

How Do Residential Recycling Programs Affect the Ratio of  
Secondary to Virgin Production in the Paper Industry?

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# 1 Introduction

Today an increasing number of paper products including packaging paper and office supply papers are produced with secondary fibers recovered from consumer waste, which helps to conserve the forestry resources. American Forest and Paper Association (AF&PA) reported in 2010 that 77% of paper and paperboard mills used some recovered paper and 115 mills used only recovered paper.<sup>1</sup>

Another phenomenon we have simultaneously observed is that more and more different kinds of residential recycling programs are carried out on various scales, enabling more post-consumer products to be collected and recycled, and making possible the proliferation of secondary fibers. EPA (2011) reported that 34.1% of all municipal solid waste generated in 2010 was recycled and composted; this figure rose from 16% in 1990 to 25.7%, 28.6% and 31.6%, in 1995, 2000 and 2005, respectively, and reaches a twofold increase at the current level compared to 1990.<sup>2</sup> Moreover, the U.S. paper recovery rate saw a record-high 66.8% in 2011, up from 46% in 2000 and 33.5% in 1990 (AF&PA, 2012).

With these two phenomena at hand, here comes my research question: how are these two facts related with each other? Is there a casual relationship between the popularization of residential recycling programs and the increase in recycled paper production? To answer this, I theoretically examine the effect of residential recycling programs on the ratio of recycled to virgin production in the paper industry.

In the next section, I construct a simple model to capture the economic dynamics between the waste recovery industry and the paper production industry. In Section 3, I assume certain functional forms so that the model becomes tractable in comparative static analysis, and then proceed to answer my research question. Finally, I conclude and make recommendations for further studies in Section 4.

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<sup>1</sup>Facts are retrieved from the official website of AF&PA, <http://www.afandpa.org/FunFacts.aspx>.

<sup>2</sup>All these percentage numbers come from EPA's *2010 Municipal Solid Waste Characterization Reports* published in November, 2011, which can be accessed online at [http://www.epa.gov/osw/nonhaz/municipal/pubs/msw\\_2010\\_rev\\_factsheet.pdf](http://www.epa.gov/osw/nonhaz/municipal/pubs/msw_2010_rev_factsheet.pdf).

## 2 A Simple Model

Recycled paper production demands secondary fibers recovered in the upstream industry as input; thus my model has to incorporate the dynamics of upstream and downstream industry productions so that it will be able to further illustrate the impacts of residential recycling programs. In this section, I develop a simple upstream-downstream interactional model based on Oz Shy's textbook prototype of vertical merger (*Industrial Organization: Theory and Applications*, 1995, pp. 176-179), by making different assumptions about market structures in the downstream industry, and generalizing firms' production and cost functions.

In the paper production industry, I assume  $N$  firms engage in recycled paper production and purchase secondary fibers recovered from consumer waste as input. In addition, one other firm produces paper completely from virgin fiber, and thus does not purchase any recycled fiber from the upstream waste recovery industry. The recycled paper produced from secondary fiber is assumed to have as high quality as that produced from virgin fiber does, and thus paper produced by these  $(N + 1)$  firms is homogeneous.<sup>3</sup> As for the market structure of the paper industry, I further assume the  $(N + 1)$  firms act as Cournot competitors,<sup>4</sup> where they simultaneously make decisions on how much paper to produce, taking prices and other firms' production quantities as given.

In the upstream waste recovery industry, I assume there is only one monopoly firm that converts post-consumer products into a secondary input that can be used by downstream firms that engage in recycled paper production. There might also be firms engaging in processing virgin wood and selling the pulp or other intermediate products to the only

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<sup>3</sup>When browsing through different brands of recycled copy papers on Amazon.com, I found statements like “[o]ffers the same hardworking characteristics and brightness as a non-recycled sheet,” (Boise Aspen 100) and “[t]his recycled paper matches the standard non-recycled (virgin fiber) paper performance for visual quality and print performance” (Mohawk 100% Recycled) in product features. Even for other papers of different brands that do not guarantee equivalent performance to non-recycled paper, they usually advocate their recycled products are perfect “for everyday use” (Printworks), or “[a] good choice for general copying, proofing, internal memos and faxes” (Hammermill), indicating high substitutabilities. These are just for office-use copy papers; if taking lower-end products such as packaging paper into consideration whose quality is much lower and less cared by consumers, I deem it reasonable to assume the recycled paper and paper made from virgin fibers have comparable quality and thus homogeneous by nature, which leads them to share a common demand function.

