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Demographics and the Evolution of Global Imbalances*

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Abstract

The working age share of the population has evolved, and will continue to evolve, asymmetrically across countries. I develop a dynamic, multicountry, Ricardian trade model with endogenous labor supply to quantify how these asymmetries systematically affect the pattern of trade imbalances across 28 countries from 1970-2014. Changes in both domestic and foreign working age shares impact a country's net exports directly through the demand for net saving and indirectly through relative labor supply and population growth. Counterfactually removing demographic-induced changes to saving unveils a strong negative contemporaneous relationship between net exports and productivity growth. Demographics, thus, alleviate the allocation puzzle, and do so to a greater degree than investment distortions. Neither labor market distortions nor trade distortions systematically reconcile the puzzle.

JEL codes: F11, F21, J11

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1 Introduction

As of 2014, the absolute value of net exports summed across 182 countries amounted to 5.2 percent of world GDP (version 9.0 of the Penn World Table). These imbalances entailed net inflows of resources for some countries and net outflows for others. For instance, for the United States imports exceeded exports by 4.2 percent of GDP, while for China imports fell short of exports by 3.7 percent of GDP. In most countries both the direction and the magnitude of trade imbalances are highly persistent over time, yet, little is known about what factors systematically determine imbalances across the world in the long run. Since trade imbalances reflect both intertemporal and intratemporal resource allocations, implications for policies that target trade and current accounts hinge on understanding the drivers.

Existing research has focussed on the effects of cross-country differences in factors such as productivity growth, trade barriers, institutions, and various distortions; few papers have studied the role of demographics. Demographics offer a promising candidate since they have direct implications for aggregate saving and are persistent over time. General equilibrium analyses linking demographics to trade imbalances have used two-country, or at most three-country, models (e.g., Ferrero, 2010; Krueger and Ludwig, 2007). Multicountry analyses have been empirical (e.g., Alfaro, Kalemli-Ozcan, and Volosovych, 2008; Higgins, 1998).

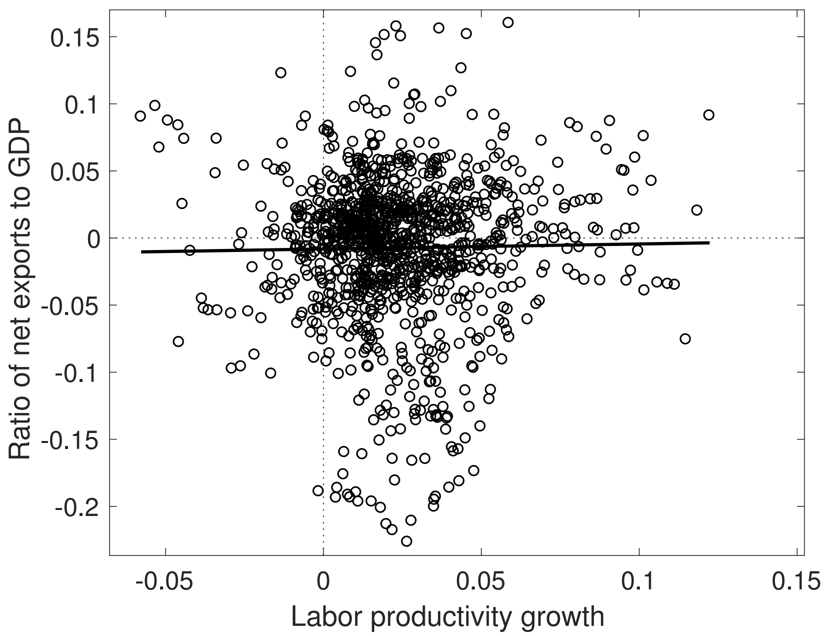
This paper builds a dynamic, multicountry, Ricardian trade model to study how demographics systematically affect the pattern of trade imbalances across 28 countries since 1970. Countries are integrated via bilateral trade linkages and can borrow and lend with each other. Dynamics are driven by saving in one-period international bonds and investment in physical capital. Labor supply is endogenous. Both contemporaneous and projected demographics affect trade imbalances directly through saving demand and indirectly through labor supply and population growth. To my knowledge, this is the first model to combine all of these features in a multicountry environment and deliver exact transitional dynamics.

I find that cross-country differences in changes to the working age share helps systematically explain both the direction and magnitude of trade imbalances. The findings shed new light on the allocation puzzle. The puzzle begins with a clear prediction from economic theory: Slow-growing countries should run trade surpluses and, fast-growing countries, trade deficits.[1] However, a great deal of empirical evidence stands in contrast to this prediction (see Gourinchas and Jeanne, 2013; Prasad, Rajan, and Subramanian, 2007). In my sample

[1]The allocation puzzle is often discussed in terms of the current account. All of the facts and findings that I describe in terms of the trade balance are also true in terms of the current account balance.

of 28 countries from 1970-2014, the elasticity of the ratio of net exports to GDP with respect to contemporaneous labor productivity growth is 0.04 (measured as the slope of the line in Figure 1). Not only is this elasticity small, it is positive, in line with the allocation puzzle.

Figure 1: Ratio of net exports to GDP against labor productivity growth



Notes: Horizontal axis is the average annual growth in labor productivity during five year windows. Vertical axis is the average ratio of net exports to GDP during five year windows. Windows run from [1970,1974]-[2010,2014]. The line corresponds to the best fit curve using OLS.

The main result is unveiled by computing a counterfactual in which the working age share in every country is simultaneously held constant at 1970 levels. Statistically analyzing data from the the counterfactual, I find that the elasticity of the ratio of net exports to GDP with respect to contemporaneous labor productivity growth is -0.90, compared to 0.04 in both the baseline model and in the data. That is, the economic prediction called into question by the allocation puzzle is indeed present but merely masked by demographic forces.

As an illustration, China's observed working age share increased at a much higher rate than the world average. In particular, from 1990-2014 China experienced a *demographic window*—a period in which the support ratio is favorable for saving—and capitalized through high national saving rates, resulting in positive net exports. In the counterfactual with the working age share held fixed in every country, China runs a large trade deficit in tandem with its high productivity growth. Conversely, since 1970 the United States' working age share increased at a slower rate compared to the world average. In the counterfactual the U.S. trade balance fluctuates around zero.

The model is calibrated to 27 countries and a rest-of-world aggregate from 1970-2060. Data from 1970-2014 are observed, while data for 2015-2060 are based on long-term projections. Incorporating projections provides external discipline to households' expectations in formulating saving decisions during the period of interest: 1970-2014. It also significantly reduces the impact that terminal conditions impose on the saving behavior during that period. I employ a wedge-accounting procedure that assigns parameter values to rationalize both the observed and the projected data as a solution to a perfect-foresight equilibrium.[2]

In the model each country is populated by a representative household, so there is no explicit notion of heterogeneity in age either within or across countries. Instead, information about the age distribution is embedded in three parameters: (i) saving wedges (changes in the discount factor), (ii) labor wedges (marginal utility of leisure), and (iii) population growth rates. I provide microfoundations to illustrate the mapping between the parameters and changes in the age distribution arising from an overlapping-generations framework. The microfoundations guide an empirical decomposition of these parameters into a demographic component and a non-demographic (distortionary) component.[3] The decomposition serves as a disciplining device to study counterfactuals in which alterations to the working age share are manifested in alterations to the three parameters.

Each parameter provides a distinct channel through which demographics affect trade imbalances. First, a higher working age share implies a higher saving wedge, i.e., a greater demand for future consumption relative to current consumption. Resultantly there will be higher contemporaneous demand for saving and higher net exports.

Second, a higher working age share implies a lower labor wedge, which in turn implies higher labor supply and productive capacity. As a result, a country will borrow less to fund its liabilities and will have contemporaneously higher demand for saving and net exports.

Third, a higher working age share implies lower population growth. When population growth is relatively low, agents will save less to ensure that consumption is smoothed out

[2]Wedge accounting has its roots in business cycle accounting (see Chari, Kehoe, and McGrattan, 2007). Eaton, Kortum, Neiman, and Romalis (2016) modified the procedure to encompass a dynamic trade model. One key difference from them is that my solution procedure solves for the decentralized competitive allocations, whereas their's assumes fixed Pareto weights, which forces each country's share in global consumption to be counterfactually invariant with respect to the baseline. In my model these weights endogenously change in each counterfactual.

[3]Since the wedges fully account for each country's saving behavior, the distortionary component captures non-demographic aspects pointed to by the literature, including financial market distortions.

over time on a per capita basis. Novel to the open economy model is that population size affects the terms of trade and real interest rate differentials across countries, a channel that is under appreciated in the literature. A relatively lower population level implies a higher real exchange rate, all else equal. Therefore, lower population growth implies relatively improving terms of trade over time and, in turn, a relatively higher real interest rate, which supports the greater demand for saving and higher net exports.

Each country's net exports is influenced by both domestic and foreign demographics. I find that net exports respond more to domestic demographics, relative to foreign demographics, in countries in which the working age share evolved more differently from the world average. For example, China's trade balance was driven more by changes in its own demographics and China's working age share increased significantly faster than that of the world. Conversly, the U.S. trade balance was influenced more by changes in foreign demographics and the U.S. working age share increased slightly slower than, but at a fairly similar rate as, that of the world.

Because trade imbalances encompass both intertemporal and intratemporal margins, it is useful to consider the ratio of net exports to GDP as the product of (i) the ratio of net exports to trade (imports plus exports) and (ii) the ratio of trade to GDP. Loosely speaking, the ratio of net exports to trade as reflects intertemporal margins, i.e., saving, that govern direction of *net* trade and capital flows. The ratio of trade to GDP reflects intratemporal margins, i.e., trade distortions, that govern the volume of *gross* trade flows and openness. Two distinct literatures have emerged studying each margin.

My paper connects, and builds on, these literatures by systematically addressing both the intertemporal margin (direction of net trade) as well as the intratemporal margin (volume of trade) by matching the observed bilateral trade flows and, hence, bilateral and aggregate trade balances across many countries in a general equilibrium framework.

The literature on imbalances and capital flows can be traced to Lucas (1990), who questions why capital does not flow from rich to poor countries. That is, in the long run, the marginal product of capital (MPK) should equalize, requiring similar capital-labor ratios. While this is still an open question, Caselli and Feyrer (2007) show that after adjusting for differences in the relative price of capital across countries, real MPKs are not very different. Because of this, my model is disciplined to match the observed relative prices.

Recently, attention has focussed on understanding why capital does not flow into fast growing countries, i.e., the allocation puzzle. A number of explanations have been put forward. Carroll, Overland, and Weil (2000) posit that habits can account for the fact that a boom income is not immediately met with a boom in consumption. Instead, fast growing countries will tend to save in the short run. However, this answer does not address why saving winds up in net exports as opposed to investment. Along these lines Aguiar and Amador (2011) argue that fast growing economies with high debt will not invest capital due to the risk of expropriation. Instead, governments in these economies have an incentive to pay down debt. Buera and Shin (2017) argue that financial frictions restrict the extent that investment can respond to fast GDP growth, implying that a fast growing country will tend to run a current account surplus in the short run. Ohanian, Restrepo-Echavarria, and Wright (2017) point to labor market frictions as being an important ingredient. They argue that, after World War II, labor wedges in Latin America impeded equilibrium supply of labor, thereby reducing the marginal product of capital and repelling foreign investment.

Each of these explanations sheds light on idiosyncratic episodes of abnormally high growth, but none systematically consider a large number of countries over a long time period. Looking across 28 countries from 1970-2014, I find that investment distortions partially alleviate the allocation puzzle, but are less important than demographics. Labor market distortions do not reconcile the puzzle in my sample. The underlying economics are straightforward: trade imbalances appear in the current account, which embodies net saving. Gourinchas and Jeanne (2013) argue that the allocation puzzle is one about saving—not investment—and more specifically about public saving. Public saving is driven in large part by pensions, where current and projected assets and liabilities depend on current and projected demographics. Asymmetries in demographic changes provide an incentive for intertemporal trade to balance assets and liabilities.

A separate, albeit related, literature explicitly investigates the role of trade openness in explaining imbalances. Using a multicountry model, Reyes-Heroles (2016) argues that the increased ratio of global trade to global GDP accounts for the rise in the ratio of global imbalances to global GDP. He argues declining trade costs over time generate larger trade volumes, thereby amplifying the net trade positions as a share of GDP. His analysis does not focus on explaining heterogeneity in country-level imbalances.

Alessandria and Choi (2017) document that the ratio of U.S. net exports to GDP increased in absolute value because trade increased as a share of GDP, while the ratio of net exports to trade has been relatively stationary. They argue that U.S. import barriers declined faster than U.S. export barriers, and that this asymmetry helps explain both the magnitude and the direction of the U.S. trade imbalance. I find that asymmetries in bilateral trade barriers help explain the temporary widening of the U.S. trade deficit since 2000, while demographics capture the persistent widening since the 1980s. Changes in bilateral trade barriers do not systematically reconcile the allocation puzzle across many countries.

China is well-cited example of a country with rapid growth and large trade surplus. Wei and Zhang (2011) argue that male-biased gender ratios encourage men to save by purchasing real estate to attract scarce female partners. Yang, Zhang, and Zhou (2012) argue that successive cohorts of young Chinese workers face increasingly flat life-cycle earnings profiles, thereby reducing household borrowing in the face of higher future aggregate productivity. İmorohoğlu and Zhao (2017a) argue that, due to China's one-child policy, the elderly rely more on personal savings to replace lacking family support during retirement. İmorohoğlu and Zhao (2017b) argue that financial constraints faced by firms is equally important as the one-child policy in accounting for the rise in China's saving rate. In Song, Storesletten, and Zilibotti (2011), financial market imperfections imply that private firms in China finance the adaptation of technology through internal savings. My findings do not rule out any of these theories but, instead, offer a systematic assessment of the role of demographic-induced saving behavior across many countries over a long time period.

My modeling approach differs slightly from those found in previous studies on demographics and trade imbalances. Krueger and Ludwig (2007) study demographics in an open economy with overlapping generations. However, their model considers only three countries. Ferrero (2010) incorporates demographics into a two-country model with two types of agents: workers and retirees. Workers transition into retirement with a time-varying probability making the problem mimic that of a representative agent. Their approach is similar to that developed by Blanchard (1985), who shows that a representative-agent model can mimic outcomes from an overlapping generations model where agents face a constant probability of death in each period. My paper abstracts from the overlapping generations structure and instead uses a representative household whose preferences change over time based on changes in the age distribution. This enables me to avoid value functions, which are far too intractable in a general equilibrium environment with a large number of countries.

Methodologically, this paper contributes to a recent strand of literature incorporating dynamics into multicountry trade models (Alvarez, 2017; Caliendo, Dvorkin, and Parro, 2015; Eaton, Kortum, Neiman, and Romalis, 2016; Ravikumar, Santacreu, and Sposi, 2017; Reyes-Heroles, 2016; Sposi, 2012; Zylkin, 2016). I extend the algorithm developed in Ravikumar, Santacreu, and Sposi (2017) to incorporate endogenous labor supply.[4]

[4]Adao, Arkolakis, and Esposito (2017) incorporate endoegnous labor supply into a static multicountry trade model.

2 Model

There are I countries, indexed by $i = 1, \ldots, I$, and time is discrete, running from $t = 1, 2, \ldots$. There is one sector consisting of a continuum of tradable varieties. Countries differ in comparative advantage across the varieties and trade is subject to bilateral iceberg costs. Varieties are purchased from the least-cost supplier and aggregated into a composite good that can be converted into consumption, investment, or intermediate inputs. Each country admits a representative household that owns its country's labor, capital, and net-foreign assets. Labor supply is endogenous is hired by domestic firms. Capital is rented inelastically to domestic firms. Income from capital, labor, and the net-foreign asset position is spent on consumption, investment in physical capital, and net purchases of one-period bonds.

2.1 Endowments

In the initial period each country is endowed with a stock of capital, K_{i1}, and an initial net-foreign asset (NFA) position, $\mathcal{A}_{i1}$. In each period population is denoted by N_{it}.

2.2 Technology

There is a unit interval of potentially tradable varieties indexed by $v \in [0, 1]$. All varieties are combined to construct a composite good,

$$Q_{it} = \left[\int_0^1 q_{it}(v)^{1-1/\eta} dv \right]^{\eta/(\eta-1)},$$

where η is the elasticity of substitution between any two varieties. The term $q_{it}(v)$ is the quantity of variety v used to construct the composite good in country i at time t.

Each country has access to a technology to produce any variety v using capital, labor, and the composite intermediate good.

$$Y_{it}(v) = z_{it}(v) \left(A_{it} K_{it}(v)^{\alpha} L_{it}(v)^{1-\alpha} \right)^{\nu_{it}} M_{it}(v)^{1-\nu_{it}}.$$

The term $M_{it}(v)$ ss the quantity of the composite good used as an input to produce $Y_{it}(v)$ units of variety v, while $K_{it}(v)$ and $L_{it}(v)$ are the quantities of capital labor employed.

The parameter $\nu_{it} \in [0, 1]$ denotes the share of value added in total output in country i at time t, while α denotes capital's share in value added.

The term A_{it} denotes country i's value-added productivity at time t while the term $z_{it}(v)$ is country i's idiosyncratic productivity draw for producing variety v at time t, which scales gross output. Idiosyncratic productivity in each country is drawn independently from a Fréchet with cumulative distribution function $F(z) = \exp(-z^{-\theta})$.

2.3 Trade

International trade is subject to iceberg trade barriers. At time t, country i must purchase $d_{ijt} \geq 1$ units of any intermediate variety from country j in order for one unit to arrive; $d_{ijt} - 1$ units *melt* away in transit. As a normalization I assume that $d_{iit} = 1$ for all i and t.

2.4 Preferences

The representative household's preferences are defined over consumption per capita, $\frac{C_{it}}{N_{it}}$, and labor supply per capita (hours), $\frac{L_{it}}{N_{it}}$, over the lifetime.

$$\sum_{t=1}^{\infty} \beta^{t-1} \psi_{it} N_{it} \mathcal{U}\left(\frac{C_{it}}{N_{it}}, \frac{L_{it}}{N_{it}}\right).$$

Utility between adjacent periods is discounted by $\beta \in (0, 1)$. The parameter ψ_{it} is a discount-factor shock in country i at time t, meaning that ψ_{it}/ψ_{it-1}—"saving wedge" from now on—is a shock to the marginal rate of substitution between consumption in adjacent time periods.

The period-utility function is given by

$$\mathcal{U}\left(\frac{C_{it}}{N_{it}}, \frac{L_{it}}{N_{it}}\right) = \frac{(C_{it}/N_{it})^{1-1/\sigma}}{1-1/\sigma} + \zeta_{it} \frac{(1 - L_{it}/N_{it})^{1-1/\phi}}{1-1/\phi}.$$

The term σ denotes the intertemporal elasticity of substitution for consumption with respect to the real interest rate, while ϕ denotes the elasticity of labor supply with respect to the real wage. Both parameters are constant across countries and over time. The term ζ_{it} is a shock to the marginal utility of leisure—"labor wedge" from now on—in country i at time t.

