

# The Size Effect and the Capital Asset Pricing Model

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## I. Introduction

The Capital Asset Pricing Model (CAPM) is one of the most widely used models in finance. Under assumptions of a perfect capital market, the CAPM predicts that all investors will hold a combination of the market portfolio of risky assets and a portfolio whose returns are uncorrelated with market returns. The central assertion of the CAPM is that the market portfolio is mean-variance efficient, implying that there exists a linear relationship between a portfolio's expected return and its market beta and that no other factors are necessary to explain expected returns. Given the model's prevalence, it has been one of the most empirically scrutinized models in finance, and several contradictions have been revealed, one of which is the marginal explanatory power of market equity (stock price times shares outstanding) on security returns – the size effect of Banz (1981). This paper tests the hypothesis that market equity significantly adds to explanations in the variability of portfolio returns over market betas.

## II. Theory of the CAPM

The CAPM builds upon the model of portfolio selection outlined by Markowitz (1952). Seeking to describe a law of investor behavior that would lead investors to prefer portfolio diversification, he proposed the mean-variance maxim: the investor considers the mean-variance frontier (efficient set) of available portfolios – portfolios that for a given level of risk maximize return or for a given level of return minimize risk – and chooses a portfolio from the efficient set that best matches their individual risk preferences.

Tobin (1958) builds upon Markowitz's work while developing the portfolio theory of money demand. Making the following assumptions on investor behavior: 1) All investors are risk-averse; 2) Investors select among stocks by utilizing a two-parameter subjective probability distribution and these two parameters are the only parameters relevant to the investor; 3) Investors can, based on values of these two parameters, rank portfolios according to a cardinal utility function; 4) Investor's portfolio decisions are made period by period; Tobin develops the *separation theorem*. The theorem states that each investor holds a combination of the risk-free asset and a single, investor-specific portfolio of riskier, interest-bearing assets, labeled the tangent portfolio  $T$ .

The CAPM, developed by Sharpe (1964) and Lintner (1965), follows easily from the separation theorem. Since Tobin only considered cash and monetary assets, he left implicit an assumption that needs to be made explicit for the CAPM: the existence of unrestricted borrowing and lending at a rate that is the same as the rate of return on a risk-free asset and is independent of both the investor and the amount borrowed or lent. Additionally, we need a few more assumptions about capital market structure and investor behavior: there are no taxes or transaction costs in financial markets, financial claims on underlying assets have perfect divisibility, and investors have homogenous and correct expectations of the probability distributions governing the future value of any portfolio. Our final assumption ensures that all investors see the same opportunities. Thus, all investors combine portfolio  $T$  with the risk-free asset (in combinations dependent upon their individual risk preferences). Since all investors hold the portfolio  $T$ , it must be the market portfolio (each risky asset's weight in  $T$  must be its total market value divided by the total market value of all risky assets), henceforth denoted by an  $M$ . Thus, we arrive at the central prediction of the CAPM: that the market portfolio is efficient. The Sharpe-Lintner CAPM equation follows from the mathematical conditions of an efficient portfolio:

$$E(R_i) = R_f + [E(R_M) - R_f]\beta_{iM}, \quad i = 1, \dots, N \quad (1)$$

where  $R_i$  is the rate of return on stock  $i$ ,  $R_f$  is the risk-free rate,  $R_M$  is the rate of return on the market portfolio and

$$\beta_{iM} = \frac{\text{Cov}(R_i, R_M)}{\text{Var}(R_M)}$$

The  $\beta$  term is labeled the “market beta of asset  $i$ ,” and is interpreted as the “sensitivity of the asset's return to variation in the market return,” [Fama & French 2004].

The final theoretical consideration is the version of the CAPM proposed in Black (1972). Motivated by the empirical work he conducted with Jensen and Scholes (1972), Black suggested an alternate formulation of the CAPM that suppressed the existence of a riskless asset, but allowed for an investor to “take long or short positions of any size in any risky asset.” By Black’s own admission, “this assumption is not realistic,” but the substitution of one unrealistic assumption for another allowed Black to reformulate the CAPM in a manner that better matched early empirical evidence. In Black’s derivation,  $E(R_{ZM})$  is substituted for  $R_f$  in equation (1), where  $R_{ZM}$  is the rate of return on a portfolio whose returns are uncorrelated with that of the market portfolio (equivalently, the portfolio’s market beta is zero-valued), and  $R_f < E(R_{ZM}) < E(R_M)$ .

