

Earnings Inequality and the Minimum Wage: Evidence from Brazil

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Earnings Inequality and the Minimum Wage: Evidence from Brazil*

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Abstract

We show that an increase in the minimum wage can have large effects throughout the earnings distribution, using a combination of theory and evidence. To this end, we develop an equilibrium search model featuring empirically relevant worker and firm heterogeneity. The minimum wage induces firms to adjust their equilibrium wage and vacancy policies, leading to spillovers on higher wages. We use the estimated model to evaluate the effects of a 119 percent increase in the real minimum wage in Brazil from 1996 to 2012. The policy change explains a large decline in earnings inequality, with spillovers reaching up to the 80th percentile of the earnings distribution. At the same time, employment and output fall only modestly as workers relocate to more productive firms. Using administrative linked employer-employee data and two household surveys, we find reduced-form evidence in support of the model predictions.

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1 Introduction

To what extent do minimum wage policies shape earnings inequality? We show that—contrary to competitive labor market theories—a change in the wage floor can have large effects throughout the earnings distribution. Using the case of Brazil, where the real minimum wage increased by 119 percent from 1996 to 2012, we find that the policy change induced a notable decline in earnings inequality, with spillovers reaching up to the 80th percentile of the earnings distribution.

Our analysis proceeds in three steps. In the first step, we develop a version of the canonical [Burdett and Mortensen \(1998\)](#) model with worker and firm heterogeneity to assess the equilibrium consequences of the minimum wage. Workers who differ in ability and value of leisure engage in undirected job search, both from unemployment and on the job, in labor markets segmented by worker type. Firms that differ in productivity post wages and vacancies in each market separately. The equilibrium wage equation includes as a special case the specification due to [Abowd, Kramarz, and Margolis \(1999\)](#), henceforth AKM). This framework nests two important benchmark models of the labor market: that of perfect competition where workers are paid their marginal product, and the monopsony outcome where workers are paid their outside option. In between those two extremes, the minimum wage induces spillovers on higher earnings percentiles, as all firms within affected markets adjust their wage and vacancy policies in equilibrium. Hence, the strength of spillovers depends on the microstructure of the labor market.

Our main contribution is to quantify the spillover effects of a minimum wage increase in Brazil. A difficulty with studying the effects of the minimum wage in previous work has been the small, transient nature of policy variation and data limitations. The apparent size of the minimum wage increase and the availability of administrative linked employer-employee data make Brazil a natural testing ground for our theory. Thus, in the second step, we estimate the structural model via a mix of nonparametric identification and the method of simulated moments, using the AKM specification as an auxiliary framework. By estimating the parameters guiding labor market fluidity and heterogeneity among workers and firms, we pin down labor market competition and hence the strength of spillovers in our model. The estimated model replicates several untargeted features of the wage distribution and wage dynamics in the data. We then simulate the effects of the observed minimum wage increase on the distribution of wages and macroeconomic outcomes.

We find that the minimum wage induces a 14 log points fall in the variance of wages, with over

half of the total impact due to equilibrium effects. While wage compression is most pronounced at the bottom of the distribution, spillovers reach up to the 80th percentile. In line with recent empirical findings by [Alvarez et al. \(2018\)](#), the policy leads to a sizable fall in frictional wage dispersion for identical workers across employers by reducing the firm productivity pay premium. At the same time, we find a muted negative employment and output response. The minimum wage squeezes firm profits, leading firms to post fewer vacancies, but more so at low-productivity firms. This effect is counteracted, however, by lower labor market congestion. Overall, this results in equilibrium relocation of workers to more productive firms and associated efficiency gains.

In the third step, we confront our model with novel empirical facts on the impact of the minimum wage in Brazil using administrative data and two household surveys. Consistent with our model predictions, we estimate compression up to the 80th wage percentile due to spillovers, which we identify off variation in the effective bindingness of the minimum wage across Brazilian regions over time ([Lee, 1999](#); [Autor et al., 2016](#)). These results are striking given that only around 2 percent of workers earn the minimum wage. Our data also allow us to test for effects on employment, including formal and informal sector jobs, as well as on firm exit and entry. Extending our methodology, we confirm mild negative effects on employment, formality, and firm dynamics. We corroborate key predictions of the model, including the absence of a mass point at the minimum wage and worker relocation induced by the policy. Finally, we suggest a simple model-consistent test for the reach of minimum wage spillovers, which confirms our previous findings.

Related literature. We contribute to three strands of the literature. The first provides a structural assessment of minimum wage effects in frictional labor markets. [Eckstein and Wolpin \(1990\)](#) estimate a generalization of the [Albrecht and Axell \(1984\)](#) model with a minimum wage but abstract from within-firm wage differences. [Koning et al. \(1995\)](#) and [van den Berg and Ridder \(1998\)](#) use a wage posting model with on-the-job search to assess minimum wage effects on unemployment. [Burdett and Mortensen \(1998\)](#) and [Bontemps et al. \(1999\)](#) are the first to formalize the idea that minimum wage spillovers may affect higher wages in an equilibrium search model. [Flinn \(2006\)](#) highlights the importance of endogenous contact rates for optimal minimum wage levels in a search and bargaining framework. Most recently, [Flinn et al. \(2017\)](#) analyze minimum wage effects in a framework where firms endogenously choose whether or not to renegotiate wages as in [Postel-Vinay and Robin \(2002\)](#), [Dey and Flinn \(2005\)](#), and [Cahuc et al. \(2006\)](#). Relative to these papers, our contribution is to quantify the equilibrium effects of the minimum wage on wage

inequality, for which we provide supporting evidence from a large policy change in Brazil.

A second literature is concerned with reduced-form estimates of the impact of a minimum wage. A long list of papers has focused on employment effects, with summaries contained in [Card and Krueger \(1995\)](#) and [Neumark and Wascher \(2008\)](#). Most findings point to small negative effects on the number of jobs, though less is known for a minimum wage change as large as that in Brazil. Fewer studies examine the effects on wage inequality, although notable exceptions include [Grossman \(1983\)](#), [DiNardo et al. \(1996\)](#), and [Machin et al. \(2003\)](#). In a seminal contribution, [Lee \(1999\)](#) uses variation in the effective bindingness of the minimum wage across US states to estimate spillovers reaching high up in the distribution. In contrast, [Autor et al. \(2016\)](#) conclude that spillovers cannot be distinguished from measurement error due to data limitations in the Current Population Survey. Using administrative data and sizable policy variation, we document widespread wage effects and little displacement due to the minimum wage in Brazil—striking findings that we reconcile through the lens of our structural model.¹

Finally, our findings speak to the literature on changes in between-firm pay differences as a driver of inequality trends.² While the econometric framework by AKM has been widely used in applied empirical research, structural interpretations have proven problematic ([Gautier and Teulings, 2006](#); [Eeckhout and Kircher, 2011](#); [Lopes de Melo, 2018](#)). Consequently, the fundamental causes behind observed changes in the wage anatomy remain largely unexplored. A small number of papers have provided different microfoundations for the AKM specification in the cross section, including [Barlevy \(2008\)](#), [Bagger et al. \(2014a\)](#), and [Burdett et al. \(2011, 2016\)](#). We complement these works by using an equilibrium model that nests the AKM wage equation to quantify the effects of the minimum wage on compression in worker and firm pay components over time.

Outline. The paper proceeds as follows. Section 2 introduces the datasets, motivating facts, and background on the minimum wage in Brazil. Section 3 develops our equilibrium search model and characterizes the effects of the minimum wage in this environment. Section 4 estimates the model, which we use in Section 5 to quantify the equilibrium effects of the minimum wage. Section 6 provides empirical evidence in support of the model predictions. Finally, Section 7 concludes.

¹While we estimate minimum wage spillovers due to monopsony power in labor markets, similar mechanics arise in the context of comparative advantage in skill assignments ([Teulings, 1995, 2000, 2003](#)), fairness considerations ([Card et al., 2012](#)), hierarchical matching ([Lopes de Melo, 2012](#)) substitutability across tasks/goods ([Stokey, 2016](#)), educational investment ([Bárány, 2016](#)), and endogenous union formation ([Taschereau-Dumouchel, 2017](#)).

²See [Davis and Haltiwanger \(1991\)](#), [Dunne et al. \(2004\)](#), [Song et al. \(2016\)](#), [Barth et al. \(2016\)](#), and [Abowd et al. \(2018\)](#) for the US; [Cardoso \(1999\)](#) for Portugal; [Iranzo et al. \(2008\)](#) for Italy; [Nordström Skans et al. \(2009\)](#), [Akerman et al. \(2013\)](#), and [Lindqvist et al. \(2015\)](#) for Sweden; [Faggio et al. \(2010\)](#) for the UK; [Eriksson et al. \(2013\)](#) for the Czech Republic; [Card et al. \(2013\)](#) and [Kantenga and Law \(2016\)](#) for Germany; and [Helpman et al. \(2017\)](#) for Brazil.

2 Data and motivating facts

2.1 Data description

To examine a decline in earnings inequality in relation to a concurrent rise in the minimum wage in Brazil from 1996 to 2012, we combine an administrative dataset with two household surveys. An introduction to these data follows, with further details relegated to Appendix A.1.

Our main data source is the *Relação Anual de Informações Sociais (RAIS)*, a linked employer-employee register by the Brazilian Ministry of Labor and Employment. Firms' survey response is mandatory, and misreporting is deterred through audits and threat of fines. Collection started in 1986, with coverage becoming near universal from 1994 onward. The data contain detailed information on job characteristics, with 73 million formal sector employment spells recorded in 2012. Although reports are annual, we observe for every job spell the date of accession and separation in addition to average monthly earnings. We keep for each worker the highest-paid among each year's longest employment spells. As Brazil's minimum wage is set in terms of monthly earnings, henceforth we interchangeably refer to this income concept as "earnings" or "wages."³

A substantial fraction of Brazil's working-age population is not formally employed and hence not covered by the RAIS. To address this gap, we complement our analysis using data from the *Pesquisa Nacional por Amostra de Domicílios (PNAD)*, a nationally representative annual household survey. Respondents are asked to produce a formal work permit (*Carteira de Trabalho e Previdência Social assinada*). Following Meghir et al. (2015), we classify as informal all self-employed and those in remunerated employment without a work permit.

We also use a second household survey, the *Pesquisa Mensal de Emprego (PME)*, conducted in Brazil's six largest metropolitan regions. The advantage of these data is that they feature for every respondent two four-month interview spells separated by eight months. Starting in 2002, this short panel component allows us to compute transition rates of workers between all employment states. For presentation purposes, we label formal sector workers as "employed," and pool informal sector workers and the unemployed under the label "nonemployed." We distinguish between the disaggregated categories in our empirical analysis of minimum wage effects later.

While each of these three datasets is geared at slightly different subpopulations and labor

³We also observe contractual hours, although we find little cross-sectional dispersion and changes over time along this margin, see Section 6.2, plausibly due to Brazil's rigid labor laws restricting part-time work arrangements.

market questions, together they provide a holistic picture of Brazil’s labor market. We restrict attention to male workers of age 18–49 to avoid issues related to female labor force participation and retirement. Table 1 presents summary statistics for this worker group. The RAIS data show that between 1996 and 2012, Brazil experienced an 18 log points increase in mean formal sector wages while the standard deviation declined by 19 log points—a striking compression visualized in Figure 19 of Appendix A.2. While the age distribution remained stable, there was a significant increase in educational attainment over this period. Using the PNAD survey data, we confirm congruent trends in the formal sector wage distribution. Relative to the formal sector, informal wages are initially characterized by lower levels but similar dispersion. Throughout 2012, the informal sector wage distribution saw an increase in its mean accompanied by mild compression. At the same time, the employment rate remained stable while the formal employment share rose by eight percentage points. Consistent with the increase in formality, the longitudinal PME data show a slight rise in the inflow rate into formal employment and a decline in the outflow rate.

Table 1. Summary statistics from three main datasets, 1996 and 2012

<i>Panel A. Linked employer-employee data (RAIS)</i>				
	1996		2012	
	Mean	St.d.	Mean	St.d.
Age	31.69	8.37	32.05	8.51
Years of education	7.78	3.92	10.73	3.18
Real wage (log BRL 2012, formal sector)	7.02	0.86	7.20	0.67
Observations	16,308,762		28,578,057	
<i>Panel B. Cross-sectional household survey (PNAD)</i>				
	1996		2012	
	Mean	St.d.	Mean	St.d.
Real wage (log BRL 2012, formal sector)	7.01	0.81	7.13	0.62
Real wage (log BRL 2012, informal sector)	6.26	0.81	6.56	0.78
Employment rate	0.95		0.95	
Formal employment share	0.68		0.76	
Observations	74,487		86,031	
<i>Panel C. Longitudinal household survey (PME)</i>				
	2002		2012	
	Mean	St.d.	Mean	St.d.
Transition rate nonemployed-employed	0.08		0.10	
Transition rate employed-nonemployed	0.05		0.04	
Observations	94,280		121,211	

Notes: Statistics are for males of age 18–49. Real wage is average (RAIS) or usual (PNAD) monthly earnings. Respondents are classified as employed if they are a domestic worker, employee, or self-employed. Formal employment is defined as being employed and having a legal work permit. Transition rates are conditional on initial labor market status, divided into employed (formal) and nonemployed (unemployed + informal). See Figures 17–18 and Tables 8–9 in Appendix A.1 for further details. Source: RAIS, PNAD, PME.

2.2 Motivating facts about Brazil's inequality decline

What explains Brazil's inequality decline over this period? [Alvarez et al. \(2018\)](#) advance a statistical decomposition of the inequality evolution in Brazil's formal sector. Noting that seemingly identical workers experience large pay differences across firms, they use the RAIS data to decompose wage differences into worker and firm heterogeneity. Specifically, they estimate a two-way fixed effects framework due to AKM, decomposing log wages w_{ijt} of individual i working at firm j in year t within five-year periods as

$$w_{ijt} = \alpha_i + \alpha_j + \gamma_t + \varepsilon_{ijt}, \quad (1)$$

where α_i denotes an individual fixed effect, α_j denotes a firm fixed effect, γ_t is a year dummy, and ε_{ijt} a residual subject to the strict exogeneity condition $\mathbb{E}[\varepsilon_{ijt} | i, j, t] = 0$.⁴

Table 2 presents the variance decomposition that results from estimating equation (1) over repeated time windows. In the initial period 1996–2000, half of the total variance of wages of 69 log points is due to worker pay heterogeneity, while one quarter is due to the same individual getting paid differently across different employers. Between 1996–2000 and 2008–2012, the total variance dropped by 23 log points, primarily due to a decline in between-firm pay dispersion, which constitutes 40 percent of the overall inequality decline over this period.⁵

Table 2. AKM variance decomposition, 1996–2000 and 2008–2012

	1996–2000	2008–2012	Change
Total variance of log wages, $Var(w_{ijt})$	0.69	0.47	-0.23
Variance of worker fixed effects, $Var(\hat{\alpha}_i)$	0.34	0.27	-0.07
Variance of firm fixed effects, $Var(\hat{\alpha}_j)$	0.16	0.07	-0.09
2×Covariance b/w workers and firms, $2 \times Cov(\hat{\alpha}_i, \hat{\alpha}_j)$	0.14	0.09	-0.05
Residual variance, $Var(\hat{\varepsilon}_{ijt})$	0.06	0.04	-0.02
Observations	81,504,144	132,219,648	
R^2	0.92	0.92	

Notes: Predicted variances (shares) due to components in log wage decomposition $w_{ijt} = \alpha_i + \alpha_j + \gamma_t + \varepsilon_{ijt}$. Omitted are variance terms involving year dummies γ_t , which account for a negligible share of the total variance. Source: [Alvarez et al. \(2018\)](#) using RAIS.

⁴Equation (1) is identified off workers switching employers across years for the largest set of individuals at firms connected through worker flows. There has been a fruitful debate around the merits and potential biases of this framework, including recent work by [Andrews et al. \(2008\)](#); [Eeckhout and Kircher \(2011\)](#); [Bonhomme et al. \(2017\)](#); [Lopes de Melo \(2018\)](#); [Card et al. \(2018\)](#); and [Borovičková and Shimer \(2018\)](#). [Alvarez et al. \(2018\)](#) present a battery of specification tests and robustness checks, and conclude that the model describes well the Brazilian data during this period.

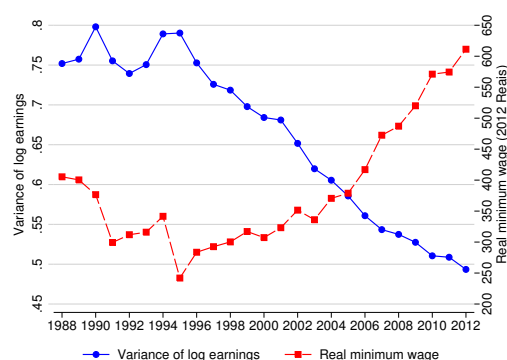
⁵In an alternative decomposition, firm heterogeneity accounts for $Var(\hat{\alpha}_j) / (Var(\hat{\alpha}_i) + Var(\hat{\alpha}_j)) = 33\%$ of initial wage dispersion and $\Delta Var(\hat{\alpha}_j) / (\Delta Var(\hat{\alpha}_i) + \Delta Var(\hat{\alpha}_j)) = 58\%$ of the change between periods.