Both the saving wedge and the labor wedge can equivalently be modeled as distortions to net-foreign income and labor income, respectively. I include these wedges in the preferences to emphasize the idea that they incorporate demographic forces that influence the relative demand for saving and the optimal labor supply, even in the absence of distortions. Later on I provide micro foundations to justify this assumption. At which point, I decompose the wedges into distinct demographic and distortionary components.

Demographics The model does not explicitly include heterogeneity in age. Instead, variation in demographics across countries and over time is manifested in (i) the saving wedge, (ii) the labor wedge, and (iii) population growth. A main goal of the quantitative exercise is to isolate the variation in these wedges that comes from variation in demographics.

Net-foreign asset accumulation The representative household enters period t with NFA position $\mathcal{A}_{it}$. If $\mathcal{A}_{it} < 0$ then the household has a net debt position. It is augmented by net purchases of one-period bonds (the current account balance), $\mathcal{B}_{it}$. With $\mathcal{A}_{i1}$ given:

$$\mathcal{A}_{it+1} = \mathcal{A}_{it} + \mathcal{B}_{it}.$$

Capital accumulation The household enters period t with K_{it} units of capital. A fraction δ depreciates during the period. Investment, X_{it}, adds to the stock of capital subject to an adjustment cost. Thus, with $K_{i1} > 0$ given, the capital accumulation technology is

$$K_{it+1} = (1-\delta)K_{it} + \delta^{1-\lambda} X_{it}^{\lambda} K_{it}^{1-\lambda}.$$

The depreciation rate, δ, and the adjustment cost elasticity, λ, are constant both across countries and over time. The term $\delta^{1-\lambda}$ ensures that there are no adjustment costs to replace depreciated capital; for instance, in a steady state $X^\star = \delta K^\star$. The adjustment cost implies that the return to capital investment is not invariant to the quantity of investment. In turn, the household always chooses a unique portfolio of capital and bonds at any prices.

Budget constraint Capital and labor are compensated at the rates r_{it} and w_{it}, respectively. Capital income is subject to a distortionary tax, τ_{it}^{k}, while current investment expenditures are tax deductible. Tax revenue is returned in lump sum to the household, T_{it}.

The world interest rate on outstanding debt at time t is denoted by q_t. If the household has a positive NFA position at time t, then net foreign income, $q_t\mathcal{A}_{it}$, is positive. Otherwise net foreign income is negative as resources must be spent to service existing liabilities.

The composite good has price P_{it} and can be transformed into χ_{it}^{c} units of consumption or into χ_{it}^{x} units of investment. These transformation costs pin down relative prices and are needed to reconcile the Lucas (1990) paradox (see Caselli and Feyrer, 2007). The budget constraint in each period is given by

$$\frac{P_{it}}{\chi_{it}^{c}} C_{it} + \mathcal{B}_{it} = \left(r_{it}K_{it} - \frac{P_{it}}{\chi_{it}^{x}} X_{it} \right)(1-\tau_{it}^{k}) + w_{it}L_i + q_t\mathcal{A}_{it} + T_{it}.$$

2.5 Equilibrium

A competitive equilibrium satisfies the following conditions: (i) taking prices as given, the representative household in each country maximizes its lifetime utility subject to its budget constraint and technologies for accumulating physical capital and assets, (ii) taking prices as given, firms maximize profits subject to the available technologies, (iii) intermediate varieties are purchased from their lowest-cost provider, and (iv) markets clear. At each point in time world GDP is the numéraire: $\sum_{i=1}^{I} r_{it}K_{it} + w_{it}L_i = 1$. That is, all prices are expressed in units of current world GDP. Appendix D describes the equilibrium conditions in more detail.

3 Calibration

The model is applied to 28 countries (27 individual countries plus a rest-of-world aggregate) from 1970-2060. Data from 1970-2014 are realized, while data from 2015-2060 are based on projections. Incorporating projections serves two purposes. First, it imposes the terminal conditions as of 2060, which has minimal effects on the saving behavior from 1970-2014, which is the period of interest. Second, it provides external discipline to the agents expectations in formulating saving decisions prior to 2014. Appendix C provides a description of the data along with a list of the countries and their 3-digit ISO codes.

The calibration involves two parts. The first part assigns values to parameters that are common across countries and constant over time: $(\theta, \eta, \delta, \lambda, \alpha, \beta, \sigma, \phi)$. These are taken off the shelf from the literature. The second part assigns values to country-specific and time-varying parameters: $(K_{i1}, \mathcal{A}_{i1})$ and $\{N_{it}, \chi^c_{it}, \chi^x_{it}, \nu_{it}, \psi_{it}, \zeta_{it}, \tau^k_{it}, d_{ijt}, A_{it}\}_{t=1}^{T}$ for all (i, j, t). These parameters are inferred to rationalize both the observed and projected data as a solution to a perfect foresight equilibrium. The data targets, roughly in order of how they map into the model parameters, are (i) initial stock of capital, (ii) initial NFA position, (iii) population, (iv) price level of consumption using PPP exchange rates, (v) price level of investment using PPP exchange rates, (vi) ratio of value added to gross output, (vii) real consumption, (viii) ratio of employment to population, (ix) real investment, (x) bilateral trade flows, and (xi) price level of tradables using PPP exchange rates.

The saving wedge, the labor wedge, and population growth each play a role in tying demographic forces to the model. The remaining parameters are used to match other important aspects of data to ensure internal consistency with national accounts.

3.1 Common parameters

The values for the common parameters are reported in Table 1. The discount rate is set to $\beta = 0.96$. I set the intertemporal elasticity of substitution to $\sigma = 1$, which corresponds to log utility over consumption, to be consistent with long run balanced growth. The Frisch elasticity of labor supply is set to $\phi = 2$, based on Peterman (2016).

Capital's share in value added is set to 0.33, based on evidence in Gollin (2002). In line with the literature, I set the depreciation rate for capital to $\delta = 0.06$. The adjustment cost elasticity is set to $\lambda = 0.76$, which is the midpoint between 0.52 and 1. The value 0.52 corresponds to the median value used by Eaton, Kortum, Neiman, and Romalis (2016) who work with quarterly data. The value 1 corresponds to no adjustment costs.

I set the trade elasticity to $\theta = 4$ as in Simonovska and Waugh (2014). The parameter η plays no role in the model other than satisfying $1 + \frac{1}{\theta}(1 - \eta) > 0$; I set $\eta = 2$.

Table 1: Common parameters

β	Annual discount factor	0.96
σ	Intertemporal elasticity of substitution	1
ϕ	Elasticity of labor supply	2
α	Capital's share in value added	0.33
δ	Annual depreciation rate for stock of capital	0.06
λ	Adjustment cost elasticity	0.76
θ	Trade elasticity	4
η	Elasticity of substitution between varieties	2

3.2 Country-specific and time-varying parameters

Some of the country-specific parameters are observable. For the ones that are not observable, I invert structural equations from the model to link them with data.

Initial conditions For each country the initial stock of capital, K_{i1}, is taken directly from the data in 1970, while the initial net-foreign asset position is set to $\mathcal{A}_{i1} = 0$.

Population, relative prices, and value added shares Population, N_{it}, is observable. The parameter ν_{it} is the ratio of aggregate value added to gross production.

The (inverse) relative price of consumption, χ^c_{it}, is computed as the price of intermediates relative to consumption. Similarly, χ^x_{it} is the price of intermediates relative to investment.

Saving wedges The saving wedges are only identified up to $I-1$ countries at each point in time. As such, I normalize $\psi_{Ut}=1$ for all t (subscript U denotes the United States). Appealing to the Euler equation for bonds in the United States, the implied world interest rate is recovered using data on the paths for U.S. population, prices, and consumption:

$$\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}} = \beta^{\sigma}\left(\frac{\psi_{it+1}}{\psi_{it}}\right)^{\sigma}\left(\frac{1+q_{t+1}}{\frac{P_{it+1}/\chi^{c}_{it+1}}{P_{it}/\chi^{c}_{it}}}\right)^{\sigma}.$$

Given the world interest rate, q_t, saving wedges in every other country are recovered from the respective Euler equations for bonds. I normalize $\psi_{i1}=1$ in every country.

Labor wedges The labor wedges are pinned down by exploiting the optimal labor-supply condition and utilizing data on wages, prices, employment, and consumption:

$$1-\frac{L_{it}}{N_{it}} = (\zeta_{it})^{\phi}\left(\frac{w_{it}}{P_{it}/\chi^{c}_{it}}\right)^{-\phi}\left(\frac{C_{it}}{N_{it}}\right)^{\phi/\sigma}. \tag{1}$$

The wage rate, w_{it}, is recovered from GDP in current U.S. dollars as: $w_{it}=(1-\alpha)\left(\frac{GDP_{it}}{L_{it}}\right)$.

Investment distortions Without loss of generality, I initialize $\tau_{i1}=0$. The remaining investment distortions require measurements of the capital stock in every period. Given capital stocks in period 1, K_{i1}, and data on investment in physical capital, X_{it}, I construct the sequence of capital stocks iteratively using $K_{it+1}=(1-\delta)K_{it}+\delta^{1-\lambda}X_{it}^{\lambda}K_{it}^{1-\lambda}$.

Using the notation, $\Phi(K',K)\equiv X\equiv \delta^{\frac{\lambda-1}{\lambda}}\left(\frac{K'}{K}-(1-\delta)\right)^{\frac{1}{\lambda}}K$, let Φ_1 and Φ_2 denote the partial derivatives with respect to the first and second arguments respectively. Given the constructed sequence of capital stocks, I recover τ^{k}_{it} iteratively using the Euler equation for investment in physical capital:

$$\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}} = \beta^{\sigma}\left(\frac{\psi_{it+1}}{\psi_{it}}\right)^{\sigma}\left(\frac{\frac{r_{it+1}}{P_{it+1}/\chi^{c}_{it+1}}-\left(\frac{\chi^{c}_{it+1}}{\chi^{x}_{it+1}}\right)\Phi_2(K_{it+2},K_{it+1})}{\left(\frac{\chi^{c}_{it}}{\chi^{x}_{it}}\right)\Phi_1(K_{it+1},K_{it})}\right)^{\sigma}\left(\frac{1-\tau^{k}_{it+1}}{1-\tau^{k}_{it}}\right)^{\sigma}.$$

Trade barriers The trade barrier for any given country pair is computed using data on prices and bilateral trade shares using the following structural equation:

$$\frac{\pi_{ijt}}{\pi_{iit}} = \left(\frac{P_{jt}}{P_{it}}\right)^{-\theta} d_{ijt}^{-\theta}, \tag{2}$$

where π_{ijt} is the share of country i's absorption that is sourced from country j and P_{it} is the price of tradables in country i. I set $d_{ijt} = 10^8$ for observations in which $\pi_{ijt} = 0$ and set $d_{ijt} = 1$ if the inferred value is less than 1. As a normalization, $d_{iit} = 1$.

Productivity I back out productivity, A_{it}, using price data and home trade shares,

$$P_{it} = \left(\frac{\gamma \pi_{iit}^{1/\theta}}{A_{it}^{\nu_{it}}}\right) \left(\frac{r_{it}}{\alpha \nu_{it}}\right)^{\alpha \nu_{it}} \left(\frac{w_{it}}{(1-\alpha)\nu_{it}}\right)^{(1-\alpha)\nu_{it}} \left(\frac{P_{it}}{1-\nu_{it}}\right)^{1-\nu_{it}}. \tag{3}$$

Measured productivity, $\left(\frac{A_{it}^{\nu_{it}}}{\gamma \pi_{iit}^{1/\theta}}\right)$, encompasses fundamental productivity, A_{it}, and a selection effect through the home trade share, π_{iit}.[5] The rental rate for capital is $r_{it} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w_{it} L_{it}}{K_{it}}\right)$.

3.3 Decomposing wedges

While the model does not explicitly incorporate heterogeneity with respect to age, I take the position that differences in the age distribution are manifested in a subset of the parameters: the saving wedge (for life cycle reasons), the labor wedge (for life cycle reasons), and population growth (for biological reasons). Appendix A provides microfoundations in an overlapping generations environment that support this position. I implement an empirical exercise to decompose the variation in wedges due to demographics and that due to distortions.

3.3.1 Isolating the demographic component within the wedges

Define s_{it} as the share of country i's population at time t that is between the ages of 15-64 (working age share from now on, as defined by the World Bank). To isolate the variation in preference wedges stemming from variation in demographics, I project variation in the calibrated wedges onto variation in the observed working age share. I do this for the saving wedge, the labor wedge, and population growth; all of the other parameters are assumed to be invariant to the age distribution. I assume that the log-wedges are linear in the working age share. Appendix F considers higher-degree polynomial specifications and also shows that the results hold under different variants of projected working age shares.

[5] $\gamma = \Gamma(1 + (1-\eta)/\theta)^{1/(1-\eta)}$, where $\Gamma(\cdot)$ is the Gamma function.

Isolating demographic content of saving wedges Consider the saving wedge as the product of both the discount factor change that depends on the age distribution, $\phi(s_{it})$, and a distortion to the return on bonds, $(1-\tau^a_{it})$: $\frac{\psi_{it}}{\psi_{it-1}} = (1-\tau^a_{it}) \times \phi(s_{it})$. Most wedge accounting exercises attribute the entirety of the saving wedge to the distortionary component (see Ohanian, Restrepo-Echavarria, and Wright, 2017). Taking logs implies $\ln\left(\frac{\psi_{it}}{\psi_{it-1}}\right) = \varphi(s_{it}) + \ln(1-\tau^a_{it})$. Since neither the demographic nor the distortionary component is directly observable, I decompose the labor wedge by projecting it onto data on the age distribution across countries and over time as follows.

$$\ln\left(\frac{\psi_{it}}{\psi_{it-1}}\right) = \gamma^\psi_i + \mu^\psi \times (s_{it-1} - s_{Ut-1}) + \varepsilon^\psi_{it-1}; i = 1, \dots I \text{ (ex. U.S.)}; t = 2, \dots T. \tag{4}$$

I exclude time-specific fixed effects and also exclude the United States (indexed by U) because $\psi_{Ut} = 1$ for all t. Using OLS, this projection exercise attributes $\hat{\mu} \times (s_{it} - s_{Ut})$ to the demographic component and attributes $\hat{\gamma^\psi_i} + \hat{\kappa^\psi_t} + \hat{\varepsilon^\psi_{it}}$ to the distortionary component. The distortionary component captures country-specific and time-varying factors that affect the consumption-saving tradeoff such as monetary policy, fiscal policy, social safety nets, currency risk, and uncertainty surrounding business cycle conditions.

Isolating demographic content of labor wedges Consider the labor wedge as the product of both labor market distortions and a shock to the marginal utility of labor that depends on the age distribution: $\zeta_{it} = (1-\tau^\ell_{it}) \times \phi(s_{it})$. I decompose the labor wedge by projecting it onto data on the age distribution across countries and over time as follows.

$$\ln(\zeta_{it}) = \gamma^\zeta_i + \kappa^\zeta_t + \mu^\zeta \times s_{it} + \varepsilon^\zeta_{it}; i = 1, \dots I; t = 1, \dots T. \tag{5}$$

Using OLS, this projection exercise attributes $\hat{\mu} \times s_{it}$ to the demographic component and attributes $\hat{\gamma^\zeta_i} + \hat{\kappa^\zeta_t} + \hat{\varepsilon^\zeta_{it}}$ to the distortionary component. Distortions capture female labor force participation differences, change in labor income taxation, and other factors that vary across countries and (or) over time that are independent from the age distribution.

Isolating the demographic component of population growth Consider isolating the contribution of demographics to population growth.

$$\ln\left(\frac{N_{it}}{N_{it-1}}\right) = \gamma^N_i + \kappa^N_{t-1} + \mu^N \times s_{it-1} + \varepsilon^N_{it-1}; i = 1, \dots I; t = 2, \dots T. \tag{6}$$

Empirical relationship between demographics and wedges Table 2 reports the estimated elasticities of each wedge with respect to the working age share. The saving wedge has a positive elasticity, while both the labor wedge and population growth have negative elasticities. The R^2 is quite low for the saving wedge, reflecting the fact that the wedges are fairly noisy over time (see Figure 2a), whereas working age shares are fairly smooth. Hence, the positive coefficient is capturing the long-run behavior. For instance, China's saving wedge is uniformly higher than that of the U.S. after 1990, the same period in which China's working age share was larger and growing rapidly. The labor wedges in all countries tend to decline over time relatively smoothly (see Figure 2b); the R^2 for the labor wedge regression is over 75 percent.

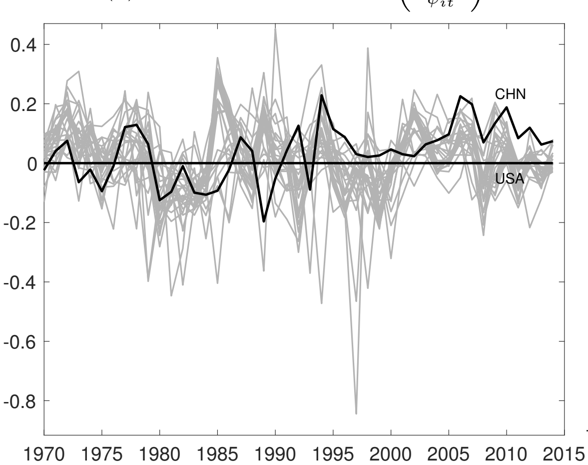
Table 2: Estimated elasticity of wedges w.r.t. the working age share

Left-hand side variable (wedge)	Coefficient on working age share	Point estimate (S.E.)	R^2
Saving wedge $\left(\frac{\psi_t}{\psi_{t-1}}\right)$	$\widehat{\mu^\psi}$	0.147 (0.044)	0.013
Labor wedge (ζ_t)	$\widehat{\mu^\zeta}$	-1.740 (0.060)	0.759
Population growth $\left(\frac{N_t}{N_{t-1}}\right)$	$\widehat{\mu^N}$	-0.044 (0.002)	0.787

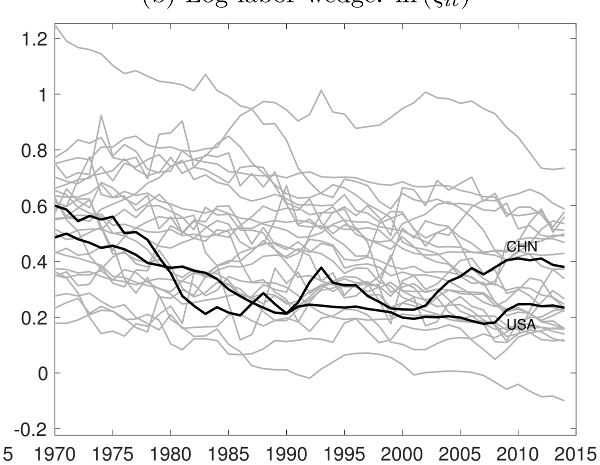
Notes: The estimates are based on OLS regressions using equation (6)-(5).

