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Abstract

We analytically characterize the aggregate productivity loss from allocative distortions in a setting that accounts for the sectoral linkages of production. We show that the effects of distortions and the role of sectoral linkages depend crucially on how substitutable inputs are. We find that the productivity loss is smaller if input substitutability is low. Moreover, with low input substitutability, sectoral linkages do not systematically amplify the effects of distortions. In addition, the impact of the sectors that supply intermediate inputs becomes smaller. We quantify these effects in the context of the distortions caused by market power, using industry-level data for 35 countries. With our benchmark calibration, which accounts for low input substitutability, the median aggregate productivity loss from industry-level markups is 1.3%. To assume instead unit elasticities of substitution (i.e., to use a Cobb-Douglas production function) would lead to overestimating the productivity loss by a factor of 1.8. Sectoral linkages do amplify the cost of markups, but the amplification factor is considerably weaker than with unit elasticities.

Keywords: Aggregate productivity, Input-output, Production network, Misallocation, CES production function, market power. *JEL Classifications*: D57, D61, O41, O47

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1 Introduction

The production of goods and services involves a complex web of firms connected through supply chains. For example, most of the items found in grocery stores are produced by food manufacturers using farm products and products from other food manufacturers, as well as various other intermediate inputs such as energy, transportation, and business services. These production linkages across firms and sectors are key to understanding aggregate productivity, especially when the firms' input decisions may be distorted by market frictions. Frictions in one sector may ripple outward through the economy, as the frictions not only affect the distorted firms' input choice but may also indirectly affect the decisions of the other firms along the production chain. In this paper, we study how aggregate productivity is affected by frictions that distort the allocation of inputs across firms, taking into account production chains and the sectoral linkages of production. Findings from recent work, which highlight how sectoral linkages amplify and propagate various sector- or firm-level shocks (e.g., Jones, 2011; Acemoglu et al., 2012; Bigio and La'o, 2017; Atalay, 2017), suggest that the interconnection between sectors amplifies the consequences of such distortions. Do sectoral linkages always amplify the productivity loss from allocative distortions? Do sectors supplying intermediate inputs have a larger impact than other sectors? Which types of linkages are relevant to understanding the effects of distortions on aggregate productivity?

We show that the answers to all these questions depend crucially on how substitutable inputs are. We provide a theoretical characterization and a quantitative evaluation of how input substitutability shapes the effect of distortions and the role of the linkages between sectors. We find that the complementarities in the production process (i.e., low input substitutability) mitigate the effects of distortions on aggregate productivity, reduce the role of sectoral linkages, and reduce the impact of intermediate-input suppliers. Furthermore, we find that sectoral linkages do not systematically amplify the effects of distortions; we derive the conditions under which the productivity loss from distortions is smaller than in an otherwise identical economy without sectoral linkages. Our results indicate that abstracting from input complementarity leads to overestimating the effects of distortions and the strength of the amplification from sectoral linkages.

We investigate the effects of distortions and the role of input substitutability using a multi-sector model with intersectoral linkages of production. The model builds on Long and Plosser (1983) and is similar to Jones (2011) but allows for more flexible production functions and a richer input-output structure. Each sector combines primary inputs with intermediate inputs using a nested constant-elasticity-of-substitution production function, with possibly non-unitary elasticity of substitution between the intermediate inputs and between the primary input and the intermediate-input bundle. The input-output structure is general in that each sector may use the various inputs in different proportions. We consider distortions in the use of primary or intermediate inputs. These distortions could be the result of financial frictions, labor-market frictions, contract-enforcement frictions, or imperfect competition. Following Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), we do not model the sources of the distortions. Rather, we model distortions, in a reduced-form manner, as wedges that lead firms to deviate from the frictionless input allocation. The allocation may be distorted along the different stages of production. First, producers may purchase an inefficient mix of intermediate inputs. Second, the relative price of the intermediate-input bundle may differ from the opportunity cost and hence may lead to an inefficient use of the intermediate-input bundle. Finally, goods with similar total resource costs may not have the same price, which introduces further inefficiencies in the consumption allocation. Input substitutability shapes the overall effect of distortions on aggregate productivity by influencing how firms' input choices respond to their own distortions and to the distorsions in their suppliers' price, as well as how the changes in the firms' inputs affect their output.

We provide an analytical characterization of the effects of distortions on aggregate total factor productivity (TFP) in the presence of sectoral linkages. We separately study two types of distortions, depending on whether they affect only primary inputs (like labor-market frictions) or all inputs uniformly (like markups). The analytical characterizations highlight the role of the interaction between the firms' distortions, their linkages, and the distortions of their suppliers. How this interaction shapes the TFP loss depends on the degree of input substitutability. We derive three main results concerning how input substitutability determines (i) the TFP loss from distortions, (ii) the amplification from sectoral linkages, and (iii) the impact of each sector.

Our first theoretical result is that input complementarity mitigates the effects of the two types of distortions (primary and all-input) on aggregate productivity. This result may seem counter-intuitive. With less flexibility to substitute less-distorted for distorted inputs, one could expect distortions to have a larger impact on aggregate productivity. On the contrary, we find that distortions have a smaller impact on aggregate productivity when inputs are more complementary. Two opposing forces are at work. On the one hand, higher complementarity amplifies the effect of a change in the firms' inputs on their output. On the other hand, higher complementarity reduces the effect of the distortions on the firms' input decision because firms respond less to the (distortion-induced) change in the relative prices when the elasticity of substitution is low. The second force, which is reminiscent of Ramsey's (1927) result on the optimal taxation of commodities, dominates, and higher complementarity leads to a smaller aggregate productivity loss. Low elasticities' mitigating effect for distortions stands in stark contrast with their implication for negative productivity shocks. In line with previous work, we show that a lower elasticity amplifies negative sectoral productivity shocks: when inputs are more complementary, a productivity drop in one sector leads to a larger fall in aggregate productivity.[1] Input substitutability's opposite effect for distortions and productivity shocks makes it important to be clear on the nature of the frictions when studying their effects on aggregate productivity. For frictions that distort relative prices (like the ones considered in this paper), accounting for input complementarity reduces the TFP loss, whereas for frictions that entail a real resource cost (and hence are akin to lower productivity), accounting for input complementarity increases the TFP loss.

Our second theoretical result is that sectoral linkages do not systematically amplify the effects of distortions. We establish the conditions under which sectoral linkages dampen the effects of distortions and find that these conditions are more likely to be satisfied when the distortions affect only primary inputs. With all-input distortions, the dampening effect occurs if inputs are highly complementary and if the correlation between the sectors' distortions and their suppliers' distortions (or their total exposure to intermediate inputs) is negative. With primary-input distortions, the inputs need not be highly complementary, and no condition on correlations needs to be satisfied; the dampening effect occurs if inputs are less substitutable than Cobb-Douglas (and provided sectoral linkages do not affect too much the size of each sector). Therefore, distortions that affect all inputs, such as markups, are more likely to be amplified by the sectoral linkages than distortions that affect only primary inputs, such as labor-market frictions. Our result suggests that markups are also more likely to be amplified than financial frictions, since financial frictions bear more on the firms' primary inputs than on their intermediate-input decisions.

Our third theoretical result also concerns the role of sectoral linkages. We show how the impact of each sector varies with the value of the elasticity of substitution. When the input elasticity is lower, the aggregate impact of distortions in sectors that supply intermediate inputs is attenuated; as a result, the relative impact of final-output suppliers becomes larger. Moreover, we find that the interconnection between distorted sectors plays a smaller role when the elasticity of substitution in lower.

We use the model, calibrated on industry-level data from the World Input-Output Database

[1] The role of complementarity has been studied by Kremer (1993) and, more recently, by Jones (2011) and Baqaee and Farhi (2019). Another example is Atalay (2017), who shows that sectoral productivity shocks are more important when inputs are less substitutable than Cobb-Douglas.

for 35 countries, to study the aggregate TFP losses from the sectoral distortions caused by market power. Markups create a wedge between the firms' price and their marginal cost and are hence isomorphic to a distortion that uniformly affects all inputs. We measure industry-level markups using the price-cost margins, and we calibrate the production function parameters for each country separately, accounting for the presence of markups and for the value of the elasticities of substitution. Estimates of these elasticities point to complementarities between inputs. Following Atalay's (2017) estimates, in our benchmark calibration we set the elasticity of substitution between intermediates to 0.01 and the one between labor and the intermediate-input bundle to 0.7. We compute the aggregate productivity gain from reducing industry-level markups to zero, and we find a productivity gain of 1.3% for the median country and one higher than 5% for Turkey, Indonesia, and Mexico. These numbers are conservative estimates of the TFP loss from markups, since they abstract from the *within*-sector misallocation caused by markups.

The quantitative analysis, performed on a large number of countries, allows us to validate and quantify our three theoretical results. In addition, we complement our analysis of the role of input substitutability by studying a question closely related to but conceptually different from the one asked in the theoretical analysis. Whereas we vary the value of the elasticity of substitution in the theoretical analysis, *holding fixed the other production-function parameters*, we vary the value of the elasticities of substitution in the quantitative analysis, *holding fixed the calibration targets*. In other words, we study the consequences of calibrating the model using different values of the elasticities of substitution, which implies recalibrating the production-function parameters each time the value of one of the elasticities of substitution is modified. From a quantitative perspective, this question is the most relevant one. We find that all the insights obtained with the theoretical results hold for this question as well.

First, the quantitative analysis shows that the TFP gains are lower when the model is calibrated with a lower input substitutability than the benchmark, and the TFP gains are higher when the model is calibrated assuming a higher input substitutability. In particular, the Cobb-Douglas specification (i.e., unit elasticity of substitution between all inputs) leads to overestimating the TFP gain for the median country by a factor of 1.8 relative to the benchmark calibration. Second, we find that the strength of the amplification from sectoral linkages increases with the degree of input substitutability. The ratio of the TFP gain in the economy with and without sectoral linkages is equal to 3.3 under Cobb-Douglas and to 1.8 with our benchmark calibration. The Cobb-Douglas specification leads also to the overestimation of the amplification from sectoral linkages. Accounting for complementarities in the production process still leads to a sizable amplification from sectoral linkages, although not as large as suggested by the Cobb-Douglas specification. Furthermore, had the distortions affected only primary inputs, there would have been no amplification effect. In that case, the TFP loss is actually a fifth lower relative to the otherwise identical economy without sectoral linkages. Finally, we find that assuming Cobb-Douglas production functions leads to the overestimation of the role of sectors that supply primarily intermediate inputs, such as mining, the non-metallic mineral products industry, and the renting-of-material-and-equipment industry. A central message of the paper is that the Cobb-Douglas specification, ubiquitous in the literature on sectoral linkages of production, leads to greatly overestimating the effects of distortions on aggregate productivity as well as the role of sectoral linkages.

Our paper contributes to the literature on the consequences of allocative distortions on aggregate productivity. This literature, initiated by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), focuses on the allocation of labor and capital; usually, intermediate inputs are not explicitly modeled.[2] By deriving analytical expressions for the effects of distortions, we complement Osotimehin (2019), who derives similar expressions in a setting without intermediate inputs.

Our paper is closely related to Jones (2011, 2013) and several related papers such as Bigio and La'o (2017), Bartelme and Gorodnichenko (2015), Fadinger et al. (2015), Caliendo et al. (2017), Grassi (2017), Liu (2017), Luo (n.d.), and Leal (2017), which investigate the effects of distortions in the presence of intersectoral linkages.[3] Like these papers, we highlight how the linkages between firms can give rise to an additional mechanism through which distortions reduce aggregate TFP. We complement these papers by showing that this mechanism is weaker when input complementarities are accounted for and can even lead to a smaller TFP loss than in an otherwise identical economy without intersectoral linkages.

With its focus on input complementarity, our paper is tightly connected to Jones (2011).[4] In contrast with that paper, we show that complementarity actually reduces the effect of

[2]For a more extensive description of the literature on misallocation, see the surveys by Hopenhayn (2014) or Restuccia and Rogerson (2017).

[3]Bartelme and Gorodnichenko (2015) estimate industry-level distortions and compute the implications for aggregate TFP, while Fadinger et al. (2015) investigate how the productivity and tax rates across sectors interact with the input-output structure of the economy. Bigio and La'o (2017) study how distortions are amplified by input-output linkages when firms use Cobb-Douglas production functions. Leal (2017) quantifies the gain from removing distortions and closing sectoral productivity gaps in Mexico. Luo (n.d.) analyzes the role of trade credit in the propagation of financial shocks. Grassi (2017) studies, in an oligopolistic setting, how productivity shocks lead to endogenous changes in distortions. Liu (2017) analyzes policy intervention in the presence of distortions. Caliendo et al. (2017) estimate the effects of domestic and external distortions on the world GDP. (unlike our approach, in which distortions are relative price gaps, Liu's (2017) and Caliendo et al.'s (2017) distortions are actual resource costs).

[4]Other recent papers emphasize non-unitary elasticity of substitution between inputs, but with a focus on the propagation of productivity shocks. See for example, Atalay (2017), Carvalho et al. (2015), Miranda-Pinto (2018), Miranda-Pinto and Young (2018), and Baqaee and Farhi (2019).

distortions on aggregate TFP. Jones (2011) considers the effect of higher dispersion in productivity and in distortions jointly. We find that the degree of complementarity has opposite implications for productivity and distortions, and jointly analyzing them may mask the mitigating effect of complementarity on distortions.[5] Contemporaneous papers by Boehm and Oberfield (2018) and Baqaee and Farhi (forthcoming) also study the effects of distortions in the context of a production network with input complementarity, but their focus is different than ours. Boehm and Oberfield (2018) consider the effects of enforcement frictions in the market for inputs. Baqaee and Farhi (forthcoming) focus on providing non-parametric formulas for the aggregation of production functions. They also analyze the TFP loss from distortions in a similar setting. We complement their study by deriving, using a distinct approach, analytical characterizations that make transparent the interaction between the distortions and the production-function parameters governing the linkages between sectors and by analyzing how input substitutability shapes the effects of distortions and the role of sectoral linkages.

The paper also contributes to the literature on the macroeconomic implications of market power. The interest in the topic, which goes back at least to Harberger (1954), has recently surged with the debate on the decline in competition and the rise in market power in the US and other countries.[6] Several recent papers have computed the welfare costs of markups and the consequences of market power on macroeconomic outcomes in various settings (Basu, 1995; Epifani and Gancia, 2011; Dhingra and Morrow, forthcoming; Behrens et al., 2018; Edmond et al., 2019, 2015; Peters, 2014). Most of these papers focus on the heterogeneity of markups across firms. Exceptions are Basu (1995), who shows that a uniform markup reduces aggregate productivity in the presence of intermediate inputs, and Edmond et al. (2019), who highlight that most of the welfare loss from markups is similar to that of a uniform output tax.[7] We complement this literature by studying the role of sectoral linkages and by analyzing the interaction between the firms' markups and their linkages. Moreover,

[5]Another notable difference with Jones (2011) is that our framework allows for complementarity not only between intermediate inputs, but also between the intermediate-input bundle and primary inputs. However, this does not explain the difference in our results. In our quantitative analysis, we separately study the effects of the two degrees of complementarity. Our results indicate that a higher degree of complementarity reduces the effect of distortions on aggregate TFP, whether we consider the complementarity between intermediate inputs (as Jones) or between the intermediate-input bundle and primary inputs.

[6]Harberger (1954) finds removing the dispersion of markups across US manufacturing industries (observed in the late 1920s) would yield a negligible welfare gain. For the recent debate on the rise of market power, see, for example, de Loecker et al. (forthcoming).

[7]Basu (1995) derives the implications for the cyclicality of aggregate productivity. He shows that the procyclicality of aggregate productivity can be attributed to a countercyclical markup. Edmond et al. (2019) find that in the US two-thirds of the welfare loss from markups are due to an aggregate markup that acts like a uniform output tax.

whereas the estimates found in the literature are typically for the US, we provide estimates for 35 countries of the cost of sectoral distortions caused by market power.

The paper is organized as follows. We present the model and the general characterization of aggregate productivity in Section 2. Then, in Section 3, we characterize the aggregate productivity loss from distortions and study the role of input substitutability. In Section 4, we present the results of the quantitative application, in which we quantify our theoretical results and compute the effects of markups on aggregate productivity in 35 countries. We offer concluding remarks in Section 5.

2 Aggregate productivity in an economy with sectoral linkages and distortions

We consider a multi-sector model in which firms buy and sell intermediate inputs to each other. The sectoral linkages, which result from the firms' purchase and sale of intermediate inputs, are a key feature of the model. In this section, we describe the model and characterize the aggregate productivity in the presence of distortions in the allocation of inputs.

2.1 Model

The model, which is a generalization of the production side of Long and Plosser (1983), shares similarities with Jones (2011).

Production. The economy consists of n sectors. In each sector, a representative firm produces goods using labor and intermediate goods with the production function

$$Q_i = A_i \left[(1-\alpha_i)^{1-\sigma}(B_i L_i)^{\sigma} + \alpha_i^{1-\sigma} X_i^{\sigma}\right]^{\frac{1}{\sigma}}, \tag{1}$$

where B_i is the labor-augmenting productivity component, A_i is the Hicks-neutral productivity component, and L_i is the labor input.[8] The intermediate-input bundle is given by

$$X_i = \left(\sum_j v_{ij}^{1-\rho} X_{ij}^{\rho}\right)^{\frac{1}{\rho}}, \tag{2}$$

where X_{ij} is the quantity of intermediate goods from sector j used by sector i. We impose

[8] For simplicity, we do not explicitly model capital. The primary input L_i can be thought of as a capital-labor bundle.

$\alpha_i \in [0,1)$, $v_{ij} \in [0,1]$, and $\sum_{j=1}^{n} v_{ij} = 1$ for all $i = 1, ..., n$.[9] With the heterogeneity in α_i and v_{ij}, sectors can differ in both their overall use of intermediate inputs and their mix of intermediate inputs. Note also that many different combinations of A_i, B_i, α_i, and v_{ij} represent the same production function; in the next section, we will exploit this multiplicity to derive a useful normalization. The key parameters of our analysis are $\rho \in (-\infty, 1)$, which determines the elasticity of substitution across intermediate goods; and $\sigma \in (-\infty, 1)$, which determines the elasticity of substitution between labor the intermediate-input bundle. The corresponding elasticities are given by $\varepsilon_\rho \equiv 1/(1-\rho)$ and $\varepsilon_\sigma \equiv 1/(1-\sigma)$. The Cobb-Douglas production function corresponds to $\sigma = \rho = 0$ (or equivalently, $\varepsilon_\rho = \varepsilon_\sigma = 1$).[10]

Final output (consumption) is an aggregate of the goods from the different sectors,

$$Y = \prod_{i=1}^{n} \beta_i^{-\beta_i} \prod_{i=1}^{n} C_i^{\beta_i}, \tag{3}$$

with $\beta_i \in [0,1]$, $\sum_{i=1}^{n} \beta_i = 1$ and where $\prod_{i=1}^{n} \beta_i^{-\beta_i}$ is a convenient normalization that simplifies the expressions of the model's solution without affecting the results.

In this economy, the sectoral linkages are governed by two sets of parameters, collected in the vector $\alpha = (\alpha_1, ..., \alpha_n)'$ and in the matrix $V = (v_{ij})$. Together with the final good parameters, $\beta = (\beta_1, ..., \beta_n)'$, α, and V shape the production network.

Resource constraints. The resource constraints are given by

$$\sum_{i=1}^{n} X_{ij} + C_j = Q_j, \forall j = 1, ..., n \tag{4}$$

$$\sum_{i=1}^{n} L_i = \bar{L}. \tag{5}$$

The first equation states that the gross output of each sector must be equal to the sum of all its uses as an intermediate (including by itself) and of its use in final consumption. The second equation is the resource constraint for labor, with $\bar{L}$ the total (exogenous) supply of labor.

Distortions. The firms' input choices may be distorted by market frictions. Following Restuccia and Rogerson (2008) and Hsieh and Klenow (2009), we model the distortions as

[9] Note that in our specification, we have $\alpha_i^{1-\sigma}$ and $(1-\alpha_i)^{1-\sigma}$, rather than α_i and $1-\alpha_i$, and similarly $v_{ij}^{1-\rho}$ instead of v_{ij}. This specification implies that measuring inputs at different levels of aggregation has no artificial effect on productivity. Also, we assume that every sector uses a positive amount of labor. This assumption simplifies the proofs and can be relaxed.

[10] The Cobb-Douglas production function is $Q_i = A_i(1-\alpha_i)^{-(1-\alpha_i)}(\alpha_i)^{-\alpha_i}(B_i L_i)^{1-\alpha_i} X_i^{\alpha_i}$ with $X_i = \prod_{j=1}^{n} v_{ij}^{-v_{ij}} X_{ij}^{v_{ij}}$.

wedges in the firms' first-order conditions. We define the distortions as deviations from the social-planner (first-best) allocation. We denote the Lagrange multiplier associated with the resource constraint of product i by λ_i and the one associated with labor by η. The allocation of inputs is characterized by the following distorted first-order conditions:

$$\beta_i \frac{Y}{C_i} = \lambda_i \tag{6}$$

$$\lambda_i A_i^{\sigma} \left(\frac{\alpha_i Q_i}{X_i} \right)^{1-\sigma} \left(\frac{\upsilon_{ij} X_i}{X_{ij}} \right)^{1-\rho} = \lambda_j (1 + \tau_{Xij}) \tag{7}$$

$$\lambda_i A_i^{\sigma} B_i^{\sigma} \left(\frac{(1-\alpha_i) Q_i}{L_i} \right)^{1-\sigma} = \eta (1 + \tau_{Li}). \tag{8}$$

Any allocation that satisfies the production and resource constraints can be rationalized by some collection of distortions.[11] Production efficiency is achieved when $\tau_{Xij} = 0$ and $\tau_{Li} = \bar{\tau}_L$ for all $i, j = 1, ..., n$. Note that the distortions τ_{Xij}, τ_{Li} are not isomorphic to productivity shocks; they reduce aggregate productivity because they modify the perceived marginal cost of inputs and thereby distort the allocation of inputs across firms.

The distortions encompass various sources of frictions. We allow the distortions for intermediate goods τ_{Xij} to vary by both purchasing industry i (taxes or subsidies, financial frictions) and supplying industry j (for example, the ease of contract enforcement may vary across commodities, as explored by Nunn 2007). While we solve the model with these general distortions, our analysis will focus on two types of distortions: distortions on labor only, τ_{Li}; and distortions that uniformly affect all the inputs, $\tau_{Xij} = \tau_{Li} = \tau_i$, $\forall j = 1, ..., n$. In the quantitative application, we study the consequences of market power and the distortions caused by markups, which are isomorphic to all-input distortions.

Aggregate production function. The aggregate production function relates the maximum final output that can be obtained given the distortions defined in equations (6)–(8) and the production possibilities determined by equations (1)–(3) and by the resource constraints given by equations (4)–(5). We call this problem *the distorted planner's problem.*[12]

[11] We show in Appendix A.1 that the absence of distortion in equation (6) is without loss of generality: for any allocation characterized by distortions in equation (6), we can find a transformation of the distortions and multipliers that allows us to rewrite the distorted first-order conditions in the form of equations (6) to (8). We also show that there exists a unique set of distortions (up to a linear transformation of the labor distortions) that can rationalize a feasible allocation.

[12] Solving the distorted planner's problem is equivalent to solving the competitive equilibrium in which $\{\tau_{Li}, \tau_{Xij}\}_{i,j=1,...,n}$ are taxes rebated lump sum to the final consumers. We prefer the distorted planner's formulation because it makes it clear that whether distortions reduce profits (like taxes) or increase profits (like markups) is irrelevant for aggregate TFP.

2.2 Aggregate productivity

We solve the distorted planner's problem and characterize aggregate productivity in both the general case and the case where the two elasticities of substitution (between intermediates, and between labor and the intermediate-input bundle) are equal. The general solution is used in the quantitative analysis, whereas the equal-elasticity case, which yields a closed-form expression for aggregate productivity, is used in the theoretical analysis. All the proofs are in Appendix B.

2.2.1 The general case

The following proposition gives the aggregate production function and the aggregate productivity of the distorted economy.

Proposition 1 *Suppose that a solution to the distorted planner's problem exists. Then final output is given by $Y = TFP\ \bar{L}$, aggregate productivity is $TFP = [d'(I-M')^{-1}c]^{-1}$, the vector of gross output is given by $Q = (I - M')^{-1}cY$, final consumption of good i is $C_i = c_iY$, and labor inputs are given by $L_i = d_iQ_i$, and intermediate inputs by $X_{ij} = m_{ij}Q_i$; where I denotes the identity matrix and*

i. d is a column vector with

$$d_i = (1-\alpha_i)(A_iB_i)^{\frac{\sigma}{1-\sigma}} \left(\frac{\lambda_i}{(1+\tau_{Li})\eta}\right)^{\frac{1}{1-\sigma}};$$

ii. M is a matrix with elements

$$m_{ij} = A_i^{\frac{\sigma}{1-\sigma}} \alpha_i v_{ij} \left[\sum_j v_{ij} \left(\frac{\lambda_i}{(1+\tau_{Xij})\lambda_j}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{\sigma-\rho}{\rho(1-\sigma)}} \left(\frac{\lambda_i}{(1+\tau_{Xij})\lambda_j}\right)^{\frac{1}{1-\rho}};$$

iii. c is a column vector with $c_i = \beta_i/\lambda_i$;

iv. λ is a column vector that contains the Lagrange multiplier and which is given by $\lambda = \exp(\log(\lambda/\eta) - \beta' \log(\lambda/\eta))$; and

v. the Lagrange multipliers solve

$$(\lambda_i/\eta) = A_i^{-1} \left[(1-\alpha_i)B_i^{\frac{\sigma}{1-\sigma}}(1+\tau_{Li})^{-\frac{\sigma}{1-\sigma}} + \alpha_i \left[\sum_{j=1}^{n} v_{ij}\left((1+\tau_{Xij})\lambda_j/\eta\right)^{-\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}\frac{\sigma}{1-\sigma}}\right]^{-\frac{1-\sigma}{\sigma}}. \tag{9}$$

The distortions affect aggregate productivity through several channels. In addition to the well-known effects on the allocation of labor (or any primary inputs) across firms, distortions also modify the allocation of intermediate inputs across firms, both directly and indirectly, by modifying the relative shadow price of goods.

Normalization. The model can be normalized such that all productivity parameters are equal to one. This normalization, which will be useful for mapping the model to the data, implies that A_i and B_i should be interpreted as deviations from a baseline.

Proposition 2 *Suppose that* $\alpha_i A_i^{\frac{\sigma}{1-\sigma}} < 1, \forall i$. *Then, there exists a set of positive constants* $\{k_i\}$ *such that* $Q_i' = k_i Q_i$ *has production function in the form of (1) with some parameters* $\{A_i', B_i', \alpha_i'\}_i$ *and* $\{v_{ij}'\}_{i,j}$ *that satisfy the constraints on these parameters, and* $A_i' = B_i' = 1, \forall i = 1, ..., n$.

In the absence of distortions, Proposition 1, together with the productivity normalization of Proposition 2, imply that aggregate productivity is equal to one. In that case, the shadow prices of all the goods are identical ($\lambda_i = 1, \forall i = 1, ..., n$), $\alpha_i' = \sum_j \lambda_j X_{ij} / (\lambda_i Q_i')$, and $v_{ij}' = \lambda_j X_{ij}' / \sum_k \lambda_k X_{ik}'$. Thus, the normalization implies that in an economy without distortions, we can choose units so that physical output is equal to the nominal value of output.

Existence and uniqueness. A solution to the distorted planner's problem does not always exist. In particular, when the elasticities of substitution are smaller than one, sufficiently large positive intermediate-goods distortions would lead to infinitely large relative prices (λ_i/η). On the other hand, sufficiently large negative distortions (in absolute value) can lead the demand for intermediate goods to exceed the feasible gross output.[13] We can however show existence and uniqueness when the vector of distortions is sufficiently small.

Proposition 3 *Suppose that* $\alpha_i A_i^{\frac{\sigma}{1-\sigma}} < 1, \forall i = 1, ..., n$. *Then, a solution to the distorted planner's problem exists and is unique in an open neighborhood of distortions containing the zero-distortions vector.*

In the quantitative application, we find a solution exists even for the sizable distortions observed in the data.

[13] This is easiest to see in a simple example with only one sector and Cobb-Douglas production. In this case, $\lambda/\eta = (1 + \tau_X)^{\frac{1}{1-\alpha}}$ and $X/Q = \alpha/(1 + \tau_X)$. Then, if $\tau_X < -(1 - \alpha)$, $X/Q > 1$, demand for intermediate goods exceeds gross output, which is clearly impossible. Note that in this example, the Inada conditions on the production function hold and an equilibrium still may not exist.

2.2.2 The equal-elasticity case

When the two elasticities of subsitution are equal, $\sigma = \rho$, we obtain a closed-form solution for aggregate productivity. We present here the case $\sigma = \rho \neq 0$ and describe the Cobb-Douglas case ($\sigma = \rho = 0$) in Appendix A.2.

Proposition 4 *Suppose that $\sigma = \rho$ and $\sigma \neq 0$. Then, aggregate output is given by*

$$\log Y = \log \bar{L} + \log TFP,$$

aggregate productivity is given by

$$\log TFP = \frac{1-\sigma}{\sigma}\beta' \log(\hat{\lambda}) - \log(\hat{c}'[I - \tilde{\alpha}\hat{A}(\Delta^{Xq} \circ V)]^{-1}\hat{A}\hat{B}, \Delta^{Lq}(\mathbf{1} - \alpha)) \tag{10}$$

and the vector of Lagrange multipliers is given by

$$\hat{\lambda} = [I - \tilde{\alpha}\hat{A}(\Delta^{Xp} \circ V)]^{-1}\hat{A}\hat{B}\Delta^{Lp}(\mathbf{1} - \alpha); \tag{11}$$

where $\circ$ denotes the Hadamard (entrywise) product, I is the identity matrix, $\mathbf{1}$ is a vector of ones, $\tilde{\alpha}, \hat{A}, \hat{B}, \Delta^{Xp}, \Delta^{Xq}$, Δ^{Lp} , Δ^{Lq} are square matrices and $\hat{\lambda}, \hat{c}$ are column vectors defined as follows: $\tilde{\alpha}_{ii} = \alpha_i$ and $\tilde{\alpha}_{ij} = 0$ $\forall i \neq j$; $\hat{A}_{ii} = A_i^{\frac{\sigma}{1-\sigma}}$ and $\hat{A}_{ij} = 0$ $\forall i \neq j$, $\hat{B}$ is defined similarly; $\Delta^{Xp}_{ij} = (1 + \tau_{Xij})^{-\frac{\sigma}{1-\sigma}}$, $\Delta^{Xq}_{ij} = (1 + \tau_{Xij})^{-\frac{1}{1-\sigma}}$; $\Delta^{Lp}_{ii} = (1 + \tau_{Li})^{-\frac{\sigma}{1-\sigma}}$, $\Delta^{Lp}_{ij} = 0, \forall i \neq j$; $\Delta^{Lq}_{ii} = (1 + \tau_{Li})^{-\frac{1}{1-\sigma}}$, $\Delta^{Lq}_{ij} = 0, \forall i \neq j$; $\hat{\lambda}_i = (\lambda_i/\eta)^{-\frac{\sigma}{1-\sigma}}$; $\hat{c}_i = \beta_i \hat{\lambda}_i^{-1}$.

As shown by equations (10) and (11), the aggregate TFP is determined by a complex interplay of the distortions (Δ^{Xq}, Δ^{Lq}, Δ^{Xp}, Δ^{Lp}) and the production linkages (α, β, V). In particular, the inverse matrices $[I - \tilde{\alpha}\hat{A}(\Delta^{Xq} \circ V)]^{-1}$ and $[I - \tilde{\alpha}\hat{A}(\Delta^{Xp} \circ V)]^{-1}$ play a key role in the interaction between the distortions and the production linkages. Our theoretical results, which we present in the next section, are based on these two equations.

