Coke or Pepsi? An Inflationary Debate:

Accounting for Product Variety in the New Keynesian Phillips Curve

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Abstract

When product variety impacts welfare, changes in the number of firms can influence the price level. By using the estimation method found in Campbell and Shiller (1987), and a modification of the model in Sbordone (2002), I test whether or not series on firm entrance imply that product variety helps to determine the path of the price level. The method is general enough so that no assumptions must be made about firm entry conditions, and simply takes the data on firm entry as given. The model predicts the evolution of the price level extremely well and provides evidence supporting recent theories on how product variety impacts the Phillips Curve.

1 Introduction

This paper seeks to estimate a variant of the New Keynesian Phillips Curve (NKPC), modeling capital formation as the development of new product lines. Assuming that additional product variety impacts household welfare, the dynamics of firm entry and exit can influence the price setting of firms. When households are willing to pay more for variety, there is a wedge between the money price of individual varieties and the price level. Since it is this real price which matters in the price setting decisions of firms, changes in variety competition influence firm pricing decisions, and therefore firm entry and exit impacts the price level.

Several recent papers explore the empirical and theoretical plausibility of this variety effect on prices. Bilbiie, Ghironi, and Melitz (2007) builds a dynamic stochastic general equilibrium model, modeling both firm entry and Phillips Curve. The Phillips Curve specification used in this paper is a slight modification of the theory in their paper. Their study finds interesting results for regulatory, and monetary theory and policy assuming that firm dynamics are important. A empirical confirmation of the theory would lend support to the relevance of their conclusions.

Modifying the theory of the NKPC to account for factors beyond sticky prices (and sticky information) amounts to finding explanations for so called “cost push shocks” – often used to help the fit of dynamic stochastic general equilibrium models. Models such as the much discussed Smets Wouters model of the European Union (Smets and Wouters (2003)) have been criticized for relying too much on extraneous shock variables (such as in Chari, Kehoe and McGrattan(2008)) – you can make anything fit the data if you assume enough dimensions of error. Recent work seeks to find refinement in the theory and thus address these criticisms. Instead of adding new shocks to medium scale macro models we can instead find new theories of cost push shocks and add new data series to the estimation.

Empirical studies like Lewis (2006) have used vector auto-regression to reveal that firm entry is linked to both nominal and real aggregate variables. Lewis constructs a theoretical dynamic stochastic general equilibrium model including firm dynamics from which she finds sign restrictions to identify the VAR. The result is strong evidence that firm dynamics matter.

In this paper, I construct a simple model of sticky pricing and firm dynamics in the same flavor as the previously discussed literature. The key mechanism to connect nominal firm prices to the price level is the concept of ‘love of variety.’ If product variety did not impact welfare, changes in the number of products available would not impact pricing – households must be willing to pay more for greater variety. For discussion of this idea in the context for CES preferences see Dixit and Stiglitz (1975) or Benassy (1996). In the model of this paper, the degree of LOV determines the degree to which the pricing decisions of firms respond to the entrance and exit of competitors.

Combining this relationship between firm prices and the amount of product variety with the NKPC from the sticky price portion of the model, I get the central behavioral equation to be explored. This resulting augmented NKPC has terms which depend on the measure of firms – and for parameter values where there are no welfare effects from firm entry and exit, these terms drop out and the equation reduces to the benchmark NKPC, as in Woodford (2003). The goal is then to estimate the parameter determining the degree of LOV, or equivalently the degree to which firm entry and exit impacts pricing.

The estimation strategy follows the method used by Sbordone (2002) and originates from the procedure used in papers by Campbell and Shiller (1987, 1988) to test present-value models of stock prices. The method minimizes the variance of the difference between the model's predicted path for the price level and that observed in the data, using vector auto-regression to estimate expectations at each point in time.

Section 2 presents a simple model of firm pricing when firm entry and exit effect product demand and firms are subject to price adjustment costs. Section 3 discusses the data set. Section 4 presents the estimation strategy used towards determining if firm entry and exit help to predict the price level process. Section 5 analyzes results from estimation of the benchmark model. Section 6 explores the robustness of the results to an alternative assumption over the relevant price level. Finally, section 7 concludes.

2 Model

2.1 Firm Price Setting

Consider the pricing decision of a some firm facing an exogenous chance of death between periods, and price stickiness a la Rotemberg (1982). Let the set of firms producing at time t be $\mathcal{N}_t$. Then some firm $\eta \in \mathcal{N}_t$ seeks to maximize the valuation of the stream of its dividends based on a sequence of stochastic discount factors $\{\mathcal{Q}_{t,\tau}\}_{\tau=t}^{\infty}$ and discounted for the chance of exit:

$$v_t(\eta) = \mathbb{E}_t \sum_{\tau=t}^{\infty} (1-\delta)^{\tau-t} \mathcal{Q}_{t,\tau} d_\tau(\eta) \tag{1}$$

where $d_t(\eta)$ are real dividends of the firm at time t, and δ is the probability that the firm exits in each period.

The firm faces a demand curve for its product stemming from the CES output aggregate over product varieties $\eta \in \mathcal{N}_t$:

$$y_t = \eta_t^{\xi - \frac{1}{\vartheta - 1}} \left(\int_{\mathcal{N}_t} y_t(\eta)^{\frac{\vartheta - 1}{\vartheta}} d\eta \right)^{\frac{\vartheta}{\vartheta - 1}}$$

where η_t is the measure of firms at time t, $y_t(\eta)$ is the level of output of firm η, ϑ is the elasticity of substitution of product varieties, and $\xi \geq 0$ parameterizes the degree of 'love of variety'. This aggregator is a simple modification of the standard CES aggregator but where ξ detangles LOV from the elasticity of substitution ϑ. The price level of output comes from minimizing the cost of a unit of the output aggregate:

$$p_t = \eta_t^{\frac{1}{\vartheta - 1} - \xi} \left(\int_{\mathcal{N}_t} p_t(\eta)^{1 - \vartheta} d\eta \right)^{\frac{1}{1 - \vartheta}}$$

The demand for η's product is then

$$y_t(\eta) = \eta_t^{\xi(\vartheta - 1) - 1} \left(\frac{p_t(\eta)}{p_t} \right)^{-\vartheta} y_t$$

Note that given symmetric firms, I then have

$$\rho_t = \eta_t^{\xi} \tag{2}$$

where $\rho_t \equiv \frac{p_t(\eta)}{p_t}$ $\forall \eta \in \mathcal{N}_t$. This equation reveals that ξ is the key parameter determining whether the measure of active firms impacts pricing. For $\xi = 0$, there is no LOV, and as a result pricing is independent of the measure of firms.

