HAPPINESS AND TIME PREFERENCE:
AN EMPIRICAL ANALYSIS OF THE EFFECTS OF INDIVIDUAL HAPPINESS AND
INFORMATION ON INTERTEMPORAL CHOICE AND DELAYING GRATIFICATION

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# Table of Contents

1. Abstract 4
2. Introduction 5
   a. Literature Review 7
      i. Research Question 1 12
      ii. Research Question 2 13
      iii. Research Question 3 15
      iv. Research Question 4 17
         1. Review of Discounting Models 17
         2. Proposed Model 22
3. Method 24
   a. Participants 24
   b. Measures 24
      i. Subjective Happiness Scale 24
      ii. Discounting Measure 25
   c. Procedure 26
4. Results 28
   a. Data Overview 28
   b. Research Question 1 32
   c. Research Question 2 33
   d. Research Question 3 34
   e. Research Question 4 36
5. Discussion 37
   a. Research Question 1 37
   b. Research Question 2 40
   c. Research Question 3 43
   d. Research Question 4 47
   e. Limitations and Future Research 55
   f. Conclusions 58
Abstract

Individuals tend to discount the future, meaning they value a reward more in the present than the same reward in the future. Research indicates there are several consequences to the rate one discounts the future, suggesting those who value the future more have better life outcomes. Because of the ramifications of such decisions, this study explores the dynamic of the relationship between discounting and one’s happiness. Participants indicated an indifference point for given present values for given time periods in the future, using both money and periods happiness as hypothetical rewards. These indifference points allow for the calculation of various discounting parameters. Furthermore, the research analyzes if knowing about the positive effects of valuing the future more affected how people made their decisions. The study also includes a further analysis of different discounting functions, as well as proposes functions which incorporate happiness into the model, and discusses the implications of the different models. Although only two tests returned statistically significant results, most measures aligned with the hypotheses.
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Many of life’s choices revolve around questions of time: work now and socialize later or socialize now and work later; eat healthy now and relax later or indulge now and workout later; browse the internet now and wake up tired or go to sleep now and wake up rested; spend money now and budget more tightly later or save now and indulge later. These types of decisions all largely condense to one main idea: does one do something now that has an immediate positive consequence associated with it, or does one engage in an activity that has a potentially larger, later reward associated with it? Although there are evaluative decisions, such as career paths and interpersonal issues, the day-to-day decision making largely focuses on whether or not one delays gratification. Different people face these types of decisions differently; some tend to prefer the instant gratification, whereas others tend to hold off for the larger reward later.

The extent to which people prefer sooner rewards rather than later, larger rewards reflects what is known as time preference, temporal discounting, and delay of gratification, depending on the domain. In general, people tend to prefer sooner, smaller rewards over the delayed rewards. The rate at which one discounts rewards over time is known as the intertemporal discount rate (Doyle, 2013). Discount rates are typically measured by deriving an equilibrium, or indifference point, between a smaller, immediate reward and a later, larger reward. Then, one can calculate the rate at which one discounts the future.
The tone of research is that the more one can opt for the later, larger reward, the better (Frederick, Loewenstein, & O’Donoghue, 2002). Often, the same holds for advice the advice-givers in life dole out: do homework before going out, save money now for more money in the future, be healthy now to be healthier in the future. However, people do not always follow such advice, and often people prefer the lesser, immediate reward (Mischel, Ebbesen, & Zeiss, 1972). Because of the vast array of intertemporal choices one must make, an exploration of the interplay of discounting and one’s overall well-being, or happiness, could provide meaningful insight into human behavior.

Many consider happiness to be the ultimate goal. The philosopher Aristotle writes, “Happiness is something final and self-sufficient, and is the end of an action.” He argues people strive for happiness for its own sake, because it is inherently good and desirable. According to Schoch (2006), utilitarian philosophers Jeremy Bentham and John Stuart Mill assert the optimal society is the one which maximizes happiness, i.e., creating the most happiness for the most people. Even the Declaration of Independence (1776) cites the “pursuit of happiness” as one of the premiere rights of individuals. Although many disagree about what truly begets happiness, the underlying theme that happiness is good and an important, if not supreme, part of human existence is consistent across an array of philosophies.