3 Theoretical analysis

In this section, we analytically characterize the effects of distortions on aggregate productivity. We find that the effects of distortions and the role of sectoral linkages depend crucially on how substitutable inputs are. All the proofs and all the derivations of the expressions given in this section are in Appendix B.

3.1 Definitions

We start by defining several useful variables. These sectoral variables, $(x_{ij}, x_i, s_i, s_{ij}, \gamma_i)$, which are all functions of the production network parameters, (α, β, V), will help analytically characterize the effects of distortions. We also define the β-weighted variance and covariance, which will be useful for studying the role of input substitutability.

1. *Leontief inverse.* The Leontief inverse, which appears commonly in input-output analysis, captures the direct and indirect linkages between sectors. We denote $\Omega \equiv (I - \tilde{\alpha}V)^{-1}$ with elements ω_{ij}, the Leontief inverse of the frictionless economy (with normalized productivity). The element ω_{ki} is equal to the percentage reduction in the total labor input required to produce good k following a percentage increase in productivity A_i, including the effect of sector i on the suppliers of sector k, and the suppliers of its suppliers, and so on.

2. *Connections between sectors and intermediate-input intensity.* We define the connection of sector j to sector i as x_{ij}, with $x_{ij} \equiv \omega_{ij}$ if $j \neq i$, and $x_{ii} \equiv \omega_{ii} - 1$. The sectoral connections satisfy the following recursive relationship: $x_{ij} = \alpha_i v_{ij} + \sum_r \alpha_i v_{ir} x_{rj}$. The variable x_{ij} governs the direct and indirect use of input j by i. We define the intermediate-input intensity of sector i as $x_i \equiv \sum_j x_{ij}$.

3. *Importance of a sector.* We define the importance of a sector as $s_i \equiv \sum_k \beta_k \omega_{ki}$.[14] This variable corresponds to the sales-to-GDP ratio of the undistorted economy (and hence sums to more than one).[15] The sector's importance can be decomposed in two terms as follows: $s_i = \beta_i + \gamma_i$. The first term, β_i, measures the *importance of the sector in final output*, and the second term, $\gamma_i \equiv \sum_k \beta_k x_{ki}$, captures the *importance of the sector as an input supplier.* The second component is related but not equivalent to upstreamness. The importance of the sector as an input supplier is high if the sector is more upstream *and* if the sector is supplying inputs to sectors important for final output.

4. *Importance of the connection between sectors.* We define the importance of the connection between sector i and j as $s_{ij} \equiv \sum_k \beta_k \omega_{ki} \omega_{kj}$, which can be rewritten as

[14] For some parameter combinations, there could be sectors with $s_i = 0$—that is, they are neither final-goods producers nor intermediate-goods suppliers (direct or indirect) to final-goods producers. We assume away such sectors (which do not affect the results) and hence assume $s_i > 0$ for all i.

[15] We show the connection with the sales-to-GDP ratio in Appendix A. The sales-to-GDP ratio is sometimes referred to as the *Domar weight* (Hulten, 1978).

$s_{ii} = \beta_i(1+2x_{ii}) + \sum_k \beta_k x_{ki}^2$ and $s_{ij} = \beta_i x_{ij} + \beta_j x_{ji} + \sum_k \beta_k x_{ki} x_{kj}$ for $i \neq j$. This indicator measures the connection of both i to j and j to i, weighted by their final-output importance as well as how similarly concentrated the two sectors' suppliers are.

5. *β-weighted covariance and variance.* For arbitrary vectors p and q, we define $\text{cov}_\beta(p,q) \equiv \sum_k \beta_k p_k q_k - \left(\sum_k \beta_k p_k\right)\left(\sum_k \beta_k q_k\right)$. The β-weighted variance is then $\text{var}_\beta(p) = \text{cov}_\beta(p,p)$. For arbitrary matrices P and Q, the β-weighted covariance of two column vectors is written $\text{cov}_\beta(P_{.i}, Q_{.j}) \equiv \sum_k \beta_k P_{ki} Q_{kj} - \left(\sum_k \beta_k P_{ki}\right)\left(\sum_k \beta_k Q_{kj}\right)$.

3.2 Analytical characterization

We provide a characterization of aggregate TFP for two types of distortions: distortions that affect only labor (or more generally, primary inputs) and distortions that symmetrically affect labor and all intermediate inputs.[16] We highlight the fundamental differences between these two types of distortions and their consequences for aggregate TFP. The derivations are obtained by approximating aggregate TFP to the second-order around its non-distorted value, under the assumption that the two elasticities of substitution are equal (that is, $\sigma = \rho$). We use the expressions of Proposition 4, and we normalize productivities to one, following Proposition 2. As explained in section 2.2.1, aggregate TFP is equal to one in the frictionless economy when the sectoral productivities are normalized to one. The expressions of aggregate TFP given in this section can therefore be interpreted as the gap between the TFP and its frictionless level.

3.2.1 Distortions on labor

When distortions affect only labor, the source of the misallocation is the dispersion in the distortions. How the dispersion in distortions reduces aggregate productivity varies with the sectoral linkages as well as with the degree of input substitutability.

The following proposition gives the approximation of TFP when only labor is distorted.

Proposition 5 *Suppose that $\sigma = \rho$, $\alpha_i \in (0,1)$ for all $i = 1,...,n$, $A_i = B_i = 1$ and $\tau_{Xij} = 0, \forall i,j = 1,...,n$.*

[16] We have derived a similar approximation for distortions on only intermediate goods. We do not include this case in the paper because it does not yield additional insights.

For small distortions, aggregate TFP is approximatively equal to

$$\log TFP \approx -\frac{1}{2}\left[\sum_i \sum_j s_{ij}(1-\alpha_i)(1-\alpha_j)\tau_{Li}\tau_{Lj} - \left(\sum_i s_i(1-\alpha_i)\tau_{Li}\right)^2\right]$$
$$-\frac{1}{2}\frac{1}{1-\sigma}\left[\sum_i s_i(1-\alpha_i)\tau_{Li}^2 - \sum_i \sum_j s_{ij}(1-\alpha_i)(1-\alpha_j)\tau_{Li}\tau_{Lj}\right], \quad (12)$$

where $\sum_i s_i(1-\alpha_i) = 1$ *and* $\sum_i \sum_j s_{ij}(1-\alpha_i)(1-\alpha_j) = 1$.

To see the fundamental role of the dispersion, consider the case where distortions are identical across firms, $\tau_{Li} = \bar{\tau}_L$. In that case, Proposition 1 implies that aggregate productivity would be equal to its frictionless level—that is, $\log TFP = 0$ (recall that the sectoral productivities are normalized to one). Note that this result holds in general and does not hinge on the approximation used in Proposition 5.[17] The dispersion in distortions is therefore the sole source of misallocation when only primary inputs are distorted. Cross-sectional dispersion, which is the main focus of the misallocation literature, remains the relevant statistics for studying primary-input distortions even when production linkages are accounted for.

However, the channels through which a given dispersion of distortions translates into a TFP loss depends on both the sectoral linkages and the value of the elasticity of substitution. The first line of equation (12) gives the TFP loss when inputs are perfect complements, $\varepsilon_\sigma = 1/(1-\sigma) = 0$, and the second line gives the additional effect that results when $\varepsilon_\sigma > 0$. Both components depend on the share of labor in the production process, $(1-\alpha_i)$, but also on the interaction between the distortions and the sectoral linkages, since linkages determine the importance of each sector s_i and that of the connection between sectors s_{ij}. When inputs are perfect complements, the distortions reduce aggregate productivity only through their effect on final consumption. In that case, both labor and intermediate inputs are chosen in fixed proportion, and the production efficiency of each sector is therefore unaffected by the distortions and their consequences on relative prices. When inputs are not perfect complements, intermediate inputs choices are affected by both the distortions and their consequence on relative prices. In addition to distorting final consumption, labor distortions also reduce the production efficiency of each sector.

[17] See Appendix A for the derivation of the result on the irrelevance of uniform labor distortions. More precisely, we show that scaling up the labor distortions has no effect on aggregate TFP.

3.2.2 Distortions on all inputs

When distortions affect all inputs, additional sources of misallocation are present, and the TFP loss does not depend only on the dispersion of distortions.

The following proposition gives the approximation of TFP when all inputs are distorted.

Proposition 6 *Suppose that $\sigma = \rho$, $\alpha_i \in (0,1)$ for all i, $A_i = B_i = 1$ and $\tau_{Xij} = \tau_{Li} = \tau_i, \forall i, j = 1, ..., n$.*

For small distortions, aggregate TFP is approximatively equal to

$$\log TFP \approx -\frac{1}{2}\left[\sum_i \sum_j s_{ij}\tau_i\tau_j - \left(\sum_i s_i\tau_i\right)^2\right] - \frac{1}{2}\frac{1}{1-\sigma}\left[\sum_i s_i\tau_i^2 - \sum_i\sum_j s_{ij}\tau_i\tau_j + 2\sum_i s_i x_i \tau_i \bar{\tau}^i_{suppliers}\right], \quad (13)$$

where $\bar{\tau}^i_{suppliers} = \sum_j (x_{ij}/x_i)\tau_j$.

Here as well, the TFP loss is shaped by the sectoral linkages. The linkages come into play through the importance of the sectors, s_i, and that of the connection between sectors, s_{ij}. The crucial difference is that now the TFP loss is affected also by the interaction between the sectors' distortions and the average distortion of their suppliers, $\bar{\tau}^i_{suppliers}$, and by the interaction between the sectors' importance s_i and their intermediate-input intensity x_i (which includes direct and indirect linkages). The TFP loss will be larger if highly distorted sectors tend to purchase intermediate inputs from highly distorted suppliers, and all the more so if the distorted sectors are important sectors.

Moreover, the dispersion in distortions is no longer the only source of misallocation. Let us consider again the case where distortions are identical across sectors—that is, $\tau_i = \bar{\tau}$ for all sectors. In that case,

$$\log TFP \approx -\frac{1}{2}\bar{\tau}^2\left[\sum_i\sum_j s_{ij} - \left(\sum_i s_i\right)^2 + \frac{1}{1-\sigma}\left(\sum_i s_i - \sum_i\sum_j s_{ij} + 2\sum_i s_i x_i\right)\right].$$

Distortions that are identical across firms also reduce aggregate TFP. First, since sectors differ in their intermediate-good intensity x_i, prices can be dispersed even when distortions are symmetric, which leads to inefficiencies in the production of the final good and of the different sectoral goods. Second, the total intermediate-goods usage is reduced because the user cost of intermediate goods relative to labor is higher, which leads to further inefficiencies in the production of the final good. This result, also highlighted in Jones (2011), contrasts with the labor-distortions case for which only the dispersion of distortions matters.[18] The key difference between the labor and all-input distortions cases is that all-input distortions affect the total quantity of intermediate inputs used, whereas labor distortions do not affect the total quantity of labor used (since aggregate TFP is, by definition, computed for a given quantity of labor). Hence, symmetric labor distortions have no impact on aggregate productivity because their only potential effect is on the total quantity of labor used.

3.3 How does input substitutability shape the effects of distortions?

As shown in the previous section, the degree of input substitutability is a central determinant of aggregate productivity. In this section, we use the analytical characterizations of Propositions 5 and 6 to explore the role of input substitutability in more detail. In our three main results, we show how the degree of input substitutability modifies the aggregate TFP loss, the amplification from sectoral linkages, and the impact of each sector.

3.3.1 Aggregate TFP loss

Our first result is that the TFP loss is larger when inputs are more substitutable (and smaller when inputs are more complementary). This result, which we state more formally below, holds for both labor and all-input distortions.

Result 1 *Suppose that $\sigma = \rho$, $A_i = B_i = 1$ $\forall i = 1, ..., n$. For small distortions (on either primary or all inputs),*

$$\frac{d \log TFP}{d\sigma} \leq 0.$$

The inequality is strict in the case of random or symmetric all-input distortions and in the presence of sectoral linkages ($\alpha_i > 0$ for some i).

At first, the result that distortions have less impact when firms cannot easily substitute between inputs could seem counterintuitive. Why isn't the outcome worse when firms are stuck with distorted inputs? The intuition behind the result is that the firms' input decision is less distorted when inputs are less substitutable, precisely because it is more difficult for firms to substitute away from the distorted input. When the elasticity of substitution is low, firms respond less to the distorted prices, hence the firms' input decision does not deviate

[18] This result is related to Diamond and Mirrlees's (1971) no-taxation-on-intermediate-inputs result.

much from the first-best allocation. At the limit, with perfectly complementary inputs, the input decision is unaffected by relative price distortions.[19] Smaller changes in the allocation lead to smaller changes in the quantity of labor directly and indirectly used for the production of each sectoral good and hence to a smaller decline in aggregate productivity. This is not, however, the only mechanism at play. A lower elasticity of substitution tends to amplify the consequences that an inefficient mix of inputs can bring about on production, thus amplifying the TFP loss. We show (for small distortions) that the first effect dominates, and a higher degree of complementary leads to a smaller TFP loss.

The way input complementarity modifies the consequences of distortions contrasts sharply with how it modifies the consequences of sectoral productivity shocks. Whereas input complementarity dampens the consequences of distortions, it amplifies the consequences of sectoral productivity shocks. We now state this result more formally.

Result 2 *Suppose that $\sigma = \rho$, $\alpha_i \in (0, 1)$ for all i, and set $A = \mathbf{1}$. Let $Y(B, \sigma)$ be the output of an economy without distortions with productivity vector B and parameter σ. Then, $Y(B, \sigma)$ is increasing in σ. Suppose there exist some k, j_1, j_2 such that $B_{j_1} \neq B_{j_2}$ and $\omega_{kj_1} > 0, \omega_{k,j_2} > 0$. Then, $Y(B, \sigma)$ is strictly increasing in σ.*

The intuition is most apparent when we consider the effect of a negative productivity shock in some sector j. When the elasticity of substitution is low, switching to other inputs would disrupt production; therefore, firms do not change much their use of sector j's products. This limited reallocation reinforces the TFP decline, since the economy must devote additional resources to a low-productivity sector. On the other hand, when the elasticity of substitution is high, switching to other inputs is not as disruptive. As a result, the economy does not need to devote as many resources to the low-productivity sector, which attenuates the TFP decline.

Results 1 and 2 highlight an interesting contrast between the effects of different inefficiencies. Frictions that distort the relative prices of goods will be less harmful in economies with stronger input complementarities, whereas frictions that reduce firm-level productivity will be less harmful in economies with stronger input substitutability.

[19]The absence of distortionary effects under perfect complementarity case has been exploited by Rotemberg and Woodford (1993) in their study of business cycles with imperfect competition. They show that there exists a value-added production function that is independent of the value of the markups only if primary inputs and intermediates are perfect complements, since in that case the firms' intermediate-inputs choice does not depend on the value of the markups.

3.3.2 Amplification from sectoral linkages

We now turn to the role of sectoral linkages and focus here on their potential amplification effect. Contrary to intuition, we find that sectoral linkages do not systematically amplify the effects of distortions. We present the conditions under which sectoral linkages dampen the effects of the two types of distortions (labor and all-input distortions). We find the dampening conditions are more likely to hold if the distortions affect only labor.

3.3.2.1 TFP in the absence of sectoral linkages

We study the possibility of a dampening effect by comparing TFP to its level in the absence of sectoral linkages (that is, $\alpha_i = 0$ for all sectors), which is given by

$$\log TFP|_{\alpha=0} \approx -\frac{1}{2} \operatorname{var}_\beta(\tau). \tag{14}$$

In the absence of sectoral linkages, TFP is simply the β-weighted variance of distortions. Sectoral linkages amplify the effects of distortions if $\log TFP < \log TFP|_{\alpha=0}$ and dampen them if $\log TFP > \log TFP|_{\alpha=0}$.

3.3.2.2 Distortions on labor

Sectoral linkages can dampen the effects of labor distortions when inputs are complements. The main condition under which the dampening effect occurs is when each sector's weight is the same in the economy with and without sectoral linkages.

Result 3 *Consider an economy with only labor distortions. The TFP in an otherwise identical economy without sectoral linkages,* $\log TFP|_{\alpha=0}$*, is given by equation* (14)*.*

1. *If* $v_{ij} = 0$ *for* $i \neq j$*, then* $\log TFP = \log TFP|_{\alpha=0}$*.*
2. *If inputs are complements (*$\sigma < 0$*) and the distortions are uncorrelated with sectoral characteristics, then* $\log TFP \geq \log TFP|_{\alpha=0}$*. The inequality is strict if* $\alpha_i v_{ij} > 0$ *for some* $i \neq j$*.*
3. *If inputs are complements (*$\sigma < 0$*) and* $(1-\alpha_i)s_i = \beta_i$*, then* $\log TFP \geq \log TFP|_{\alpha=0}$*.*

In this result, we establish three separate conditions under which sectoral linkages do not amplify the effects of distortions. First, we consider a special but instructive case in which the economy consists of "island" sectors: firms purchase only intermediate goods from their own sector. In that economy, where all linkages are *intra*-sectoral, the efficiency of sectoral production is not affected because the distortions equally affect all labor used (directly or indirectly) in producing each product. Therefore, the TFP loss of that economy is identical to that of the no-sectoral-linkages economy. This special case highlights the crucial role that *inter*-sectoral linkages play in the amplification of labor distortions.

Next, we consider the case where distortions are random, in the sense that they are distributed independently from any other sectoral parameter.[20] In that case, we can further approximate TFP by assuming a large number of sectors, and we obtain

$$\log TFP \approx -\frac{1}{2}\operatorname{var}(\tau_L) - \frac{1}{2}\frac{\sigma}{1-\sigma}\left[1 - \sum_i s_{ii}(1-\alpha_i)^2\right]\operatorname{var}(\tau_L), \tag{15}$$

where $\sum_i s_{ii}(1-\alpha_i)^2 = 1$ if $\alpha_i = 0, \forall i$ or if $v_{ij} = 0\ \forall i \neq j$, and $\sum_i s_{ii}(1-\alpha_i)^2 < 1$ otherwise. The connections between sectors do not matter for the effect of random labor distortions when the production function is Cobb-Douglas. In that case, $\sigma = 0$, equation (15) simplifies to $\log TFP \approx -(1/2)\operatorname{var}(\tau_L)$, and aggregate TFP is then identical to that of the economy without sectoral linkages. The intermediate-input linkages generate two counteracting effects. On the one hand, intermediate inputs constitute an additional channel through which distortions reduce aggregate productivity. The labor distortions modify the production of each sector by distorting (indirectly) the intermediate-input decisions. On the other hand, the use of intermediate inputs reduces the effect of labor distortions, as labor then represents a smaller share of the firms' inputs. When the production function is Cobb-Douglas, the two offsetting effects cancel out exactly. When inputs are complements ($\sigma < 0$), the intermediate-input channel is weaker; the second effect dominates, and sectoral linkages dampen the effects of labor distortions.

Finally, we consider the general case in which distortions can be correlated with the sectoral characteristics. Here as well, the Cobb-Douglas production function is a useful starting point. With Cobb-Douglas production functions, aggregate productivity is

$$\log TFP \approx -\frac{1}{2}\left[\sum_i s_i(1-\alpha_i)\tau_{Li}^2 - \left(\sum_i s_i(1-\alpha_i)\tau_{Li}\right)^2\right] \tag{16}$$

and can be easily compared to that of the no-sectoral-linkages economy, since in both equations (14 and 16) TFP is equal to a weighted variance of distortions, with weights β_i in the economy without sectoral linkages and with weights $s_i(1-\alpha_i)$ in the economy with sectoral

[20]See Appendix A.5 for details of the construction.

linkages.[21] In both economies, the weights are equal to the undistorted value-added share of each sector. The TFP loss can then be smaller or larger than in equation (14), depending on how sectoral linkages modify the value added of each sector. The TFP loss would be smaller if highly distorted sectors account for a lower value-added share in the presence of sectoral linkages. However, if the undistorted value added of each sector is the same in the two economies ($s_i(1-\alpha_i) = \beta_i$), then the TFP loss is unaffected by sectoral linkages. Assuming $s_i(1-\alpha_i) = \beta_i$ with the general CES production function, we get

$$\log TFP \approx \log TFP|_{\alpha=0} - \frac{1}{2}\frac{\sigma}{1-\sigma}\left[\sum_i s_i(1-\alpha_i)\tau_{Li}^2 - \sum_i\sum_j s_{ij}(1-\alpha_i)(1-\alpha_j)\tau_{Li}\tau_{Lj}\right],$$

where the term between brackets is positive. Hence, sectoral linkages dampen the effects of distortions when $\sigma < 0$—that is, when inputs are less substitutable than Cobb-Douglas (and provided the value-added share of each sector remains the same). The dampening effect stems from the distortions' weaker impact on the efficiency of sectoral production.

3.3.2.3 Distortions on all inputs

When intermediate inputs also are distorted, the presence of sectoral linkages does not reduce the weight of the distorted inputs in production (as they did in the case of labor distortions), and sectoral linkages are therefore less likely to dampen the effects of distortions. We find that sectoral linkages can still dampen the effect of all-input distortions, but the dampening effect occurs under stronger conditions than when only labor is distorted.

Result 4 *Consider an economy with sectoral linkages ($\alpha_i > 0$ for some i) and with all-input distortions. The TFP in an otherwise identical economy but without sectoral linkages, $\log TFP|_{\alpha=0}$, is given by equation (14).*

For a vector of distortions that is not too large, if $Cov_\beta(\tau_i, x_i\bar{\tau}^i_{suppliers}) < -(1/2)\,\mathrm{var}_\beta(x_i\bar{\tau}^i_{suppliers})$; then, for all σ sufficiently small, $\log TFP > \log TFP|_{\alpha=0}$.[22]

The result states that sectoral linkages dampen the effects of distortions if $x_i\bar{\tau}^i_{suppliers}$ is sufficiently negatively correlated to τ_i—that is, if distortions are high in sectors with a

[21] Note that the weights in the sectoral-linkages economy do sum to one: $\sum s_i(1-\alpha_i) = 1$.

[22] Note that for simplicity, we slightly changed the notation relative to the definition of Section 3.1. Here $\mathrm{var}_\beta(\tau_i + x_i\bar{\tau}^i_{suppliers})$ refers to the variance of the vector whose element is $\tau_i + x_i\bar{\tau}^i_{suppliers}$. We use a similar notation for the covariance.

low intermediate-input intensity or in sectors purchasing intermediates from low-distortion sectors. To see the intuition behind the result, let us consider the case of perfect complementarity. In that case, aggregate productivity is given by

$$\log TFP \quad \approx \quad -\frac{1}{2}\,\mathrm{var}_{\beta}\left(\tau_i + x_i \bar{\tau}^{i}_{suppliers}\right). \tag{17}$$

With perfect complementarity, sectoral production is efficient, and the TFP loss depends only on the consequences of the price dispersion, caused by distortions, for final consumption. Sectoral linkages can therefore amplify or dampen the TFP loss, depending on how they affect the price dispersion, which depends in turn on the interaction between the distortions and the sectoral linkages. If all the firms purchase inputs from the same sectors and in the same proportion, then the TFP loss would be identical in the economies with and without sectoral linkages.[23] If firms in high-distortion sectors purchase inputs from low-distortion sectors, sectoral linkages will tend to reduce the price dispersion.

3.3.3 Impact of each sector

As shown in the previous section, whether sectoral linkages amplify or dampen the effects of distortions depends crucially on the value of the elasticity of substitution. In this section, we further study the role of sectoral linkages by focusing on the role of each sector. Which sectors have a large aggregate impact when distorted? What type of linkages lead to a larger impact? Again, the answers to these questions depend on the value of the elasticity of substitution. In particular, intermediate-input suppliers have a smaller impact if inputs are less substitutable. To study the determinants of the impact of each sector, we first consider the case in which only one sector is distorted and then the case in which two sectors are distorted.

3.3.3.1 When only one sector is distorted

We find that the aggregate impact of a distortion in only one sector depends primarily on the importance of the sector for final output and on its importance as an intermediate-input supplier.

Result 5 *The effect of a distortion in only one sector is given by*

$$\frac{d\log TFP}{d\tau_{Li}} \approx -\left[(1-\alpha_i)^2(s_{ii}-s_i^2) + \frac{1}{1-\sigma}[(1-\alpha_i)s_i - (1-\alpha_i)^2 s_{ii}]\right]\tau_{Li},$$

[23] In that case, $x_i = x$ and $\bar{\tau}^{i}_{suppliers} = \bar{\tau}_{suppliers}$ $\forall i = 1, ..., n$; therefore, $\mathrm{var}_{\beta}(\tau_i + x_i\bar{\tau}^{i}_{suppliers}) = \mathrm{var}_{\beta}(\tau_i)$.

when only labor is distorted; and by

$$\frac{d\log TFP}{d\tau_i} \approx -\left[s_{ii} - s_i^2 + \frac{1}{1-\sigma}(s_i - s_{ii} + 2s_i x_{ii})\right]\tau_i,$$

when all inputs are distorted.

We derive the results for both types of distortions but focus our discussion on the case where all inputs are distorted.[24] Note that even in this simple case, where only one sector is distorted, the aggregate impact of the distortion cannot be summarized by a single statistics. Furthermore, Result 5 shows that the relevant sectoral linkages depend on the elasticity of substitution. With perfect input complementarity ($1/(1-\sigma) = 0$), the relevant sectoral linkages are given by the first term of the expression, $s_{ii} - s_i^2$. The additional effect—given by the second term, $s_i - s_{ii} + 2s_i x_{ii}$—matters only when inputs can be substituted. Both components depend on s_i, the importance of the sector, and on s_{ii}, which is not as easily interpretable as s_{ij} (which is the importance of the connection between i and j). To give more intuition, we approximate the two components and obtain

$$\frac{d\log TFP}{d\tau_i} \approx -\left(\beta_i + \frac{1}{1-\sigma}\gamma_i\right)\tau_i.$$

The key characteristics that determine the sector's impact are therefore β_i, the importance of the sector for final output, and γ_i, the importance of the sector as an intermediate-input supplier. The distortion has a more detrimental impact if it affects a sector that is important for final output, whatever the value of the elasticity of substitution. By contrast, a distortion on an important intermediate-input supplier leads to a larger TFP loss only if inputs can be substituted; the impact of the intermediate-input suppliers increases with the elasticity of substitution.

To develop intuition further and better understand which sectoral linkages are behind γ_i, let us consider the case of a simple input-output economy in which all the firms use the same intermediate-input bundle (that is, $v_{ij} = v_j \ \forall i = 1, ..., n$). In this simple economy, $\gamma_i = (\bar{\alpha}_\beta v_i)/(1-\bar{\alpha}_v)$, where $\bar{\alpha}_\beta = \sum_i \beta_i \alpha_i$, and $\bar{\alpha}_v$ is similarly defined. Thus, the importance of a sector as an intermediate-input supplier increases with the direct intensity with which the sector's product is used in the intermediate-input bundle's production, v_i; and with the direct intensity with which the intermediate-input bundle is used, $(\bar{\alpha}_\beta, \bar{\alpha}_v)$.

[24] The labor-distortions case includes similar terms, with two essential differences: the effect of the distortion is scaled down by $1-\alpha_i$ (the weight of labor in the production function); and the double-marginalization term $2s_i x_{ii}$ is absent.

3.3.3.2 Distortions on multiple sectors

How is the impact of a sectoral distortion modified when the distortion in another sector increases? To study this interaction effect and hence better explain how linkages between distorted sectors shape the effects of distortions, we consider the case in which only two sectors are distorted. The two sectors' distortions interact via two channels. First, when the two sectors are in each other's production chains, the distortions are on top of each other, leading to a cumulative impact. Second, the two distortions alter the variance of consumer and input prices, which brings about additional effects that may attenuate or reinforce the impact of the two distortions. The strength of these two channels depends on the value of the elasticity of substitution.

Result 6 *The effect of distortions in two sectors i and j, with $i \neq j$, is given by*

$$\frac{d^2 \log TFP}{d\tau_{Li} d\tau_{Lj}} \approx - \left[(s_{ij} - s_i s_j)(1-\alpha_i)(1-\alpha_j) - \frac{1}{1-\sigma} s_{ij} (1-\alpha_i)(1-\alpha_j) \right],$$

when only labor is distorted; and by

$$\frac{d^2 \log TFP}{d\tau_i d\tau_j} \approx - \left[s_{ij} - s_i s_j + \frac{1}{1-\sigma} (s_i x_{ij} + s_j x_{ji} - s_{ij}) \right],$$

when all inputs are distorted.

As for Result 5, we discuss here the case in which all inputs are distorted. The interaction effect depends not only on the importance of the two sectors, s_i and s_j, but also on the strength of the interconnection between the two distorted sectors, through s_{ij}, x_{ij}, and x_{ji}. Here as well, the relevant sectoral linkages depend on the value of the elasticity of substitution. With perfect input complementarity ($1/(1-\sigma) = 0$), the relevant sectoral linkages are captured by $s_{ij} - s_i s_j$, which corresponds to the distortion's effect on the final consumption allocation, holding the efficiency of sectoral production fixed. We can rewrite this component as follows:

$$s_{ij} - s_i s_j = -\beta_i \beta_j + \beta_i (x_{ij} - \gamma_j) + \beta_j (x_{ji} - \gamma_i) + \text{cov}_\beta(x_{.i}, x_{.j}).$$

The first term, $-\beta_i \beta_j$, is the direct effect of the interaction on final consumption in the absence of sectoral linkages. In that case, the interaction of the two distortions tends to attenuate the distortions' effect on aggregate productivity: a higher distortion on sector j reduces the impact of sector i's distortion because it makes more similar the prices of sectors

i and j, which reduces the final-consumption misallocation. The second and third terms, $\beta_i (x_{ij} - \gamma_j) + \beta_j (x_{ji} - \gamma_i)$, capture the role of sectoral linkages. These terms indicate that the connection between the two sectors reinforces the impact of the distortions only if the two sectors use each other's products more than the average sector does. Sectoral linkages also affect the final consumption allocation through the term $\text{cov}_\beta(x_{.i}, x_{.j})$, which indicates that the inefficiencies are reinforced if the use of the products from the two distorted sectors is concentrated in the same subset of the economy.

When inputs can be substituted, an additional effect, captured by $(s_i x_{ij} + s_j x_{ji} - s_{ij})$, arises. This expression, which gives the TFP loss caused by the inefficient mix of inputs in each sector, can be rewritten as follows:

$$s_i x_{ij} + s_j x_{ji} - s_{ij} = [-\gamma_i \gamma_j + \gamma_i x_{ij} + \gamma_j x_{ji} - \text{cov}_\beta(x_{.i}, x_{.j})].$$

Similar to the direct effect on consumption discussed above, the first term, $-\gamma_i \gamma_j$, measures the interaction's direct effect on the intermediate-input prices. Multiple distortions tend to reduce the dispersion of intermediate-input prices, which limits the productivity loss. The second and third terms, $\gamma_i x_{ij} + \gamma_j x_{ji}$, capture the role of the interconnection between the two sectors. A stronger connection between the two sectors heightens the effect of the distortions because the distortions are imposed on top of each other (double marginalization effect). Combining the first three terms, we get $\gamma_i \gamma_j (x_{ij}/\gamma_j + x_{ji}/\gamma_i - 1)$. Thus, the double marginalization effect dominates the direct effect if one of the two sectors is a more important supplier to the other sector than to the average sector. The last term, $\text{cov}_\beta(x_{.i}, x_{.j})$, captures another consequence of sectoral linkages. When sector i and j tend to supply inputs to the same sectors (high covariance in the use of the two sectors' products), the effect of the distortions is reduced. In that case, a higher distortion in sector j falls mainly on purchasing sectors with already-high input prices, so the additional increase in the variance of input prices will be smaller (and the variance could even fall), which attenuates the impact of the distortions. The covariance has therefore opposite effects for the production allocation and for the consumption allocation described above. When two sectors tend to supply inputs to the same customers, the distortions on these two sectors tend to increase the final price of their common customers and hence the variance of final prices, which reinforces the impact of the distortions. On the other hand, when inputs can be substituted, the similarity in the use of the two sectors' products tends to improve the allocation. Products that are important for the same industries have similar prices, so the weighted variance of input prices increases by less. Which effect is stronger depends crucially on the elasticity of substitution.

