# Emissions Policy with Endogenous Technological Change and Knowledge Spillovers

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## Abstract

This paper constructs a stylized model of firm output and emissions that accounts for the two externalities that characterize environmental markets when technological change is endogenous. The first externality concerns emissions, the external damages of which are not accounted for by the firm, and the second concerns knowledge spillovers, which increase the social return to innovation above the private return. We show that when firms can reduce emissions separately through both output and abatement, the existence of knowledge spillovers implies that a single tax on emissions is no longer capable of inducing the firm to emit at the socially optimal level. This is due to the different effects that spillovers have on the firm output and abatement decisions. We then derive the output tax and abatement subsidy required to correct the emissions externality and the investment subsidy required to correct the spillovers externality. When spillovers exist, the output tax should be set below marginal external damages and the abatement subsidy above marginal external damages.

# I. Introduction

Endogenous technological change has become a key component of models of environmental policy during the past two decades. During that time, many studies have demonstrated the importance of including technological change as an endogenous determinant of emissions.[1] When technological change is considered endogenous, environmental policies not only provide incentives for firms to reduce emissions directly through emissions abatement, but also change the incentives that firms have to invest in developing new technologies that reduce emissions intensity. Several studies have shown that accounting for this endogeneity reduces the cost associated with achieving a given level of emissions reduction, underscoring its importance for environmental policy.[2]

However, the public nature of technology introduces a second externality into models of environmental policy.[3] The inability of firms to appropriate all of the gains from investment in innovation means that the private returns to investment will be lower than the social returns to investment. Thus, investment in innovation may be sub-optimal in the absence of government intervention.

This paper draws on many of these studies in order to construct a stylized model of firm behavior that can be used to calculate numerical estimates of socially optimal taxes and subsidies that correct these two market failures. While previous studies have examined the importance of endogeneity and knowledge spillovers in setting emissions policy, none have incorporated them into a single stylized model of firm behavior in order to derive these socially optimal taxes and subsidies. This paper shows that modeling complete firm behavior, which includes the firm output decision in addition to

[1] Clarke and Weyant (2002); Jaffe et al. (2003); Loschel (2002)
[2] Goulder and Schneider (1999); Nordhaus (1998); Parry (1995); Parry et al. (2000)
[3] Griliches (1992); Jaffe et al. (2004); Jaffe et al. (2003)

the more commonly studied abatement and investment decisions, yields interesting results for setting optimal emissions reduction policy. Due to the interactions between these two externalities, it is not possible to implement a single tax on emissions that induces firms to emit at the socially optimal level. Instead, a tax and subsidy to output and abatement, respectively, are required to induce firms to set output and abatement, and thus emissions, at the socially optimal levels. This is due to the different effects that spillovers have on the output and abatement decisions. The existence of spillovers implies that the output tax should be set below the marginal external damages of emissions and the abatement subsidy should be set above the marginal external damages of emissions. The tax and subsidy together correct the emissions externality, and a subsidy to investment corrects the spillovers externality. The policies correct the externalities by causing the firm to internalize the external costs or benefits associated with the three firm decisions. These three policies combined induce the firm to invest in innovation and emit at the socially optimal level.

This result might explain the mixed findings of empirical studies of the effects of emissions trading programs on technological innovation. These studies have shown that tradable emissions permit programs instituted in the U.S. have often resulted in less innovation than expected.[4] These results can be explained in part by low permit prices and changes in relative input prices, both of which make abatement more attractive than innovation, but they might also be a consequence of the investment disincentive spillovers create. If so, our results might reveal how existing permit prices, which are directly analogous to emissions taxes, might be set at levels incapable of inducing socially optimal innovation because they ignore the effects of spillovers.