2.3 The minimum wage in Brazil

Motivated by Brazil's inequality decline between 1996 and 2012, we turn to a salient change in the labor market over this period: the rise in the minimum wage.⁶ Brazil's statutory minimum wage is set at the federal level and stated in terms of a monthly earnings floor. There are no provisions for legal subminimum or differentiated minimum wages across demographics or economic subdivisions (Lemos et al., 2004).⁷ The nominal minimum wage is customarily adjusted once a year according to a predetermined formula that depends on realized inflation from last year plus realized GDP growth from two years prior. In practice, under various governments the calculation has been subject to discretionary adjustments in consultation with Brazil's tripartite body.

Brazil's real minimum wage had deteriorated under high inflation before 1996 when a switch in government ignited a gradual ascent of the wage floor by 119 percent in real terms, reaching 622 BRL or 410 PPP-adjusted USD per month by 2012. Accounting for aggregate real wage growth, this corresponds to a 56.8 log points rise in the minimum wage relative to mean wages in the formal sector. To put these numbers into context, the minimum wage as a fraction of median wages increased from around 34 percent in 1996 to 60 percent in 2012. The negative comovement of the minimum wage and the variance of log earnings over the preceding 25-year period, shown in Figure 1, suggests that the minimum wage may be related to inequality dynamics in Brazil.⁸

Figure 1. Evolution of earnings inequality and the real minimum wage in Brazil, 1988–2012



Notes: Statistics are for males of age 18–49. Real minimum wage is the annual average of the monthly time series. Source: RAIS, IPEA.

⁶While Brazil enacted other social policies during the mid-2000s, such as the *Bolsa Família* transfer program for needy families launched in 2003, the minimum wage predates many of these policies and coincides with the timing of Brazil's inequality decline starting in 1996.

⁷The minimum wage is set for full-time workers with 44-hour contracts and adjusted proportionately for part-time workers. Using information on hours in the RAIS and PNAD data, we find a small initial share of such workers and no significant changes related to the minimum wage over time. Special labor contracts allow for parts of the minimum wage to be paid in-kind in the form of accommodation and food, although in the PNAD data only 0.8 percent of workers report receiving nonmonetary remuneration in 1996, and 0.3 percent of workers in 2012.

⁸By exploiting cross-sectional variation in the data, our empirical methodology will identify effects of the minimum wage net of aggregate trends, so the fact that inequality declined in Brazil over this period is not crucial to our analysis.

To what extent can the rise in the minimum wage account for Brazil’s concurrent inequality decline from 1996 to 2012? Evaluating the effects of the minimum wage in Brazil over this period, as well as designing such policies in other contexts, requires a model that is consistent with the key roles of both worker heterogeneity and pay dispersion for identical workers across firms.

3 Equilibrium model

This section develops a version of the [Burdett and Mortensen \(1998\)](#) equilibrium model with worker and firm heterogeneity that we use to assess the effects of a minimum wage increase. In line with salient empirical facts, this framework can generate endogenous wage dispersion for identical workers across employers. The Burdett-Mortensen model is widely used to study wage determination, and our exposition closely follows that of [Bontemps et al. \(1999, 2000\)](#); [Mortensen \(2003\)](#); and [Jolivet et al. \(2006\)](#). Our contribution is to allow for lots of empirically relevant heterogeneity in a tractable manner in order to use the estimated framework for quantitative analysis.

3.1 Environment

We study a stationary economy cast in continuous time that consists of a unit mass of infinitely-lived workers and a mass M_0 of firms who meet in a frictional labor market.

Workers. Workers differ in ability level $\theta \stackrel{\mathcal{L}}{\sim} H(\cdot)$ over support $[\underline{\theta}, \bar{\theta}]$. They can be employed or nonemployed, the latter of which we map to the pool of unemployed plus informally employed in the data later. Workers value a stream of consumption equal to their wage when employed or b_θ when nonemployed, discounted at rate ρ . In both states, workers search for jobs within markets segmented by ability type, as in [van den Berg and Ridder \(1998\)](#), which can be thought of as a continuum of separate Burdett-Mortensen economies with parameters indexed by θ .

Let λ_θ^u denote the job offer arrival rate for the nonemployed and $\lambda_\theta^e = s_\theta \lambda_\theta^u$, for fixed s_θ , the arrival rate for the employed. A job offer entails a wage draw $w \sim F_\theta(\cdot)$ over support $[\underline{w}_\theta, \bar{w}_\theta]$. Although workers take arrival rates and the wage offer distribution as given, both are determined endogenously through firms’ equilibrium vacancy and wage posting decisions, possibly subject to a minimum wage. Matches dissolve exogenously at rate δ_θ , leading a share $u_\theta = \delta_\theta / (\delta_\theta + \lambda_\theta^u)$ of workers to be frictionally nonemployed. As employed workers gradually find higher-paying jobs

through on-the-job search, the realized wage distribution G_θ first-order stochastically dominates the wage offer distribution F_θ . Indeed, $G_\theta(w) = F_\theta(w) / (1 + \kappa_\theta(1 - F_\theta(w)))$, where $\kappa_\theta \equiv \lambda_\theta^e / \delta_\theta$ governs the effective speed of climbing up the job ladder.

The values of nonemployed workers, W_θ , and of workers employed at wage w , $S_\theta(w)$, satisfy

$$\begin{aligned}\rho W_\theta &= b_\theta + \lambda_\theta^u \int_{w_\theta}^{\bar{w}_\theta} \max\{S_\theta(w) - W_\theta, 0\} dF_\theta(w) \\ \rho S_\theta(w) &= w + \lambda_\theta^e \int_w^{\bar{w}_\theta} [S_\theta(w') - S_\theta(w)] dF_\theta(w') + \delta_\theta [W_\theta - S_\theta(w)].\end{aligned}$$

The optimal strategy of a nonemployed worker involves a reservation threshold ϕ_θ equal to the flow value of nonemployment plus the forgone option value of remaining nonemployed:

$$\phi_\theta = b_\theta + (\lambda_\theta^u - \lambda_\theta^e) \int_{\phi_\theta}^{\bar{w}_\theta} \frac{1 - F_\theta(w)}{\rho + \delta_\theta + \lambda_\theta^e(1 - F_\theta(w))} dw.$$

In contrast to [Albrecht and Axell \(1984\)](#), our model features heterogeneity in the reservation threshold across, but not within, θ -markets. We say the minimum wage is *binding* in market θ whenever $w^{\min} > \phi_\theta$ so that workers' reservation wage is $R_\theta = \max\{\phi_\theta, w^{\min}\}$.

Firms. Firms are characterized by a productivity level $p \stackrel{c}{\sim} \Gamma_0$ over support $[p_0, \bar{p}]$. They operate a linear production technology combining l_θ workers of each ability type θ to produce flow output

$$y(p, \{l_\theta\}_{\theta \in \Theta}) = p \int_{\theta \in \Theta} \theta l_\theta d\theta.$$

Motivated by [Flinn \(2006\)](#)'s insight that the endogeneity of contact rates has important implications for minimum wage effects, firms attract type- θ workers by posting v_θ job openings subject to cost $c_\theta(v_\theta) : c'_\theta, c''_\theta > 0$. The firm commits to a wage w_θ for its vacancies in market θ . Its wage rank $1 - F_\theta(w_\theta)$ together with its recruiting intensity v_θ jointly determine a firm's employment level $l_\theta(w_\theta, v_\theta)$. As production and the recruitment process are independent across markets, a productivity p firm's problem coincides with separate profit maximization in each market:

$$\forall \theta : \max_{w_\theta \geq w^{\min}, v_\theta} \{(p\theta - w_\theta) l_\theta(w_\theta, v_\theta) - c_\theta(v_\theta)\}.$$

A firm makes positive profits in market θ only if it posts a wage between workers' reservation

wage R_θ and its productivity p . Hence, there is an active mass of firms $M_\theta = M_0(1 - \Gamma_0(\underline{p}_\theta))$ distributed $\Gamma_\theta(p) = \Gamma_0(p | p > \underline{p}_\theta)$ with lower bound $\underline{p}_\theta = R_\theta/\theta$. Given optimal wage and vacancy posting policies $(w_\theta(p), v_\theta(p))$ in market θ , the wage offer distribution for a given aggregate vacancy mass $V_\theta = M_\theta \int_{\underline{p}_\theta}^{\bar{p}} v_\theta(p') d\Gamma_\theta(p')$ is simply $F_\theta(w_\theta(p)) = M_\theta \int_{\underline{p}_\theta}^p v_\theta(p') d\Gamma_\theta(p') / V_\theta$.

Matching. The effective pool of searching workers, $u + s(1 - u)$, and vacancy mass, V , together produce matches according to the Cobb-Douglas function $\chi[u + s(1 - u)]^{1-\alpha} V^\alpha$, where χ is a matching efficiency parameter and α governs the elasticity of matches with respect to vacancies. We can then express the nonemployed job finding rate as $\lambda_\theta^u = \chi(V_\theta / (u_\theta + s_\theta(1 - u_\theta)))^\alpha$, the employed job finding rate as $\lambda_\theta^e = s_\theta \lambda_\theta^u$, and firms' contact rate as $q_\theta = \chi((u_\theta + s_\theta(1 - u_\theta)) / V_\theta)^{1-\alpha}$.

3.2 Equilibrium effects of the minimum wage

We define, characterize, and outline a solution algorithm for a *search equilibrium with a minimum wage* in Appendix B.1–B.3. We illuminate here the model's mechanism giving rise to wage dispersion for identical workers across firms and the effects of the minimum wage in this environment.

We focus first on a single θ -market. Job-to-job mobility renders firms' wage and vacancy policies interdependent. In choosing a wage, firms take as given the distribution of competing wage offers F_θ and weigh two opposing forces. On the one hand, a lower wage increases per-worker profits. On the other hand, a higher wage rank raises steady-state employment through increased poaching and decreased voluntary quits. As has been well known since [Burdett and Mortensen \(1998\)](#), this trade-off leads more productive firms to post higher wages, leading to equilibrium wage dispersion for identical workers.⁹ Perturbations to this environment lead to *spillovers* between all employers in a θ -market, even if only a subset of firms is directly affected.

Concretely, let us consider the effects of a minimum wage raise between steady states. A set of firms will adjust their wage offers to comply with the new wage floor. As firm optimization induces the equilibrium wage offer distribution to be continuous and wages to be strictly increasing in productivity, other firms adjust wages in equilibrium to retain their pay rank. Such competitive pressure leads the minimum wage to spill over to higher-paying firms. Finally, fewer vacancy postings due to lower profit margins and firm exit will result in higher frictional unemployment.

⁹We focus here on the empirically relevant case of firm heterogeneity, which gives rise to a unique pure strategy equilibrium. In contrast, the model with homogeneous firms has a unique mixed strategy equilibrium with an upward-sloping wage density, different from the heterogeneous firms equilibrium considered here.

We now turn to the economy with a continuum of segmented markets. In markets where the minimum is binding, the strength of the above mechanism depends on the labor market configuration. Minimum wage bindingness is determined by workers' reservation wages. Markets where the minimum wage is nonbinding remain unaffected. In this sense, labor market segmentation puts a cap on the spillover effects that are omnipresent in the original [Burdett and Mortensen \(1998\)](#) framework. Evidently then, the mix of worker versus firm heterogeneity will be a crucial input to our estimated model when considering equilibrium effects of the minimum wage.

The assumption of perfect market segmentation shuts down spillovers between markets for the sake of tractability. Specifically, it allows us to characterize minimum wage effects that would plausibly extend to more general formulations. Furthermore, it delivers as an equilibrium outcome a structural wage equation resembling the AKM worker and firm fixed effects regression, which permits us to interpret the data through the lens of our model. We will present evidence in support of the assumption holding in our data, at least approximately. Consequently, we consider equilibrium spillovers between firms within each θ -market but abstract from any spillovers across θ -markets.¹⁰

Our framework relates closely to the empirical literature on pay decompositions into worker and firm heterogeneity started by AKM. To see this, with exogenous contact rates, $(\lambda_\theta^u, \lambda_\theta^e, \delta_\theta)$ constant across θ -markets, $b_\theta \propto \theta$ and a non-binding minimum wage, equilibrium wages in our model coincide with the log additive specification (1) that has been popular in empirical studies of the wage distribution and its changes over time:

$$\log w(p, \theta) = \alpha_i(\theta) + \alpha_j(p), \quad (2)$$

where the “worker effect” $\alpha_i(\theta) = \log \theta$ is an increasing function of ability, while the “firm effect” or piece rate $\alpha_j(p) = p - \int_{\underline{p}}^p [(1 - \Gamma_0(\underline{p}) + \kappa(1 - \Gamma_0(p)))/(1 - \Gamma_0(\underline{p}) + \kappa(1 - \Gamma_0(x)))]^2 dx$ is independent of worker ability and strictly increasing in firm productivity. Under more general parameterizations or a binding minimum wage, the exact decomposition in equation (2) is perturbed but the wage function $w(p, \theta; w^{\min})$ retains its important monotonicity properties.

How does the minimum wage affect wage inequality in this equilibrium framework? It is instructive to characterize the spillover effects of the minimum wage for a special case of the

¹⁰More generally, the degree of complementarity or substitutability between different worker types in the production function guides the direction and strength of intermarket linkages between wages and job openings.

model. We later confirm the generality of these results in numerical simulations.

Proposition. Assume exogenous contact rates, constant $(\lambda_\theta^u, \lambda_\theta^e, \delta_\theta) \in \mathbb{R}_{++}^3$ for all θ , and $b_\theta \propto \theta$. Then for markets where the minimum wage binds, $\{\theta | w^{\min} \geq \phi_\theta\}$, a marginal increase in the minimum wage

1. increases wages at all firms: $\partial w(p, \theta; w^{\min}) / \partial w^{\min} > 0 \forall p$;
2. decreases the productivity pay premium across firms: $\partial [\partial w(p, \theta; w^{\min}) / \partial p] / \partial w^{\min} < 0$; and
3. decreases the returns to worker ability: $\partial [\partial w(p, \theta; w^{\min}) / \partial \theta] / \partial w^{\min} < 0$.

Proof. See Appendix B.4. □

We interpret the proposition as follows. Part 1 states that wages increase for all workers with a reservation wage below the minimum wage. In the presence of search frictions, rent sharing is an equilibrium outcome, and the minimum wage acts as a transfer from firms to workers in the markets it affects. Part 2 characterizes the nature of spillovers between firms within a market. Wage increases at the initially lowest-paying firms are one-for-one with the minimum wage but gradually decline for higher-paying firms, leading to a flattening of the firm productivity-pay gradient. Finally, part 3 shows that among all affected markets, lower ability workers gain more from the minimum wage, leading to a flattening of the worker ability-pay gradient.

Our model nests two important benchmarks: that of perfectly competitive labor markets with workers paid their marginal product ($\lambda_\theta^e / \delta_\theta \rightarrow +\infty$), and the monopsony outcome where all observed wage heterogeneity reflects differences in workers' outside option ($\lambda_\theta^e / \delta_\theta = 0$). In both cases, though for different reasons, there is no "frictional wage dispersion" across firms so that the minimum wage induces no spillovers. For the intermediate range, the parameterization of the model determines the strength of equilibrium spillovers. Hence, the model's predictions for minimum wage effects depend crucially on estimates of the heterogeneous labor market parameters.

4 Estimation

We discipline the previous section's model with Brazil's RAIS linked employer-employee data. For our core exercise, we estimate structural model parameters to the "pre-period" 1996–2000 through a mix of nonparametric identification and the method of simulated moments via indirect inference (MSM-II) (Gourieroux et al., 1993; Smith, 1993). We use the estimated framework for the quantitative analysis of a rise in the minimum wage.

4.1 Estimation strategy

Our procedure exploits the model architecture to estimate key parameters in two stages, extending [Bontemps et al. \(2000\)](#), [Jolivet et al. \(2006\)](#), and [Bagger et al. \(2014a,b\)](#) to our environment with lots of heterogeneity. We pre-set a small number of “deep parameters.” The first estimation stage then uses ordinal information on unobserved worker and firm types to nonparametrically identify labor market parameters off worker flows. The second stage takes these parameters as given and estimates via MSM-II the distributions of worker ability and firm productivity, using the AKM two-way fixed effects specification in equation (1) as an auxiliary model.¹¹

Pre-set parameters. We set the elasticity of the aggregate matching function to $\alpha = 0.3$ ([Petrongolo and Pissarides, 2001](#)) and normalize match efficiency to $\chi \equiv 1$. We use the vacancy cost function $c_\theta(v) = c_\theta v^{1+c_1} / (1 + c_1)$ with curvature parameter $c_1 = 1$ ([Shephard, 2017](#)). We set the discount rate to $\rho = 0.0041$, corresponding to a 5 percent annual interest rate ([Hornstein et al., 2011](#)). The relative mass of firms, $M_0 = 0.05$, replicates a mean firm size of about 20 in the data.