Firms produce differentiated products via the production function

$$y_t(\eta) = z_t k^{\alpha} l_t(\eta)^{1 - \alpha}$$

where z_t is total factor productivity, k is the (invariant) amount of capital used to produce a single variety[1], $\alpha \in (0, 1)$, and $l_t(\eta)$ is the labor used by firm η. Minimizing the cost of labor given the

[1]In other words, each entering firm must invest in k units of capital to produce in the following period. I implicitly assume that there is a constant proportion of product variety to capital.

real wage $\frac{w_t}{p_t}$ and the level of production $y_t(\eta)$ implies real marginal cost of

$$mc_t(\eta) = \frac{1}{1-\alpha}\frac{w_t}{p_t}\frac{l_t(\eta)}{y_t(\eta)}$$

With prices subject to the price adjustment function,

$$pac_t(\eta) = \frac{\kappa}{2}\left(\frac{p_t(\eta)}{p_{t-1}(\eta)} - 1\right)^2 \frac{p_t(\eta)}{p_t} y_t(\eta)$$

firm dividends can be expressed as a function of pricing, marginal cost, and aggregate variables:

$$d_t(\eta) = \left\{\left[1 - \frac{\kappa}{2}\left(\frac{p_t(\eta)}{p_{t-1}(\eta)} - 1\right)^2\right]\left(\frac{p_t(\eta)}{p_t}\right)^{1-\vartheta} - mc_t(\eta)\left(\frac{p_t(\eta)}{p_t}\right)^{-\vartheta}\right\}\eta_t^{\xi(\vartheta-1)-1} y_t \tag{3}$$

Maximizing firm value (1) subject to the dividend equation (3), firms set prices as a markup over marginal cost where – imposing symmetry across firms – the markup is

$$\frac{\rho_t}{mc_t} \equiv \mu_t = \frac{\vartheta}{(\vartheta-1)\left(1-\frac{\kappa}{2}\pi_t^2\right) + \kappa\pi_t(1+\pi_t) - \kappa(1-\delta)\mathbb{E}_t\left[\mathcal{Q}_{t,t+1}\pi_{t+1}(1+\pi_{t+1})\frac{y_{t+1}}{y_t}\left(\frac{\eta_{t+1}}{\eta_t}\right)^{2\xi(\vartheta-1)-1}\right]} \tag{4}$$

where $\pi_t \equiv \frac{p_t(\eta)}{p_{t-1}(\eta)} - 1 \ \forall \eta \in \mathcal{N}_t$. Note that when there is symmetry across firms, the output aggregator and clearing in the labor market ($l_t = \eta_t l_t(\eta)$, $\forall \eta \in \mathcal{N}_t$) imply

$$mc_t = \frac{1}{1-\alpha}\frac{w_t}{p_t}\frac{l_t}{y_t}\eta_t^{\xi}$$

Now using equation (2) we get

$$\mu_t = \left(\frac{1}{1-\alpha}\frac{w_t}{p_t}\frac{l_t}{y_t}\right)^{-1}$$

In a zero inflation steady state, the markup is $\bar{\mu} = \frac{\vartheta}{\vartheta-1}$. Log linearizing equation (4) around this steady state gives the pricing equation:

$$\pi_t = (1-\delta)\beta\mathbb{E}_t\pi_{t+1} - \frac{\vartheta-1}{\kappa}(\ln\mu_t - \ln\bar{\mu}) \tag{5}$$

where β is the steady state value of $\mathcal{Q}_{t,t+1}\frac{y_{t+1}}{y_t}\left(\frac{\eta_{t+1}}{\eta_t}\right)^{2\xi(\vartheta-1)-1}$.

2.2 Inflation Measurement and Unit Labor Costs

The inflation in equation (5) is inflation of producer prices, and not inflation of the welfare consistent consumption price index. Equation (2) implies (up to log-linearization) that consumption price inflation is

$$\Delta \ln p_t = \pi_t - \xi \Delta \ln \eta_t$$

Additionally, the series on unit labor costs are defined as compensation per hour times hours per output – labor's share of national income. In the aggregate, due to the cobb-douglas production function assumption, the markup is inversely proportional to unit labor cost:

$$\mu_t = (1-\alpha)\left(\frac{ulc_t}{p_t}\right)^{-1}$$

I can then express the phillips curve as a difference equation in the log of the price level:

$$\ln p_t - \phi_1 \mathbb{E}_t \Delta \ln p_{t+1} + \phi_2 \Delta \ln p_t = \ln ulc_t + \phi_0 + \xi \left[\phi_1 \Delta \ln \eta_{t+1} - \phi_2 \Delta \ln \eta_t\right] \tag{6}$$

where $\phi_0 = \ln \frac{\vartheta}{(1-\alpha)(\vartheta-1)}$, $\phi_1 \equiv (1-\delta)\beta\kappa(\vartheta-1)^{-1}$, and $\phi_2 \equiv \frac{\kappa}{\vartheta-1}$. Note that η_{t+1} is in the information set at t since it depends non-stochastically on the current number of firms and the current number of entering firms which are known at t. Here, ξ is the parameter of interest – I want to know if there is a product variety effect in the price level process.