As a conclusion to the above theoretical discussion, it should be noted that both the Black and Sharpe-Lintner CAPM are predicated upon highly unrealistic assumptions of investor behavior and market structure. However, as Sharpe writes, “the proper test of a theory is not the realism of its assumptions but the acceptability of its implications,” [Sharpe 1964], and testing the implications of the CAPM is at the heart of all empirical work conducted on the model.

### III. Empirical Testing of the CAPM

Ideal empirical tests of the CAPM would test whether the market portfolio is mean-variance efficient. Unfortunately, the market portfolio “is theoretically and empirically elusive,” [Fama & French 2004]. The literature, therefore, tests different hypotheses. Fama and Macbeth (1973) identify three testable implications: 1) the expected return of a security is linearly related to its risk in any efficient portfolio; 2) No measure of risk other than market beta is necessary to explain variations in expected returns of a portfolio or security; and 3) Higher risk is associated with higher returns:  $R_M - R_{ZM} > 0$ . In tests specifically of the Sharpe-Lintner model, a fourth testable implication is that the intercept in the regression of security returns excess of the returns of a risk-free asset on the security’s market beta is zero. The size effect, which posits that market equity has significant marginal explanatory power on security returns, is a test of the second hypothesis. The Fama-Macbeth hypotheses were criticized extensively by Roll (1977) for what has come to be known as the market-proxy problem. We will revisit these criticisms after examining relevant empirical tests of the CAPM.

The standard regression technique in empirically testing the CAPM is to “regress a cross-section of average asset returns on estimates of asset betas,” [Fama & French (2004)], and test our hypotheses on the coefficients of the regression. To do this, however we need to rewrite equation (1) in terms of actualized security and portfolio returns instead of expected returns. The earliest empirical work focused on testing the Sharpe-Lintner version of the CAPM. Researchers utilized the regression equation of Jensen (1968) in their tests:

$$R_{it} - R_{ft} = \alpha_j + \beta_j[R_{Mt} - R_{ft}] \quad (2)$$

where  $R_{ft}$  measures the actualized rate of return of a risk-free asset during period  $t$ , and  $R_{Mt}$  measures actualized market returns during period  $t$ . Hypothesis tests would then be performed to determine whether  $\alpha_j$  was significantly different from zero. To test hypotheses (1) - (3) from above, Fama & Macbeth (1973) formulate a generalized asset pricing model of period-by period returns of the form:

$$R_{it} = \gamma_{0t} + \gamma_{1t}\beta_{it} + \gamma_{2t}\nu_{it} + \cdots + \gamma_{jt}\zeta_{it} + \varepsilon_{it} \quad (3)$$

where  $R_{it}$  is the one-period percentage return of security  $i$ ,  $\nu_{it} \dots \zeta_{it}$  measure additional risk factors tested in the model, and the regression coefficients are permitted to vary stochastically. The aforementioned hypotheses can be tested based on the values of the regression coefficients. For example, testing that higher risk is associated with higher returns amounts to testing whether the coefficient  $\gamma_{1t}$  is significantly positive.

Two additional methodological problems arise when conducting regressions using equations (2) and (3). The first is the error-in-variables bias that occurs because the true values of market betas used in the regression are unknown, and using estimates of market betas instead can result in substantial

measurement error. Since “estimates of beta for diversified portfolios are more precise than for individual securities,” [Fama & French (2004)], and, if the CAPM holds, it must also apply to portfolio returns, researchers suggested performing the cross-sectional regressions utilizing portfolios instead of individual assets. Unfortunately, using grouped data reduces the range of the betas and, therefore, statistical power. The key insight of Black, Jensen & Scholes (1972) was to construct ranked portfolios based on beta values to preserve the range of the betas. However, if the same estimates of beta are used to rank our portfolios as are used in the regression, then securities selected for the high-beta portfolio would tend to have positive measurement error and securities selected for low-beta portfolios would tend to have negative measurement error, biasing our estimates of the regression coefficients. To overcome this, BJS (1972) suggest the “use of an instrumental variable that is highly correlated with” the betas in the regression, but “can be observed independently,” of these betas. The usual procedure is to use past period’s data (normally the last 2 – 5 years) of an individual security to estimate the security’s beta for the current period (called pre-ranking betas). Pre-ranking beta values are then used to place securities into beta-ranked portfolios. The next 2 – 5 years of data are used to estimate post-ranking beta values for individual stocks. The portfolio-weighted average of post-ranking betas is assigned as the beta value of the portfolio. Cross-sectional regressions are then evaluated on these portfolios for a time period after the period used to estimate betas. The procedure of forming pre-ranking beta portfolios is normally performed yearly. Fama & French (2004) identify this grouping methodology as becoming standard in the literature.

