# Social Interactions and Inequality

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## Outline

1. Basic ideas

2. Theory

3. Public Policy

# Social Influences and Individual Outcomes: Basic Ideas

1. Individual beliefs, preferences, and opportunities are conditioned by group memberships. This dependence typically takes the form of complementarities, so the likelihood or level of an action by one person increases with respect to the behavior (or certain characteristics) of others.

2. Memberships evolve in response to these interactions. Groups (nonoverlapping subsets of the population) stratify along characteristics which affect outcomes. Economic and social (typically ethnic) segregation result in neighborhoods, schools, etc.

3. Persistent intergenerational inequality and poverty result as individuals face different interactions environments over their lives as well as well as persistent intergenerational inequality and poverty as stratification of society affects both parents and children.

Social interaction models thus study the interplay of social forces which influence individual outcomes and individual decisions which determine group memberships and hence social forces.

Leads to a “memberships theory of inequality” in which segregation is source of persistent inequality.

## Key Features of this Approach

1. Individual incentives and social structure meld into a more general explanation of individual behavior. From the perspective of economics, introduction of better sociology; from the perspective of sociology, better economics!

2. Approach explicitly incorporates incomplete markets and other deviations from baseline neoclassical theory of choice.

3. Aggregate behaviors such as crime or nonmarital fertility rates emerge through the interactions within a heterogeneous population

## Examples of Social Influences

1. Peer group effects

2. Role models

3. Social norms

4. Social learning

## Phenomena Where Social Interactions Plausibly Matter

1. Fertility

2. Education

3. Employment

4. Health

5. Language

## Types of Groups

1. Endogenous

-Neighborhoods

-Firms

-Schools

2. “Exogenous”

- Ethnicity

- Gender

## Basic Structure of Social Interactions Theories

## "Standard" Model of Individual Choice

$\omega_i =$ choice of behavior of individual *i*

$\Omega_i =$ constraint set

$X_i =$ observable individual characteristics,

$\varepsilon_i$ = unobservable individual characteristics (to the modeler)

Algebraically, the individual choices represent solutions to

$$\max_{\omega_i \in \Omega_i} V(\omega_i, X_i, \varepsilon_i)$$

$$\text{such that } \Omega_i = \Omega(X_i, \varepsilon_i)$$

## Social Interactions Approach

$g(i) =$ group of individual $i$

$y_{g(i)} =$ characteristics of $g(i)$

$\mu_i^e(\omega_{-i}) =$ subjective beliefs individual $i$ has concerning behavior of others in his group, where

$$\omega_{-i} = (\omega_1, \ldots \omega_{i-1}, \omega_{i+1}, \ldots \omega_I)$$

In this case, choice is described by

$$\max_{\omega_i \in \Omega_i} V\left(\omega_i, x_i, y_{g(i)}, \mu_i^e\left(\omega_{-i} \middle| y_{g(i)}\right), \varepsilon_i\right)$$

$$\text{such that } \Omega_i = \Omega\left(x_i, y_{g(i)}, \mu_i^e\left(\omega_{-i}\right), \varepsilon_i\right)$$

In words, preferences, constraints, beliefs depend on memberships.

# Key Theoretical Properties

1. Multiple Equilibria

2. Social Multipliers

3. Phase Transition

– The properties are “universal,” although they of course depend on parameter values.

– Deeply nonlinear

## A Multinomial Logit Approach to Social Interactions

1. Each agent faces a common choice set with $L$ discrete possibilities, i.e. $\Omega_i = \{0, 1, \ldots, L-1\}$.

2. Each choice $l$ produces a payoff for $i$ according to:

$$V_{i,l} = h_{i,l} + Jp^{e}_{i,l} + \varepsilon_{i,l}$$

3. Random utility terms $\varepsilon_{i,l}$ are independent across $i$ and $l$ and are doubly exponentially distributed with index parameter $\beta$,

$$\mu\left(\varepsilon_{i,l} \le \varsigma\right) = \text{exp}\left(-\text{exp}\left(-\beta\varsigma + \gamma\right)\right)$$

where $\gamma$ is Euler's constant.

## Characterizing Choices

These assumptions may be combined to produce a full description of the choice probabilities for each individual.

$$\mu\left(\omega_i = l \middle| h_{i,j}, p^e_{i,j}\ \forall j\right) =$$
$$\mu\left(\operatorname{argmax}_{j\in\{0\ldots L-1\}} h_{i,j} + Jp^e_{i,j} + \varepsilon_{i,j} = l \middle| h_{i,j}, p^e_{i,j}\ \forall j\right)$$

The double exponential assumption for the random payoff terms leads to the canonical multinomial logit probability structure

$$\mu\left(\omega_i = l \middle| h_{i,j}, p_{i,j}^e \ \forall j\right) = \frac{\exp\left(\beta h_{i,l} + \beta J p_{i,l}^e\right)}{\sum_{j=0}^{L-1} \exp\left(\beta h_{i,j} + \beta J p_{i,j}^e\right)}$$