2.3 Variety Dynamics

Since the data used is on firm entry and not directly the 'number' of firms in existence, I must posit a law of motion for the measure of firms. First, since we have implicitly assumed that there is a fixed amount of capital used for each variety, k, it seems reasonable to assume a law of motion for firm entry built from a typical capital law of motion. For example, the typically law of motion for capital takes the form

$$k_{t+1} = (1-\delta)k_t + i_t$$

where k_t is aggregate capital at t and i_t is aggregate investment. Although it has become fairly common in the literature to assume some sort of capacity utilization and investment adjustment costs to this specification, I do not explore those ideas here. Instead, I assume the most simple form possible since the goal is not to explain firm entry dynamics but to explain the consequences of them for the price level

Given a constant k per individual product type, the equation then becomes

$$\eta_{t+1} = (1-\delta)\eta_t + \eta_t^E \tag{7}$$

This equation, given parameter δ and an initial value of the measure of firms $\ln \eta_0$, allows us to use data on firm entry to construct an approximate series for the measure of firms over time.

2.4 Model Price Level

Following Sbordone (2002), the optimal path of prices implied by the difference equation (6) is[2]

$$\ln p_t = \lambda_1 \ln p_{t-1} + (1-\lambda_1)(1-\lambda_2^{-1}) \sum_{\tau=t}^{\infty} \lambda_2^{t-\tau} \mathbb{E}_t[\ln ulc_\tau + \phi_0 + \xi(\phi_1 \Delta \ln \eta_{\tau+1} - \phi_2 \Delta \ln \eta_\tau)] \tag{8}$$

where λ_1, λ_2 are the roots of $P(\lambda) = \phi_1\lambda^2 - (1+\phi_1+\phi_2)\lambda + \phi_2$ and $0 < |\lambda_1| < 1 < |\lambda_2|$. Given estimates for the expectations held by firms of the future state of the economy, this equation gives us the model's predicted path for the price level.

However, for ease of computation I restate this equation in a form which uses only variables which are included in the VAR specification used for estimating expectations and only the parameters $\phi_0, \lambda_1, \lambda_2, \xi, \ln \eta_0$, and δ[3]. I rewrite equation (8) in the form[4]

$$\ln\left(\frac{p_t}{ulc_t}\right) = \lambda_1 \ln\left(\frac{p_{t-1}}{ulc_{t-1}}\right) - \Delta \ln ulc_t + (1-\lambda_1)\phi_0 + \sum_{\tau=t}^{\infty} \lambda_2^{t-\tau} \mathbb{E}_t[\chi_\tau] \tag{9}$$

where $\chi_t \equiv (1-\lambda_1)\Delta \ln ulc_t + \xi\left(\lambda_2^{-1}\Delta \ln \eta_{t+1} - \lambda_1 \Delta \ln \eta_t\right)$.

3 Dataset

The price level and unit labor cost data is standard. For the main results, I use the implicit GDP deflator data series (GDPDEF) published by the Bureau of Economic Analysis (BEA) on a quarterly basis. Since deflating nominal variables in the appropriate welfare consistent way is so essential to studies involving firm dynamics, I provide a robustness test by using the alternative

[2]See the appendix for derivation of (8).

[3]The last two parameters enter the equation indirectly through the generation of the firm measure series via equation (7).

[4]See the appendix for derivaton of equation (9).

Table 1: Dataset, Series Codes and Sources

Dataset – (Quarterly, 1948:Q1 to 1993:Q4)		
$\{ulc_t\}$	ULCNFB	Nonfarm Business Unit Labor Costs (BEA)
$\{p_t\}$	GDPDEF	Implicit GDP Deflator (BEA)
	CPIAUCSL	Consumer Price Index For All Urban Consumers (BEA)
$\{\eta_t^E\}$	NBF	Index of Net Business Formation (BEA Survey of Current Business, Nov. 1994)
	NBI	New Business Incorporations (BEA Survey of Current Business, Nov. 1994)

of the Consumer Price Index (CPIAUCSL). This series also is standard and comes from the BEA. Unit labor costs come from the BEA as well and as is typical in the literature[5] I choose the non-farm business series (ULCNFB).

Data on firm dynamics is much more difficult to come by. For many years, the BEA's Survey of Current Business published two series which attempted to measure the amount of firm entrance in each month. Both series appeared in the Business Cycle Indicators subsection of the Survey of Current Business, but now have been discontinued. The Index of Net Business Formation (NBF) combines a variety of indicators into an approximate index, while the New Business Incorporations (NBI) series simply counts the total number of new corporations chartered in the United States each month. I was able to find data on these series from the first quarter of 1948 through the last month of 1993 in the November 1994 issue of the Survey of Current Business. Using these data series as two possible measures of firm entry, I restrict my sample to 1948:Q1 through 1993:Q4, and convert these series into quarterly data. Table 1 summarizes the data set.

At the end of the paper, Figure 1 shows the natural log of the NBF series over the sample and Figure 2 shows the natural log of the NBI series. Although the data series appear to have a slight trends, they pass standard unit root tests (i.e. Dickey-Fuller). They are used in equation (7) to generate the series $\{\eta_t\}$ which is then differenced for use in the VAR and the model equation (9).

4 Estimation Strategy

Estimation proceeds by choosing the parameter values which minimize the variance of the difference between the model's predicted path for the price level, and the path actually observed in the data. Since unit labor costs are taken as given – the model uses this data to generate its

[5]For example, Sbordone (2002) – which this paper is an extension of – uses the non-farm business series for unit labor costs.

prediction – the model residuals can be expressed as

$$e_t = \ln p_t^{data} - \ln p_t^{model} = \ln\left(\frac{p_t}{ulc_t}\right)^{data} - \ln\left(\frac{p_t}{ulc_t}\right)^{model}$$

where the 'data' superscript denotes the values observed in the data and the 'model' superscript denotes the predicted values from the model. Letting $\theta = \begin{pmatrix}\phi_0 & \lambda_1 & \lambda_2 & \xi & \ln\eta_0 & \delta\end{pmatrix}'$, the estimated model parameters are simply

$$\begin{aligned}\hat{\theta} = \arg\min_{\theta} \;\; & var(e_t) \\ \text{s.t.} \;\; & 0 < |\lambda_1| < 1 < |\lambda_2| \\ & 0 \leq \xi \\ & 0 \leq \delta \leq 1\end{aligned}$$

The deep parameter β can be found by the equation $\beta = [(1-\delta)\lambda_1\lambda_2]^{-1}$. By assuming a value for one of κ and θ, the other comes from the relation $\frac{\kappa}{\theta-1} = [(1-\delta)\beta(\lambda_1+\lambda_2)-1]^{-1}$. I do not impose any restrictions on ϕ_0 or use it to find any deep parameters as it absorbs any scale differences over the various indices used.