<sup>4</sup>To depict competition in the markets of homogeneous paper products, I choose Cournot quantity competition for several reasons: first, Cournot model is able to predict the perfectly competitive results when  $N$  gets arbitrarily large; second, assuming price competitions, say a Bertrand price competition, may require strong assumptions about whether the marginal cost of producing recycled paper is higher than that of virgin production, for which I do not have enough evidence to make judgement; last, dynamic strategic behaviors like those in the leader-follower model introduce more complexity in calculation and do not necessarily correspond to the real-world situation, and thus for simplicity, I use static Cournot model instead of the dynamic leader-follower one.

downstream firm engaged in virgin production. However, since the downstream recycled paper producers do not purchase from the firms processing virgin wood, those virgin fiber manufacturers are not directly competing with the monopoly firm producing secondary fibers, at least in the short run. Therefore, I do not separately make assumptions about firms processing virgin wood; the cost of acquiring pulp for the virgin paper producers is integrated into a total cost function, which will be discussed in the following subsection.

## 2.1 The Downstream Paper Production Industry

The paper production firms face an inverse industry demand function  $P(Q_1 + Q_2)$ , where  $Q_1$  is the output of virgin paper, and  $Q_2$  is the summed output of all firms engaging in recycled production. The only firm producing paper from virgin fiber bears the total cost of  $c_1(Q_1)$  when producing  $Q_1$  unit of paper. Thus the virgin production firm's objective function is:

$$\max_{Q_1} \pi_1 = P(Q_1 + Q_2)Q_1 - c_1(Q_1).$$

As a Cournot competitor,<sup>5</sup> this firm optimally chooses its output  $Q_1$ , holding quantities produced by the other  $N$  firms constant. The first-order condition is:

$$P'Q_1 + P - c_1' = 0. \tag{1}$$

Implicitly, the FOC represents a reaction function, where the optimal quantity  $\hat{Q}_1$  changes with the other Cournot firms' production level,  $Q_2$  if in the aggregate sense.

The  $N$  firms engaging in recycled paper production have identical cost structures and production functions: using  $q_i$  units of secondary fiber in production, the  $i^{\text{th}}$  firm ( $i = 1, 2, \dots, N$ ) bears the total cost of  $[c_2(q_i) + \phi q_i]$ , where  $\phi$  is the price of secondary fiber purchased from the upstream monopoly, and  $c_2(q_i)$  represents all costs associated with the production process other than the direct expenditure on purchasing input;  $f(q_i)$  is the common production

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<sup>5</sup>Note that although this firm is the only one engaging in primary production, it is by nature a Cournot behavior, since the virgin paper and recycled paper are assumed to be homogeneous and share a common demand function, so this firm is competing with the other  $N$  firms producing recycled paper in the same market. What makes this firm different from others is just that it has a different production function and cost structure.

function shared by all secondary production firms, and thus  $Q_2 = \sum_i f(q_i)$ . Let  $\pi_{2i}$  be the profit of the  $i^{\text{th}}$  firm producing recycled paper, and we get the following objective function for them:

$$\max_{q_i} \pi_{2i} = P \left( Q_1 + f(q_i) + \sum_{j \neq i} f(q_j) \right) \cdot f(q_i) - c_2(q_i) - \phi q_i.$$

Like the optimization of the non-recycled paper production firm, each firm engaging in secondary production chooses production quantity optimally, holding quantities of the other competitors' constant. Note that the production function is already assumed, so that I set  $q_i$ , the amount of input, as the decision variable, instead of the quantity of paper  $f(q_i)$ . Hence, the first-order condition for each firm  $i$  is:

$$P' f'(q_i) f(q_i) + P f'(q_i) - c_2'(q_i) - \phi = 0. \quad (2)$$

Again, equation (2) is also implicitly a reaction function: optimal input amount  $\hat{q}_i$  is determined by those of other secondary production firms and the quantity of virgin paper, i.e.  $q_j (j \neq i)$  and  $Q_1$ . An important assumption should be noted here: when firms producing recycled paper purchase secondary fibers from the upstream monopoly, they are price-takers. Thus  $\phi$  is a constant when they maximize their profit in (2).