The second problem arises because “correlation of the residuals in cross-section regressions,” [Fama & French (2004)] causes underestimates of the standard errors of coefficients when using OLS, leading to inference problems. A solution to the problem was proposed in Fama & Macbeth (1973). They perform month-by-month cross-sectional regressions of monthly returns on the betas and use the resultant “time-series of month-by-month values of the regression coefficients ... as inputs,” in tests of the hypotheses. More explicitly, equation (3) is estimated for every month of returns, and test statistics are calculated using the resulting time-series of regression coefficients. For the coefficient  $\gamma_{jt}$ , the test statistic is:

$$\omega(\hat{\gamma}_j) = \frac{\hat{\gamma}_j}{\hat{\sigma}_{\gamma_j}}, \quad \text{where } \hat{\gamma}_j = \frac{1}{T} \sum_{t=1}^T \hat{\gamma}_{jt}, \quad \text{and } \hat{\sigma}_{\gamma_j} = \frac{1}{T(T-1)} \sum_{t=1}^T (\hat{\gamma}_{jt} - \hat{\gamma}_j)^2 \quad (4)$$

and where the test statistic is distributed according to a Student’s  $t$ -distribution with  $T - 1$  degrees of freedom and  $T$  denotes the number of periods considered.<sup>1</sup> Note that the monthly estimations of equation (3) can be computed utilizing either a cross-section of portfolio returns or a cross-section of individual stock returns. The choice to use one or the other depends upon whether or not it is possible to utilize firm-specific data for the additional risk factors in equation (3). Fama & French (2004) identify the methodology of conducting tests on a time series of cross-sectional regression coefficients, known as the Fama-Macbeth (FM) regressions, as becoming standard in the literature.

Two important early empirical studies of the CAPM are BJS (1972) and FM (1973). The main focus of BJS (1972) is testing the Sharpe-Lintner version of the CAPM. The authors utilize regression equation (2) to produce a time-series of regression coefficients, and test whether the time-series average of the intercept is significantly different from zero and whether the time-series average of the coefficient on beta differs from time-series average of  $R_{Mt} - R_{ft}$  (the theoretical prediction) utilizing the t-tests outlined in (4). To accurately estimate market betas, the authors use the grouping procedure explained above with both pre-ranking and post-ranking betas computed from five years worth of data. BJS use all stocks listed on the New York Stock Exchange (NYSE) from January 1926 to March 1966 to form their asset portfolios, use an equal-weighted portfolio of NYSE stocks that is updated monthly as their proxy for the market portfolio, and use the 30-day rate on U.S. Treasury bills as a proxy for the risk-free rate of return. They find strong evidence to reject the Sharpe-Lintner model – the time-series average of the intercept was significantly different from zero with a test statistic of 6.52 and the time series intercept of the coefficient on market

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<sup>1</sup>The formulas above are found in Campbell, Lo, & MacKinlay (1997).

betas was significantly different from the time-series average value of  $R_{Mt} - R_{ft}$  with a test statistic of 6.53. These empirical results lead Black to formulate his version of the CAPM later that year.

In FM (1973), the authors test that the relationship between a portfolio's return and its corresponding market beta is linear and positive, and that no other risk factors have explanatory power on the portfolio's returns. To test linearity they include  $\beta^2$  as an explanatory variable in equation (3); they include as their measure of an additional risk factor the "standard deviation of the least-squares residuals from the so-called market model."<sup>2</sup> They regress equation (3) upon portfolios for each month in their period, and conduct tests upon the time-series test statistics for the regression coefficients as outlined in (4). To test the robustness of their inferences on regression coefficients to the inclusion of other independent variables, they perform the monthly cross-sectional regression using just beta as an explanatory variable, both beta and beta-squared as explanatory variables, and beta, beta-squared, and their additional risk factor as independent variables. Fama & Macbeth use the grouping procedure outlined above with both pre-ranking and post-ranking betas estimated from five-years worth of data. The authors use all common stocks traded on the NYSE from January 1926 through June 1968 to form asset portfolios, and use Fisher's Arithmetic Index (returns of an equal-weight portfolio updated monthly of all stocks on the NYSE) as a proxy for market returns. They find that over 1935 - 1968, the coefficients on beta-squared and the additional risk factor are insignificant at the five-percent level of confidence (test statistics of -.86 and 1.11 respectively) and there is a positive relation between beta and portfolio return that is robust to the inclusion of the other two independent variables. From these results, Fama & Macbeth conclude they cannot reject the CAPM.