The predicted path of $\ln(p_t/ulc_t)$ comes from equation (9) which is a function of expectations of the process $\{\ln(p_t/ulc_t), \Delta\ln ulc_t, \Delta\ln\eta_{t+1}\}$[6]. Thus, the first step is to specify a forecasting model for this process.

4.1 Forecasting Model and Finding e_t

Assume that there is vector Z_t which captures all relevant information for projecting the future values of unit labor costs and the measure of firms in the economy. Moreover, suppose that $\{Z_t\}$ is a markov process given by $Z_{t+1} = \Phi Z_t + \epsilon_{t+1}$ for some mean zero shock ϵ_{t+1}. To estimate Φ, I use a VAR model. If S_t is an $k \times 1$ vector containing information on the current state of the economy, and we use p lags in the VAR, then $Z_t = (1, S_t', S_{t-1}', \ldots, S_{t-p+1}')'$, and the matrix Φ will take the

[6] Note that $\ln\eta_{t+1}$ is predictable at time t.

form

$$\Phi = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 0 \\ \gamma_0 & \Gamma_1 & \Gamma_2 & \Gamma_3 & \dots & \Gamma_{p-1} & \Gamma_p \\ 0 & \mathcal{I}_k & \mathcal{O}_k & \mathcal{O}_k & \dots & \mathcal{O}_k & \mathcal{O}_k \\ 0 & \mathcal{O}_k & \mathcal{I}_k & \mathcal{O}_k & \dots & \mathcal{O}_k & \mathcal{O}_k \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \mathcal{O}_k & \mathcal{O}_k & \mathcal{O}_k & \dots & \mathcal{O}_k & \mathcal{O}_k \\ 0 & \mathcal{O}_k & \mathcal{O}_k & \mathcal{O}_k & \dots & \mathcal{I}_k & \mathcal{O}_k \end{pmatrix}$$

where γ_0 is a $k \times 1$ vector of constants from the VAR, Γ_i is the $k \times k$ matrix of coefficients on the ith lagged vector in the VAR, $\mathcal{I}_j$ denotes a $j \times j$ identity matrix, and $\mathcal{O}_j$ denotes a $j \times j$ matrix of zeros. Then expectations over future unit labor costs and the future measure of firms can be recovered from the vector $\mathbb{E}_t Z_\tau = \Phi^{\tau-t} Z_t$. Finally, the discounted sum in equation (9) is a linear combination (based on the coefficients on the terms in χ_t) of the elements of the vector

$$\zeta_t = \sum_{\tau=t}^{\infty} \lambda_s^{t-\tau} \Phi^{\tau-t} Z_t = (\mathcal{I}_{(pk+1)} - \lambda_2^{-1}\Phi)^{-1} Z_t \tag{10}$$

In the benchmark model we use $S_t = \begin{pmatrix} \ln(p_t/ulc_t) & \Delta \ln ulc_t & \Delta \ln \eta_{t+1} \end{pmatrix}'$. To calculate $\sum_{\tau=t}^{\infty} \lambda_s^{t-\tau} \mathbb{E}_t \chi_t$, I must extract the expected discounted sums of $\{\Delta \ln ulc_\tau\}$, $\{\Delta \ln \eta_{\tau+1}\}$, and $\{\Delta \ln \eta_\tau\}$. Given the setup outlined above these sums would be in the third, fourth, and seventh rows of the vector ζ_t assuming two lags in the VAR (and would be in the same columns for longer lag lengths). Therefore for $p = 2$

$$\sum_{\tau=t}^{\infty} \lambda_s^{t-\tau} \mathbb{E}_t \chi_\tau = Z_t'[(\mathcal{I}_{(pk+1)} - \lambda_2^{-1}\Phi)^{-1}]' \begin{pmatrix} 0 & 0 & 1-\lambda_1 & \xi\lambda_2^{-1} & 0 & 0 & -\xi\lambda_1 \end{pmatrix}'$$

Next, since

$$e_t = \ln\left(\frac{p_t}{ulc_t}\right) - \lambda_1 \ln\left(\frac{p_{t-1}}{ulc_{t-1}}\right) + \Delta \ln ulc_t - (1-\lambda_1)\phi_0 - \sum_{\tau=t}^{\infty} \lambda_s^{t-\tau} \mathbb{E}_t \chi_\tau$$

and the non-summed terms can also be selected from Z_t, I can express the model's error at t as

$$e_t = Z_t' \begin{pmatrix} -(1-\lambda_1)\phi_0 & 1 & 1 & 0 & -\lambda_1 & 0 & 0 \end{pmatrix}' - \sum_{\tau=t}^{\infty} \lambda_s^{t-\tau} \mathbb{E}_t \chi_\tau$$

Combining these, the error is

$$e_t = Z_t'\omega \tag{11}$$

where the weighting vector ω is defined as

$$\begin{aligned}\omega = &\begin{pmatrix} -(1-\lambda_1)\phi_0 & 1 & 1 & 0 & -\lambda_1 & 0 & 0 \end{pmatrix}' \\ &- [(\mathcal{I}_{(pk+1)} - \lambda_2^{-1}\Phi)^{-1}]' \begin{pmatrix} 0 & 0 & 1-\lambda_1 & \xi\lambda_2^{-1} & 0 & 0 & -\xi\lambda_1 \end{pmatrix}'\end{aligned}$$

5 Benchmark Results

For each variation of the model, I conduct a search over the parameter space for local minimizers. Then taking the minimum of these local minimizers, I reject those with implausible values for the the firm hazard rate. Any value of $\delta > 0.2$ would imply that 20% of firms go out of business each quarter. This cutoff is fairly conservative considering that quarterly rates of capital depreciation are typically in the ballpark of 0.025. Then, taking the minimum of the remaining plausible model variance minimizers, I iterate until the parameter estimate doesn't change, to ensure a true minimum.