Since equation (2) represents  $N$  first-order conditions, the  $(N + 1) \times (N + 1)$  system summarized by (1) and (2) solves the optimal quantities  $\hat{Q}_1$  and  $\hat{q}_i$ , if the demand, production and cost functions jointly ensure a unique set of solutions. Under this existence assumption, apparently,  $\hat{q}_i$  is the same across all secondary production firms due to symmetry, so that the optimal quantities are fully determined by parameters (temporarily including  $\phi$ ).<sup>6</sup> The rest of this paper is based on the existence of equilibrium quantities and the symmetry.

## 2.2 The Upstream Waste Recovery Industry

As the only firm recovering consumer waste, the monopoly determines the optimal price for its product, the secondary fiber. Demand faced by the monopoly firm is identical to

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<sup>6</sup>The specific functional form conditions that ensure a unique set of solutions to (1) and (2) are still being worked. I leave out the formal mathematical proof of symmetric optimal input amount chosen by the  $N$  secondary production firms here so as to keep this paper short.

$\sum_i \hat{q}_i = N\hat{q}_i$ , since the monopoly firm behaves upon optimal choices of the downstream firms. The cost of recovery is also comprised of two parts: one directly from expenditures on acquiring waste paper  $r$ , at the price of  $p_r$ , and the other from the costs occurring during the process of recovery,  $c(r)$ . Thus the recovery firm's objective function is:

$$\begin{aligned} \max_{\phi} \quad & \pi_R = \phi \cdot N\hat{q}_i - c(r) - p_r \cdot r, \\ \text{s.t.} \quad & r = r(N\hat{q}_i) \end{aligned}$$

where  $r$  and  $N\hat{q}_i$  are linked by a production function  $r$ , which represents the recovery firm's technology. As a monopoly in recovering post-consumer waste into secondary input, either price or quantity can be the decision variable in optimization, because once price or quantity is determined, the other one can be computed from the demand function revealed in the downstream competition. Unlike the common practice of setting monopolies as quantity players, I here set the price  $\phi$  as the monopoly firm's decision variable, in order to give a sense that the upstream recovery industry determines the price of secondary fiber, while downstream firms producing recycled paper act as price takers. Thus the first-order condition should follow:<sup>7</sup>

$$N\hat{q}_i + \phi N \cdot \frac{\partial \hat{q}_i}{\partial \phi} - c' r' N \cdot \frac{\partial \hat{q}_i}{\partial \phi} - p_r \cdot r' N \cdot \frac{\partial \hat{q}_i}{\partial \phi} = 0 \quad (3)$$

Clearly, the optimal price  $\hat{\phi}$  is determined in (3), and thus plugging the calculated  $\hat{\phi}$  into (1) and (2), completely parameterized  $\hat{Q}_1$  and  $\hat{q}_i$  can be attained. Thus the equilibrium is fully characterized by these three equations.

### 3 Comparative Statics

#### 3.1 Choosing Functional Forms: a Tractable Example

To simplify the derivation when examining the influences of residential recycling programs, I hereby choose forms for some functions presented in Section 2. Let  $P(Q_1 + Q_2) = a - b \cdot$

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<sup>7</sup>It can be easily shown mathematically that monopoly's quantity decision equals to the price decision: equation (3) is equivalent to the FOC with respect to  $r$ ,  $\frac{\partial \phi}{\partial \hat{q}_i} \frac{\partial \hat{q}_i}{\partial r} N\hat{q}_i + \phi N \frac{\partial \hat{q}_i}{\partial r} - c'(r) - p_r = 0$ , noting  $\frac{\partial \hat{q}_i}{\partial r} N = \frac{1}{r}$ .