[4] Burtraw (2000); Burtraw et al. (2005); Gagelmann (2003); Kemp (2000); Lange and Bellas (2006)

The rest of this paper is divided into five sections. The first section explains the three different ways that firms can reduce emissions when technological change is endogenous. The second section considers the basic case where technology is exogenous and derives the single socially optimal tax on emissions that corrects the emissions externality. Section three builds on this basic model by allowing for the endogeneity of technological change and the existence of knowledge spillovers. It shows that when spillovers are assumed to be zero, the emissions tax still induces firms to emit at the socially optimal level. However, when spillovers are present in addition to the emissions externality, the optimal tax on emissions is no longer capable of inducing the optimal level of emissions. It also derives a research subsidy that corrects the spillovers externality. Section four shows that a tax on output and a subsidy to abatement can instead be used to correct the emissions externality when spillovers are present. Section fives concludes.

## I. The Firm Decision

Previous studies of the relationship between the emissions and knowledge spillovers externalities have focused only on the abatement and investment decisions. This paper extends such analyses by accounting for the relationship between output and emissions. This is a more realistic characterization of firm behavior, and future studies can extend this model to examine how emissions policies affect welfare through price changes that result from output changes. This paper, however, will focus only on the implications of this relationship between output and emissions for setting socially optimal emissions policy. Firms can reduce emissions in three separate ways: 1) They can innovate, which increases abatement efficiency, while holding output and level of abatement constant; 2) They can reduce output, holding abatement and abatement efficiency constant; and 3) They can increase abatement, holding output and abatement efficiency constant. The basic firm profit function that accounts for these possible decisions is specified below:[5]

$$\pi = px - c_x x - (1 - z(H))c_a((A^2)/2)x - t(M - A)x - R(h) \qquad (0)$$

Firms produce a good, *x*, that is sold in a perfectly competitive market at price *p*. Production is associated with baseline emissions per unit output *M*. Baseline emissions is the level of emissions per unit output in the absence of government intervention. Firms face a tax per unit emissions of *t*, and they can reduce emissions per unit output by increasing abatement per unit output, *A*. Abatement might include installing additional units of an existing technology, such as "scrubbers," or substituting less polluting inputs, such as natural gas, into a given production process.[6] These abatement technologies already exist, and the firm simply purchases more of them to increase abatement. The cost of increasing abatement is $c_a$. The abatement cost function is quadratic in form, resulting in an upward-sloping marginal abatement cost curve.

Firms can also reduce emissions per unit output by investing in innovation. *R* represents expenditures on innovation and is an increasing function of innovation, *h*. The knowledge stock increases over time according to the equation $H_t = H_{t-1} + h_{t-1}$. $z(H)$ is an increasing function of the firm knowledge stock and has values $0 \leq z \leq 1$, where $z(0) = 0$. It is the rate at which increases in the knowledge stock, or innovation, translate into lower abatement costs. Innovation decreases the cost of abatement as follows. Innovation

[5] The functions in this paper are adapted from Fischer and Newell (2004), Parry (1997), and Parry et al. (2000).
[6] Parry (1997); Parry et al. (2000)

increases the knowledge stock, which leads to an improvement of existing technology, thus making each unit of abatement more efficient at reducing emissions. Improvements in abatement efficiency might involve creating a more efficient scrubber or improving the production process for given inputs. These improvements reduce the cost of abatement by requiring fewer units of the improved technology—units of abatement—to reduce emissions a certain amount.

Firms can also reduce emissions by simply reducing output. By expressing emissions and abatement in per unit output terms, it is possible to account for the fact that emissions are a result of the production process and that firms can change their level of emissions by changing their level of output. This is a more realistic or complete specification than many models that focus only on the abatement and investment decisions. Previous studies have shown how this output effect can affect consumer surplus.[7] We acknowledge that the output effect might not be significant in all cases, especially if abatement is relatively costless.[8] However, different emissions will have different associated relative costs and benefits of reducing output, increasing abatement, and increasing investment in innovation. Ours is a generalized model that can be applied to any emissions externality. Next, we explore the general implications of including the output decision for setting emissions policy that maximizes social welfare.