Following this early research, empirical tests of the CAPM focused on testing that no other risk factor has explanatory power on security returns. Banz (1981) was the first to suggest market equity explains significant variation in portfolio returns above market betas, implying a misspecification of the CAPM. He utilized equation (3) for his regression:

$$R_{it} = \gamma_{0t} + \gamma_{1t}\beta_{it} + \gamma_{2t} \left[ \frac{(\phi_{it} - \phi_{mt})}{\phi_{mt}} \right] + \varepsilon_{it} \quad (5)$$

where  $\phi_{it}$  is the average market value of security  $i$  in period  $t$  and  $\phi_{mt}$  is the average market value in period  $t$ . The fraction  $(\phi_{it} - \phi_{mt})/\phi_{mt}$  that is included as an independent variable measures the fraction of total market value attributable to a single security or portfolio, and is thus a measure of size. In order to control for potential correlation between beta and market equity, Banz altered the grouping procedure of BJS (1972) slightly so that securities were first placed into five size-ranked portfolios, and these portfolios were then subdivided into five beta-ranked portfolios utilizing pre-ranking beta values of the stocks. Thus, inferences drawn from the time-series of regression coefficients for the cross-sectional regressions estimated using these size-beta portfolios will be able to distinguish between variability in portfolio returns due to size and variability in portfolio returns attributable to beta. Banz utilized all common stocks listed on the NYSE for at least five years between 1926 and 1975 to form asset portfolios, and uses three different proxies for market returns: an equal-weighted NYSE portfolio, a value-weighted NYSE portfolio, and a value-weighted combination of government and corporate bonds and the value-weighted NYSE portfolio. He found that during 1936 - 1975, there was a significant negative coefficient on his measure of size that was robust for all three proxies of the market (test statistic of -2.92 for the most general market proxy). He concludes that "the market value of the equity of a firm," was "an additional factor relevant for asset pricing," [Banz (1981)]. Specifically, "small NYSE firms have had significantly larger risk-adjusted returns than large NYSE firms over a forty-year period," a phenomenon that has come to be known as the size effect. However, Banz concludes that since there is "no theoretical foundation for such an effect," he could not discern whether market equity was itself the relevant factor in explaining expected returns or whether market equity was "just a proxy for one or more true but unknown factors correlated with size."

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<sup>2</sup>The author of this paper is unfamiliar with the market model and does not understand the motivation behind Fama & Macbeth choosing to utilize this variable as an additional risk factor.

The work of Fama & French (1992) strengthens the arguments advanced by Banz (1981). The authors utilize a similar regression equation as Banz (1981), except they substitute  $\ln(\phi_{it})$  for the fraction in equation (5). They also utilize the FM regression techniques, but group portfolios differently than BJS (1972). Since market equity is measured at the level of the individual stock, they saw “no reason to smear the information in these variables by using portfolios in the FM regressions,” [Fama & French (1992)]. Instead, after forming ten portfolios based on size and then subdividing each size portfolio into ten portfolios based on pre-ranking beta-values, they compute twelve-month post-ranking returns for each size-beta portfolio. This sorting process is conducted in June of every year. Thus, they have a time-series of monthly returns for each size-beta portfolio.<sup>3</sup> They estimate the portfolio beta by regressing the entire time-series of monthly returns against the time-series of market returns and assign the portfolio beta to the individual stocks in the portfolio. Consequently, they conduct the monthly cross-sectional regressions upon a cross-section of individual stock returns. To test the robustness of their inferences on regression coefficients to the inclusion of other independent variables, the authors perform the monthly cross-sectional regressions with only the cross-section of betas as an independent variable, only the cross-section of the natural log of market equity as an independent variable, the cross-section of both beta and size as independent variables, and the cross-section of beta, size, and other ratios (book-to-market, earnings-to-price, etc.) as independent variables. The authors include common stocks for all non-financial firms listed on all of the NYSE, AMEX and NASDAQ exchanges from 1962-1990 in their analysis, and use a value-weighted portfolio of all NYSE, AMEX, and NASDAQ stocks as a proxy for the market.