Figure 2: Calibrated wedges

(a) Log saving wedge: $\ln\left(\frac{\psi_{it+1}}{\psi_{it}}\right)$



(b) Log labor wedge: $\ln(\zeta_{it})$



Notes: The U.S. saving wedge is normalized to 1 in every period.

4 Counterfactual analysis

To quantify the importance of demographics, I exploit the estimates from equations (4)-(6) to construct counterfactual processes for the wedges by removing variation stemming from the demographic component. Given the counterfactual processes for the wedges, I compute the dynamic equilibrium under perfect foresight from 1970-2060. Appendix E provides the details of the algorithm for solving for the transition, which builds on Sposi (2012) and Ravikumar, Santacreu, and Sposi (2017).

4.1 Freezing each country's working age share as of 1970

In order to quantify the importance of differential changes in working age shares across countries, this counterfactual assumes that the working age share of the population is constant from 1970-2060. I construct counterfactual sequences for saving wedges, labor wedges, and population by removing the variation that stems from the working age share, but allowing the distortionary component to vary over time, as follows.

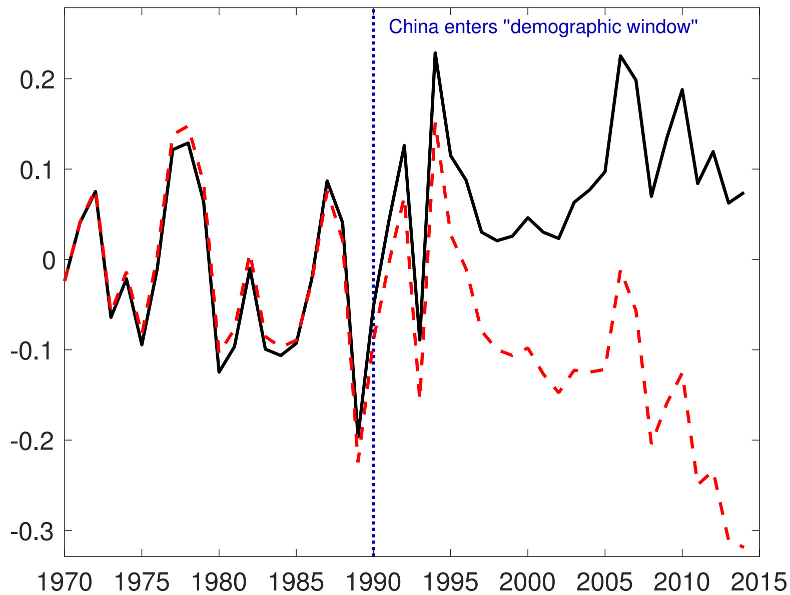
$$\psi_{it}^{s_{i70}} = \begin{cases} \psi_{it}, & t = 1970 \\ \exp\left(\widehat{\gamma_i^{\psi}} + \widehat{\mu^{\psi}} \times (s_{i1970} - s_{U1970}) + \widehat{\varepsilon_{it}^{\psi}}\right) \times \psi_{it-1}^{s_{i70}}, & t \geq 1971 \end{cases}, \tag{7a}$$

$$\zeta_{it}^{s_{i70}} = \exp\left(\widehat{\gamma_i^{\zeta}} + \widehat{\kappa_t^{\zeta}} + \widehat{\mu^{\zeta}} \times s_{i1970} + \widehat{\varepsilon_{it}^{\zeta}}\right), \; t \geq 1970. \tag{7b}$$

$$N_{it}^{s_{i70}} = \begin{cases} N_{it}, & t = 1970 \\ \exp\left(\widehat{\gamma_i^{N}} + \widehat{\kappa_t^{N}} + \widehat{\mu^{N}} \times s_{i1970} + \widehat{\varepsilon_{it}^{N}}\right) \times N_{it-1}^{s_{i70}}, & t \geq 1971 \end{cases}, \tag{7c}$$

Figure 3 illustrates how the assumption of constant working age shares is manifested in the saving wedge for China. Prior to 1990, when China's working age share was not dramatically different from that of the world, the counterfactual saving wedge is similar to that in the baseline. However, after China enters its demographic window in 1990, China's counterfactual saving wedge diverges well below that in the baseline. By stripping out the dynamics of the saving wedge due to the high and rapidly increasing working age share, the counterfactual path for China's counterfactual demand for saving is dramatically reduced. Note that the short-run movements in the saving wedge are similar to those the baseline, but the overall downward shift reflects persistent increases in China's working age share.

Figure 3: Calibrated and counterfactual saving wedge in China



Notes: Solid line refers to the calibrated saving wedge. Dashed line refers to the counterfactual saving wedge with its working age share held fixed at the 1970 level. The vertical bar at 1990 indicates the year that China entered its demographic window.

I feed in the processes for $\{N_{it}^{s_{i70}}, \psi_{it}^{s_{i70}}, \zeta_{it}^{s_{i70}}\}$ and leave all other parameters at their baseline values. Note that these wedges still vary throughout time due to the distortionary component, but the variation stemming from demographics is removed.

Removing demographic forces unveils a strong negative relationship between net exports and productivity growth as shown in Figure 4. In the baseline model the slope of the best fit curve is flat, at 0.04. In the counterfactual the slope is significantly negative, at -0.90.

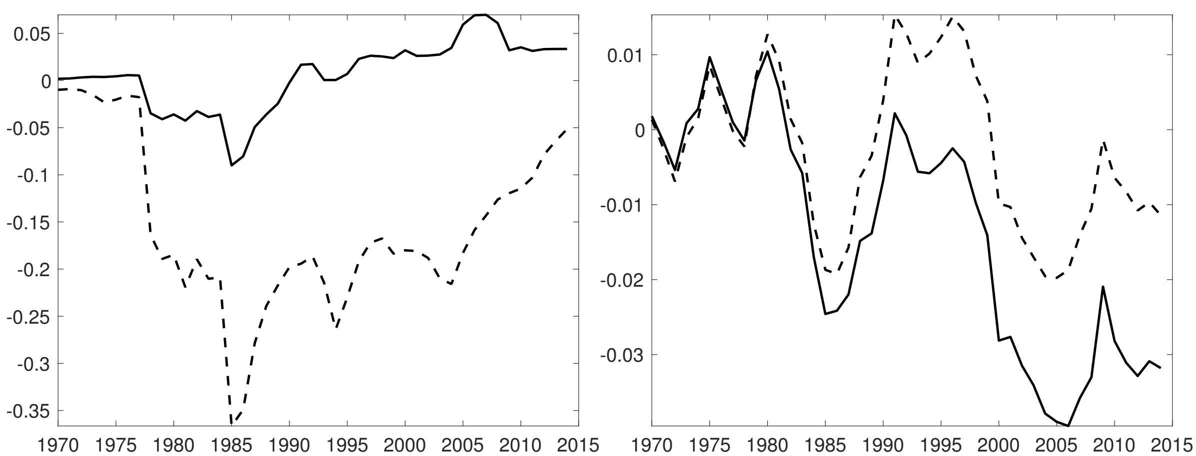
To put this into context, consider a contrast between China and the United States. China experienced a rapid increase in its working age share, while the U.S. experienced only a mild increase. Figure 5 shows that if working age shares were held constant in all countries, then China would have run a large trade deficit. On the flip side, the U.S. trade balance would have diminished and been more or less stationary around zero. This finding generalizes Ferrero (2010) beyond the United States. He argues, in a two-country model of the U.S. and G6 countries, that demographics are responsible for the persistent increase in the U.S. trade deficit. The baseline and counterfactual trade imbalances for all countries are in Figure G.4 in Appendix G. Countries that experienced relatively fast increases in their working age share prior to 2014, have counterfactually lower net exports relative to the baseline.

Figure 4: Ratio of net exports to GDP against labor productivity growth

Notes: Horizontal axis is the average annual growth in labor productivity during five year windows. Vertical axis is the average ratio of net exports to GDP during five year windows. Windows run from [1970,1974]-[2010,2014]. The lines correspond to the best fit curves using OLS.

Figure 5: Ratio of net exports to GDP from 1970-2014

(a) China (b) United States



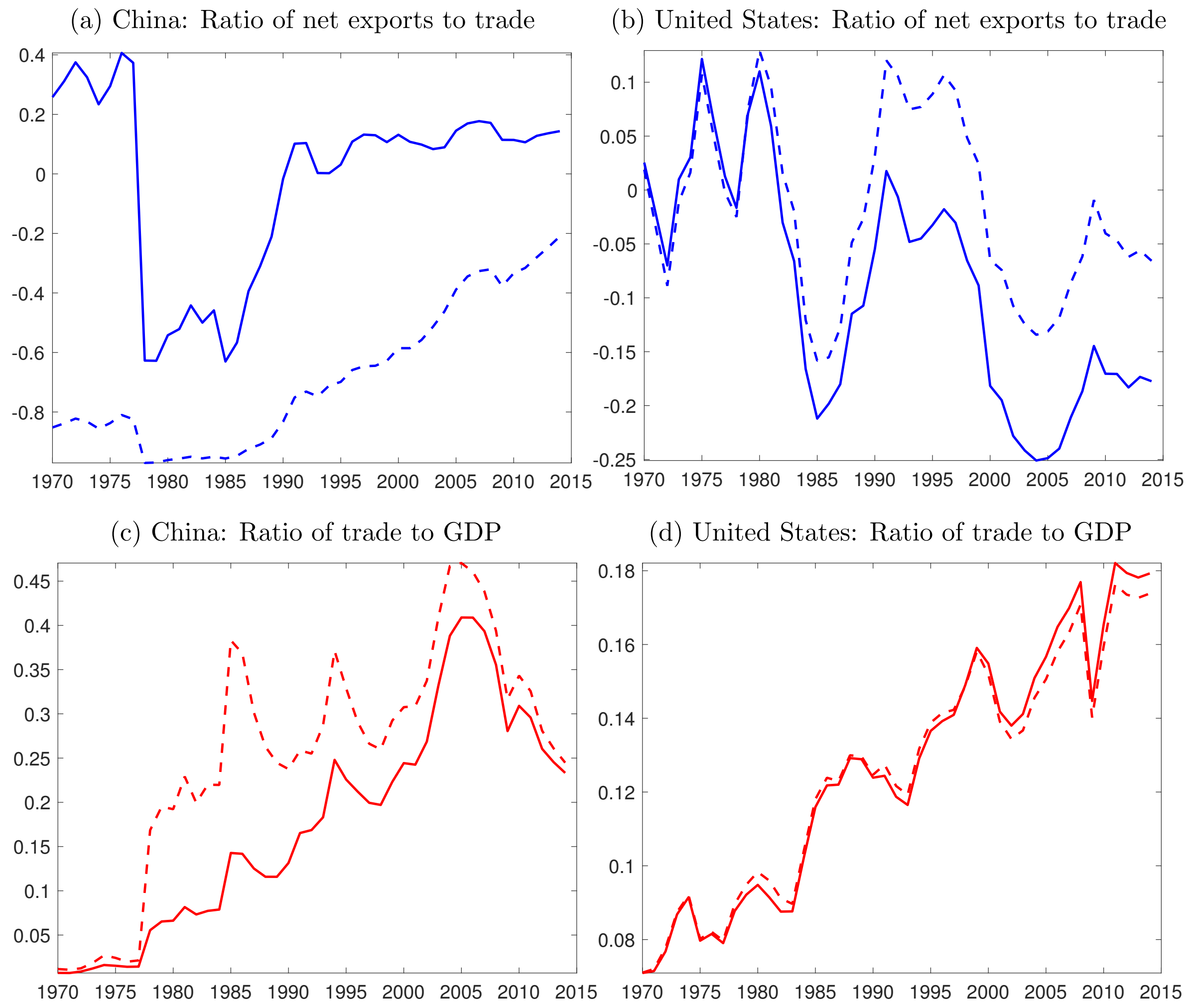
Notes: Solid lines refer to the baseline model. Dashed lines refer to the counterfactual with working age shares simultaneously held fixed at 1970 levels.

Consider the ratio of net exports to GDP as the product of (i) the ratio of net exports to trade (intertemporal margin) and (ii) the ratio of trade to GDP (intratemporal margin):

$$\frac{EXP - IMP}{GDP} = \left(\frac{EXP - IMP}{EXP + IMP}\right) \times \left(\frac{EXP + IMP}{GDP}\right) \tag{8}$$

In Figure 6a, China's counterfactual ratio of net exports to trade is uniformly lower compared to the baseline. The ratio of trade to GDP (Figure 6b) is slightly higher, amplifying the contribution of the ratio of net exports to trade to the ratio of net exports to GDP.

Figure 6: Ratio of net exports to trade and ratio of trade to GDP from 1970-2014



Notes: Solid lines refer to the baseline model. Dashed lines refer to the counterfactual with working age shares simultaneously held fixed at 1970 levels.

For the United States there is essentially no change in the ratio of trade to GDP relative to baseline (Figure 6c). All changes in the ratio of net exports to GDP stems from an increase in the ratio of net exports to trade (Figure 6d). Demographics impacted the U.S. trade balance through the direction of trade flows and not the volume of trade.

Because of the assumption of perfect foresight, projected demographics—post-2014—matter for saving decisions prior to 2014. Appendix B considers the implications of freezing projected demographics as of 2014. Projected demographic changes alone do not systematically reconcile the allocation puzzle. However, projected changes in demographics have important implications for the observed pattern of trade imbalances.[6] The effects are primarily driven by higher anticipated demand for future consumption through the saving wedge. The effects through the labor wedge and population growth are quantitatively small.

Domestic versus foreign demographic forces What is the relative importance of domestic versus foreign demographics in shaping the each country's trade balance? To answer this question I consider a series of counterfactuals where demographics are unilaterally frozen as of 1970 in only one country. In country i, I feed in counterfactual processes $\{N_{it}^{s_{i70}}, \psi_{it}^{s_{i70}}, \zeta_{it}^{s_{i70}}\}$, and keep parameters in every other country at their calibrated values.

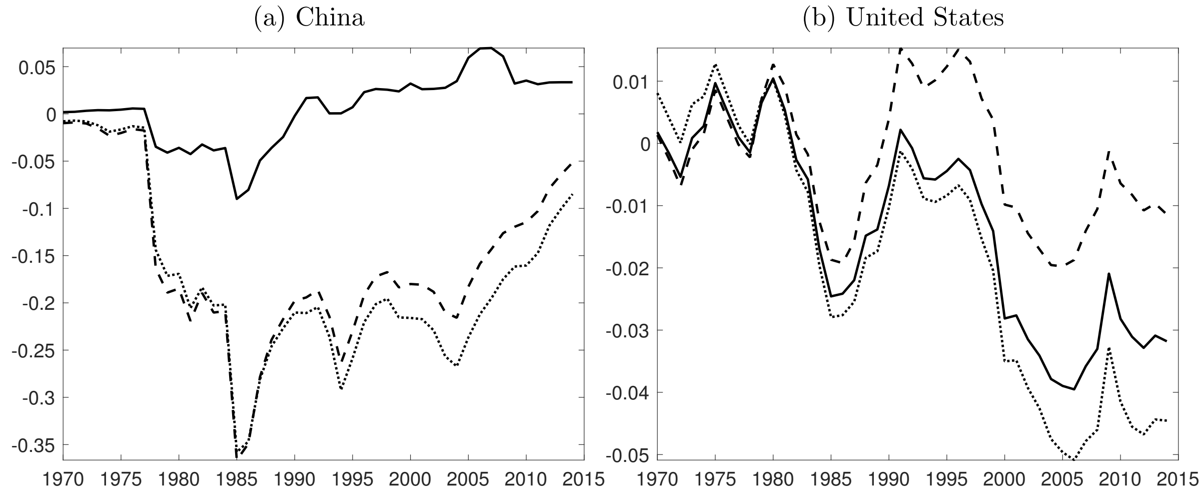
Figure 7a shows that China's net exports behave similarly to the counterfactual in which every country's working age share is simultaneously held constant; China runs a large trade deficit. That is, China's trade balance is influenced more by changes in its own demographics relative to changes in foreign demographics. In contrast, the U.S. trade deficit slightly widens relative to the baseline model, as depicted in Figure 7b. This outcome dramatically differs from that in the counterfactual in which every country's working age share is simultaneously held constant and the U.S. trade deficit effectively vanishes. That is, foreign demographic changes contribute more to the U.S. trade deficit than domestic demographics do.

This result has a lot to do with how each country's demographics evolve relative to the world. Figure 8 illustrates that China's working age share rose far more rapidly than that of the world, while the U.S. working age share increased at a slightly slower rate than that of the world.[7] This finding speaks to the results in Steinberg (2016). Using a two-country model of the United States and the rest of world, he finds that most of the increase in the U.S. trade deficit since 1992 can be accounted for by increasing demand for saving by non-U.S. countries, while a smaller portion is explained by decreasing U.S. demand for saving.

[6] Gagnon, Johannsen, and Lopez-Salido (2016) show that both observed and projected demographic trends in the U.S. affect the real interest rate in a closed economy setting.

[7] Figure G.1 in Appendix G shows the evolutions for every country in the sample.

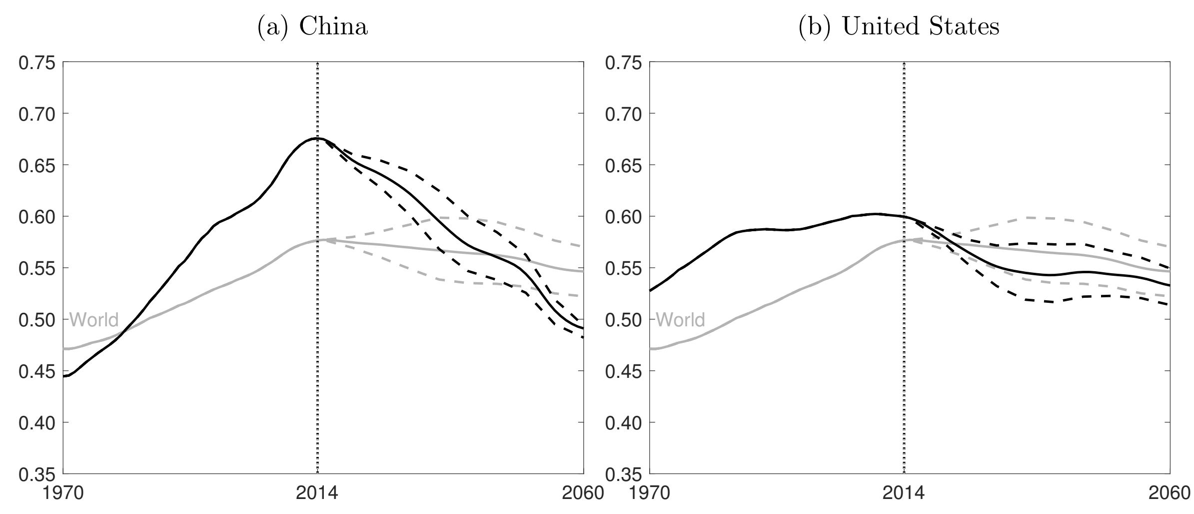
Figure 7: Ratio of net exports to GDP from 1970-2014



Notes: Solid lines refer to the baseline model. Dashed lines refer to the counterfactual with every country's age distribution simultaneously frozen at 1970 levels. Dotted lines refer to the series of counterfactuals with each country's age distribution unilaterally frozen at its 1970 level.