## II. The Basic Model with Exogenous Technological Change

In order to examine the importance of accounting for the endogeneity of technological change and knowledge spillovers in calculating a socially optimal tax on

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[7] Parry (1997)

[8] Schmalensee et al. (1998) show that almost all of the sulfur dioxide emissions reductions related to the Clean Air Act were due to substitution toward less polluting inputs and increased use of scrubbers.

emissions, we first derive a model that does not account for these factors. Calculating a socially optimal tax involves specifying two equations: one for the firm and one for the social planner. In the first, the firm maximizes its own profit without realizing that it produces an externality, which in this case is emissions. The firm is subject to a tax on the externality that is imposed by the social planner. The social planner seeks to maximize social welfare by recognizing the damage that the externality, emissions, does to society. By maximizing each of these equations with respect to the variables that the firm controls, the social planner can find the socially optimal tax on the externality that causes the firm to internalize the damages of the externality, inducing the firm to emit at the socially optimal level.[9]

The model without technology and spillovers developed here will be referred to throughout this paper as the basic model. The model specifies a firm profit function and a social welfare function, derives the first order conditions for each equation, and equates the corresponding first order conditions of each equation in order to find the tax on emissions that causes the firm to internalize the emissions externality and thus emit at the socially optimal level. Technology is assumed to increase independently of the firm and social planner decisions, and thus is not included in either of the two equations. The firm profit function is:

$$\pi = px - c_x x - c_a((A^2)/2)x - t(M - A)x \quad (1)$$

The firm maximizes profit by selecting levels of $x$ and $A$. Setting the derivative of (1) with respect to $x$ equal to zero yields:

$$p = c_x + c_a(A^2)/2 + t(M - A) \quad (2)$$

[9] Fullerton and West (2002) provide an example of this technique.

This equation shows that the firm will increase output to the point where the price of $x$ equals the private cost of producing the marginal unit of $x$. This marginal private cost is the production cost of $x$ plus the abatement cost per unit of $x$ plus the tax paid on the marginal unit of $x$. Taking the derivative of (1) with respect to $A$ and setting it equal to zero yields:

$$c_a A = t \tag{3}$$

This result shows that the firm will increase abatement up until the point at which the marginal cost of abatement, $c_a A$, equals the marginal benefit of abatement, which is the tax not paid on the marginal unit of emissions abatement.

In order to calculate the socially optimal tax, we must now find the levels of firm output and abatement that maximize social welfare. To do this, we express social welfare in the following way:

$$W = px - c_x x - c_a((A^2)/2)x - d(M - A)x \tag{4}$$

where all of the variables are the same as above and $d$ represents the marginal external damages caused by emissions per unit of output. Social welfare depends on the decisions of the single representative firm described above. In this basic model, the firm profit function and social welfare function differ only in that the firm is subject to a tax on emissions, imposed by the social planner, and the social planner recognizes the external damages caused by emissions. In order to set the tax on emissions at the socially optimal level, the social planner first maximizes social welfare with respect to the variables that the firm can control, $x$ and $A$. Setting the derivative of (4) with respect to $x$ and $A$ equal to zero yields equations (5) and (6), respectively.

$$p = c_x + c_a(A^2)/2 + d(M - A) \tag{5}$$

$$c_a A = d \tag{6}$$

Equation (5) shows that to maximize social welfare, the firm should produce *x* up until the point at which the marginal social benefit of production, *p*, equals the marginal social cost of production.[10] This marginal social cost is equal to the sum of the cost of production, the cost of abatement, and the damages from emissions. This equation is identical to equation (2) except for the last term. Whereas the firm only accounts for the cost of the emissions tax, the social planner accounts for the cost of emissions damages. In order to induce the firm to account for the damages that its emissions have on society and thus emit at the socially optimal level, the social planner sets the tax at the level of marginal emissions damages.

Comparing equations (3) and (6) in a similar way yields the same result. In order to equate these two first order conditions, the social planner sets the tax on emissions equal to the external damages of emissions. This induces the firm to increase abatement up until the point at which the marginal private cost of abatement equals the marginal benefit to society (the marginal external damages avoided), which maximizes social welfare.