Fama & French find a significant negative relationship between a security’s returns and the natural log of its market equity that “persists no matter what other explanatory variables are in the regressions,” and conclude that the size effect is robust. This strong result eliminates the possibility that market equity is a proxy for another variable included in their regressions. The authors also find that in the presence of size, the theorized significant positive relationship between market betas and average returns disappears – the coefficient on a security’s beta becomes negative and insignificant. The strength of their results is identified by Fama & French (2004) as “marking the turning point when it is generally acknowledged that the CAPM has potentially fatal problems.”

To conclude our discussion of empirical literature on the CAPM, we must address the market proxy problem posed by Roll (1977). The central assertion of Roll’s paper is that inferences on the CAPM could be highly sensitive to the market proxy used, and this could invalidate all existing empirical work on the CAPM. Roll criticized the Fama-Macbeth hypotheses extensively, arguing that “there is only a single testable hypothesis associated with [the CAPM]: the market portfolio is mean-variance efficient,” and that all other implications “follow from the market portfolio’s efficiency and are not independently testable.” Using the mathematics of efficient sets, Roll shows that the testable implications identified by Fama & Macbeth are definitionally equivalent to stating that the market portfolio is efficient. Thus, if the market portfolio is “known to be efficient, these relations are ... tautological.” Roll identifies another assumption of the CAPM that is often left unstated: the market portfolio is identifiable. As Roll writes, this assumption is crucial because,

There will always be some portfolio which is ex-post efficient and will bring about exact observed linearity among ex-post sample mean returns and ex-post sample betas. If we do not know the composition of the market portfolio, we might by chance select a portfolio that is close to mean-variance efficient. In fact, it may be hard to find a highly-diversified portfolio that is sufficiently far inside the ex-post efficient frontier to permit the detection of statistically significant departures from mean return/beta exact linearity.

Consequently, when testing the CAPM, the researcher is always testing the joint hypothesis that the CAPM is correct *and* that the portfolio used as the market proxy was the true market portfolio. Unfortunately, “the trouble with joint hypotheses [is that] one never knows what to conclude.” For example, if evidence suggests rejection of the joint hypothesis, one’s ability to reject the CAPM depends upon how close the

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<sup>3</sup>The composition of these portfolios change on a yearly basis.

market proxy is to the true market portfolio. A further cause for consternation is that since the market portfolio is unidentifiable,<sup>4</sup> Roll is pessimistic that the CAPM can ever be tested empirically.

In response to Roll’s criticisms, Stambaugh (1982) conducted a sensitivity analysis of inferences drawn on the CAPM to different proxies of the market portfolio. Stambaugh constructed market proxies including corporate and government bonds, real estate, and consumer durables in addition to common stocks. The weight of common stocks in these different market proxies ranged from 15% to 34%. Stambaugh then tests the hypotheses that there is an exact positive, linear relationship between portfolio returns and market betas and that the intercept of this relation is the rate of return on the risk-free asset. To avoid assuming that estimates for market beta are equal to actual beta values, Stambaugh tests the CAPM using the likelihood ratio test and the Lagrangian multiplier test.<sup>5</sup> Stambaugh finds that for all market portfolios, the tests he conduct do not reject a positive, linear relationship between portfolio returns and market betas but do reject that the intercept is the rate of return on riskless assets. Thus, Stambaugh shows existing empirical results are robust to the market proxy used. While this sensitivity analysis does not preclude the possibility that other market proxies can result in contradictory inferences, it does “indicate that such an occurrence is less likely than Roll’s (1977) arguments suggest.” More specific to the size effect, Banz (1981) uses three different proxies for the market portfolio, and finds that his results are robust to the market proxy used. Furthermore, as cited in Fama & French (2004), existing inferences on the CAPM are robust to the inclusion of international financial assets in the market proxy. In light of this work, Roll’s criticisms have become less salient concerns in empirical testing of the CAPM.<sup>6</sup>