$(Q_1 + Q_2)$ , which is the simplest possible linear demand,  $c_1(Q_1) = c_1Q_1$ ,  $c_2(q_i) = c_2q_i$ , and  $c(r) = c_0r$ , where  $c_1$ ,  $c_2$ , and  $c_0$  are constants, representing linear cost functions. I choose these simplest possible functional forms in order to guarantee the unique existence of optimal quantities and symmetry among secondary production firms, and to make the life easier in the derivation.

For the two production functions, I assume both of them exhibit constant returns to scale, and carry the specific forms of  $N\hat{q}_i = \theta_1r$  and  $f(q_i) = \theta_2q_i$ , where  $\theta_1, \theta_2$  are constant in the unit interval  $(0, 1)$ . They can be interpreted as the “survival rates” during the production processes, as Martin (1982) notes in his work, where  $r(z)$  is a scrap survival function of  $z$ , the expenditure on scrap recovery. Unlike his specification, I treat the survival rate as constant so that mathematical presentation will be in its simplest form. Although derivation is simplified under these functional form choices, I need to impose extra restrictions on the parameters so as to ensure interior solutions of optimal quantities:

$$\begin{aligned} (a + c_1)\theta_1\theta_2 &> 2(c_0 + p_r + c_2\theta_1) \\ a(N + 4)\theta_1\theta_2 + 2N\theta_1c_2 + 2N(c_0 + p_r) &> c_1\theta_1\theta_2(3N + 4). \end{aligned}$$

Thus the first-order conditions shown in (1) and (2) now take the specific form of (4) and (5) shown below:

$$a - 2bQ_1 - b\theta_2 \sum_i q_i - c_1 = 0, \quad (4)$$

$$a\theta_2 - b\theta_2Q_1 - b\theta_2^2 \cdot \left( q_i + \sum_{j \neq i} q_j \right) - c_2 - \phi = 0. \quad (5)$$

Solving equations (4) and (5), we get:

$$\hat{q}_i = \frac{(a + c_1)\theta_2 - 2(c_2 + \phi)}{b\theta_2^2(N + 2)}, \quad (6)$$

$$\hat{Q}_1 = \frac{a\theta_2 - (N + 1)c_1\theta_2 + N(c_2 + \phi)}{b\theta_2(N + 2)}. \quad (7)$$

The above two equations imply that Cournot firms’ optimal quantity increase with competi-

tors' costs, while decrease with their own costs.

Substituting  $r'$  for  $\frac{1}{\theta_1}$ , equation (3) takes the specific form as shown below:

$$N\hat{q}_i + \phi N \cdot \frac{\partial \hat{q}_i}{\partial \phi} - c_0 \frac{N}{\theta_1} \cdot \frac{\partial \hat{q}_i}{\partial \phi} - p_r \cdot \frac{N}{\theta_1} \cdot \frac{\partial \hat{q}_i}{\partial \phi} = 0. \quad (8)$$

The intuition behind this equation is rather simple: the monopoly firm chooses the optimal price to equate marginal revenue and marginal cost, so that profit is maximized.

### 3.2 Accounting for the Existence of Residential Recycling Programs

If there were no residential recycling program, firms in the upstream recovery industry would merely be able to purchase waste paper from other paper producing firms, most of which should be from firms engaging in virgin production, rather than those in recycled paper production, because fibers for paper production cannot be recycled for more than 4-7 times.<sup>8</sup> When residential recycling programs start up, these waste recovery firms can get post-consumer papers from a broad range of sources and are able to get waste at very low prices, if not completely free. The reason lies in the different natures of paper producing firms and recycling programs.

Private firms in the paper industry are profit-maximizing entities, and therefore will price the waste paper at least as high as the marginal production cost if they are going to sell it; however, recycling programs can be viewed as public enterprises, so post-consumer waste should not be priced higher than the average cost that occurs during the collecting, transporting and sorting processes. Thus it is reasonable to say that the increasing prevalence of residential recycling programs will cause an exogenous decrease in  $p_r$  in the model presented above. And now it is worth examining how this exogenous change will affect the profits of downstream paper production firms.