First stage. The goal of the first stage is to estimate four labor market parameters by worker type: $\{\delta_\theta, \lambda_\theta^u, \lambda_\theta^e, R_\theta\}$. We begin by estimating on the 1996–2000 RAIS data a version of the worker and firm fixed effects model due to AKM as in equation (1): $y_{ijt} = \alpha_i + \alpha_j + \gamma_t + \varepsilon_{ijt}$. Recalling the strictly monotonic equilibrium wage mapping from our model, we group workers by estimated AKM worker fixed effects decile and rank firms continuously according to their estimated AKM firm effect. We nonparametrically identify labor market parameters by worker group off a monthly panel of worker flows, making use of the structural restrictions implied by the model:

1. We estimate the monthly separation rate as the average rate of leaving formal employment for at least one month: $\hat{\delta}_\theta = \mathbb{E}(\text{nonemployed}_{t+1} | \text{employed}_t, \theta)$.
2. We estimate the job hazard from nonemployment, $\hat{\lambda}_\theta^u$, by tracking workers for up to 24 months after leaving a formal sector job, and estimating via nonlinear least squares the following proportional hazard model: $\log \mathbb{P}(\# \text{ months until reentry} \geq t | \theta) = t \times \log(1 - \lambda_\theta^u)$.

¹¹An insightful strand of the literature has abandoned the assumption implicit here that workers and firms are globally rankable and instead allows for more flexible wage functions. See, for example, [Shimer and Smith \(2000\)](#); [Lise et al. \(2016\)](#); [Hagedorn et al. \(2017\)](#); [Bagger and Lentz \(2018\)](#). Absent monotonicity in the wage function, estimated AKM coefficients can be misleading vis-à-vis the respective economic models used by those authors. We show below that AKM, albeit misspecified, corresponds closely to the economic model from the previous section, making it an informative auxiliary model in our MSM-II procedure.

3. We map the rate of upward mobility across AKM firm fixed effects ranks into the effective speed of climbing up the job ladder, κ_θ . To this end, we exploit the model restriction $G_\theta(w) = F_\theta(w)/(1 + \kappa_\theta(1 - F_\theta(w)))$ using nonparametric density estimates of the AKM firm effects distribution, $\mathcal{G}_\theta(\hat{\alpha}_j)$, and AKM firm effects starting distribution from nonemployment, $\mathcal{F}_\theta(\hat{\alpha}_j)$, to estimate $\hat{\kappa}_\theta$.¹² Combined with our estimate of the separation rate, we obtain the job-to-job mobility parameter of interest using the model relation $\hat{\lambda}_\theta^e = \hat{\kappa}_\theta^e \times \hat{\delta}_\theta$ and hence $\hat{s}_\theta = \hat{\lambda}_\theta^e / \hat{\lambda}_\theta^u$.
4. We infer workers' reservation wage as the smallest accepted wage, $\mathcal{R}_\theta = \min_i \{w_\theta^i\}$, which [Flinn and Heckman \(1982\)](#) show is a strongly consistent estimator for the reservation wage R_θ in our model. To limit the influence of measurement error, we trim the lowest percentile of the starting wage distribution.

To simulate our model with more types, we feed a linear interpolation of the above labor market parameters into the computer, using 50 grid points in practice.

Second stage. The goal of the second stage is to estimate the distributions of worker ability and firm productivity. We assume that worker ability is distributed $\theta \sim \log \mathcal{N}(\mu, \sigma^2)$ with mean μ and standard deviation σ , and that firm productivity is distributed, $p \sim \text{Pareto}(\zeta)$ with tail parameter ζ and scale parameter normalized to one. While we could presumably improve the model fit by being more flexible, we find the parametric families describe the data reasonably well.

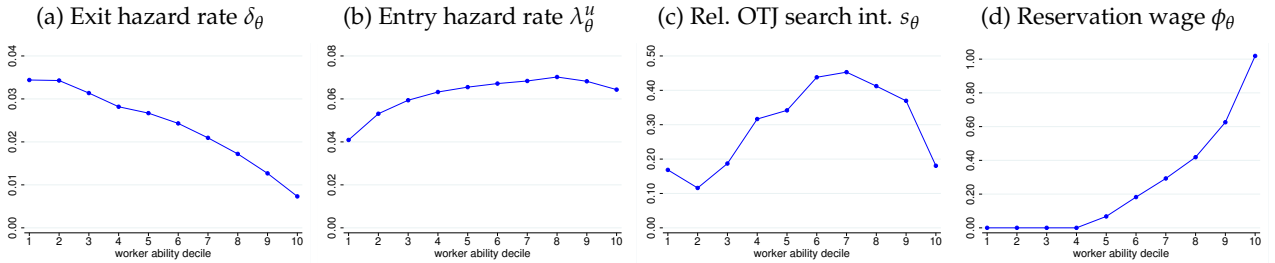
We simulate from our model a large number of worker histories. The indirect inference step consists of estimating the auxiliary model in equation (1) on simulated data, as we did on the RAIS microdata. We choose structural parameters to minimize the sum of equally weighted squared log differences between empirical and simulated moments. Heuristically, the following moments inform these parameters in the indirect inference step: The distance between mean wages and the minimum wage informs average worker ability, μ ; the dispersion in AKM worker fixed effects informs the standard deviation of worker ability, σ ; the dispersion in AKM firm effects informs the shape parameter of the firm productivity distribution, ζ ; and the pre-estimated value of λ_θ^u informs the vacancy cost intercept, c_θ . Although each of these moments is particularly informative about one particular parameter, all parameters are jointly determined.

¹²In a previous version of this paper, we obtained similar estimates for κ_θ using two additional model-consistent methods: one from a job duration regression and the other by comparing the wage distribution of new hires to that of the population.

4.2 Parameter estimates

First stage. Figure 2 shows the 40 estimated first-stage labor market parameters by worker ability decile. While the employment-to-nonemployment (EN) hazard in Brazil is similar to that in the US, the nonemployment-to-employment (NE) hazard is lower and more in line with continental Europe (Engbom, 2017). We find substantial heterogeneity in parameter estimates across worker ability groups. The EN hazard of the lowest worker decile is more than four times that for the highest decile,¹³ while the NE hazard is 32 percent lower and relative on-the-job search intensity is 53 percent lower. Reservation wages equal the minimum wage for the lowest four deciles of the ability distribution, and convex increasing thereafter. As we will see, this heterogeneity in labor market parameters implies substantial sorting of higher-paid workers to higher-paying firms.

Figure 2. Estimated labor market parameters by worker ability decile, 1996–2000



Notes: Each worker ability decile contains around 9 million observations. Source: RAIS.

Second stage. Table 3 shows estimated second-stage model parameters guiding worker and firm heterogeneity. A log-normal worker ability mean value of $\mu = 1.85$ and standard deviation of $\sigma = 0.48$ together with a firm productivity Pareto tail index of $\zeta = 7.70$ minimize the MSM-II criterion.¹⁴ In Appendix C.1, we vary two parameters at a time to verify that our criterion function is well behaved around the estimates. Appendix C.2 shows the estimated vacancy cost schedule.

Table 3. Estimated worker ability and firm productivity parameters, 1996–2000

Description	Parameter	Value	Target
Log-normal mean worker ability	μ	1.85	Min-to-mean wage ratio
Log-normal st.d. of worker ability	σ	0.48	Variance of AKM worker effects
Pareto tail index of firm productivity	ζ	7.70	Variance of AKM firm effects

Notes: Mean and variance of worker ability refer to log-normal distribution parameters. Tail index of firm productivity refers to shape parameter of the Pareto distribution, with mean firm productivity normalized to 1. Source: simulations.

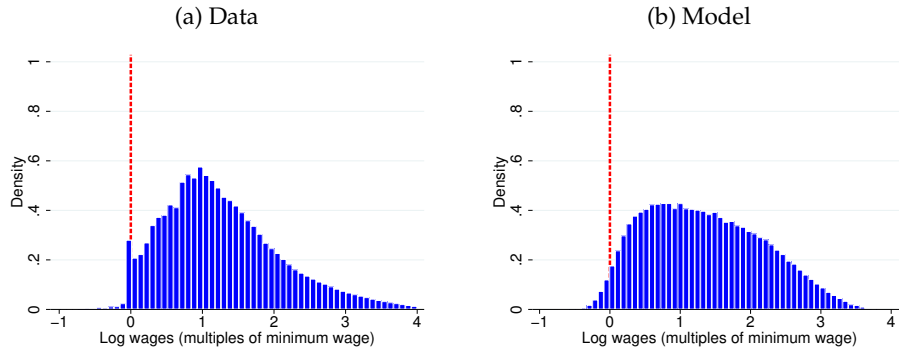
¹³Pessoa Araujo (2017) also finds a negative relation between separation rates and wages in Brazil’s RAIS data.

¹⁴Market segmentation ameliorates some challenges highlighted by previous work, as “frictional wage dispersion” or AKM firm effects constitute only 24 percent of total wage variance. Our model produces a sizable mean-min ratio of 2.62 for a mildly negative mean flow value of nonemployment, $\mathbb{E}_\theta[b_\theta] = -0.27$ (Hornstein et al., 2011). Similarly, we need no implausibly large values of productivity to match the right tail of the wage distribution (Bontemps et al., 2000).

4.3 Model fit

The estimated model successfully matches both targeted and untargeted moments of the cross-sectional wage distribution and labor market dynamics for Brazil’s “pre-period” 1996–2000. Figure 3 plots the empirical and simulated wage distributions in panels (a) and (b).¹⁵ The model overpredicts the mass of workers in the upper half of the distribution but overall does a good job despite only targeting three moments of the underlying worker ability and firm productivity distributions. While not targeted, the simulated model produces a labor share—defined as wage payments divided by output net of vacancy costs—of 0.62, which is close to its empirical counterpart of 0.55 from the Penn World Tables (Restrepo-Echavarria and Reinbold, 2017).

Figure 3. Data vs. model: Wage distribution in the model and data, 1996–2000



Notes: Histograms of wages constructed using 60 equi-spaced bins. For this figure only, model distribution includes added measurement error $\varepsilon \sim \log \mathcal{N}(0, 0.036)$ to match residual variance from AKM regression. Source: RAIS and simulations.

Table 4 shows that the model matches separately the variances of AKM worker and firm effects from the auxiliary regression. It also replicates the positive covariance between worker and firm components due to higher-ability workers’ faster speed of climbing the job ladder, κ_β . Overall, the model generates 98 percent of the variance of empirical log wages net of the residual.

Table 4. Data vs. model: AKM variance decomposition, 1996–2000

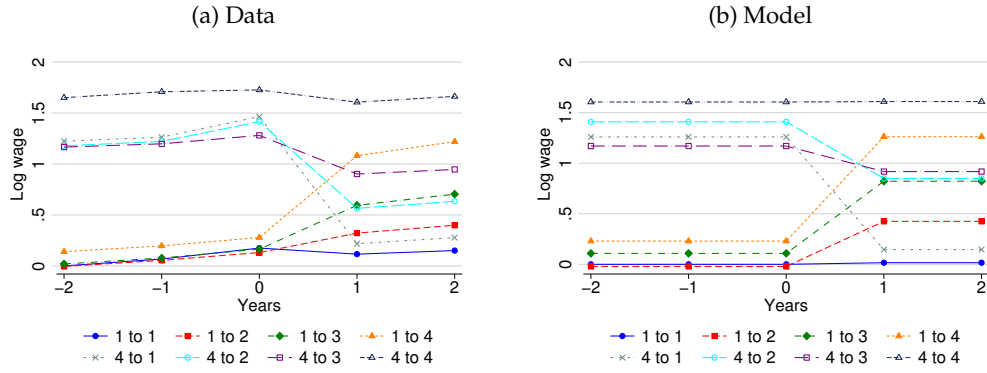
	Data	Model
Total variance of log wages, $Var(w_{ijt})$	0.69	0.62
Variance of worker fixed effects, $Var(\hat{\alpha}_i)$	0.34	0.33
Variance of firm fixed effects, $Var(\hat{\alpha}_j)$	0.16	0.16
2×Covariance b/w workers and firms, $2 \times Cov(\hat{\alpha}_i, \hat{\alpha}_j)$	0.14	0.12
Residual variance, $Var(\hat{\varepsilon}_{ijt})$	0.06	0.00

Notes: Predicted variances (shares) due to components in log wage decomposition $w_{ijt} = \alpha_i + \alpha_j + \gamma_t + \varepsilon_{ijt}$. Omitted are variance terms involving year dummies γ_t , which account for a negligible share of the total variance. Source: RAIS and simulations.

¹⁵For this figure only, we added a small amount of measurement error $\varepsilon \sim \log \mathcal{N}(0, 0.036)$ to simulated wages to match the residual variance from the AKM regression below. We have experimented with including the noise variance as a parameter in the estimation routine with similar results. Of course no subminimum wages would occur otherwise.

We also confront our model with two exercises that the literature has proposed as AKM diagnostic tools (Card et al., 2013). We view these as specification checks for the AKM regression on our model-generated data, and also validating our model as a good description of the Brazilian data. The first diagnostic tool is to check symmetry between wage gains and losses of switchers across the firm effects distribution. Figure 4 shows an event study of average wages for workers starting in the first and fourth firm effect quartiles. The qualitative and quantitative features of the empirical event study in panel (a) are captured well by the model equivalent in panel (b).

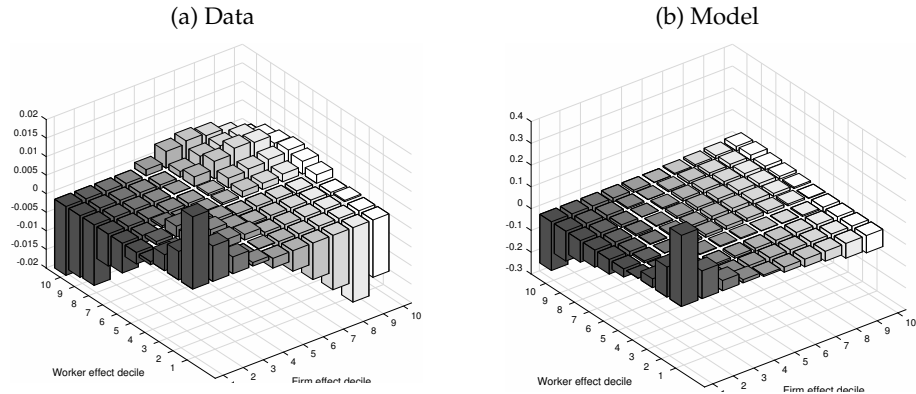
Figure 4. Data vs. model: Event study graph of wage gains from switching firms, 1996–2000



Notes: Figure plots changes in mean log wage upon switching employers between year 0 and year 1. Different lines show transitions from first and fourth quartiles of AKM firm fixed effects distribution for the period 1996–2000. Source: RAIS and simulations.

The second diagnostic tool is a residual plot to detect systematic deviations from AKM’s additive separability assumption. Comparing panels (a) and (b) of Figure 5 shows that the model, while generating a smaller magnitude of systematic residual variance, reproduces the pattern of residual variation across worker and firm effects groups found in the data. The minimum wage induces low-low matches to be associated with a positive residual, indicative of the nonlinear nature by which the wage floor affects pay schedules across worker and firm types.

Figure 5. Data vs. model: AKM wage residuals, 1996–2000



Notes: Figure shows estimated AKM residual $\hat{\varepsilon}_{ijt} = w_{ijt} - \hat{\alpha}_i - \hat{\alpha}_j - \hat{\gamma}_t$ by worker and firm effect deciles. Source: RAIS and simulations.

5 Simulated policy experiment

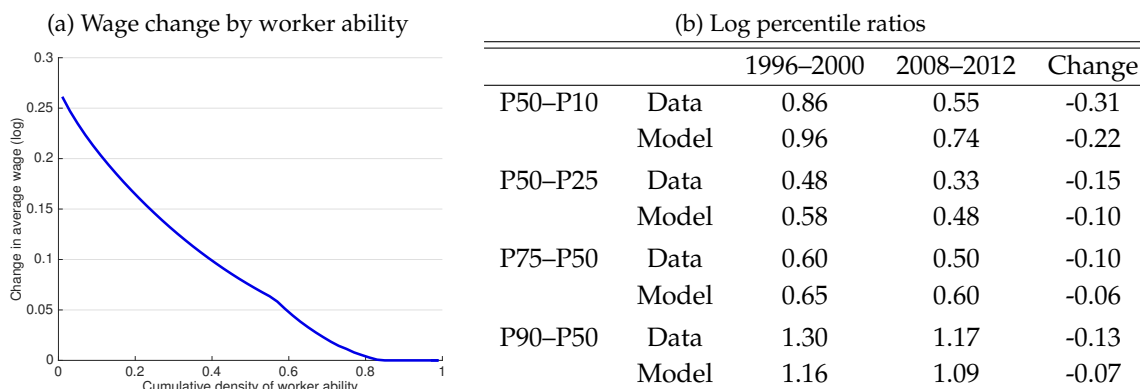
We use the estimated model to simulate the following policy experiment: what are the steady-state effects of a 44 log points increase in the real productivity-adjusted minimum wage between 1996–2000 and 2008–2012 on the wage distribution and macroeconomic variables, holding fixed all structural parameters?