As the model is non-linear, I must use a local approximation of the Variance-Covariance matrix to conduct hypothesis testing. The method for finding this matrix is standard and can be seen as a local linear approximation

5.1 Estimating the Variance-Covariance Matrix of the Parameters

Before reporting the results, I quickly outline the method for constructing the variance-covariance matrix ofthe parameter estimates. Let the function $f(Z_t, \theta)$ represent the model's prediction for $\ln(p_t/ulc_t)$. I then use numerical derivatives to find the jacobian matrix:

$$F = \frac{\partial f(Z,\theta)}{\partial \theta} = \begin{pmatrix} \frac{\partial f(Z_1,\theta)}{\partial \theta} \\ \frac{\partial f(Z_2,\theta)}{\partial \theta} \\ \vdots \\ \frac{\partial f(Z_{pk+1},\theta)}{\partial \theta} \end{pmatrix} = \begin{pmatrix} \frac{\partial f(Z_1,\theta)}{\partial \theta_1} & \frac{\partial f(Z_1,\theta)}{\partial \theta_2} & \cdots & \frac{\partial f(Z_1,\theta)}{\partial \theta_5} \\ \frac{\partial f(Z_2,\theta)}{\partial \theta_1} & \frac{\partial f(Z_2,\theta)}{\partial \theta_2} & \cdots & \frac{\partial f(Z_2,\theta)}{\partial \theta_5} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f(Z_{pk+1},\theta)}{\partial \theta_1} & \frac{\partial f(Z_{pk+1},\theta)}{\partial \theta_2} & \cdots & \frac{\partial f(Z_{pk+1},\theta)}{\partial \theta_5} \end{pmatrix}$$

The Variance-Covariance estimate is then

$$\Sigma = s^2(F'F)^{-1}$$

and the standard errors of the model parameters are the vector

$$SE = diag(\Sigma)^{\frac{1}{2}}$$

5.2 Estimates

The baseline model has 5 lags in the forecasting variable, uses the first observation on the series of firm entry as the initialization for constructing the firm measure series, and uses the implicit GDP deflator as the measure for the price level. Since results are nearly identical whether using the Net Business Formation index of the New Business Incorporations index, I present results for the baseline model and both these indices. Results are in Table 1.

Table 2: Benchmark Model Estimates

Baseline Model ($\eta_1 = \eta_1^E$ and 5 lags in the forecasting VAR)

Data Series:	GDPDEF, NBF	GDPDEF, NBI
ϕ_0	-0.4963	-0.7816
	(-6.7365)	(-1.6613)
λ_1	0.9002	0.9095
	(116.6770)	(137.7234)
λ_2	1.0412	1.0268
	(105.1290)	(59.9590)
ξ	**0.4851**	**0.5036**
	(3.3066)	**(3.7973)**
δ^{**}	0.0382	0.0175
	(1.8161)	(0.9644)
		Note: t-stats in parenthesis
SSE	0.0025	0.0018
s^2	1.9564×10^{-5}	1.4313×10^{-5}

For both firm entry series, the model fits the data quite well. Using the Net Business Formation series for firm entry, the hypotheses that each parameter other than the firm hazard rate is zero is rejected at a significance of 1%. The firm hazard rate parameter δ is significant at the 10% level. When using the New Business Incorporations series, all parameters are significant at the 1% level apart from the constant term ϕ_0 which is significant at 10% and the firm hazard rate δ which is insignificant.

These results suggest that firm entry does play a key role in explaining the price level over time. Particularly when examining the fit of the model visually, this fairly simple theoretical account of the relationship between firm entry and pricing seems to explain the data remarkably well. Figure 3 shows the actual data series and the model's prediction.

The estimates for δ are a potential concern. This parameter essentially determines the smoothness of the η_t series. With a high δ and thus a high degree of smoothing, the parameter reduces the amount of variation that firm entry can explain.

However, both estimates of δ are close to the typically assumed rate of capital depreciation of about 2.5% per quarter. Because of the proximity of the estimates – particularly when the model is not meant to explain this parameter – the low t-stat on the NBI estimate of δ should not be of particular concern.

Another possible issue comes from autocorrelation and heteroskedasticity in the residuals. Figure 4 shows the residuals from the NBF model. Just from visual inspection, both possible violations of the Gauss-Markov classical assumptions seem to be an issue. Running a Durbin-Watson test for autocorrelation gives a DW statistic of 0.9028. Additionally regressing the residuals of a lag of themselves implies an auto-correlation coefficient of about 0.5487. Autocorrelation is definitely present – which is not surprising considering we are working with time series.

I do not run formal tests for heteroskedasticity since the model is non-linear and there is no intuitive relation between the data and the apparent heteroskedasticity. Looking at Figure 4 which plots the residuals from the benchmark model using the NBF series, there appear to be two periods of heightened volatility. Approximately from 1950 through 1956 and from 1973 through 1982. Interestingly, these periods coincide with two extremely inflationary periods in the macroeconomic history of the United States. It appears that additional unexplained factors may be at work in these periods.

However, the model's variance is extremely small. Neither correcting for autocorrelation nor heteroskedasticity is likely to appreciably impact the hypothesis tests and regardless our estimates are still unbiased without any correction. Additionally, doing a simple weighted least squares regression using the inverse of the residual squared for each observation as a weight does not change estimates appreciably. Also of note, in Sbordone (2002), no correction is made for autocorrelation or heteroskedasticity. In that paper, the residuals are reported graphically and have similar characteristics to the residuals from this model. Based on preliminary tests for non-spherical errors and since similar empirical studies have not bothered to account for such issues, I do not proceed with further corrections and accept the benchmark model's estimates.

6 Result Robustness to Price Series

Concerns expressed in the empirical and theoretical literature on firm entry and the NKPC[7] suggest that the model could potentially be improved by or be susceptible to the choice of the price level series. How nominal variables – such as unit labor costs in our model – get deflated when dealing with firm dynamics depends crucially on the index used. A variety of indices exist and a variety of methodologies get used in constructing these indices. The most important point for this study is that many chained quantity type indices are simply unable to account for short term changes in product variety. They will indeed tend to be biased in this dimension over high frequency movements.