This basic model thus shows that in the absence of endogenous technological change and knowledge spillovers, the socially optimal tax is simply set at the level of marginal external emissions damages. This is a typical finding in the literature: social planners can maximize social welfare in the presence of an externality by setting a tax on the externality equal to the marginal external damages of the externality.[11] The more interesting question that this paper hopes to address is how and by how much the socially

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[10] We assume there are no external benefits of production.
[11] Pigou (1932)

optimal tax on emissions is affected by the existence of endogenous technological change and knowledge spillovers. The next section develops a model that accounts for these two concepts in order to derive the new socially optimal emissions tax. As will be shown, this single tax on emissions fails to induce optimal output and abatement—and thus optimal emissions—in the presence of two externalities.

## III. The Model with Endogenous Technological Change and Knowledge Spillovers

As in the basic model, this model will define both a firm profit function and social welfare function. Both functions account for the endogeneity of abatement technology. In contrast to the previous model, the social planner attempts to implement two policies in order to cause the firm to internalize the two externalities. These are a tax on emissions, as in the basic model, and an additional subsidy to investment in innovation. The social planner recognizes the negative emissions externality and imposes a tax on the firm to correct the externality, as described above. The social planner also recognizes the positive knowledge spillovers externality. This positive externality implies that the social return to innovation is greater than the private return to innovation. In order to induce the firm to increase its investment in innovation to the socially optimal level, the social planner imposes a subsidy to investment in innovation that decreases the cost of investment to the firm.

Following Fischer and Newell (2004), optimization of the two functions occurs over two periods. In the first period, the firm can choose to invest in technology that decreases the cost of abatement in the second period. The firm can choose different levels of output and abatement in each period. The firm faces a tax on emissions and a subsidy to investment that are set at the beginning of the first period and do not change in the second period. The social planner recognizes both the external damages of emissions and the existence of knowledge spillovers. The firm profit function is expressed below:

$$\pi = n[p_1x_1 - c_xx_1 - (1 - z(H_1))c_a((A_1^2)/2)x_1 - t(M - A_1)x_1 - (1 - s)R(h_1)]$$
$$+ \delta m[[p_2x_2 - c_xx_2 - (1 - z(H_2))c_a((A_2^2)/2)x_2 - t(M - A_2)x_2] \quad (7)$$

Previously defined variables have the same definitions. $n$ and $m$ are the number of years in the first and second periods, respectively, and profits in the second period are discounted at a rate $\delta$. $s$ is the rate at which investment in innovation is subsidized and has value $0 \leq s \leq 1$. This profit function now captures the fact that firms can respond to a tax on emissions in three ways: by decreasing output, by increasing abatement directly, and by investing in innovation that will reduce the cost of abatement in the second period. Whereas in the basic model technological change is assumed to be exogenous, here it is a function of firm investment, which itself responds to environmental policy.[12]

The firm maximizes profit by choosing output, abatement, and innovation investment in the first period. Taking the derivative of (7) with respect to $x$ in period $t$ and setting it equal to zero yields:

$$p_t = c_x + (1 - z(H_t))c_a(A_t^2)/2 + t(M - A_t) \quad (8)$$

[12] The basic model includes only one period because when technological change is exogenous, the firm is unable to invest in innovation and thereby reduce the costs of abatement; abatement costs are the same in both periods. As prices, production costs, and taxes/subsidies are assumed to be the same in both periods, the firm will choose the same levels of output and abatement in both periods, and thus one period is sufficient. The results of these two models are directly comparable because in the two-period model, derivatives are taken with respect to output and abatement in a given period, and thus the results are for a given period and are a function of the abatement costs in that period. Only the derivative with respect to investment in innovation is a function of variables from both period, and there is no analogous result in the basic model due to its assumption of exogenous technological change.