The following paper is an attempt to study both the psychological and economics components of the preceding issues. Through experimental observation and measurement of individual discount rates and levels of happiness, I examine the underlying psychological components of temporal discounting and its relationship with happiness, as well as incorporating that into different models of economic behavior.
Additionally, I propose a new model incorporating happiness. Specifically, I seek answers to the following questions:

- **Research Question One**: Do individuals discount periods of happiness differently than money?
- **Research Question Two**: Is there a connection between the extent to which someone discounts the future and their happiness?
- **Research Question Three**: Does being told about the positive effects of low discount rates and the adverse effects of high discount rates change individuals’ indicated temporal preferences?
- **Research Question Four**: How does the type of discounting model affect the interpretations and economics importance of the results?

From here, I review the relevant literature in economics and psychology to provide an overview of the study. In the next section, I then go further through each of research questions, posing hypotheses when appropriate. For Research Question Four, I also outline the economic framework and discounting functions through which I analyze the results, as well as pose a new model for analysis. The methods section explains the experimental procedure used to gather data, presented subsequently in the results section. Finally, a discussion section highlights the implications of the findings of this study, as well as faults with the study, future research, and other conclusions.

**Literature Review**

Classical economic thought holds that people are utility maximizers; individuals select the array of goods and activities, known as a consumption bundle, which gives them the greatest utility, subject to budget, time, and other constraints. Every good or
activity has some inherent value or utility tied to it, and different individuals may derive different utilities from the same good. Thus, different people may have entirely different consumption bundles, yet still remain “rational,” where rationality is indicative of choosing the consumption bundle which maximizes utility.

This paradigm also holds for choice over time. For what economists consider rational beings, the ideal choice bundle for the present takes into account both options available in the present and options available in the future, as well as the trade-offs associated with each option. However, as discussed, individuals discount the future, and thus the nominal value of the future option is typically reduced in value in the present. Taking into considerations these discounted future values, perfectly rational consumers maximize their utility by selecting the optimal consumption bundle, given constraints such as income and time, as well as expectations for future income, expenditures, and risk. Finding this optimal point indicates how much a person should consume in the present and how much a person should save for the future, as well as play a large role in non-fiscal decisions, such as exercising and social activities.

However, research indicates people tend to systematically deviate from the basic model of temporal discounting in a wide variety of ways, as well more general decision making models in economics. Kahneman and Tversky (1974) asserted people often use heuristics, or “rules of thumb” to make decisions. They argued people can be influenced to make decisions in certain directions based on the presentation of data. For example, Ariely (2008) argued that many of the values we assign to different goods or experiences are largely relative based on comparisons to other goods or experiences close in mental proximity. Thus, people determined how much something is worth based on related (or even unrelated) stimuli. In more concrete terms, Ariely gave an example of a restaurant
menu. People tended to avoid the most expensive item on the menu but were willing to purchase the second most expensive. Thus, by adding a high-cost item to the menu, and structuring the prices such that the second most expensive item had a high profit margin, businesses increased profits, as people bought more of the high-profit item, even though the food had no inherently higher value; it only changed relative to the other items on the menu. This lack of ability to assess values in absolute terms may well impact how people view components of the consumption bundle at different points in the future.

Prelec and Loewenstein (1991) reviewed these concepts as they apply to discounting to identify several anomalies not predicted by the normative model. Prelec and Loewenstein described asymmetric discounting between gains and losses, whereas losses are devalued less over time relative to gains. The magnitude effect refers to the tendency of individuals to discount small values more than large values. The framing effect simply signifies the importance of the manner in which the choice is presented; some wordings, priming effects, and other factors all play a role in how people perceive intertemporal choices. Finally, there is a delay effect, or dynamic inconsistency, which is explained in detail in the Review of Discounting Models section. However, none of these are factored into traditional models of discounting.

Temporal discounting is subject to other factors as well. According to Metcalfe and Mischel (1999), at any given time, people’s mindset falls on a spectrum of hot and cool. Hot states are dictated by emotion and desires, whereas cold states are more calculating, rational, and objective. The researchers found people put in a hot state became more likely to opt for immediate gratification rather than be able to delay gratification, indicating a higher discount rate for hot states than cool states.
Furthermore, Baumeister, Vohs, and Tice (2007) argued that self-control, and thus the ability to discount at a low rate, is like a muscle, which can get overworked. Although people have the capability of strengthening their self-control, if it is overworked, they may exhibit less self-control in the short run. This led people with a “depleted ego,” as the researchers call it, to favor short-run decisions over choices which are better in the long run. Finally, other researchers have examined the effects on other variables such as age and health and those effects on discount rates (Chao, Szrek, Pereira, & Pauly, 2009).