So the joint probabilities for all choices may be written as

$$\mu\left(\omega_1 = l_1,\ldots,\omega_I = l_I \,\middle|\, h_{i,j}, p^e_{i,j}\ \forall i,j\right) = \prod_i \frac{\exp\left(\beta h_{i,l_i} + \beta J p^e_{i,l_i}\right)}{\sum_{j=0}^{L-1} \exp\left(\beta h_{i,j} + \beta J p^e_{i,j}\right)}$$

## Self-Consistency of Beliefs

Self-consistent beliefs imply that the subjective choice probabilities $p_l^e$ equal the objective expected values of the percentage of agents in the group who choose $l$, $p_l$, the structure of the model implies that

$$p_{i,l}^e = p_l = \int \frac{\exp\left(\beta h_{i,l} + \beta J p_l\right)}{\sum_{j=0}^{L-1} \exp\left(\beta h_{i,j} + \beta J p_j\right)} dF_h$$

where $F_h$ is the empirical probability distribution for the vector of deterministic terms $h_{i,l}$.

It is straightforward to verify that the model always has at least one equilibrium set of self-consistent aggregate choice probabilities.

## Characterizing Equilibria

To understand the properties of this model, it is useful to focus on the special case where $h_{i,l} = 0\ \forall i, l$. For this special case, the choice probabilities (and hence the expected distribution of choices within a group) are completely determined by the compound parameter $\beta J$.

An important question is whether and how the presence of interdependencies produces multiple equilibria for the choice probabilities in a group.

In order to develop some intuition as to why the number of equilibria is connected to the magnitude of $\beta J$, it is helpful to consider two extreme cases for the compound parameter, namely $\beta J = 0$ and $\beta J = \infty$.

For the case $\beta J = 0$, one can immediately verify that there exists a unique equilibrium for the aggregate choice probabilities such that $p_l = \frac{1}{L}$ $\forall l$.

This follows from the fact that under the assumption that all individual heterogeneity in choices come from the realizations of $\varepsilon_{i,l}$, a process whose elements are independent and identically distributed across choices and individuals. Since all agents are ex ante identical, the aggregate choice probabilities must be equal.

The case $\beta J = \infty$ is more complicated. The set of aggregate choice probabilities $p_l = \frac{1}{L}$ is also an equilibrium if $\beta J = \infty$ since conditional on these probabilities, the symmetries in payoffs associated with each choice that led to this equilibrium when $\beta J = 0$ are preserved as there is no difference in the social component of payoffs across choices.

However, this is not the only equilibrium. To see why this is so, observe that for any pair of choices $l$ and $l'$ for which the aggregate choice probabilities are nonzero, it must be the case that

$$\frac{p_l}{p_{l'}} = \frac{\exp\left(\beta J p_l\right)}{\exp\left(\beta J p_{l'}\right)}$$

for any $\beta J$. This follows from the fact that each agent is ex ante identical. Thus, it is immediate that any set of equilibrium probabilities that are bounded away from 0 will become equal as $\beta J \Rightarrow \infty$.

This condition is necessary as well as sufficient, so any configuration such that $p_l = \frac{1}{b}$ for some subset of $b$ choices and $p_l = 0$ for the other $L - b$ choices is an equilibrium. Hence, for the case where $J = \infty$, there exist

$$\sum_{b=1}^{L} \binom{L}{b} = 2^L - 1$$

different equilibrium probability configurations.

Recall that $\beta$ indexes the density of random utility and $J$ measures the strength of interdependence between decisions.

Why do large $J$ and $\beta$ lead to multiple equilibria?

$J$ is intuitive: strong complementarity facilitates bunching.

$\beta$ less obvious. Large values mean thin tails for $\varepsilon$'s. When tails are thick, too many agents make choices that are determined by shocks, reduced scope for self-consistent bunching.

**Theorem. Multiple equilibria in the multinomial logit model with social interactions**

Assume that $h_{i,l} = k \ \forall i, l$. Then there will exist at least three self-consistent choice probabilities if $\frac{\beta J}{L} > 1$, otherwise choices probabilities are unique.

## Messages

Complex interplay of private incentives, social incentives, heterogeneity determine aggregate configurations, number of equilibria, etc.

Segregation creates potential for multiple equilibria in disadvantaged neighborhoods.

# An Intertemporal Great Gatsby Curve

## a. demography

The population possesses a standard overlapping generations structure. Agents live 2 periods.

In period 1 of life, agent is born and receives human capital investment from the neighborhood in which she grows up. In period 2, adulthood, the agent receives income, becomes a member of a neighborhood, has one child, consumes and pays taxes.