5.1 Equilibrium effects of the minimum wage

Effects on wage levels and wage inequality. In line with part 1 of our model proposition, the minimum wage leads to higher wage growth at the bottom of the ability distribution. By construction, wages of the lowest-paid workers rise by 44 log points due to the minimum wage increase. Since there is no mass of workers employed at the minimum wage, however, the gains from the policy change are lower than that, even for the lowest-skill group. Panel (a) of Figure 6 shows that workers in the lowest skill group, conditional on remaining employed, experience average wage gains of 26 log points. The gains remain positive and significant, though gradually fading out, until around the 80th percentile of the ability distribution.

The unequal incidence of the minimum wage induces a decline in the variance of wages of 14 log points, or 61 percent of the empirical decline. Panel (b) compares log percentile ratios in the model and in the data over time. The P50–P10 declines by 22 log points in the model and 31 points in the data, while the P90–P50 declines by 7 log points in the model and 13 points in the data. It is worth highlighting that the share of workers employed at the minimum wage is far below 10 percent throughout this period, suggesting far-reaching spillover effects of the minimum wage.

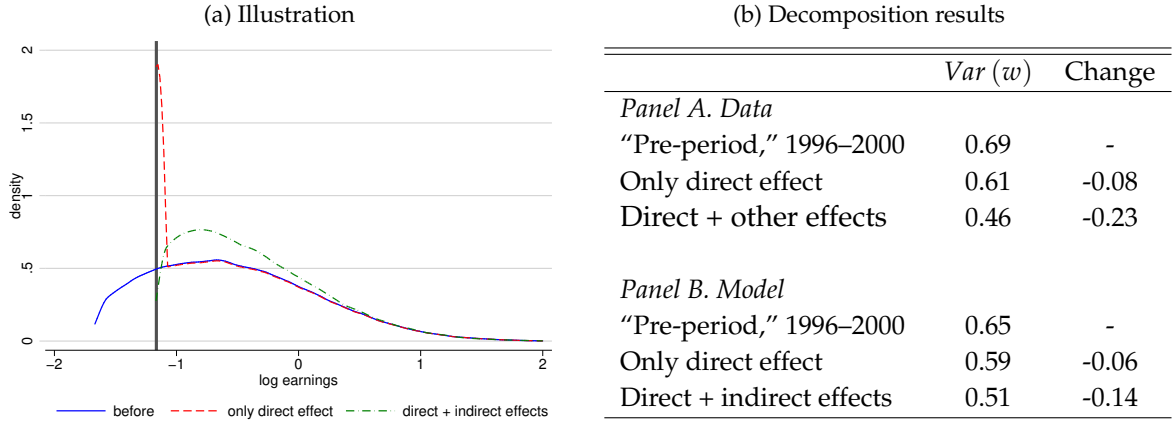
Figure 6. Data vs. model: Effects on wage distribution, 1996–2000 and 2008–2012



Notes: Panel ?? plots the change in log average wages between 1996–2000 and 2008–2012 from the model. Panel ?? shows log percentile ratios of wages in the data and in the model. Source: RAIS and simulations.

To quantify the importance of minimum wage spillovers, we propose a two-step decomposition. Let us define the “direct effect” as moving workers up to the new wage floor and the “indirect effect” as the additional adjustment due to workers’ and firms’ equilibrium responses, as panel (a) of Figure 7 illustrates. Panel (b) shows that around 40 percent of the total change in the variance of log wages is due to the direct effect, both in the data and in the model. This leads us to conclude that indirect effects or spillovers lead to sizable propagation of the minimum wage.

Figure 7. Data vs. model: Direct and indirect effects on wages, 1996–2000 and 2008–2012

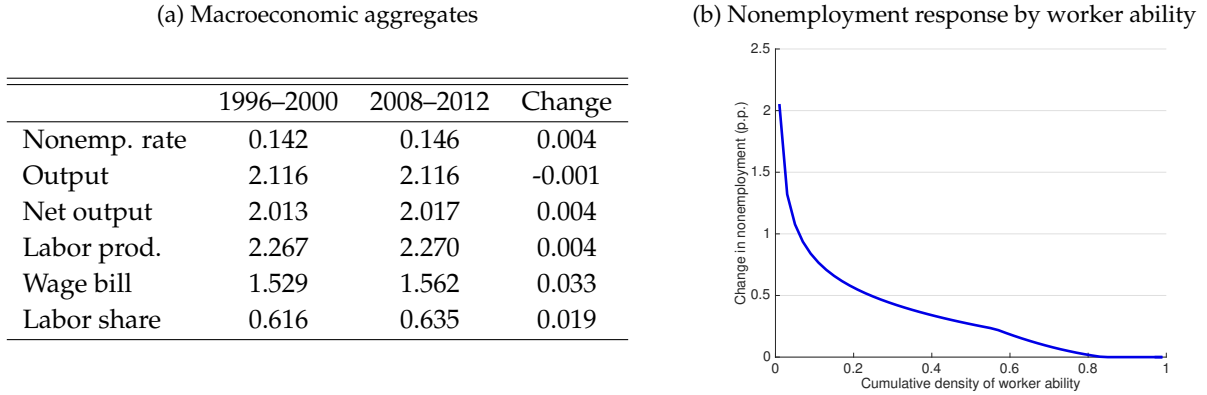


Notes: Panel (a) illustrates direct and indirect minimum wage effects. Panel (b) implements the decomposition, where in the data “other effects” include indirect minimum wage effects and other effects outside of our model. Source: RAIS and simulations.

Macroeconomic consequences. Panel (a) of Figure 8 summarizes the impact of the minimum wage on macroeconomic outcomes. As in Flinn (2006), firms respond by creating fewer jobs, leading to a 0.4 percentage points increase in the aggregate nonemployment rate but sevenfold that for the lowest-skill workers, as shown in panel (b). Gross output declines by a modest 0.1 log points, while output net of hiring costs and labor productivity increase by 0.4 log points. Combined with a wage bill increase of 3.3 log points, the model predicts a labor share increase of 1.9 percentage points, broadly in line with the empirical 0.9 log points increase over this period.

What explains the muted negative response of the macroeconomy to the minimum wage? We show in Appendix B.2 that firms’ optimal vacancy posting policy can be written as $v_\theta(p) = \{q_\theta \pi_\theta(p) / c_\theta\}^{1/c_1}$, where $\pi_\theta(p)$ is a per-vacancy profit function that is increasing in p . The minimum wage squeezes per-vacancy profits for all firms, $\partial \pi_\theta(p) / \partial w^{min} < 0$, but relatively less so for high-productivity firms. The dashed red line in Figure 9 plots the resulting distribution of vacancy cuts in partial equilibrium, that is, for a fixed worker-finding rate q_θ . In general equilibrium, however, the fall in aggregate vacancies increases the worker-finding rate, encouraging

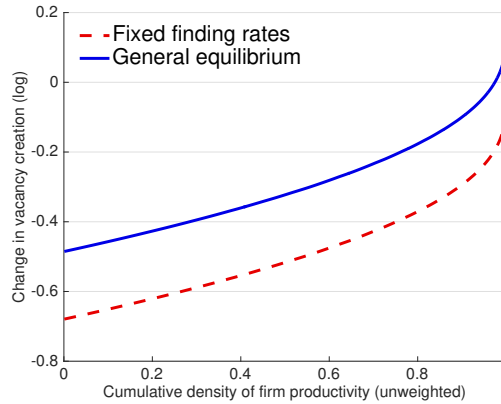
Figure 8. Model: Effects on macroeconomic outcomes, 1996–2000 and 2008–2012



Notes: Panel (a) shows the effects on macroeconomic aggregates, where (net) output is the log of the sum of total output (minus vacancy costs); labor productivity is the log of output per worker; wage bill is the log of the sum of wages; and labor share is the total wage bill divided by net output. Panel (b) plots the response of nonemployment across worker ability levels. Source: simulations.

firms to create vacancies due to lower congestion externalities (Shimer and Smith, 2001). The solid blue line shows that this general equilibrium force is strong enough that high-productivity firms on net increase vacancy creation. Consequently, the negative effects of the minimum wage are moderated by efficient reallocation of workers toward more productive firms.

Figure 9. Model: Effects on vacancy creation, 1996–2000 and 2008–2012



Notes: Figure shows vacancy response by firm productivity in lowest θ -market. “Fixed finding rates” plots the counterfactual effects of the minimum wage on vacancy creation for fixed worker-finding rate q_θ . “General equilibrium” plots the full equilibrium change allowing the finding rate to adjust. Source: simulations.

5.2 The anatomy of minimum wage effects

Worker and firm wage components. Our investigation was motivated by Brazil’s inequality decline featuring lower between-firm pay dispersion for identical workers. We find that the rise in the minimum wage can help us understand this trend. Table 5 presents results from the AKM

wage decomposition (1) over time in the data versus the model. The variance of firm effects falls by 5 log points, or 52 percent of the observed decline. Through the lens of our model, we interpret this as a reduction in frictional wage dispersion. The variance of worker effects also falls by 7 log points, close to its empirical counterpart. Our model interprets this as an increase in relative bargaining power among low-skill groups. The correlation between worker and firm pay components declines by a small amount as a result of negative assortative matching induced at the bottom of the wage distribution. Naturally, our analysis leaves room for explanations other than the minimum wage to account for the remaining changes in the wage structure over time.

Table 5. Data vs. model: AKM variance decomposition, 1996–2000 and 2008–2012

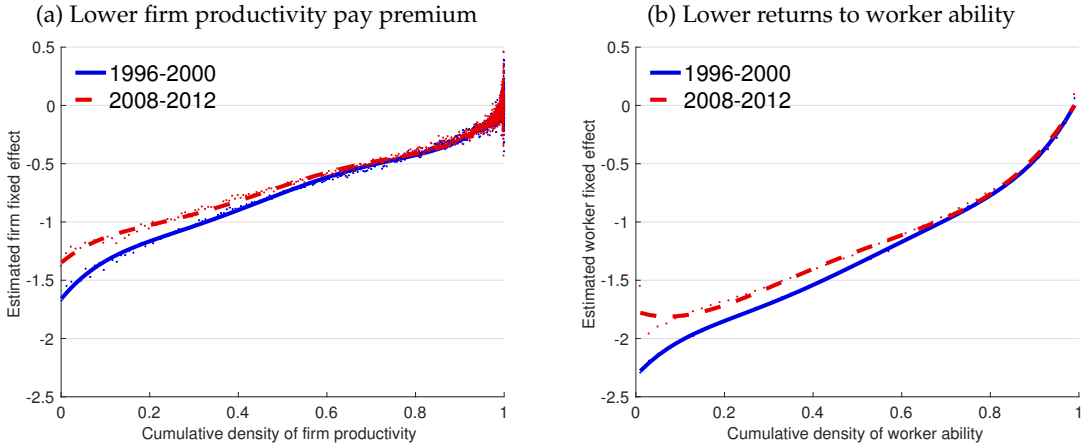
	1996–2000		2008–2012		Change	
	Data	Model	Data	Model	Data	Model
Total variance of log wages	0.69	0.62	0.47	0.47	-0.23	-0.14
Variance of worker fixed effects	0.34	0.33	0.27	0.26	-0.07	-0.07
Variance of firm fixed effects	0.16	0.16	0.07	0.12	-0.09	-0.05
2×Covariance b/w workers and firms	0.14	0.12	0.09	0.09	-0.05	-0.03
Residual variance	0.06	0.00	0.04	0.01	-0.02	0.00

Notes: Predicted variances (shares) due to components in log wage decomposition $w_{ijt} = \alpha_i + \alpha_j + \gamma_t + \varepsilon_{ijt}$. Omitted are variance terms involving year dummies γ_t , which account for a negligible share of the total variance. Source: RAIS and simulations.

Changes in returns as the main driver. Although all structural parameters are held constant in our policy experiment, changes in worker and firm wage components arise for two reasons: First, the minimum wage changes the match distribution due to the unequal incidence of nonemployment across worker abilities and vacancy adjustments across firm productivities. In line with the data, such compositional changes account for a small share of the total minimum wage impact.

Second, as predicted by parts 2 and 3 of our model proposition, the minimum wage reduces the returns to pay-relevant worker and firm characteristics. Figure 10 shows a flattening of AKM firm effects across productivity in panel (a), and of AKM worker effects across ability in panel (b). Changes in returns, rather than compositional changes, explain most of the inequality decline due to the minimum wage. This is in line with Alvarez et al. (2018)’s finding that lower productivity pay premia and lower returns to worker ability were the key drivers behind compression of AKM firm and worker fixed effects, and hence overall wage inequality, in the data.

Figure 10. Model: Gradient effects of the minimum wage, 1996–2000 and 2008–2012



Notes: Estimated firm and worker pay components from log wage decomposition $w_{ijt} = \alpha_i + \alpha_j + \gamma_t + \varepsilon_{ijt}$. Source: simulations.

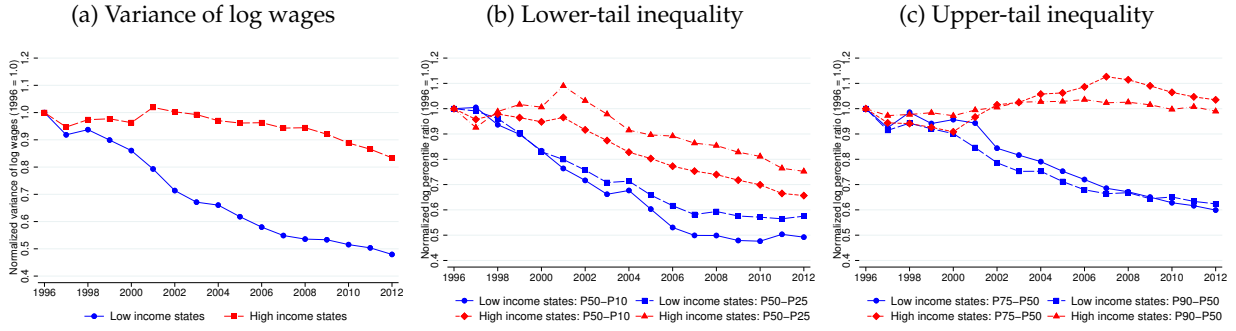
6 Evidence in support of the model predictions

We confront our model with novel empirical facts on the impact of the minimum wage in Brazil. Our starting observation is that wage inequality, while declining overall in Brazil between 1996 and 2012, fell disproportionately in initially low-income regions for which the minimum wage was more binding. Figure 11 groups into “low income” and “high income” the three lowest and three highest among Brazil’s 27 states ranked by mean log wage in 1996, and plots normalized wage inequality measures between 1996 and 2012. Panel (a) shows that the variance of log wages drops by more than half in initially low-income states, but by less than one-fifth in initially high-income states. Panel (b) shows that lower-tail inequality drops especially in initially low-income states, with the P50–P10 and P50–P25 for this group declining by 50 and 40 percent, respectively, but by markedly less for initially high-income states. In contrast, upper-tail inequality, measured by the P75–P50 or the P90–P50, declines only in initially low-income states, as shown in panel (c).