The Consumer Price Index is well known to have such a bias. Using the implicit GDP deflator in the benchmark case does not necessarily ensure that the benchmark model accounts for true product variety movements. As a test, I re-estimate the model using the CPI data instead of the GDP deflator data. Because the CPI does not account for short term product entry, if the benchmark model is really picking up product variety variation, then we would expect to *not* find as high a value for ξ in this new regression when compared to the benchmark. Table 3 gives the estimation results.

The estimates for ξ plunge towards zero when we estimate the model using the CPI. Again, this doesn't mean that the model is invalid – it is a test if the GDP deflator series reflect changes in product variety. This robustness test solidifies the main findings, and improves confidence in the benchmark estimates.

7 Conclusion

In all, this empirical exercise provides some simple evidence that new theoretical developments in the Phillips Curve literature may be onto something. The concept of 'love of variety' – where

[7] Such as Lewis (2006), and Bilbiie, Ghironi, and Melitz (2007).

Table 3: Model estimates with the Consumer Price Index used as the price level series.

Baseline Model ($\eta_1 = \eta_1^E$ and 5 lags in the forecasting VAR)

Data Series:	CPIAUCSL, NBF	CPIAUCSL, NBI
ϕ_0	-0.9108	-7.4878
	(-7.8437)	(-0.5227)
λ_1	0.8642	0.9391
	(136.1896)	(247.2305)
λ_2	1.0091	1.0015
	(1.4274×10^3)	(373.3744)
ξ	**0.0000**	**0.1474**
	$(\mathbf{1.6750 \times 10^{-6}})$	**(2.1180)**
δ^{**}	0.0000	0.1000
	(9.7877×10^{-5})	(3.6088)
		Note: t-stats in parenthesis
SSE	0.0016	0.0033
s^2	1.2707×10^{-5}	2.6050×10^{-5}

greater product diversity makes households better off – provides a simple yet elegant link between pricing and firm entry and competition. It appears this mechanism may be alive and well in the data.

Reconsidering the residuals from the benchmark model, I observed that the model's residual variance appears to increase rapidly in periods of high inflation. The model seems to (very slightly) break down during such episodes. Perhaps this observation is a clue as to the costs of inflation. Is this evidence that the true costs of inflation come from information issues? If it were only sticky pricing and firm dynamics that cause real effects from inflation, then one should not see the model have a more difficult time matching the data during such inflationary episodes.

This study also demonstrates quite cleanly how the theory of the New Keynesian Phillips Curve can be adapted to address small deviations in the benchmark theoretical NKPC. Augmenting the NKPC proves to be a useful method for understanding increasingly complex pricing dynamics. Although the 'cost push shock' concept has been criticized in medium scale dynamic stochastic general equilibrium models (see Chari, Kehoe and McGrattan (2008)), this paper shows that there exists room in the theory of the NKPC to account for the determinants of these shocks. It appears to be a flexible workhorse from which to continue research.

Accordingly, constructing more specific tests for the nature of the firm entry dynamic may be useful. Is the relationship non-linear? Should the firm measure law of motion include adjustment costs as in standard neoclassical investment theory? This paper suggests that these are valid questions, and future studies regarding firm dynamics and the Phillips Curve may prove useful.

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A Derivation of Equations (8) and (9)

A.1 Equation (8)

Rewriting equation (6)

$$\ln p_t - \phi_1 \mathbb{E}_t \Delta \ln p_{t+1} + \phi_2 \Delta \ln p_t = \ln ulc_t + \phi_0 + \xi \left[\phi_1 \Delta \ln \eta_{t+1} - \phi_2 \Delta \ln \eta_t \right]$$

I first restate it as

$$\mathbb{E}_t[(-\phi_1 + (1+\phi_1+\phi_2)L - \phi_2 L^2) \ln p_{t+1}] = \ln ulc_t + \phi_0 + \xi \left[\phi_1 \Delta \ln \eta_{t+1} - \phi_2 \Delta \ln \eta_t \right]$$

Factoring the lag polynomial and adding on expectational error u_{t+1} where $\mathbb{E}_t u_{t+1} = 0$

$$-\phi_1 (1 - \lambda_1 L)(1 - \lambda_2 L) \ln p_{t+1} = \ln ulc_t + \phi_0 + \xi \left[\phi_1 \Delta \ln \eta_{t+1} - \phi_2 \Delta \ln \eta_t \right] + u_{t+1}$$

where λ_1, λ_2 are the roots of $P(\lambda) = \phi_1 \lambda^2 - (1+\phi_1+\phi_2)\lambda + \phi_2$ and $0 < |\lambda_1| < 1 < |\lambda_2|$. Dividing through by the second factor of the lag polynomial gives

$$\begin{aligned}
-\phi_1 (1 - \lambda_1 L) \ln p_{t+1} &= \frac{1}{1 - \lambda_2 L} \left[\ln ulc_t + \phi_0 + \xi \left(\phi_1 \Delta \ln \eta_{t+1} - \phi_2 \Delta \ln \eta_t \right) + u_{t+1} \right] \\
&= -\frac{(\lambda_2 L)^{-1}}{1 - (\lambda_2 L)^{-1}} \left[\ln ulc_t + \phi_0 + \xi \left(\phi_1 \Delta \ln \eta_{t+1} - \phi_2 \Delta \ln \eta_t \right) + u_{t+1} \right] \\
&= -(\lambda_2 L)^{-1} \sum_{\tau=t}^{\infty} (\lambda_2^{-1} L)^{t-\tau} \left[\ln ulc_t + \phi_0 + \xi \left(\phi_1 \Delta \ln \eta_{t+1} - \phi_2 \Delta \ln \eta_t \right) + u_{t+1} \right]
\end{aligned}$$