In the end, the result shows that the effects of the two distortions reinforce each other if the two distorted sectors are highly interconnected, whatever the value of the elasticity of substitution. The interaction effect increases with the importance of the two sectors as input suppliers but only if the interconnection's strength and the degree of input substitutability are sufficiently high. Finally, the covariance between the use of the two sectors' product (by other sectors) tends to reinforce the interaction effect, but to a lesser degree if inputs are more substitutable.

4 Quantitative analysis: the TFP loss from market power

In this section, we quantify the role of input substitutability in the context of the distortions caused by market power. We compute the TFP gains from eliminating industry-level markups in 35 countries and show how the gains vary with the degree of input substitutability. In addition to providing estimates of the cost of industry-level markups for a large number of countries, this section complements our theoretical results by showing that they are empirically relevant and by presenting their quantitative implications.

4.1 Markups across industries and countries

We measure the degree of market power in each industry and each country by computing price-cost margins, which, as we will see in Section 4.2.2, map one-to-one with the firms' markups. Given our focus on input-output linkages, our analysis requires data covering all sectors of the economy. We use industry-level data from the World Input-Output Database's (WIOD) 2013 Socio-Economic Accounts.[25] The data set gives the sales, labor compensation, intermediate input costs, and real capital stock by industry (at roughly two-digit ISIC level) for 40 countries over the period 1995–2011.[26] The comprehensive sectoral coverage, the availability of the capital-stock measure at the industry level, the large selection of countries, and the inclusion of several middle-income countries are key advantages of this dataset. We restrict our attention to market activities, and we therefore drop sectors L to P ("Public Admin and Defence," "Compulsory Social Security," "Education Health and Social Work," "Other Community," "Social and Personal Services," "Private Households with Employed Persons"). In Appendix C.2, we provide more details on the variables as well as the list of

[25]The data are publicly available at: http://www.wiod.org/home. We use the 2013 release instead of the more recent release of the WIOD (2016) because the latter does not contain capital-stock data.

[26]We drop Italy from the sample because the data suggest there could be some classification issues: in Italy, "real estate" has an unusually low capital–output ratio and "renting of material and equipments" has unusually high capital–output ratio (factor of 0.09 and 21 relative to the industry median). In the next section, we drop four other countries for technical reasons. The TFP gains are therefore computed for 35 countries.

countries and sectors.

For each country-industry pair, we measure the price-cost margin, pcm, as sales minus total costs over sales:

$$\mathsf{pcm} = (\mathsf{sales} - \mathsf{labor\ cost} - \mathsf{intermediate\ input\ cost} - \mathsf{capital\ cost})/\mathsf{sales}.$$

To reduce the influence of outliers, for each country-industry pair, we take the median price-cost margin over the period 1995–2007; pcm hence refers to the median.[27] We use the median price-cost margin in the quantitative exercise of the next section as well.

In the WIOD dataset, the variables are constructed to reflect the sectors' income and costs: the sales are measured at basic prices, the labor cost includes social contribution as well as the compensation of self-employed labor, and the intermediate-input cost includes trade and transportation margins. All these variables (sales, labor cost, intermediate-input cost) can therefore be used directly, without any adjustments, in the computation of the pcm. The only variable not directly available is the capital cost. Following the approach of Jorgenson (1967), we construct the capital cost as follows:[28]

$$\mathsf{capital\ cost} = (i + \delta)p_I K,$$

where i is the nominal interest rate, δ the industry-specific depreciation rate, p_I is the price index for investment goods, and K is the real capital stock. Both p_I and K are obtained from the WIOD. We set $i = 0.04$, and δ is computed for each sector using US data from the BEA Fixed Assets Accounts of 2000. The rate of depreciation varies with the type of assets and ranges from about 2.5% (buildings) to 33% (computers). Accounting for the variation in the depreciation rate across industries is essential when measuring the pcm, since each sector's depreciation rate varies with the composition of its capital stock. Using the same depreciation rate across industries would bias the comparison of pcm across industries. For example, using a depreciation rate of 33% instead of 2.5% would lead to overestimating the capital cost by a factor of five and to substantially underestimating the pcm.

Figure 1 shows the distribution of pcm across sectors and countries. For each sector, we report the 25th, the median and 75th percentile of pcm. The pcm is highly dispersed across

[27] We compute the median over the period 1995–2007 because the real capital stock variable is not at all available in 2010 and 2011 and not available for many countries in 2008 and 2009.

[28] The general expression proposed by Jorgenson also depends on taxes and on the inflation of investment goods, from which we have abstracted.

both countries and sectors, with the lowest 25th percentile at -40% and the highest 75th percentile at 25%. We find that in a few sectors—most notably in real estate, agriculture and transportation—the price-cost margin is negative for a large share of the observations. Although we cannot rule out that these negative pcm reflect an overestimation of the firms' costs, an overestimation that could be caused by our simple capital-cost measure, we believe they may reflect actual economic losses in some sectors. For a high proportion of countries, the firms' revenue in the agricultural and transportation sectors is not sufficient to cover their labor and intermediate-input costs and the depreciation of capital. This finding strongly suggests that the negative pcms reflect economic losses in those sectors. In other words, if the nominal interest rate were equal to zero, many countries would still have a negative pcm in the agriculture and transportation sectors.[29]

The pcm implies markups values on the lower range of the values that have been found using other methods. For example, the recent paper by de Loecker et al. (forthcoming) finds the average markup among publicly listed firms to be 40–50% over 1995-2007, whereas we find an average markup of 16% in the US.[30] The key difference between the two measures is that our measure gives the ratio of price over *average* costs, whereas de Loecker et al. (forthcoming) aim to capture the ratio of the price over *marginal* costs. Although identical in our theoretical framework, the two measures can in reality be quite different. Because our measure underestimates the markup over marginal costs, our analysis will underestimate the productivity cost of markups (since the latter depends on the markup over marginal costs).

We report, in Figure 2, each country's median pcm as a function of the country's income per capita, which we compute from the Penn World Table (Feenstra et al., 2015). As shown in the figure, price-cost margins tend to be higher in lower-income than in high-income countries and higher in North America than in Europe. By construction, the price-cost margin is sensitive to the quality of the measures of sales and costs. A potential issue with the cross-country comparison is that the quality of the measurement of sales and costs may vary across countries. In particular, tax evasion and the size of the informal sector could bias the pcm because they lead to incorrect measures of both sales and costs. Tax evasion and informality constitute an issue inasmuch as the sales and costs are misreported to different extents. If firms underreport their sales but report their full costs to reduce their corporate tax or sales tax, the measured pcm will underestimate the actual price-cost margin. Our measure may therefore underestimate the price-cost margin in lower-income countries,

[29] Mismeasurement of the capital stock is more likely in the real-estate sector. For that sector, the pcms become positive for all countries when the nominal interest rate is set to zero.

[30] We use labor costs as weights and compute our average markup over all the sectors included in our sample. See Section 4.2.2 for more details on the connection between the markup and the pcm.

where tax enforcement is often weaker. A related issue is the measure of the labor income of self-employed workers. In the national accounts, labor costs are computed as employee compensation, and the income of self-employed workers is usually not included. For our purpose, we need to fully deduct all labor costs and therefore need a measure that adjusts for the labor compensation of self-employed workers. The WIOD labor-compensation variable includes such an adjustment.[31] As noted by Gollin (2002), adjusting for the labor-income of self-employed workers is important when measuring the labor share (of GDP) in lower-income countries, since a large fraction of the workforce is self-employed in those countries. The adjustment brings the labor share in developing countries closer to the levels observed for high-income countries. In Appendix C.3, we report each country's aggregate labor share as well as the aggregate capital stock over total value added. Insofar as the adjustment does not fully account for the labor income of self-employed workers, the pcm will overestimate the price-cost margin in lower-income countries.

To check the validity of our measure, we plot each country's median pcm against the country's product-market-regulation indicator. The product-market-regulation indicator, published by the OECD (2019), measures the intensity of barriers to competition such as state intervention and impediments to trade, investment, and firm entry.[32] As shown in Figure 3, and in line with intuition, pcm tends to be higher in countries in which regulations are the least competition friendly (that is, with a high product-market-regulation indicator).

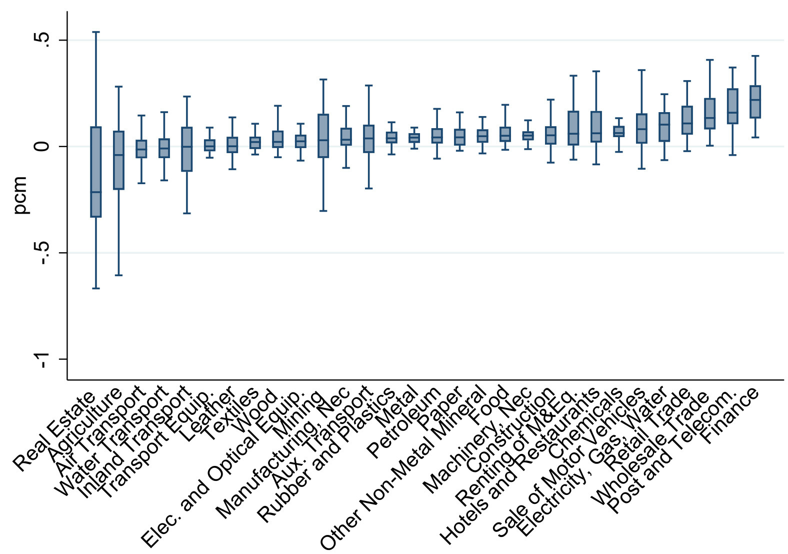
In Figure 4, we report the distribution of pcm in the three countries with the highest median pcm. As shown in the figure, Mexico, Turkey, and Indonesia have not only a higher median pcm but also more dispersion across sectors than countries with lower pcms, such as Belgium.[33] Moreover, Figure 5 shows that the interaction between the sectors' markups and their suppliers' is higher in these three countries. The aggregate-productivity implications of these patterns are investigated in the next section.

[31] The adjustment for self-employed labor income is based on a compensation per hour equal to the compensation per hour of employees in advanced economies, and for emerging economies, the compensation of self-employed workers is imputed using additional information to infer the gap between the earnings of self-employed and employed workers.

[32] The data are publicly available at https://doi.org/10.1787/pmr-data-en. For each country, we use the mean of the overall product market regulation indicator over available years (1998, 2003, 2008 and/or 2013). Data are available for all the countries of the WIOD sample, except Taiwan.

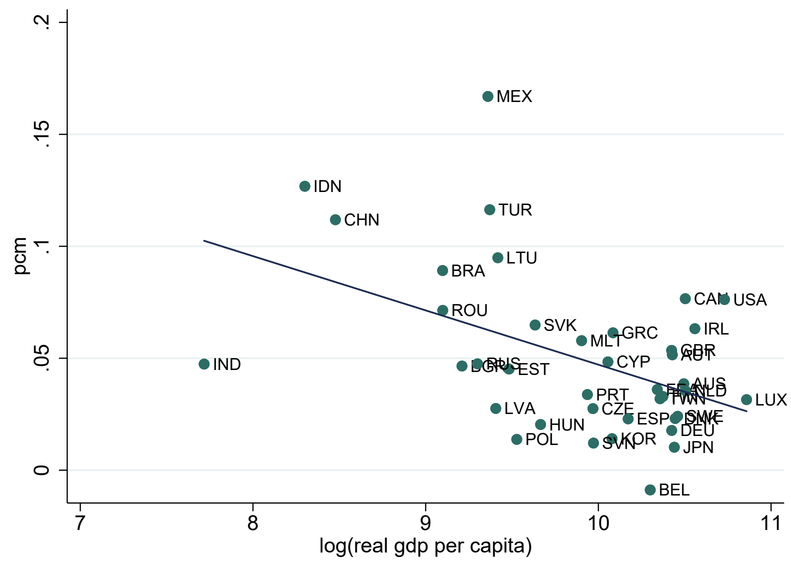
[33] In the full sample, the median and standard deviation of pcm are only weakly positively correlated. See Appendix C.

Figure 1: The price-cost margin across industries



Note: The graph represents the distribution of pcm by industry. The upper and lower sides of the box give the 25th and 75th percentiles, and the bar in the box represents the median. The industries are ranked from lowest to highest median pcm .

Figure 2: The price-cost margin across countries

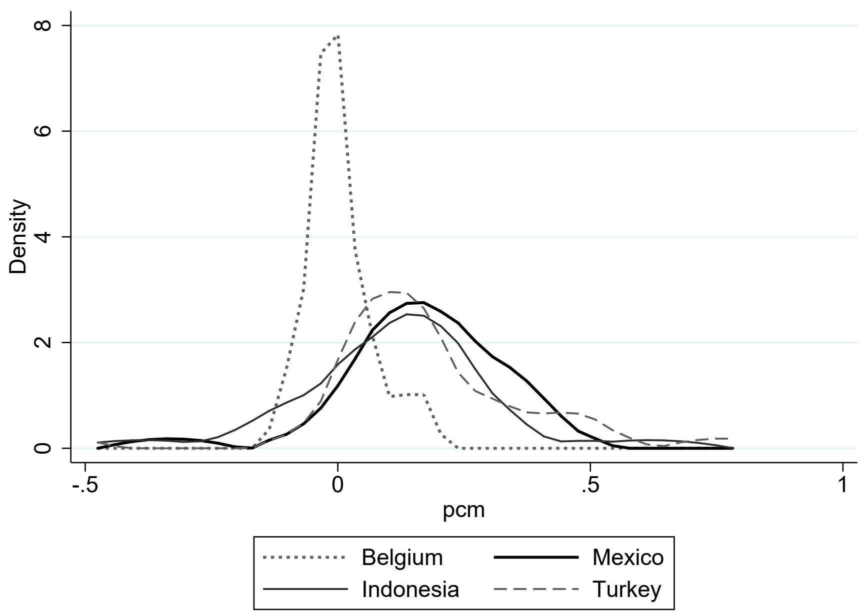


Note: The graph shows the median pcm (across industries) by country as a function of each country's real gdp per capita (1995–2007 mean).

Figure 3: The price-cost margin and product market regulation across countries

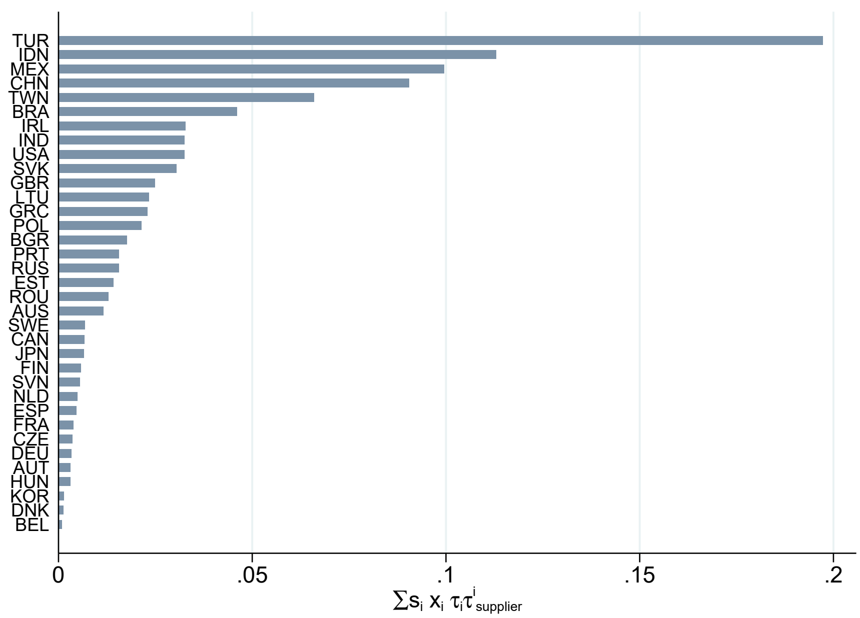
Note: The graph shows the median pcm (across industries) by country as a function of the product market regulation indicator, pmr. A higher pmr captures a less competition-friendly environment.

Figure 4: The distribution of markups across sectors in Belgium and in a selection of high-pcm countries



Note: The graph shows the density of pcm across industries in Belgium, Mexio, Indonesia and Turkey.

Figure 5: Interaction between the sectors' markups and their suppliers'



Note: The graph shows the strength of the interaction between the sectors' markups and their suppliers' for all the countries of the sample.

4.2 Bringing the model to the data

We show how to use the model to quantify the productivity losses caused by market power. First, we present the connection between the model and an environment where firms charge a markup on their marginal costs. Then, we explain how the model is calibrated.

4.2.1 Incorporating markups into the model

We describe an economic environment with markups that is isomorphic to the distorted planner's problem of Section 2 with distortions common to all inputs. We sketch here the economic environment. In Appendix C.1, we give more details on the equivalence between the environment and the model.

In this economy, firms sell their output at an exogenous markup. The firms sell their output to firms in the different sectors of the economy and to the final-good sector, which is assumed to be perfectly competitive.[34] We do not specify the source of the markups, which could be the result of product differentiation, the market structure, or (the lack of) competition regulation.[35] Given the framework considered, the productivity loss would be the same whatever the sources of the markup.

More precisely, we assume that each firm is committed to selling unlimited quantities at a constant markup over marginal cost. Prices are then

$$p_i = \mu_i mc_i,$$

where μ_i is the sector-specific markup, and mc_i is the marginal cost of production. Under this assumption, markups are isomorphic to all-input distortions. Moreover, the firms' cost minimization implies that the allocation satisfies the equations (7) and (8) with $\tau_{Xij} = \tau_{Li} = \mu_i - 1$, with $p_i = \lambda_i = \mu_i mc_i$ given by (9). Similarly, cost minimization in the final-goods market implies (6), and the market-clearing conditions imply the resource constraints. The solution of this environment with markups is therefore given by Proposition 1.

[34] The perfect-competition assumption in the final good sector is without loss of generality.

[35] In the case of product differentiation, the markups could depend on the elasticity of substitution across firms; we assume that the markups are exogenous to the elasticity of substitution *between* sectors (but they could depend on the elasticity of substitution *within* sectors).

4.2.2 Calibration

The model is calibrated on industry-level data from 35 countries, separately for each country.[36] The data used in the calibration also are obtained from the World Input-Output Database (WIOD). For each country-industry pair, we use median values (over 1995–2007) as targets for the calibration. We describe here the calibration strategy and, in Appendix C.2, provide more details on the data sources and the construction of the variables. To simplify the exposition, we use price notations, $p_i = \lambda_i$ and $w = \eta$, instead of the multipliers.

Markups. We derive markups from the price-cost margin measure, pcm, which is described in Section 4.1. In our framework, the markup maps one for one into the price-cost margin. To see the link between the price-cost margin and the markup, let us write the price-cost margin in terms of the model variables (country subscripts are omitted)

$$\mathsf{pcm}_i = \left(p_i Q_i - wL_i - \sum p_j X_{ij} - (i + \delta_i) q_i K_i\right) / (p_i Q_i),$$

where i is the interest rate, and δ_i and q_i are the depreciation rate and the price of the sector-specific capital, respectively. Note that although the model described in Section 2 does not explicitly incorporate capital, we do take into account the capital cost when measuring the price-cost margin. Using the first-order conditions (equations [7] and [8] with $\tau_{Li} = \tau_{Xij} = \mu_i - 1$) and the corresponding equation for capital, Euler's theorem implies that $\eta L_i + \sum_j p_j X_{ij} + (i + \delta_i) q_i K_i = (p_i Q_i)/\mu_i$, and hence

$$\mu_i = \frac{1}{1 - \mathsf{pcm}_i}.$$

Implicit here is the assumption that other distortions average out at the industry level.

Final demand. The final expenditure shares can be used to pin down the parameter β_i. From equation (6), we obtain

$$\beta_i = p_i C_i / (\sum_j p_j C_j),$$

where $p_i C_i$ is the final output of sector i.

Technology. With markups, the parameters of the production function $\{\alpha_i, v_{ij}\}_{i,j=1\ldots n}$ cannot be estimated directly from intermediate-input cost shares because the markups create

[36] Data for 40 countries are available in the WIOD. We excluded four small countries (Latvia, Malta, Cyprus, and Luxembourg) because too many zeros in the input-output table prevent us from solving the model at the same level of disaggregation as for the other countries. We also excluded Italy because the data patterns make us suspect classification discrepancies with respect to other countries.

a wedge between the cost shares and the technology parameters. However, given the values of the industry-level markups and those of the elasticities of substitution, we can derive from the first-order conditions of the firms an estimate for these parameters.[37] From Proposition 1 and $p_i = \mu_i mc_i$, we have

$$n_{ij} \equiv \alpha_i v_{ij} = \frac{p_j X_{ij}}{p_i Q_i/\mu_i} \left(\sum_j v_{ij} [p_i/p_j]^{\frac{\rho}{1-\rho}} \right)^{-\frac{\sigma-\rho}{\rho(1-\sigma)}} \left(\frac{p_i}{p_j} \right)^{-\frac{\rho}{1-\rho}} \mu_i^{\frac{\sigma}{1-\sigma}},$$

where $p_j X_{ij}/(p_i Q_i/\mu_i)$ is the cost share of intermediate good j for sector i.[38] The cost share reflects not only the parameters α_i, v_{ij} but also the entire price vector p, which is itself a function of the parameters v_{ij} and α_i. The calibration of the parameters α_i, v_{ij} hence involves finding a fixed point.[39] A simple iterative procedure quickly finds the fixed point and is robust to different initial guesses. With a solution for $\{n_{ij}\}$, we obtain $\alpha_i = \sum_j n_{ij}$ and then $v_{ij} = n_{ij}/\alpha_i$.

The remaining two parameters—ρ and σ, which govern the elasticity of substitution between intermediate and primary inputs and the one between intermediate inputs—ε_σ and ε_ρ, are not straightforward to calibrate. A few empirical papers have estimated these elasticities of substitution.[40] The elasticity between primary inputs and the intermediate-input bundle has been estimated to be between 0.4 and 0.9. Rotemberg and Woodford (1996) find a value of 0.7, Oberfield and Raval (2014) find a range between 0.6 and 0.9, and Atalay (2017) finds estimates ranging between 0.4 and 0.8. Atalay (2017) estimates also the elasticity of substitution between intermediates and finds an estimate close to zero.[41] We set $(\varepsilon_\sigma, \varepsilon_\rho) = (0.7, 0.01)$ in our benchmark calibration, and we report the results for other combinations of the elasticities' values within the set $(0.01, 0.70, 1.00)$. Note that since the calibration of the technology parameters $\{\alpha_i, v_{ij}\}_{i,j=1...n}$ relies on the value of the elasticities of substitution, these parameters need to be recalibrated whenever the value of the elasticities

[37] With our specification, $\alpha_i v_{ij}$ could be read directly off the cost shares (for any value of the elasticity) if there were no distortions. With distortions, the cost shares can be used directly only in the Cobb-Douglas case ($\sigma = \rho = 0$).

[38] As justified by Proposition 2, we can choose the units in such a way to ensure $A_i = 1$, $B_i = 1$ for all i.

[39] In Appendix C.3, we plot the estimated α against the "naive" estimate $\sum_j p_j X_{ij}/p_i Q_i$. As we might expect, in a majority of observations the estimated parameters are higher than the observed shares because of the presence of markups. However the markups cause the price of intermediates in some sectors to increase so much that the observed intermediate share is higher than the underlying technical parameter.

[40] In a recent paper, Miranda-Pinto and Young (2018) show that the degree of input substitutability varies across sectors, with higher elasticities in services than in manufacturing industries. We leave the implications of this sectoral heterogeneity for further research.

[41] These estimates are short-run elasticities. Most likely, the elasticities of substitution are higher when the horizon considered is longer. We view these short-run elasticities as plausible lower bounds for the long-run elasticities.

of substitution is modified.

4.3 Quantitative results

We compute the aggregate productivity gains from removing industry-level markups in each of the 35 countries. In addition to quantifying the cost of industry-level markups, the quantitative analysis allows us to show that the theoretical results of Section 3 are empirically relevant. Here, we focus on the effects of varying the elasticities of substitution between intermediates and between labor and the intermediate-input bundle, holding fixed the calibration targets. This approach implies that the technology parameters $\{\alpha_i, v_{ij}\}_{i,j=1\ldots n}$ are recalibrated whenever the elasticities are let to vary. The results of the quantitative analysis with fixed technology parameters (which corresponds more closely to the theoretical analysis) are reported in Appendix C.4.

To quantify the TFP gains from removing markups, we use the general solution described in Proposition 1 to compute each country's TFP with and without markups. The numerical solution of the model can easily be computed by iterating on the Lagrange multipliers. As noted in Section 4.1, the markups are negative for some country-industry pairs. In the counterfactual, we set all positive markups to zero and leave the negative markups unchanged. In contrast to the theoretical analysis, the elasticity between intermediates and that between labor and the intermediate-input bundle need not be set to the same value with the numerical solution. We therefore compute the results for different values of the two elasticities of substitution. For ease of exposition, we present here the results for the median country, as well as for the 10 countries with the largest TFP gains. The complete tables of results, for all 35 countries and for additional values of the elasticities, are reported in Appendix C.4.

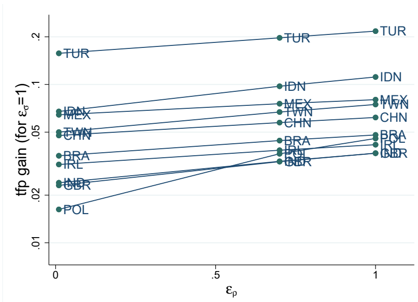
Figure 6 reports the aggregate TFP gain from removing markups in the 10 countries with the largest gains. In these 10 countries, the gain ranges from about 1% to 22%, depending on the country and on the value of the elasticities of substitution. We view this quantification as a lower bound of the productivity losses caused by markups. Our results capture the consequences of the misallocation only *across* sectors because the nature of the data does not permit measuring *within*-sector misallocation. Moreover, our markup measure, which is based on the firms' average costs rather than on their marginal costs, most likely underestimates markups, and our results therefore underestimate the productivity loss.

As shown in Figure 6, the TFP gain typically increases with the two input elasticities (this is also the case for the 25 other countries not shown in the figure). The result that a higher elasticity leads to a larger cost of distortions, derived in Proposition 1, therefore holds empirically as well. We find that the TFP gain also increases separately with each elasticity.[42] Furthermore, with the quantitative approach, we can now compare the role of the two elasticities. The picture that emerges is not clear-cut. For some countries (Turkey, Taiwan, Mexico, Brazil), the TFP gain is more sensitive to the value of the elasticity of substitution between labor and the intermediate-input bundle than to that of the elasticity between intermediates, whereas the opposite holds true in other countries (Poland, Romania); the effects of the two elasticities are quite similar for the rest of the countries. The left panel of Table 1 reports the TFP gains for the benchmark value of the elasticities $(\varepsilon_\sigma, \varepsilon_\rho) = (0.7, 0.01)$—as well as for the two polar cases of (almost) perfect complementarity, $(\varepsilon_\sigma, \varepsilon_\rho) = (0.01, 0.01)$, and Cobb-Douglas, $(\varepsilon_\sigma, \varepsilon_\rho) = (1.00, 1.00)$—for the median country and for the same 10 countries considered in Figure 6. In the end, the quantification of the cost of markups hinges crucially on the value of the elasticities of substitution between inputs. With the benchmark value of the elasticities, the TFP gain is equal to 1.3% in the median country. A unit elasticity of substitution between all inputs multiplies the TFP gain for the median country by 1.8 relative to the benchmark calibration, and by 2.5 relative to an elasticity of substitution of 0.01 across all inputs.

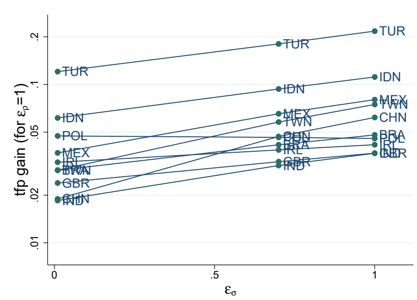
The value of the elasticities of substitution also determines the strength of the sectoral linkages' amplification effect. In fact, the larger TFP gain obtained when the elasticities of substitution are higher than the benchmark comes from a stronger amplification effect. The right panel of Table 1 reports the value of the amplification factor—that is, the ratio of the TFP gain with and without sectoral linkages—for the same 10 countries. For these countries, the amplification factor can be as high as 7.7; if China is excluded, it can be as high as 2.9. A higher value of either elasticity of substitution leads typically to a stronger amplification effect. This result holds also for the other countries whose amplification factors are reported in Appendix C.4. The amplification factor of the median country is equal to 1.8 for the benchmark calibration but would be equal to 3.3 if the elasticity of substitution between all inputs was equal to one. We find also that, in line with Section 3's theoretical results, sectoral linkages do not always amplify the effects of markups on aggregate productivity. In India, when inputs are highly complementary, the amplification factor is below one. When inputs are highly complementary, there is no room for inefficiency in the firms' inputs decisions, and the only source of misallocation comes from the effect of the (inefficient) price dispersion on final consumption. The dampening effect then comes from the negative correlation between the

[42] In a minority of cases, the TFP gain is not monotone relative to the elasticity ε_σ (for Poland and Romania). This result is due to the recalibration (the model is recalibrated each time the value of one of the elasticities of substitution is modified). When the model parameters (other than the elasticities of substitution) are kept constant, the TFP gain is monotonic (see Appendix C.4).

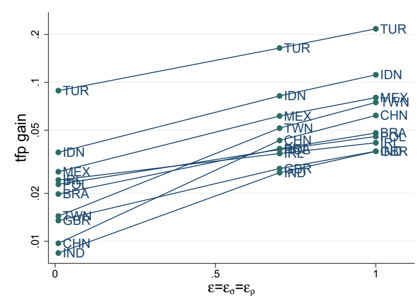
Figure 6: The TFP gain and input substitutability



(a) varying the intermediate input elasticity



(b) varying the labor-intermediate input bundle elasticity



(c) varying the (identical) input elasticity

Note: The graphs report the TFP gain from removing markups as a function of the elasticities of substitution ϵ_ρ and ϵ_σ for the 10 countries with the largest gains. In panel (a), we set $\varepsilon_\sigma = 1$ and vary ε_ρ; in panel (b), we set $\varepsilon_\rho = 1$ and vary ε_σ; in panel (c), we assume that $\epsilon_\sigma = \epsilon_\rho$.

sector's markups and either the sector's intermediate-input intensity or the average markup of the sector's suppliers.