The firm increases output until the private marginal benefit of production in period $t$ equals the private marginal cost of production in period $t$, but now the cost of abatement for the marginal unit of output can decrease by increasing the knowledge stock. Differentiating (7) with respect to $A$ in period $t$ and setting it equal to zero yields:

$$(1 - z(H_t))c_a A_t = t \quad (9)$$

Here the firm chooses the level of abatement at which the tax, the marginal private benefit of abatement in period $t$, equals the marginal private cost of abatement in period $t$, where this cost is lowered by increasing the knowledge stock.

When technological change is endogenous, the firm can also maximize profits by choosing the level of investment in innovation that reduces abatement costs. Setting the derivative of (7) with respect to $h$ equal to zero reveals the optimal level of investment in innovation:

$$n[(1 - s)R_{h1}(h_1)] = \delta m[z_{h1}(H_2)c_a((A_2^2)/2)x_2] \quad (10)$$

To maximize profits over both periods, the firm chooses to invest in innovation in the first period until the marginal private cost of investment in the first period equals the discounted marginal private benefit of reducing abatement costs in the second period.

In order to calculate the socially optimal taxes on emissions, we must now specify the social welfare function that accounts for the emissions and spillovers externalities.

$$W = n[p_1x_1 - c_xx_1 - (1 - (1 + \sigma)z(H_1))c_a((A_1^2)/2)x_1 - d(M - A_1)x_1 - R(h_1)]$$
$$+ \delta m[p_2x_2 - c_xx_2 - (1 - (1 + \sigma)z(H_2))c_a((A_2^2)/2)x_2 - d(M - A_2)x_2] \quad (11)$$

As in the basic model, it includes a term that represents the total external damages of unabated emissions, whereas the firm function shows the total tax paid on those emissions. It now also includes a term that represents the positive spillovers externality,

$\sigma$. It is the rate at which the gains from investment in innovation spill over to other firms and has value $0 \leq \sigma \leq 1$. When spillovers are present, they increase the social return to innovation above the private return to innovation because some of the gains from innovation spill over to other firms in the industry, reducing their abatement costs by some amount.[13] The firm accounts only for its own gains from investment, the private return, while the social planner accounts for the gains to all firms, the social return. Taking the derivatives of (11) with respect to output and abatement in period *t* and investment in the first period and setting them equal to zero yields the following first order conditions:

$$p_t = c_x + (1 - (1 + \sigma)z(H_t))c_a(A_t^2)/2 + d(M - A_t) \quad (12)$$

$$(1 - (1 + \sigma)z(H_t))c_aA_t = d \quad (13)$$

$$nR_{h1}(h_1) = \delta m[(1 + \sigma)z_{h1}(H_2)c_a((A_2^2)/2)x_2] \quad (14)$$

Equation (12) reveals that the socially optimal level of output is the point at which the marginal social benefit of production equals the marginal social cost of production. This cost is the cost of production per unit output plus the cost of abatement per unit output plus the external emissions damages per unit output. In the presence of spillovers, the socially optimal level of production is greater than the level at which the firm chooses to produce. This is because spillovers cause the firm to invest in innovation less than is socially optimal. As a result, the cost of abatement in the second period remains higher than is socially optimal. This leads the firm to abate less than is socially optimal in the second period and instead reduce emissions by decreasing output. Thus, firm output is

[13] Innovation might benefit other firms when firms invest in basic research that can be applied by all firms or when other firms are able to partially replicate new technologies due to imperfect patent processes or reverse engineering.

lower than the socially optimal level of output. This is important for understanding the optimal emissions tax and output tax derived below.

Equation (13) shows that to maximize social welfare, abatement should increase until the point at which the marginal social cost of abatement equals the marginal social benefit of abatement (the marginal external damages avoided). In this case, the existence of knowledge spillovers decreases the marginal social cost of abatement. By solving for the level of abatement, $A$, it is easy to see that as the marginal social cost of abatement decreases, the equilibrium level of abatement increases. Thus, when spillovers are present, the socially optimal level of abatement is greater than the level of abatement that the firm chooses. The existence of spillovers implies that firm investment in innovation will be lower than is socially optimal. Because of this, the cost of abatement in the second period remains higher than is socially optimal. Firms will thus abate less in the second period than is socially optimal. We will see how this spillover effect changes the optimal emissions tax and abatement subsidy below.