My results suggest that demographic forces are a key driver for saving by non-U.S. countries.

Figure 8: Evolution of working age shares



Notes: Black lines – country level. Gray line – world aggregate. Data from 1970-2014 are observed. Data from 2015-2060 are based on projections: high-variant (upper), medium-variant (middle), and low-variant (lower). High- and low-variants assume plus and minus one-half child per woman, respectively, relative to the medium-variant.

This logic can be formalized to construct a metric to measure the relative importance of domestic demographics. Appealing to Figure 7, define the metric as the distance between the dashed and solid lines relative to the sum of (i) the distance between the dashed and solid lines and (ii) the distance between the dashed and dotted lines:

$$\frac{\sum_{t=1970}^{2014} |nx_{it}^{sim} - nx_{it}^{base}|}{\sum_{t=1970}^{2014} |nx_{it}^{sim} - nx_{it}^{base}| + |nx_{it}^{sim} - nx_{it}^{uni}|},$$

where nx denotes the ratio of net exports to GDP. Superscript *base* refers to the baseline model, *sim* refers to the counterfactual with every country's demographics simultaneously frozen, and *uni* refers to the series of counterfactuals with each country's demographics unilaterally frozen. The metric is 1 when the trade balance is the same in both counterfactuals.

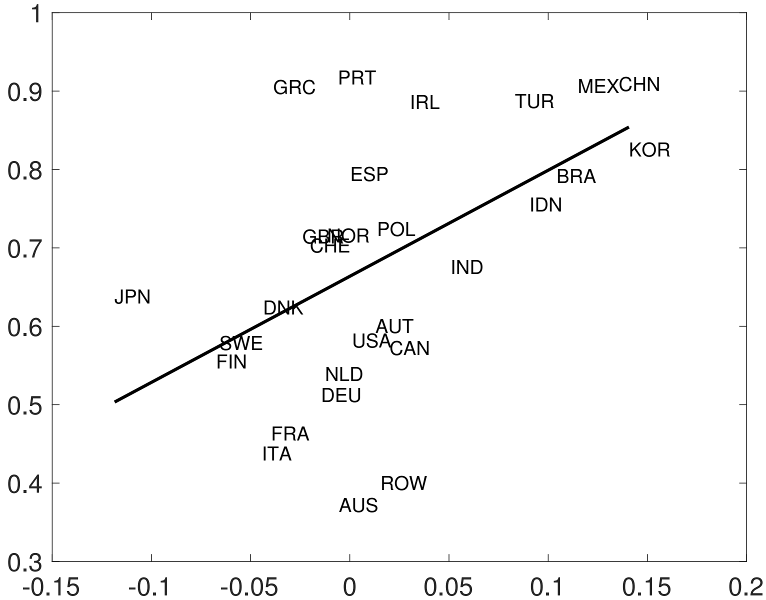
Figure 9 shows that this metric covaries positively with how differently a country's working share share evolved compared to the world. The horizontal axis is the change in a country's working age share between 1970 and 2014, relative to the world. China's working age share increased by 15 percentage points more than that in the world as a whole; About 90 percent of China's observed trade balance is attributed to domestic demographics.

4.2 Effect of asymmetric population growth

Aside from the age distribution of the population, population size itself is an important demographic characteristic. Figure G.2 in Appendix G shows that there are substantial differences in population growth rates across countries. These differences could potentially have meaningful implications for intertemporal trade.

To investigate the role of differential population growth, this counterfactual freezes every country's population, simultaneously, at 1970 levels, holding all other parameters at their calibrates values. In the counterfactual, the relationship between the ratio of net exports to GDP and productivity growth is practically unchanged relative to the baseline, meaning that differences in population growth across countries do not help reconcile the allocation puzzle. Nonetheless, asymmetric population growth does affect the pattern of trade imbalances through real exchange rates and real interest rate differentials. Uncovered interest parity holds in this model, so changes in the real exchange rate between any two countries equals the real interest rate differential between them. Because of this, I focus exclusively on the real exchange rate channel.

Figure 9: Importance of domestic relative to foreign demographics on trade imbalances



Notes: Horizontal axis is $(s_{i2014} - s_{i1970}) - (s_{W2014} - s_{W1970})$, where subscript W refers to the world aggregate. Vertical axis is $\frac{\sum_{t=1970}^{2014} |nx_{it}^{sim} - nx_{it}^{base}|}{\sum_{t=1970}^{2014} |nx_{it}^{sim} - nx_{it}^{base}| + |nx_{it}^{sim} - nx_{it}^{uni}|}$, where nx denotes the ratio of net exports to GDP. Superscript *base* refers to the baseline model. Superscript *sim* refers to the counterfactual with every country's age distribution simultaneously frozen at 1970 levels. Superscript *uni* refers to the series of counterfactuals with each country's age distribution unilaterally frozen at its 1970 level.

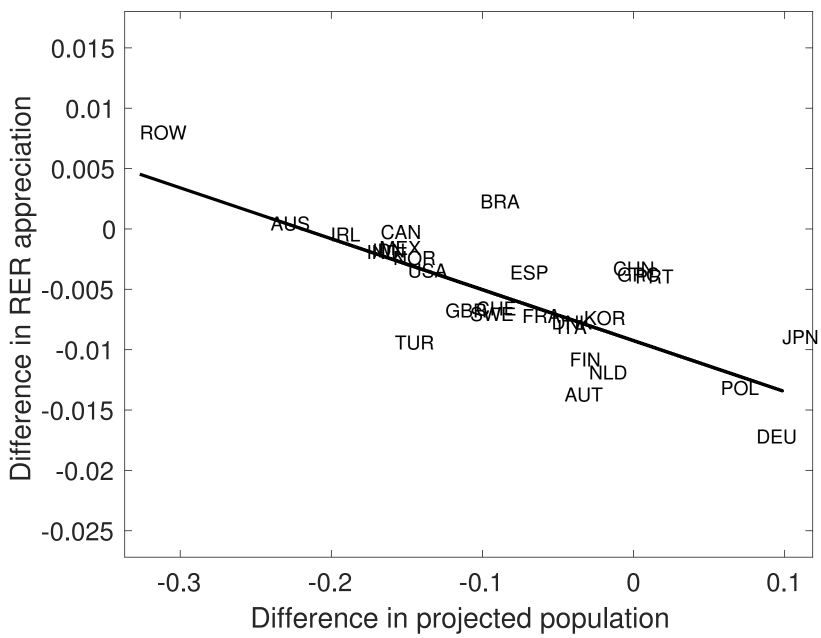
Asymmetric population growth and real exchange rates Asymmetric population growth has implications for real exchange rate (RER) behavior. To see this, consider simultaneously freezing every country's population level as of 2014 and leave all other parameters at their baseline values, including the population from 1970-2014. Agents are forward looking, so prices and allocations prior to 2014 differ from those in the baseline.

All else equal, larger countries have larger home trade shares and, hence, lower measured productivity, $A_{it}^{\nu_{it}}(\pi_{iit})^{-1/\theta}$. So if a country expects fast population growth, it also expects to have a deterioration in its terms of trade i.e., declining RER. Country i's RER is the trade-weighted average of all of its bilateral RERs, $\frac{P_i}{P_j}$, for all $j \neq i$:

$$RER_{it} = \frac{\sum_{j=1, j\neq i}^{I} \left(\frac{P_{it}}{P_{jt}}\right)(Trd_{ijt} + Trd_{jit})}{\sum_{j=1, j\neq i}^{I} (Trd_{ijt} + Trd_{jit})}. \tag{9}$$

On the vertical axis of Figure 10 is the average log difference between the future RER (2015-2060) relative to the current RER (1970-2014) in the counterfactual and that same log difference in the baseline. Countries near the bottom of the vertical axis realize a future depreciation in their RER in the counterfactual compared to the baseline. On the horizontal axis is the average log difference, from 2015-2060, between population in the counterfactual and that in the baseline. Countries on the far right of the horizontal axis have counterfactual projected populations that are higher than their baseline projections. The negative relationship indicates that higher projected population growth causes the path for the RER to tilt downward over time, relative to the baseline.

Figure 10: Difference in real exchange rate path against difference in projected population



Notes: Horizontal axis is $\left(\frac{1}{46}\right)\sum_{t=2015}^{2060}\ln(N_{i2014}/N_{it})$. Vertical axis is $\left(\frac{1}{46}\right)\sum_{t=2015}^{2060}\ln(RER_{it}^{cf}/RER_{it}^{base})-\left(\frac{1}{45}\right)\sum_{t=1970}^{2014}\ln(RER_{it}^{cf}/RER_{it}^{base})$. RER denotes the real exchange rate given by equation (9). Superscript cf refers to the counterfactual. Superscript *base* refers to the baseline model.

This effect is similar to the trade-barrier-induced tilting effect discussed in Obstfeld and Rogoff (2001) and Reyes-Heroles (2016). That is, a downward tilt in a country's RER is akin to a decrease in its real interest rate vis-à-vis the rest of the world, inducing the country to save more early on. This outcome is consistent with the incentive to save in order to smooth consumption *per capita* when faced with high population growth, a channel that operates solely through investment in closed economy models.

4.3 Trade barriers as a driver of imbalances

Trade barriers are directly related to bilateral trade flows and, hence, have immediate implications for the trade balance. I decompose the bilateral trade barriers into a trend component and a bilateral distortionary component as follows

$$\ln(d_{ijt} - 1) = \ln(\bar{d}_t - 1) + \varepsilon^d_{ijt}.$$

I decompose $(d - 1)$—the portion that melts away—as opposed to d. The trend component of trade barriers is simply the geometric mean of all bilateral trade barriers:

$$\widehat{\bar{d}}_t = 1 + \exp\left(\sum_{i=1}^{I}\sum_{\substack{j=1\\ j\neq i}}^{I} \ln(d_{ijt} - 1)\right).$$

The estimated trend component, $\widehat{\bar{d}}_t$, steadily declined from 4.5 in 1970 to 2.6 in 2014. This decline captures global reductions shipping costs as well as tariff reductions. The distortionary component captures bilateral changes in trade barriers that are specific to bilateral trading pairs beyond the trend component.

Freezing the bilateral distortions I quantify the effect of changes in the distortionary component by constructing a counterfactual path for bilateral trade barriers with the distortionary component held constant at 1970 values, but allow the trend component to vary over time:

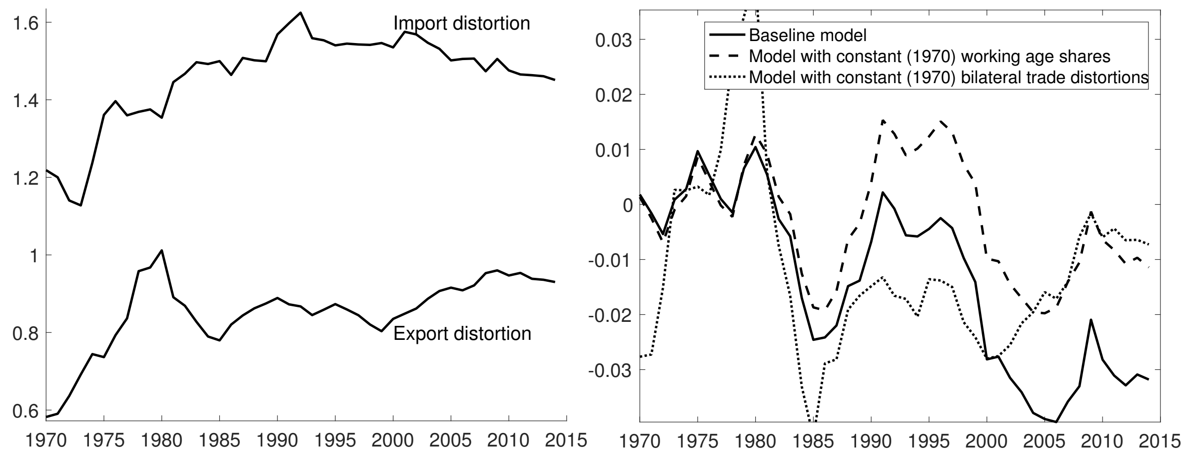
$$d_t^{\varepsilon^d_{ij70}} = 1 + \left(\widehat{\bar{d}}_t - 1\right) \times \exp\left(\widehat{\varepsilon^d_{ij1970}}\right). \tag{10}$$

In this counterfactual there is a strong positive relationship between the ratio of net exports to GDP and labor productivity growth: The elasticity is 0.80, compared to 0.04 in the baseline. Controlling for changes in the distortionary component of trade barriers makes the allocation puzzle even more puzzling.

That is not to say that asymmetries in trade barriers are unimportant in driving each country's trade balance. For instance, Figure 11 suggests that the U.S. trade distortions played a role in shaping the U.S. trade imbalance. The late 1970s and early 2000s are precisely the periods that the average U.S. export distortion rose, and the counterfactual U.S. net exports exceeded the observed (baseline) U.S. net exports (dotted line compared to solid line).

Figure 11: U.S. trade distortions and trade imbalance

(a) Average bilateral trade distortion (b) Ratio of net exports to GDP



Notes: Panel (a) - The average distortions are computed as the average of $\exp(\widehat{\varepsilon^{d}_{ijt}})$ in equation (10), weighted by bilateral imports and exports, respectively.

Alessandria and Choi (2017) argue that this asymmetry after 2000—decreasing import distortion and increasing export distortion—is responsible for the widening U.S. deficit. My model reproduces this result, but clearly, the distortionary component of trade barriers has a much milder impact on the long-run widening of the U.S. deficit than demographic-induced changes to savings do. This can be seen clearly in Figure 11b. That is, demographics have persistently pushed U.S. net exports lower since at least the 1980s, whereas asymmetries in U.S. trade distortions appear to explain shorter-run fluctuations in the deficit.

Freezing both the trend component and the bilateral distortions Reyes-Heroles (2016) argues that even the trend component of trade barriers is important for explaining global imbalances. That is, as trade costs decline, it becomes less costly for countries to borrow and lend, so country-level imbalances increase in absolute value. In this counterfactual I freeze both the trend component of trade barriers and the bilateral distortions. That is, the bilateral trade barriers are held fixed at their 1970 levels:

$$d^{70}_{ijt} = d_{ij1970}, t \geq 1970.$$

In the baseline model, the sum of the absolute value of trade imbalances across countries increases from 1.5 percent of world GDP in 1970 to 3 percent in 2014, an increase of 1.5

percentage points. In the counterfactual with the working age share held fixed this ratio *increases* by 2.3 percentage points. In the counterfactual with trade barriers held fixed this ratio *decreases* by 2.1 percentage points (from 5.3 percent to 3.2 percent). As such, variation in the working age share does not help explain the overall increase in the magnitude of global imbalances over time, while variation in trade barriers does.

4.4 Investment and labor market distortions

Existing literature has emphasized distortions to investment and labor markets as important drivers of imbalances. I study each of these in the context of the allocation puzzle.

Investment distortions Intertemporal choices are affected by investment distortions, which directly affect real rates of return and can have profound implications for capital flows and trade imbalances. These distortions capture financial frictions pertaining to investment, such as those in Buera and Shin (2017), as well as institutional problems that discourage investment, such as those discussed in Aguiar and Amador (2011). I consider a counterfactual in which the investment distortions are held fixed at 1970 levels in every country:

$$\tau_{it}^{k,70} = \tau_{i1970}^{k}, t \geq 1970.$$

In this counterfactual the elasticity of the ratio of net exports to GDP with respect to labor productivity growth is -0.16, compared to 0.04 in the baseline. Therefore, removing variation in investment distortions partly alleviates the allocation puzzle but does so to a far lesser extent than removing variation in working age shares.

Labor market distortions The calibrated labor wedges contain a demographic component and a distortionary component. This counterfactual holds fixed the variation stemming from the distortionary component as of 1970. The reduced form nature of the decomposition implies that these distortions are captured by the time-fixed effect and the residual in equation (5). The fixed effects capture common global factors that affect labor market dynamics, such as technological change and large scale GATT/WTO reforms. The residual captures country-specific distortions that vary over time, such as fiscal policy and other factors emphasized by Ohanian, Restrepo-Echavarria, and Wright (2017). As such, I set

$\kappa_t^\zeta = \kappa_{1970}^\zeta$ and $\varepsilon_{it}^\zeta = \varepsilon_{i1970}^\zeta$ and compute counterfactual process for the labor wedges.

$$\zeta_{it}^{\varepsilon_{i1970}^\zeta} = \exp\left(\widehat{\gamma_i^\zeta} + \widehat{\kappa_{1970}^\zeta} + \widehat{\mu^\zeta} \times s_{it} + \widehat{\varepsilon_{i1970}^\zeta}\right), t \geq 1970$$

In this counterfactual the elasticity of the ratio of net exports to GDP with respect to labor productivity growth is 0.15. This elasticity is greater than that in the baseline, implying that labor market distortions do not systematically explain the allocation puzzle, but actually make it more puzzling.

4.5 Summary

Table 3 reports the elasticity between the ratio of net exports to GDP and productivity growth for the baseline and counterfactual specifications. Changes in demographics are by far the most important for reconciling the allocation puzzle. Investment distortions partially alleviate the puzzle, but to a far less degree than demographics. Neither bilateral trade distortions nor labor market distortions help reconcile the allocation puzzle.

This is not to say that trade barriers and labor market distortions are irrelevant for trade imbalances. Bilateral trade distortions help shape short-run behavior of country-level imbalances, but cannot account for long-run persistence. The continual decline in trade barriers across the world do help account for the absolute rise in global imbalances, but cannot account for the pattern of imbalances across countries.

Table 3: Elasticity between ratio of net exports to GDP and productivity growth

Specification	Elasticity
Baseline model (data)	0.04
Counterfactual: Fixed working age shares	−0.90
Counterfactual: Fixed bilateral trade distortions	+0.81
Counterfactual: Fixed investment distortions	−0.16
Counterfactual: Fixed labor market distortions	+0.15

The main findings suggest that there is an intimate relationship between changes in the working age share and the rate of labor productivity growth. Countries that experienced fast productivity growth, but did not run trade deficits, also tend to be countries that experienced relatively fast increases in their working age shares. By counterfactually holding fixed the working age share and removing demographic-induced changes to saving, the correlation between net exports and productivity growth becomes negative.