We now turn to the impact of each sector. The degree of input substitutability determines which network statistics is the most relevant to measure the sectors' impact. We report in Figure 7 the median value (across countries) of each sector's overall importance, $s_i = \beta_i + \gamma_i$, together with its importance for final output, β_i. The importance of the sector as an intermediate-input supplier, γ_i, also appears on the graph as the difference between s_i and β_i. As explained in Section 3.3.3, the sector's impact is close to s_i when the two input elasticities are equal to one, and it is close to β_i when inputs are perfect complements. The comparison between these two polar cases nicely illustrates how central the value of the elasticity of substitution is. The figure shows that assuming a unit elasticity of substitution

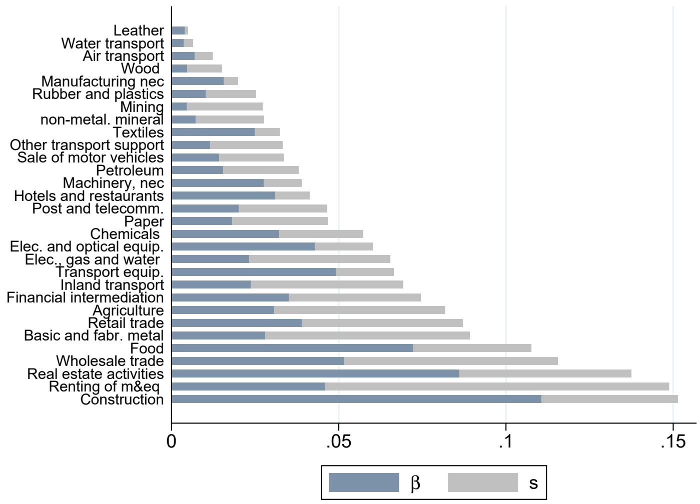
Table 1: TFP gain and IO amplification factor

	TFP gain			IO amplification factor		
$(\varepsilon_\sigma, \varepsilon_\rho)$	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)
median	0.009	0.013	0.023	1.5	1.8	3.3
IND	0.008	0.018	0.036	0.7	1.6	3.1
GBR	0.014	0.020	0.038	1.6	2.3	4.3
IRL	0.024	0.029	0.042	1.3	1.6	2.3
POL	0.023	0.019	0.047	1.9	1.6	3.9
BRA	0.020	0.031	0.049	1.6	2.5	3.9
CHN	0.010	0.035	0.062	2.1	7.7	13.7
TWN	0.014	0.037	0.074	1.1	2.9	5.9
MEX	0.027	0.052	0.081	1.4	2.7	4.2
IDN	0.036	0.057	0.111	1.6	2.5	4.9
TUR	0.089	0.132	0.217	1.2	1.8	2.9

Note: This table presents the TFP gain and the Input-Output amplification factor for the median country and for the 10 countries with the largest TFP gain. Results are shown for the benchmark calibration, $(\varepsilon_\sigma, \varepsilon_\rho) = (0.70, 0.01)$ and for two other combinations of the elasticities. The TFP gain is computed as (TFP without markups)/(TFP with markups) -1. The IO amplification factor is the ratio of the TFP gain in the baseline economy over the TFP gain in the economy with $\alpha_i = 0$, $\forall i = 1, ..., n$. See Appendix C.4 for the full table of results.

leads to overestimating the impact of all the sectors, particularly the sectors that supply primarily intermediate inputs, such as the basic metal, the material-and-equipment rentals, the wood products, the non-metallic mineral, and the mining industries. In half of the sectors, the impact is overestimated by a factor of 2.2 relative to the perfect complement case. The overestimation is not uniform across sectors and hence also modifies the sectors' ranking. Among the 10 most important sectors (highest s_i), basic and fabricated metals' and inland transportation's ranks are the most overestimated when using s_i instead of β_i, whereas transport equipment's and electrical equipment's ranks are the most underestimated. On the other hand, sectors that are important final-output suppliers—such as construction, real estate, and the food and beverage industry—rank high whatever the value of the elasticities of substitution. For our benchmark value of the elasticities of substitution, the impact of each sector is likely to be in between the two polar cases shown in Figure 7. We believe that assuming a unit elasticity of substitution leads to overestimating the impact of intermediate-input suppliers, albeit to a smaller extent than is suggested by the comparison with the perfect complementary case discussed above.

Figure 7: The impact of each sector



Note: This graph represents the median value (across countries) of the sectors' overall importance, s_i, and that their importance in final output, β_i. The underlying technology parameters are estimated under the benchmark elasticities values, $(\varepsilon_\sigma, \varepsilon_\rho) = (0.7, 0.01)$.

4.4 Additional results and robustness checks

We undertake additional exercices and counterfactuals, and we verify that the results are robust to outliers. Detailed results are reported in Appendix C.4. Here, we give a summary of the main results.

We first consider the TFP gain that would obtain if the distortions affected only labor (and not all inputs). As highlighted in Section 3, distortions' effects vary depending on whether they affect both labor and intermediates or only labor. In line with the theoretical results, we find that the TFP loss is substantially smaller when the distortions affect only labor, and the sectoral linkages are more likely to dampen the effect of distortions. In fact, sectoral linkages dampen the effect of the distortions in the median country. As shown in Table 2, the median TFP loss is equal to 0.5% and is a fifth smaller relative to an otherwise identical economy without sectoral linkages (in the baseline calibration). If we further assume that the sectors' sizes are the same in the two economies (that is, $\beta_i|_{\alpha=0} = (1-\alpha_i)s_i$), the median TFP loss would be even smaller, about two-fifths smaller than in the economy without sectoral linkages.

We then compute the TFP gain of removing (all-input) markups for two alternative

Table 2: TFP gain and IO amplification factor - markups on labor only

	TFP gain			IO amplification factor		
$(\varepsilon_\sigma, \varepsilon_\rho)$	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)
median	0.003	0.005	0.008	0.5	0.8	1.3
IND	0.005	0.008	0.015	0.4	0.7	1.3
BRA	0.007	0.010	0.015	0.6	0.8	1.2
ROU	0.007	0.009	0.015	0.6	0.8	1.3
IRL	0.010	0.012	0.017	0.5	0.7	0.9
SVN	0.012	0.013	0.017	0.7	0.8	1.0
TWN	0.007	0.012	0.020	0.6	1.0	1.6
POL	0.008	0.010	0.021	0.6	0.9	1.8
MEX	0.013	0.017	0.027	0.7	0.9	1.4
IDN	0.018	0.023	0.036	0.8	1.0	1.6
TUR	0.031	0.041	0.054	0.4	0.6	0.7

Note: This table presents the TFP gain and the Input-Output amplification factor for the median country and for the 10 countries with the largest TFP gain when markups are only on labor. Results are shown for the benchmark calibration, $(\varepsilon_\sigma, \varepsilon_\rho) = (0.70, 0.01)$ and for two other combinations of the elasticities. The TFP gain is computed as (TFP without markups)/(TFP with markups) -1. The IO amplification factor is the ratio of the TFP gain in the baseline economy over the TFP gain in the economy with $\alpha_i = 0$, $\forall i = 1, ..., n$. All the production parameters (except for ε_σ and ε_ρ) are set at their benchmark values (which accounts for all-input markups) and are kept constant when varying the elasticities of substitution. See Appendix C.4 for the full table of results.

counterfactuals. In the baseline counterfactual, only the positive markups are set to zero; here, we recompute the results in the case in which both negative and positive markups are set to zero. In addition, we consider the counterfactual in which all the parameters of the production function are held constant (instead of recalibrating them when the elasticities of substitution vary, as we did in the baseline exercise). These two counterfactuals better correspond to the approach followed in the theoretical analysis. We find that the predictions of the theoretical results hold more systematically in these two cases.

Finally, we check the robustness to outliers in the values of the markup. In a first robustness check, we compute the gap between the price-cost margin and the industry's median, and we winsorize the price-cost margin using the 1st and 99th percentile of the gap (computed on pooled data). In the second robustness check, we compute the counterfactual TFP obtained when setting positive markups to zero in all sectors except real estate. Given the challenges associated with measuring the capital stock and the large capital-output ratio in that sector, the likelihood of mismeasuring the markup is higher in that sector. As shown in Section 4.1, the real-estate sector displays a tremendous dispersion in pcm across countries; for most countries, the price-cost margin in real estate is negative, but in the few countries in which the price-cost margin is positive, it has very high values. In both robustness checks, the TFP gain from reducing markups is only slightly lower for the median country but considerably smaller for Taiwan, Turkey, and Indonesia, whose TFP gain is about half as large without the outliers. Removing outliers does not affect our main result concerning the role of the elasticity. We find that the TFP gain and the amplification factor typically increase with the two elasticities of substitution.

5 Conclusion

We study how the sectoral linkages of production shape the aggregate TFP loss from distortions and we shed light on the crucial role played by input substitutability.

We show, analytically and quantitatively, that the TFP loss from distortions is smaller when input substitutability is lower. We find that the smaller effect of distortions is related to the smaller role played by sectoral linkages. When input substitutability is lower, the amplification from sectoral linkages is weaker, and sectors that supply intermediate inputs have a smaller impact. Moreover, we find that sectoral linkages do not systematically amplify the effect of distortions. We derive the conditions under which sectoral linkages dampen the effect of distortions. The dampening effect occurs if the elasticity of substitution is smaller than one and if additional conditions, which are more likely to hold when the distortions affect only primary inputs than when the firms' intermediate-input decisions are directly affected, are satisfied.

For our quantitative analysis, we focus on the sectoral distortions caused by market power. Using sectoral-level data from 35 countries, we find that the median TFP gain from removing sectoral-level markups is equal to 1.3%. These estimates as a lower bound of the cost of markups because they do not account for the cost of *firm*-level markups. An important message of the quantitative analysis is that using a unit elasticity of substitution (i.e., the Cobb-Douglas specification), as is commonly done in the literature, would have led to overestimating the cost of industry-level markups by a factor of 1.8. The large quantitative implications documented in our analysis call for caution in the choice of the specification of the production function as well as for more empirical evidence on the values of the elasticities of substitution.

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APPENDIX
(Not for publication)

A Additional Propositions

We provide 7 additional propositions. We show that we can assume, without loss of generality, that there are no direct distortions on the consumption allocation. We also present the solution of the model with Cobb-Douglas production functions, the expression of the Domar weight in the absence of distortions. In addition, we show that uniform labor distortions have no effect on aggregate productivity and we provide the expression of TFP when distortions are random. We also derive additional results on the propagation of distortions.

A.1 Allocation with consumption frictions

If a planner maximizes output (and welfare) subject to the production functions (1)-(3) and resource constraints (4) - (5), the optimal allocation will satisfy:

$$\frac{dY}{dC_i} = \lambda_i$$

$$\lambda_i \frac{dQ_i}{dX_{ij}} = \lambda_j$$

$$\lambda_i \frac{dQ_i}{dL_i} = \eta.$$

This implies that conditions (6)- (8) can be considered as deviations from the frictionless allocation. There may be distortions that do not occur during the production process but change the consumption mix; for example consumption taxes or subsidies. Therefore, we may want to modify (6) as follows:

$$\frac{dY}{dC_i} = \lambda_i(1+\tau_{Ci}),$$

with τ_{Ci} being the consumption distortion.

The following proposition implies that we can assume $\tau_{Ci} = 0$ without loss of generality. Moreover, any allocation that satisfies the physical feasibility constraints can be rationalized by a set of distortions in our framework.

Proposition A.1 *Suppose that an allocation* $\{C_i, Q_i, X_{ij}, L_i\}$ *satisfies* (7), (8) *and:*

$$\frac{dY}{dC_i} = \lambda_i(1+\tau_{Ci})$$

for some vectors $(\lambda, \tau_X, \tau_L, \tau_C)$*. Then there exist vectors* $(\lambda', \tau'_X, \tau'_L)$ *such that the allocation satisfies* (6)- (8)*. Suppose that the allocation* $\{C_i, Q_i, X_{ij}, L_i\}$ *satisfies* (1)-(3) *and* (4)-(5)*. Then there exist* $\{\lambda_i, \tau_{Xij}, \tau_{Li}\}$ *such that the allocation satisfies* (6)-(8).

A.2 Solution with Cobb-Douglas production functions

Proposition A.2 *Suppose that $\sigma = \rho = 0$. Then λ_i/η is given by*

$$\begin{aligned}\log(\lambda/\eta) &= -(I-\tilde{\alpha}V)^{-1}((I-\tilde{\alpha})\log B + \log A) + \\ &\quad (I-\tilde{\alpha}V)^{-1}[(I-\tilde{\alpha})\Delta^{Lp} + \tilde{\alpha}(V \circ \Delta^{Xp})]\mathbf{1},\end{aligned} \tag{A.1}$$

and output by

$$\begin{aligned}\log Y &= \log \bar{L} + \beta'(I-\tilde{\alpha}V)^{-1}((I-\tilde{\alpha})\log B + \log A) - \beta'(I-\tilde{\alpha}V)^{-1}[(I-\tilde{\alpha})\Delta^{Lp} + \tilde{\alpha}(V \circ \Delta^{Xp})]\mathbf{1} \\ &\quad - \log(\beta'(I-\tilde{\alpha}(V \circ \Delta^{Xq}))^{-1}\Delta^{Lq}(\mathbf{1}-\alpha)),\end{aligned}$$

where $\Delta^{Xp}_{ij} = \log(1+\tau_{Xij})$, $\Delta^{Lp}_{ii} = \log(1+\tau_{Li})$; $\Delta^{Lp}_{ij} = 0, i \neq j$ and $\tilde{\alpha}$ is a square matrix with $\tilde{\alpha}_{ii} = \alpha_i$ and $\tilde{\alpha}_{ij} = 0$ if $i \neq j$ and $\Delta^{Xq}_{ij} = (1+\tau_{Xij})^{-1}$; $\Delta^{Lq}_{ii} = (1+\tau_{Li})^{-1}, \Delta^{Lq}_{ij} = 0, i \neq j$ are matrices of distortions, and $\circ$ denotes the Hadamard (entrywise) product.

A.3 Domar weight of the frictionless allocation

Proposition A.3 *Consider an economy with $A_i = B_i = 1, \forall i$ and no distortions. Then $Y = L, L_i/Q_i = 1-\alpha_i, X_{ij}/Q_i = \alpha_i v_{ij}$ and $s_i = Q_i/Y = \lambda_i Q_i/Y$.*

A.4 Irrelevance of uniform labor distortions

Proposition A.4 *Consider two distorted economies, denoted 1 and 2, which are otherwise identical except that $\tau_i^{L2} = (1+z)(1+\tau_i^{L1}) - 1$, $z > -1$. Then $\eta^2 = \eta^1/(1+z)$ and all the other equilibrium variables are the same.*

A.5 Random distortions

If the number of sectors is sufficiently large and the distortions are independent of the sector characteristics (its importance as a supplier of final or intermediate goods; its use of intermediate goods; similar characteristics of its suppliers and customers) it becomes easy to characterize the TFP loss in terms of the mean and the variance of the distortions. We present the approximation of TFP in the case of random distortions for the two types of distortions.

Proposition A.5 *Suppose that $\sigma = \rho$, $\alpha_i \in (0,1)$ for all $i = 1, ..., n$, $A_i = B_i = 1$ and $\tau_{Xij} = 0, \forall i, j = 1, ..., n$. Suppose also that τ_{Li} is identically and independently distributed across sectors and independent from any other model parameters, with mean $\bar{\tau}_L$ and cross-sectional variance $\mathrm{var}(\tau_L)$. For small random distortions, aggregate TFP is approximatively equal to*

$$\log TFP \approx -\frac{1}{2}Var(\tau_L) - \frac{1}{2}\frac{\sigma}{1-\sigma}\psi(\alpha, \beta, V)\,\mathrm{var}(\tau_L), \tag{A.2}$$

where $\psi(\alpha, \beta, V) \geq 0$. If $\alpha_i v_{ij} > 0$ for some $i \neq j$, then $\psi(\alpha, \beta, V) > 0$.

Proposition A.6 *Suppose that $\sigma = \rho$, $\alpha_i \in (0,1)$ for all i, $A_i = B_i = 1$ and $\tau_{Xij} = \tau_{Li} = \tau_i, \forall i, j$. Suppose also that τ_i is identically and independently distributed across sectors and independent from any other model parameters, with mean $\bar{\tau}$ and cross-sectional variance $\operatorname{var}(\tau)$.*
For small random distortions, aggregate TFP is approximatively equal to

$$\log TFP \approx -\Gamma_0(\alpha,\beta,V)\bar{\tau}^2 - \chi_0(\alpha,\beta,V)\operatorname{var}(\tau) - \frac{\sigma}{1-\sigma}\left[\Gamma_1(\alpha,\beta,V)\bar{\tau}^2 + \chi_1(\alpha,\beta,V)\operatorname{var}(\tau)\right], \quad \text{(A.3)}$$

where $\Gamma_0(\alpha,\beta,V), \Gamma_1(\alpha,\beta,V), \chi_0(\alpha,\beta,V), \chi_1(\alpha,\beta,V) \geq 0$ and $\Gamma_0(\alpha,\beta,V)+\frac{\sigma}{1-\sigma}\Gamma_1(\alpha,\beta,V) \geq 0, \chi_0(\alpha,\beta,V)+\frac{\sigma}{1-\sigma}\chi_1(\alpha,\beta,V) \geq 0$.
If $\alpha_i > 0$ for some i, then $\chi_1 > 0, \Gamma_1 > 0$.

A.6 Upstream and downstream propagation of distortions

Do distortions on sectors early in the production chain matter more or less? As we move down the production process, substitution possibilities should attenuate the effect of distortions, but on the other hand, they have effects on multiple sectors. Acemoglu et al. (2016) tackle this question in the context of productivity shocks in a Cobb-Douglas framework. They find that productivity shocks travel downstream, while demand shocks travel upstream.
In the same spirit, we consider an interesting special case in which we can separate the production network into upstream and downstream components. We define sector i to be a purchaser of sector j if $[\tilde{\alpha}V]_{ij}^{(n)} > 0$ for some n, that is, if sector j is a direct or indirect supplier of sector i. Sector k is upstream of j it is not a purchaser of sector ℓ.[43] It is trivial to relabel the sectors in such a way that the upstream sectors are numbered $1, 2 \dots \ell - 1$, the sector in question to be ℓ and the purchaser sectors are $\ell + 1 \dots n$.

Lemma A.1 *A necessary and sufficient condition for sectors $1, \dots \ell-1$ to be upstream from ℓ is that $\alpha_i v_{ij} = 0$, $\forall i \leq \ell - 1, j \geq \ell$.*

Our definition of upstreamness is ordinal and cannot necessarily compare any two sectors, but when it does, it agrees with the measure that Antràs et al. (2012) propose.
Given that lemma and proposition 4, we can characterize the effect of imposing distortions on sector ℓ. We simplify and focus on the case with markups.

Proposition A.7 *Suppose that $\tau_{Li} = \tau_{Xij} = \tau_i, \forall i$ and that sectors $1, \dots, \ell - 1$ are upstream from sector ℓ. Then:*

1. $\frac{\partial}{\partial \tau_\ell}\frac{\lambda_i}{\eta} = 0$ *if* $i < \ell$ *and* $\frac{\partial}{\partial \tau_\ell}\frac{\lambda_i}{\eta} > 0$ *if* $i \geq \ell$.
2. $\frac{\partial}{\partial \tau_\ell}\lambda_i < 0$ *if* $i < \ell$.

[43] Sector ℓ can itself be a purchaser of a purchases sector, hence we do not use the term "downstream" sector.

3. $\frac{\partial}{\partial \tau_\ell} \frac{C_i}{Y} > 0$ *if* $i < \ell$ *and* $\frac{\partial}{\partial \tau_\ell} C_i > 0$ *if* $i < \ell$.

4. $\frac{\partial}{\partial \tau_\ell} \frac{\sum_j (1+\tau_{Xij})\lambda_j X_{ij}}{\lambda_i Q_i} = 0$ *if* $i < \ell$; $\frac{\partial}{\partial \tau_{\ell,x}} \frac{\sum_j \lambda_j X_{ij}}{\lambda_i Q_i} > 0$ *if* $i > \ell$ *and* $\sigma < 0$; $\frac{\partial}{\partial \tau_\ell} \frac{\sum_j \lambda_j X_{ij}}{\lambda_i Q_i} < 0$ *if* $i > \ell$ *and* $\sigma > 0$.

The division of labor in the upstream sectors is unaffected by the distortion and hence their price in terms of labor is unaffected. Since imposing distortions lowers the real wage, this implies that upstream industries become cheaper and expand in a relative and absolute sense. This conclusion does not depend on the elasticity of substitution, and is in stark contrast with the predictions for the productivity shocks case. Another interesting implication of this special case is that, except for the sector directly affected by the distortion, it leads to higher intermediate shares in downstream sectors if the elasticity of substitution is low. The reason is that the distortion has mainly price effects. Therefore, high intermediate shares are consistent with output loss and distortion on intermediate goods.

B Proofs

B.1 Model solution and normalization

Proof of Proposition 1 (General CES case). Part (i), (ii) and (iii) of the proposition, which give the expressions of m_{ij}, d_i and c_i, are obtained from the first-order conditions. Equation (6) implies expression (iii). Equation (7) implies:

$$X_{ij} = A_i^{\frac{\sigma}{1-\rho}} v_{ij} X_i \left[\frac{\alpha_i Q_i}{X_i}\right]^{\frac{1-\sigma}{1-\rho}} \left(\frac{\lambda_i}{(1+\tau_{Xij})\lambda_j}\right)^{\frac{1}{1-\rho}}, \tag{B.1}$$

which then gives

$$X_i = A_i^{\frac{\sigma}{1-\sigma}} \alpha_i Q_i \left[\sum_j v_{ij} \left(\frac{\lambda_i}{(1+\tau_{Xij})\lambda_j}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}\frac{1}{1-\sigma}}. \tag{B.2}$$

Combining the two equations gives expression (ii). Expression (i) is obtained from equation (8):

$$L_i = (A_i B_i)^{\frac{\sigma}{1-\sigma}} (1-\alpha_i) Q_i \left(\frac{\lambda_i}{(1+\tau_{Li})\eta}\right)^{\frac{1}{1-\sigma}}. \tag{B.3}$$

Then plugging equations (B.2) and (B.3) in the production function (1) yields

$$1 = A_i^{\frac{1}{1-\sigma}} \lambda_i^{\frac{1}{1-\sigma}} \left[(1-\alpha_i) B_i^{\frac{\sigma}{1-\sigma}} [(1+\tau_{Li})\eta]^{-\frac{\sigma}{1-\sigma}} + \alpha_i \left[\sum_j v_{ij} \left(\frac{1}{(1+\tau_{Xij})\lambda_j}\right)^{\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}\frac{\sigma}{1-\sigma}}\right]^{\frac{1}{\sigma}}, \tag{B.4}$$

which gives expression v of the proposition.

From equation (6), $\log C_i = \log \beta_i + \log Y - \log \lambda_i$ and from equation (3), $\log Y = \sum_i \beta_i \log C_i - \sum_i \beta_i \log \beta_i$. Combining the two equations, we get $\sum_i \beta_i \log \lambda_i = 0$ and hence $\log \eta = -\sum_i \beta_i \log(\lambda_i/\eta)$, therefore

$$\log \lambda_i = \log(\lambda_i/\eta) - \sum_i \beta_i \log(\lambda_i/\eta),$$

which yields expression (iv) of the Proposition.
The resource constraint for good i can be written as

$$\sum_j m_{ji} q_j + c_i = q_i, \tag{B.5}$$

where $q_i = Q_i/Y$. In matrix form, this is $M'q + c = q$, so $q = (I - M')^{-1}c$. Since $L_i = d_i Q_i = d_i q_i Y$, the resource constraint on labor can be written as $d'(I - M')^{-1}cY = \bar{L}$, which implies $Y = [d'(I - M')^{-1}c]^{-1}\bar{L}$ as stated in the proposition. ■

Proof of Proposition 2. First, we show that we can always reparameterize the production function in such a way to have $B_i = 1$ for all i. Let $\{A_i, B_i, \alpha_i, v_{ij}\}_{i,j}$ be some arbitrary parameters of the production function.
Then for all i set

$$B_i' = 1$$

$$\alpha_i' = 1 - \frac{(1-\alpha_i) B_i^{\frac{\sigma}{1-\sigma}}}{(1-\alpha_i) B_i^{\frac{\sigma}{1-\sigma}} + \alpha_i}$$

$$A_i' = A_i [(1-\alpha_i) B_i^{\frac{\sigma}{1-\sigma}} + \alpha_i]^{\frac{1-\sigma}{\sigma}}$$

Inspection shows that for all i and $L_i > 0, X_{ij} > 0$,

$$A_i \left[(1-\alpha_i)^{1-\sigma} (B_i L_i)^\sigma + \alpha_i^{1-\sigma} \left(\sum_j v_{ij}^{1-\rho} X_{ij}^\rho \right)^{\frac{\sigma}{\rho}} \right]^{\frac{1}{\sigma}} = A_i' \left[(1-\alpha_i')^{1-\sigma} L_i^\sigma + \alpha_i'^{1-\sigma} \left(\sum_j v_{ij}^{1-\rho} X_{ij}^\rho \right)^{\frac{\sigma}{\rho}} \right]^{\frac{1}{\sigma}}.$$

Then B_i', α_i' and A_i' represent the same production function as B_i, α_i and A_i. Hence without loss of generality, we can assume $B_i = 1, \forall i$.
Next, we show that by choosing the units appropriately, we can have $A_i = 1$ for all i. Let $k \in \mathbf{R}^n_{++}$ be an arbitrary vector of unit changes. Define $Q_i' = k_i Q_i$. Then $X_{ij} = X_{ij}'/k_j$. Substituting in the production function we obtain:

$$Q_i' = k_i A_i \left[(1-\alpha_i)^{1-\sigma} L_i^\sigma + \alpha_i^{1-\sigma} \left(\sum_j v_{ij}^{1-\rho} k_j^{-\rho} X_{ij}'^{\rho} \right)^{\frac{\sigma}{\rho}} \right]^{\frac{1}{\sigma}}$$

The right-hand side of this expression looks the original production function with v_{ij}/k_j^ρ playing the role of the parameters in the intermediate good aggregator. However, in general $\sum_j v_{ij}/k_j^\rho \neq 1$. To get around this problem, define the new parameters $v_{ij}(k) = \frac{v_{ij}k_j^{-\frac{\rho}{1-\rho}}}{\sum_j v_{ij}k_j^{-\frac{\rho}{1-\rho}}}$, $m_i(k) = \left(\sum_j v_{ij}k_j^{-\frac{\rho}{1-\rho}}\right)^{-\frac{1-\rho}{\rho}}$, $\alpha_i(k) = 1 - \frac{1-\alpha_i}{1-\alpha_i+\alpha_i m_i(k)^{-\frac{\sigma}{1-\sigma}}}$ and

$$A_i(k) = k_i A_i \left[1 - \alpha_i + \alpha_i m_i(k)^{-\frac{\sigma}{1-\sigma}}\right]^{\frac{1-\sigma}{\sigma}}.$$

This parameterization satisfies all the constraints for the production function ($A_i(k) > 0, \alpha_i(k) \in [0,1], v_{ij}(k) \in [0,1], \sum_j v_{ij}(k) = 1$). Again we can verify that

$$k_i A_i \left[(1-\alpha_i)^{1-\sigma} L_i^\sigma + \alpha_i^{1-\sigma}\left(\sum_j v_{ij}^{1-\rho} k_j^{-\rho} {X'_{ij}}^\rho\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1}{\sigma}} =$$

$$A_i(k)\left[(1-\alpha_i(k))^{1-\sigma} L_i^\sigma + \alpha_i(k)^{1-\sigma}\left(\sum_j v_{ij}(k)^{1-\rho} {X'_{ij}}^\rho\right)^{\frac{\sigma}{\rho}}\right]^{\frac{1}{\sigma}}$$

Hence, the new parameters represent the same production function.
The proposition is equivalent to showing that there exists a vector k such that $A_i(k) = 1\ \forall i$. Then given the expression of $A_i(k)$ this is:

$$k_i = A_i^{-1}\left[(1-\alpha_i) + \alpha_i m_i(k)^{-\frac{\sigma}{1-\sigma}}\right]^{-\frac{1-\sigma}{\sigma}},\ i = 1, \dots n \tag{B.6}$$

Denote the mapping defined by the right-hand side of equation (B.6) as $g(k)$, where k and g are $n \times 1$ vectors. Then (B.6) is simply a fixed point. Since g is monotone and continuous, a sufficient condition for it to have fixed points is that if $k \le \bar{k}$ then $g(k) \le \bar{k}$ and if $k \ge \underline{k}$ then $g(k) \ge \underline{k}$ for some positive vectors $\underline{k}, \bar{k}$, $\underline{k} \le \bar{k}$ and the inequalities are elementwise.
If $k(s) = (s, s, \dots s)'$, then $g_i(k(s))/s = A_i^{-1}\left[(1-\alpha_i)s^{\frac{\sigma}{1-\sigma}} + \alpha_i\right]^{-\frac{1-\sigma}{\sigma}}$.
There are two cases to consider.

1. $\sigma < 0$. Since $\lim_{s\to 0} g_i(k(s))/s = \infty$, $g(k(\underline{s})) > \underline{s}$ for some sufficiently small $\underline{s}$. Set $\underline{k} = (\underline{s}, ...\underline{s})'$. Therefore $g(\underline{k}) > \underline{k}$. Let $k \ge \underline{k}$. Then

$$g(k) \ge g(\min_i k_i, \min_i k_i, \dots) \ge g(\underline{k}) > \underline{k}$$

   On the other hand, $\lim_{s\to\infty} g_i(k(s))/s = A_i^{-1}[\alpha_i]^{-\frac{1-\sigma}{\sigma}}$, so if $A_i > \alpha_i^{-\frac{1-\sigma}{\sigma}}$ for all i, $g(k(s)) < k(s)$ for s sufficiently large. Then set $\bar{k} = (s, \dots s)'$ for s sufficiently large. Showing that $k \le \bar{k}$ implies $g(k) < \bar{k}$ is the same as the case above.

2. $\sigma > 0$. $\lim_{s\to 0} g_i(k(s))/s = A_i^{-1}\alpha_i^{-\frac{1-\sigma}{\sigma}}$, so if $A_i < \alpha_i^{-\frac{1-\sigma}{\sigma}}$ for all i, $g(k(s)) > k(s)$ for all $s < \underline{s}$ for some $\underline{s}$ sufficiently small. Set $\underline{k} = (\underline{s}, \dots, \underline{s})'$

   On the other hand, $\lim_{s\to\infty} g_i(k(s))/s = 0$, so if $s \le \bar{s}$, $g(k(s)) < k(s)$ for some $\bar{s}$ sufficiently large. Set $\bar{k} = (\bar{s}, \dots, \bar{s})'$

Showing that $\underline{k} \leq k \leq \bar{k}$ implies $\underline{k} < g(k) < \bar{k}$ is the same as before.

We can unify the conditions for the two cases as $A_i^{-\frac{\sigma}{1-\sigma}} > \alpha_i,\ i = 1, \dots n$. Let k^* be a fixed point. Then we can set $A'_i = A_i(k^*), \alpha'_i(k^*), v'_{ij} = v_{ij}(k^*)$.

For the redefined variables $A'_i = B_i = 1$. It is immediate that $\lambda_i/\eta = 1$ satisfy v of Proposition 1. Hence (iv) of proposition 1 implies that $\lambda_i = 1$ for all i. The last two statements of the proposition follow from condition ii of Proposition 1. ■

Proof of Proposition 3 . As Proposition 2 allows us to do, we assume that $A_i = B_i = 1\ \forall i$.

First, we consider the case of zero distortions. It is immediate $\lambda/\eta = \mathbf{1}$ is a solution to the system (9). We will show that it is the only solution. Suppose that $\bar{\lambda} \equiv \max_k\{\lambda_k/\eta\} > 1$. Then

$$\frac{\lambda_i}{\eta} \leq \left[(1-\alpha_i)1^{-\frac{\sigma}{1-\sigma}} + \alpha_i \bar{\lambda}^{-\frac{\sigma}{1-\sigma}}\right]^{-\frac{1-\sigma}{\sigma}} < \bar{\lambda}.$$

Since this holds for all i, we get that $\max_k\{\lambda_k/\eta\} < \max_k\{\lambda_k/\eta\}$, a contradiction, so $\max_k\{\lambda_k/\eta\} \leq 1$. In the same way we get that $\min_k\{\lambda_k/\eta\} \geq 1$, so $\lambda_i/\eta = 1$ for all i. Since the rest of the solution is a function of λ/η, this proves uniqueness. The solution will exist if the matrix $(I - M')^{-1}$ exists with nonnegative elements. It is immediate that $M = \tilde{\alpha}V$, so existence is guaranteed by lemma B.1.