To see the influence of  $p_r$  on equilibrium profit of recycled paper production firms,  $\hat{\pi}_{2i}$ , I use the chain rules to take derivatives as used extensively in industrial economics literature,

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<sup>8</sup>“[W]ood fibers can be recycled only four to seven times before they become too short and brittle to be made into new paper.” Thus virgin pulp is always needed. source: <http://www.neenahpaper.com/NeenahGreen/EnvironmentalFAQs>.



e.g. Martin (1982), to examine the partial derivative of  $\hat{\pi}_{2i}$  with respect to  $p_r$ :

$$\frac{\partial \hat{\pi}_{2i}}{\partial p_r} = \frac{\partial \hat{\pi}_{2i}}{\partial \phi} \cdot \frac{\partial \phi}{\partial p_r}. \quad (9)$$

Based on equations (6), (7), (9), and the original objective function  $\pi_{2i}$ , it can be calculated

$$\frac{\partial \hat{\pi}_{2i}}{\partial \phi} = \frac{-2}{b\theta_2^2(N+2)} \left[ \theta_2 \left( a - b\hat{Q}_1 - b\hat{Q}_2 \right) + b\theta_2^2\hat{q}_i - c_2 - \phi \right].$$

Noticing that  $a - b\hat{Q}_1 - b\hat{Q}_2 = \hat{P}$ , the above equation can be further rewritten as

$$\frac{\partial \hat{\pi}_{2i}}{\partial \phi} = \frac{-2}{b\theta_2^2(N+2)} \left( \frac{\hat{\pi}_{2i}}{\hat{q}_i} + b\theta_2^2\hat{q}_i \right). \quad (10)$$

From (10), we see apparently that  $\frac{\partial \hat{\pi}_{2i}}{\partial \phi} < 0$ , since  $\hat{\pi}_{2i}$ ,  $\hat{q}_i$ , and  $b$  are all positive. As for the latter part of the right-hand side of equation (9), I differentiate equation (8) on both sides with respect to  $p_r$ , and it yields:

$$N \cdot \frac{\partial \hat{q}_i}{\partial \phi} \left( 2 \frac{\partial \phi}{\partial p_r} - \frac{1}{\theta_1} \right) = 0.$$

Since  $\frac{\partial \hat{q}_i}{\partial \phi} < 0$  and  $\theta_1 > 0$ , obviously we have:

$$\frac{\partial \phi}{\partial p_r} = \frac{1}{2\theta_1} > 0 \quad (11)$$

Thus from (10) and (11), we easily arrive at

$$\frac{\partial \hat{\pi}_{2i}}{\partial p_r} < 0 \quad (12)$$

Economically, it means higher prices of waste paper will cause recycled paper production firms to profit less. In the context of residential recycling program, it implies that the adoption and increasing prevalence of recycling programs will help firms producing recycled paper earn more profit through driving down  $p_r$ .

### 3.3 On the Ratio of Recycled to Virgin Production

Since recycling programs drive down the price of waste/post-consumer paper, the downstream paper market will become more competitive: price of paper products will decline, as indicated by the following equations:

$$\hat{P} = a - b(\hat{Q}_1 + \hat{Q}_2) = \frac{(a + c_1)\theta_2 + (c_2 + \phi)N}{(N + 2)\theta_2},$$

$$\frac{\partial \hat{P}}{\partial p_r} = \frac{\partial \hat{P}}{\partial \phi} \cdot \frac{\partial \phi}{\partial p_r} > 0. \quad (13)$$

If there is free entry into the downstream recycled paper production industry, then recycling programs will induce potential firms' entry since there is an observed increase in the profit of incumbent secondary production firms (see equation (12)). Then an increase in the number of secondary production firms will also drive down the equilibrium profit for each firm:

$$\hat{\pi}_{2i} = b(\theta_2 \hat{q}_i)^2,$$

$$\frac{\partial \hat{\pi}_{2i}}{\partial N} = 2b\theta_2^2 \hat{q}_i \cdot \frac{\partial \hat{q}_i}{\partial N} < 0 \quad (14)$$

If simply taking the ratio of numbers of firms in secondary production to virgin production, here comes the first measure of ratio, or rather, the long-run measure:

$$R_1 = \frac{N}{1} = N, \quad (15)$$

in the equation above, I assume the monopoly power is hard to threaten, and thus the number of firms producing virgin paper remains one. Clearly, with an increase in  $N$ ,  $R_1$  will increase consequently.