Life-cycle models could generate the link between saving and productivity growth. Older workers have higher income than young workers and also save more as they near retirement. If worker-level earnings reflect productivity, then increases in the working age share due to an aging population can yield higher productivity growth alongside higher saving.

5 Conclusion

The paper builds a multicountry, dynamic, Ricardian model of trade, where dynamics are driven by international borrowing and lending and capital accumulation. Trade imbalances arise endogenously as the result of relative shifts in technologies, in trade barriers, in factor market distortions, and in demographics. All of the exogenous forces are calibrated using a wedge accounting procedure so that the model rationalizes past and projected national accounts data and bilateral trade flows across 28 countries from 1970-2060.

Demographics directly affect imbalances through the relative demand for national saving and indirectly impact imbalances through labor supply and population growth. By counterfactually holding fixed the working age share in each country as of 1970, a strong negative relationship between each country's ratio of net exports to GDP and productivity growth emerges. In other words, demographics alleviate the allocation puzzle. Net exports respond more to domestic demographics, relative to foreign demographics, in countries in which the working age share evolved more differently from the world average.

Differences in population growth do not help reconcile the allocation puzzle. However, countries with relatively fast projected population growth experience relative declines in their real exchange rate over time and finance higher rates of consumption growth by borrowing early and lending late. Investment distortions shed some light on the allocation puzzle, but do so to a lesser degree than demographics. Neither labor market distortions nor trade distortions help reconcile the puzzle.

The results in this paper allude to a relationship between productivity growth and changes in demographics. Countries that experienced fast productivity growth, but did not run trade deficits, tend to be the same countries that experienced relatively fast increases in their working age shares. Digging into this relationship is beyond of the scope of this paper. Future work should aim at developing methods that explicitly incorporate heterogeneity in age, such as an overlapping generations environment, into a multicountry dynamic model of trade. Such a framework can be used to more carefully study how demographics shape productivity, comparative advantage, and trade imbalances in a unified framework.

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A Microfounding the wedges

Imagine data being generated by a small open economy with overlapping generations (OLG). The econometrician views the world through the lens of a representative household and only observes aggregate consumption, aggregate labor supply, prices, and the age distribution. This example shows how the econometrician can calibrate preferences for the representative household such that (i) the representative household's decisions yield the same aggregate outcomes as the OLG economy and (ii) variation in the wedges depends only on variation in the age distribution.

In the OLG economy agents live for two periods. At time t a cohort arrives with N_t^w working age agents that coexist with the existing N_t^r retirees. Retirees disappear after one period. Working age agents choose their labor supply. One unit of labor generates one unit of output. There a risk-free international bond through which output can be saved.

Normalize the wage to 1 in every period, implying that the price of consumption is 1. Take the constant world interest rate on bonds, q, as given. Agents born at time t solve

$$\max_{c_t^w, c_{t+1}^r, \ell_t^w} \left\{ \ln\left(c_t^w\right) + \beta \ln\left(c_{t+1}^r\right) + \frac{\left(1-\ell_t^w\right)^{1-1/\phi}}{1-1/\phi} \text{ s.t. } c_t^w + \frac{c_{t+1}^r}{1+q} = \ell_t^w \right\}.$$

The solution is characterized (implicitly) by $(1-\ell^w) = \left(\frac{\ell^w}{1+\beta}\right)^\phi$, $c^w = (1-\ell^w)^{1/\phi}$, and $c^r = \beta(1+q)c^w$. While each agent's decision is constant over time and is unaffected by demographics, aggregate consumption, $C_t = N_t^w c_t^w + N_t^r c_t^r$, and aggregate labor supply, $L_t = N_t^w \ell_t^w$, can vary over time with demographics.

Let $n_t^w = \frac{N_t^w}{N_t}$ denote the working age share in the population at time t. Then, aggregate consumption per capita growth and the nonemployment-population ratio, respectively, are

$$\frac{C_{t+1}/N_{t+1}}{C_t/N_t} = \left(\frac{1 + n_{t+1}^w \left(\frac{1-\beta(1+q)}{\beta(1+q)}\right)}{\beta(1+q) + n_t^w(1-\beta(1+q))} \right) \beta(1+q), \tag{A.1}$$

$$\left(1 - \frac{L_t}{N_t}\right) = \left((1-n_t^w) + \frac{n_t^w}{\left(n_t^w(1-\beta(1+q)) + \beta(1+q)\right)^\phi} \right) \left(\frac{C_t}{N_t}\right)^\phi. \tag{A.2}$$

The econometrician interprets aggregate data through the lens of a representative-agent model and, introduces shocks to the discount factor, ψ, and marginal utility of leisure, ζ:

$$\max\left\{\sum_{t=1}^{\infty}\beta^{t-1}\psi_t N_t\left[\ln\left(\frac{C_t}{N_t}\right)+\zeta_t\frac{\left(1-\frac{L_t}{N_t}\right)^{1-1/\phi}}{1-1/\phi}\right]\text{ s.t. }\sum_{t=1}^{\infty}\frac{C_t}{(1+q)^{t-1}}=\sum_{t=1}^{\infty}\frac{L_t}{(1+q)^{t-1}}\right\}$$

The solution to the representative household's problem is

$$\frac{C_{t+1}/N_{t+1}}{C_t/N_t}=\left(\frac{\psi_{t+1}}{\psi_t}\right)\beta(1+q), \tag{A.3}$$

$$\left(1-\frac{L_t}{N_t}\right)=\zeta_t^{\phi}\left(\frac{C_t}{N_t}\right)^{\phi}. \tag{A.4}$$

Assume that the change in the working age share is a function of its current level:

$$\frac{n_{t+1}^w}{n_t^w}=\omega(n_t^w). \tag{A.5}$$

Combining equation (A.1) with equations (A.3) and (A.5) shows that variation in the saving wedge depends only on variation in the working age share. Similarly, combining (A.2) with equation (A.4) shows that variation in the labor wedge depends only on variation in the working age share. The micro-founded expressions for the calibrated wedges are

$$\frac{\psi_{t+1}}{\psi_t}=\left(\frac{1+\omega(n_t^w)n_t^w\left(\frac{1-\beta(1+q)}{\beta(1+q)}\right)}{\beta(1+q)+n_t^w(1-\beta(1+q))}\right), \tag{A.6}$$

$$\zeta_t^{\phi}=\left((1-n_t^w)+\frac{n_t^w}{\left(n_t^w(1-\beta(1+q))+\beta(1+q)\right)^{\phi}}\right). \tag{A.7}$$

B The role of projected demographic changes

The following counterfactual exercise isolates how much of the *observed* trade imbalances are due to differences in the rate that *future* demographics evolve. I assume that from 1970-2014 all exogenous parameters are equal to their baseline values. Beginning in 2014, each country's working age share is permanently frozen. From 2015 forward each country will face counterfactual paths for saving wedges, labor wedges, and population. All variation in saving wedges, labor wedges, and population after 2014 can be attributed to changes in the distortionary component, i.e., the time fixed effects and residuals in equations (6)-(5). I construct counterfactual paths for the saving wedge, the labor wedge, and population growth as follows.

$$\psi_{it}^{s_{i14}} = \begin{cases} \psi_{it}, & t \leq 2014 \\ \exp\left(\widehat{\gamma_i^{\psi}} + \widehat{\mu^{\psi}} \times (s_{i2014} - s_{U2014}) + \widehat{\varepsilon_{it-1}^{\psi}}\right) \times \psi_{it-1}^{s_{i14}}, & t \geq 2015 \end{cases}, \tag{B.1a}$$

$$\zeta_{it}^{s_{i14}} = \begin{cases} \zeta_{it}, & t \leq 2014 \\ \exp\left(\widehat{\gamma_i^{\zeta}} + \widehat{\kappa_{t-1}^{\zeta}} + \widehat{\mu^{\zeta}} \times s_{i2014} + \widehat{\varepsilon_{it-1}^{\zeta}}\right), & t \geq 2015 \end{cases}, \tag{B.1b}$$

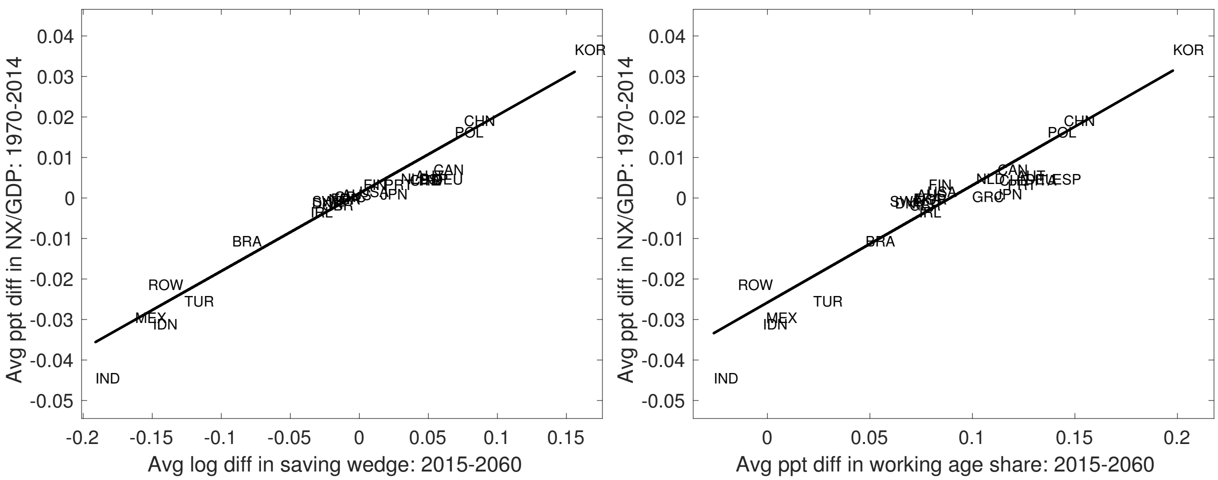
$$N_{it}^{s_{i14}} = \begin{cases} N_{it}, & t \leq 2014 \\ \exp\left(\widehat{\gamma_i^{N}} + \widehat{\kappa_{t-1}^{N}} + \widehat{\mu^{N}} \times s_{i2014} + \widehat{\varepsilon_{it-1}^{N}}\right) \times N_{it-1}^{s_{i14}}, & t \geq 2015 \end{cases}, \tag{B.1c}$$

The role of projected demographics through saving wedges The following counterfactual exercise isolates how much of the *observed* trade imbalances are due to differences in the rate that *future* demographics evolve through the saving wedge only. I assume that from 1970-2014 all exogenous parameters are equal to their baseline values. Beginning in 2014, each country's working age share is permanently frozen. From 2015 forward each country will face counterfactual paths for the saving wedge, $\{\psi_{it}^{s_{i14}}\}$, while the paths for the labor wedge and population growth are the same as in the baseline. All of variation in the saving wedge after 2014 can be attributed to changes in non-demographic factors.

Higher projected working age shares imply higher projected saving wedges, which directly impact the demand for saving and imply higher current net exports. Figure B.1 illustrates this finding. The left panel shows a positive relationship between the difference in the future saving wedge and the difference in the current trade balance, where the differences are with respect to the counterfactual and the baseline. The right panel depicts a positive relationship between the difference in a country's projected working age share and the difference in its current trade balance.

The role of projected demographics through labor wedges The following counterfactual exercise isolates how much of the *observed* trade imbalances are due to differences in the rate that *future* demographics evolve through the labor wedge only. I assume that from 1970-2014 all exogenous parameters are equal to their baseline values. Beginning in 2014, each country's working age share is permanently frozen. From 2015 forward each country will face counterfactual paths for the labor wedge, $\{\zeta_{it}^{s_{i14}}\}$, while the paths for the saving

Figure B.1: Realized difference in ratio of net exports to GDP against projected difference in saving wedge



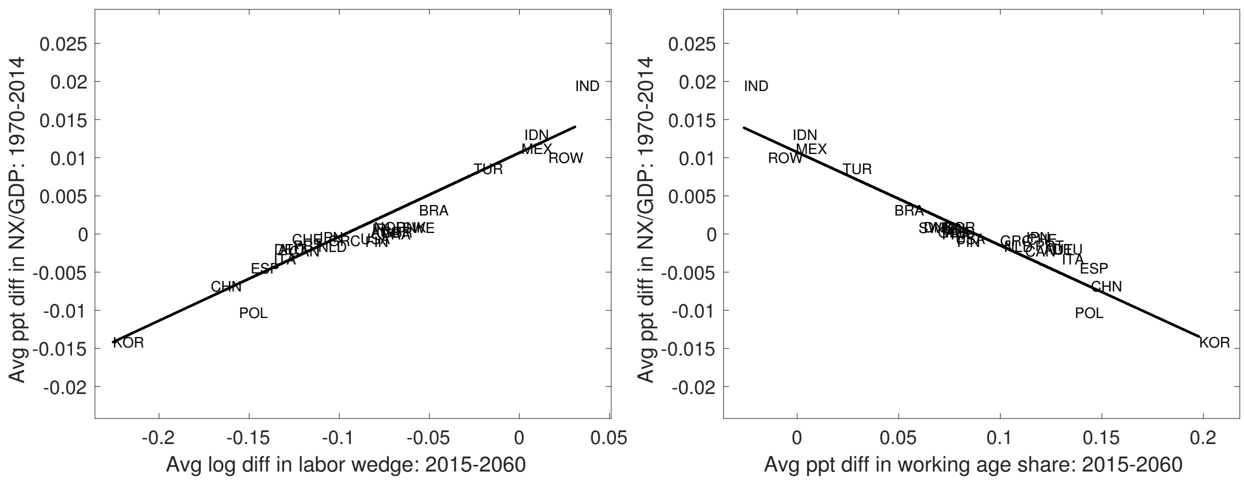
Notes: Horizontal axis (left)is $\left(\frac{1}{46}\right)\sum_{t=2015}^{2060}\ln(\psi_{it}^{s_{i14}}/\psi_{it})$. Horizontal axis (right) is $\left(\frac{1}{46}\right)\sum_{t=2015}^{2060}(s_{i2014}-s_{it})$. Vertical axis is $\left(\frac{1}{45}\right)\sum_{t=1970}^{2014}\left(nx_{it}^{cf}-nx_{it}^{base}\right)$. nx denotes the ratio of net exports to GDP. Superscript cf refers to the counterfactual. Superscript $base$ refers to the baseline model.

wedge and population are the same as in the baseline. All of variation in the labor wedge after 2014 can be attributed to changes in non-demographic factors.

Higher projected working age shares imply lower projected labor wedges and, in turn, higher labor supply implying higher future productive capacity. Therefore, a country will rely less on external finance to fund its future liabilities and will save less today, i.e., lower current net exports. Figure B.2 illustrates this result. The left panel shows a positive relationship between the difference in the future labor wedge and the difference in the current trade balance, where the differences are with respect to the counterfactual and the baseline. The right panel depicts a negative relationship between the difference in a country's projected working age share and the difference in its current trade balance.

The role of projected demographics through population The following counterfactual exercise isolates how much of the *observed* trade imbalances are due to differences in the rate that *future* demographics evolve through the population growth only. I assume that from 1970-2014 all exogenous parameters are equal to their baseline values. Beginning in 2014, each country's working age share is permanently frozen. From 2015 forward each country will face counterfactual paths for population, $\{N_{it}^{s_{i14}}\}$, while the paths for the saving

Figure B.2: Realized difference in ratio of net exports to GDP against projected difference in labor wedge

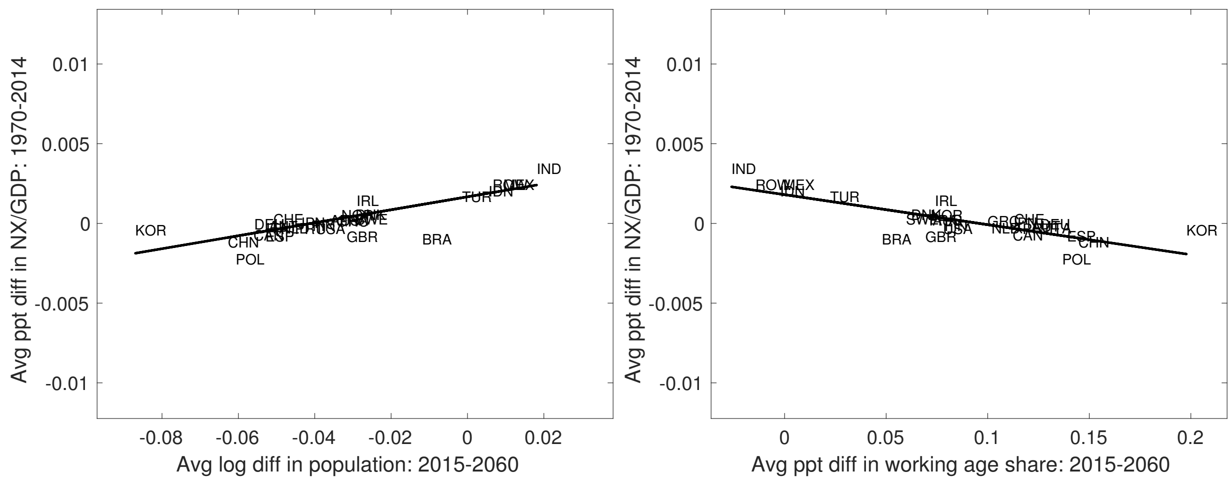


Notes: Horizontal axis (left)is $\left(\frac{1}{46}\right)\sum_{t=2015}^{2060}\ln(\zeta_{it}^{s_{i14}}/\zeta_{it})$. Horizontal axis (right) is $\left(\frac{1}{46}\right)\sum_{t=2015}^{2060}(s_{i2014}-s_{it})$. Vertical axis is $\left(\frac{1}{45}\right)\sum_{t=1970}^{2014}\left(nx_{it}^{cf}-nx_{it}^{base}\right)$. nx denotes the ratio of net exports to GDP. Superscript cf refers to the counterfactual. Superscript $base$ refers to the baseline model.

wedge and labor wedge are the same as in the baseline. All of variation in population after 2014 can be attributed to changes in non-demographic factors.

Higher projected working age shares imply lower future population growth and affects saving through competing channels. On the one hand, lower future population means lower future productive capacity, which alone would encourage current saving. On the other hand, the desire to smooth consumption per capita would encourage current borrowing. Both of these forces are present in a closed economy model. What is novel to the open economy model is that population size affects the terms of trade. Lower population implies relatively stronger RER in the future and, in turn, encourages borrowing today. The net result is that lower future population is, on average, associated with lower current net exports. Figure B.3 illustrates this finding. The left panel shows a positive relationship between the difference in the future population and the difference in the current trade balance, where the differences are with respect to the counterfactual and the baseline. The right panel depicts a negative relationship between the difference in a country's projected working age share and the difference in its current trade balance.