To account for many of these effects, Fudenberg and Levine (2006) argue they can be included in a dual-self model. They illustrate by splitting the choice into a dual-self decision framework, one can account for short-sighted or impulsive actions. They argue that there are multiple short-run selves who are interested in the present, and a long-run self that tries to dictate the short-run selves. Thaler and Shefrin (1981) posed a similar idea, where the long-run self is the “planner” and the short-run self is the “doer.” In this paradigm, the effects described above influence the extent to which the short-run self is able to execute the plan of the long-run, planning self. The short-run self usually intends on fulfilling the long-run self’s wishes to delay gratification, but certain influences cause a person to deviate from one’s intended path. For example, a person may plan to go on a diet to lose weight, a plan determined by the long-run self. The short run self wants to follow this plan. However, if the person has a stressful day, which depletes one’s ego (Baumeister, Vohs, & Tice 2007), the myopic short run self may be more willing to give in and eat an unhealthy dessert violating the diet. This type of effect can be integrated into a dual-self model analyses.

Evidence also suggests discount rates vary with age. Hartwick (2012) indicated people’s discount rate changes over time, particularly getting lower as one goes through
middle age. Dynan, Skinner, and Zeldes (2002) credited this lowering of discount rates to bequest motives, whereas Shmanske (1997) credited the accumulation of memories as a driving force of utility in old age, and thus one does not rely as heavily on consumption goods for utility. These factors all play into the notion that simply comparing nominal values of rewards over different time frames is an insufficient method of describing how people discount the future. An area that has yet to be explored is the possible interplay of happiness on discount rates.

Traditionally in economics, utility acts as a proxy for happiness. Goods and activities from which people derive value and enjoy comprise positive utility, which theoretically leads to happiness. Because money is a means for obtaining methods of producing utility, it is often assumed that increased income leads to increased happiness. However, research indicates this is not true.

According to Perez-Truglia (2010) and Lee (2009), people acclimate to changes in income and other life circumstances through a phenomenon known as hedonic adaptation. This process explains why winners of the lottery only experience elevated happiness for brief periods of time, and after an accident, new paraplegics generally only suffer a decrease in overall happiness for a relatively brief period of time. Eventually both lottery winners and paraplegics regress back to their set point of happiness, despite the objectively different circumstances.

Frank (2012) argues that an individual’s income relative to one’s peers is far more important than absolute wealth. Furthermore, Deaton and Kahneman (2010) argue people’s subjective well-being does not increase based on wealth past a threshold of $75,000. Using a more advanced data set, Stevenson and Wolfers (2013) contradict Deaton and Kahneman and state happiness does in fact increase with increases in
wealth. However, Stevenson and Wolfers assert the relationship between happiness and wealth is logarithmic in nature. Thus, there are decreasing marginal utilities to increasing wealth; as income increases, each additional incremental increase in wealth leads to a smaller increase in happiness.

Many positive psychologists argue there are other components to one’s happiness than external circumstances. Lyobomirsky (2008) argues that there are three pieces which make up one's level of happiness. According to her research, ten percent of differences in individuals’ level of happiness is accounted for by external circumstances, fifty percent of differences is accounted for by happiness habits, i.e., the mental strategies one uses, and forty percent of differences are determined by a set point. The set point means simply that people are predisposed to a certain level of happiness, and ceteris paribus some people will simply be happier than others. Fujita and Diener (2005) also provide evidence for set-point theory, and argue the set point is partially responsible for hedonic adaptation.

This background to the relevant issues of both happiness and discounting establishes the foundation of my research. Given the above information, I develop my research questions and hypotheses below.