Equation (14) shows the socially optimal level of investment in technology. Firms should invest in technology until the marginal social cost of investment in the first period equals the discounted marginal social benefits in the second period that are gained through the decreased cost of abatement in that period. When spillovers are present, the socially optimal level of investment in the first period is higher than it would be without spillovers, and it is higher than the level at which the firm will choose to invest.

Deriving the Optimal Investment Subsidy

In order to calculate the optimal investment subsidy, which induces the firm to internalize the benefits of the positive spillover externality and thus invest at the socially level, we equate the two first order conditions related to the investment decision, equations (10) and (14). Doing so yields the following equation:

$$n[sR_{h1}(h_1)] = \delta m[\sigma z_{h1}(H_2)c_a((A_2^2)/2)x_2] \qquad (15)$$

The subsidy is set so that the marginal benefit of investment in the second period that does not accrue to the firm but instead spills over to other firms—the difference between the marginal social benefit and the marginal private benefit of investment—equals the marginal private cost of investment that is subsidized by the social planner. In other words, the subsidy reduces the marginal private cost of investment by the same amount that spillovers increase the marginal social benefit of investment above the marginal private benefit. Because the firm cannot appropriate all of the gains of investment in innovation, it must be subsidized so that it will increase investment to the socially optimal level. As the rate of spillovers increases, so does the subsidy rate.

Attempting to Derive the Single Optimal Emissions Tax

Attempting to derive the single optimal emissions tax shows that such a tax fails to induce socially optimal firm output and abatement decisions when knowledge spillovers are present. This is because spillovers affect the output and abatement decisions in different ways. Equating equations (8) and (12) yields the tax on emissions that induces the firm to set output at the socially optimal level:

$$t = d - [\sigma z(H_t))c_a(A_t^2)/2] / (M - A_t) \qquad (16)$$

When spillovers are assumed to be zero, the tax is simply set at the level of marginal external damages of emissions, *d*, as described before. However, when spillovers exist, the tax should be set lower than marginal external damages. This result coincides with the interpretation of the effect of spillovers on output stated above. Spillovers imply that the firm invests in innovation less than is socially optimal and thus chooses a level of output below the social optimum. To correct this spillover effect on output, the tax on emissions that induces optimal output should be lower than marginal external damages when spillovers exist. The second term on the right-hand side of (16) shows how much lower it should be. It is interpreted as the difference between the social and private returns to investment in innovation as it relates to decreasing the cost of abatement per unit output. Decreasing the tax by this amount causes the firm to internalize the spillover externality as it relates to output. By setting the tax on emissions lower by this amount, the firm will reduce output less, resulting in the socially optimal level of output. This response can be seen in the firm profit function, (7).

The tax on emissions that induces the firm to abate at the socially optimal level is found by equating equations (9) and (13):

$$t = d + \sigma z(H_t) c_a A_t \qquad (17)$$

In the absence of spillovers, the second term equals zero and the tax on emissions is again set at the level of marginal external damages of emissions. When spillovers are present, the tax is set higher than marginal external damages. As described above, spillovers imply that the socially optimal level of abatement is greater than the level the firm chooses. In order to induce the firm to increase abatement to the socially optimal level, the emissions tax is set higher than marginal external damages alone. The second term on the right-hand side of (17) shows by just how much the tax should be set. This term represents the difference between the social and private returns to investment in innovation as it relates to decreasing the marginal abatement cost per unit output. Increasing the tax by this amount causes the firm to internalize the spillover externality as it relates to abatement, and the firm will respond by increasing abatement to the socially optimal level, as can be seen in the firm profit function (7).

