Research Memo: Adding Nonfarm Employment to the Mixed-Frequency VAR Model*

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Abstract

This memo describes a revision to the mixed-frequency vector autoregression (MF-VAR) model originally constructed by Schorfheide and Song (2012) and subsequently revised by Beauchemin (2013). In this most recent version, the 14-variable model is expanded to include nonfarm payroll employment. The forecast performance of the augmented model is compared with that of its predecessor.

*The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

1 Introduction

This memo describes a revision to the mixed-frequency vector autoregression (MF-VAR) model originally constructed by Schorfheide and Song (2012) and subsequently revised by Beauchemin (2013). In this model version, the 14-variable model is expanded to include total employment at nonfarm businesses.[1]. I compare the forecast performance of the augmented model with that of its 14-variable predecessor (heretofore "benchmark model"). The results show that nonfarm employment improves overall forecast performance. Forecasts of variables included in the Federal Reserve's Summary of Economic Projections (SEP) also show a general improvement in accuracy: PCE price forecasts are substantially better; real GDP forecasts are more accurate in the medium to longer terms while conceding only moderate to small amounts of accuracy in the immediate short-term; and the forecast performance for the unemployment rate and the federal funds rate is essentially unchanged.

2 Model Overview and Notation

This section provides a brief overview of the MF-VAR model, mostly to establish the notation used in evaluating forecast performance. Those interested in a detailed treatment of the econometric foundations of the MF-VAR model (i.e., the likelihood function, the prior and posterior distributions, and the two-step Gibbs sampler, which implements the mixed-frequency feature of the model) should refer to Schorfheide and Song (2012).

Let $Y_t = (y_{1,t}, y_{2,t}, \ldots, y_{n,t})'$ be the data vector of n random variables following a VAR(p) process of the form

$$y_t = \Phi_1 y_{t-1} + \cdots + \Phi_p y_{t-p} + \Phi_c + u_t, \qquad u_t \sim iid\, N\left(0, \Sigma\right) \tag{1}$$

for $t = 1, \cdots, T$. In this expression, $\Phi_1, \ldots, \Phi_p$ are $n \times n$ matrices of VAR coefficients, $\Phi_c = (c_1, c_2, \ldots, c_n)'$ is an n-dimensional vector of constants, and $\Sigma = Eu_t u_t'$. Each equation in this n-variable system has $k = np + 1$ regressors. For the benchmark 14-variable, 6-lag model, $k = 85$; expanding the model to include nonfarm employment, $k = 91$. Like other

[1] Data are from the "B" tables (Current Employment Statistics survey) of the monthly Employment Situation release: http://www.bls.gov/news.release/empsit.b.htm .

nonstationary variables in the model, nonfarm employment ($LEMP$) is subject to a natural log transformation. The complete list of model variables and transformations is given in Table 1.

Prediction in a Bayesian framework is based on the posterior predictive density. Letting $Y_{T+1:T+H} = \left(y'_{T+1}, y'_{T+2,}, \ldots, y'_{T+H}\right)'$ represent an arbitrary forecast path in the set of all possible future paths, the predictive density assigns a probability to each path:

$$p\left(Y_{T+1:T+H} \mid Y^T\right) = \int p\left(Y_{T+1:T+H}, \Theta \mid Y_{1-p:T}\right) d\Theta \tag{2}$$

where $p\left(Y^{T+1,T+H}, \theta \mid Y^T\right)$ is the joint density of model parameters and future variable observations. Using the rules of probability, the integrand can be written as

$$p\left(Y_{T+1:T+H}, \Theta \mid Y^T\right) = p\left(Y_{T+1:T+H} \mid Y_{1-p:T}, \Theta\right) p\left(\Theta \mid Y_{1-p:T}\right), \tag{3}$$

where $\Theta = (\Phi, \Sigma)$. The two sources of forecast uncertainty are highlighted by this expression. The first term on the right-hand side of (3) describes the uncertainty on future observables given the observed data and model parameters, or equivalently, the forecast uncertainty due to future VAR disturbances. The second term is the model posterior distribution describing parameter uncertainty. Estimation of the posterior distribution of the VAR coefficients and the variance-covariance matrix of disturbance terms, as well as the inference of the latent monthly observations of the quarterly variables, require a two-step Gibbs sampling algorithm described in Schorfheide and Song (2012).

3 Comparing Forecast Performance

The forecasting performance of the updated model relative to the current model focuses on the five model variables that are part of the SEP (*GDPR*, *UR*, *PC*, *PCXFE*, and *RFF*), but results are reported for all model variables.