The last line is well defined since $|\lambda_2| > 1$. When multiplying by a lag, dividing through by $-\phi_1$, and solving out all lag operators, I get

$$\ln p_t = \lambda_1 \ln p_{t-1} + \frac{\lambda_2^{-1}}{\phi_1} \sum_{\tau=t}^{\infty} \lambda_2^{t-\tau} \left[\ln ulc_\tau + \phi_0 + \xi \left(\phi_1 \Delta \ln \eta_{\tau+1} - \phi_2 \Delta \ln \eta_\tau \right) + u_{\tau+1} \right]$$

Finally, noting that $\lambda_1 \lambda_2 = \frac{\phi_2}{\phi_1}$ and $\lambda_1 + \lambda_2 = \frac{1+\phi_1+\phi_2}{\phi_1}$ so that $\frac{\lambda_2^{-1}}{\phi_1} = (1-\lambda_1)(1-\lambda_2^{-1})$, by taking time t conditional expectations we get equation (8):

$$\ln p_t = \lambda_1 \ln p_{t-1} + (1-\lambda_1)(1-\lambda_2^{-1}) \sum_{\tau=t}^{\infty} \mathbb{E}_t \left[\ln ulc_\tau + \phi_0 + \xi \left(\phi_1 \Delta \ln \eta_{\tau+1} - \phi_2 \Delta \ln \eta_\tau \right) \right] \tag{8}$$

A.2 Equation (9)

To get to equation (9) from (8) first note that for $d \in (0,1)$ and any process $\{\mathcal{X}_t\}$

$$
\begin{aligned}
(1-d)\sum_{j=0}^{\infty} d^j \mathbb{E}_t \mathcal{X}_\tau &= \sum_{j=0}^{\infty} d^j \mathbb{E}_t \mathcal{X}_{t+j} - d\sum_{j=0}^{\infty} d^j \mathbb{E}_t \mathcal{X}_{t+j} = \sum_{j=0}^{\infty} d^j \mathbb{E}_t \mathcal{X}_{t+j} - \sum_{j=-1}^{\infty} d^{j+1} \mathbb{E}_t \mathcal{X}_{t+j} + \mathcal{X}_{t-1} \\
&= \sum_{j=0}^{\infty} d^j \mathbb{E}_t \mathcal{X}_{t+j} - \sum_{j=0}^{\infty} d^j \mathbb{E}_t \mathcal{X}_{t+j-1} + \mathcal{X}_{t-1} = \mathcal{X}_{t-1} + \sum_{j=0}^{\infty} d^j \mathbb{E}_t \Delta \mathcal{X}_{t+j}
\end{aligned}
$$

Therefore by pulling ϕ_0 out of the sum and using the above fact, I can rewrite (8) as

$$
\ln p_t = \lambda_1 \ln p_{t-1} + (1-\lambda_1)\left\{\phi_0 + \ln ulc_{t-1} + \sum_{\tau=t}^{\infty} \lambda_2^{t-\tau} \mathbb{E}_t \left[\Delta \ln ulc_\tau + \tilde{\chi}_\tau\right]\right\}
$$

where

$$
\tilde{\chi}_t \equiv \xi(1-\lambda_2^{-1})\left(\phi_1 \Delta \ln \eta_{t+1} - \phi_2 \Delta \ln \eta_t\right) = \xi \frac{1}{1-\lambda_1}\left(\lambda_2^{-1} \Delta \ln \eta_{t+1} - \lambda_1 \Delta \ln \eta_t\right)
$$

since $\lambda_1 \lambda_2 = \frac{\phi_2}{\phi_1}$ and $\lambda_1 + \lambda_2 = \frac{1+\phi_1+\phi_2}{\phi_1}$ imply that $(1-\lambda_2^{-1})\phi_1 = \frac{\lambda_2^{-1}}{1-\lambda_1}$ and $(1-\lambda_2^{-1})\phi_2 = \frac{\lambda_1}{1-\lambda_1}$. Rearranging the equation and adding and subtracting a $\ln ulc_t$ gives equation (9):

$$
\ln\left(\frac{p_t}{ulc_t}\right) = \lambda_1\left(\ln \frac{p_{t-1}}{ulc_{t-1}}\right) - \Delta \ln ulc_t + (1-\lambda_1)\phi_0 + \sum_{\tau=t}^{\infty} \lambda_2^{t-\tau} \mathbb{E}_t \chi_\tau \tag{9}
$$

where $\chi_t \equiv (1-\lambda_1)\Delta \ln ulc_t + \xi\left(\lambda_2^{-1} \Delta \ln \eta_{t+1} - \lambda_1 \Delta \ln \eta_t\right)$.

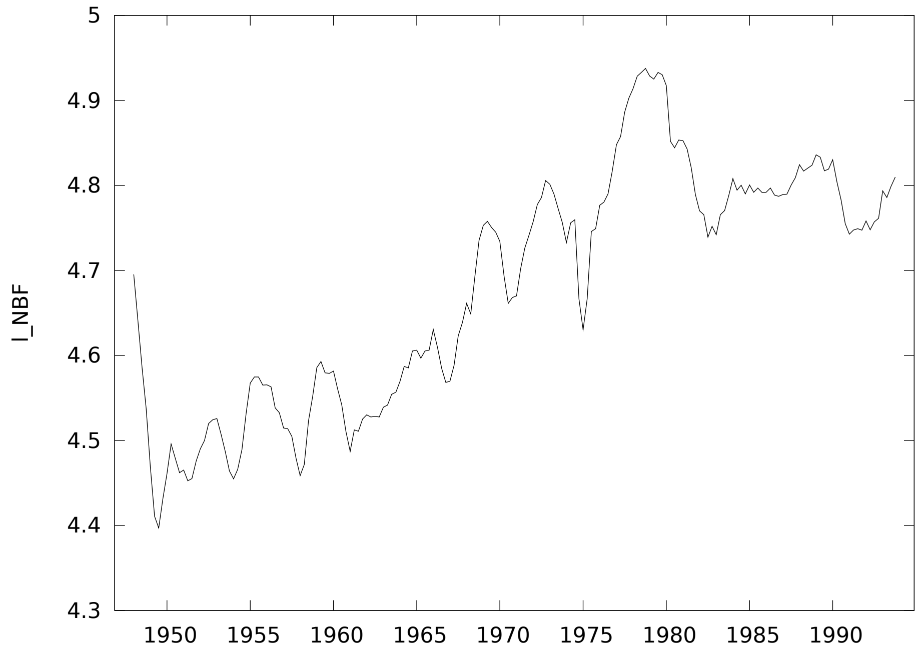

Figure 1: The natural log of the Index of Net Business Formation (Discontinued) from the Survey of Current Business by the Bureau of Economic Analysis.