Figure B.3: Realized difference in ratio of net exports to GDP against projected difference in population



Notes: Horizontal axis (left)is $\left(\frac{1}{46}\right)\sum_{t=2015}^{2060}\ln(N_{it}^{s_{i14}}/N_{it})$. Horizontal axis (right) is $\left(\frac{1}{46}\right)\sum_{t=2015}^{2060}(s_{i2014}-s_{it})$. Vertical axis is $\left(\frac{1}{45}\right)\sum_{t=1970}^{2014}\left(nx_{it}^{cf}-nx_{it}^{base}\right)$. nx denotes the ratio of net exports to GDP. Superscript cf refers to the counterfactual. Superscript $base$ refers to the baseline model.

C Data

This section of the Appendix describes the sources of data as well as adjustments made to the data. Sources include the 2016 release of the World Input-Output Database (Timmer, Dietzenbacher, Los, Stehrer, and de Vries, 2015, (WIOD)), version 9.0 of the Penn World Table (Feenstra, Inklaar, and Timmer, 2015, (PWT)), Organization for Economic Cooperation and Development (2014) Long-Term Projections Database (OECD), 2015 revision of the United Nations (2015) World Population Prospects (UN), and the International Monetary Fund Direction of Trade Statistics (IMFDOTS). Table C.1 summarizes the data raw data.

Selection of countries is based on constructing a panel with data spanning 1970-2060. The countries (3-digit isocodes) are: Australia (AUS), Austria (AUT), Brazil (BRA), Canada (CAN), China (CHN), Denmark (DNK), Finland (FIN), France (FRA), Germany (DEU), Greece (GRC), India (IND), Indonesia (IDN), Ireland (IRL), Italy (ITA), Japan (JPN), South Korea (KOR), Mexico (MEX), Netherlands (NLD), Norway (NOR), Poland (POL), Portugal (PRT), Spain (ESP), Sweden (SWE), Switzerland (CHE), Turkey (TUR), United Kingdom (GBR), United States (USA), and Rest-of-world (ROW). Below I provide descriptions of how the data from 1970-2014 are constructed, and separately for 2015-2060.

Table C.1: Model variables and corresponding data sources

Variable description	Model counterpart	Data source 1970-2014	Data source 2015-2060
Working age share	s_{it}	UN	UN
Population	N_{it}	PWT	UN
Employment	L_{it}	PWT	OECD
Value added*	$w_{it}L_{it} + r_{it}K_{it}$	PWT & WIOD	OECD
Investment***	X_{it}	PWT	OECD
Price of composite intermediate**	P_{it}	PWT	Imputed
Price of consumption**	P_{it}/χ_{it}^c	PWT	Imputed
Price of investment**	P_{it}/χ_{it}^x	PWT	Imputed
Initial capital stock***	K_{i1}	PWT	N/A
Gross output*	$P_{it}Y_{it}$	Imputed & WIOD	Imputed
Bilateral trade flow*	$P_{it}Q_{it}\pi_{ijt}$	IMFDOTS & WIOD	Imputed
Absorption*	$P_{it}Q_{it}$	Imputed & WIOD	Imputed

Notes: *Values are measured in current prices using market exchange rates. **Prices are measured using PPP exchange rates. ***Quantities are measured as values deflated by prices.

Constructing realized data from 1970-2014

- Age distribution data from 1970-2014 come from the UN. For the ROW aggregate I take the age distribution data for the "world" aggregate that the UN reports, and subtract the sum of the data for the countries in my sample.

- Population data from 1970-2014 come directly from PWT. For the ROW aggregate, the population is computed as the sum of the entire population across all countries in PWT, minus the sum of the population across countries in my sample.

- Employment data from 1970-2014 come directly from PWT. For the ROW aggregate, employment is computed as the sum of across all countries in PWT, minus the sum across countries in my sample.

- Value added in current U.S. dollars is taken from various sources. From 2000-2014, these data are obtained from WIOD and are computed as the sum of all value added across every industry in each country-year. From 1970-2000 these data are obtained from PWT and computed as output-side real GDP at current PPP times the price level of GDP at current PPP exchange rate (relative to the U.S.). The data from PWT are multiplicatively spliced to WIOD as of the year 2000.

- Price of consumption from 1970-2014 is computed directly from PWT. For ROW, it is computed as the ratio of consumption in current prices relative to consumption in PPP prices. Consumption in ROW is computed as the sum across all countries in PWT minus the sum across countries in my sample.

- Price of investment is computed analogously to the price of consumption.

- Price of the composite intermediate good from 1970-2014 is constructed using various data in PWT. I take a weighted average of the price level of imports and of exports, each of which come directly from PWT. The weight applied to imports is the country's import share in total absorption, and the weight applied to exports is the country's home trade share in total absorption. Trade and absorption data are described below. For ROW, I compute the price of imports as the ratio of ROW imports in current prices divided by ROW imports in PPP prices. ROW imports is computed as the sum of imports across all countries in PWT minus the sum across countries in my sample.

- Investment quantities from 1970-2014 is computed from various data in the PWT. I begin by computing the nominal investment rate (ratio of expenditures on investment as a share of GDP in current prices). I then multiply the nominal investment rate by GDP in current U.S. dollars to arrive at total investment spending in current U.S. dollars. Finally, I deflate the current investment expenditures by the price of investment.

- Initial capital stock is taken directly from PWT. Capital stock in ROW is computed as the sum across all countries in PWT minus the sum across countries in my sample.

- Gross output in current U.S. dollars from 2000-2014 is obtained from WIOD and are computed as the sum of all gross output across every industry in each country-year. Prior to 2000, I impute these data using the ratio of value added to gross output in 2000, and applying that ratio to scale value added in each year prior to 2000.

- Bilateral trade in current U.S dollars from 2000-2014 is computed directly from WIOD as the sum of all trade flows (intermediate usage and final usage) across all industries. Prior to 2000, bilateral trade flows are obtained from the IMFDOTS. Bilateral trade flows with ROW are computed as imports (exports) to (form) the world minus the sum of imports (exports) to (from) the countries in my sample. These data are multiplicatively spliced to the WIOD data as of the year 2000.

- Absorption in current U.S. dollars from 2000-2014 is computed using WIOD data as gross output minus net exports, summed across all industries.

Constructing projected data from 2015-2060

- Age distribution data from 2015-2060 come from the UN. For the ROW aggregate I take the age distribution data for the "world" aggregate that the UN reports, and subtract the sum of the data for the countries in my sample.

- Population data from 2015-2060 are taken directly from UN and spliced to the PWT levels as of the year 2014.

- Employment data from 2015-2060 are taken directly from OECD and spliced to the PWT levels as of the year 2014.

- From 2015-2060, data on value added in current U.S. dollars these data are obtained from OECD projections and computed as real GDP per capita (in constant, local currency units) times the population times the price level (in local currency units) times the PPP level (relative to the U.S.) times the nominal exchange rate (local currency per U.S. dollar at current market prices). The data from OECD are multiplicatively spliced to WIOD as of the year 2014.

- Price of consumption from 2015-2060 is imputed by assuming equal growth rates to the price deflator for aggregate GDP, where the price deflator for aggregate GDP is directly computed using data in the OECD projections.

- Price of investment (P^x) from 2015-2060 is imputed using information on its co-movement with the price of consumption (P^c). In particular, I estimate the relationship between growth in the relative price against a constant and a one-year lag in the relative price growth, for the years 1972-2014.

$$\ln\left(\frac{P^c_{it}/P^x_{it}}{P^c_{it-1}/P^x_{it-1}}\right) = \beta_0 + \beta_1 \ln\left(\frac{P^c_{it-1}/P^x_{it-1}}{P^c_{it-2}/P^x_{it-2}}\right) + \varepsilon_{it}. \tag{C.1}$$

  I use the estimates from equation (C.1) to impute the sequence of prices for investment from 2015-2060, given the already imputed data for the price of consumption during these years.

- Price of the composite intermediate good from 2015-2060 is imputed by first constructing data for the price of imports of exports. Prices of imports and exports are computed analogously to the price of investment by estimating equation (C.1) for each series. I then take a weighted average of the price levels of imports and of exports to determine the price of the composite good. The weight applied to imports is the country's import share in total absorption, and the weight applied to exports is the country's home trade share in total absorption. Trade and absorption data are described below.

- Investment quantities from 2015-2060 are computed from various variables in the OECD projections. I begin by imputing the nominal investment rate (ratio of expenditures on investment to GDP in current prices) using information on its co-movement with the relative prices. I estimate the relationship between the investment rate against a country-fixed effect, the lagged investment rate, the contemporaneous and lagged relative price of investment, and the contemporaneous and lagged real GDP per capita for the years 1972-2014. Letting $\rho_{it} = \frac{P_{xit}X_{it}}{GDP_{it}}$ denote the investment rate,

$$\begin{aligned}\ln\left(\frac{\rho_{it}}{1-\rho_{it}}\right) &= \alpha_i + \beta_1 \ln\left(\frac{\rho_{it-1}}{1-\rho_{it-1}}\right) \\ &\quad + \beta_2 \ln\left(\frac{P^x_{it}}{P^c_{it}}\right) + \beta_3 \ln\left(\frac{P^x_{it-1}}{P^c_{it-1}}\right) + \beta_4 \ln(y_{it}) + \beta_5 \ln(y_{it-1}) + \varepsilon_{it}. \qquad \text{(C.2)}\end{aligned}$$

  Using $\ln(\rho/(1-\rho))$ to ensures that the imputed values of ρ are bounded between 0 and 1. I use the estimated coefficients from equation (C.2) together with projections on the relative price and income per capita to construct projections for the investment rate. I then multiply the nominal investment rate by GDP in current U.S. dollars (available in the OECD projections) to arrive at total investment spending in current U.S. dollars. Finally, I deflate the investment expenditures by the price of investment.

- Gross output in current U.S. dollars from 2015-2060 is imputed using the ratio of value added to gross output in 2014, and applying that ratio to scale value added in each year after 2014. The value added data (GDP in current U.S. dollars) after 2014 is obtained directly from OECD projections.

- Bilateral trade in current U.S dollars from 2015-2060 are constructed in multiple steps. First, let $x_{ijt} = \frac{X_{ijt}}{GO_{jt}-EXP_{jt}}$ be the ratio of country j's exports to country i, relative to country j's gross output net of its total exports. I then estimate how changes in this trade share co-moves with changes in the importer's aggregate import price index, changes in the exporter's aggregate export price index, and changes in both the importer's and exporter's levels of GDP:

$$\ln\left(\frac{x_{ijt}}{x_{ijt-1}}\right) = \beta_1 \ln\left(\frac{P^m_{it}}{P^m_{it-1}}\right) + \beta_2 \ln\left(\frac{P^x_{jt}}{P^x_{jt-1}}\right) + \beta_3 \ln\left(\frac{GDP_{it}}{GDP_{it-1}}\right) + \beta_4 \ln\left(\frac{GDP_{jt}}{GDP_{jt-1}}\right) + \varepsilon_{it}. \tag{C.3}$$

I use the estimated coefficients from equation (C.3) together with projections of prices of imports and of exports and levels of GDP to construct trade shares from 2015-2060.

In the second step, I use the fact that country i's domestic sales is determined by

$$X_{iit} = \frac{GO_{it}}{1 + \sum_{j \neq i} x_{jit}},$$

where gross output and trade shares from 2015-2060 have already been constructed.

Finally, given projected domestic sales and trade shares, the bilateral trade flow is

$$X_{ijt} = X_{jjt} x_{ijt}.$$

- Absorption in current U.S. dollars from 2015-2060 is gross output minus net exports.

D Equilibrium conditions

This section describes the solution to a perfect foresight equilibrium.

D.1 Household optimization

The representative household's optimal path for consumption satisfies two Euler equations:

$$\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}} = \beta^\sigma \left(\frac{\psi_{it+1}}{\psi_{it}}\right)^\sigma \left(\frac{1+q_{t+1}}{\frac{P_{it+1}/\chi^c_{it+1}}{P_{it}/\chi^c_{it}}}\right)^\sigma, \tag{D.1}$$

$$\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}} = \beta^\sigma \left(\frac{\psi_{it+1}}{\psi_{it}}\right)^\sigma \left(\frac{\frac{r_{it+1}}{P_{it+1}/\chi^c_{it+1}} - \left(\frac{\chi^c_{it+1}}{\chi^x_{it+1}}\right)\Phi_2(K_{it+2}, K_{it+1})}{\left(\frac{\chi^c_{it}}{\chi^x_{it}}\right)\Phi_1(K_{it+1}, K_{it})}\right)^\sigma \left(\frac{1-\tau^k_{it+1}}{1-\tau^k_{it}}\right)^\sigma. \tag{D.2}$$

The first Euler equation describes the trade-off between consumption and saving in one-period bonds, while the second Euler equation describes the trade-off between consumption and investment in physical capital. Recall that $X_{it} \equiv \Phi(K_{it+1}, K_{it})$ is investment, where Φ_1 and Φ_2 are the derivatives with respect to the first and second arguments, respectively.

Labor supply in each period is chosen to satisfy

$$\frac{L_{it}}{N_{it}} = 1 - (\zeta_{it})^{\phi} \left(\frac{w_{it}}{P_{it}/\chi^c_{it}} \right)^{-\phi} \left(\frac{C_{it}}{N_{it}} \right)^{\phi/\sigma}. \tag{D.3}$$

Given K_{i1} and $\mathcal{A}_{i1}$, the paths of consumption, net-purchases of bonds, investment in physical capital, and labor supply must satisfy the budget constraint and the accumulation technologies for capital and net-foreign assets:

$$\frac{P_{it}}{\chi^c_{it}} C_{it} + \mathcal{A}_{it+1} = \left(r_{it} K_{it} - \frac{P_{it}}{\chi^x_{it}} \Phi(K_{it+1}, K_{it}) \right) (1 - \tau^k_{it}) + w_{it} L_{it} + (1 + q_t) \mathcal{A}_{it} + T_{it}. \tag{D.4}$$

D.2 Firm optimization

Markets are perfectly competitive so firms set prices equal to marginal costs. Omitting time subscripts for now, denote the price of variety v, produced in country j and purchased by country i, as $p_{ij}(v)$. Then $p_{ij}(v) = p_{jj}(v) d_{ij}$, where $p_{jj}(v)$ is the marginal cost of producing variety v in country j. Since country i purchases each variety from the country that can deliver it at the lowest price, the price in country i is $p_i(v) = \min_{j=1,\ldots,I} [p_{jj}(v) d_{ij}]$. The price of the composite intermediate good in country i at time t is then

$$P_{it} = \gamma \left[\sum_{j=1}^{I} \left(A_{jt}^{-\nu_{jt}} u_{jt} d_{ijt} \right)^{-\theta} \right]^{-\frac{1}{\theta}}, \tag{D.5}$$

where $u_{jt} = \left(\frac{r_{jt}}{\alpha \nu_{jt}} \right)^{\alpha \nu_{jt}} \left(\frac{w_{jt}}{(1-\alpha)\nu_{jt}} \right)^{(1-\alpha)\nu_{jt}} \left(\frac{P_{jt}}{1-\nu_{jt}} \right)^{1-\nu_{jt}}$ is the unit cost for a bundle of inputs for intermediate-goods producers in country j at time t.

Next I define total factor usage (K, L, M) and output (Y) by summing over varieties.

$$K_{it} = \int_0^1 K_{it}(v) dv, \qquad L_{it} = \int_0^1 L_{it}(v) dv,$$
$$M_{it} = \int_0^1 M_{it}(v) dv, \qquad Y_{it} = \int_0^1 Y_{it}(v) dv.$$

The term $L_{it}(v)$ denotes the quantity of labor employed in the production of variety v at time t. If country i imports variety v at time t, then $L_{it}(v) = 0$. Hence, L_{it} is the total quantity of labor employed in country i at time t. Similarly, K_{it} is the total quantity of capital used, M_{it} is the total quantity of the composite good used as an intermediate input in production, and Y_{it} is the total quantity of output produced.

Cost minimization by firms implies that factor expenses exhaust the value of output.

$$r_{it}K_{it} = \alpha\nu_{it}P_{it}Y_{it}, \qquad w_{it}L_{it} = (1-\alpha)\nu_{it}P_{it}Y_{it}, \qquad P_{it}M_{it} = (1-\nu_{it})P_{it}Y_{it}.$$

That is, $\alpha\nu_{it}$ is the fraction of the value of production that compensates capital services, $(1-\alpha)\nu_{it}$ is the fraction that compensates labor services, and $1-\nu_{it}$ is the fraction that covers the cost of intermediate inputs; there are zero profits.

Trade flows The fraction of country i's expenditures allocated to intermediate varieties produced by country j is given by

$$\pi_{ijt} = \frac{\left(A_{jt}^{-\nu_{jt}}u_{jt}d_{ijt}\right)^{-\theta}}{\sum_{j=1}^{I}\left(A_{jt}^{-\nu_{jt}}u_{jt}d_{ijt}.\right)^{-\theta}}. \tag{D.6}$$

D.2.1 Market clearing conditions

Revenues from distortionary capital taxes are returned in lump sum to the household:

$$\tau_{it}^{k}r_{it}K_{it} = T_{it}.$$

The supply of the composite good, which is an aggregate of all imported and domestic varieties, must equal the demand., which consists of consumption, investment, and intermediate input.

$$\frac{C_{it}}{\chi_{it}^{c}} + \frac{X_{it}}{\chi_{it}^{x}} + M_{it} = Q_{it}.$$

Finally, the balance of payments must hold in each country: the current account equals net exports plus net-foreign income. With net exports equal to gross output less gross absorption, this condition implies

$$\mathcal{B}_{it} = P_{it}Y_{it} - P_{it}Q_{it} + q_t\mathcal{A}_{it}.$$

D.3 Remark

The world interest rate is strictly nominal. As such, the value plays essentially no role other than pinning down a numéraire. Since my choice of numéraire is world GDP in each period, the world interest rate reflects the relative valuation of world GDP at two points in time. This interpretation is useful in guiding the solution procedure and also makes for straightforward mapping between model and data. That is, in the model the prices map into current units, as opposed to constant units. In other words, the model can be rewritten so that all prices are quoted in time-1 units (like an Arrow-Debreu world) with the world interest rate of zero and the equilibrium would yield identical quantities.

E Solution algorithm

In this section of the Appendix I describe the algorithm for computing the equilibrium transition path. Before going further into the algorithm, I introduce some notation. I denote the cross-country vector of a given variable at a point in time using vector notation, i.e., $\vec{K}_t = \{K_{it}\}_{i=1}^{I}$ is the vector of capital stocks across countries at time t.

