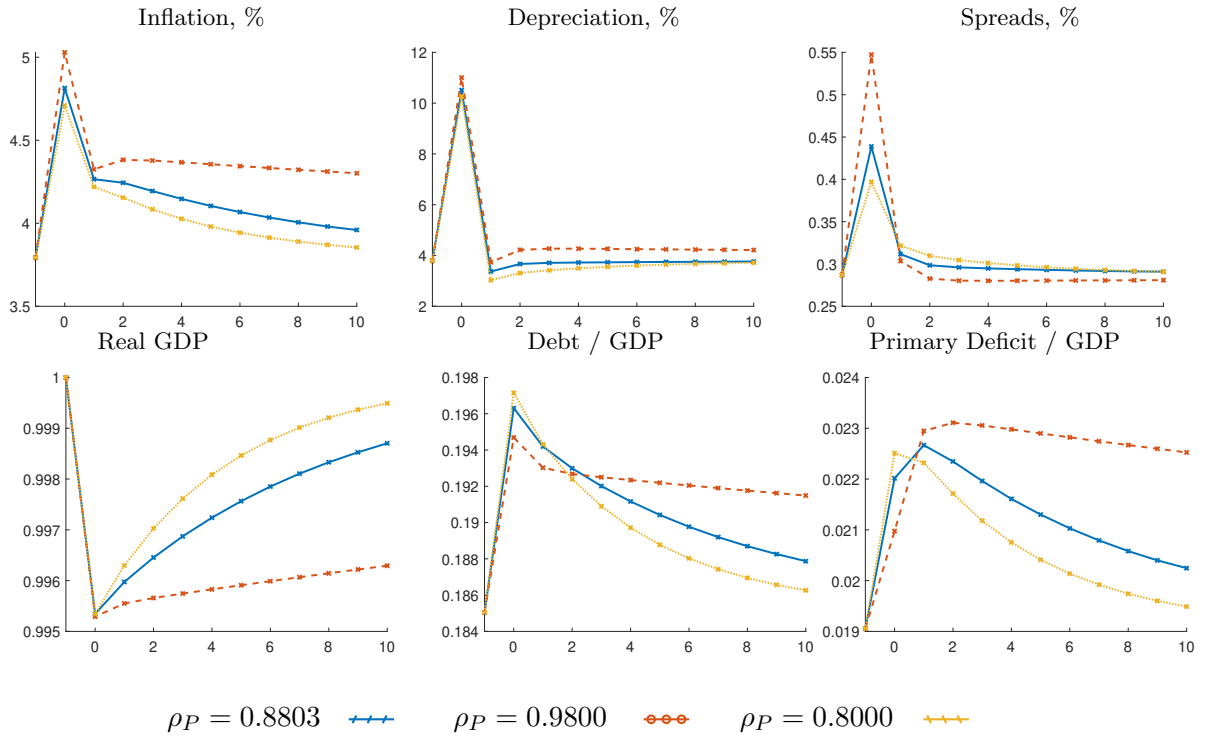
In Figure 4, we plot the coefficients $\beta^\delta$ multiplied by $-10$ to represent a 10 percent decline in the terms of trade. The standard errors showed by the shaded area in the figure are robust standard errors.

The time period for the regressions in 1980 to 2019 or the latest available observation. The most important exception is the regression for the EMBI spread, which starts in 1997 due to data availability of this variable.

We also conduct this comparison using contemporaneous regressions between these three variables and the terms of trade. The estimated semi-elasticities, similar to those in Drechsel and Tenreyro (2017), are quite similar in the model and the data and resemble the effect at time zero in the analysis presented here.