6.1 Spillover effects identified off regional variation

These patterns lead us to ask: to what extent can the rise in the minimum wage rationalize the observed heterogeneity in wage inequality in the cross section and over time? As the policy is set at the federal level, it is hard to find exogenous variation in its treatment intensity. To identify minimum wage effects in this environment, we follow Lee (1999) and Autor et al. (2016) in exploiting heterogeneous exposure across subpopulations that differ in initial bindingness with

Figure 11. Data: Evolution of wage inequality measures across state groups, 1996–2012



Notes: All panels group into “low income” and “high income” the three lowest and three highest states ranked by mean log wage in 1996, and plot between 1996 and 2012 different wage inequality measures normalized to 1.0 in 1996. Panel (a) shows the variance of log wages, panel (b) shows lower-tail wage percentile ratios, and panel (c) shows upper-tail wage percentile ratios. Source: RAIS.

respect to the federal minimum wage. We define the “effective minimum wage” or Kaitz index for subpopulation s at time t , $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, as the log difference between the prevailing minimum wage, w_t^{min} , and the median wage of subpopulation s , w_{st}^{median} .¹⁶

Figure 12 plots the relation between different wage percentile ratios and the Kaitz index for the data compared to the model.¹⁷ Panel (a) plots empirical lower-tail inequality, measured by the P50–P10, against the Kaitz index across Brazilian states over time. The negative 45 degree line marks states where the minimum wage is binding for the lower 10 percent of workers. Panel (b) repeats the same exercise on our simulated data. Both plots show a negative relationship between the P50–P10 and the Kaitz index that grows more pronounced for more binding states in the cross section and over time. For comparison, panels (c)–(d) show a weaker relationship between top inequality, measured by the P90–P50, and the Kaitz index.¹⁸

Following Lee (1999) and Autor et al. (2016), we regress an outcome variable $y_{st}(p)$ for wage percentile p of state s in year t on the effective bindingness of the minimum wage and controls:

$$y_{st}(p) = \sum_{n=1}^N \beta_n(p) kaitz_{st}^n + \gamma_{st}(p) + \varepsilon_{st}(p) \quad (3)$$

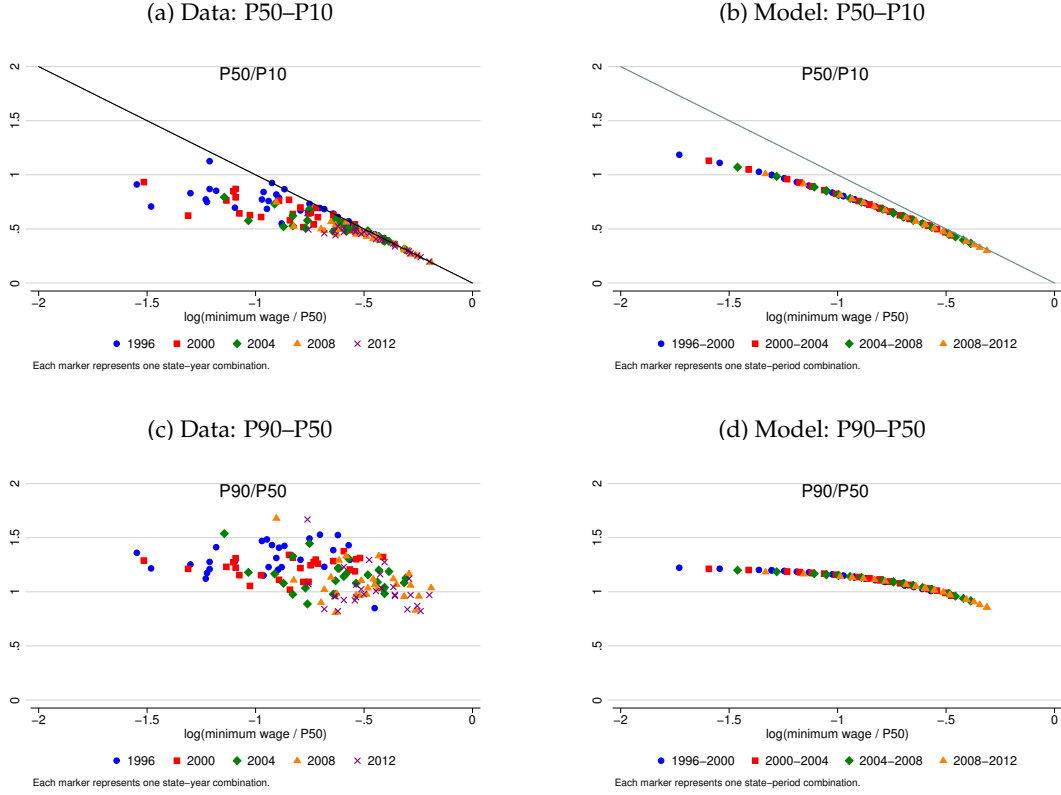
where N is the polynomial order of the Kaitz index, $\gamma_{st}(p)$ denotes either year dummies or linear

¹⁶Figure 22 in Appendix D.1 shows that variation in the Kaitz index across Brazilian states is large initially and declines as the minimum wage increases, while roughly preserving the ranking of states over time.

¹⁷We produce data from our model by simulating 27 separate economies with mean worker ability, μ , and the reservation threshold, ϕ_θ , scaled by a common factor to match the empirical Kaitz index distribution across Brazil’s states. The simplifying assumption that each state is a separate labor market is stark but motivated by the fact that only 3–5 percent of all workers switch jobs between states in a given year.

¹⁸Figure 25–26 in Appendix D.3 show that our conclusions are robust to considering a broader set of earnings percentile ratios and to running the analysis at a more granular level for Brazil’s 559 microregions.

Figure 12. Data vs. model: Wage percentile ratios across Brazilian states over time, 1996–2012



Notes: Each marker represents one state-year combination for each of Brazil's 27 states in the data, and 27 separate model simulations with estimated mean wages for the model to match the empirical Kaitz index distribution. Source: RAIS and simulations.

state-time trends plus year dummies, and $\varepsilon_{st}(p)$ is an error term that we assume satisfies the strict exogeneity assumption $\mathbb{E}[\varepsilon_{st}(p)|kaitz_{st}, \dots, kaitz_{st}^n, \gamma_{st}(p)] = 0$. After estimating equation (3) separately by wage percentile p , we compute the marginal effect of the minimum wage as $\rho_p \equiv \sum_{n=1}^N n\beta_n(p) kaitz_{st}^{n-1}$. Allowing for polynomials of order $N \geq 2$ is important to capture the nonlinear effects of the minimum wage as it becomes more binding.

We first consider as an outcome variable in equation (3) the log ratio between wage percentile p and the median wage, $y_{st}(p) = \log w_{st}(p) - \log w_{st}^{median}$, for various values of p .¹⁹ To the extent that the minimum wage leads to higher wage growth at lower percentiles, we expect the estimated marginal effect ρ_p to be weakly decreasing across wage percentiles p . We interpret positive (negative) point estimates of ρ_p for $p < 50$ ($p > 50$) as an increase in the minimum wage leading to compression in the lower (upper) tail of the wage distribution. We interpret as *spillovers* the downward-sloping range of ρ_p estimates.

¹⁹The identifying assumption then becomes that conditional on controls $\gamma_{st}(p)$, the “centrality measure,” or Kaitz index, is not systematically correlated with “underlying” wage dispersion across states and over time. Appendix D.2 provides evidence in support of this identifying assumption holding in Brazil.

Table 6 shows results from estimating equation (3) with polynomial order $N = 2$ in the RAIS microdata and in our model-simulated data.²⁰ Specification (1) is a variant of that in Lee (1999) with only year effects estimated across Brazil’s 27 states. We find that the minimum wage has significant marginal effects up to the 80th percentile of the wage distribution, with spillovers (i.e., negative gradient of marginal effect estimates) reaching up to just below the 80th percentile. For example, a ten percent increase in the minimum wage increases the tenth wage percentile by five percent relative to the median. Specification (2) is the preferred OLS specification from Autor et al. (2016) and adds a set of linear state-time trends, leading to qualitatively similar conclusions although slightly higher point estimates. As additional robustness checks, we run specification (1) across Brazil’s 556 microregions and across 54 2-digit industries in specifications (3) and (4), respectively. The estimated spillover effects under these two specifications reach between the 70th and the 90th wage percentile, though standard errors make them hard to distinguish from our modal estimate of spillovers up to the 80th percentile.

Table 6. Data vs. model: Marginal effects of the minimum wage on wage percentile ratios

	$p = 10$	$p = 20$	$p = 30$	$p = 40$	$p = 60$	$p = 70$	$p = 80$	$p = 90$
<i>Panel A. Data</i>								
Specification (1)	0.467*** (0.014)	0.283*** (0.011)	0.149*** (0.008)	0.057*** (0.005)	-0.063*** (0.006)	-0.117*** (0.013)	-0.092*** (0.022)	0.006 (0.027)
Specification (2)	0.748*** (0.032)	0.492*** (0.028)	0.392*** (0.023)	0.185*** (0.020)	-0.111*** (0.023)	-0.143*** (0.038)	0.023 (0.047)	-0.013 (0.051)
Specification (3)	0.466*** (0.024)	0.313*** (0.022)	0.182*** (0.016)	0.070*** (0.007)	-0.041*** (0.008)	-0.064*** (0.017)	-0.047* (0.026)	0.054 (0.035)
Specification (4)	0.558*** (0.035)	0.353*** (0.026)	0.222*** (0.020)	0.109*** (0.011)	-0.110*** (0.014)	-0.225*** (0.028)	-0.356*** (0.046)	-0.483*** (0.065)
<i>Panel B. Model</i>								
Specification (5)	0.467*** (0.001)	0.263*** (0.001)	0.147*** (0.001)	0.063*** (0.001)	-0.046*** (0.001)	-0.079*** (0.002)	-0.102*** (0.003)	-0.116*** (0.003)

Notes: Table shows predicted marginal effects evaluated at the worker-weighted mean across years 1992–2012. * = significant at the 10% level, ** = 5%, *** = 1%. Underlying regressions are variants of equation (3) with polynomial degree $N = 2$. Specification (1) includes year effects and is run across 27 states. Specification (2) includes additional linear state-time trends. Specification (3) is the same as specification (1) run across 556 microregions with cluster-robust standard errors. Specification (4) is the same as specification (1) run across 54 2-digit industries with cluster-robust standard errors. Model specification (5) is the same as specification (1) run on computer-simulated data across 27 states differing in their distance to the minimum wage. Source: RAIS and simulations.

Applying the same regression model as in specification (1) to our model-generated state-level data, the estimated marginal effects for lower-tail wage percentile ratios are strikingly congruent between the model and the data, suggesting that we can interpret the empirical estimates as due to

²⁰We tried polynomials of order $N > 2$ without obtaining significantly different results to those presented below.

minimum wage spillovers. The model also predicts upper-tail elasticities in line with the data up to the 90th percentile. Above that, the model shows a more negative point estimate relative to the data, although little evidence of spillovers (i.e., a small gradient of marginal effect estimates). We interpret this as relative wages in the top decile comoving with the minimum wage bindingness for reasons outside of our model.

In some specifications, we find estimated marginal effects that are upward sloping in the uppermost range of the wage distribution, usually above the 90th percentile. This could be a sign of transitory shocks affecting the upper tail and the Kaitz index simultaneously, possibly due to concurrent changes in Brazilian labor markets. Nevertheless, by cutting the data along different dimensions and at levels of aggregation, as well as using alternative specifications, we consistently find significant minimum wage effects up to the 80th percentile of the wage distribution.

6.2 Employment effects identified off regional variation

So far, our analysis has been silent on the issue of informality in Brazil. The distinction between informality and unemployment is important to the extent that each may represent a separate margin of adjustment. We now extend our regression framework to investigate the effects of the minimum wage on both formal and informal employment in Brazil between 1996 and 2012. To this end, we combine administrative data with two household surveys to estimate variants of the specification in equation (3) with the dependent variable, y_{st} , at the state-year or metropolitan area-year level.

Panel A of Table 7 shows that the minimum wage has precisely estimated zero effects on the population size, labor force participation rate, employment rate, and formal employment share. These estimates accord well with our model prediction of muted employment effects due to the minimum wage.²¹ Results from the PME data in panel B show significant but moderate negative effects on transition rates from nonformal to formal as well as from formal to nonformal employment. The mild slowdown of recruitment of workers from outside the formal sector matches our model prediction of fewer aggregate vacancies.²² Finally, panel C shows that the estimated effects on mean hours worked, as well as firm entry and exit rates, defined as the employment share at new firms relative to the previous year and the employment share at firms that exit in the fol-

²¹In the PNAD data, the formal employment share rose by 16 percentage points over this period, largely accounted for by changes in educational composition of the workforce but little movement in within-group formality rates.

²²The reduction in labor market exit, while beyond the confines of our model, could be consistent with less voluntary exit from the formal sector due to the rise in its average wage level.

lowing year, respectively. Our estimates indicate that the effects of the minimum wage on these outcome variables are indistinguishable from zero.²³

Table 7. Data: Employment effects of the minimum wage, 1996–2012

	Marginal effect (s.e.)
<i>Panel A. Cross-sectional household survey (PNAD)</i>	
Log population size	-0.050 (0.041)
Labor force participation rate	-0.001 (0.016)
Employment rate	-0.003 (0.010)
Formal employment share	-0.005 (0.024)
<i>Panel B. Longitudinal household survey (PME)</i>	
Transition rate nonformal-formal	-0.045* (0.024)
Transition rate formal-nonformal	-0.026** (0.012)
<i>Panel C. Linked employer-employee data (RAIS)</i>	
Log mean hours worked	-0.011 (0.031)
Firm entry rate	0.002 (0.017)
Firm exit rate	0.001 (0.007)

Notes: Table shows predicted marginal effects with standard errors in parentheses evaluated at the worker-weighted mean across Brazil’s 27 states from 1996 to 2012. Underlying regressions are variants of equation (3) with polynomial degree $N = 2$ including year effects and linear state-year trends. * = significant at the 10% level, ** = 5%, *** = 1%. Results for the PNAD and PME data are weighted by the appropriate sample weights. Source: PNAD, PME, RAIS.

6.3 Other testable implications of the model

Our theoretical framework in Section 3 built on some key assumptions about the mechanics of Brazilian labor markets. Among those were the importance of wage dispersion for identical workers across firms, strategic wage posting, substitutability between worker abilities in the production function, and labor market segmentation by worker type. We already motivated the first ingredient in Section 2.2. Here, we briefly discuss the remaining points vis-à-vis the Brazilian data.

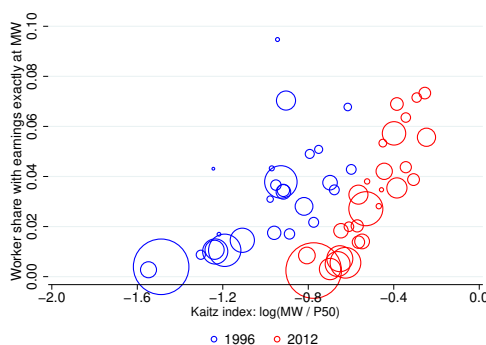
Little mass point at the minimum wage. A feature of the [Burdett and Mortensen \(1998\)](#) model that carries over to our framework is the continuity of the wage distribution. This prediction is the direct result of strategic wage posting by firms, which in equilibrium rules out mass points anywhere, in particular at the minimum wage. We Many authors have noted that this feature of the model is at odds with the data in certain contexts, such as [Manning \(2003\)](#) for the US, suggesting

²³Our results match evidence on minimum wage effects on job flows and employment in the US ([Dube et al., 2016](#); [Cengiz et al., 2017](#)) as well as firm dynamics in the UK and Hungary ([Draca et al., 2011](#); [Harasztosi and Lindner, 2017](#)).

that the benchmark wage posting model is not a good description of those labor markets.

Although we find substantial heterogeneity in the bindingness of the minimum wage in Brazil, our administrative data show a small spike at the wage floor for male workers of age 18–49. Figure 13 shows the share of workers earning exactly the minimum wage by state in 1996 and 2012 against the relative bindingness of the minimum wage measured by the Kaitz index, with the area of circles proportional to their population size. We find that a stable average of 2 percent, ranging across states from 0.5 to 9.5 percent, of workers in the population earn the minimum wage, although most heterogeneity appears to be state-specific and varying little over time.²⁴

Figure 13. Data: Worker share earning exactly the minimum wage across states, 1996 and 2012



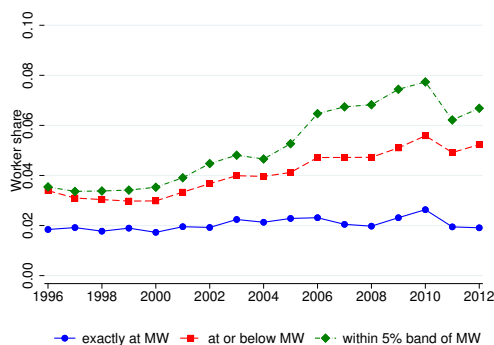
Notes: Figure shows share of male workers of age 18–49 earning exactly the minimum wage against the Kaitz index, $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, across states in 1996 and 2012. Area of circles is proportional to population size. Source: RAIS.

We can broaden our definition of “mass point” to three measures whose evolution from 1996 to 2012 is depicted in Figure 14. The share of workers earning exactly the minimum wage, shown by the blue line, remains flat at two percent. A little more than 1.5 percent of workers in 1996 and around three percent of workers in 2012 report earning less than the minimum wage, shown by the red line. These observations are likely due to a mix of legal exceptions, misreporting, and illegal employment. Our most generous definition includes workers within a 5 percent band around the minimum wage, shown in green. This most generous measure rises from 3.5 to 7 percent over this period, not far from our model equivalent of three percent. Against the backdrop of a 119 percent rise in the real minimum wage, we interpret these numbers as broadly in line with our model predictions and the institutional constraints on individual bargaining embedded in Brazilian labor market institutions. While some degree of bargaining plausibly takes place in Brazil, we view the absence of a large mass point as an informative distinction between our wage posting model and

²⁴For comparison, in the US around 3.3 percent of hourly paid workers earned the prevailing federal minimum wage or less in 2015 (U.S. Bureau of Labor Statistics, 2017).

the bargaining approach embedded in [Flinn \(2006\)](#), [Flinn \(2010\)](#), and [Flinn et al. \(2017\)](#).