#### IV. Conceptual Model & Ideal Data

Our conceptual model centers around the Fama and Macbeth regressions. For each time period in our sample, we estimate the regression equation:

$$R_{it} = \gamma_{0t} + \gamma_{1t}\beta_{it} + \gamma_{2t} \ln(\phi_{it}) + \varepsilon_{it} \quad (6)$$

for a cross-section of individual securities, where the variables are defined as above. We conduct tests upon the time-series of regression coefficients using standard time-series test statistics outlined in (4). Specifically, we determine the values of  $\omega(\hat{\gamma}_1)$  and  $\omega(\hat{\gamma}_2)$ , the time-series test statistics of the coefficients on beta and size respectively, and test:

1. Is  $\omega(\hat{\gamma}_2)$  significantly negative?<sup>7</sup>
2. Is  $\omega(\hat{\gamma}_1)$  significantly positive?

The first implication tests whether market equity has significant marginal explanatory power on security returns, while the second implication tests whether there exists a positive relationship between beta and portfolio returns.

In an ideal world, to conduct tests on the CAPM we would be able to observe the true market portfolio and have perfect measures of expected returns, realized returns, and market betas. As described above, a failure to observe the true market portfolio results in empirical tests of the CAPM testing a joint hypothesis, making it difficult to draw unambiguous conclusions from the tests. Furthermore, the actual statement of the CAPM, equation (1), is written in terms of expected returns. An inability to measure investors expectations results in errors-in-variables bias given that we are forced to proxy expected returns with realized returns. Additionally, our measures of realized returns are themselves subject to contention. For

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<sup>4</sup>The market portfolio would need to include every risky asset available to investors.

<sup>5</sup>Methodologies entirely opaque to the author of this paper.

<sup>6</sup>Owing to time and data constraints facing the author, this paper makes no attempt to test the robustness of our results to the market proxy used. Instead, we defer to the existing literature’s suggestion that inferences drawn on the CAPM are robust to the market proxy utilized.

<sup>7</sup>We utilize a one-sided test here because the literature suggests that market equity is negatively related to portfolio return.

example, it is unclear whether to include dividend payments in stock returns, or whether to use percentage change in price or the difference in logged prices to measure realized returns. Another pressing concern is the inability to observe market betas. As described in detail above, the impossibility of measuring market betas (indeed, of truly quantifying the risk of a security) results in severe errors-in-variable bias. While methodologies have been developed to mitigate this bias, any result of tests of the CAPM are still subject to the measurement error arising from using estimates for betas.

As a final consideration, we would ideally be able to match market capitalization data from a given point in time to the point in time when it is reflected in security returns – if investors are unaware of the data, it cannot factor into explaining expected returns. However, time it takes for market capitalization information to be reflected in prices is difficult to measure.<sup>8</sup> Additionally, since returns are calculated over a period of time, and market capitalization fluctuates through time, it is unclear whether to measure size using average market capitalization from a past period or snapshots of market capitalization from a specific point in time.

## V. Data & Methodology

Due to constraints on time and data availability, the author of this paper cannot conduct tests on all NYSE stocks over a forty-year time period as found in the literature. Instead, the author considers returns on twenty stocks from 1996 to the present: the ten largest firms by market capitalization listed on the S&P 500 index in 2008 and ten firms selected from the Russell 2000 small market capitalization index. An attempt was made to select the stocks from the Russell 2000 based on the firms in which the index had the largest holding as of 2008. However, five of the ten firms in which the index invests the most heavily do not have historical stock price data extending until 1996.<sup>9</sup> Consequently, other firms were selected from the index to complete the list of ten small market capitalization firms necessary for testing.<sup>10</sup> Note that the manner in which we selected stocks, where we explicitly searched for stocks that have existed since 1996, introduces survivor bias – we will be unable to distinguish whether the results we derive reflect a property of successful firms that have survived for lengthy periods of time rather than the size effect itself.

For the twenty firms in question, monthly stock close prices, adjusted for dividends and stock splits, were taken from Yahoo! Finance. Market capitalization data was recorded from the twenty firms' 10-K filings, available at the EDGAR database maintained by the SEC. Market equity was calculated by taking the average shares outstanding<sup>11</sup> reported in the annual report for year  $t$ , assigning it to the twelve-month period following the filing of the annual report by the firm,<sup>12</sup> and multiplying this value by adjusted close price for each month in this twelve-month period. Monthly returns were computed by taking the difference in adjusted close price at the end of each month. We utilized the S&P 500 index stock (a value-weighted portfolio of S&P 500 stocks) as our market proxy, and computed market returns as differences in monthly close price for this index stock. At this point, it must be noted that we are utilizing a very narrowly defined market proxy, subjecting our results to a rather severe case of the joint hypothesis problem outlined in Roll (1977).