**b. preferences**

The utility of adult *it* is determined in adulthood and depends on consumption $C_{it}$ and income of her offspring, $Y_{it+1}$. Offspring income is not known at $t$, so each agent is assumed to maximize expected utility that has a Cobb-Douglas specification.

$$EU_{it} = \pi_1 \log(C_{it}) + \pi_2 E\left(\log(Y_{it+1}) \middle| F_t\right)$$

## c. income and human capital

Adult *it*'s income is determined by two factors, level of human capital that is determined in childhood, $H_{it-1}$ and a shock experienced in adulthood $\xi_{it}$.

$$Y_{it} = \phi H_{it-1} \xi_{it}$$

This functional form matters as it will allow the model to generate endogenous long term growth in dynasty-specific income.

## d. family expenditures

A parent's income decomposes between consumption and taxes.

$$Y_{it} = C_{it} + T_{it}$$

**e. educational expenditure and educational investment in children**

Taxes are linear in income and are neighborhood- and time-specific

$$\forall i \in nt,\ T_{it} = \tau_{nt} Y_{it} \ .$$

The total expenditure available for education in neighborhood $n$ at $t$

$$TE_{nt} = \sum_{j \in nt} T_{jt}$$

and so constitutes the resources available for educational investment.

We assume that the education process exhibits non-convexities with respect to population size, i.e. there exists a type of returns to scale (with respect to student population size) in the educational process.

Some evidence for this; can be replaced with assumption of preferences for larger neighborhoods

Let $p_{nt}$ denotes the population size of $n$ at time $t$. The educational investment provided by the neighborhood to each child, $ED_{nt}$ (equivalent to educational quality), requires total expenditures

$$ED_{nt} = \frac{TE_{nt}}{\nu\left(p_{nt}\right)}$$

where $\nu(p_{nt})$ is increasing such that that for some positive parameters $\lambda_1$ and $\lambda_2$, $0 < \lambda_1 < \frac{\nu(p_{nt})}{p_{nt}} < \lambda_2 < 1$

**f. human capital**

The human capital of a child is determined by two factors: the child's skill level $s_{it}$ and the educational investment level $ED_{nt}$

$$H_{it} = \theta(s_{it}) ED_{nt},$$

where $\theta(\cdot)$ is positive and increasing. The term "skills" is used as a catch-all to capture the class of personality traits, preferences, and beliefs that transform a given level of educational investment into human capital.

The linear structure is extremely important as it will allow dynasty income to grow over time. Together model has an AK-type growth structure relating educational investment and human capital, which can lead family dynasties to exhibit income growth because of increasing investment over time.

Entry level skills are determined by an interplay of family and neighborhood characteristics

$$s_{it} = \zeta\left(Y_i, \bar{Y}_{-i}\right)$$

where $\zeta$ is increasing and exhibits complementarities. Dependence on $Y_i$ is a placeholder for the role of families in skill formation. Dependence on $\bar{Y}_{-i}$ is readily motivated by a range of social interactions models.

## g.neighborhood formation

Neighborhoods reform every period, i.e. there is no housing stock. As such, neighborhoods are like clubs. Neighborhoods are groupings of families, i.e. all families who wish to form a common neighborhood and set a minimum income threshold for membership. This is a strong assumption. That said, we would emphasize that zoning restrictions matter in neighborhood stratification, so the core assumption should not be regarded as obviously inferior to a neighborhood formation rule based on prices.

## h.political economy

The equilibrium tax rate in a neighborhood is one such that there does not exist an alternative one preferred by a majority of adults in the neighborhood.

The Cobb-Douglas preference assumption renders existence of a unique majority voting equilibrium trivial because, under these preferences, there is no disagreement on the preferred tax rate. o desired budget share allocation.

### i. borrowing constraints

Neither families nor neighborhoods can borrow. This extends the standard borrowing constraints in models of this type. With respect to families, we adopt Loury (1981) idea that parents cannot borrow against future offspring income. Unlike his case, the borrowing constraint matters for neighborhood membership, not because of direct family investment. In addition, in our analysis, communities cannot entail children who grow up as members to pay off debts accrued for their education. Both assumptions follow legal standards, and so are not controversial.

**Neighborhood formation and intergenerational income dynamics: model properties**

**Proposition 1. Equilibrium neighborhood structure**

i. At each $t$ for every cross-sectional income distribution, there is at least one equilibrium configuration of families across neighborhoods.

ii. In any equilibrium, neighborhoods are segregated.

Proposition 1 does not establish that income segregation will occur. Clearly it is possible that all families are members of a common neighborhood. If all families have the same income, complete integration into a single neighborhood will occur because of the nonconvexity in the education investment process. Income inequality is needed for segregation. Proposition 2 follows immediately from the form of the education production function nonconvexity we have assumed.