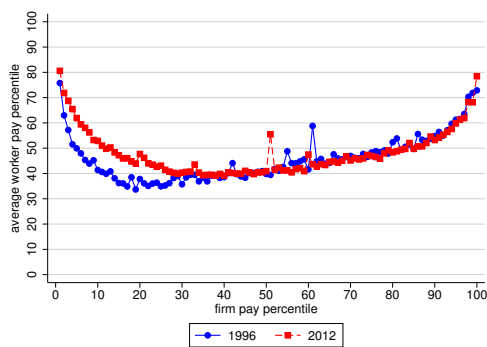
Figure 14. Data: Worker share at, below, or around the minimum wage, 1996–2012



Notes: Blue line shows share of workers earning exactly the minimum wage. Red line shows share at or below the minimum wage. Green line plots share within 5 percent of the minimum wage. Source: RAIS.

Negative sorting at lowest skill levels. An important part of our argument rested on the assumption that workers are perfectly substitutable in firms' production function. As a consequence, low-ability workers displaced from their jobs by the minimum wage could relocate to higher productivity firms, thereby buffering the adverse employment effects of the policy. A prediction of this model is that the worker input mix at low-productivity firms will tilt toward higher mean worker abilities, giving rise to negative sorting at the bottom of the wage distribution. Figure 15 confirms that in spite of the overall positive sorting pattern we reported before, there is a negative correlation between AKM worker and firm fixed effects at the bottom of the firm effects distribution. Although we interpret this correlation cautiously in the light of recent econometric critiques by [Andrews et al. \(2008\)](#), [Bonhomme et al. \(2017\)](#), and [Borovičková and Shimer \(2018\)](#), we observe, in line with our model prediction, a strengthening of this negative correlation as the minimum wage becomes more binding in Brazil between 1996 and 2012.

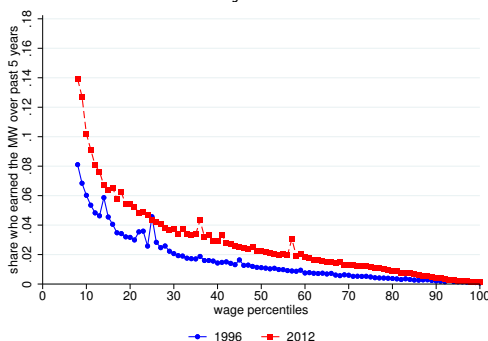
Figure 15. Data: Negative sorting at the bottom of the wage distribution, 1996 and 2012



Notes: Figure shows mean worker fixed effect across firm fixed effect percentiles, based on AKM log wage decomposition $w_{ijt} = \alpha_i + \alpha_j + \gamma_t + \varepsilon_{ijt}$, in 1996 and 2012. Source: RAIS.

A simple test for the reach of spillovers. Our empirical strategy, in line with our model predictions, identified spillover effects of the minimum wage reaching up to the 80th wage percentile. In the data, these effects were estimated off systematic comovement of the wage distribution with the Kaitz index. In our model, these effects were disciplined by the microstructure of worker and firm heterogeneity that we estimated. We now provide a model-consistent empirical test of the reach of spillovers that can be easily implemented in longitudinal worker data. Our proposed test ranks workers by their current wage and computes for each wage rank the share of individuals who previously, say over the past five years, earned the minimum wage. Through the lens of our model, we expect spillovers to reach up to the highest wage rank at which a positive mass of workers have been previously employed at the minimum wage. Figure 16 shows the results of implementing our test on the Brazilian data for 1996 and 2012. Confirming our previous results, we find a positive mass of workers previously earning the minimum wage up to around the 80th percentile of the wage distribution in 1996 and up to the 90th percentile in 2012.²⁵

Figure 16. Data: Worker share who recently earned the minimum wage, 1996 and 2012



Notes: Figure shows share of workers who have held a job paying the minimum wage over the past five years across percentiles of the current wage distribution. Source: RAIS.

7 Conclusion

What are the effects of the minimum wage on inequality? The answer to this question depends crucially on the microstructure of the labor market. We developed a flexible equilibrium model in the spirit of [Burdett and Mortensen \(1998\)](#) to quantify the effects of a minimum wage increase on wage inequality and other macroeconomic outcomes in Brazil between 1996 and 2012. Our analysis was disciplined by empirically relevant dimensions of worker and firm heterogeneity

²⁵ Another validation of our findings comes from inspection of Figure 17 in Appendix A.1, which shows a pronounced elevation of average wages and a number of pay-relevant worker covariates above the 80th wage percentile. Reassuringly, the minimum wage appears to have had little effect on the highest skill groups including college graduates whose population shares increase steeply after the 80th wage percentile, as shown in Figure 18 in Appendix A.1.

estimated through the AKM framework. Conversely, our structural model could be mapped into the AKM decomposition of a large decline in wage inequality in Brazil, a significant share of which is explained by the rise of the minimum wage.

In contrast to competitive labor market theories, we find that the change in the wage floor had large effects throughout the distribution, inducing a 14 log points decline in the variance of wages and affecting workers up to the 80th percentile. We also find a muted nonemployment response and small efficiency gains from the policy. The effects of the minimum wage are mediated by a lower firm productivity pay premium and lower returns to worker ability. We present empirical evidence from administrative linked employer-employee data and two household surveys in support of these findings. We conclude that the minimum wage can have large effects on inequality.

These insights point to fruitful avenues for future research. First, it would be interesting to quantify spillovers of Brazil's formal sector minimum wage into the informal economy, which is not directly constrained by the policy. Second, our finding of large spillovers poses a challenge to recent empirical work attempting to identify minimum wage effects by comparison to a control group. Third, it may be worth revisiting the contribution of labor market policies and institutions including unions, unemployment benefits, non-compete agreements, and antidiscrimination laws—all of which affect only a small worker share directly but may lead to sizable equilibrium effects—towards inequality trends in other countries. Finally, while we have stopped short of optimal policy analysis, our results will be an important ingredient for any such venture.

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Appendix

We structure the additional materials as follows: Data (Appendix A), Theory (Appendix B), Estimation (Appendix C), and Empirics (Appendix D).

A Data Appendix

This appendix provides details on the datasets used in Section 2 and throughout the paper, including subsections on data sources, cleaning procedures, variable construction, and summary statistics (Appendix A.1), and the wage distribution by year (Appendix A.2).

A.1 Data sources, cleaning procedures, variable construction, and summary statistics

Linked employer-employee data (RAIS). We use the RAIS microdata with person and firm identifiers covering the period 1992–2012 available to us under a confidentiality agreement with the Brazilian Ministry of Labor (*Ministério do Trabalho*, or *MTb*). We also use a version of the same data going back to 1988, which we accessed through Brazil’s Institute of Applied Economic Research (*Instituto de Pesquisa Econômica Aplicada*, or *IPEA*) at the Brazilian Institute of Geography and Statistics (*Instituto Brasileiro de Geografia e Estatística*, or *IBGE*). We devise our own cleaning procedure for these data, starting with the raw text files, benefiting from guidance by the data team at IPEA.

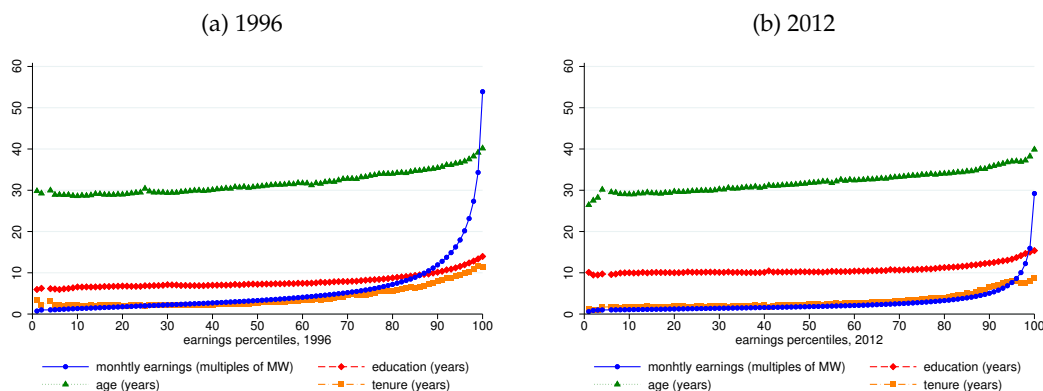
Our cleaning procedure consists of three stages. The first stage reads in and standardizes the format of the raw data files that were transmitted to us at the region-year level, saving a set of files at the region-year level. The second stage reads in all region files within a year and applies a set of cleaning and recoding procedures to the data to make them consistent within each year, saving a set of yearly files. The third stage reads in all yearly files and applies a set of cleaning procedures to the data to make them consistent across years.

Whenever possible, we use the official crosswalks provided by IBGE to convert industry codes (IBGE, CNAE 1.0, and CNAE 2.0 classifications) and municipality codes (IBGE classification).

Cross-sectional and longitudinal household surveys (PNAD and PME). The raw microdata are publicly available for download starting from 1996 for PNAD and starting from March 2002 for PME at ftp://ftp.ibge.gov.br/Trabalho_e_Rendimento/. For basic cleaning, starting with

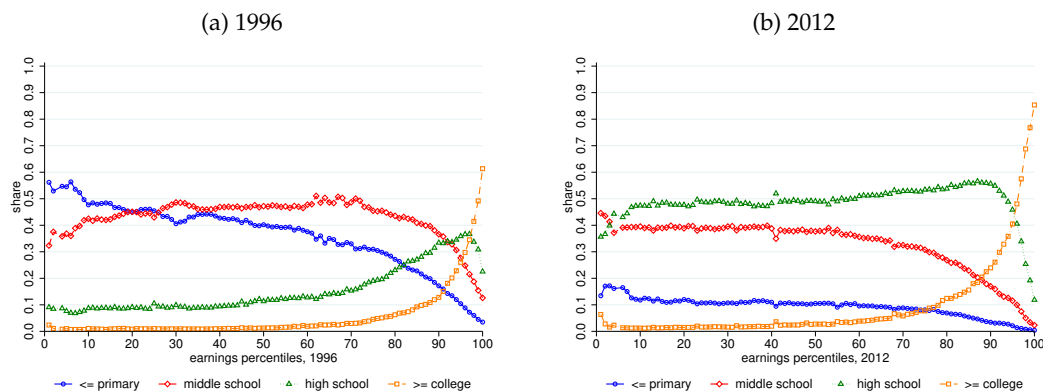
the raw data in text format, we use the standardized cleaning procedures adopted from the Data Zoom suite developed at PUC-Rio and available for replication online at <http://www.econ.puc-rio.br/datazoom/english/index.html>. From there, we apply a set of procedures to clean and recode key variables used in our analysis.

Figure 17. RAIS cross-sectional summary statistics, 1996 and 2012



Notes: Figure shows mean monthly earnings (“wages”), years of education, age, and tenure across wage percentiles for 1996 in panel (a) and for 2012 in panel (b). All statistics are for adult male workers of age 18–49. Source: RAIS.

Figure 18. RAIS education degree shares, 1996 and 2012



Notes: Figure shows shares of education degrees across wage percentiles for 1996 in panel (a) and for 2012 in panel (b). All statistics are for adult male workers of age 18–49. Source: RAIS.

Table 8. Summary statistics for cross-sectional household survey (PNAD)

	# Workers	Real wage (formal)		Real wage (informal)		Employment rate	Formal share
		Mean	Std. dev.	Mean	Std. dev.		
1996	74,487	7.01	0.81	6.26	0.81	0.95	0.68
1997	78,731	7.02	0.79	6.26	0.82	0.94	0.68
1998	79,060	7.03	0.78	6.26	0.81	0.93	0.67
1999	81,230	6.97	0.77	6.21	0.79	0.93	0.66
2000	--	--	--	--	--	--	--
2001	89,102	6.93	0.74	6.20	0.81	0.93	0.66
2002	90,855	6.90	0.73	6.19	0.81	0.93	0.66
2003	91,490	6.84	0.71	6.12	0.77	0.92	0.67
2004	94,526	6.85	0.69	6.15	0.77	0.94	0.68
2005	97,348	6.89	0.67	6.19	0.77	0.93	0.68
2006	97,757	6.94	0.66	6.25	0.76	0.94	0.69
2007	95,598	6.97	0.65	6.30	0.78	0.94	0.71
2008	93,677	7.00	0.65	6.35	0.76	0.95	0.72
2009	95,170	7.02	0.63	6.36	0.76	0.94	0.73
2010	--	--	--	--	--	--	--
2011	84,910	7.07	0.62	6.51	0.75	0.95	0.76
2012	86,031	7.13	0.62	6.56	0.78	0.95	0.76

Notes: Table shows summary statistics on wages, employment rates, and formal employment shares between 1996 and 2012. All statistics are for adult male workers of age 18–49. Real wages are measured in 2012 BRL and in logs. Surveys are not available for census years 2000 and 2010. Source: PNAD.

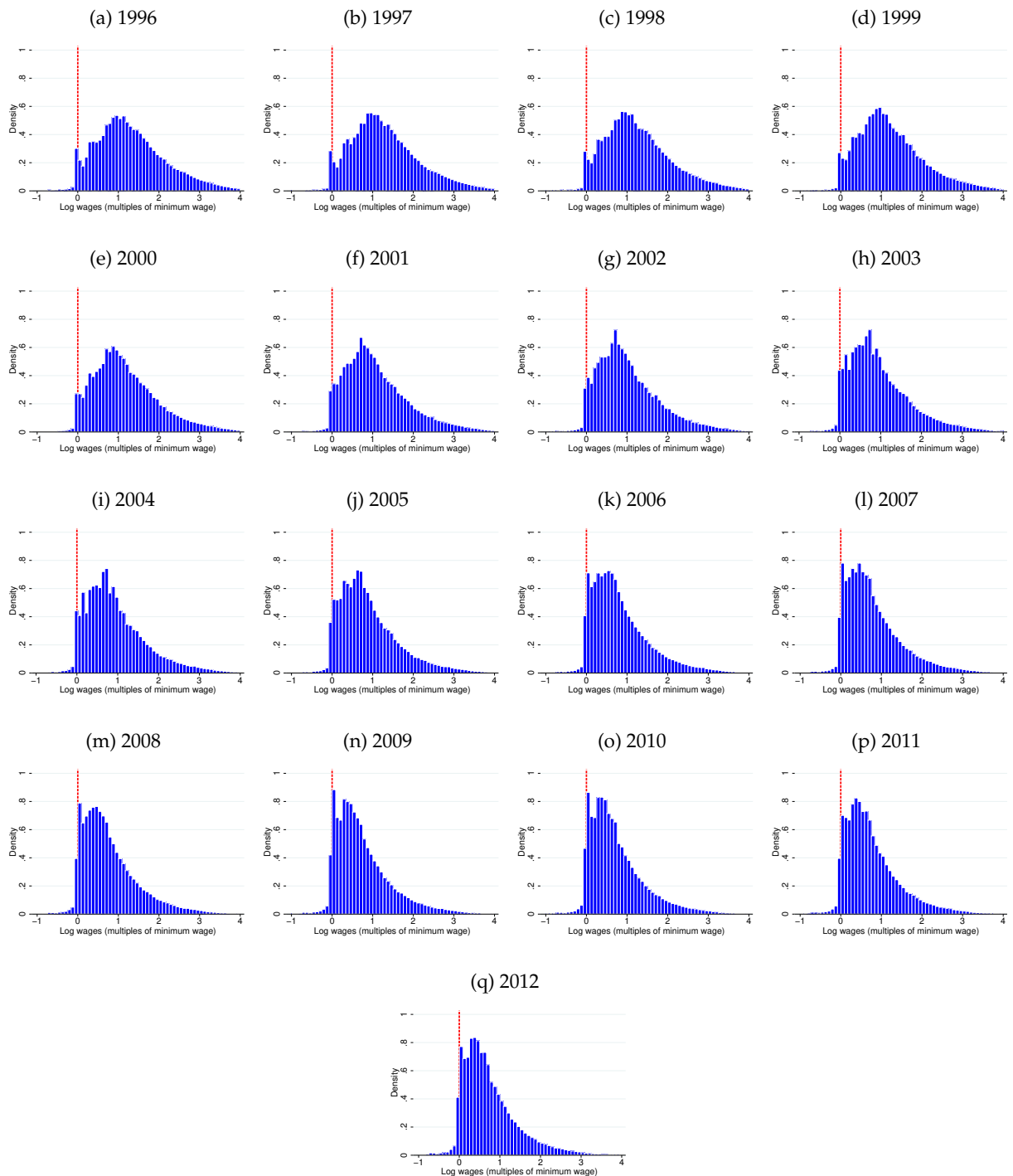
Table 9. Summary statistics for longitudinal household survey (PME)

	# Workers	Transition rate	Transition rate
		employed-nonemployed	employed-nonemployed
2002	94,280	0.08	0.05
2003	140,734	0.09	0.06
2004	146,847	0.08	0.05
2005	154,159	0.08	0.05
2006	153,646	0.08	0.04
2007	154,338	0.09	0.05
2008	150,104	0.10	0.05
2009	149,762	0.10	0.04
2010	150,443	0.10	0.04
2011	145,012	0.11	0.04
2012	121,211	0.10	0.04

Notes: Table shows summary statistics on transition rates between employment (formal) and nonemployment (nonformal + unemployed) between 1996 and 2012. All statistics are for adult male workers of age 18–49. Surveys start in March 2002. Source: PME.