Forecast accuracy metrics for point forecasts are generated using the pseudo-iterated approach in which parameter uncertainty is integrated out and future disturbances are set to zero. The h-step-ahead forecast is obtained using the posterior mean coefficient matric $\overline{\Phi}$ and is computed by recursive substitution:

$$\widehat{y}_{T+h} = \overline{\Phi}_c + \overline{\Phi}_1 \widehat{y}_{T+h-1} + \overline{\Phi}_2 \widehat{y}_{T+h-2} + \ldots + \overline{\Phi}_p \widehat{y}_{T+h-p}$$

for $h = 1, \ldots, H$ where $\widehat{y}_{T+h} = y_{T+h-p}$ for $h \leq p$.

Forecast evaluations are conducted on a recursive basis in which the sample period is lengthened by one observation for each forecast. The initial sample period runs from 1968M1 to 1986M12, with the 1967M7–1967M12 observations serving as the pre-sample to accommodate the six lags. A 36-step (month) ahead forecast is then computed covering the 1987M1–1989M12 period. In the next recursion, the sample is updated to 1968M1–1987M1 and the point forecast is computed for the 1987M2–1990M1 period. The process continues until the last forecast that accommodates a three-year interval covering the end of the data sample can be constructed. That recursion uses the 1968M1–2011M6 sample to forecast the 2011M7–2014M6 period.

Before proceeding with the forecast evaluations, I obtain the optimal set of model hyperparameters at each recursion. This requires 414 consecutive maximizations of the marginal data density over the five-dimensional hyperparameter space, implying (potentially) different prior means for all forecasts. Monthly inferences for the three quarterly series are first obtained (for each model) by running the full two-step Gibbs sampler over the entire sample period 1968M1–2014M6. The three inferred monthly series are subsequently treated as data in computing the optimal hyperparameter series and evaluating the point forecasts.[2]

Because the actual quarterly variables are only observed at that native frequency, forecasts of the quarterly averages are evaluated even though the model is solved at the underlying monthly frequency. We abuse notation slightly so that $h = 1, \ldots, H$ is counted in quarters rather than months. Forecasts are initially evaluated over the 1987Q1–2014Q2 period, which allows for roughly a 20-year evaluation window. This period is also frequently identified as one characterized by a single monetary policy regime.

Forecast evaluations are based on the mean squared forecast error (MSFE) statistic. Let T_0 denote the beginning of the evaluation period minus one period (1986Q4 or 2007Q4) and T_1

[2] See Schorfheide and Song (2012) for details.

the end period (2014Q2). The MSFE is defined as

$$MSFE_{i,h} = \frac{\sum_{t=T_0}^{T_1-h}(y_{i,t+h}^{data} - \widetilde{y}_{t+h})^2}{T_1 - h - T_0 + 1}$$

for each forecast variable i and forecast horizon $h = 1, \ldots, H$. Before MSFEs are computed, the simulated projections $\widehat{y}_{t+h}$ are transformed back to original units $\widetilde{y}_{t+h}$ according to the transformations indicated in Table 1. Defining $MSFE_{i,h}^{NEW}$ as the MSFEs for the model augmented with payroll employment and $MSFE_{i,h}^{OLD}$ as the ones for the 14-variable benchmark model, the relative mean squared forecast error (RMSFE) statistic is expressed as the ratio of the former to the latter,

$$RMSFE_{i,h} = \frac{MSFE_{i,h}^{NEW}}{MSFE_{i,h}^{OLD}} \ ,$$

so that values less than one imply better forecasts from the model augmented by nonfarm employment.

Table 2 presents the forecast accuracy comparison. Overall improvements, as measured by the number of smaller relative MSFEs recorded by the revised model over all variables and forecast horizons, are more than half at 54.8 percent (bottom panel). Segmenting the results by forecast horizon shows that overall improvement is best in the near term ($h = 1, ..., 4$) with 64.3 percent of cases showing improvement, followed by the medium term ($h = 5, ..., 8$) with 57.1 percent. The benchmark model does better in the longer term ($h = 9, ..., 12$) with only 42.9 percent of cases improved upon by the augmented model.

With respect to the SEP variables, the results are mixed but on the whole positive for the augmented model. Beginning with the longer evaluation period (see Table 2), real GDP ($GDPR$) is better predicted by the benchmark model in the near term—particularly for the first two quarters. The advantage disappears as the forecast horizon lengthens, with more accurate real GDP forecasts produced by the augmented model in the medium and longer term. Skill at forecasting the unemployment rate (UR) is nearly equal in the two models, with the exception of the one-quarter horizon where the benchmark model has the advantage. Forecasts of PCE prices generated by the augmented model, in contrast, display substantial improvement at all time horizons. Note in particular that core consumer price ($PCXFE$) forecast accuracy improves between 8 and 10 percent. The accuracy gains achieved by the augmented model for overall consumer prices (PC) are smaller (roughly half of those for core prices) but nevertheless large. This result is to be expected in consideration of the difficult-to-predict energy price component in overall PCE prices. Finally, the federal funds rate (RFF) forecast accuracy is comparable between the two models.