The second measure of the ratio is the ratio of equilibrium quantities produced by secondary production firms and the virgin production monopoly, respectively, which can be

regarded as a short-run measure:<sup>9</sup>

$$R_2 = \frac{\hat{Q}_2}{\hat{Q}_1}. \quad (16)$$

I here hold  $N$  unchanged, that is, there are strong entry barriers in the downstream paper industry, or we only focus on the short run. Now examine the effect of  $p_r$  on  $R_2$ : since  $\frac{\partial \hat{Q}_2}{\partial p_r} < 0$  and  $\frac{\partial \hat{Q}_1}{\partial p_r} > 0$ ,  $R_2$  will increase when the existence of recycling programs drive down  $p_r$ .

## 4 Conclusion

In this paper, I investigate how residential recycling programs influence recycled and virgin paper production. My comparative statics results are based on a simple example using constant return to scale production functions and linear cost functions, together with the monopoly structure in the upstream recovery industry and the Cournot structure in the downstream paper production industry. I find the inception and the continued prevalence of residential recycling programs will increase the profit of firms producing recycled paper, thus encouraging potential firms to enter the recycled paper production sector, if there is little entry barrier. Consequently, there will be more firms engaged in producing recycled paper, which fits well with current environmental protection needs. Even if we merely focus on the short run, a decrease in waste/post-consumer paper price associated with the advent of recycling programs will also drive incumbent firms to produce more recycled paper. Hence, in both measures of the ratio of recycled to virgin production, number of firms and paper produced, there will be an increase in the ratio.

However, all these results summarized above are derived from a model with functions specified in the simplest manner. If further studies are interested in investigating results in a more generalized form, more realistic and complex functions could be assumed. Further research can also explore different assumptions about market structures, and check if the same result still holds. Moreover, this paper does not theorize the relationship between

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<sup>9</sup>After parameterization,  $R_2 = -2 + \frac{2(n+2)(a-c_1)\theta_1\theta_2}{2[(n+4)a-(3n+4)c_1]\theta_1\theta_2+n(c_0+p_r+c_2\theta_1)}$ , the marginal effect of  $p_r$  can also be determined in this explicit form. Other parameters except  $c_0$  will have more complex and ambiguous effects.

the decrease of waste/post-consumer paper price and the advent of residential recycling programs; instead, I use economic logic expressed in the natural language to justify this assumption. This is a shortcoming of this paper, and shall be addressed in a better and more formal fashion in the future.

## References

- [1] Aadland, D., and Caplan, A. J. 2003. Willingness to pay for curbside recycling with detection and mitigation of hypothetical bias. *American Journal of Agricultural Economics* 85(2), 492–502.
- [2] Carroll, W. 1995. The organization and efficiency of residential recycling services. *Eastern Economic Journal* 21(2), 215–225.
- [3] Duggal, V. G., Saltzman, C., and Williams, M. L. 1991. Recycling: An economic analysis. *Eastern Economic Journal* 17(3), 351–358.
- [4] Gaudet, G., and Long, N. V. 2003. Recycling redux: A nash-cournot approach. *Japanese Economic Review* 54(4), 409–419.
- [5] Hong, S., and Adams, R. M. 1999. Household responses to price incentives for recycling: Some further evidence. *Land Economics* 75(4), 505–514.
- [6] Kinnaman, T. C. 2006. Policy watch: Examining the justification for residential recycling. *The Journal of Economic Perspectives* 20(4), 219–232.
- [7] Martin, R. E. 1982. Monopoly power and the recycling of raw materials. *The Journal of Industrial Economics* 30(4), 405–419.
- [8] Matsueda, N., and Nagase, Y. 2008. Economic instruments and resource use in a recyclable product market. *Economics Bulletin* 17(21), 1–10.
- [9] Shy, O. *Industrial organization: Theory and applications*. Cambridge and London: MIT Press, 1995.
- [10] Swan, P. L. 1980. Alcoa: The influence of recycling on monopoly power. *Journal of Political Economy* 88(1), 76–99.
- [11] Tiller, K. H., Jakus, P. M., and Park, W. M. 1997. Household willingness to pay for dropoff recycling. *Journal of Agricultural and Resource Economics* 22(2), 310–320.