A.2 Wage distribution by year

Figure 19. Data: Wage distribution by year, 1996–2012



Notes: Each panel shows the wage histogram based on 60 equi-spaced bins for population of male workers aged 18–49 for one year between 1996 and 2012. Source: RAIS.

B Theory Appendix

This appendix provides details on the model outlined in Section 3, including subsections on the equilibrium definition (Appendix B.1), equilibrium characterization (Appendix B.2), the solution algorithm (Appendix B.3), and proof of the proposition (Appendix B.4).

B.1 Equilibrium definition

Definition. A search equilibrium with a minimum wage consists of a set of firms' wage and vacancy posting policies $(w_\theta(p), v_\theta(p))$, wage offer distribution $F_\theta(w)$, employment level function $l_\theta(w, v)$, unemployment rate u_θ , aggregate vacancy mass V_θ , contact rate q_θ , reservation wage policy R_θ , transition rates $(\lambda_\theta^u, \lambda_\theta^e)$, separation rate δ_θ , and flow value of nonemployment b_θ subject to a minimum wage w^{\min} :

1. *Worker optimality:* Given transition rates, the separation rate, the flow value of nonemployment, and the offer distribution, the reservation wage policy solves workers' problem;
2. *Firm optimality:* Taking as given the employment level function and wage offer distribution, wage and vacancy policies solve firms' problem;
3. *Enforcement:* No worker accepts, or else no firm posts, a wage below the minimum wage;
4. *Labor market aggregation:* The unemployment rate is consistent with transition rates and the separation rate, the aggregate vacancy mass is consistent with firms' vacancy posting policies, transition rates are determined through the aggregate matching function, and the employment level function and the wage offer distribution are consistent with wage and vacancy posting policies.

B.2 Equilibrium characterization

The size of a firm, $l_\theta(w, v)$, evolves according to the Kolmogorov forward equation:

$$\dot{l}_\theta(w, v) = -[\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(w))] l_\theta(w, v) + v q_\theta \left[\frac{u_\theta}{u_\theta + (1 - u_\theta) s_\theta} + \frac{(1 - u_\theta) s_\theta}{u_\theta + (1 - u_\theta) s_\theta} G_\theta(w) \right]. \quad (4)$$

The separate terms are explained as follows. Out of a firm's current workforce l_θ , a fraction δ_θ exit to nonemployment and a fraction $s_\theta \lambda_\theta^u (1 - F_\theta(w))$ quit to better-paying employers. At rate q_θ ,

each of the firm's vacancies contacts a worker, and a share $u_\theta / (u_\theta + s_\theta(1 - u_\theta))$ of those workers is nonemployed while the remainder are currently employed. Nonemployed workers accept all wage offers in equilibrium, while a fraction $G_\theta(w)$ of employed workers accept an offer of wage w . In the stationary equilibrium, equation (4) must equal zero such that

$$l_\theta(w, v) = vq_\theta \frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(w))} \left[\frac{u_\theta}{u_\theta + (1 - u_\theta)s_\theta} + \frac{(1 - u_\theta)s_\theta}{u_\theta + (1 - u_\theta)s_\theta} G_\theta(w) \right].$$

Recall also that $u_\theta = \delta_\theta / (\delta_\theta + \lambda_\theta^u)$ and $G_\theta(w) = F_\theta(w) / (1 + \kappa_\theta(1 - F_\theta(w)))$ in the stationary economy. Substituting and simplifying, for given vacancy and wage policies (w, v) , a firm's stationary equilibrium size is given by

$$l_\theta(w, v) = vq_\theta \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(w))} \right)^2 \frac{\delta_\theta (\delta_\theta + s_\theta \lambda_\theta^u)}{\delta_\theta + \lambda_\theta^u}. \quad (5)$$

Define the piece rate \tilde{w}_θ such that $w_\theta = \theta \tilde{w}_\theta$. We can write the problem of firm p in market θ as

$$\max_{\tilde{w} \geq R_\theta / \theta, v} \left\{ vq_\theta \frac{\delta_\theta (\delta_\theta + s_\theta \lambda_\theta^u)}{\delta_\theta + \lambda_\theta^u} \theta (p - \tilde{w}) \left(\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(\tilde{w}))} \right)^2 - c_\theta(v) \right\}.$$

Imposing our assumed functional form for the cost of vacancy creation, $c_\theta(v) = c_\theta v^{1+c_1} / (1 + c_1)$, the first-order conditions with respect to vacancies and piece rates are

$$v_\theta(p) = \left\{ \frac{q_\theta}{c_\theta} [p - \tilde{w}_\theta(p)] \frac{\delta_\theta (\delta_\theta + s_\theta \lambda_\theta^u)}{\delta_\theta + \lambda_\theta^u} \theta \left[\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta[\tilde{w}_\theta(p)])} \right]^2 \right\}^{\frac{1}{c_1}} \quad (6)$$

$$1 = [p - \tilde{w}_\theta(p)] \frac{2s_\theta \lambda_\theta^u f_\theta(\tilde{w}_\theta(p))}{\delta_\theta + s_\theta \lambda_\theta^u [1 - F_\theta(\tilde{w}_\theta(p))]} \quad (7)$$

Recall that the distribution of wage offers is given by

$$F_\theta(w_\theta(p)) = M_\theta \int_{\underline{p}_\theta}^p \frac{v_\theta(p')}{V_\theta} d\Gamma_\theta(p'), \quad (8)$$

where total vacancies equal

$$V_\theta = M_\theta \int_{\underline{p}_\theta}^{\bar{p}} v_\theta(p') d\Gamma_\theta(p'), \quad (9)$$

and finally the job finding rate of workers is implicitly defined by

$$\lambda_\theta^u = \left(\frac{\delta_\theta}{\delta_\theta + \lambda_\theta^u} + \frac{s_\theta \lambda_\theta^u}{\delta_\theta + \lambda_\theta^u} \right)^{-\alpha} V_\theta^\alpha, \quad (10)$$

while firms' contact rate is given by $q_\theta = (\lambda_\theta^u)^{\frac{\alpha-1}{\alpha}}$. Note that we can write equation (6) as

$$v_\theta(p) = \left\{ \frac{q_\theta}{c_\theta} \pi_\theta(p) \right\}^{\frac{1}{c_1}},$$

where

$$\pi_\theta(p) = \max_{\tilde{w} \geq R_\theta/\theta} \left\{ (p - \tilde{w}) \frac{\delta_\theta (\delta_\theta + s_\theta \lambda_\theta^u)}{\delta_\theta + \lambda_\theta^u} \left[\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - F_\theta(\tilde{w}))} \right]^2 \right\}.$$

Since $\pi_\theta(p)$ is increasing in productivity and $c_\theta, c_1 > 0$, it follows that $v'_\theta(p) > 0$. That is, more productive employers create more jobs. As in [Burdett and Mortensen \(1998\)](#), a single-crossing property of the profit function with respect to productivity and wages for a given vacancy policy implies that the optimal wage policy, $\tilde{w}_\theta(p)$, is strictly increasing in productivity. Similarly, the equilibrium wage offer distribution has no mass points.

B.3 Solution algorithm

Define $h_\theta(p) = F_\theta(w_\theta(p))$ so that $f_\theta(w_\theta(p)) = h'_\theta(p)/w'_\theta(p)$ and $v_\theta(p) = \frac{V_\theta}{M_\theta} \frac{h'_\theta(p)}{\gamma_\theta(p)}$. Substituting this into the first-order conditions (6)–(7), we have

$$h'_\theta(p) = \frac{M_\theta \gamma_\theta(p)}{V_\theta} \left\{ \frac{(\lambda_\theta^u)^{-\frac{1-\alpha}{\alpha}}}{c_\theta} [p - w_\theta(p)] \frac{\delta_\theta (\delta_\theta + s_\theta \lambda_\theta^u)}{\delta_\theta + \lambda_\theta^u} \theta \left[\frac{1}{\delta_\theta + s_\theta \lambda_\theta^u (1 - h_\theta(p))} \right]^2 \right\}^{\frac{1}{c_1}} \quad (11)$$

and

$$w'_\theta(p) = [p - w_\theta(p)] \frac{2s_\theta \lambda_\theta^u h'_\theta(p)}{\delta_\theta + s_\theta \lambda_\theta^u [1 - h_\theta(p)]} \quad (12)$$

We have estimates of δ_θ , s_θ and λ_θ^u from the data, imposing a restriction on the total mass of vacancies, V_θ , from equation (10), $V_\theta = (\lambda_\theta^u)^{\frac{1}{\alpha}} \left(\frac{\delta_\theta}{\delta_\theta + \lambda_\theta^u} + \frac{s_\theta \lambda_\theta^u}{\delta_\theta + \lambda_\theta^u} \right)$. Taking V_θ and the finding rates as given, we can solve the system of first-order ordinary differential equations (11)–(12) subject to the boundary conditions

$$w_\theta(\underline{p}_\theta) = \frac{\max\{\phi_\theta, w^{min}\}}{\theta}, \quad \text{and} \quad \lim_{p \rightarrow \underline{p}_\theta} h_\theta(p) = 0$$

to obtain $h'_\theta(p)$ and $w_\theta(p)$. For the solution to be sensible, we require that $\lim_{p \rightarrow \bar{p}} h_\theta(p) = 1$, that is, that $F_\theta(w_\theta(p))$ is a valid cumulative distribution function. This amounts to finding the cost parameter c_θ that ensures that this condition holds.²⁶ We find such a c_θ by guessing an initial c_θ , solving the problem, and checking whether the condition $\lim_{p \rightarrow \bar{p}} h(p) = 1$ holds. If it does not, we update c_θ until convergence.

In order to subsequently evaluate the impact of a rise in the minimum wage, we take the estimated cost parameter c_θ as given and instead find the job finding rate λ_θ^u that ensures that $\lim_{p \rightarrow \bar{p}} h(p) = 1$ holds.

B.4 Proof of the proposition

Under the assumption that labor market parameters are the same across worker types and that the minimum wage is initially low enough to be nonbinding, equilibrium wages can be written as

$$w(p, \theta) = \theta \left[p - \int_{\underline{p}_\theta}^p \left[\frac{1 + \kappa_\theta^e (1 - \Gamma_0(p))}{1 + \kappa_\theta^e (1 - \Gamma_0(x))} \right]^2 dx \right] \quad (13)$$

Part 1. We want to show that an increase in the minimum wage raises all wages in markets where it becomes binding. Differentiating equation (13) with respect to the minimum wage gives that

$$\frac{\partial w(p, \theta)}{\partial w^{\min}} = \left[\frac{1 + \kappa_\theta^e (1 - \Gamma_0(p))}{1 + \kappa_\theta^e \left(1 - \Gamma_0 \left(\frac{w^{\min}}{\theta} \right) \right)} \right]^2 > 0,$$

which establishes the first part of the proposition.

Part 2. Consider a market where the minimum wage is binding. Differentiating equation (13) with respect to productivity gives that the productivity-pay gradient is given by

$$\frac{\partial w(p, \theta)}{\partial p} = \theta 2 \kappa_\theta^e \gamma_0(p) [1 + \kappa_\theta^e (1 - \Gamma_0(p))] \int_{\underline{p}_\theta}^p \left(\frac{1}{1 + \kappa_\theta^e (1 - \Gamma_0(x))} \right)^2 dx.$$

²⁶Note that this condition ensures that vacancy creation aggregates to the total amount of vacancies in the economy since $M_\theta \int_{\underline{p}_\theta}^{\bar{p}} v_\theta(p) d\Gamma_\theta(p) = M_\theta \int_{\underline{p}_\theta}^{\bar{p}} \frac{V_\theta}{M_\theta} \frac{h'_\theta(p)}{\gamma_\theta(p)} \gamma_\theta(p) dp = V_\theta \int_{\underline{p}_\theta}^{\bar{p}} h'_\theta(p) dp = V_\theta [h_\theta(\bar{p}_\theta) - h_\theta(\underline{p}_\theta)] = V_\theta$.

Differentiating this equation with respect to the minimum wage gives that

$$\frac{\partial \left(\frac{\partial w(p, \theta)}{\partial p} \right)}{\partial w^{\min}} = \theta 2 \kappa_{\theta}^e \gamma_0(p) [1 + \kappa_{\theta}^e (1 - \Gamma_0(p))] \left(-\frac{1}{\theta} \right) \left(\frac{1}{1 + \kappa_{\theta}^e \left(1 - \Gamma_0 \left(\frac{w^{\min}}{\theta} \right) \right)} \right)^2 < 0.$$

Hence, the firm productivity-pay gradient falls with the minimum wage.

Part 3. Consider markets where the minimum wage is binding. Differentiating equation (13) with respect to ability gives that the ability-pay gradient is given by

$$\frac{\partial w(p, \theta)}{\partial \theta} = p - \int_{p_{\theta}}^p \left[\frac{1 + \kappa_{\theta}^e (1 - \Gamma_0(p))}{1 + \kappa_{\theta}^e (1 - \Gamma_0(x))} \right]^2 dx - \frac{w^{\min}}{\theta} \left[\frac{1 + \kappa_{\theta}^e (1 - \Gamma_0(p))}{1 + \kappa_{\theta}^e \left(1 - \Gamma_0 \left(\frac{w^{\min}}{\theta} \right) \right)} \right]^2$$

Differentiating this equation with respect to the minimum wage gives that

$$\begin{aligned} \frac{\partial \left(\frac{\partial w(p, \theta)}{\partial \theta} \right)}{\partial w^{\min}} &= \frac{1}{\theta} \left[\frac{1 + \kappa_{\theta}^e (1 - \Gamma_0(p))}{1 + \kappa_{\theta}^e \left(1 - \Gamma_0 \left(\frac{w^{\min}}{\theta} \right) \right)} \right]^2 - \frac{1}{\theta} \left[\frac{1 + \kappa_{\theta}^e (1 - \Gamma_0(p))}{1 + \kappa_{\theta}^e \left(1 - \Gamma_0 \left(\frac{w^{\min}}{\theta} \right) \right)} \right]^2 \\ &\quad - \frac{w^{\min}}{\theta} [1 + \kappa_{\theta}^e (1 - \Gamma_0(p))]^2 (-2) \left(-\kappa_{\theta}^e \gamma_0 \left(\frac{w^{\min}}{\theta} \right) \right) \frac{1}{\theta} \left[\frac{1}{1 + \kappa_{\theta}^e \left(1 - \Gamma_0 \left(\frac{w^{\min}}{\theta} \right) \right)} \right]^3 \\ &= -\frac{2w^{\min}}{\theta^2} [1 + \kappa_{\theta}^e (1 - \Gamma_0(p))]^2 \kappa_{\theta}^e \gamma_0 \left(\frac{w^{\min}}{\theta} \right) \left[\frac{1}{1 + \kappa_{\theta}^e \left(1 - \Gamma_0 \left(\frac{w^{\min}}{\theta} \right) \right)} \right]^3 < 0. \end{aligned}$$

Hence, in markets where the minimum wage is binding, the worker ability-pay gradient falls with the minimum wage. \square

C Estimation Appendix

This appendix provides details on the estimation procedure described in Section 4, including subsections on the estimation criterion (Appendix C.1) and the estimated vacancy cost profile (Appendix C.2).

C.1 Estimation criterion

As outlined in Section 4, we estimate the average worker ability, the dispersion in worker ability, the shape of the productivity distribution, and the cost of creating jobs by indirect inference. We implement this by repeatedly solving the model over a pre-specified grid for the first three parameters and recording the model-predicted values for the targeted moments. Within each loop, we iteratively solve for the cost of creating jobs that allows the model to match the UE hazard rate estimated in the first step of our estimation.