**Research question one: Do individuals discount periods of happiness differently than money?** Research has replaced money as the decision item with other types of goods and services that produce utility. In these studies, researchers have generally found that non-monetary incentives are discounted more steeply than dollar values (Doyle, 2013). For example, one study found consumable goods, such as candy, beer, and soda are discounted at a greater rate than monetary incentives (Estle, Green, Myerson, & Holt, 2007). The same study found that people who abuse drugs discount
drugs more heavily than money as well. Odum and Rainaud (2003) found similar results with drug-addicted individuals. That research also found, for non-drug-addicted people, individuals discounted food and alcohol similarly and both more so than money. Chapman (1995) found that people discount health and money at different rates, and although there was a low correlation between the rates, the difference in discounting was not due to different utility functions for health and money.

Consumables are more directly transferable to pleasure and ergo happiness than money. When one thinks of chocolate, one can more readily envision the utility received from the chocolate, whereas the specific utility from an amount of cash is more nebulous. Further demonstrating this idea, people tended to be happier when they receive a material gift over money of the same value, even though the recipient could hypothetically use the money to buy something more valuable to himself (Kube, Marechal & Puppe, 2012). Thus, consumables are an easier transaction to happiness than is money. The temptation is therefore greater with consumables than with money, making it more difficult to delay gratification. Thus, the hypothesis is that periods of happiness will be discounted more highly than money, i.e., people will delay gratification more for money than for happiness.

**Research question two: Is there a connection between the extent to which someone discounts the future and their happiness?** There are two ways to view a potential relationship between happiness and discounting: first, happiness affects discounting, and second, discounting affects happiness. Both perspectives are considered for this research.

Ifcher and Zarghamee (2011) provided much of the basis for the happiness affects discounting perspective. According to their analysis of the General Social Survey (1973,
1974, 1976) there is correlational evidence that people who agreed more with “live for today” statements (suggesting a higher discount rate) tended to be less happy, even when controlled by factors such as age, gender, income, education, and health. However, the General Social Survey only has one item for happiness and one than appears to indicate discounting tendencies, rather than true attempts to quantify either variable.

Ifcher and Zarghamee (2011) provided some additional evidence of the effects of positive affect on temporal discounting, as they tested participants by inducing a positive mood using comedy clips and found that those in the positive affect group had lower discount rates than those in the control group. Furthermore, according to Ifcher and Zarghamee, a study by Baumeister, Bratslavsky, and Tice (1998) found that positive affect increased time spent studying for a test with no rewards. Additionally, Isen and Reeve (2005) found that positive affect boosted intrinsic motivation. Furthermore, Guven (2012) suggested that happiness affects discounting. According to that research, individual happiness impacted economic decision-making. Through using the positive correlation between sunshine and happiness, Guven found that when people saved more and spent less when happier, as caused by the good weather. Furthermore, happy people evaluated decisions more, and exhibited more self-control. However, since Guven uses sunshine as a proxy for happiness, the research may have measured more positive affect, or a state of mind rather than the trait of happiness into which the current study delves.

Isen and Erez (2002) suggested people in positive moods better mustered motivation to hold out for the larger rewards in the future. Furthermore, Tice and Bratslavsky (2000) explained the phenomenon with the cycle that people in poor moods
give in to impulses in order to make oneself feel better; the short-term reward is an attempt to buoy one’s mood. These all provide evidence that when studied, there will be a negative correlation between happiness and discount rate.

From the discounting affects happiness viewpoint, high temporal discount rates have been linked with negative life outcomes, such as obesity (Epstein, Salvy, Carr, Dearing, & Bickel, 2010; Sellitto, Ciaramelli, & di Pellegrino, 2011; Weller, Cook, Avsar, & Cox, 2008), gambling (Holt, Green, & Myerson, 2003), impulsive consumer behavior (Dittmar & Bond, 2010), drug addiction (Bickel, Koffarnus, Moody & Wilson, 2014) drunk driving (Sloan, Eldred & Xu, 2014), as well as other health behaviors (Daugherty & Brase, 2010). According to Chapman and Elstein (1995), some discount so heavily they neglect to practice preventative health measures to protect against future complications and illness. Additionally, low discounting rates have been linked with greater savings rates (Thaler, 1994), higher SAT scores and greater amounts of education received (Mischel, Ebbesen, & Zeiss, 1972). With these connections between negative consequences and high rates of discounting, one could expect people who discount the future heavily to be less happy than those who discount the future less.

