Another case against floating exchange rates when prices are flexible: some examples

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Abstract

Fixed and floating exchange-rates are compared against the background of a model of a pure-currency economy with flexible prices. Using a mechanism that insures that all transactions are in the form of the producer's home currency, examples are produced that display the following property: the set of discount factors consistent with achievement of the first-best outcome is larger under fixed exchange rates than it is under floating exchange rates.

One case against floating exchange-rates when prices are flexible rests on exchange-rate indeterminacy (see Kareken and Wallace [4] and King *et. al.*[5]. Such indeterminacy arises when the currencies of different countries are perfect substitutes, at least over some range. Indeterminacy implies that there are equilibria in which the exchange rate is sunspot-random, randomness which in some settings reduces welfare. Here, I present another case against floating exchange rates when prices are flexible, one that is consistent with no substitution among currencies. This case arises from the different ways that fixed- and floating exchange-rate systems respond to country-specific aggregate real shocks in a model of two symmetric countries. In the examples, the set of parameters for which the first-best outcome is implementable under floating exchange rates is smaller than that set under fixed rates. And because fixed exchange rates and a unified currency are identical in the model, the examples constitute a new argument in favor of a unified currency.

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In order to have a straightforward representation of a foreign-exchange market, one which lends itself to the standard way of modeling floating- and fixed exchange-rates (and more general foreign-exchange intervention schemes), the model contains home and foreign currencies. The simplest way to have currencies in a model and to have it be *essential* is to adopt a *pure-currency* model, one in which currencies are the only assets and in which all trade is spot trade.[1] Moreover, because country-specific aggregate shocks are crucial for the analysis, it is extremely helpful to use a specification in which the distribution of currency holdings that results from prior trades is *not* a state variable of the model. That leads me to use a version of the Lagos-Wright (LW) model (see [6]). In order to produce no-substitution among currencies in a way that is consistent with my notion of implementability, I rely on the ideas in Hu *et. al.* (HKW) (see [1]) and in Zhu and Wallace (ZW) (see [10]).[2] In particular, consumers are induced to pay in the form of the producer's home currency in the consumer-producer two-person meetings of the model because they would otherwise be sacrificing all of their bargaining power. Given my focus on currencies, the model is very different from models of nominal exchange rates which do not contain currencies. A recent and leading example of such a model is Itskhoki and Muzhin [3]. They study a model of floating exchange rates with flexible prices, but one in which it is far from obvious how to study fixed exchange rates.[3]

In my version of LW, welfare depends entirely on the trades that occur in two-person meetings between producers and consumers, what is often described as the decentralized stage. The first-best outcome in each such meeting is an amount of production (and consumption) that equates the marginal utility of consumption to the marginal disutility of production, an amount that depends on the realization of the shock to the marginal utility of consumption, but does not depend on the discount factor. In the examples, the interval of discount factors for which the first-best is implementable is smaller under floating exchange rates than it is under fixed exchange rates.

[1] For a discussion of essentiality of money, see Wallace ([8]).

[2] Nosal and Rocheteau ([7]) used ZW, but only to get a determinate exchange rate under floating exchange rates.

[3] Itskhoki and Muzhin [3] mention exchange-rate intervention only in the conclusion of their paper. Their remarks suggest that they do not view their model as suitable for the study of such intervention.

1 The environment

There are two symmetric countries.[4] Each is populated by a nonatomic unit measure of infinitely-lived people and each has its own divisible money, the amount of which is nomalized to be unity per home resident. The common discount factor is denoted $\beta \in (0,1)$.

There are two stages at each discrete date and each stage has a produced and perishable good. The sequence of actions at a date is as follows:

$$\begin{array}{ccccccc} \text{state of a} & & \text{three} & & \textbf{Stage-1} & & \textbf{Stage-2} \\ \text{person} & \rightarrow & \text{shocks} & \rightarrow & \text{linear good} & \rightarrow & \text{consumer-producer} \\ (m_1, m_2) & & \text{realized} & & \text{and two monies} & & \text{(two-person) meetings} \end{array}.$$