Now consider the case with distortions. We use the implicit function theorem to show the existence of solution of system (9) in some neighborhood of the zero vector. Let's rewrite it as $F(\lambda/\eta, \tau) = \mathbf{0}$. This is a continuous function of both sets of parameters. The Jacobian of F with respect to λ/η evaluated at zero distortions is $\frac{d}{d\lambda/\eta}F(\lambda/\eta, \mathbf{0}) = I - \tilde{\alpha}V$ and lemma B.1 ensures that $\det\left|\frac{d}{d\lambda/\eta}F(\lambda/\eta, \mathbf{0})\right| \neq 0$. So by the implicit function theorem there is a unique continuous solution of λ/η as a function of the distortions. Lastly the invertibility of the matrix $I - M'$ comes again from lemma B.1 and continuity.

■

Proof of Proposition 4 (Equal-elasticity case). With $\sigma = \rho$, equation (B.1) gives $m_{ij} = \alpha_i v_{ij} A_i^{\frac{\sigma}{1-\sigma}} [\lambda_i/((1+\tau_{Xij})\lambda_j)]^{\frac{1}{1-\sigma}}$, and equation (B.4) is then:

$$\left(\frac{\lambda_i}{\eta}\right)^{-\frac{\sigma}{1-\sigma}} = (1-\alpha_i)(A_i B_i)^{\frac{\sigma}{1-\sigma}}(1+\tau_{Li})^{-\frac{\sigma}{1-\sigma}} + \alpha_i A_i^{\frac{\sigma}{1-\sigma}} \sum_{j=1}^{n} v_{ij}(1+\tau_{Xij})^{-\frac{\sigma}{1-\sigma}} \left(\frac{\lambda_j}{\eta}\right)^{-\frac{\sigma}{1-\sigma}}$$

Expressing this in matrix form,

$$\hat{\lambda} = \hat{A}\hat{B}\Delta^{Lp}(1-\alpha) + \tilde{\alpha}\hat{A}(\Delta^{Xp} \circ V)\hat{\lambda},$$

where $\hat{\lambda}_i = (\lambda_i/\eta)^{-\frac{\sigma}{1-\sigma}}$, the square matrixes $\hat{A}$, $\hat{B}$, Δ^{Lp}, Δ^{Xp} and $\tilde{\alpha}$ have diagonal elements equal to $\hat{A}_{ii} = A_i^{\frac{\sigma}{1-\sigma}}$, $\hat{B}_{ii} = B_i^{\frac{\sigma}{1-\sigma}}$, $\Delta^{Lp}_{ii} = (1+\tau_{Li})^{-\frac{\sigma}{1-\sigma}}$, $\Delta^{Xp}_{ij} = (1+\tau_{Xij})^{-\frac{\sigma}{1-\sigma}}$, $\tilde{\alpha}_{ii} = \alpha_i$, and nondiagonal elements equal to zero. The equation implies (11). Plugging the expression for m_{ij} in the resource constraint for good i (B.5), we have

$$\sum_j \alpha_j v_{ji} A_j^{\frac{\sigma}{1-\sigma}} (1+\tau_{ji})^{-\frac{1}{1-\sigma}} \lambda_j^{\frac{1}{1-\sigma}} q_j + \lambda_i^{\frac{1}{1-\sigma}} c_i = \lambda_i^{\frac{1}{1-\sigma}} q_i.$$

Denote $\hat{q}_i = q_i \lambda_i^{\frac{1}{1-\sigma}}$ and $\hat{c}_i = \lambda_i c_i \hat{\lambda}_i^{-1}$. We obtain in matrix form

$$[\tilde{\alpha}\hat{A}(\Delta^{Xq} \circ V)]'\hat{q} + \hat{c}\eta^{\frac{\sigma}{1-\sigma}} = \hat{q}.$$

Therefore,

$$\hat{q} = [I - (\tilde{\alpha}\hat{A}(\Delta^{Xq} \circ V))']^{-1}\hat{c}\eta^{\frac{\sigma}{1-\sigma}} \tag{B.7}$$

Finally, using $d_i = (1-\alpha_i)(A_iB_i)^{\frac{\sigma}{1-\sigma}}\lambda_i^{\frac{1}{1-\sigma}}[(1+\tau_{Li})\eta]^{-\frac{1}{1-\sigma}}$ in the resource constraint on labor written as $\sum d_i q_i Y = \bar{L}$, gives $(\mathbf{1}-\alpha)'\hat{A}\hat{B}\Delta^{Lq}\hat{q}\eta^{-\frac{1}{1-\sigma}}Y = \bar{L}$. Then substituting for the expression of $\hat{q}$ and using $\log \eta = -\beta' \log(\lambda/\eta)$, we get equation (10). ■

B.2 Technical Results

Lemma B.1 *Suppose* $\max\{\alpha_i\} = \bar{\alpha} < 1$. *Then* $I - \tilde{\alpha}V$ *is invertible, i.e.* Ω *exists;* $\omega_{ij} \geq 0$ *for all* i, j*;* $\omega_{ii} \geq 1$ *and for all* $i \neq j$*,* $\omega_{ij} < \omega_{jj}$*. Finally,* $\Omega = I$ *if and only if* $\alpha_i = 0, \forall i$*.*

Proof of Lemma B.1. Let $A = \tilde{\alpha}V$. From the definition it follows that $a_{ij} \geq 0$ and $\sum_j a_{ij} \leq \bar{\alpha}$. Also $\Omega = [I - A]^{-1}$ (if this inverse exists) which can be rewritten $\Omega = I + A\Omega$.
Let $\Omega^{(0)} = I$ and let

$$\Omega^{(p+1)} = I + A\Omega^{(p)}$$

Note that by induction

$$\Omega^{(p)} = I + A + A^2 + \ldots A^p,$$

hence $\omega_{ij}^{(p+1)} \geq \omega_{ij}^{(p)}$ and $\omega_{ij}^{(p)} \geq 0$. The fact that $\sum_k a_{ik} \leq \bar{\alpha} < 1$ implies (by induction) that $a_{ij}^p \leq \bar{\alpha}^p$ and $0 \leq \omega_{ij}^{(p)} \leq \frac{1-\bar{\alpha}^{p+1}}{1-\bar{\alpha}}$. So $\Omega \equiv \lim_{p\to\infty} \Omega^{(p)}$ exists and is finite.

$$(I-A)\Omega = (I-A)\lim_{p\to\infty}\Omega^{(p)} = \lim_{p\to\infty}[(I-A)\Omega^{(p)}] = \lim_{p\to\infty}[I - A^{p+1}] = I,$$

which implies that $\Omega = (I-A)^{-1}$.
By construction $\omega_{ij}^{(p)} \geq 0$, so $\omega_{ij} = \lim_{p\to\infty}\omega_{ij}^{(p)} \geq 0$, which proves the first statement. Similarly $\omega_{ii}^{(p)} \geq 1$ implies that $\omega_{ii} \geq 1$.
Next, we show by induction that $\omega_{ij}^{(p)} < \omega_{jj}^{(p)}$ for all $i \neq j$ and p. This is true by construction for $p = 0$. Suppose it is true for some p. Consider $p+1$. Suppose that $i \neq j$. Then by the definition of

$\Omega^{(p+1)}$:

$$\begin{aligned} \omega_{ij}^{(p+1)} &= \sum_k a_{ik}\omega_{kj}^{(p)} \\ &\leq \omega_{jj}^{(p)} \sum_k a_{ik} \\ &< \omega_{jj}^{(p)} \\ &\leq \omega_{jj}^{(p+1)}. \end{aligned}$$

The second line comes from the inductive step; the third line from the fact that $\sum_k a_{ik} < 1$ and $\omega_{jj}^{(p)} \geq 1$; the last one from the fact that the sequence of matrices is monotone.
Then taking limits with respect to p, we establish that $\omega_{ij} \leq \omega_{jj}$. Finally, since $\Omega = I + A\Omega$, by the same chain of inequalities as above we establish that if $i \neq j$, $\omega_{ij} < \omega_{jj}$.
If $\alpha_i = 0\ \forall i$, then A is a square matrix of zeros, so $\Omega = (I - A)^{-1} = I^{-1} = I$. Suppose that $\alpha_i > 0$ for some i. Then $a_{ij} > 0$ for some j. If $i \neq j$ $\omega_{ij} \geq \omega_{ij}^1 = a_{ij} > 0$; if $i = j$, then $\omega_{ii} \geq \omega_{ii}^1 > 1$. So in either case $\Omega \neq I$. Hence the last statement is proved by contrapositive. ■

Lemma B.2 $\Omega(\mathbf{1} - \alpha) = \mathbf{1}$.

Proof of Lemma B.2. Since $V\mathbf{1} = \mathbf{1}$, and $\alpha = \tilde{\alpha}\mathbf{1} = \tilde{\alpha}V\mathbf{1}$ we have that

$$\mathbf{1} - \alpha = I\mathbf{1} - \tilde{\alpha}V\mathbf{1} = (I - \tilde{\alpha}V)\mathbf{1}.$$

Premultiplying the equality above by $(I - \tilde{\alpha}V)^{-1}$ implies the result. ■

Lemma B.3 *Let* $g_{ij} \equiv \sum_r v_{ir}\omega_{rj}$. *Then* $\alpha_i g_{ij} = \omega_{ij}$ *for* $i \neq j$ *and* $\alpha_i g_{ii} = \omega_{ii} - 1$.

Proof of Lemma B.3. Let G be a matrix with elements (i, j) equal to $\alpha_i g_{ij}$. Direct inspection shows that $G = \tilde{\alpha}V\Omega$. Since $\Omega = (I - \tilde{\alpha}V)^{-1}$, we have that $I = (I - \tilde{\alpha}V)\Omega = \Omega - G$, so $G = \Omega - I$, which proves the claim. ■

Lemma B.4 *Suppose that the distortions on intermediate goods are sector-specific, so* $\tau_{Xij} = \tau_{Xik} = \tau_{Xi}, \forall i, j, k = 1, ..., n$. *We will denote the vector of intermediate-good distortions by* τ_X, *the vector of labor distortions by* τ_L, *and the vector of all distortions by* τ. *Suppose that* $A = B = \mathbf{1}$. *Let* $\mathbf{0}$ *be a* *matrix of zeros with dimensions clear from context. Then,*

$$\begin{aligned}
\frac{d^2}{d\tau_{Li}\tau_{Lj}}\log TFP(\mathbf{0}) &= \left(\frac{\sigma}{1-\sigma}s_{ij}+s_is_j\right)(1-\alpha_i)(1-\alpha_j), \quad \textit{if } i\neq j \\
\frac{d^2}{d\tau_{Li}\tau_{Li}}\log TFP(\mathbf{0}) &= \left(\frac{\sigma}{1-\sigma}s_{ii}+s_i^2\right)(1-\alpha_i)^2-\frac{1}{1-\sigma}s_i(1-\alpha_i) \\
\frac{d^2}{d\tau_{Xi}\tau_{Lj}}\log TFP(\mathbf{0}) &= \left[\frac{\sigma}{1-\sigma}s_{ij}+s_is_j-\frac{1}{1-\sigma}s_i\left(\sum_s v_{is}\omega_{sj}\right)\right]\alpha_i(1-\alpha_j) \\
\frac{d^2}{d\tau_{Xi}\tau_{Xj}}\log TFP(\mathbf{0}) &= \left[\frac{\sigma}{1-\sigma}s_{ij}+s_is_j-\frac{1}{1-\sigma}\left(s_i\sum_s v_{is}\omega_{sj}+s_j\sum_s v_{js}\omega_{si}\right)\right]\alpha_i\alpha_j \quad \textit{if } i\neq j \\
\frac{d^2}{d\tau_{Xi}\tau_{Xi}}\log TFP(\mathbf{0}) &= \left[\frac{\sigma}{1-\sigma}s_{ii}+s_i^2-\frac{2}{1-\sigma}s_i\left(\sum_s v_{is}\omega_{si}\right)\right]\alpha_i^2-\frac{1}{1-\sigma}s_i\alpha_i
\end{aligned}$$

Proof of Lemma B.4. If there is no superscript on the distortion, we will mean that it can be either on labor or on intermediates.

Using proposition 4 with the assumption that intermediate goods distortions are purchaser-specific and $A_i = B_i = 1\ \forall i = 1, ..., n$, we can write aggregate productivity as

$$\log TFP = \frac{1-\sigma}{\sigma}\beta'\log(\hat{\lambda}) - \log(\hat{c}'[I-\tilde{\alpha}(\Delta^{Xq}\circ V)]^{-1}\Delta^{Lq}(\mathbf{1}-\alpha)),$$

with

$$\hat{\lambda} = [I-\tilde{\alpha}(\Delta^{Xp}\circ V)]^{-1}\Delta^{Lp}(\mathbf{1}-\alpha)$$

Denote the first term $N^p(\tau) \equiv \frac{1-\sigma}{\sigma}\beta'\log(\hat{\lambda})$ and the second term $N^q(\tau) \equiv -\log(\hat{c}'[I-\tilde{\alpha}(\Delta^{Xq}\circ V)]^{-1}\Delta^{Lq}(\mathbf{1}-\alpha))$. Let us define $\Omega^p(\tau) \equiv [I-\tilde{\alpha}(\Delta^{Xp}\circ V)]^{-1}$ and $\Omega^q(\tau) \equiv [I-\tilde{\alpha}(\Delta^{Xq}\circ V)]^{-1}$. Note that $\Omega^p(\mathbf{0}) = \Omega^q(\mathbf{0}) = \Omega$.

For ease of exposition, we derive the expressions in a sequence of 13 steps. To simplify the expressions, we will use the notation f_{τ_i} to denote the partial derivative of f with respect to τ_i at point τ for any real-valued or matrix function f. Similarly we will use the notation $f_{\tau_i\tau_j}$ to denote the partial derivative of f_{τ_i} with respect to τ_j.

1. For any distortions, τ_i, τ_j, we have that:

$$N^p_{\tau_i} = \frac{1-\sigma}{\sigma}\sum_k \beta_k\frac{\hat{\lambda}_{k\tau_i}}{\hat{\lambda}_k}$$

and

$$N^p_{\tau_i\tau_j} = \frac{1-\sigma}{\sigma}\sum_k \beta_k\left[\frac{\hat{\lambda}_{k\tau_i\tau_j}}{\hat{\lambda}_k} - \frac{\hat{\lambda}_{k\tau_i}\hat{\lambda}_{k\tau_j}}{\hat{\lambda}_k^2}\right]$$

Proof This follows from the definition of N^p.

2. Denote $G = \hat{c}'\Omega^q\Delta^{Lq}(\mathbf{1}-\alpha)$. For any distortions τ_i, τ_j:

$$N^q_{\tau_i} = -\frac{\hat{c}'_{\tau_i}\Omega^q\Delta^{Lq} + \hat{c}'\Omega^q_{\tau_i}\Delta^{Lq} + \hat{c}'\Omega^q\Delta^{Lq}_{\tau_i}}{G}(\mathbf{1}-\alpha)$$

$$\begin{aligned} N^q_{\tau_i\tau_j} &= -\frac{\hat{c}'_{\tau_i\tau_j}\Omega^q\Delta^{Lq} + \hat{c}'\Omega^q_{\tau_i\tau_j}\Delta^{Lq} + \hat{c}'\Omega^q\Delta^{Lq}_{\tau_i\tau_j}}{G}(\mathbf{1}-\alpha) \\ &\quad -\frac{\hat{c}'_{\tau_i}\Omega^q_{\tau_j}\Delta^{Lq} + \hat{c}'_{\tau_i}\Omega^q\Delta^{Lq}_{\tau_j} + \hat{c}'_{\tau_j}\Omega^q_{\tau_i}\Delta^{Lq}}{G}(\mathbf{1}-\alpha) \\ &\quad -\frac{\hat{c}'\Omega^q_{\tau_i}\Delta^{Lq}_{\tau_j} + \hat{c}'_{\tau_j}\Omega^q\Delta^{Lq}_{\tau_i} + \hat{c}'\Omega^q_{\tau_j}\Delta^{Lq}_{\tau_i}}{G}(\mathbf{1}-\alpha) + N^q_{\tau_i}N^q_{\tau_j} \end{aligned}$$

Proof This follows from the fact

$$N^q_{\tau_i} = -\frac{G_{\tau_i}}{G}$$

$$N^q_{\tau_i\tau_j} = -\frac{G_{\tau_i\tau_j}}{G} + \frac{G_{\tau_i}G_{\tau_j}}{G^2}.$$

3. Denote $Z^i = \tilde{\alpha}I^iV$, where I^i is a square matrix with $I^i_{ii} = 1$ and 0 elsewhere. Then

$$\Omega^p_{\tau_{Xi}} = -\frac{\sigma}{1-\sigma}(1+\tau_{Xi})^{-\sigma/(1-\sigma)-1}\Omega^pZ^i\Omega^p,$$

$$\Omega^p_{\tau_{Xi}\tau_{Xj}} = \left(\frac{\sigma}{1-\sigma}\right)^2[(1+\tau_{Xi})(1+\tau_{Xj})]^{-\frac{\sigma}{1-\sigma}-1}[\Omega^pZ^i\Omega^pZ^j\Omega^p + \Omega^pZ^j\Omega^pZ^i\Omega^p] \text{ if i} \neq \text{j}.$$

$$\Omega^p_{\tau_{Xi}\tau_{Xi}} = \frac{\sigma}{(1-\sigma)^2}(1+\tau_{Xi})^{-\frac{\sigma}{1-\sigma}-2}\Omega^pZ^i\Omega^p + 2\left(\frac{\sigma}{1-\sigma}\right)^2(1+\tau_{Xi})^{-\frac{2\sigma}{1-\sigma}-2}\Omega^pZ^i\Omega^pZ^i\Omega^p.$$

Similarly,

$$\Omega^q_{\tau_{Xi}} = -\frac{1}{1-\sigma}(1+\tau_{Xi})^{-1/(1-\sigma)-1}\Omega^qZ^i\Omega^q,$$

$$\Omega^q_{\tau_{Xi}\tau_{Xj}} = \left(\frac{1}{1-\sigma}\right)^2[(1+\tau_{Xi})(1+\tau_{Xj})]^{-\frac{1}{1-\sigma}-1}[\Omega^qZ^i\Omega^qZ^j\Omega^q + \Omega^qZ^j\Omega^qZ^i\Omega^q] \text{ if i} \neq \text{j}.$$

$$\Omega^q_{\tau_{Xi}\tau_{Xi}} = \frac{2-\sigma}{(1-\sigma)^2}(1+\tau_{Xi})^{-\frac{\sigma}{1-\sigma}-2}\Omega^qZ^i\Omega^q + 2\left(\frac{1}{1-\sigma}\right)^2(1+\tau_{Xi})^{-\frac{2}{1-\sigma}-2}\Omega^qZ^i\Omega^qZ^i\Omega^q.$$

Proof. Let $D(x)$ be a square matrix that depends on the parameter x and is differentiable in x. Then standard linear algebra implies that on the interior of the set where $D(x)$ is invertible, we have that

$$\frac{d}{dx}D^{-1}(x) = -D^{-1}(x)\frac{d}{dx}D(x)D^{-1}(x).$$

This fact and the definitions of Ω^p, Ω^q implies the results.

4. Let $(\mathbf{1}-\alpha)^i \equiv I^i(\mathbf{1}-\alpha)$, which is a column vector that is zero everywhere, except for the i-th row which is $1-\alpha_i$. We have

$$\hat{\lambda}_{\tau_{Xi}} = -\frac{\sigma}{1-\sigma}(1+\tau_{Xi})^{-\frac{\sigma}{1-\sigma}-1}\Omega^p Z^i \Omega^p \Delta^{Lp}(\mathbf{1}-\alpha)$$

$$\hat{\lambda}_{\tau_{Lj}} = -\frac{\sigma}{1-\sigma}(1+\tau_{Lj})^{-\frac{\sigma}{1-\sigma}-1}\Omega^p(\mathbf{1}-\alpha)^j,$$

$$\hat{\lambda}_{\tau_{Lj}\tau_s^L} = \mathbf{0}, \text{ if } s \neq j$$

$$\hat{\lambda}_{\tau_{Lj}\tau_{Lj}} = \frac{\sigma}{(1-\sigma)^2}(1+\tau_{Lj})^{-\frac{\sigma}{1-\sigma}-2}\Omega^p(\mathbf{1}-\alpha)^j,$$

$$\hat{\lambda}_{\tau_{Xi}\tau_{Lj}} = \left(\frac{\sigma}{1-\sigma}\right)^2(1+\tau_{Lj})^{-\frac{\sigma}{1-\sigma}-1}(1+\tau_{Xi})^{-\frac{\sigma}{1-\sigma}-1}\Omega^p Z^i \Omega^p(\mathbf{1}-\alpha)^j.$$

$$\hat{\lambda}_{\tau_{Xi}\tau_{Xj}} = \left(\frac{\sigma}{1-\sigma}\right)^2[(1+\tau_{Xj})(1+\tau_{Xi})]^{-\frac{\sigma}{1-\sigma}-1}[\Omega^p Z^i \Omega^p Z^j + \Omega^p Z^j \Omega^p Z^i]\Omega^p \Delta^{Lp}(\mathbf{1}-\alpha), \text{ if } i \neq j$$

$$\begin{aligned}\hat{\lambda}_{\tau_{Xi}\tau_{Xi}} &= \frac{\sigma}{(1-\sigma)^2}(1+\tau_{Xi})^{-\frac{\sigma}{1-\sigma}-2}\Omega^p Z^i \Omega^p \Delta^{Lp}(\mathbf{1}-\alpha) \\ &\quad +2\left(\frac{\sigma}{1-\sigma}\right)^2(1+\tau_{Xi})^{-\frac{2\sigma}{1-\sigma}-2}\Omega^p Z^i \Omega^p Z^i \Omega^p \Delta^{Lp}(\mathbf{1}-\alpha).\end{aligned}$$

Proof These expressions follow from the fact that $\hat{\lambda} = \Omega^p \Delta^{Lp}(\mathbf{1}-\alpha)$ and the derivatives of the matrix Ω^p derived in Step 3.

5. Define $C^i = \Omega Z^i \Omega(\mathbf{1}-\alpha), D^{ij} = \Omega Z^i \Omega Z^j \Omega(\mathbf{1}-\alpha), E^i = \Omega(\mathbf{1}-\alpha)^i, F^{ij} = \Omega Z^i \Omega(\mathbf{1}-\alpha)^j$. (Note that they are all column vectors.) Then:

$$\hat{\lambda}_{\tau_{Xi}}(\mathbf{0}) = -\frac{\sigma}{1-\sigma}C^i$$

$$\hat{\lambda}_{\tau_{Lj}}(\mathbf{0}) = -\frac{\sigma}{1-\sigma}E^j,$$

$$\hat{\lambda}_{\tau_{Lj}\tau_s^L}(\mathbf{0}) = \mathbf{0}, \text{ if } s \neq j$$

$$\hat{\lambda}_{\tau_{Lj}\tau_{Lj}}(\mathbf{0}) = \frac{\sigma}{(1-\sigma)^2}E^j,$$

$$\hat{\lambda}_{\tau_{Xi}\tau_{Lj}}(\mathbf{0}) = \left(\frac{\sigma}{1-\sigma}\right)^2 F^{ij}.$$

$$\hat{\lambda}_{\tau_{Xi}\tau_{Xj}}(\mathbf{0}) = \left(\frac{\sigma}{1-\sigma}\right)^2[D^{ij}+D^{ji}] \text{ if } i \neq j$$

$$\hat{\lambda}_{\tau_{Xi}\tau_{Xi}}(\mathbf{0}) = \frac{\sigma}{(1-\sigma)^2}C^i + 2\left(\frac{\sigma}{1-\sigma}\right)^2 D^{ii}.$$

Proof This follows from the fact that for $\tau = \mathbf{0}$, $\Delta^{Lp} = I$, $\Omega^p = \Omega$ and the expressions found in Step 4.

6.
$$\hat{c}_{\tau_{Xi}}(\mathbf{0}) = \frac{\sigma}{1-\sigma}\beta \circ C^i$$
$$\hat{c}_{\tau_{Lj}}(\mathbf{0}) = \frac{\sigma}{1-\sigma}\beta \circ E^j,$$
$$\hat{c}_{\tau_{Li}\tau_{Lj}}(\mathbf{0}) = 2\left(\frac{\sigma}{1-\sigma}\right)^2 \beta \circ E^i \circ E^j, \text{ if } i \neq j$$
$$\hat{c}_{\tau_{Lj}\tau_{Lj}}(\mathbf{0}) = -\frac{\sigma}{(1-\sigma)^2}\beta \circ E^j + 2\left(\frac{\sigma}{1-\sigma}\right)^2 \beta \circ E^j \circ E^j$$
$$\hat{c}_{\tau_{Xi}\tau_{Lj}}(\mathbf{0}) = -\left(\frac{\sigma}{1-\sigma}\right)^2 \beta \circ F^{ij} + 2\left(\frac{\sigma}{1-\sigma}\right)^2 \beta \circ C^i \circ E^j$$
$$\hat{c}_{\tau_{Xi}\tau_{Xj}}(\mathbf{0}) = -\left(\frac{\sigma}{1-\sigma}\right)^2 \beta \circ [D^{ij} + D^{ji}] + 2\left(\frac{\sigma}{1-\sigma}\right)^2 \beta \circ C^i \circ C^j \text{ if } i \neq j$$
$$\hat{c}_{\tau_{Xi}\tau_{Xi}}(\mathbf{0}) = -\frac{\sigma}{(1-\sigma)^2}\beta \circ C^i - 2\left(\frac{\sigma}{1-\sigma}\right)^2 \beta \circ D^{ii} + 2\left(\frac{\sigma}{1-\sigma}\right)^2 \beta \circ C^i \circ C^i.$$

Proof By definition, $\hat{c}_k = \beta_k/\hat{\lambda}_k$. Then $\hat{c}_{k,\tau_i} = -\beta_k\hat{\lambda}_{k,\tau_i}/\hat{\lambda}_k^2$; $\hat{c}_{k,\tau_i\tau_j} = -\beta_k\hat{\lambda}_{k,\tau_i\tau_j}/\hat{\lambda}_k^2 + 2\beta_k\hat{\lambda}_{k,\tau_i}\hat{\lambda}_{k,\tau_j}/\hat{\lambda}_k^3$. Then the fact that $\hat{\lambda}(\mathbf{0}) = \mathbf{1}$ and the expressions in Step 5 imply the result.

7.
$$\Delta^{Lq}_{\tau_{Li}}(\mathbf{0}) = -\frac{1}{1-\sigma}I^i$$
$$\Delta^{Lq}_{\tau_{Li}\tau_{Lj}}(\mathbf{0}) = \mathbf{0}, \text{ if } i \neq j$$
$$\Delta^{Lq}_{\tau_{Li}\tau_{Li}}(\mathbf{0}) = \frac{2-\sigma}{(1-\sigma)^2}I^i$$

and all derivatives involving τ_{Xi} are zero matrices.

Proof Follows directly from the definition.

8. $C^i_k = \alpha_i\omega_{ki}$; $D^{ij}_k = \omega_{ki}\sum_s v_{is}\omega_{sj}\alpha_i\alpha_j$; $E^i_k = (1-\alpha_i)\omega_{ki}$; $F^{ij}_k = \omega_{ki}\sum_s v_{is}\omega_{sj}\alpha_i(1-\alpha_j)$

   Proof This follows from the definition of these vectors and the facts that $\Omega(\mathbf{1}-\alpha) = \mathbf{1}, V\mathbf{1} = \mathbf{1}$.

9. We can now derive the first expression of Lemma B.4 $\frac{d}{d\tau_{Li}\tau_{Lj}}\log Y(\mathbf{0}),\ i \neq j$.

   From Step 1, 5 and 8, we have

$$N^p_{\tau_{Li}\tau_{Lj}}(\mathbf{0}) = -\frac{\sigma}{1-\sigma}\sum_k \beta_k E^j_k E^i_k = -\frac{\sigma}{1-\sigma}(1-\alpha_i)(1-\alpha_j)s_{ij} \text{for } i \neq j$$

   Since $\Omega^q(\mathbf{0}) = \Omega, G(\mathbf{0}) = 1, \Omega(\mathbf{1}-\alpha) = \mathbf{1}$ and some of the terms in the expression for $\frac{d^2}{d\tau_i d\tau_j}N$

are zero matrices, Step 2 implies

$$\begin{aligned} N^q_{\tau_{Li}\tau_{Lj}}(\mathbf{0}) &= -\left(\hat{c}'_{\tau_{Li}\tau_{Lj}}\Omega + \hat{c}'_{\tau_{Li}}\Omega\Delta^{Lq}_{\tau_{Lj}} + \hat{c}'_{\tau_{Lj}}\Omega\Delta^{Lq}_{\tau_{Li}}\right)(\mathbf{1}-\alpha) \\ &\quad + \left[\left(\hat{c}'_{\tau_{Li}}\Omega + \hat{c}'\Omega\Delta^{Lq}_{\tau_{Li}}\right)(\mathbf{1}-\alpha)\right]\left[\left(\hat{c}'_{\tau_{Lj}}\Omega + \hat{c}'\Omega\Delta^{Lq}_{\tau_{Lj}}\right)(\mathbf{1}-\alpha)\right] \text{ for } i \neq j \end{aligned}$$

Then substituting the various expressions, we get

$$\begin{aligned} N^q_{\tau_{Li}\tau_{Lj}}(\mathbf{0}) &= -2\left(\frac{\sigma}{1-\sigma}\right)^2 \sum_k \beta_k E^i_k E^j_k + 2\frac{\sigma}{(1-\sigma)^2}\sum_k \beta_k E^i_k E^j_k \\ &\quad + \left(\sum_k \beta_k E^i_k\right)\left(\sum_k \beta_k E^j_k\right) \\ &= \frac{2\sigma}{1-\sigma} s_{ij}(1-\alpha_i)(1-\alpha_j) + s_i s_j (1-\alpha_i)(1-\alpha_j) \text{for } i \neq j \end{aligned}$$

Then summing the derivatives of N^p and N^q, we get that

$$\frac{d^2}{d\tau_{Li}\tau_{Lj}} \log Y(\mathbf{0}) = \left(\frac{\sigma}{1-\sigma} s_{ij} + s_i s_j\right)(1-\alpha_i)(1-\alpha_j) \text{for } i \neq j$$

This proves the first statement of the lemma.