The substantial evidence of positive effects of low discount rates and delaying gratification sets the foundation for Research Question 3. Assuming the ramifications of discounting described above affect utility, one may wonder how information about discounting affects decision-making in terms or intertemporal choice.

**Research question three: Does being told about the positive effects of low discount rates and the adverse effects of high discount rates change individuals’ indicated temporal preferences?** Several paradigms exist to analyze this question, often with contradictory results, i.e., in some circumstances, when
informed of possible repercussions of an action, people adhere to the prescribed advice, and in other circumstances, people tended to ignore or even do the opposite of what is good for them. For example, financial literacy education tended to increase savings rates (Deuflhard, Georgarakos, & Inderst, 2014; Grinstead, Mauldin, Sabia, Koonce, & Palmer, 2011). However, encouragement efforts on the part of the government have been largely insufficient in getting people to save an appropriate amount (Ellen, Wiener, & Fitzgerald, 2012). Furthermore, research by Bushman (1998) indicated warning labels on high-fat products actually increased a person’s preference for the high-fat item. Similarly, Green and Armstrong (2012) found that disclaimers in advertising tended to have no or an adverse effect relative to their intended purpose. Furthermore, a meta-analysis by Cox, Wogalter, Stokes, and Tipton (2012) found that of the 79 cases studied, product warnings increased safe behavior in 53 cases, had no effect in 11 cases, and in 15 cases, the warning led to higher rates of unsafe behavior. In a different meta-analysis, Gallagher and Updegraff (2012) found that messages stated in terms of gains encouraged illness preventing behaviors, whereas messages framed as losses did not have as strong of an effect. Although none of these studies explicitly address the notion of temporal discounting, all the listed issues had a temporal dimension to them; make a decision in the present that has the potential to impact one’s life in the future. Thus, there is room to examine whether or not an individual knowing about the aforementioned consequences of high discount rates will affect individuals’ indicated preferences, even in the short run.

In the studies analyzed, people followed the intention of the information more when the information was presented as educational rather than an attempted to change behavior. Thus, this study addresses this research question using education rather than
persuasion as the main method of investigating potential changes. Therefore, I hypothesize when informed of the positive effects of being able to delay gratification and the adverse effects of preferring immediate gratification, individuals will reduce their discount rates.

Within this question, this study looks at an exploratory analysis of whether or not one’s happiness affects how being told about the repercussions of discounting affects the change in their indications about their discounting. The study investigates the possibility that how happy one is may influence how one follows the “doing what one is told is good for oneself” principle. Little to no research exists regarding this question, but it is nonetheless a relationship worth exploring. A person who is more capable of integrating information into one’s lifestyle may be more likely to be happy, as that person may have better life outcomes than those who disregard information.

The previous three research questions all deal with overall effects of discounting. However, there are different ways in which to measure discounting, and the model imposed on individuals impacts the interpretations of the results. Furthermore, any measurable effects in the previous research question begs the question of economic significance, which lead to the final research question, which overarches the previous three.

Research question four: How does the type of discounting model affect the interpretations and economics importance of the results?

Review of discounting models. Theoretically, temporal decision making is the way people value outcomes at different time horizons (Wittman & Paulus, 2009). A temporal discount rate can be determined by finding the indifference point for a certain immediate reward, future reward, and time frame (Critchfield & Kollins, 2001).
However, based on the assumptions made about how people discount, different models obtain different discounting parameters, which impact how the model predicts behavior.

There are several mathematical approaches to model temporal discounting (Doyle, 2013). The most basic of approaches is using a constant discounting formula, in which the rate at which one discounts is not dependent on time, modeled by the following formula

\[ P = F - Td \] 

(1)

where \( P \) is the present value, \( F \) is the future value, \( T \) is the time delay, and \( d \) is the discount rate. This model suggests that for every time period, a constant value is subtracted from the nominal future value of the reward. Although this can be useful for instantaneous problems, using it to model multiple time-frames does not model the data well (Frederick, Loewenstein, & O’Donoghue, 2002), as people tend not to discount at a constant rate.