E.1 Computing the equilibrium transition path

Given the initial conditions—$\vec{K}_1, \vec{\mathcal{A}}_1$—the equilibrium transition path consists of 15 objects: $\{\vec{w}_t\}_{t=1}^{T}$, $\{\vec{r}_t\}_{t=1}^{T}$, $\{q_t\}_{t=1}^{T}$, $\{\vec{P}_t\}_{t=1}^{T}$, $\{\vec{Y}_t\}_{t=1}^{T}$, $\{\vec{Q}_t\}_{t=1}^{T}$, $\{\vec{C}_t\}_{t=1}^{T}$, $\{\vec{X}_t\}_{t=1}^{T}$, $\{\vec{K}_t\}_{t=1}^{T+1}$, $\{\vec{L}_t\}_{t=1}^{T}$, $\{\vec{M}_t\}_{t=1}^{T}$, $\{\vec{\mathcal{B}}_t\}_{t=1}^{T}$, $\{\vec{\mathcal{A}}_t\}_{t=1}^{T}$, $\{\vec{T}_t\}_{t=1}^{T}$, and $\{\vec{\vec{\pi}}_t\}_{t=1}^{T}$ (I use the double-arrow notation on $\vec{\vec{\pi}}_t$ to indicate that this is an $I \times I$ matrix in each period t). Table E.1 provides a list of 15 equilibrium conditions that these objects must satisfy.

The solution procedure is boils down to two loops. The outer loop consists of iterating on the labor supply decision and the rate of investment in physical capital. The inner loop consists of iterating on a set of excess demand equations as in Sposi (2012), given the guess for labor supply and investment rates. First, for the outer loop, guess at the sequence of labor supply and investment rate for every country in every period. Given the labor supply and investment rate, within the inner loop start with an initial guess for the entire sequences of wage vectors and the world interest rate on bonds. Form these two objects, recover all remaining prices and quantities, across countries and throughout time, using optimality conditions and market clearing conditions, excluding the balance of payments condition. Then use departures from the the balance of payments condition to update the

Table E.1: Equilibrium conditions

1	$r_{it}K_{it} = \alpha\nu_{it}P_{it}Y_{it}$	$\forall(i,t)$
2	$w_{it}L_{it} = (1-\alpha)\nu_{it}P_{it}Y_{it}$	$\forall(i,t)$
3	$P_{it}M_{it} = (1-\nu_{it})P_{it}Y_{it}$	$\forall(i,t)$
4	$\frac{C_{it}}{\chi^c_{it}} + \frac{X_{it}}{\chi^x_{it}} + M_{it} = Q_{it}$	$\forall(i,t)$
5	$\sum_{j=1}^{I} P_{jt}Q_{jt}\pi_{jit} = P_{it}Y_{it}$	$\forall(i,t)$
6	$P_{it} = \gamma\left[\sum_{j=1}^{I}\left(A_{jt}^{-\nu_{jt}}u_{jt}d_{ijt}\right)^{-\theta}\right]^{-\frac{1}{\theta}}$	$\forall(i,t)$
7	$\pi_{ijt} = \frac{\left(A_{jt}^{-\nu_{jt}}u_{jt}d_{ijt}\right)^{-\theta}}{\sum_{j=1}^{I}\left(A_{jt}^{-\nu_{jt}}u_{jt}d_{ijt}\right)^{-\theta}}$	$\forall(i,j,t)$
8	$\frac{P_{it}}{\chi^c_{it}}C_{it} + \mathcal{B}_{it} = \left(r_{it}K_{it} - \frac{P_{it}}{\chi^x_{it}}X_{it}\right)(1-\tau^k_{it}) + w_{it}L_{it} + q_t\mathcal{A}_{it} + T_{it}$	$\forall(i,t)$
9	$\mathcal{A}_{it+1} = \mathcal{A}_{it} + \mathcal{B}_{it}$	$\forall(i,t)$
10	$K_{it+1} = (1-\delta)K_{it} + \delta^{1-\lambda}\left(X_{it}\right)^{\lambda}K_{it}^{1-\lambda}$	$\forall(i,t)$
11	$\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}} = \beta^{\sigma}\left(\frac{\psi_{it+1}}{\psi_{it}}\right)^{\sigma}\left(\frac{\frac{1+q_{t+1}}{P_{it+1}/\chi^c_{it+1}}}{P_{it}/\chi^c_{it}}\right)^{\sigma}$	$\forall(i,t)$
12	$\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}} = \beta^{\sigma}\left(\frac{\psi_{it+1}}{\psi_{it}}\right)^{\sigma}\left(\frac{\frac{r_{it+1}}{P_{it+1}/\chi^c_{it+1}} - \left(\frac{\chi^c_{it+1}}{\chi^x_{it+1}}\right)\Phi_2(K_{it+2},K_{it+1})}{\left(\frac{\chi^c_{it}}{\chi^x_{it}}\right)\Phi_1(K_{it+1},K_{it})}\right)^{\sigma}\left(\frac{1-\tau^k_{it+1}}{1-\tau^k_{it}}\right)^{\sigma}$	$\forall(i,t)$
13	$\frac{L_{it}}{N_{it}} = 1 - (\zeta_{it})^{\phi}\left(\frac{w_{it}}{P_{cit}}\right)^{-\phi}\left(\frac{C_{it}}{N_{cit}}\right)^{\phi/\sigma}$	$\forall(i,t)$
14	$\mathcal{B}_{it} = P_{it}Y_{it} - P_{it}Q_{it} + q_t\mathcal{A}_{it}$	$\forall(i,t)$
15	$\tau^k_{it}\left(r_{it}K_{it} - \frac{P_{it}}{\chi^x_{it}}X_{it}\right) = T_{it}$	$\forall(i,t)$

Notes: $u_{jt} = \left(\frac{r_{jt}}{\alpha\nu_{jt}}\right)^{\alpha\nu_{jt}}\left(\frac{w_{jt}}{(1-\alpha)\nu_{jt}}\right)^{(1-\alpha)\nu_{jt}}\left(\frac{P_{jt}}{1-\nu_{jt}}\right)^{1-\nu_{jt}}$. Φ_1 and Φ_2 denote the derivatives of the function $\Phi(K', K) = \delta^{1-1/\lambda}\left(\frac{K'}{K} - (1-\delta)\right)^{1/\lambda}K$ w.r.t. the first and second arguments, respectively.

wages, and use deviations from intertemporal price relationships to update the world interest rate. Iterate on this until the wages and world interest rate satisfy the balance of payments condition and the intertemporal condition for prices. Then back up to the outer loop and check if the labor supply and investment rate decisions salsify optimality. If not, update the guess at labor supply and investment rate and solve the inner loop again. The details to this procedure follow.

1. Guess at a sequence of labor supply choices in every country, $\{\vec{h}_t\}_{t=1}^{T}$ with

$$0 < h_{it} \equiv L_{it}/N_{it} < 1,$$

guess a sequence of nominal investment rates, $\{\vec{\rho}_t\}_{t=1}^T$ with

$$0 < \rho_{it} \equiv \frac{(P_{it}/\chi_{it}^x)X_{it}}{w_{it}L_{it}/(1-\alpha)} < 1,$$

and guess at a terminal NFA position in each country, $\mathcal{A}_{iT+1}$, with $\sum_{i=1}^I \mathcal{A}_{iT+1} = 0$. Take these as given for the next sequence of steps.

(a) Guess the entire path for wages, $\{\vec{w}_t\}_{t=1}^T$, across countries and the entire path for the world interest rate, $\{q_t\}_{t=2}^T$, such that $\sum_i \frac{w_{it}L_{it}}{1-\alpha} = 1\,(\forall t)$.

(b) In period 1 set $\vec{r}_1 = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\vec{w}_1\vec{H}_1}{\vec{K}_1}\right)$, where the initial stock of capital is predetermined. Compute prices $\vec{P}_1$ using condition 6 in Table E.1. Solve for physical investment, $\vec{X}_1$, using the guess for the nominal investment rate together with prices: $\vec{X}_1 = \frac{\vec{\rho}_1\vec{w}_1\vec{L}_1}{(1-\alpha)(\vec{P}_1/\vec{\chi}_1^x)}$ and for the next-period capital stock, $\vec{K}_2$, using condition 10. Repeat this set of calculations for period 2, then for period 3, and all the way through period T.

(c) Compute the bilateral trade shares $\{\vec{\vec{\pi}}_t\}_{t=1}^T$ using condition 7.

(d) This step is slightly more involved. I show how to compute the path for consumption and bond purchases by solving the intertemporal problem of the household. This is done in three parts. First I derive the lifetime budget constraint, second I derive the fraction of lifetime wealth allocated to consumption at each period t, and third I recover the sequences for bond purchases and the stock of NFAs.

Deriving the lifetime budget constraint To begin, compute the lifetime budget constraint for the representative household (omitting country subscripts for now). Begin with the period budget constraint from condition 8 and combine it with the NFA accumulation technology in condition 9 and the balanced tax revenue in condition 15:

$$\mathcal{A}_{t+1} = r_tK_t + w_tL_t - \frac{P_t}{\chi^c}C_t - \frac{P_t}{\chi_t^x}X_t + (1+q_t)\mathcal{A}_t.$$

Iterate the period budget constraint forward through time and derive a lifetime budget constraint. Given $A_{i1} > 0$, compute the NFA position at time $t = 2$:

$$\mathcal{A}_2 = r_1K_1 + w_1L_1 - \frac{P_1}{\chi_1^c}C_1 - \frac{P_1}{\chi_1^x}X_1 + (1+q_1)\mathcal{A}_1.$$

Similarly, compute the NFA position at time $t = 3$:

$$\begin{aligned}
\mathcal{A}_3 &= r_2 K_2 + w_2 L_2 - \frac{P_2}{\chi_2^c} C_2 - \frac{P_2}{\chi_2^x} X_2 + (1+q_2)\mathcal{A}_2 \\
\Rightarrow \mathcal{A}_3 &= r_2 K_2 + w_2 L_2 - \frac{P_2}{\chi_2^x} X_2 + (1+q_2)\left(r_1 K_1 + w_1 L_1 - \frac{P_1}{\chi_1^x} X_1\right) \\
&\quad - \frac{P_2}{\chi_2^c} C_2 - (1+q_2)\frac{P_1}{\chi_1^c} C_1 + (1+q_2)(1+q_1)\mathcal{A}_{i1}.
\end{aligned}$$

Continue to period 4 in a similar way

$$\begin{aligned}
\mathcal{A}_4 &= r_3 K_3 + w_3 L_3 - P_3 C_3 - \frac{P_3}{\chi_3^x} X_3 + (1+q_3)\mathcal{A}_3 \\
\Rightarrow \mathcal{A}_4 &= r_3 K_3 + w_3 L_3 - \frac{P_3}{\chi_3^x} X_3 \\
&\quad + (1+q_3)\left(r_2 K_2 + w_2 L_2 - \frac{P_2}{\chi_2^x} X_2\right) + (1+q_3)(1+q_2)\left(r_1 K_1 + w_1 L_1 - \frac{P_1}{\chi_1^x} X_1\right) \\
&\quad - \frac{P_3}{\chi_3^c} C_3 - (1+q_3)\frac{P_2}{\chi_2^c} C_2 - (1+q_3)(1+q_2)\frac{P_1}{\chi_1^c} C_1 + (1+q_3)(1+q_2)(1+q_1)\mathcal{A}_1.
\end{aligned}$$

Before proceeding, it is useful to define $(1 + \mathcal{Q}_t) = \prod_{n=1}^{t}(1 + q_n)$.

$$\begin{aligned}
\Rightarrow \mathcal{A}_4 &= \frac{(1+\mathcal{Q}_3)\left(r_3 K_3 + w_3 L_3 - \frac{P_3}{\chi_3^x} X_3\right)}{(1+\mathcal{Q}_3)} \\
&\quad + \frac{(1+\mathcal{Q}_3)\left(r_2 K_2 + w_2 L_2 - \frac{P_2}{\chi_2^x} X_2\right)}{(1+\mathcal{Q}_2)} \\
&\quad + \frac{(1+\mathcal{Q}_3)\left(r_1 K_1 + w_1 L_1 - \frac{P_1}{\chi_1^x} X_1\right)}{(1+\mathcal{Q}_1)} \\
&\quad - \frac{(1+\mathcal{Q}_3)\frac{P_3}{\chi_3^c} C_3}{(1+\mathcal{Q}_3)} - \frac{(1+\mathcal{Q}_3)\frac{P_2}{\chi_2^c} C_2}{(1+\mathcal{Q}_2)} - \frac{(1+\mathcal{Q}_3)\frac{P_1}{\chi_1^c} C_1}{(1+\mathcal{Q}_1)} + (1+\mathcal{Q}_3)\mathcal{A}_1.
\end{aligned}$$

By induction, for any time t,

$$\begin{aligned}
\mathcal{A}_{t+1} &= \sum_{n=1}^{t} \frac{(1+\mathcal{Q}_t)\left(r_n K_n + w_n L_n - \frac{P_n}{\chi_n^x} X_n\right)}{(1+\mathcal{Q}_n)} - \sum_{n=1}^{t} \frac{(1+\mathcal{Q}_t)\frac{P_n}{\chi_n^c} C_n}{(1+\mathcal{Q}_n)} + (1+\mathcal{Q}_t)\mathcal{A}_1 \\
\Rightarrow \mathcal{A}_{t+1} &= (1+\mathcal{Q}_t)\left(\sum_{n=1}^{t} \frac{r_n K_n + w_n L_n - \frac{P_n}{\chi_n^x} X_n}{(1+\mathcal{Q}_n)} - \sum_{n=1}^{t} \frac{\frac{P_n}{\chi_n^c} C_n}{(1+\mathcal{Q}_n)} + \mathcal{A}_1\right).
\end{aligned}$$

Observe the previous expression as of $t = T$ to derive the lifetime budget constraint:

$$\sum_{n=1}^{T} \frac{\frac{P_n}{\chi_n^c} C_n}{(1+\mathcal{Q}_n)} = \underbrace{\sum_{n=1}^{T} \frac{r_n K_n + w_n L_n - \frac{P_n}{\chi_n^x} X_n}{(1+\mathcal{Q}_n)} + \mathcal{A}_1 - \frac{\mathcal{A}_{T+1}}{(1+\mathcal{Q}_T)}}_{W}. \tag{E.1}$$

In the lifetime budget constraint (E.1), W denotes the net present value of lifetime wealth, taking both the initial and terminal NFA positions as given.

Solving for the path of consumption Next, compute how the net-present value of lifetime wealth is optimally allocated throughout time. The Euler equation for bonds (condition 11) implies the following relationship between consumption in any two periods t and n:

$$C_n = \left(\frac{N_n}{N_t}\right) \beta^{\sigma(n-t)} \left(\frac{\psi_n}{\psi_t}\right)^{\sigma} \left(\frac{1+\mathcal{Q}_n}{1+\mathcal{Q}_t}\right)^{\sigma} \left(\frac{P_t/\chi_t^c}{P_n/\chi_n^c}\right)^{\sigma} C_t$$

$$\Rightarrow \frac{\frac{P_n}{\chi_n^c} C_n}{1+\mathcal{Q}_n} = \left(\frac{N_n}{N_t}\right) \beta^{\sigma(n-t)} \left(\frac{\psi_n}{\psi_t}\right)^{\sigma} \left(\frac{1+\mathcal{Q}_n}{1+\mathcal{Q}_t}\right)^{\sigma-1} \left(\frac{P_t/\chi_t^c}{P_n/\chi_n^c}\right)^{\sigma-1} \frac{\frac{P_t}{\chi_t^c} C_t}{1+\mathcal{Q}_t}.$$

Since equation (E.1) implies that $\sum_{n=1}^{T} \frac{\frac{P_{in}}{\chi_{in}^c} C_{in}}{1+\mathcal{Q}_n} = W$, rearrange the previous expression (putting country subscripts back in) to obtain

$$\frac{\frac{P_{it}}{\chi_{it}^c} C_{it}}{1+\mathcal{Q}_{it}} = \underbrace{\left(\frac{N_{it} \beta^{\sigma t} \psi_{it}^{\sigma} (1+\mathcal{Q}_{it})^{\sigma-1} \left(\frac{P_{it}}{\chi_{it}^c}\right)^{1-\sigma}}{\sum_{n=1}^{T} N_{in} \beta^{\sigma n} \psi_{in}^{\sigma} (1+\mathcal{Q}_{in})^{\sigma-1} \left(\frac{P_{in}}{\chi_{in}^c}\right)^{1-\sigma}} \right)}_{\xi_{it}} W_i. \tag{E.2}$$

That is, in period t the household in country i spends a share ξ_{it} of lifetime wealth on consumption, with $\sum_{t=1}^{T} \xi_{it} = 1$ for all i. Note that ξ_{it} depends only on prices.

Computing bond purchases and the NFA positions In period 1 take as given consumption spending, investment spending, capital income, labor income, and net income from the initial NFA position; each of which have already been computed in previous steps. Then solve for net bond purchases $\{\vec{B}_t\}_{t=1}^{T}$ using the period budget constraint in condition 8. Use condition 9 to solve for the

NFA position in period 2. Repeat this set of calculations iteratively for periods $2, \ldots, T$.

Trade balance condition I compute an excess demand equation as in Alvarez and Lucas (2007), but instead of imposing that net exports equal zero in each country, I impose that net exports equal the current account less net foreign income from assets.

$$Z_{it}^w\left(\{\vec{w}_t, q_t\}_{t=1}^T\right) = \frac{P_{it}Y_{it} - P_{it}Q_{it} - B_{it} + q_t\mathcal{A}_{it}}{w_{it}}.$$

Condition 14 requires that $Z_{it}^w\left(\{\vec{w}_t, q_t\}_{t=1}^T\right) = 0$ for all (i,t). If this is different from zero in at least some country at some point in time update the wages as follows.

$$\Lambda_{it}^w\left(\{\vec{w}_t, q_t\}_{t=1}^T\right) = w_{it}\left(1 + \kappa\frac{Z_{it}^w\left(\{\vec{w}_t, q_t\}_{t=1}^T\right)}{L_{it}}\right)$$

is the updated wages, where κ is chosen to be sufficiently small so that $\Lambda^w > 0$.

Normalizing model units The last part of this step is updating the equilibrium world interest rate. Recall that the numéraire is defined to be world GDP at each point in time: $\sum_{i=1}^I(r_{it}K_{it} + w_{it}L_{it}) = 1$ $(\forall t)$. For an arbitrary sequence of $\{q_{t+1}\}_{t=1}^T$, this condition need not hold. As such, update the the world interest rate as

$$1 + q_t = \frac{\sum_{i=1}^I(r_{it-1}K_{it-1} + \Lambda_{it-1}^w L_{it-1})}{\sum_{i=1}^I(r_{it}K_{it} + \Lambda_{it}^w L_{it})} \text{ for } t = 2, \ldots, T. \tag{E.3}$$

The values for capital stock and the rental rate of capital are computed in step 2, while the values for wages are the updated values Λ^w above. I set $q_1 = \frac{1-\beta}{\beta}$ (the interest rate that prevails in a steady state) and chose A_{i1} so that $q_1 A_{i1}$ matches the desired initial NFA position in current prices.

Having updated the wages and the world interest rate, return to step 2a and perform each step again. Iterate through this procedure until the excess demand is sufficiently close to zero. In the computations I find that my preferred convergence metric:

$$\max_{t=1}^{T}\left\{\max_{i=1}^{I}\left\{\left|Z_{it}^w\left(\{\vec{w}_t, q_t\}_{t=1}^T\right)\right|\right\}\right\}$$

converges roughly monotonically towards zero.