That is, each person enters a period with a portfolio consisting of amounts of the two monies, money 1 (country 1's money) and money 2 (country 2's money). Then three shocks are realized, two idiosyncractic shocks and one aggregate shock, all of which pertain to stage-2 trade. One idiosyncratic shock determines whether a person is a *producer* or a *consumer* at stage-2 and each outcome has probability one-half. The other idiosyncratic shock determines for each consumer whether they consume the home good or the foreign good at stage-2. (One interpretation is that those who consume the foreign good become tourists at stage-2 before returning home.) A constant fraction consume the foreign good, a fraction whose magnitude does not matter and which could even be zero. Producers stay in their home countries. Everyone is in a stage-2 meeting. The aggregate shock determines two marginal utilities: that of consuming the country-1 good and that of consuming the country-2 good at stage 2, marginal utilites which are common to both home and foreign consumers.

Everyone can produce and consume the stage-1 good, a good that gives additively-separable linear period utility. When the current aggregate state is $i \in \{1, ..., I\}$, the period utility of a consumer who consumes q at stage-1 and who consumes $y \geq 0$ of the stage-2, country-k good is $q + \theta_i^k u(y)$, while the period utility of a producer who consumes q at stage 1 and who produces $y \geq 0$ at stage 2 is $q - c(y)$. We assume that $u(0) = c(0) = 0$, $c' > 0$, $c'' \geq 0$, $u'' < 0 < u'$, and $u'(0) > c'(0)$. The aggregate shock, (θ_i^1, θ_i^2) in state-i, follows an I-state Markov chain in which π_{ij} is the transition probability

[4] This model was formulated jointly with Tao Zhu. In a separate paper, he is pursuing different aspects of the model.

from state i to state j. The only private information is about histories. In particular, in a stage-2 meeting, both the current state and the portfolios held are common knowledge in the meeting.

This setting lends itself to a simple description of the optimum if we consider welfare at the start of any stage-1 before shocks are realized and weight people equally. Such welfare depends only on what happens at stage-2 and has an upper bound that is achieved if production and consumption are given by

$$\arg\max_{y\geq 0}[\theta_i^k u(y) - c(y)] = \hat{y}_i^k. \tag{1}$$

2 First-best equilibria

We assume that a trade in a meeting is implementable if it is in the meeting-specific core when people take future trades as given. For stage-1, it follows that trade is competitive. Moreover, as is well-known, the presence of the linear good at stage-1 implies that the wealth with which a person enters stage-1 affects only their consumption of the linear good and, therefore, is not part of the state of the economy. The exchange-rate regime is modeled as stage-1 intervention in the standard way. When the exchange-rate is floating, the quantity of each money is fixed and the exogenous quantities of both monies are held at the end of stage-1. When it is fixed, the government of each country supplies unlimited amounts of its own money at a fixed and constant nominal exchange rate normalized to be unity. Such intervention does not affect the *total* quantity of money and that total is held at the end of stage-1.

The requirement that trade in the stage-2 two-person meetings be in the meeting-specific core gives us a lot of leeway in choosing the trades and how they depend on the portfolio of monies that the consumer brings into the meeting. In order to implement $\hat{y}_i^k$ while having all stage-2 transactions be in the home currency of the producer, we rely on the closely related ideas in ZW, HKW, and Hu and Rocheteau [2]. In those papers, the selection from the core depends on the portfolio of the consumer. That selection incentivizes the consumer to enter the meeting with a particular kind of portfolio. Here, it will be a portfolio that has an amount of the producer's home money that is large enough to induce production of the first-best level of output at stage-2. But whether there is an equilibrium in which that can happen depends on the discount factor. The condition on the discount factor has the form

$\beta \in [\beta_{\min}, 1)$, where $\beta_{\min}$ can depend on the exchange-rate regime.

There is a game, adapted from that in HKW, whose equilibrium outcome is the first-best stage-2 trade. The game is defined relative to a planner proposal. The planner proposal has two parts. One part is a suggested value of country-i money that the consumer who will consume in country-i acquires at stage-1, x_i^k. The other part is a stage-2 trade: trade z_i^k for $\hat{y}_i^k$, where $z_i^k \leq x_i^k$. The stage-2 game is as follows. If actual holdings of the consumer are less than x_i^k, then the producer makes a take-it-or-leave-it offer. Otherwise, the consumer suggests a trade and the producer chooses from $\{yes, no\}$. If the producer says *yes*, then that suggested trade is carried out; if the producer says *no*, then the planner's suggested trade is carried out.