**Proposition 2. Segregation and inequality**

There exist income levels $\bar{Y}^{high}$ and $\bar{Y}^{low}$ such that families with $Y_{it} > \bar{Y}^{high}$ will not form neighborhoods with families with incomes $\bar{Y}^{low} > Y_{it}$.

Intuitively, if family incomes are sufficiently different, then more affluent families do not want neighbors whose tax base and social interactions effects are substantially lower than their own. Benefits to agglomeration for the affluent can be reversed when families are sufficiently poorer.

**Income dynamics**

Along an equilibrium path for neighborhoods, dynasty income dynamics follow the transition process

$$\Pr\left(Y_{it+1} \middle| F_t\right) = \Pr\left(Y_{it+1} \middle| \bar{Y}_{nt}, p_{nt}\right)$$

This equation illustrates the primary difficulty in analyzing income dynamics in this framework: one has to forecast the neighborhood composition. This leads us to focus on the behavior of families in the tails of the income distribution, in particular the highest and lowest income families at a given point in time.

**Proposition 3. Equilibrium income segregation and its effect on the highest and lowest income families**

i. Conditional on the income distribution at $t$, the expected offspring income for the highest family in the population is maximized relative to any other configuration of families across neighborhoods.

ii. Conditional on the income distribution at $t$, the expected offspring income of the lowest income family in the population is minimized relative to any other configuration of families across neighborhoods that does not reduce the size of that family's neighborhood.

**Proposition 4. Expected average growth rate for children in higher income neighborhoods than for children in lower income neighborhoods**

Let $g_{nt+1}$ denote the average expected income growth between parents and offspring in neighborhood $n,t$. For any two neighborhoods $n$ and $n'$ if $\bar{Y}_{nt} < \bar{Y}_{n't}$ $p_{nt} \geq p_{nt'}$, then $g_{nt+1} - g_{n't+1} > 0$.

Proposition 4 does not speak to the sign of $g_{nt}$. Under the linear assumptions of this model, there exists a formulation of $\Theta(\cdot)$ and $\xi(\cdot,\cdot,\cdot)$ such that neighborhoods exhibit positive expected growth in all time periods, i.e. $\forall nt \quad g_{nt} > g_{\min} > 0$. In essence, this will hold when educational investment is sufficiently productive relative to the preference-determined equilibrium tax rates so that investment levels grow (this is the AK growth model requirement as modified by the presence of social interactions). We assume positive growth in what follows.

**Proposition 5. Decoupling of upper and lower tails from the rest of the population of family dynasties**

i. If $\forall nt \;\; g_{nt} > 0$, then there exists a set of time $t$ income distributions such that the top $\alpha$% of families in the distribution never experience a reduction in the ratios of their incomes compared to any outside this group

ii. If $\forall nt \;\; g_{nt} > 0$, then there exists a set of time $t$ income distributions such that the bottom $\beta$% of families in the distribution never experience an increase in the ratios their incomes compared to any dynasty outside this group

**Proposition 6. Intergenerational Great Gatsby curve**

There are skill formation technologies such that there exists a set of time $t$ income distributions such that the intergenerational elasticity of parent/offspring income will be increased by a mean preserving increase in the variance of logarithm of initial income.

# Messages

Social factors can produce Great Gatsby Curve

Mobility behavior in growing economies can be qualitatively different from stationary ones.

# Public Policy

-Associational redistribution

-Nonlinearity

# Associational Redistribution

**Examples**

-affirmative action

-busing for integration

-charter schools/magnet schools

## Normative Issues

**-**competing ethical claims

-political feasibility

-supply side approach

## Ethics of Associational Redistribution

Following ideas due to John Roemer and others, one objective of public policy is to reduce the dependence of individual outcomes on factors for which an individual is not responsible.

Many group memberships fall into this category, therefore the government may be justified in redistributing group memberships.

# Competing Ethical Claims

-Meritocracy

-Self Actualization

## Politics of Associational Redistribution

Bottom Line: Such policies are immensely unpopular**.**

Possible alternative: implement policies that only indirectly redistribute memberships. One way to do this is to invest differentially in individuals to alter chances of admission, etc.

## Nonlinearities

Neighborhoods models strongly suggest that policy effects may be highly nonlinear.

This means is that one cannot evaluate a large policy intervention by a proportional scaling up of the effects found from a small policy intervention.  This nonlinearity can cut in more than one direction.

It is possible that a large scale expansion of the MTO demonstration could be far less efficacious than the small scale program has been. On the other hand, it is possible for large scale interventions to be far more efficacious than small scale ones. One reason is that a large scale intervention may alter the number of possible self-consistent aggregate behaviors for a given group.

Nonlinearity produces new issues associated with optimal policy design. Should resources be concentrated on a few of the disadvantaged in order to exploit nonlinearities? How does one deal with fairness issues?

Bottom line: equity and efficiency tradeoffs