10. Similarly, we can derive the second expression of Lemma B.4 $\frac{d}{d\tau_{Li}\tau_{Li}} \log Y(\mathbf{0})$.

$$N^p_{\tau_{Li}\tau_{Li}} = \frac{1}{1-\sigma}\sum_k \beta_k E^i_k - \frac{\sigma}{1-\sigma}\sum_k \beta_k E^i_k E^i_k = \frac{1}{1-\sigma} s_i(1-\alpha_i) - \frac{\sigma}{1-\sigma}(1-\alpha_i)^2 s_{ii}$$

$$\begin{aligned} N^q_{\tau_{Li}\tau_{Li}}(\mathbf{0}) &= -\left(\hat{c}'_{\tau_{Li}\tau_{Li}}\Omega + 2\hat{c}'_{\tau_{Li}}\Omega\Delta^{Lq}_{\tau_{Li}} + \hat{c}'\Omega\Delta^{Lq}_{\tau^L_i\tau^L_i}\right)(\mathbf{1}-\alpha) + \left[\left(\hat{c}'_{\tau_{Li}}\Omega + \hat{c}'\Omega\Delta^{Lq}_{\tau_{Li}}\right)(\mathbf{1}-\alpha)\right]^2 \\ &= \frac{\sigma}{(1-\sigma)^2}\sum_k \beta_k E^i_k - 2\left(\frac{\sigma}{1-\sigma}\right)^2 \sum_k \beta_k E^i_k E^i_k + 2\frac{\sigma}{(1-\sigma)^2}\sum_k \beta_k E^i_k E^i_k \\ &\quad - \frac{2-\sigma}{(1-\sigma)^2}\sum_k \beta_k E^i_k + s_i^2(1-\alpha_i)^2 \\ &= \frac{2\sigma}{1-\sigma} s_{ii}(1-\alpha_i)^2 - \frac{2}{1-\sigma} s_i(1-\alpha_i) + s_i^2(1-\alpha_i)^2 \end{aligned}$$

Then summing the derivatives of N^p and N^q, we get that

$$\frac{d^2}{d\tau_{Li}\tau_{Li}} \log Y(\mathbf{0}) = \left(\frac{\sigma}{1-\sigma} s_{ii} + s_i^2\right)(1-\alpha_i)^2 - \frac{1}{1-\sigma} s_i(1-\alpha_i)$$

This proves the second statement of the lemma.

11. Find $\frac{d^2}{d\tau_{Xi} d\tau_{Lj}} \log Y(\mathbf{0})$.

In the same way as Step 9, we see that

$$N^p_{\tau_{Xi}\tau_{Lj}}(\mathbf{0}) = \frac{\sigma}{1-\sigma}\sum_k \beta_k F_k^{ij} - \frac{\sigma}{1-\sigma}\sum_k \beta_k C_k^i E_k^j = \frac{\sigma}{1-\sigma}\alpha_i(1-\alpha_j)[s_i \sum_s v_{is}\omega_{sj} - s_{ij}]$$

Taking into account the zero terms,

$$\begin{aligned}
N^q_{\tau_{Xi}\tau_{Lj}}(\mathbf{0}) &= -\left[\hat{c}'_{\tau_{Xi}\tau_{Lj}}\Omega + \hat{c}'_{\tau_{Xi}}\Omega\Delta^{Lq}_{\tau_{Lj}} + \hat{c}'_{\tau_{Lj}}\Omega_{q,\tau_{Xi}} + \hat{c}'\Omega_{q,\tau_{Xi}}\Delta^{Lq}_{\tau_{Lj}}\right](\mathbf{1}-\alpha) \\
&\quad + \left[\hat{c}'_{\tau_{Xi}}\Omega + \hat{c}'\Omega_{q,\tau_{Xi}}\right](\mathbf{1}-\alpha)\left[\hat{c}'_{\tau_{Lj}}\Omega + \hat{c}'\Omega\Delta^{Lq}_{\tau_{Lj}}\right](\mathbf{1}-\alpha) \\
&= \left(\frac{\sigma}{1-\sigma}\right)^2 \sum_k \beta_k F_k^{ij} - 2\left(\frac{\sigma}{1-\sigma}\right)^2 \sum_k \beta_k C_k^i E_k^j \\
&\quad + \frac{\sigma}{(1-\sigma)^2}\sum_k \beta_k C_k^i E_k^j + \frac{\sigma}{(1-\sigma)^2}\sum_k \beta_k C_k^i E_k^j \\
&\quad - \frac{1}{(1-\sigma)^2}\sum_k \beta_k F_k^{ij} + \left(\sum_k \beta_k C_k^i\right)\left(\sum_k \beta_k E_k^j\right) \\
&= \left\{-\frac{1+\sigma}{1-\sigma}s_i\left(\sum_s v_{is}\omega_{sj}\right) + \frac{2\sigma}{1-\sigma}s_{ij} + s_i s_j\right\}\alpha_i(1-\alpha_j)
\end{aligned}$$

Then summing the derivatives of N^p and N^q, we get that

$$\frac{d^2}{d\tau_{Xi}\tau_{Lj}}\log Y(\mathbf{0}) = \left[\frac{\sigma}{1-\sigma}s_{ij} + s_i s_j - \frac{1}{1-\sigma}s_i\left(\sum_s v_{is}\omega_{sj}\right)\right]\alpha_i(1-\alpha_j)$$

This proves the third statement of the lemma.

12. Find $\frac{d^2}{d\tau_{Xi}\tau_{Xj}}\log Y(\mathbf{0})$ for $i \neq j$. In the same way as Step 9, we see that

$$\begin{aligned}
N^p_{\tau_{Xi}\tau_{Xj}}(\mathbf{0}) &= \frac{\sigma}{1-\sigma}\sum_k \beta_k[D_k^{ij} + D_k^{ji}] - \frac{\sigma}{1-\sigma}\sum_k \beta_k C_k^i C_k^j \\
&= \frac{\sigma}{1-\sigma}\left(s_i \sum_s v_{is}\omega_{sj} + s_j \sum_s v_{js}\omega_{si} - s_{ij}\right)\alpha_i\alpha_j
\end{aligned}$$

Taking into account the zero terms,

$$\begin{aligned}
N^q_{\tau_{Xi}\tau_{Xj}}(\mathbf{0}) &= -\left[\hat{c}'_{\tau_{Xi}\tau_{Xj}}\Omega + \hat{c}'\Omega_{q,\tau_{Xi}\tau_{Xj}} + \hat{c}'_{\tau_{Xi}}\Omega_{q,\tau_{Xj}} + \hat{c}'_{\tau_{Xj}}\Omega_{q,\tau_{Xi}}\right](\mathbf{1}-\alpha) \\
&\quad + \left(\hat{c}'_{\tau_{Xi}}\Omega + \hat{c}'\Omega_{q,\tau_{Xi}}\right)(\mathbf{1}-\alpha)\left(\hat{c}'_{\tau_{Xj}}\Omega + \hat{c}'\Omega_{q,\tau_{Xj}}\right)(\mathbf{1}-\alpha) \\
&= \left(\frac{\sigma}{1-\sigma}\right)^2 \sum_k \beta_k [D_k^{ij} + D_k^{ji}] - 2\left(\frac{\sigma}{1-\sigma}\right)^2 \sum_k \beta_k C_k^i C_k^j \\
&\quad - \left(\frac{1}{1-\sigma}\right)^2 \sum_k \beta_k [D_k^{ij} + D_k^{ji}] + \frac{\sigma}{(1-\sigma)^2}\sum_k \beta_k C_k^i C_k^j + \frac{\sigma}{(1-\sigma)^2}\sum_k \beta_k C_k^i C_k^j \\
&\quad + \left(\sum_k \beta_k C_k^i\right)\left(\sum_k \beta_k C_k^j\right) \\
&= \left[-\frac{1+\sigma}{1-\sigma}\left(s_i \sum_s v_{is}\omega_{sj} + s_j \sum_s v_{js}\omega_{si}\right) + \frac{2\sigma}{1-\sigma}s_{ij} + s_i s_j\right]\alpha_i\alpha_j
\end{aligned}$$

Then summing the derivatives of N^p and N^q, we get that

$$\frac{d^2}{d\tau_{Xi}\tau_{Xj}}\log Y(\mathbf{0}) = \left[\frac{\sigma}{1-\sigma}s_{ij} + s_i s_j - \frac{1}{1-\sigma}\left(s_i \sum_s v_{is}\omega_{sj} + s_j \sum_s v_{js}\omega_{si}\right)\right]\alpha_i\alpha_j$$

This proves the fourth statement of the lemma.

13. Find $\frac{d^2}{d\tau_{Xi}d\tau_{Xi}}\log Y(\mathbf{0})$.

In the same way as Step 9, we see that

$$\begin{aligned}
N^p_{\tau_{Xi}\tau_{Xi}}(\mathbf{0}) &= \frac{1}{1-\sigma}\sum_k \beta_k C_k^i + \frac{2\sigma}{1-\sigma}\sum_k \beta_k D_k^{ii} - \frac{\sigma}{1-\sigma}\sum_k \beta_k C_k^i C_k^i \\
&= \frac{\sigma}{1-\sigma}\left(2s_i \sum_s v_{is}\omega_{si} - s_{ii}\right)\alpha_i^2 + \frac{1}{1-\sigma}\alpha_i s_i
\end{aligned}$$

Taking into account the zero terms,

$$\begin{aligned}
N^q_{\tau_{Xi}\tau_{Xi}}(\mathbf{0}) &= -\left[\hat{c}'_{\tau_{Xi}\tau_{Xi}}\Omega + \hat{c}'\Omega_{q,\tau_{Xi}\tau_{Xi}} + 2\hat{c}'_{\tau_{Xi}}\Omega_{q,\tau_{Xi}}\right](\mathbf{1}-\alpha) + \left[\left(\hat{c}'_{\tau_{Xi}}\Omega + \hat{c}'\Omega_{q,\tau_{Xi}}\right)(\mathbf{1}-\alpha)\right]^2 \\
&= \frac{\sigma}{(1-\sigma)^2}\sum_k \beta_k C_k^i + 2\left(\frac{\sigma}{1-\sigma}\right)^2 \sum_k \beta_k D_k^{ii} - 2\left(\frac{\sigma}{1-\sigma}\right)^2 \sum_k \beta_k C_k^i C_k^i \\
&\quad -2\left(\frac{1}{1-\sigma}\right)^2 \sum_k \beta_k D_k^{ii} - \frac{2-\sigma}{(1-\sigma)^2}\sum_k \beta_k C_k^i \\
&\quad + \frac{2\sigma}{(1-\sigma)^2}\sum_k \beta_k C_k^i C_k^i + \left(\sum_k \beta_k C_k^i\right)^2 \\
&= \left[-2\frac{1+\sigma}{1-\sigma}s_i\left(\sum_s v_{is}\omega_{si}\right) + \frac{2\sigma}{1-\sigma}s_{ii} + s_i^2\right]\alpha_i^2 - \frac{2}{1-\sigma}s_i\alpha_i
\end{aligned}$$

Then summing the derivatives of N^p and N^q, we get that

$$\frac{d^2}{d\tau_{Xi}\tau_{Xi}}\log Y(\mathbf{0}) = \left[\frac{\sigma}{1-\sigma}s_{ii} + s_i^2 - \frac{2}{1-\sigma}s_i\left(\sum_s v_{is}\omega_{si}\right)\right]\alpha_i^2 - \frac{1}{1-\sigma}s_i\alpha_i$$

This concludes the proof. ■

Lemma B.5 *Suppose* $\max\{\alpha_i\} = \bar{\alpha} < 1$. *Let z be some $n \times 1$ vector such that $\mathbf{0} \leq z \leq \mathbf{1}$. Let $a = \Omega z$ and $c = \Omega\Omega z$. Then, $2c_i - a_i - a_i^2 \geq 0$.*

Proof of Lemma B.5. Let $\bar{V} = \tilde{\alpha}V$. Using $\Omega = (I + \bar{V} + \bar{V}^2 + ...)$, we can write

$$\begin{aligned} a &= \left(\sum_{\ell=0}^{\infty}\bar{V}^\ell\right)z \\ a_i &= \sum_{\ell=0}^{\infty}\sum_{k=1}^{n}\bar{V}_{ik}^\ell z_k = \sum_{\ell=0}^{\infty}\hat{a}_i^\ell, \end{aligned}$$

where $\hat{a}_i^\ell \equiv \sum_{k=1}^n \bar{V}_{ik}^\ell z_k$.

$$\begin{aligned} c &= \left(\sum_{\ell=0}^{\infty}\bar{V}^\ell\right)^2 z = \left(\sum_{\ell=1}^{\infty}\ell\bar{V}^{\ell-1}\right)z \\ c_i &= \sum_{\ell=1}^{\infty}\ell\hat{a}_i^{\ell-1} \end{aligned}$$

We then have

$$\begin{aligned} 2c_i - a_i - a_i^2 &= 2\sum_{\ell=1}^{\infty}\ell\hat{a}_i^{\ell-1} - \sum_{\ell=0}^{\infty}\hat{a}_i^\ell - \left(\sum_{\ell=0}^{\infty}\hat{a}_i^\ell\right)^2 \\ &= 2\sum_{\ell=0}^{\infty}(\ell+1)\hat{a}_i^\ell - \sum_{\ell=0}^{\infty}\hat{a}_i^\ell - \sum_{\ell=0}^{\infty}(\hat{a}_i^\ell)^2 - 2\sum_{\ell=1}^{\infty}\sum_{s=0}^{\ell-1}\hat{a}_i^\ell a_i^s \\ &= 2\sum_{\ell=1}^{\infty}\ell\hat{a}_i^\ell + \sum_{\ell=0}^{\infty}\hat{a}_i^\ell - \sum_{\ell=0}^{\infty}(\hat{a}_i^\ell)^2 - 2\sum_{\ell=1}^{\infty}\sum_{s=0}^{\ell-1}\hat{a}_i^\ell\hat{a}_i^s \end{aligned}$$

Given the equation above, to establish the inequality it will be sufficient to show that $0 \leq \hat{a}_i^\ell < 1$, if $\ell \geq 1$. We will show by induction that $0 \leq \hat{a}_i^\ell \leq \bar{\alpha}^\ell < 1$. From the definition, $\hat{a}_i^0 = z_i \in [0, \bar{\alpha}^0]$. Then $\hat{a}^\ell = \bar{V}\hat{a}^{\ell-1} \leq \bar{V}\bar{\alpha}^{\ell-1}\mathbf{1} \leq \bar{\alpha}^\ell\mathbf{1}$. (We used the fact that $\bar{V}\mathbf{1} \leq \alpha\mathbf{1}$.) Showing that $\hat{a}^\ell \geq \mathbf{0}$ is similar. ■

Lemma B.6 *Suppose that the hypothesis of lemma B.5 holds, $z = \alpha$, all sectors are nontrivial and there are positive input-output linkages in the economy. Then for some i, $\beta_i[2c_i - a_i - a_i^2] > 0$.*

Proof of Lemma B.6. We will use the same construction as the proof of lemma B.5. Let i be such that $\alpha_i > 0$. Since $\hat{a}_i^0 = \alpha_i \in (0,1)$, $\hat{a}_i^0 - (\hat{a}_i^0)^2 > 0$, so $2c_i - a_i - a_i^2 > 0$. Then if $\beta_i > 0$, we are done. On the other hand, suppose that $\beta_i = 0$. Since $s_i > 0$, there is some $k \neq i$ such that $\omega_{ki} > 0$ and $\beta_k > 0$. If $\alpha_k = 0$, then $\omega_{ki} = 0$ for all $i \neq k$, so $\alpha_k > 0$. Then by the same logic as above, k satisfies the conclusion of the lemma. ■

Lemma B.7 *Suppose that the hypothesis of lemma B.5 holds,* $\mathbf{0} < z \leq \mathbf{1}$, *all sectors are nontrivial and there are positive input-output linkages in the economy. Then for some* i, $\beta_i[2c_i - a_i - a_i^2] > 0$.

Proof of Lemma B.7. Using the same construction as the proof of lemma B.5, it is sufficient to show that for some i it holds that $\beta_i > 0$ and $\hat{a}_i^\ell > 0$, $\ell \geq 1$.
Since there are positive IO linkages, $\bar{V}_{ij} > 0$ for some i, j. Note that $\hat{a}_i^1 \geq \bar{V}_{ij} z_j > 0$, so if $\beta_i > 0$, we are done. Suppose not. Since $s_i > 0$, there exists some $k \neq i$ with $\beta_k > 0$ and $\omega_{ki} > 0$. We have that $\omega_{ki} = \sum_{\ell=1}^{\infty} \bar{V}_{ki}^\ell$, so $\bar{V}_{ki}^\ell > 0$ for some ℓ. Then now $\hat{a}_k^\ell \geq \bar{V}_{ki}^\ell z_i > 0$, so we are done. ■

Lemma B.8 *The square symmetric matrix* $(s_{ij} r_i r_j - s_i s_j r_i r_j)_{i,j}$, *where* $r_i \neq 0$ *are arbitrary, is positive semi-definite.*

Proof of Lemma B.8. Let ξ^i be a random variable that takes the values $\omega_{ki} r_i$, $k = 1, \ldots n$ with probabilities β_k. Then $s_{ij} r_i r_j = E\{\xi^i \xi^j\}$, $s_i r_i = E\xi^i$ and $s_{ij} r_i r_j - s_i s_j r_i r_j = E\left\{(\xi^i - E\xi^i)(\xi^j - E\xi^j)\right\} = cov(\xi^i, \xi^j)$. Let $d_{ij} = (s_{ij} - s_i s_j) r_i r_j$ and $D = (d_{ij})$. Then D is a variance-covariance matrix and hence positive semi-definite. ■

B.3 Proofs of main results

Proof of Proposition 5. From lemma B.4 and the fact that $Y = 1$ when there are no distortions, a Taylor expansion around the non distorted allocation gives

$$\log TFP \approx \frac{1}{2}\left(\sum_i s_i(1-\alpha_i)\tau_{Li}\right)^2 + \frac{1}{2}\frac{\sigma}{1-\sigma}\sum_i\sum_j s_{ij}(1-\alpha_i)(1-\alpha_j)\tau_{Li}\tau_{jL} - \frac{1}{2}\frac{1}{1-\sigma}\sum_i s_i(1-\alpha_i)\tau_{Li}^2,$$

All the first-order terms are zero. ■

Proof of Proposition 6. Since $\tau_{Xi} = \tau_{Li} = \tau_i$, we have that for any i, j (including $i = j$)

$$\frac{d^2}{d\tau_i d\tau_j}\log TFP = \frac{d^2}{d\tau_{Xi} d\tau_{Xj}}\log TFP + \frac{d^2}{d\tau_{Xi} d\tau_{Lj}}\log TFP + \frac{d^2}{d\tau_{Li} d\tau_{Xj}}\log TFP + \frac{d^2}{d\tau_{Li} d\tau_{Lj}}\log TFP.$$

Then applying lemma B.4 and grouping terms, we obtain for $i \neq j$:

$$\begin{aligned}\frac{d^2}{d\tau_i d\tau_j}\log TFP(\mathbf{0}) &= \frac{\sigma}{1-\sigma}s_{ij} + s_i s_j - \frac{1}{1-\sigma}[s_i\alpha_i g_{ij} + s_j \alpha_j g_{ji}] \\ &= \frac{\sigma}{1-\sigma}s_{ij} + s_i s_j - \frac{1}{1-\sigma}[s_i\omega_{ij} + s_j\omega_{ji}],\end{aligned}$$

where we used lemma B.3 in the second line.

Similarly, if $i = j$

$$\begin{aligned}\frac{d^2}{d\tau_i d\tau_i}\log TFP(\mathbf{0}) &= \frac{\sigma}{1-\sigma}s_{ii} + s_i^2 - 2\frac{1}{1-\sigma}s_i\alpha_i g_{ii} - \frac{1}{1-\sigma}s_i \\ &= \frac{\sigma}{1-\sigma}s_{ii} + s_i^2 - 2\frac{1}{1-\sigma}s_i\omega_{ii} + \frac{1}{1-\sigma}s_i,\end{aligned}$$

Then the result follows from Taylor's expansion around $\tau_i = 0, \forall i$. ■

Proof of Result 1. Propositions 5 and 6 imply that we can represent the Hessian (in both cases) as

$$\frac{d^2}{d\tau d\tau}\log TFP(\mathbf{0}) = -M - \frac{1}{1-\sigma}N,$$

where M and N are some matrices that are functions of the undistorted allocation, but not σ.. Proving the claim is equivalent to showing that N is positive semi-definite.

Suppose that N is not positive semi-definite, so for some vector τ we have $\tau' N\tau < 0$. Then for some σ large enough, $\frac{1}{2}\tau'\frac{d^2}{d\tau d\tau}\log TFP(\mathbf{0})\tau = -\frac{1}{2}\tau' M\tau - \frac{1}{2}\frac{1}{1-\sigma}\tau' N\tau > 0$. Since $\frac{d\log TFP}{d\tau} = \mathbf{0}$, from Taylor's theorem we know that

$$\log TFP(z\tau) = -z^2\frac{1}{2}\tau'\frac{d^2\log TFP}{d\tau d\tau}(\mathbf{0} + \xi(z)z\tau)\tau,$$

where z is a scalar and $\xi(z) \in [0,1]$. Since $\tau'\frac{d^2\log TFP}{d\tau d\tau}(\mathbf{0})\tau < 0$ continuity implies that for z small enough $\tau'\frac{d^2\log TFP}{d\tau d\tau}(\mathbf{0} + \xi(z)z\tau)\tau < 0$ and hence $\log TFP(z\tau) > 0$, which is a contradiction. Hence N is positive semi-definite.

The strict inequality in the case of random distortions follows directly from Proposition A.6.

Consider uniform distortions. Since uniform labor distortions have no effect on TFP, without loss of generality we can consider uniform markups. Using Proposition 6, we can write

$$\begin{aligned}\frac{d}{d\tau^2}\log TFP(\mathbf{0}) &= -\frac{1}{1-\sigma}\left[2\sum_i\sum_j s_i\omega_{ij}\alpha_j - \sum_i \alpha_i s_i\right] + \frac{\sigma}{1-\sigma}\sum_i\sum_j \alpha_i\alpha_j s_{ij} + \left(\sum_i \alpha_i s_i\right)^2 \\ \frac{d}{d\sigma}\left(\frac{d}{d\tau^2}\right)\log TFP(\mathbf{0}) &= -\frac{1}{(1-\sigma)^2}\left[2\sum_i\sum_j s_i\omega_{ij}\alpha_j - \sum_i\alpha_i s_i - \sum_i\sum_j\alpha_i\alpha_j s_{ij}\right]\end{aligned}$$

To show that $\frac{d}{d\sigma}\left(\frac{d}{d\tau^2}\right)\log TFP(\mathbf{0}) < 0$, let us rewrite it as

$$\frac{d}{d\sigma}\left(\frac{d}{d\tau^2}\right)\log TFP(\mathbf{0}) = -\frac{1}{(1-\sigma)^2}\left[2\sum_i\sum_k \beta_k\omega_{ki}\sum_j \omega_{ij}\alpha_j - \sum_i\sum_k \alpha_i\beta_k\omega_{ki} - \sum_i\sum_k \alpha_i\beta_k\omega_{ki}\sum_j \omega_{kj}\alpha_j\right]$$

$$\frac{d}{d\sigma}\left(\frac{d}{d\tau^2}\right)\log TFP(\mathbf{0}) = -\frac{1}{(1-\sigma)^2}\left[\sum_k \beta_k\left(2c_k - a_k - a_k^2\right)\right],$$

with $a_k = \sum_j \omega_{kj}\alpha_j$ and $c_k = \sum_i \omega_{ki}a_i$, that is $a = \Omega\alpha$ and $c = \Omega\Omega\alpha$.
Using Lemma B.7, we then have $\beta_k[2c_k - a_k - a_k^2] \geq 0$, with strict inequality for at least one k, which implies $\frac{d}{d\sigma}\left(\frac{d}{d\tau^2}\right)\log TFP(\mathbf{0}) > 0$ ■

Proof of Result 2. Consider the frictionless economy with $\sigma = \rho$, and $A = \mathbf{1}$. In that economy, aggregate productivity is

$$\log TFP = \frac{1-\sigma}{\sigma}\beta'\log\hat{\lambda} - \log(\hat{c}'\Omega\hat{B}(\mathbf{1}-\alpha)),$$

with

$$\hat{\lambda} = \Omega\hat{B}(\mathbf{1}-\alpha)$$

Using the definition of $\hat{c}$, we get that $\hat{c}'\Omega\hat{B}(\mathbf{1}-\alpha) = \sum_i \beta_i\hat{\lambda}_i^{-1}\hat{\lambda}_i = 1$. Therefore,

$$\log TFP = \frac{1-\sigma}{\sigma}\beta'\log\hat{\lambda},$$

which can be rewritten

$$\log TFP = \sum_k \beta_k \log\left(\hat{\lambda}_k^{\frac{1-\sigma}{\sigma}}\right),$$

with

$$\hat{\lambda}_k^{\frac{1-\sigma}{\sigma}} = \left(\sum_j (1-\alpha_j)\omega_{kj}B_j^{\frac{\sigma}{1-\sigma}}\right)^{\frac{1-\sigma}{\sigma}}$$

Since $\sum_j(1-\alpha_j)\omega_{kj} = 1$, the expression above is a power mean with parameter $\sigma/(1-\sigma)$. Since power means are increasing in the exponent and $\sigma/(1-\sigma)$ is increasing in σ, it follows that $\lambda_k^{\frac{1-\sigma}{\sigma}}$ is increasing in σ and hence Y is increasing in σ.
Since $s_k > 0$, there exists m such that $\beta_m\omega_{mk} > 0$. This implies that $\beta_m\omega_{mj_1} > 0, \beta_m\omega_{mj_2} > 0$. From the properties of generalized means, $\lambda_m^{\frac{1-\sigma}{\sigma}}$ is strictly increasing in σ and hence Y is also strictly increasing in σ. ■

Proof of Result 3. (i) If $v_{ii} = 1$, $v_{ij} = 0$ for $i \neq j$, direct inspection shows that $\omega_{ii} = \frac{1}{1-\alpha_i}$ and $\omega_{ij} = 0$ for $i \neq j$. Then $s_i = \beta_i/(1-\alpha_i)$, $s_{ii} = \beta_i/(1-\alpha_i)^2$, $s_{ij} = 0$ if $i \neq j$. So applying Proposition 5 immediately implies that $\log TFP = \log TFP|_{\alpha=0}$.
(ii) We use Proposition 5. In the proof of Result 1 we show that

$\sum_i s_i(1-\alpha_i)\tau_{Li}^2 - \sum_i \sum_j s_{ij}(1-\alpha_i)(1-\alpha_j)\tau_{Li}\tau_{Lj} \geq 0$. Then if $\sigma \leq 0$, the proposition implies that

$$\begin{aligned}\log TFP &\geq -\frac{1}{2}\left[\sum_i s_i(1-\alpha_i)\tau_{Li}^2 - \left(\sum_i s_i(1-\alpha_i)\tau_{Li}\right)^2\right] \\ &= -\frac{1}{2}\left[\sum_i \beta_i\tau_{Li}^2 - \left(\sum_i \beta_i\tau_{Li}\right)^2\right] = \log TFP|_{\alpha=0}\,.\end{aligned}$$

(iii) Proposition A.5 implies that for $\sigma < 0$, $\log TFP > -\frac{1}{2}\operatorname{var}\tau_L$. With uncorrelated distortions, $\sum_i \beta_i\tau_{Li}^2 - \left(\sum_i \beta_i\tau_{Li}\right)^2 \, plim \operatorname{var}\tau_L$. ■

Proof of Result 4.

Let $\widehat{}$ denote second order approximation around $\mathbf{0}$. Then Proposition 6 implies that there exists $\bar{\sigma}$ such that for all $\sigma < \bar{\sigma}$, $\widehat{\log TFP} > \widehat{\log TFP}|_{\alpha=0}$ if and only if

$$-\frac{1}{2}\left[\sum_i\sum_j s_{ij}\tau_i\tau_j - \left(\sum_i s_i\tau_i\right)^2\right] > -\frac{1}{2}\left[\sum_i \beta_i\tau_i^2 - \left(\sum_i \beta_i\tau_i\right)^2\right],$$

The term on the right-hand side is simply $-\frac{1}{2}\operatorname{var}_\beta \tau$. The definitions imply that

$$\begin{aligned}\sum_i\sum_j s_{ij}\tau_i\tau_j &= \sum_i\sum_j\sum_k \beta_k\omega_{ki}\omega_{kj}\tau_j\tau_i = \sum_k \beta_k(\sum_i \omega_{ki}\tau_i)^2 \\ \sum_i s_i\tau_i &= \sum_i\sum_k \beta_k\omega_{ki}\tau_i = \sum_k \beta_k\left(\sum_i \omega_{ki}\tau_i\right).\end{aligned}$$

Then the definitions of x_i, $\bar{\tau}_{suppliers}$ and the fact that $\omega_{kk} = 1 + x_{kk}$ imply the inequality above can be written as:

$$-\frac{1}{2}\left[\sum_i \beta_i\left(\tau_i + x_i\bar{\tau}^i_{suppliers}\right)^2 - \left(\sum_i \beta_i\left(\tau_i + x_i\bar{\tau}^i_{suppliers}\right)\right)^2\right] > -\frac{1}{2}\operatorname{var}_\beta(\tau),$$

which proves the condition $Cov_\beta(\tau_i, x_i\bar{\tau}_i^{suppliers}) < -(1/2)Var_\beta(x_i\bar{\tau}_i^{suppliers})$. Finally, since the Hessian is continuous, if the vector τ is not too large, $\widehat{\log TFP} > \widehat{\log TFP}|_{\alpha=0}$ implies that $\log TFP > \log TFP|_{\alpha=0}$. ■

B.4 Proofs of Additional Propositions

Proof of Proposition A.1. Define $\lambda_i' = \lambda_i(1+\tau_{Ci})$. Then the conditions can be rewritten as:

$$\frac{dY}{dC_i} = \lambda_i'$$

$$\lambda_i' \frac{dQ_i}{dX_{ij}} = \lambda_j' \frac{1+\tau_{Ci}}{1+\tau_{Cj}}(1+\tau_{Xij})$$

$$\lambda_i' \frac{dQ_i}{dL_i} = \eta(1+\tau_{Li})(1+\tau_{Ci}).$$

Define $\tau'_{Xij} = \frac{1+\tau_{Ci}}{1+\tau_{Cj}}(1+\tau_{Xij})-1$ and $\tau'_{Li} = (1+\tau_{Li})(1+\tau_{Ci})-1$. Then $\lambda', \tau'_{Xij}, \tau'_{Li}$ and the original allocation satisfy satisfies (6)- (8).

Next, we prove the second statement of the proposition. Define λ_i by (6). Define

$$1+\tau_{Xij} = \frac{dQ_i}{dX_{ij}} \frac{dY/dC_i}{dY/dC_j}$$

and

$$\frac{1+\tau_{Li}}{1+\tau_{1L}} = \frac{dY/dC_i \frac{dQ_i}{dL_i}}{dY/dC_1 \frac{dQ_1}{dL_1}}.$$

We can set τ_{i1} at an arbitrary value greater than -1. ■

Proof of Proposition A.2. With $\rho = \sigma = 0$, we get $m_{ij} = \alpha_i v_{ij} \lambda_i / ((1+\tau_{Xij})\lambda_j)$ and $d_i = (1-\alpha_i)\lambda_i/[\eta(1+\tau_{Li})]$. Substituting in the production function:

$$Q_i = A_i(1-\alpha_i)^{-(1-\alpha_i)}(\alpha_i)^{-\alpha_i}(B_i L_i)^{1-\alpha_i}(\prod_{j=1}^{n} v_{ij}^{-v_{ij}} X_{ij}^{v_{ij}})^{\alpha_i},$$

we obtain

$$\log(\lambda/\eta) = -(I-\tilde{\alpha})\log B - \log A + [(I-\tilde{\alpha})\Delta^{Lp} + \tilde{\alpha}(V \circ \Delta^{Xp})]\mathbf{1} + \tilde{\alpha} V \log(\lambda/\eta).$$

We can use the expressions derived for the $\rho = \sigma$ case. Equation (B.7) becomes

$$\hat{q}' = \beta'(I - \tilde{\alpha}(V \circ \Delta^{Xq}))^{-1},$$

where $\hat{q}_i = \lambda_i Q_i / Y$. Using the expression for d_i in the resource constraint on labor written as $\sum d_i q_i Y = \bar{L}$, gives $(\mathbf{1}-\alpha)' \Delta^{Lq} \hat{q} \eta^{-1} Y = \bar{L}$. Then substituting for the expression of $\hat{q}$, we get

$$\log Y = \log \eta - \log(\beta'(I - \tilde{\alpha}(V \circ \Delta^{Xq}))^{-1} \Delta^{Lq}(\mathbf{1}-\alpha) + \log \bar{L}.$$

Since $\eta = -\beta' \log(\lambda/\eta)$, this yields the expression of output stated in the proposition. ■

Proof of Proposition A.3. It is immediate that in this economy the only solution to the system of equations (9) in Proposition 1 is $\mathbf{1}$. Then part (iii) of this proposition implies that $\lambda = \mathbf{1}, \eta = 1$. Plugging in we obtain: $d = \mathbf{1} - \alpha$, $c = \beta$, $M_{ij} = \alpha_i v_{ij}$. Then $[I - M']^{-1} = \Omega'$. So $TFP = [(\mathbf{1}-\alpha)'\Omega'\beta]^{-1} = [\mathbf{1}'\beta]^{-1} = 1$, where we used Lemma B.2.