Methodologically, our tests identically follow those of Fama & French (1992), with minor changes made to account for the smaller number of stocks used in our analysis. Arbitrarily, we choose to select portfolios on April 1 of each year. We utilize 24 months of prior return data to determine pre-ranking betas for each stock on April 1 for each year. Given that we only have twenty stocks, we form only two size portfolios based on market capitalization values for the end of March. Unsurprisingly, at every portfolio formation date, the ten firms selected from the S&P 500 index form the large-size portfolio, while the ten stocks

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<sup>8</sup>Though, perhaps the event studies literature would have a few suggestions.

<sup>9</sup>They were founded after 1996, for example.

<sup>10</sup>For a complete list of the twenty stocks utilized in the analysis, please refer to the Appendix.

<sup>11</sup>Used for lack of other available data.

<sup>12</sup>This is done to ensure that the average number of shares outstanding is information that is available to the investor during the time period under consideration. Note that like Fama & French (1992) we assume no lag time between the filing of the annual report and investor awareness of its contents.



selected from the Russell 2000 index form the small-size portfolio. To be able to distinguish between size effects and beta effects in our regression, we utilize the pre-ranking betas for the individual stocks to divide each size portfolio into two size-beta portfolios, and calculate the equal-weighted returns of each size-beta portfolio for the twelve months after formation. We repeat for each year in our sample. The result is a time-series of monthly returns on four size-beta portfolios from April 1998 – December 2009. We estimate the beta of the portfolio by regressing the entire time-series of monthly size-beta portfolio returns against the contemporaneous market returns and one-period lagged market returns and summing the regression coefficients on the two independent variables. This methodology is suggested by Fama & French (1992) to account for non-synchronous trading effects.<sup>13</sup> The portfolio beta values are then assigned to the individual stocks in each portfolio to be used in the FM regressions.

We conduct the FM regressions upon a cross-section of individual stocks using equation (6) as written, and versions of equation (6) having only beta and only size as independent variables.<sup>14</sup> Note that because we only have four size-beta portfolios, there are only four different values for the cross-section of security betas in each monthly regression. Such a small number of distinct observations in our cross-sectional regressions will produce high standard errors for coefficient estimates. Finally, we conduct standard time-series statistical tests using equation (4) upon the time series of regression coefficients arising from our three different cross-sectional regressions.

## VI. Results

The time-series average (and the corresponding test statistic) of the regression coefficients for our three specifications of the FM cross-sectional regressions are found in Table 1.

**Table 1:**  
1998 – 2009 Time-Series Averages of Regression Coefficients  
from the FM Regressions

Regression Specification	$\hat{\gamma}_1$	$\hat{\gamma}_2$
$R_{it} \sim \beta_{it}$	-1.764 (-0.874)	...
$R_{it} \sim \ln(\phi_{it})$	...	-0.0123 (-0.606)
$R_{it} \sim \beta_{it}, \ln(\phi_{it})$	-2.606 (-1.012)	0.0112 (0.471)

Test-statistics calculated with 139 degrees of freedom,  
corresponding to 140 monthly returns

The most striking result of Table 1 is the negative coefficient on market betas that exists both when market betas are the sole explanatory variable and when size is included in the FM regressions. This suggests that for our basket of securities and market proxy, increased risk does not lead to increased returns even when risk is taken to be the sole explanation of variations in security returns. If our market proxy is taken to be a good approximation of the true market portfolio, this is an indictment of the CAPM. The second item worth noting is our inability to reproduce the size effect – the coefficient on the natural log of market equity is negative when market equity is the sole independent variable but positive when both size and beta are independent variables. This change in the sign of the coefficient on size is particularly noteworthy because it suggests that there is not a consistent relationship between size and security return. Consequently, our data does not suggest a relationship between size and security returns that is robust to the inclusion of security betas.