According to this game, the consumer gets a gain from trade only if their money holding leaving stage-1 is as large as the planner's suggested holding. If it is, then the planner's suggested trade is in the two-person core: it maximizes the sum of the consumer's payoff and the producer's payoff. It follows that the consumer is willing to acquire the planner's suggested stage-1 holding and that at stage-2 the consumer can do no better than propose the planner's suggested trade.

Before we define equilibria that support the first best, four remarks are in order. First, an implication of the linear utility of the stage-1 good is that β^{-1} is an upper bound on the expected gross real return on either money and that at that expected return consumers are willing to hold any real amount of money as they leave stage-1. Second, those who leave stage-2 with money care only about its expected value at the next stage-1, another implication of the linear utility of the stage-1 good. Third, we are not achieving the first-best by using the Friedman rule—pay interest on money at the rate $\beta^{-1} - 1$ using lump-sum taxes to finance the interest payments—because HKW shows that such taxation may not be consistent with allowing people to skip stage-1. Fourth, although the model was not designed to have the following two properties, they make it easier to think about equilibria. Obviously, the model under fixed exchange-rates is equivalent to a model of a single country with one money and two kinds of stage-2 consumers at each date. Less obviously, perhaps, the model under floating exchange-rates is equivalent to a model of two autarkic countries, each with its own money and each with one kind of stage-2 consumer at each date. This is true because the country-of-origin of the consumers in a country at a date does not matter and because the entire exogenous stock of money of a country will at the end of stage-1 be held by those consuming in that country at stage-2.

As above, letting (x_i^k, z_i^k) be the stage-1 value of country-k money that the consumer who will consume in country-k acquires and the amount traded at stage-2, we have the following definitions of existence of stationary equilibria that support the first-best.

Definition 1. (Floating) Let

$$R_i^k = \frac{\sum_j \pi_{ij} x_j^k}{x_i^k}, \tag{2}$$

the expected gross rate of return on country k money in state-i. Then, $\{x_i^k, z_i^k\}_{i=1}^I$ with $x_i^k \geq z_i^k$ is a stationary *floating* exchange-rate equilibrium that supports the first-best if the following hold for each i and k: $R_i^k \leq \beta^{-1}$,

$$\beta R_i^k z_i^k - c(\hat{y}_i^k) \geq 0, \tag{3}$$

and

$$u(\hat{y}_i^k) - z_i^k + (x_i^k - z_i^k)(R_i^k - \beta^{-1}) \geq 0. \tag{4}$$

Definition 2. (Fixed) Let

$$R_i = \frac{\sum_j \pi_{ij}(x_j^1 + x_j^2)}{x_i^1 + x_i^2}. \tag{5}$$

Then $\{x_i^k, z_i^k\}_{i=1}^I$ for $k = 1, 2$ is a stationary *fixed* exchange-rate equilibrium that supports the first-best if the following hold for each i and k: $R_i \leq \beta^{-1}$,

$$\beta R_i z_i^k - c(\hat{y}_i^k) \geq 0 \tag{6}$$

and

$$u(\hat{y}_i^k) - z_i^k + (x_i^k - z_i^k)(R_i - \beta^{-1}) \geq 0. \tag{7}$$

In these definitions, the inequalities are participation constraints at the first-best output, the first for producers and the second for consumers.

In general, the constraints in these definitions imply restrictions on the discount factor. To see that, it is instructive to look at the analogue of the special case in HKW; namely, one money and no shocks. We can examine that case by applying definition 1 with $I = 1$ in which case $R = 1$. Then, (3) and (4) imply $\beta \geq c(\hat{y})/u(\hat{y})$, where $\hat{y} = \arg\max_{y \geq 0}[u(y) - c(y)]$, a quantity which shows up in the examples below.