Next, since $Q = [I - M']^{-1} c Y = \Omega' \beta L$, it is immediate that $Q_i = \sum_k \beta_k \omega_{ki} L = s_i L$, which implies the last result. The other results are immediate. ■

Proof of Proposition A.4. Consider the equilibrium variables for economy 1: $\{\lambda_i^1, \eta_i^1, Q_i^i, X_{ij}^1, L_i^1, d_i^1, c_i^1, M^1, Y^1, TFP^1\}$. We will show that if we set $\lambda_i^2 = \lambda_i^1$, $\eta_i^2 = \eta_i^1/(1+z)$ and all the other variables identical to economy 1, all the conditions in proposition 1 are satisfied.

First we show that condition (v) is satisfied.

$$A_i^{-1}\left[(1-\alpha_i)B_i^{\frac{\sigma}{1-\sigma}}(1+\tau_{Li}^2)^{-\frac{\sigma}{1-\sigma}}+\alpha_i\left[\sum_{j=1}^n v_{ij}\left((1+\tau_{Xij}^2)\lambda_j^2/\eta^2\right)^{-\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}\frac{\sigma}{1-\sigma}}\right]^{-\frac{1-\sigma}{\sigma}}$$

$$=(1+z)A_i^{-1}\left[(1-\alpha_i)B_i^{\frac{\sigma}{1-\sigma}}(1+\tau_{Li}^1)^{-\frac{\sigma}{1-\sigma}}+\alpha_i\left[\sum_{j=1}^n v_{ij}\left((1+\tau_{Xij}^1)\lambda_j^1/\eta^1\right)^{-\frac{\rho}{1-\rho}}\right]^{\frac{1-\rho}{\rho}\frac{\sigma}{1-\sigma}}\right]^{-\frac{1-\sigma}{\sigma}}$$

$$=(1+z)\frac{\lambda_i^1}{\eta^1}=\frac{\lambda_i^2}{\eta^2},$$

which proves the claim. Then it is immediate that the constructed λ^2, η^2, satisfy (iv). Plugging in the formulas in parts (i), (ii) and (iii) implies that $d^1 = d^2, M^1 = M^2, c^1 = c^2$ and hence $TFP^1 = TFP^2, Y^1 = Y^2$ and so on. ■

Proof of Proposition A.5. We can rewrite the approximation derived above as

$$\begin{aligned}\log TFP \;\approx\;& \frac{1}{2}\left(n\frac{1}{n}\sum_i s_i(1-\alpha_i)\tau_{Li}\right)^2+\frac{1}{2}\frac{\sigma}{1-\sigma}\left(n\frac{1}{n}\sum_i\sum_{j\neq i}s_{ij}(1-\alpha_i)(1-\alpha_j)\tau_{Li}\tau_{jL}+n\frac{1}{n}\sum_i s_{ii}(1-\alpha_i)^2\tau_{Li}^2\right)\\&-\frac{1}{2}\frac{1}{1-\sigma}n\frac{1}{n}\sum_i s_i(1-\alpha_i)\tau_{Li}^2\end{aligned}$$

With the assumptions that the distortions are independent of any other sector parameters, and are independently distributed with mean $\bar{\tau_L}$ and cross-sectional variance $V(\tau_{Li})$, and using the Law of large numbers, we have

$$\begin{aligned}\log TFP \;\approx\;& \frac{1}{2}\left((\sum_i s_i(1-\alpha_i))\bar{\tau}_L\right)^2+\frac{1}{2}\frac{\sigma}{1-\sigma}\left(\sum_i\sum_{j\neq i}s_{ij}(1-\alpha_i)(1-\alpha_j)\bar{\tau}_L^2+\sum_i s_{ii}(1-\alpha_i)^2E(\tau_{Li}^2)\right)\\&-\frac{1}{2}\frac{1}{1-\sigma}\sum_i s_i(1-\alpha_i)E(\tau_{Li}^2)\end{aligned}$$

Which can be rewritten as

$$\begin{aligned}\log TFP \;\approx\;& \frac{1}{2}\bar{\tau}_L^2\left(\sum_i s_i(1-\alpha_i)\right)^2-\frac{1}{2}\frac{1}{1-\sigma}E(\tau_{Li}^2)\sum_i s_i(1-\alpha_i)\\&+\frac{1}{2}\frac{\sigma}{1-\sigma}\left(\bar{\tau}_L^2\sum_i\sum_j s_{ij}(1-\alpha_i)(1-\alpha_j)-\bar{\tau}_L^2\sum_i s_{ii}(1-\alpha_i)^2+E(\tau_{Li}^2)\sum_i s_{ii}(1-\alpha_i)^2\right)\end{aligned}$$

After re-arranging the terms, we obtain

$$\begin{aligned}\log TFP &\approx \frac{1}{2}\bar{\tau}_L^2\left(\sum_i s_i(1-\alpha_i)\right)^2 - \frac{1}{2}\sum_i s_i(1-\alpha_i)E(\tau_{Li}^2) - \frac{1}{2}\frac{\sigma}{1-\sigma}E(\tau_{Li}^2)\sum_i s_i(1-\alpha_i)\\ &+\frac{1}{2}\frac{\sigma}{1-\sigma}\left(\bar{\tau}_L^2\sum_i\sum_j s_{ij}(1-\alpha_i)(1-\alpha_j) - \bar{\tau}_L^2\sum_i s_{ii}(1-\alpha_i)^2 + E(\tau_{Li}^2)\sum_i s_{ii}(1-\alpha_i)^2\right)\end{aligned}$$

Finally, using $\sum_i s_i(1-\alpha_i) = 1$, and noticing that $\sum_i\sum_j s_{ij}(1-\alpha_i)(1-\alpha_j) = \sum_i s_i(1-\alpha_i) = 1$, we have

$$\log TFP \approx -\frac{1}{2}V(\tau_{Li}) - \frac{1}{2}\frac{\sigma}{1-\sigma}\left(1-\sum_i s_{ii}(1-\alpha_i)^2\right)V(\tau_{Li}),$$

with $1-\sum_i s_{ii}(1-\alpha_i)^2 \geq 0$. To see this, recall that $\sum_i \omega_{ki}(1-\alpha_i) = 1$. Therefore, $\omega_{ki}(1-\alpha_i) \leq 1$ for all i, and $\omega_{ki}^2(1-\alpha_i)^2 \leq \omega_{ki}(1-\alpha_i)$ which gives $\sum_i \omega_{ki}^2(1-\alpha_i)^2 \leq 1$, hence $\sum_i s_{ii}(1-\alpha_i)^2 \equiv \sum_k \beta_k \sum_i \omega_{ki}^2(1-\alpha_i)^2 \leq 1$.
Finally, suppose that $\alpha_i v_{ij} > 0$ for some $i \neq j$. Since i is nontrivial, $\omega_{ki} > 0$ for some k with $\beta_k > 0$. This implies that $\omega_{kj} > 0$. Suppose that $k \neq j$. We have that $\omega_{kk} > 1$ and $\omega_{kk}(1-\alpha_k)+\omega_{kj}(1-\alpha_j) \leq 1$, so $\omega_{kj}(1-\alpha_j) < 1$. Also $1-\alpha_j > 1-\bar{\alpha} > 0, \omega_{kj} > 0$, so $\omega_{kj}(1-\alpha_j) \in (0,1)$ and hence $\omega_{kj}^2(1-\alpha_j)^2 < \omega_{kj}(1-\alpha_j)$, so $\sum_i \omega_{ki}^2(1-\alpha_i)^2 < 1$ and by the logic above $\psi > 0$. Now suppose that $k = j$. Then it must be that $\omega_{ki} > 0$ with $\beta_k > 0$ and $k \neq i$. Then the construction above goes through in exactly the same way. ∎

Proof of Proposition A.6.
Using (13) and the law of large numbers we obtain:

$$\begin{aligned}\log TFP &\approx -\frac{1}{1-\sigma}\left(\sum_i\sum_j s_i\omega_{ij}\right)\bar{\tau}^2 - \frac{1}{1-\sigma}\left(\sum_i s_i\omega_{ii}\right)\left(E(\tau_i^2)-\bar{\tau}^2\right)\\ &+\frac{1}{2}\frac{\sigma}{1-\sigma}\left(\sum_i\sum_j s_{ij}\right)\bar{\tau}^2 + \frac{1}{2}\frac{\sigma}{1-\sigma}\left(\sum_i s_{ii}\right)\left(E(\tau_i^2)-\bar{\tau}^2\right) + \frac{1}{2}\left(\sum_i s_i\right)^2\bar{\tau}^2 + \frac{1}{2}\left(\sum_i s_i^2\right)\left(E(\tau_i^2)-\bar{\tau}^2\right)\\ &+\frac{1}{2}\frac{1}{1-\sigma}\left(\sum_i s_i\right)\left(E(\tau_i^2)-\bar{\tau}^2\right) + \frac{1}{2}\frac{1}{1-\sigma}\left(\sum_i s_i\right)\bar{\tau}^2\end{aligned}$$

Rearranging, we get

$$\begin{aligned}
\log TFP \quad \approx \quad & -\frac{1}{2}\left[2\sum_i\sum_j s_i\omega_{ij} - \sum_i\sum_j s_i s_j - \sum_i s_i\right]\bar{\tau}^2 \\
& -\frac{1}{2}\left[2\sum_i s_i\omega_{ii} - \sum_i s_i^2 - \sum_i s_i\right]Var(\tau) \\
& -\frac{1}{2}\frac{\sigma}{1-\sigma}\left[2\sum_i\sum_j s_i\omega_{ij} - \sum_i\sum_j s_{ij} - \sum_i s_i\right]\bar{\tau}^2 \\
& -\frac{1}{2}\frac{\sigma}{1-\sigma}\left[2\sum_i s_i\omega_{ii} - \sum_i s_{ii} - \sum_i s_i\right]Var(\tau).
\end{aligned}$$

Then this gives us equation (A.3), where

$$\begin{aligned}
\Gamma_0(\alpha,\beta,\Omega) &= \frac{1}{2}\left[2\sum_i\sum_j s_i\omega_{ij} - \sum_i\sum_j s_i s_j - \sum_i s_i\right] \\
\Gamma_1(\alpha,\beta,\Omega) &= \frac{1}{2}\left[2\sum_i\sum_j s_i\omega_{ij} - \sum_i\sum_j s_{ij} - \sum_i s_i\right] \\
\chi_0(\alpha,\beta,\Omega) &= \frac{1}{2}\left[2\sum_i s_i\omega_{ii} - \sum_i s_i^2 - \sum_i s_i\right] \\
\chi_1(\alpha,\beta,\Omega) &= \frac{1}{2}\left[2\sum_i s_i\omega_{ii} - \sum_i s_{ii} - \sum_i s_i\right]
\end{aligned}$$

Notice that:

$$\begin{aligned}
2\Gamma_1 &= 2\sum_i\sum_j s_i\omega_{ij} - \sum_i\sum_j s_{ij} - \sum_i s_i \\
&= 2\sum_i\sum_j\sum_k \beta_k\omega_{ki}\omega_{ij} - \sum_i\sum_j\sum_k \beta_k\omega_{ki}\omega_{kj} - \sum_i\sum_k \beta_k\omega_{ki} \\
&= \sum_k \beta_k\left[2\sum_i\sum_j \omega_{ki}\omega_{ij} - \left(\sum_i \omega_{ki}\right)^2 - \sum_i \omega_{ki}\right]
\end{aligned}$$

The expression in the square brackets is nonnegative by Lemma B.5 with $z = \mathbf{1}$, so $\Gamma_1 \geq 0$. In the same way, when $\alpha_i > 0$ for some i, lemma B.7 implies that $\Gamma_1 > 0$.
$\Gamma_0 - \Gamma_1 = \sum_i\sum_j(s_{ij} - s_i s_j)$. Let $d_{ij} = s_{ij} - s_i s_j$ and $D = (d_{ij})$. By lemma B.8 D is positive semi-definite. Then $\Gamma_0 - \Gamma_1 = \frac{1}{2}\mathbf{1}'D\mathbf{1} \geq 0$. Therefore, $\Gamma_0 \geq \Gamma_1 \geq 0$. This also proves that $\Gamma_0 + \frac{\sigma}{1-\sigma}\Gamma_1 = \Gamma_0 - \Gamma_1 + \frac{1}{1-\sigma}\Gamma_1 \geq 0$.

$$\begin{aligned}\chi_1(\alpha,\beta,V) &= \frac{1}{2}\sum_i s_i(\omega_{ii}-1)+\sum_i\sum_k \beta_k\omega_{ki}\omega_{ii}-\sum_i\sum_k \beta_k\omega_{ki}^2 \\ &= \frac{1}{2}\sum_i s_i(\omega_{ii}-1)+\sum_i\sum_k \beta_k\omega_{ki}(\omega_{ii}-\omega_{ki}).\end{aligned}$$

Lemma B.1 states that $\omega_{ii}-1\geq 0$ and $\omega_{ii}-\omega_{ki}\geq 0$, which completes the proof that $\chi_1\geq 0$.
Suppose that $\chi_1=0$. Both terms are nonnegative. Then since the first term must be zero and $s_i>0$ then $\omega_{ii}=1$ for all i. Since $\omega_{ii}-\omega_{ki}>0$ for all $k\neq i$, then $\omega_{ki}=0$ for all $k\neq i$, that is $\Omega=I$. Then lemma B.1 implies that $\alpha_i=0$ for all i. This proves (by contrapositive) that $\chi_1>0$ if $\alpha_i>0$ for some i.
Finally, $\chi_0-\chi_1=\sum_i[s_{ii}-s_i^2]$. Lemma B.8 implies that $s_{ii}-s_i^2\geq 0,\ \forall i$. This proves that $\chi_0\geq 0$ and also that $\chi_0(\alpha,\beta,V)+\frac{\sigma}{1-\sigma}\chi_1(\alpha,\beta,V)\geq 0$. ■

Proof of Lemma A.1. Suppose that two arbitrary matrices F and G are of the form in the hypothesis of the lemma. Then from the definition of matrix multiplication, FG is of the same form. This fact implies (by mathematical induction) that $[\tilde{\alpha}V]^{(n)}$ is of the same form for all n, hence sectors $1,\ldots,\ell-1$ are upstream from ℓ.
Next we tackle the necessity condition. Suppose that for some $i<\ell, j\geq\ell$, $\alpha_i v_{ij}>0$. If $j=\ell$, then i is a purchaser of ℓ by definition. Suppose not. For some n, $[\tilde{\alpha}V]_{j\ell}^{(n)}>0$. Then $[\tilde{\alpha}V]_{i\ell}^{(n+1)}\geq \alpha_i v_{ij}[\tilde{\alpha}V]_{j\ell}^{(n)}>0$, which is a contradiction. ■

Proof of Proposition A.7.

1. Define $H\equiv\tilde{\alpha}\hat{A}(\Delta^{Xp}\circ V)]$ and $\Omega^p=[I-H]^{-1}$. It is immediate that $H_{ij}^{(n)}>0$ if and only if $[\tilde{\alpha}V]^{(n)}>0$. Moreover, $\Omega^p=I+H+H^2+\ldots$, so $\Omega_{ij}^p>0$, $i\neq j$ if and only $[\tilde{\alpha}V]_{ij}^{(n)}>0$ for some n. This implies that if $1,\ldots\ell-1$ are upstream from ℓ and $\ell+1\ldots n$ are purchasers of ℓ, we have that $\Omega_{i\ell}^p=0$ for $i<\ell$ and $\Omega_{i\ell}^p>0$ for $i\geq\ell$.

   Proposition 4 implies that
   $$\frac{d(\lambda/\eta)}{d\tau_{\ell L}}=\tilde{\lambda}\circ(\Omega^p\hat{A}\hat{B}\Delta^1(\mathbf{1}-\alpha)),$$
   where $\tilde{\lambda}$ is a $n\times 1$ vector with $\tilde{\lambda_i}=(\lambda_i/\eta)^{1/(1-\sigma)}$ and $\Delta_\ell^1=(1+\tau_{\ell L})^{-\frac{\sigma}{1-\sigma}-1}$ and $\Delta_{iL}^1=0$ for $i\neq\ell$. Then the fact that $\Omega_{i\ell}^p=0$ for $i<\ell$ and $\Omega_{i\ell}^p>0$ for $i\geq\ell$ implies the result about the sign of $\frac{d(\lambda_i/\eta)}{d\tau_{\ell L}}$.

   In a similar fashion,
   $$\frac{d(\lambda/\eta)}{d\tau_{\ell k}}=-\frac{1-\sigma}{\sigma}\tilde{\lambda}\circ(\frac{d}{d\tau_{\ell k}}\Omega^p\hat{A}\hat{B}\Delta^{Lp}(\mathbf{1}-\alpha)).$$
   From the definition of Ω^p, it follows that
   $$\frac{d}{d\tau_{\ell k}}\Omega^p=-\frac{\sigma}{1-\sigma}\Omega^p M\Omega^p,$$

where $M_{\ell k} = \alpha_\ell A_\ell^{\frac{\sigma}{1-\sigma}}(1+\tau_{\ell k})^{-\frac{1}{1-\sigma}} v_{\ell k} > 0$ and $M_{ij} = 0$ otherwise. Therefore,

$$\frac{d(\lambda/\eta)}{d\tau_{\ell k}} = \tilde{\lambda} \circ (\Omega^p M \hat{\lambda}).$$

From the structure of Ω^p and M it follows that $(\Omega^p M)_{i\ell} > 0$ if $i \geq \ell$ and $\Omega^p_{ij} = 0$ otherwise. This implies the first result.

2. Next, since $\log \eta = -\beta' \log(\lambda/\eta)$, $d\eta/d\tau_{\ell,k} < 0$. Then since $d\lambda_i/dx = d(\eta\lambda_i/\eta)/dx = (\lambda_i/\eta)d\eta/dx + \eta d(\lambda_i/\eta)/dx$, result 1 implies the result.

3. Since $C_i/Y = \beta_i/\lambda_i$, result 2 implies the first statement. Second, $\log C_i = \log \beta_i - \log \lambda_i + \log Y = \log \beta_i - \log \eta - \log(\lambda_i/\eta) + \log Y = \log \beta_i + \beta' \log(\lambda/\eta) - \log(\lambda_i/\eta) + \log Y$. Then proposition 4 implies that

$$\log C_i = \log \beta_i - \log(\lambda_i/\eta) - \log\left(\beta'[I - \tilde{\alpha}\hat{A}(\Delta^{Xq} \circ V)]^{-1}\hat{A}\hat{B}\Delta^{Lq}(\mathbf{1} - \alpha)\right).$$

Let $E = [I - \tilde{\alpha}\hat{A}(\Delta^{Xq} \circ V)]^{-1}$. The statement is equivalent to

$$\frac{d}{dx}[\beta' E\hat{A}\hat{B}\Delta^{Lq}(\mathbf{1} - \alpha)] < 0,$$

where $x = \tau_{\ell,L}$ or $x = \tau_{\ell,k}$. So,

$$\frac{d}{d\tau_{\ell,L}}[\beta' E\hat{A}\hat{B}\Delta^{Lq}(\mathbf{1} - \alpha)] = -\frac{1}{1-\sigma}\beta' E\hat{A}\hat{B}\Delta^{2}(\mathbf{1} - \alpha) < 0,$$

where $\Delta^2_\ell = (1+\tau_{Li})^{-\frac{1}{1-\sigma}-1}$ and zero elsewhere. Next we turn to $\tau_{\ell,k}$.

$$\frac{d}{d\tau_{\ell k}}E = -\frac{1}{1-\sigma}ENE,$$

where $N_{\ell k} = \alpha_\ell A_\ell^{\frac{\sigma}{1-\sigma}}(1+\tau_{\ell k})^{-\frac{1}{1-\sigma}-1} v_{\ell k} > 0$ and $N_{ij} = 0$ otherwise. Therefore,

$$\frac{d}{d\tau_{\ell,k}}[\beta' E\hat{A}\hat{B}\Delta^{Lq}(\mathbf{1} - \alpha)] = -\frac{1}{1-\sigma}\beta' ENE\hat{A}\hat{B}\Delta^{Lq}(\mathbf{1} - \alpha) < 0,$$

which concludes the proof of this result.

4. Finally, the zero-profit condition implies that

$$\frac{\sum_j (1+\tau_{Xij})\lambda_j X_{ij}}{\lambda_i Q_i} = 1 - \frac{(1+\tau_{Li})\eta L_i}{\lambda_i Q_i} = 1 - (A_i B_i)^{\frac{\sigma}{1-\sigma}}(1-\alpha_i)\left(\frac{\lambda_i}{(1+\tau_{Li}\eta)}\right)^{\frac{\sigma}{1-\sigma}}.$$

Then result 1 implies the rest.

■

C Application to markups

C.1 Model with markups

Set $\lambda_i = p_i, \eta = w$. Then it is immediate that cost minimization in the final goods sector implies that (6) holds. Next, consider the cost minimization in the various sectors. Given the prices of the different inputs, we have the following optimality condition:

$$\chi_i A_i^\sigma \left(\frac{\alpha_i Q_i}{X_i}\right)^{1-\sigma} \left(\frac{v_{ij} X_i}{x_{ij}}\right)^{1-\rho} = p_j \quad \text{(C.1)}$$

$$\chi_i A_i^\sigma B_i^\sigma \left(\frac{(1-\alpha_i) Q_i}{L_i}\right)^{1-\sigma} = w \quad \text{(C.2)}$$

$$\chi_i \left(A_i \left[(1-\alpha_i)^{1-\sigma}(B_i L_i)^\sigma + \alpha_i^{1-\sigma} X_i^\sigma\right]^{\frac{1}{\sigma}} - 1\right) = 0, \quad \text{(C.3)}$$

where χ_i is the multiplier to the constraint $Q_i \geq 1$. Since $\chi_i = mc_i = p_i/\mu_i = \lambda_i/\mu_i$, the allocation satisfies (7) and (8) with $\tau_{ij} = \tau_{iL} = \mu_i - 1$.

Plugging (C.1) and (C.2) into the complementary slackness condition (C.3) we get

$$p_i = A_i^{-1} \left[(1-\alpha_i) B_i^{\frac{\sigma}{1-\sigma}} w^{-\frac{\sigma}{1-\sigma}} + \alpha_i \left(\sum_j v_{ij} p_j^{-\frac{\rho}{1-\rho}}\right)^{\frac{1-\rho}{\rho}\frac{\sigma}{1-\sigma}}\right]^{-\frac{1-\sigma}{\sigma}} \mu_i. \quad \text{(C.4)}$$

Given the expression for mc_i, and hence λ_i, Proposition 1 pins down the equilibrium.

C.2 Data

Our main source is the World Input-Output Database. The data are publicly available at: `http://www.wiod.org/home`. The dataset gives industry-level data on sales, labor compensation, capital and intermediate input purchases for 40 countries and 35 sectors over the period 1995-2011. We use the 2013 release because 2016 release does not include data on capital. The countries in the dataset are: Australia, Austria, Belgium, Brazil, Bulgaria, Canada, China, Cyprus, Czech Republic, Denmark, Estonia, Finland, France, Germany, Greece, Hungary, India Indonesia, Ireland, Italy, Japan, Republic of Korea, Latvia, Lithuania, Luxembourg, Malta, Mexico, Netherlands, Poland, Portugal, Romania, Russia, Slovak Republic, Slovenia, Spain, Sweden, Taiwan, Turkey, United Kingdom, United States. In our quantitative evaluation of the cost of markups, we exclude Latvia, Malta, Cyprus, Luxembourg. The input-output table of these countries contains too many zeros which prevents us from solving the model at the same disaggregation level as the other countries. We also exclude Italy because the patterns in the data makes us suspect classification discrepancies relative to other countries. We conduct our analysis on 30 sectors, after removing the public administration, education, health and social work, social services and private households services (sectors c31-c35). The 30 sectors are: Agriculture, Hunting, Forestry and Fishing; Mining and Quarrying; Food, Beverages and Tobacco ;Textiles and Textile Products; Leather, Leather and Footwear; Wood and Products of Wood and Cork; Pulp, Paper, Paper Products, Printing and Publishing; Coke, Refined Petroleum and Nuclear Fuel; Chemicals and Chemical Products ;Rubber and Plastics; Other Non-Metallic Mineral; Basic Metals and Fabricated Metal; Machinery, Nec; Electrical and Optical Equipment; Transport Equipment; Manufacturing Nec, Recycling; Electricity, Gas and Water Supply; Construction; Sale, Maintenance and Repair of Motor Vehicles and Motorcycles, Retail Sale of Fuel; Wholesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles; Retail Trade, Except of Motor Vehicles and Motorcycles, Repair of Household Goods; Hotels and Restaurants; Inland Transport; Water Transport ; Air Transport; Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies; Post and Telecommunications; Financial Intermediation; Real Estate Activities; Renting of M&Eq and Other Business Activities. The variables needed for our quantitative exercise are available over the period 1995-2007, for all sectors and all countries, except for China and Indonesia, whose data on "Sale, maintenance and repair of motor vehicles and motorcycles; retail sale of fuel" (sector c19) are missing.

Data for sales, wage bill and intermediate input costs are available directly from the dataset. The variables are "Gross output by industry at current basic prices (in millions of national currency)", "Labour compensation (in millions of national currency)", and "Intermediate inputs at current purchasers' prices (in millions of national currency)".[44] We now describe how the remaining variables are constructed.

We compute capital costs as $(i+\delta)qK$, where i is the nominal interest rate, δ is the depreciation rate and K is the real capital stock and q is the investment price index. Both the capital stock and the investment price index are obtained from the WIOD SEA dataset. We set the interest rate to 4% and calibrate the industry-specific depreciation rate, δ, using the Bureau of Economic Analysis implied rates of depreciation of private nonresidential fixed assets. The implied depreciation rates are reported for each industry by asset type. We compute the industry-level depreciation rate by weighting the depreciation rates by the share of the asset in the total current-cost net capital stock of the industry. The BEA data is available at `https://apps.bea.gov/national/FA2004/Details/Index.htm`. The data is converted from the BEA to the WIOD industry classification using the concordance from NAICS 2002 to ISIC 3.1 provided by the U.S. Census, available at `https://www.census.gov/eos/www/naics/concordances/concordances.html`. We use the 2000 data.

We compute the intermediate input shares by dividing the value of intermediate inputs purchased by the sector by its sales net of the value of intermediate inputs imported by the sector.

$$\frac{p_j X_{ij}}{p_i Q_i} = \frac{\text{interm-input}_{ij}}{\text{sales}_i - \sum_j \text{imported-interm-input}_{ij}}$$

This adjustment is done to be consistent with the model in which we abstract from international trade.

[44] For more information on how those variables are constructed, see Erumban et al. (2012).

Final expenditures are computed as

$$p_i C_i = \text{sales}_i - \sum_k \text{interm-input}_{ki}$$

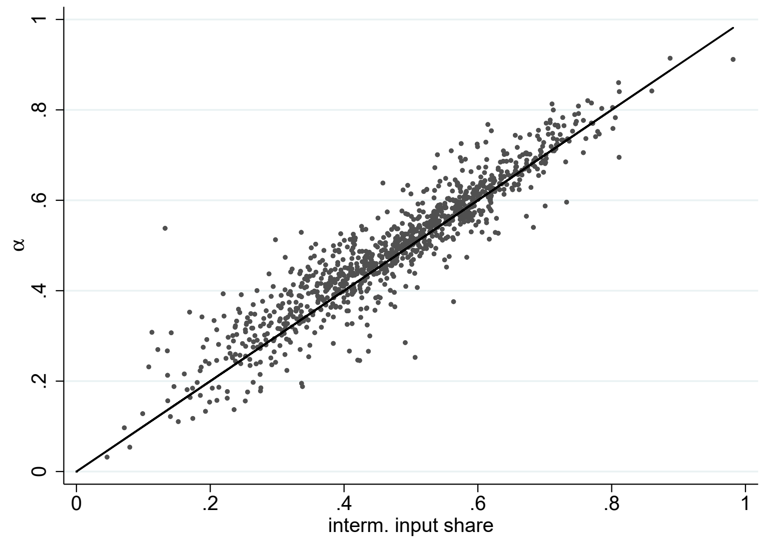
C.3 Preliminary tables and figures

Table C.1: Production function parameters, by country

country	β			α			V		
	min	med	max	min	med	max	min	med	max
AUS	0.00	0.02	0.17	0.28	0.56	0.91	0.00	0.01	0.58
AUT	0.00	0.03	0.10	0.18	0.44	0.66	0.00	0.01	0.74
BEL	0.00	0.03	0.09	0.18	0.50	0.75	0.00	0.01	0.63
BGR	0.01	0.02	0.11	0.13	0.51	0.72	0.00	0.01	0.71
BRA	0.00	0.03	0.10	0.03	0.54	0.81	0.00	0.01	0.59
CAN	0.00	0.02	0.11	0.16	0.47	0.84	0.00	0.01	0.59
CHN	0.00	0.02	0.20	0.18	0.71	0.82	0.00	0.01	0.66
CZE	0.00	0.02	0.10	0.32	0.54	0.76	0.00	0.01	0.58
DEU	0.00	0.02	0.11	0.15	0.52	0.77	0.00	0.01	0.68
DNK	0.00	0.02	0.11	0.16	0.45	0.91	0.00	0.01	0.84
ESP	0.00	0.02	0.15	0.14	0.59	0.77	0.00	0.01	0.53
EST	0.00	0.03	0.11	0.18	0.48	0.74	0.00	0.01	0.61
FIN	0.00	0.02	0.12	0.24	0.49	0.73	0.00	0.01	0.54
FRA	0.00	0.03	0.13	0.12	0.55	0.77	0.00	0.01	0.54
GBR	0.00	0.02	0.10	0.19	0.48	0.75	0.00	0.01	0.79
GRC	-0.01	0.02	0.15	0.13	0.42	0.74	0.00	0.01	0.87
HUN	0.00	0.03	0.16	0.16	0.51	0.73	0.00	0.01	0.53
IDN	0.00	0.02	0.15	0.17	0.50	0.77	0.00	0.01	0.92
IND	0.00	0.02	0.18	0.10	0.59	0.82	0.00	0.01	0.65
IRL	0.00	0.01	0.17	0.25	0.44	0.69	0.00	0.01	0.64
JPN	0.00	0.02	0.16	0.12	0.52	0.73	0.00	0.01	0.60
KOR	0.00	0.02	0.15	0.19	0.53	0.86	0.00	0.01	0.68
LTU	0.00	0.02	0.12	0.11	0.39	0.66	0.00	0.01	0.61
MEX	0.00	0.02	0.11	0.05	0.52	0.84	0.00	0.01	0.85
NLD	0.00	0.03	0.10	0.18	0.46	0.73	0.00	0.01	0.59
POL	0.00	0.03	0.12	0.25	0.55	0.77	0.00	0.01	0.66
PRT	0.00	0.02	0.14	0.19	0.54	0.77	0.00	0.01	0.69
ROU	0.00	0.02	0.12	0.21	0.48	0.72	0.00	0.01	0.82
RUS	0.00	0.02	0.12	0.16	0.52	0.75	0.00	0.01	0.55
SVK	0.00	0.03	0.09	0.20	0.55	0.70	0.00	0.01	0.76
SVN	0.00	0.03	0.11	0.23	0.48	0.71	0.00	0.01	0.59
SWE	0.00	0.02	0.12	0.26	0.48	0.69	0.00	0.01	0.49
TUR	0.00	0.02	0.10	0.29	0.55	0.80	0.00	0.01	0.80
TWN	0.00	0.02	0.22	0.24	0.51	0.75	0.00	0.01	0.57
USA	0.00	0.02	0.14	0.28	0.57	0.77	0.00	0.01	0.76

Notes: Parameters calibrated under the assumption $\varepsilon_\sigma = 0.70$ and $\varepsilon_\rho = 0.01$.