Our tests of the CAPM, however, are also concerned with the significance of the coefficients. For example, to conclude that size has marginal explanatory power in addition to market betas for security returns, we would need a significant, robust relation between size and portfolio returns. However, none

<sup>13</sup>The author of this paper understands neither the effects of non-synchronous trading upon security returns nor how the aforementioned regression technique can correct for them.

<sup>14</sup>The natural log of market equity is used in these regressions because of the very large size of market equity values. We wish to be able to distinguish more between orders of magnitude of market equity (differences between firms with large and small market capitalizations) than between market equity of the same order of magnitude (corresponding to variations within a market capitalization class). Using the natural log of market equity allows us to make such distinctions.

of the time-series averages of coefficients in any of our cross-sectional regressions are significant (none of their test statistics, calculated with 139 degrees of freedom, are greater than two in magnitude). The low significance is due to the high standard errors on coefficients in the individual cross-sectional regressions owing to small sample size. As previously mentioned, there are only four distinct values in the cross-section of security betas at each point in time, and only twenty stocks in the cross-section of returns. Such a small sample size leads to high standard error in coefficient estimation, reflected in the high time-series standard errors.<sup>15</sup>

Given the lack of significance on all the time-series averages of coefficients in our regressions, we have neither sufficient evidence to reject the CAPM nor sufficient evidence to suggest that market capitalization is a significant factor in explaining security returns. While we did not find a significant, positive relationship between market betas and security returns as predicted by the CAPM, we must be cautious in rejecting the CAPM outright given that we suffer from the joint-hypothesis problem and, potentially, selection bias. In our joint hypothesis, it is clear that we have found evidence against the S&P 500 index being the true market portfolio,<sup>16</sup> but it is unclear whether our market proxy is a sufficiently close approximation of the true market portfolio to warrant making inferences on the CAPM. Finally, we found no evidence to suggest that there was a consistent or significant relationship between size and market equity.

## VII. Concluding Remarks

This paper tested whether size, as measured by the natural log of market capitalization, significantly added to explanations of the variation of the cross-section of stock returns above what was explained by market betas. To test this, we utilized the Fama & Macbeth cross-sectional approach to testing the CAPM – we conducted month-by-month cross-sectional regressions upon individual security returns and performed statistical tests upon the resultant time-series of regression coefficients. In contrast to the predictions of the CAPM, we did not find evidence of a positive relationship between market beta and security return in the returns of twenty stocks over the period 1998 – 2009. Nor did we find evidence for the size effect in our data. While we found some evidence against the CAPM, we cannot unambiguously reject the CAPM because of uncertainty arising from the joint-hypothesis problem and the low statistical power of our tests.

The major limitation of our research is that no attempt was made to discern how accurately the market proxy used approximated the true market portfolio.<sup>17</sup> Given that such efforts were not made, the author does not know how severely the paper’s results suffer from the joint hypothesis problem, which limits the strength of the conclusions one can draw from the results. Moreover, the lack of statistical power in our tests, arising from considering too few stocks in the cross-section of security returns, further limits our ability to draw strong conclusions from our results. For both these reasons, it is perhaps best to view the results in this paper as preliminary research, to be augmented and refined in the future.

It would be presumptuous of the author, with his limited knowledge of finance, to posit directions of future research in empirical testing of the CAPM, one of the most developed research areas of empirical finance. Moreover, a degree of consensus appears to have been reached that the CAPM does not withstand empirical scrutiny when explaining the cross-section of risky asset returns, and current research focuses more on the testing of multi-factor asset pricing models than the CAPM.

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<sup>15</sup>In contrast, Fama & French examine the cross-section of returns for over 2000 stocks, and have over one-hundred distinct values in the cross-section of security betas, resulting in much lower standard errors in their estimation of regression coefficients.

<sup>16</sup>Otherwise, it follows definitionally that the perfect linear relationship between security returns and market betas would have existed.

<sup>17</sup>The actual test would be of the sensitivity of inferences to the market proxy used.

# Appendix

The ten large market capitalization firms used in the analysis are: Microsoft, Exxon Mobil, General Electric, AT&T, Citigroup, Bank of America, Proctor & Gamble, Cisco Systems, Chevron Corporation, and Johnson & Johnson. The ten small market capitalization firms used in the analysis are: Abaxis Inc., Lance Inc., 3Com, Piedmont Natural Gas, Jack Henry & Associates, Tetra Tech, Polycom, Celadon Group, HB Fuller, and Playboy Enterprises.

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