We have just concluded that in order to induce the firm to choose the socially optimal levels of output and abatement in the presence of spillovers, the emissions tax should be set below marginal external damages in the first case and above marginal external damages in the second case. Clearly, this is not possible. The social planner can choose an emissions tax that induces either optimal output or abatement, but not both simultaneously. This impossibility is due to the different effects that spillovers have on the firm output and abatement decisions, which are described above. The next section derives two separate alternative policies, an output tax and an abatement subsidy. By implementing a separate policy for each decision, it is possible for spillovers to affect each decision differently and still induce the firm to choose the socially optimal levels of output and abatement simultaneously.

## IV. Deriving the Output Tax and Abatement Subsidy

As has been described above, the firm can reduce emissions in three ways: by reducing output, by increasing abatement, and by investing in innovation. The social planner's goal is to induce the firm to choose the socially optimal levels of output, abatement, and investment in the presence of both an emissions externality and a knowledge spillovers externality. The subsidy to investment derived above induces the firm to invest at the socially optimal level. However, spillovers also affect the firm output and abatement decisions, and they do so in different and opposite ways. Thus, correcting the emissions externality requires separate policies that are capable of inducing the socially optimal levels of output and abatement when spillovers are present. The policies considered here are a tax on output, $t_x$, and a subsidy to abatement per unit output, $s_a$. The firm is subject to these two policies in addition to the investment subsidy, and the single tax on emissions is discarded. The firm profit function is then:

$$\pi = n[p_1x_1 - c_xx_1 - t_xx_1 - (1 - z(H_1))c_a((A_1^2)/2)x_1 + s_aA_1 - (1 - s)R(h_1)]$$
$$+ \delta m[[p_2x_2 - c_xx_2 - t_xx_2 - (1 - z(H_2))c_a((A_2^2)/2)x_2 + s_aA_2] \quad (18)$$

Taking the derivatives of this equation with respect to $x$ and $A$ and setting them equal to zero yields the firm first order conditions related to the output and abatement decisions, respectively.

$$p_t = c_x + t_x + (1 - z(H_t))c_a(A_t^2)/2 \quad (19)$$

$$s_a = ((1 - z(H_t))c_aA_tx_t \quad (20)$$

Equation (19) shows that the firm will increase output until the marginal private benefit of production equals the marginal private cost of production. This marginal private cost now includes the tax per unit output. The greater the tax, the lower the equilibrium level of output. Equation (20) shows that the firm will choose the level of abatement at which the marginal private benefit of abatement, the subsidy, equals the marginal private cost of abatement. The greater the subsidy is, the greater is the equilibrium level of abatement.

To derive the socially optimal output tax and subsidy, we must equate these firm first order conditions with the corresponding social planner first order conditions. The social planner recognizes the emissions and spillovers externalities just as it did before, and thus the social welfare function is specified as in equation (11). The social planner first order conditions for the output and abatement decisions are then equations (12) and (13), respectively. Equating (19) and (12) yields the socially optimal tax on output:

$$t_x = d(M - A_t) - \sigma z(H_t))c_a(A_t^2)/2 \qquad (21)$$

When spillovers do not exist, the second term equals zero and the tax on output is set equal to the marginal external damages of output. This is as theory predicts. The marginal private cost of output, the tax, should equal the marginal social cost of output in order to induce the firm to produce at the socially optimal level. When spillovers exist, the tax should be set lower. This is as we expect, given the description of the effect of spillovers on the output decision stated above. Spillovers cause the firm to produce below the socially optimal level, and thus the tax on output should be set lower in their presence.

It is interesting to compare this result to the single emissions tax that induces optimal output. It is straightforward to show from (21) and (16) that:

$$t_x = t_{ex}(\mathrm{M} - \mathrm{A}) \qquad (22)$$

where $t_{ex}$ is the tax on emissions that induces optimal output. The tax per unit output is simply the tax per unit emissions times emissions per unit output. This equivalence provides a check on our results.