A look at the second panel of Table 2 reveals a partial conjecture for the reason behind the improvements in price-level forecasting accuracy. The introduction of nonfarm employment improves forecast accuracy of both existing labor market quantity and price variables: the index of aggregate labor hours ($LHRS$) and earnings per hour ($EARNS$). The accuracy improvement in the latter is especially striking—roughly 15 to 20 percent. The addition of nonfarm employment implicitly introduces an hours-per-worker concept to the model, perhaps adding a finer statistical rendering of labor market utilization.

4 Summary

The results have shown that introducing total nonfarm employment to the MF-VAR model improves its overall forecast performance. With respect to SEP variables, the accuracy of PCE price forecasts is substantially better. Real GDP forecasts are more accurate in the medium to longer terms while conceding only moderate to small amounts of accuracy in the immediate short term. Forecast performance for the unemployment rate and the federal funds rate is essentially unchanged.

References

[1] Beauchemin, K., 2013. “A 14-Variable Mixed-Frequency VAR Model,” *Federal Reserve Bank of Minneapolis, Staff Report* 493.

[2] Schorfheide, F. and D. Song, 2012. “Real-Time Forecasting with a Mixed-Frequency VAR,” *Federal Reserve Bank of Minneapolis, Working Paper* 701.

Tables

Table 1. Updated Model Variables

Code	Series	Change	Transform
GDPR*	Real GDP (chained 2005 dollars)		log-level
UR	Unemployment rate (%)		level/100
PC	PCE price index		log-level
PCXFE	Core PCE price index		log-level
RFF	Effective federal funds rate (%)		level/100
LHRS	Index of aggregate weekly hours		log-level
LEMP	Nonfarm business payroll employment	New	log-level
EARNS	Average hourly earnings ($)		log-level
IP	Industrial production index		log-level
CONSR	Real personal consumption expenditures		log-level
IFIXR*	Real fixed investment		log-level
GOVR*	Real government purchases		log-level
RTCM10	10-year Treasury note yield (%)		level/100
RBAA	Moody's Baa corporate bond yield (%)		level/100
SP500	S&P 500 composite stock price index		log-level

*Quarterly time series.

Table 2. Relative Mean Squared Errors

y_i	1	2	3	4	5	6	7	8	9	10	11	12
GDPR	1.040	1.043	1.023	1.003	0.976	0.970	0.968	0.968	0.967	0.969	0.972	0.975
UR	1.035	0.982	0.985	0.991	0.998	1.005	1.009	1.011	1.011	1.013	1.013	1.012
PC	0.960	0.954	0.956	0.953	0.943	0.942	0.940	0.935	0.936	0.935	0.935	0.934
PCXFE	0.915	0.908	0.910	0.908	0.906	0.904	0.901	0.898	0.899	0.900	0.899	0.898
RFF	1.007	0.971	0.996	1.018	1.026	1.026	1.029	1.031	1.034	1.036	1.038	1.037
LHRS	0.954	0.943	0.951	0.963	0.974	0.980	0.983	0.985	0.986	0.987	0.988	0.988
LEMP	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
EARNS	0.848	0.796	0.784	0.784	0.783	0.780	0.781	0.781	0.781	0.781	0.779	0.775
IP	0.960	0.947	0.962	0.987	1.004	1.011	1.016	1.020	1.022	1.023	1.027	1.030
CONSR	0.966	0.908	0.918	0.942	0.953	0.963	0.958	0.971	0.976	0.981	0.988	0.994
IFIXR	1.125	1.070	1.053	1.036	1.025	1.022	1.028	1.041	1.052	1.064	1.081	1.096
GOVR	1.141	1.011	1.006	0.971	0.965	0.976	0.980	0.989	1.008	1.021	1.028	1.038
RTCM10	1.147	1.020	1.015	1.022	1.037	1.037	1.033	1.043	1.063	1.075	1.076	1.086
RBAA	1.006	0.940	0.953	0.961	0.985	1.004	1.033	1.071	1.108	1.133	1.139	1.154
SP500	1.013	0.973	0.987	0.996	0.995	1.000	1.006	1.007	1.006	1.008	1.0082	1.009

	h = 1,...,12	h = 1,...,4	h = 5,...,8	h = 9,...,12
Mean	0.982	0.975	0.975	0.996
Median	0.990	0.972	0.987	1.009
Min	0.775	0.784	0.780	0.775
Max	1.154	1.147	1.107	1.154
% < 1	0.548	0.643	0.571	0.429