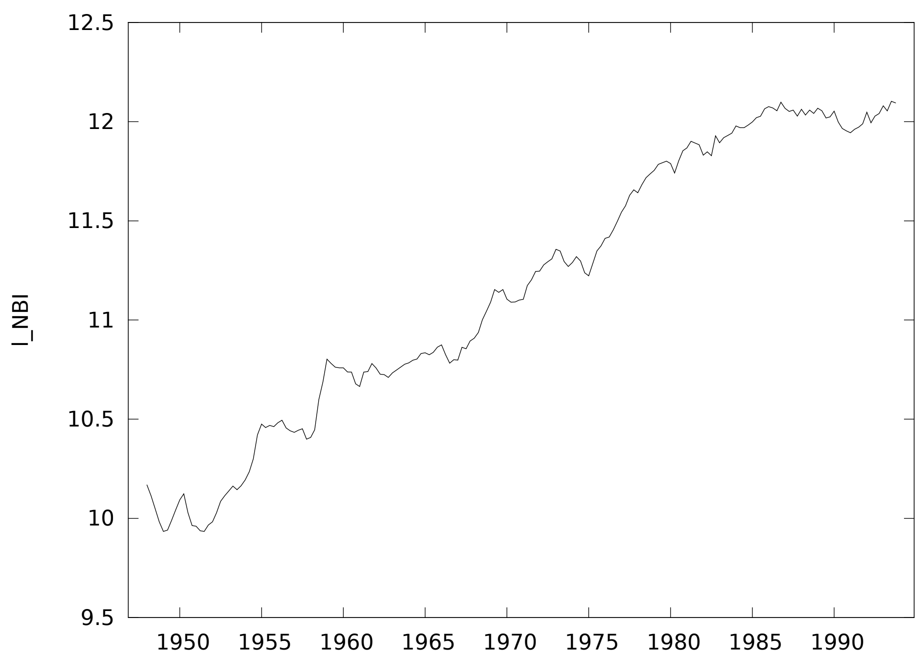

Figure 2: The natural log of New Business Incorporations (Discontinued) from the Survey of Current Business by the Bureau of Economic Analysis.

Figure 3: Benchmark model's fit using NBF for firm entry series.

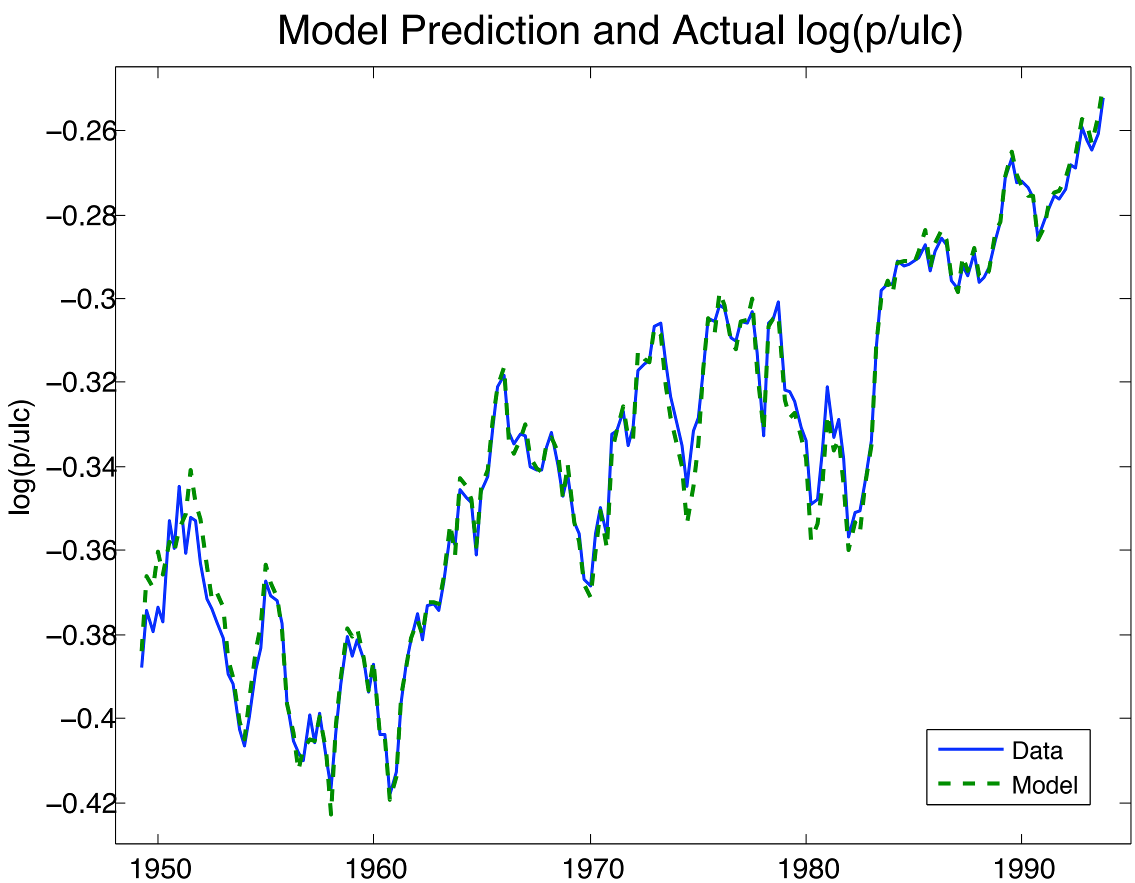

Figure 4: Benchmark model residuals using NBF for firm entry series.