2. The last step of the algorithm is to update the labor supply, investment rate, and terminal NFA position. Until now, the optimality condition 13 for the labor supply and condition 12 for the investment in physical capital have not been used. As such, compute a "residual" as to each of these first-order conditions as

$$Z_{it}^{h}\left(\{\vec{h}_{it}\}_{t=1}^{T}\right) = 1 - \zeta_{it}^{\phi}\left(\frac{w_{it}}{P_{cit}}\right)^{-\phi}\left(\frac{C_{it}}{N_{cit}}\right)^{\phi/\sigma} - \left(\frac{L_{it}}{N_{it}}\right),$$

$$Z_{it}^{\rho}\left(\{\vec{\rho}_{it}\}_{t=1}^{T}\right) = \beta^{\sigma}\left(\frac{\psi_{it+1}}{\psi_{it}}\right)^{\sigma}\left(\frac{\frac{r_{it+1}}{P_{it+1}/\chi_{it+1}^{c}} - \left(\frac{\chi_{it+1}^{c}}{\chi_{it+1}^{x}}\right)\Phi_2(K_{it+2},K_{it+1})}{\left(\frac{\chi_{it}^{c}}{\chi_{it}^{x}}\right)\Phi_1(K_{it+1},K_{it})}\right)^{\sigma}\left(\frac{1-\tau_{it+1}^{k}}{1-\tau_{it}^{k}}\right)^{\sigma} - \left(\frac{C_{it+1}/N_{it+1}}{C_{it}/N_{it}}\right).$$

Condition 13 requires that $Z_{it}^{h}\left(\{\vec{h}_{it}\}_{t=1}^{T}\right) = 0$ for all (i,t), while condition 11 requires that $Z_{it}^{\rho}\left(\{\vec{\rho}_{it}\}_{t=1}^{T}\right) = 0$. Update the labor supply and investment rate as

$$\Lambda_{it}^{h}\left(\{\vec{h}_{t}\}_{t=1}^{T}\right) = h_{it}\left(1 + \psi Z_{it}^{h}\left(\{\vec{h}_{t}\}_{t=1}^{T}\right)\right),$$
$$\Lambda_{it}^{\rho}\left(\{\vec{\rho}_{t}\}_{t=1}^{T}\right) = \rho_{it}\left(1 + \psi Z_{it}^{h}\left(\{\vec{\rho}_{t}\}_{t=1}^{T}\right)\right),$$

where ψ is a constant value assigned to ensure that the updated guesses remain positive. Given the updated sequence of labor supply and investment rate, return to step 1.

With T chosen to be sufficiently large, the turnpike theorem implies that the terminal NFA position has no bearing on the transition path up to some time $t^{\star} < T$ (see Maliar, Maliar, Taylor, and Tsener, 2015).

F Robustness

This section of the Appendix considers alternative specifications for decomposing wedges into demographic and distortionary components.

F.1 High- and low-variant demographic projections

Projections for the age distribution involve uncertainty surrounding fertility, migration flows, and death rates. The UN provides different variants of the projections, each of which yield different projected paths for each country's population and each country's age distribution of the population. The main part of the paper used the medium-variant projections. Two other readily available projections are the high-variant and low-variant projections.

According to the UN, "A medium (or standard) variant is the most likely demographic development (forecast?). High and low variants are considered the reasonable upper and lower limits of realistic projections and indicate the margin of uncertainty."

For each variant of the projections, I repeat the entire quantitative exercise as follows. First, taking the relevant variant for projected population population levels, I implement the wedge accounting procedure and calibrate all country-specific and time-varying parameters of the model. Second, taking the relevant variant for projected age distributions, I decompose the wedges into demographic and distortionary components using OLS, exactly as in the paper. Finally, I consider a counterfactual in which each country's working age share is unilaterally held fixed at its 1970 levels. I prefer to compare the unilateral counterfactuals since the high and low bounds of the projections differ across countries.

The elasticity between the ratio of net exports to GDP and labor productivity growth, using the counterfactual data from 1970-2014, is similar across all variants of the projections. In the low-variant counterfactual, the elasticity is -0.77 and in the high-variant counterfactual the elasticity is -1.03. Recall that the elasticity is the medium-variant counterfactual is -0.94.

F.2 Alternative empirical specifications for wedge decomposition

In the main paper, decomposition of the wedges assumed that the log-wedge is linear in the working age share. This was specified intentionally to impose a conservative mapping between demographics and wedges. In this robustness exercise I specify that the log-wedge is a polynomial of order $H = 3$ in the working age share.

$$\ln\left(\frac{N_{it}}{N_{it-1}}\right) = \gamma_i^N + \kappa_{t-1}^N + \sum_{h=1}^{H} \mu_h^N \times s_{it-1}^h + \varepsilon_{it-1}^N; i = 1, \dots I; t = 2, \dots T,$$

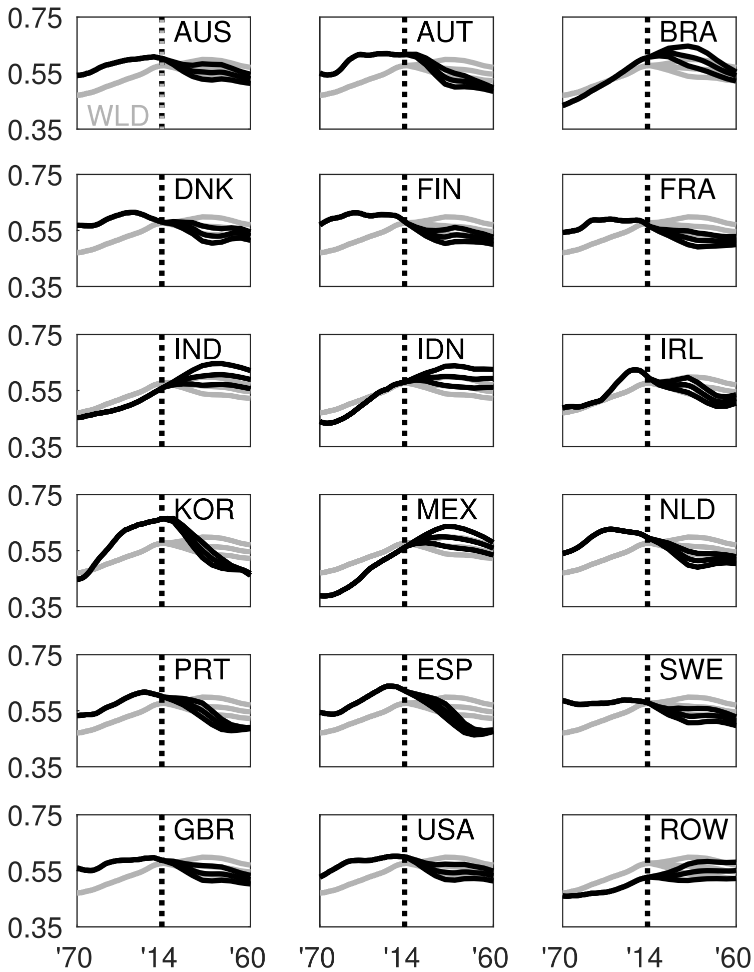
$$\ln\left(\frac{\psi_{it}}{\psi_{it-1}}\right) = \gamma_i^\psi + \sum_{h=1}^{H} \mu_h^\psi \times (s_{it-1} - s_{Ut-1})^h + \varepsilon_{it-1}^\psi; i = 1, \dots I \text{ (ex. U.S.)}; t = 2, \dots T,$$

$$\ln(\zeta_{it}) = \gamma_i^\zeta + \kappa_t^\zeta + \sum_{h=1}^{H} \mu_h^\zeta \times s_{it}^h + \varepsilon_{it}^\zeta; i = 1, \dots I; t = 1, \dots T.$$

I decompose the wedges using this specification and compute the counterfactual in which every country's age share vectors are held fixed at their 1970 levels. In the counterfactual, the elasticity between the ratio of net exports to GDP and labor productivity growth is -0.75, compared to -0.90 in the linear decomposition used in the main paper.

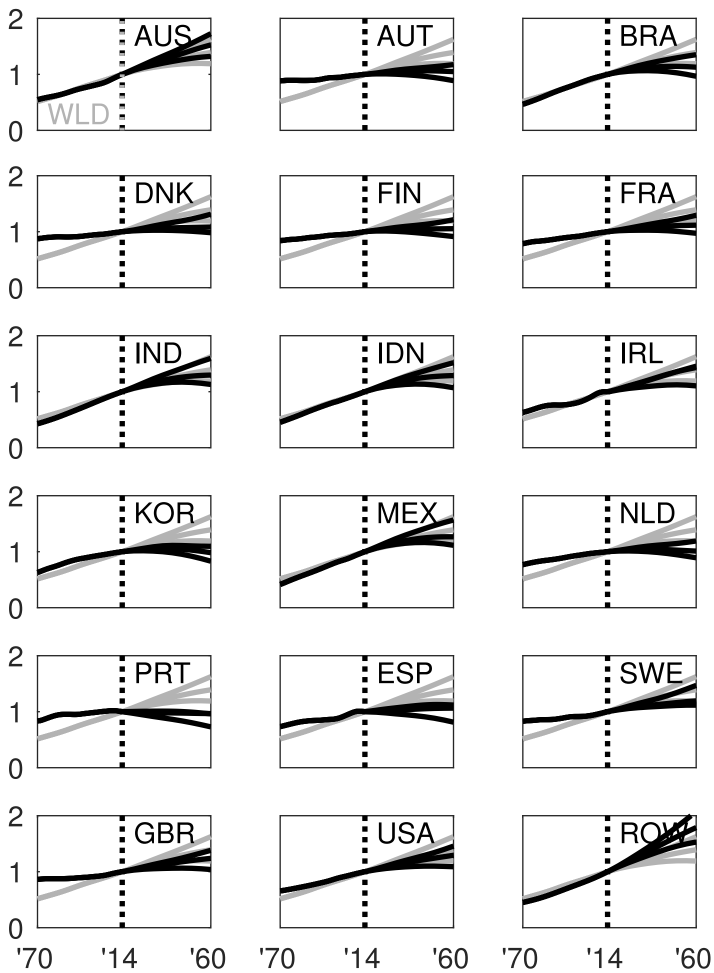
G Additional figures

Figure G.1: Share of population aged 15-64



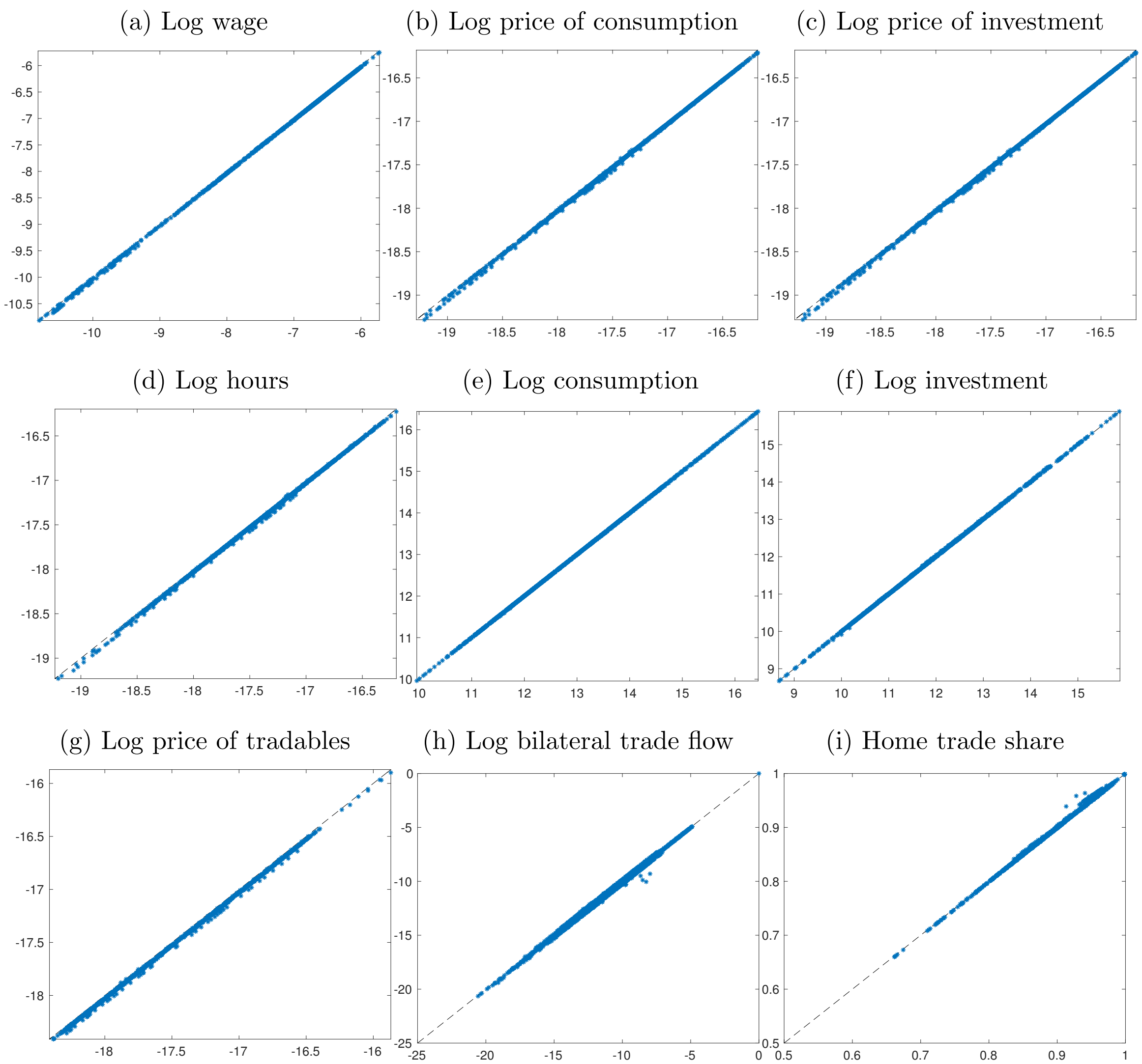
Notes: Black lines – country level. Gray line – world aggregate. Data from 1970-2014 are observed. Data from 2015-2060 are based on projections: high-variant (upper), medium-variant (middle), and low-variant (lower). High- and low-variants assume plus and minus one-half child per woman, respectively, relative to the medium-variant.

Figure G.2: Population – indexed to 1 in 2014



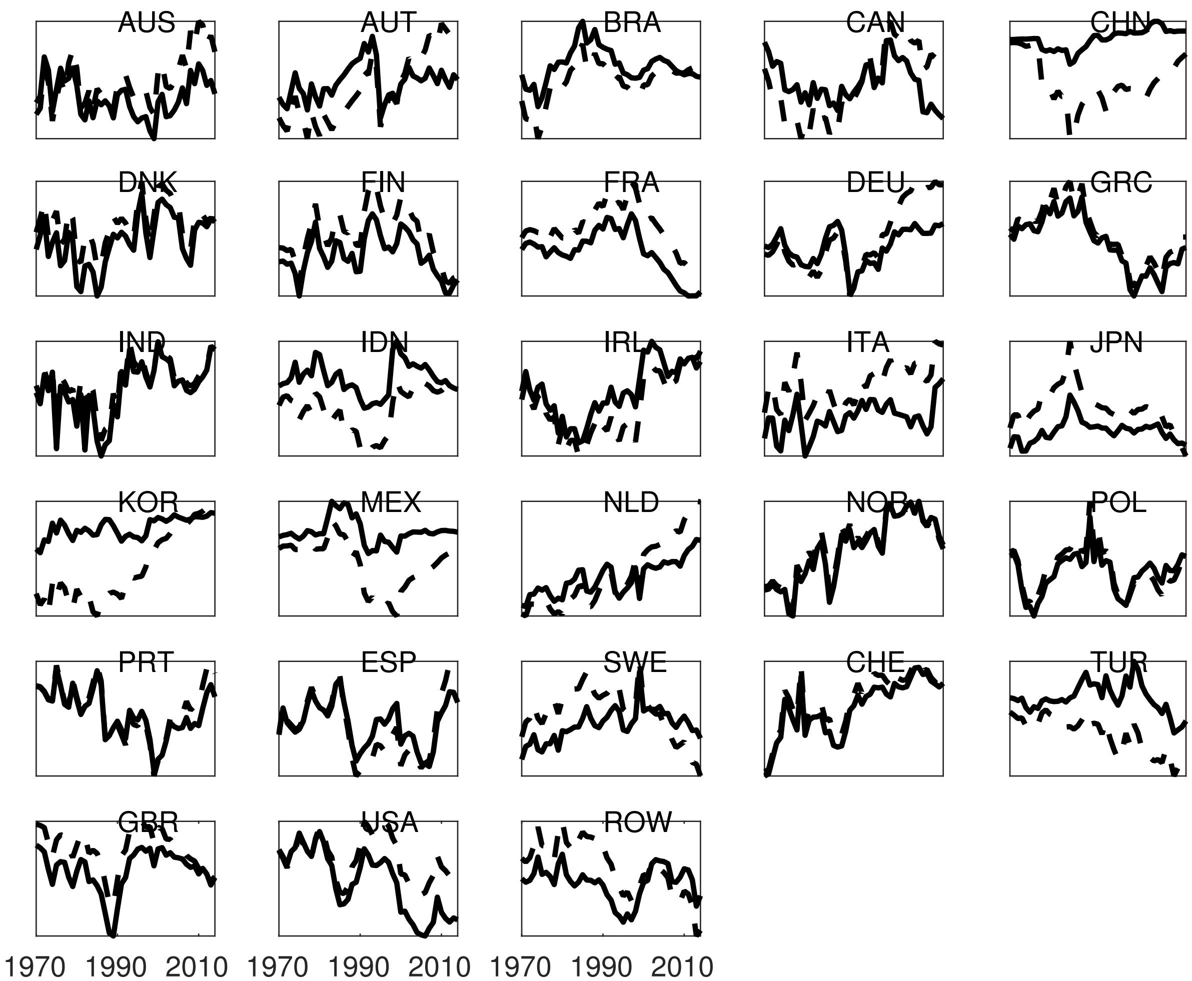
Notes: Black lines – country level. Gray line – world aggregate. Data from 1970-2014 are observed. Data from 2015-2060 are based on projections: high-variant (upper), medium-variant (middle), and low-variant (lower). High- and low-variants assume plus and minus one-half child per woman, respectively, relative to the medium-variant.

Figure G.3: Model fit for targeted data: 1970-2014



Notes: Horizontal axis - data. Vertical axis - model. Each point corresponds to one country in one year. The dashed line represents the 45^o line.

Figure G.4: Ratio of net exports to GDP



Notes: Solid lines – Baseline model (same as data). Dashed lines– Counterfactual with working age shares simultaneously fixed at 1970 levels.