Equating (20) and (13) yields the socially optimal tax on abatement:

$$s_a = dx_t + \sigma z(H_t))c_a A_t x_t \qquad (23)$$

In the absence of spillovers, the second term again equals zero and the subsidy is set equal to the marginal external benefit of abatement. Here the marginal private benefit of abatement per unit output, the subsidy, equals the marginal social benefit of abatement, which is the marginal external damages of emissions avoided by increasing abatement per unit output. When spillovers are present, the subsidy should be set higher due to the effect of spillovers on the firm abatement decision, as described above. Spillovers cause the firm to abate less than is socially optimal, therefore the subsidy to abatement should be set higher in their presence.

Comparing this subsidy with the tax on emissions that induces optimal abatement, equation (17), also lends support to our results. Equating (23) and (17) reveals that:

$$s_a = t_{ea} x_t \qquad (24)$$

where $t_{ea}$ is the tax on emissions that induces optimal abatement. This equation shows that the subsidy to abatement per unit output must equal the tax per unit emissions times output. Interpretation of this result is less straightforward than with equation (22) but is as follows. Each side of the equation represents the gain from abatement. When the firm faces an abatement subsidy, increasing abatement per unit output by one unit results in an increase in the total subsidy received equal to $s_a$, by definition. When the firm faces a single emissions tax, increasing abatement per unit output by one unit reduces the total tax paid on emissions by $t_{ea} x_t$, as can be seen from (7). Thus, the subsidy to abatement provides the same incentive to the firm as the tax on emissions that is intended to induce optimal abatement.

The output tax and abatement subsidy thus induce the firm to choose the levels of output and abatement that maximize social welfare in the presence of both an emissions externality and a knowledge spillovers externality. They cause the firm to internalize the emissions externality while also correcting for the different effects of spillovers on the output and abatement decisions. Together with the investment subsidy derived above, these three policies solve both the emissions and spillovers externalities simultaneously.

## V. Conclusion

This paper has shown that in the presence of both emissions and knowledge spillovers externalities, a single tax on emissions is incapable of inducing firms to emit at the socially optimal level. Spillovers affect the firm output and abatement decisions in different and opposite ways, and thus a separate output tax and abatement subsidy are necessary to induce the firm to choose the output and abatement levels that result in the socially optimal emissions level. We have also demonstrated, though the result is not new, that a simple subsidy to investment can correct the spillovers externality by inducing the firm to invest in innovation at the socially optimal level. These three policies solve the emissions and spillovers externalities simultaneously, thus maximizing social welfare.

The importance of these results lies in their suggestions to policymakers. By not accounting for the existence of endogenous technological change and the spillovers associated with investment in innovation, policymakers miss an opportunity to increase social welfare by subsidizing investment in innovation that reduces the cost of abatement and thus results in greater equilibrium abatement levels. As innovation has been cited as one of the most important means of reducing emissions in the long run, this would have consequences not only for the current generation but for many subsequent generations as well.

We have also shown that in the presence of emissions and spillovers externalities, ignoring the firm output decision and only considering the firm abatement and investment decisions will result in an emissions tax that will reduce firm output below the social optimum. It might be interesting to examine the implications of this result, and the results of this paper in general, when prices are endogenous. This would allow the output decision to affect the price of output and thus consumer surplus.

A numerical analysis of the results of this paper could demonstrate by just how much spillovers affect the socially optimal taxes and subsidies derived here. It could also show how the equilibrium levels of abatement, output, investment, and emissions change under different assumptions regarding endogenous technological change and knowledge spillovers. Such a study would hopefully provide a more thorough understanding of the importance of endogenous technological and the interactions between the emissions and spillovers externalities.

The ability of an output tax and abatement subsidy to correct the emissions externality means that such policies might be a better means of inducing optimal emissions even in the absence of significant spillovers. Monitoring individual firm emissions might be more costly than monitoring each firm's level of output. An abatement subsidy would push the cost of monitoring abatement onto the firm, which would report its increases in abatement in order to receive the subsidy. Thus, this two-part policy might prove a more feasible means of correcting the emissions externality than an emissions tax.

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