Figure C.1: Observed intermediate input share vs α



Note: Parameter α estimated for $(\epsilon_\sigma, \epsilon_\rho) = (0.70, 0.01)$.

Figure C.2: The price cost margin: median and standard deviation across countries

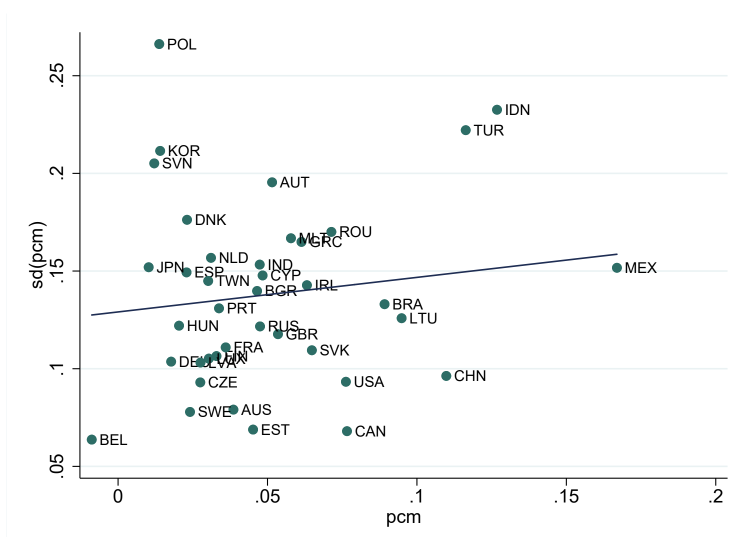
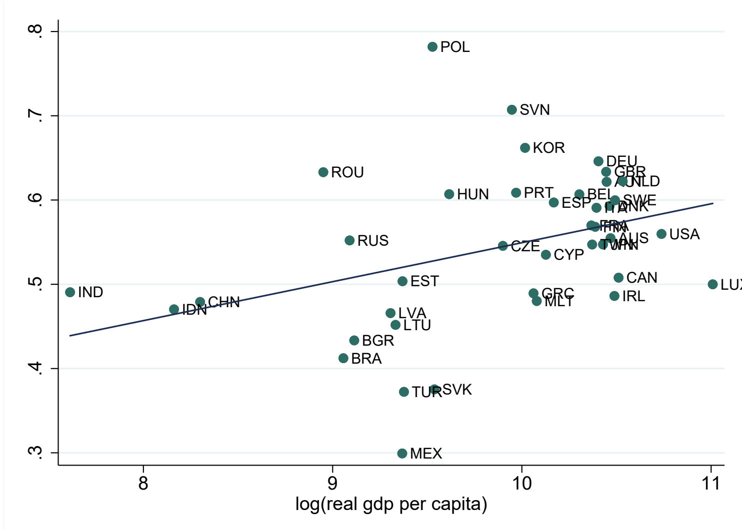
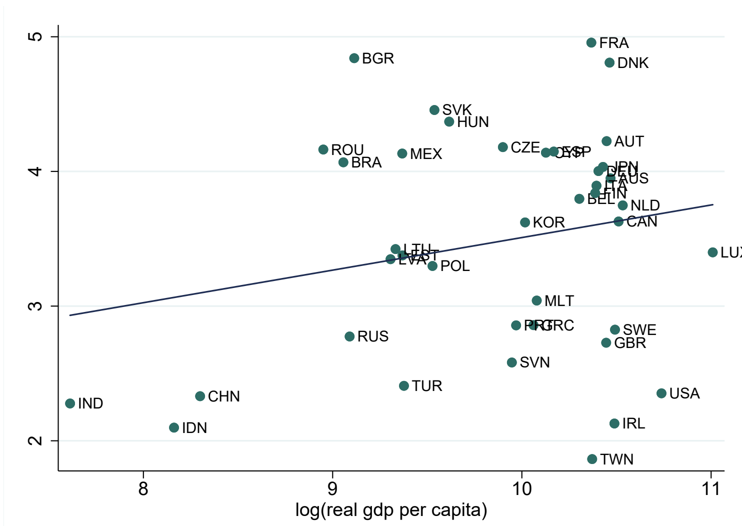


Figure C.3: The aggregate labor share



Note: The labor share is computed as the sum of labor compensation over the nominal value added. The figure display data for year 2000.

Figure C.4: The aggregate capital-ouput ratio



Note: The capital-output ratio is computed as the sum the real capital stock times the investment deflator over the nominal value added. This is for year 2000.

Table C.2: Production function parameters, by industry

industry	β			α			V		
	min	med	max	min	med	max	min	med	max
Agriculture	0.01	0.03	0.18	0.22	0.40	0.59	0.00	0.01	0.82
Mining	-0.01	0.00	0.10	0.19	0.39	0.55	0.00	0.01	0.52
Food	0.04	0.07	0.13	0.64	0.73	0.86	0.00	0.01	0.61
Textiles	0.00	0.02	0.10	0.33	0.55	0.78	0.00	0.01	0.56
Leather	0.00	0.00	0.02	0.45	0.55	0.82	0.00	0.01	0.47
Wood	0.00	0.00	0.05	0.45	0.62	0.77	0.00	0.01	0.54
Paper	0.01	0.02	0.10	0.43	0.57	0.72	0.00	0.01	0.58
Petroleum	0.00	0.02	0.08	0.25	0.61	0.91	0.00	0.01	0.92
Chemicals	0.01	0.03	0.15	0.32	0.62	0.78	0.00	0.01	0.49
Rubber and plastics	0.00	0.01	0.03	0.41	0.56	0.77	0.00	0.01	0.45
non-metal. mineral	0.00	0.01	0.02	0.43	0.55	0.70	0.00	0.01	0.36
Basic and fabr. metal	0.01	0.03	0.08	0.40	0.61	0.79	0.00	0.01	0.68
Machinery, nec	0.01	0.03	0.07	0.38	0.54	0.75	0.00	0.01	0.41
Elec. and optical equip.	0.01	0.04	0.22	0.38	0.56	0.79	0.00	0.01	0.49
Transport equip.	0.01	0.05	0.11	0.27	0.59	0.80	0.00	0.01	0.60
Manufacturing nec	0.00	0.02	0.03	0.31	0.56	0.73	0.00	0.02	0.45
Elec., gas and water	0.01	0.02	0.05	0.23	0.51	0.67	0.00	0.01	0.80
Construction	0.06	0.11	0.20	0.29	0.56	0.72	0.00	0.01	0.49
Sale of motor vehicles	0.00	0.01	0.03	0.21	0.40	0.52	0.00	0.01	0.55
Wholesale trade	0.02	0.05	0.12	0.25	0.41	0.55	0.00	0.01	0.45
Retail trade	0.01	0.04	0.10	0.16	0.34	0.53	0.00	0.01	0.55
Hotels and restaurants	0.01	0.03	0.13	0.24	0.48	0.73	0.00	0.01	0.59
Inland transport	0.01	0.02	0.09	0.24	0.39	0.60	0.00	0.01	0.51
Water transport	0.00	0.00	0.07	0.33	0.49	0.77	0.00	0.01	0.84
Air transport	0.00	0.01	0.02	0.36	0.53	0.74	0.00	0.01	0.62
Other transport support	0.00	0.01	0.06	0.24	0.47	0.70	0.00	0.01	0.80
Post and telecomm.	0.01	0.02	0.04	0.18	0.45	0.64	0.00	0.01	0.64
Financial intermediation	0.01	0.04	0.09	0.17	0.44	0.55	0.00	0.00	0.62
Real estate activities	0.01	0.09	0.14	0.03	0.18	0.54	0.00	0.00	0.87
Renting of m&eq	0.01	0.05	0.10	0.19	0.38	0.57	0.00	0.01	0.63

Notes: Parameters calibrated under the assumption $\varepsilon_\sigma = 0.70$ and $\varepsilon_\rho = 0.01$. In the last panel, the statistics are computed on $v_{i\cdot}$.

C.4 Additional tables of results

We report here additional tables of results.

Table C.3 and C.4 give the baseline results for the 35 countries and for additional combinations of $(\varepsilon_\sigma, \varepsilon_\rho)$.

In Table C.5 reports the TFP gain when the markup distort only the firms' labor. Table C.6 reports the Input-Output amplification factor. We consider two economies without sectoral linkages: in the first one, the β_i are set to the same value as in the economy with linkages and in the second one, the β_i are recalibrated so as to yield sector of identical sizes in the economy with and without linkages.

In Table C.7, we consider the TFP gain from setting all the markups, even the negative ones, to zero.

In Table C.8, we present the results when holding fixed the production-function parameters as the elastiticies are let to vary.

We then report the results of our robustness checks. In the first robustness check, we recompute the TFP gain after removing the pcm outliers. We winsorize the price-cost margin measure by capping the gap between the pcm and the industry median at the 1st and 99th percentile (computed on the pooled data). The gap is pcm - median(pcm). The results are presented in Table C.9. We also consider the TFP gains from setting all positive pcms to zero, except that of the real estate sector. We report the results in Table C.10.

Table C.3: TFP gain and the elasticity of substitution

country	$\varepsilon_\sigma = 1$			$\varepsilon_\rho = 1$			$\varepsilon_\sigma = \varepsilon_\sigma = \varepsilon$		
	$\varepsilon_\rho = 0.01$	$\varepsilon_\rho = 0.70$	$\varepsilon_\rho = 1$	$\varepsilon_\sigma = 0.01$	$\varepsilon_\sigma = 0.07$	$\varepsilon_\sigma = 1$	$\varepsilon = 0.01$	$\varepsilon = 0.07$	$\varepsilon = 1$
BEL	0.002	0.002	0.003	0.003	0.003	0.003	0.002	0.002	0.003
CZE	0.003	0.006	0.007	0.006	0.006	0.007	0.003	0.005	0.007
CAN	0.005	0.006	0.007	0.005	0.006	0.007	0.003	0.006	0.007
KOR	-0.002	0.004	0.007	0.013	0.0097	0.007	0.004	0.006	0.007
HUN	0.004	0.007	0.009	0.009	0.009	0.009	0.004	0.007	0.009
SWE	0.006	0.008	0.009	0.007	0.008	0.009	0.003	0.007	0.009
FRA	0.007	0.009	0.010	0.010	0.010	0.010	0.007	0.009	0.010
AUT	0.004	0.008	0.010	0.012	0.011	0.010	0.007	0.009	0.010
DEU	0.005	0.009	0.011	0.010	0.010	0.011	0.004	0.009	0.011
FIN	0.008	0.011	0.013	0.012	0.013	0.013	0.007	0.011	0.013
EST	0.009	0.012	0.013	0.006	0.011	0.013	0.003	0.010	0.013
AUS	0.008	0.013	0.015	0.010	0.013	0.015	0.005	0.012	0.015
ESP	0.009	0.014	0.016	0.014	0.015	0.016	0.009	0.013	0.016
JPN	0.009	0.015	0.018	0.015	0.017	0.018	0.007	0.014	0.018
NLD	0.013	0.018	0.021	0.019	0.020	0.021	0.011	0.018	0.021
USA	0.018	0.020	0.022	0.007	0.017	0.022	0.004	0.016	0.022
DNK	0.015	0.020	0.023	0.022	0.023	0.023	0.015	0.020	0.023
PRT	0.013	0.020	0.023	0.016	0.021	0.023	0.008	0.018	0.023
RUS	0.015	0.021	0.023	0.015	0.021	0.023	0.009	0.018	0.023
GRC	0.017	0.022	0.025	0.018	0.023	0.025	0.011	0.020	0.025
SVN	0.017	0.023	0.026	0.028	0.027	0.026	0.019	0.024	0.026
SVK	0.018	0.024	0.027	0.014	0.023	0.027	0.008	0.020	0.027
ROU	0.011	0.022	0.027	0.031	0.029	0.027	0.017	0.024	0.027
LTU	0.020	0.026	0.029	0.018	0.025	0.029	0.011	0.023	0.029
BGR	0.019	0.029	0.033	0.023	0.030	0.033	0.012	0.025	0.033
IND	0.024	0.032	0.036	0.018	0.030	0.036	0.008	0.026	0.036
GBR	0.023	0.033	0.038	0.024	0.033	0.038	0.014	0.029	0.038
IRL	0.032	0.039	0.042	0.032	0.039	0.042	0.024	0.036	0.042
POL	0.016	0.037	0.047	0.048	0.047	0.047	0.023	0.038	0.047
BRA	0.036	0.045	0.049	0.029	0.042	0.049	0.020	0.039	0.049
CHN	0.048	0.058	0.062	0.019	0.047	0.062	0.010	0.043	0.062
TWN	0.050	0.066	0.074	0.028	0.057	0.074	0.014	0.051	0.074
MEX	0.065	0.076	0.081	0.037	0.066	0.081	0.027	0.062	0.081
IDN	0.067	0.097	0.111	0.062	0.094	0.111	0.036	0.082	0.111
TUR	0.157	0.197	0.217	0.121	0.180	0.217	0.089	0.164	0.217
median	0.015	0.020	0.023	0.016	0.021	0.023	0.009	0.018	0.023

Notes: The model is re-calibrated for each value of the elasticities of substitution. The case ($\varepsilon_\sigma = \varepsilon_\rho = 1$) is repeated for convenience.

Table C.4: TFP gain and IO amplification

	TFP gain			IO amplification factor		
$(\varepsilon_\sigma, \varepsilon_\rho)$	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)
BEL	0.002	0.002	0.003	1.3	1.3	2.5
CZE	0.003	0.003	0.007	1.6	1.8	3.9
CAN	0.003	0.004	0.007	1.7	2.7	4.4
KOR	0.004	-0.000	0.007	1.4	-0.0	2.4
HUN	0.004	0.004	0.009	1.6	1.5	3.4
SWE	0.003	0.005	0.009	1.2	1.7	3.2
FRA	0.007	0.007	0.010	1.8	1.8	2.6
AUT	0.007	0.005	0.010	1.4	1.1	2.2
DEU	0.004	0.005	0.011	1.6	1.6	3.8
FIN	0.007	0.008	0.013	1.5	1.5	2.7
EST	0.003	0.007	0.013	1.5	4.1	8.0
AUS	0.005	0.007	0.015	1.5	2.3	4.5
ESP	0.009	0.009	0.016	1.4	1.5	2.5
JPN	0.007	0.008	0.018	1.3	1.5	3.2
NLD	0.011	0.012	0.021	1.6	1.7	3.0
USA	0.004	0.013	0.022	1.0	3.1	5.1
DNK	0.015	0.015	0.023	1.4	1.4	2.1
PRT	0.008	0.011	0.023	1.5	2.2	4.3
RUS	0.009	0.013	0.023	1.2	1.8	3.3
GRC	0.011	0.015	0.025	1.0	1.4	2.3
SVN	0.019	0.018	0.026	1.2	1.1	1.5
SVK	0.008	0.015	0.027	1.9	3.6	6.4
ROU	0.017	0.013	0.027	1.5	1.2	2.4
LTU	0.011	0.017	0.029	1.3	1.9	3.2
BGR	0.012	0.017	0.033	1.5	2.2	4.3
IND	0.008	0.018	0.036	0.7	1.6	3.1
GBR	0.014	0.020	0.038	1.6	2.3	4.3
IRL	0.024	0.029	0.042	1.3	1.6	2.3
POL	0.023	0.019	0.047	1.9	1.6	3.9
BRA	0.020	0.031	0.049	1.6	2.5	3.9
CHN	0.010	0.035	0.062	2.1	7.7	13.7
TWN	0.014	0.037	0.074	1.1	2.9	5.9
MEX	0.027	0.052	0.081	1.4	2.7	4.2
IDN	0.036	0.057	0.111	1.6	2.5	4.9
TUR	0.089	0.132	0.217	1.2	1.8	2.9
median	0.009	0.013	0.023	1.5	1.8	3.3

Table C.5: TFP gain - markups on labor only

	TFP gain		
$(\varepsilon_\sigma, \varepsilon_\rho)$	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)
BEL	0.001	0.001	0.001
CAN	0.001	0.001	0.002
EST	0.001	0.001	0.002
CZE	0.001	0.001	0.002
SWE	0.001	0.002	0.003
AUS	0.002	0.002	0.004
HUN	0.001	0.002	0.004
FRA	0.003	0.003	0.005
USA	0.002	0.003	0.005
DEU	0.002	0.003	0.005
FIN	0.002	0.003	0.005
KOR	0.001	0.002	0.006
SVK	0.002	0.003	0.006
CHN	0.002	0.004	0.007
JPN	0.002	0.004	0.007
PRT	0.002	0.004	0.007
AUT	0.003	0.004	0.007
ESP	0.003	0.005	0.008
RUS	0.003	0.005	0.008
BGR	0.005	0.007	0.010
NLD	0.006	0.007	0.011
LTU	0.005	0.007	0.012
DNK	0.009	0.010	0.013
GBR	0.005	0.007	0.013
GRC	0.005	0.008	0.014
IND	0.005	0.008	0.015
BRA	0.007	0.010	0.015
ROU	0.007	0.009	0.015
IRL	0.010	0.012	0.017
SVN	0.012	0.013	0.017
TWN	0.007	0.012	0.020
POL	0.008	0.010	0.021
MEX	0.013	0.017	0.027
IDN	0.018	0.023	0.036
TUR	0.031	0.041	0.054
median	0.003	0.005	0.008

Table C.6: IO amplification factor - markups on labor only

	no recalibration of β_i			recalibration $\beta_i = (1-\alpha_i)s_i$		
$(\varepsilon_\sigma, \varepsilon_\rho)$	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)
BEL	0.5	0.7	1.2	0.4	0.6	1.0
CAN	0.5	0.7	1.2	0.4	0.6	1.0
EST	0.5	0.7	1.4	0.3	0.5	1.0
CZE	0.4	0.7	1.5	0.3	0.5	1.0
SWE	0.4	0.6	1.0	0.4	0.6	1.0
AUS	0.5	0.7	1.2	0.4	0.6	1.0
HUN	0.5	0.8	1.7	0.3	0.5	1.0
FRA	0.7	0.8	1.2	0.6	0.7	1.0
USA	0.4	0.7	1.1	0.4	0.6	1.0
DEU	0.7	0.9	1.9	0.4	0.5	1.0
FIN	0.4	0.7	1.1	0.4	0.6	1.0
KOR	0.5	0.8	2.0	0.2	0.4	1.0
SVK	0.6	0.7	1.5	0.4	0.5	1.0
CHN	0.4	0.8	1.5	0.3	0.6	1.0
JPN	0.4	0.7	1.3	0.3	0.6	1.0
PRT	0.5	0.7	1.3	0.4	0.5	1.0
AUT	0.6	0.8	1.5	0.4	0.5	1.0
ESP	0.5	0.8	1.3	0.4	0.6	1.0
RUS	0.4	0.7	1.1	0.4	0.6	1.0
BGR	0.6	0.9	1.4	0.4	0.6	1.0
NLD	0.8	1.0	1.5	0.5	0.7	1.0
LTU	0.6	0.8	1.3	0.5	0.6	1.0
DNK	0.8	0.9	1.2	0.7	0.8	1.0
GBR	0.6	0.8	1.5	0.4	0.6	1.0
GRC	0.5	0.8	1.3	0.4	0.6	1.0
IND	0.4	0.7	1.3	0.3	0.6	1.0
BRA	0.6	0.8	1.2	0.5	0.7	1.0
ROU	0.6	0.8	1.3	0.5	0.6	1.0
IRL	0.5	0.7	0.9	0.6	0.7	1.0
SVN	0.7	0.8	1.0	0.7	0.8	1.0
TWN	0.6	1.0	1.6	0.4	0.6	1.0
POL	0.6	0.9	1.8	0.4	0.5	1.0
MEX	0.7	0.9	1.4	0.5	0.6	1.0
IDN	0.8	1.0	1.6	0.5	0.6	1.0
TUR	0.4	0.6	0.7	0.6	0.8	1.0
median	0.5	0.8	1.3	0.4	0.6	1.0

Notes: The Input-Output amplification factor is computed under two cases: when the no-linkages economy is defined as $\alpha_i = 0$, and when the no-linkages economy's β_i are recalibrated to maintain the size of each sector identical to their size in the economy with sectoral linkages.

Table C.7: TFP gain and IO amplification - setting positive and negative markups to zero

	TFP gain			IO amplification factor		
$(\varepsilon_\sigma, \varepsilon_\rho)$	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)
BEL	0.002	0.002	0.004	1.2	1.3	2.4
CAN	0.003	0.004	0.007	1.7	2.7	4.3
SWE	0.004	0.005	0.010	1.2	1.7	3.1
CZE	0.005	0.006	0.012	1.2	1.4	2.7
EST	0.003	0.007	0.013	1.5	4.1	7.9
HUN	0.007	0.007	0.015	1.3	1.4	2.7
AUS	0.005	0.008	0.016	1.5	2.2	4.3
DEU	0.007	0.008	0.016	1.2	1.4	2.7
FIN	0.010	0.011	0.019	1.3	1.5	2.5
JPN	0.008	0.010	0.021	1.2	1.5	3.1
USA	0.004	0.013	0.022	1.0	3.1	5.1
RUS	0.010	0.015	0.026	1.1	1.7	2.9
FRA	0.019	0.019	0.026	1.2	1.3	1.7
PRT	0.010	0.014	0.028	1.5	2.0	4.0
GRC	0.013	0.018	0.030	1.0	1.3	2.3
AUT	0.017	0.019	0.032	1.2	1.3	2.3
SVK	0.011	0.019	0.032	1.7	2.8	4.9
LTU	0.014	0.020	0.032	1.2	1.7	2.9
ESP	0.020	0.021	0.033	1.1	1.2	1.8
NLD	0.019	0.020	0.034	1.3	1.4	2.3
KOR	0.011	0.012	0.035	1.1	1.3	3.8
IND	0.008	0.019	0.037	0.7	1.6	3.1
DNK	0.026	0.027	0.039	1.1	1.2	1.7
BGR	0.017	0.022	0.040	1.3	1.7	3.1
GBR	0.019	0.026	0.046	1.3	1.8	3.2
SVN	0.029	0.030	0.047	1.2	1.3	2.0
IRL	0.027	0.033	0.047	1.3	1.5	2.2
BRA	0.023	0.034	0.054	1.4	2.1	3.3
CHN	0.010	0.035	0.063	2.1	7.2	12.8
ROU	0.038	0.046	0.071	1.5	1.8	2.9
TWN	0.014	0.037	0.074	1.1	2.9	5.9
MEX	0.031	0.056	0.086	1.3	2.4	3.8
POL	0.053	0.060	0.108	1.7	1.9	3.4
IDN	0.039	0.060	0.118	1.5	2.4	4.7
TUR	0.100	0.144	0.232	1.2	1.7	2.7
median	0.013	0.019	0.032	1.2	1.7	3.1

Table C.8: TFP gain and IO amplification - no recalibration

	TFP gain			IO amplification factor		
$(\varepsilon_\sigma, \varepsilon_\rho)$	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)
BEL	0.002	0.002	0.003	1.3	1.3	2.3
KOR	0.004	-0.000	0.005	1.4	-0.0	1.6
CZE	0.003	0.003	0.006	1.6	1.8	3.3
CAN	0.003	0.004	0.006	1.8	2.7	4.0
HUN	0.004	0.004	0.008	1.6	1.5	2.9
SWE	0.004	0.005	0.008	1.2	1.7	2.6
DEU	0.005	0.005	0.009	1.6	1.6	3.2
AUT	0.007	0.005	0.009	1.4	1.1	2.0
FRA	0.007	0.007	0.009	1.9	1.8	2.4
EST	0.003	0.007	0.010	1.7	4.1	6.1
FIN	0.008	0.008	0.011	1.5	1.5	2.3
AUS	0.005	0.007	0.012	1.6	2.3	3.7
ESP	0.009	0.009	0.013	1.5	1.5	2.1
JPN	0.007	0.008	0.014	1.3	1.5	2.5
NLD	0.012	0.012	0.016	1.7	1.7	2.3
DNK	0.015	0.015	0.018	1.4	1.4	1.6
PRT	0.009	0.011	0.018	1.6	2.2	3.4
USA	0.004	0.013	0.018	1.0	3.1	4.3
RUS	0.009	0.013	0.019	1.2	1.8	2.6
ROU	0.018	0.013	0.021	1.6	1.2	1.8
GRC	0.011	0.015	0.021	1.0	1.4	1.9
SVK	0.009	0.015	0.022	2.1	3.6	5.3
BGR	0.012	0.017	0.023	1.6	2.2	3.0
LTU	0.012	0.017	0.023	1.3	1.9	2.6
SVN	0.019	0.018	0.024	1.2	1.1	1.4
GBR	0.015	0.020	0.028	1.7	2.3	3.2
IND	0.008	0.018	0.028	0.7	1.6	2.5
IRL	0.026	0.029	0.036	1.4	1.6	2.0
BRA	0.022	0.031	0.039	1.8	2.5	3.1
POL	0.024	0.019	0.040	2.0	1.6	3.4
CHN	0.012	0.035	0.046	2.6	7.7	10.2
TWN	0.017	0.037	0.048	1.3	2.9	3.8
MEX	0.031	0.052	0.063	1.6	2.7	3.3
IDN	0.046	0.057	0.071	2.0	2.5	3.1
TUR	0.098	0.132	0.142	1.3	1.8	1.9
median	0.009	0.013	0.018	1.6	1.8	2.6

Table C.9: TFP gain and IO amplification - Winsorized

	TFP gain			IO amplification factor		
$(\varepsilon_\sigma, \varepsilon_\rho)$	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)
BEL	0.002	0.002	0.003	1.3	1.3	2.5
SVN	0.004	0.003	0.006	1.4	1.1	2.2
CAN	0.003	0.004	0.007	1.7	2.7	4.4
KOR	0.004	0.001	0.008	1.4	0.5	2.8
HUN	0.004	0.004	0.009	1.6	1.5	3.4
SWE	0.003	0.005	0.009	1.2	1.7	3.2
AUT	0.006	0.005	0.009	1.4	1.2	2.1
FRA	0.007	0.007	0.010	1.8	1.8	2.6
DEU	0.004	0.005	0.011	1.6	1.6	3.8
DNK	0.009	0.008	0.013	1.4	1.3	2.0
FIN	0.007	0.008	0.013	1.5	1.5	2.7
EST	0.003	0.007	0.013	1.5	4.1	8.0
NLD	0.008	0.009	0.015	1.6	1.6	2.7
AUS	0.005	0.007	0.015	1.5	2.3	4.5
ESP	0.009	0.009	0.016	1.4	1.5	2.5
JPN	0.007	0.008	0.018	1.3	1.6	3.2
USA	0.004	0.013	0.022	1.0	3.1	5.1
GRC	0.010	0.013	0.022	1.0	1.4	2.4
PRT	0.008	0.011	0.023	1.5	2.2	4.3
RUS	0.009	0.013	0.023	1.2	1.8	3.3
ROU	0.016	0.013	0.026	1.5	1.2	2.5
SVK	0.008	0.015	0.027	1.9	3.6	6.4
BGR	0.010	0.015	0.027	1.6	2.3	4.2
LTU	0.011	0.017	0.029	1.3	1.9	3.2
TWN	0.007	0.016	0.030	0.9	2.3	4.2
LVA	0.009	0.018	0.032	1.2	2.4	4.3
POL	0.015	0.017	0.037	1.6	1.8	3.9
GBR	0.014	0.020	0.038	1.6	2.3	4.3
IRL	0.024	0.029	0.042	1.3	1.6	2.3
BRA	0.020	0.031	0.049	1.6	2.5	3.9
IDN	0.020	0.031	0.056	1.6	2.6	4.5
CHN	0.010	0.035	0.062	2.1	7.7	13.7
MEX	0.027	0.052	0.081	1.4	2.7	4.2
TUR	0.034	0.067	0.121	1.5	2.9	5.3
median	0.008	0.012	0.022	1.5	1.8	3.6

Table C.10: TFP gain and IO amplification - Real estate unchanged

	TFP gain			IO amplification factor		
$(\varepsilon_\sigma, \varepsilon_\rho)$	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)	(0.01,0.01)	(0.70,0.01)	(1.00,1.00)
BEL	0.002	0.002	0.003	1.3	1.3	2.5
SVN	0.002	0.001	0.004	2.2	1.2	4.7
ITA	0.003	-0.005	0.005	3.8	-7.2	6.4
CZE	0.003	0.003	0.007	1.6	1.8	3.9
CAN	0.003	0.004	0.007	1.7	2.7	4.4
KOR	0.004	-0.000	0.007	1.4	-0.0	2.4
SWE	0.003	0.004	0.008	1.2	2.0	3.7
HUN	0.004	0.004	0.009	1.6	1.5	3.4
FRA	0.007	0.007	0.010	1.8	1.8	2.6
AUT	0.007	0.005	0.010	1.4	1.1	2.2
DEU	0.004	0.005	0.011	1.6	1.6	3.8
EST	0.002	0.006	0.012	1.7	5.1	9.9
FIN	0.007	0.008	0.013	1.5	1.5	2.7
AUS	0.005	0.007	0.015	1.5	2.3	4.5
ESP	0.009	0.009	0.016	1.4	1.5	2.5
GRC	0.006	0.010	0.017	1.0	1.7	2.9
JPN	0.007	0.008	0.018	1.3	1.5	3.2
USA	0.003	0.011	0.019	0.9	4.1	6.8
NLD	0.011	0.012	0.021	1.6	1.7	3.0
DNK	0.015	0.015	0.023	1.4	1.4	2.1
PRT	0.008	0.011	0.023	1.5	2.2	4.3
RUS	0.009	0.013	0.023	1.2	1.8	3.3
SVK	0.008	0.015	0.027	1.9	3.6	6.4
ROU	0.017	0.013	0.027	1.5	1.2	2.4
LTU	0.011	0.017	0.029	1.3	1.9	3.2
IND	0.005	0.016	0.032	0.6	1.8	3.7
BGR	0.012	0.017	0.033	1.5	2.2	4.3
TWN	0.006	0.022	0.035	1.1	3.8	6.1
GBR	0.014	0.020	0.038	1.6	2.3	4.3
IRL	0.024	0.029	0.042	1.3	1.6	2.3
POL	0.023	0.019	0.047	1.9	1.6	3.9
BRA	0.020	0.031	0.049	1.6	2.5	3.9
CHN	0.010	0.035	0.062	2.1	7.7	13.7
MEX	0.027	0.052	0.081	1.4	2.7	4.2
IDN	0.036	0.057	0.109	1.6	2.5	4.9
TUR	0.032	0.073	0.132	2.0	4.5	8.1
median	0.007	0.011	0.020	1.5	1.8	3.6