3 A two-date periodic and asynchronous example

Here, $I = 2$,

$$(\theta_i^1, \theta_i^2) = \begin{cases} (1,0) \text{ if } i = 1 \\ \\ (0,1) \text{ if } i = 2 \end{cases}, \tag{8}$$

and $\pi_{12} = \pi_{21} = 1$. That is, at each date consumers of the good produced in one country do not value consumption, while those who consume the good produced in the other country do value it—an extreme and asynchronous seasonal.

Under floating exchange rates

In accord with the remarks above, we have two separate economies which are identical. Let x_+ be the stage-1 value of money when $\theta_i = 1$ and let x_0 be that value when $\theta_i = 0$. In order to have a stationary monetary equilibrium, we must have $x_+/x_0 = \beta^{-1}$. Then, by (3), we have $\beta^2 x_+ \geq c(\hat{y})$. Finally, using (4), we get

$$\beta^2 \geq c(\hat{y})/u(\hat{y}). \tag{9}$$

Under fixed exchange rates

Using the remark above that fixed-rates is equivalent to a single economy with a single money, this is an economy that is constant over time. In particular, the presence at each date of the same mass of consumers who do not value consumption and of the producers who potentially produce for them plays no role. Hence, this reduces to the analogue of the HKW economy. The condition for achievement of the first-best is, as noted above, $\beta \geq c(\hat{y})/u(\hat{y})$, a less stringent condition than (9).

4 A two-state, *i.i.d. over-time example*

Same as example-1, but $\pi_{ij} = 1/2$ for all (i, j).

Under floating exchange rates

We have two separate economies which are identical, but now there is uncertainty in each. Let x_+ be the stage-1 value of money when $\theta_i = 1$ and let x_0 be that value when $\theta_i = 0$. Also let, R_+ be the stage-1 gross rate of return on money when $\theta_i = 1$ and let R_0 be that value when $\theta_i = 0$. It follows that

$$R_0 = \frac{(x_0 + x_1)/2}{x_0} = \beta^{-1}$$

Using the second equality, we have

$$x_1/x_0 = 2\beta^{-1} - 1.$$

Therefore,

$$R_+ = \frac{(x_0 + x_1)/2}{x_1} = \frac{x_0}{2x_1} + \frac{1}{2} = \frac{\beta^{-1}}{2\beta^{-1} - 1}.$$

(Notice that $R_+ < \beta^{-1}$.) Now, by (3)

$$\beta R_+ x_1 = \frac{1}{2\beta^{-1} - 1} x_1 = \frac{\beta}{2 - \beta} x_1 \geq c(\hat{y})$$

Then, using (4), we have

$$\frac{\beta}{2 - \beta} \geq \frac{c(\hat{y})}{u(\hat{y})}. \tag{10}$$

Under fixed exchange rates.

Using the fact that this is equivalent to a single economy with one money, nothing is random here. At each date, half the consumers do not value consumption and the other half value it according to the function u. Therefore, we have the same condition as in example 1; namely, $\beta \geq c(\hat{y})/u(\hat{y})$, a less stringent condition than (10).

5 Remarks

1. *Special features of the examples.*

The zero realizations of the shocks is not necessary. Those realizations could be replaced by sufficiently small positive realizations without affecting the results. The examples are special in that each implies that the economy under fixed exchange rates is a constant economy.

2. *Is there a somewhat general result?*

I suspect that there is. Under fixed exchange rates, there is a single economy with two types of consumers at each date. Under floating rates, there are two separate economies each with one type of consumer at each date. Under some fairly general conditions, the presence of two types of consumers at each date should require less variability of rates of return than when there is one type of consumer at each date. And less variability should translate into a weaker condition on the discount factor for achievement of the first-best.

3. *How important is the use of Lagos-Wright (2005)?*

Unfortunately, it or some other device that eliminates past trades as a state variable of the model seems important. As I have emphasized elsewhere (see Wallace [9]), such degeneracy has special consequences. My only defense of its use here is that the LW structure was not designed for the current application. By the way, it is easy to convert the model set out above to one that displays exchange-rate indeterminacy under floating exchange rates: replace stage-1 of the current model by a market in which the only objects traded are the currencies. In such a version, half of wealth is held by producers who care only about the distribution of returns of the two monies. That is sufficient to produce indeterminacy.

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