We use a 25-by-25-by-25 point grid in the three parameters of interest defined over a sufficiently large domain. Solving and parallel-simulating the model 15,625 times is relatively efficient and runs for about 16 hours on a modern quad-core desktop computer. We search on a given parameter grid for the triplet (μ, σ^2, ζ) that minimizes the sum of squared log square differences between three target moments in the data versus the model:

$$\underset{\hat{\mu}, \hat{\sigma}^2, \hat{\zeta}}{\operatorname{argmin}} \left\{ \log \left[\frac{\operatorname{Var}(\alpha_i^M)}{\operatorname{Var}(\alpha_i^D)} \right]^2 + \log \left[\frac{\operatorname{Var}(\alpha_j^M)}{\operatorname{Var}(\alpha_j^D)} \right]^2 + \log \left[\frac{mM^M}{mM^D} \right]^2 \right\}, \quad (14)$$

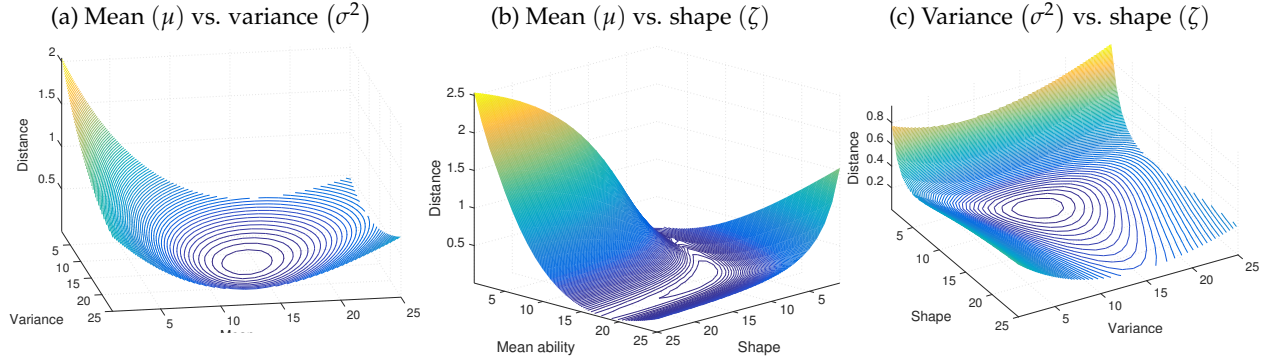
where $\operatorname{Var}(\alpha_i)$ denotes the variance of AKM worker fixed effects, $\operatorname{Var}(\alpha_j)$ denotes the variance of AKM firm fixed effects, and mM denotes the minimum-to-mean wage ratio.

To guarantee a unique interior solution, we analyzed the behavior of the objective function in two dimensions at a time, fixing the third parameter at its estimated value. This is plotted in Figure 20. The variance of worker ability appears well identified, while there is some ambiguity in the worker ability-firm productivity shape dimension. Specifically, a higher mean worker ability—implying a less binding minimum wage—can be compensated for by a higher shape parameter of the firm productivity distribution—implying less dispersed firm productivity.

We have reevaluated the impact of an increase in the minimum wage for different combinations of (μ, ζ) that produce only a slightly worse fit to the data compared to our best estimates,

and find that our results are quantitatively robust to such perturbations.

Figure 20. Distance metric from estimation procedure, 1996–2000

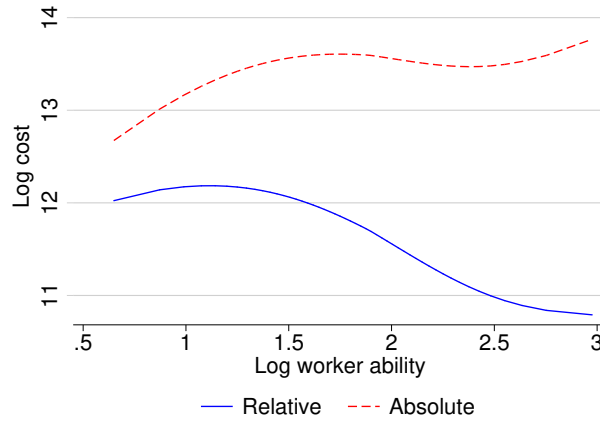


Notes: Figure plots the distance minimization criterion in equation (14). Mean (μ) and variance (σ^2) pertain to a log-Normal worker ability distribution, while shape (ζ) is the Pareto tail parameter of the firm productivity distribution. Source: simulations.

C.2 Estimated vacancy cost profile

Figure 21 shows estimated second-stage vacancy cost intercepts c_θ across worker abilities. The vacancy cost relative to worker ability (solid blue line) is downward sloping, implied by the slope of job finding rates across ability groups. The absolute vacancy cost cost (dashed red line) is upward sloping, implied by the magnitudes of job finding rates across ability groups.

Figure 21. Estimated relative and absolute vacancy posting cost c_θ across worker abilities



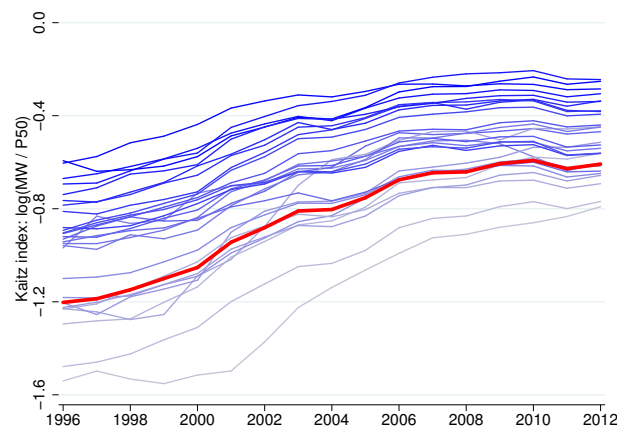
Notes: Figure plots vacancy cost intercept c_θ relative to worker ability (solid blue) and in levels (dashed red). Source: simulations.

D Empirics Appendix

This appendix provides further empirical evidence related to what is presented in Section 6, including subsections on the evolution of the Kaitz index by state over time (Appendix D.1), evidence supporting the empirical identifying assumption (Appendix D.2), and additional results on spillover effects identified off regional variation (Appendix D.3).

D.1 Evolution of the Kaitz index by state

Figure 22. Data: Evolution of the Kaitz index by state, 1996–2012



Notes: Kaitz index is defined as $kaitz = \log(\text{minimum wage}) - \log(\text{median wage})$. Each blue line markets one of Brazil's 27 states. Red line marks equally weighted mean across states. Source: RAIS.

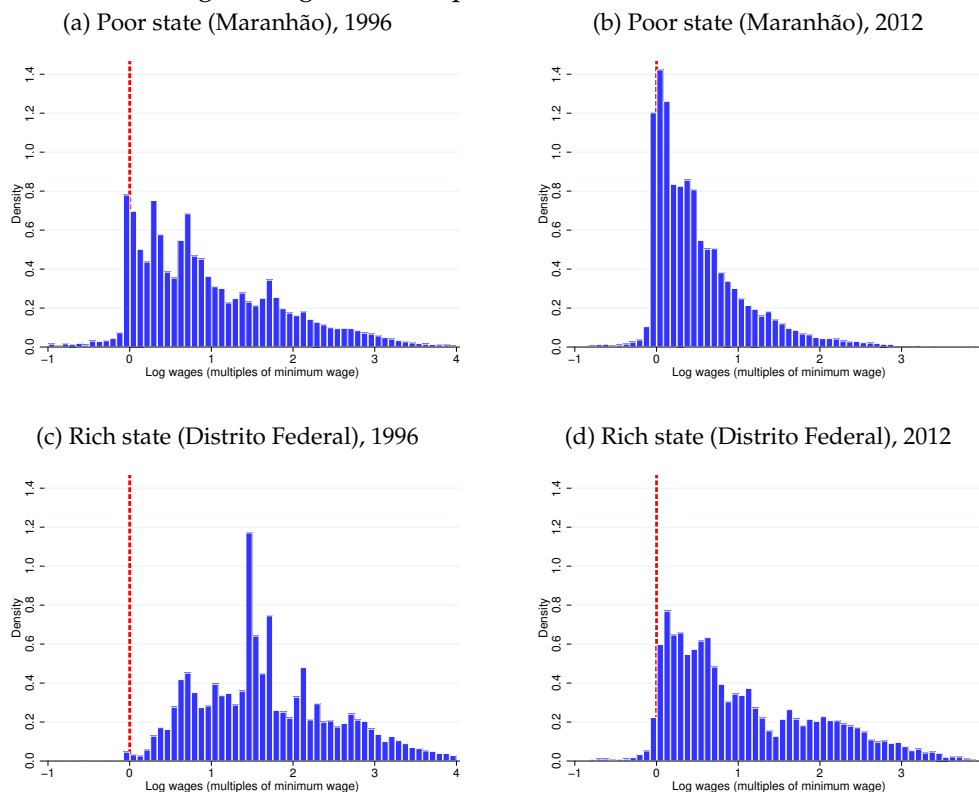
D.2 Evidence supporting the empirical identifying assumption

This section provides support for our empirical identifying assumption in Section 6, namely that cross-state variation in the “centrality” of the wage distribution is not systematically related to the shape of the “underlying” wage distribution in Brazil. Although the assumption is not literally testable, we here provide two proxy tests for the assumption.

We show as a first test of our identifying assumption, namely that Brazilian states share a “latent wage distribution,” the resemblance of the wage wage distribution in one of Brazil’s poorest states in 1996 and that of one of its richest states in 2012 under the higher minimum wage. Figure 23 shows histograms of wages for the state of *Maranhão*, the second poorest in Brazil, and for the state (federal district) of *Distrito Federal*, the richest in Brazil, in 1996 and 2012. Both states see pronounced compression in their distribution over time as the minimum wage increases. We note the striking similarity in the shape of the wage distribution between panel (a) showing the poor

state Maranhão in 1996 and panel (d) showing the rich state Distrito Federal in 2012.

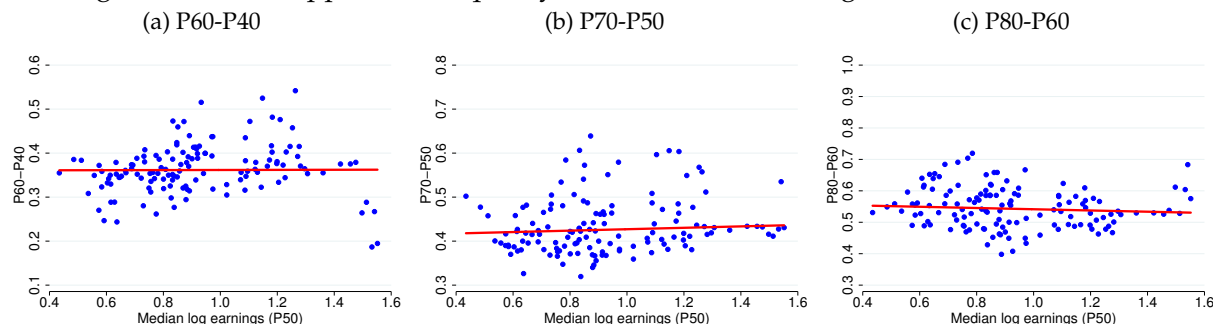
Figure 23. Data: Wage histograms for a poor versus rich state of Brazil, 1996 and 2012



Notes: Figure shows wage histograms for the state of Maranhão, the second poorest in Brazil, and in the state (federal district) of Distrito Federal, the richest in Brazil, in 1996 and 2012. Source: RAIS.

We now demonstrate as a second test of our identifying assumption that the upper tail of the wage distribution is invariant to the level of the “effective minimum wage.” Figure 24 shows that the relation between various upper-tail wage percentile ratios and the median wages across states from 1996 to 2000 is mostly flat, consistent with a shared “latent distribution” across states that is differentially uncovered by the federal minimum wage.

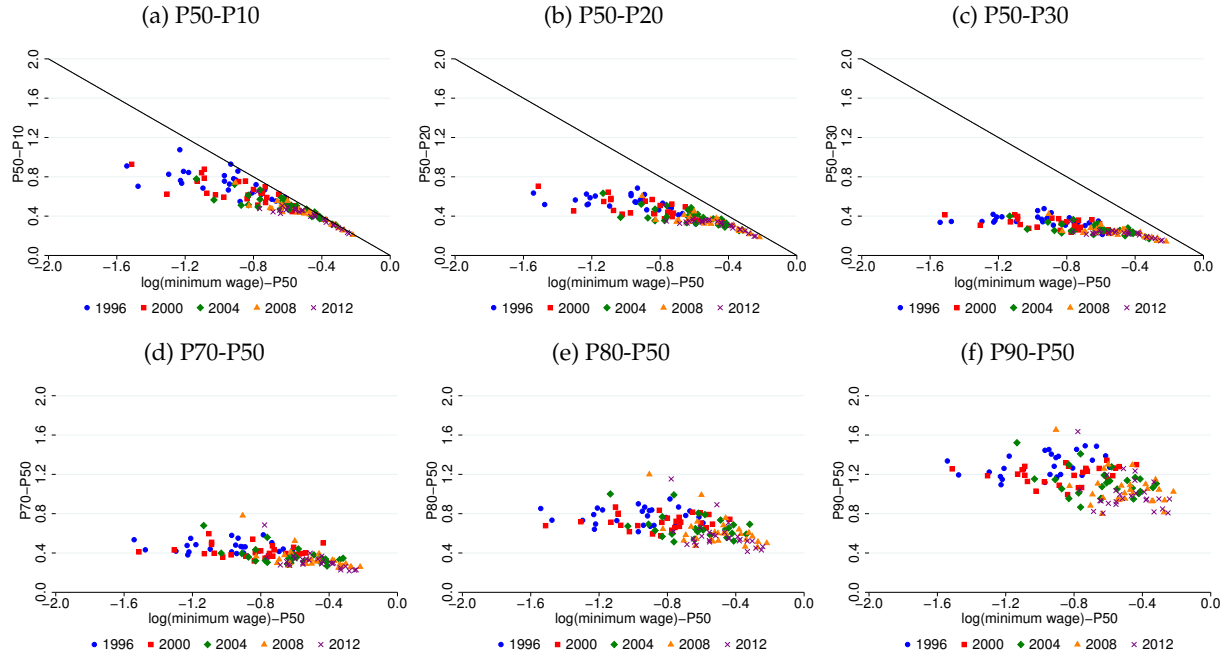
Figure 24. Data: Upper-tail inequality versus median earnings across states, 1996–2000



Notes: Blue dots represent state-year observation. Red line represents worker-weighted linear fit. Specification with no state dummies or state trends. Source: RAIS.

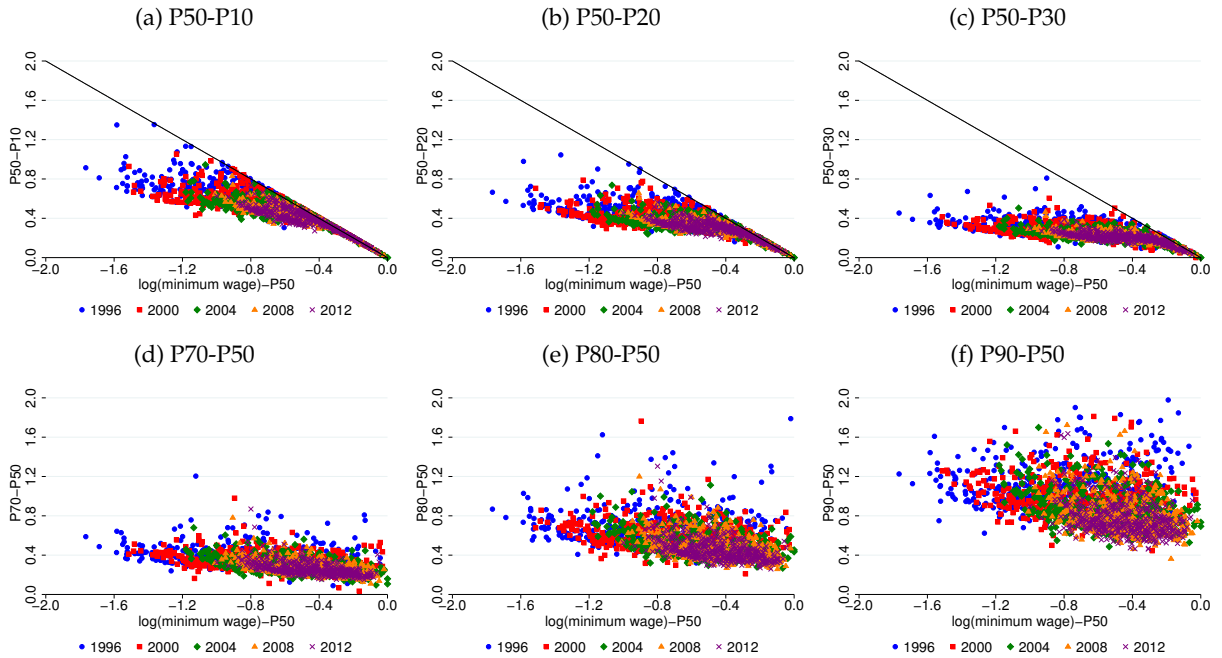
D.3 Additional results on spillover effects identified off regional variation

Figure 25. Data: Wage percentile ratios across Brazilian states over time, 1996–2012



Notes: Figure plots different wage percentile ratios against the Kaitz index, $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, with each marker representing one state-year combination for each of Brazil's 27 states. Source: RAIS.

Figure 26. Data: Wage percentile ratios across Brazilian microregions over time, 1996–2012



Notes: Figure plots different wage percentile ratios against the Kaitz index, $kaitz_{st} \equiv \log w_t^{min} - \log w_{st}^{median}$, with each marker representing one state-microregion combination for each of Brazil's 556 microregions. A small number of outliers are dropped for presentation purposes. Source: RAIS.