An improvement on the constant discounting function is an exponential function (Doyle, 2013; Frederick, Loewenstein, & O’Donoghue, 2002). This is modeled by the either the function

\[ P = F \times \left[ \frac{1}{1 + r} \right]^t \] 

(2)

or the more commonly known

\[ F = P \times (1 + r)^t \] 

(3)

where \( F \) is the future value, \( P \) is the present value, \( r \) is the discount rate, and \( t \) is the time. This is the same type of function accountants use to estimate the future value of assets and liabilities. This type of function suggests that individuals reduced the value of the future value at a rate proportional to the difference in time between the present and
the future. As time increases, the amount discounted from the future value increases, but not as much as it does in the as the discount from the previous period; although the rate remains the same, the nominal value associated with that rate decreases over each additional time period. The model was proposed by Paul Samuelson, and is commonly referred to as the discounted utility model. Although the discounted utility model has been used to estimate discounting, it has not been confirmed by scientific research. The model essentially states that the present value of a reward is a function of the nominal future value (i.e., what the value of the reward would be if it were received today) multiplied by a discount factor $\left(\frac{1}{1+r}\right)^t$. When $r$ is greater than zero, the discount factor will be a positive number less than one. Thus, as $t$ increases, the discount factor drops toward zero. This means the greater the time difference between the present and future reward, the greater the difference between the present and future value. This is how many people tend to think of discounting, and thus this model is an accessible way to conceptualize temporal choice.

However, the discounted utility model has not been shown to exhibit all the characteristics of individuals’ time preference. One such problem with the discounted utility model is that the discounting parameter $r$ gives the same amount of discounting across different intervals of the same length. For example, an $r$ of 0.25 indicates that an individual would be indifferent between an immediate reward of $800 and a future reward of $1000 delayed by one time period (for the sake of the example, consider the time period to be one year). The same $r$ of 0.25 would indicate that a person would be indifferent between the reward of $800 in five periods and $1000 in six periods. Along a similar vein, consider and individual who prefers $100 today to $105 tomorrow.
According to the discounted utility model, the same individual would prefer the $100 in one year to $105 in a year and one day. However, research indicates this is not true; the person may very well reverse preferences when both options are delayed by a year. People tend to discount the immediate future more heavily than the distant future, relative to the discounted utility model (Thaler & Loewenstein, 1989). This is known as dynamic inconsistency. Consequently, the discounted utility model underestimates utility in the near future and overestimates utility in the distant future.

One function that attempts to alleviate this disparity is the hyperbolic discounting function. In this case, the function is modeled by

$$P = \frac{1}{(1+at)^\beta} \cdot F$$

(4)

where $\alpha, \beta$ are constants greater than zero. In this case, increasing $\alpha$ causes the discount factor to increase toward one as $\alpha$ approaches infinity, whereas increasing $\beta$ causes the discount factor to decrease. Therefore, a higher $\alpha$ means a lower discounting rate, whereas a higher $\beta$ means a higher discounting rate. However, the $\alpha$ term appears twice in the formula. The $\alpha$ component of $(1+at)$ causes the discount factor to decrease as $\alpha$ gets larger, whereas the $\alpha$ component of the exponent $\beta/\alpha$ causes the discount factor to increase as $\alpha$ gets larger. Thus, the two uses of $\alpha$ in the formula to some extent counteract each other. As stated, however, letting both $\alpha$'s vary, increasing $\alpha$ causes the discount factor to increase toward one, but because of the counteracting of the $\alpha$ in the exponent, not as quickly as it would otherwise. As a result, $\alpha$ captures a more long term effect of the discounting model, whereas $\beta$ captures a more short-run perspective. This means that influencing $\beta$ has a greater effect on the value of more immediate rewards, compared to $\alpha$ which will affect the reward values in the long term. The advantage of the
hyperbolic function is that it captures individuals’ tendency to discount heavily in the short run and less so as the later reward is moved further into the future (Frederick, Loewenstein, & O’Donoghue, 2002).

This ability of the hyperbolic function to change discounting over separate time horizons helps solve the problem posed earlier in which different time intervals of the same length are preferred differently by individuals. The hyperbolic discounting function allows for a larger discount effect in the early time periods, but one that levels out over time. Thus, in the scenario above where an individual was indifferent between $800 today and $1000 after one time period, they may not demonstrate the same indifference between $800 in five time periods and $1000 in six time periods, as the person would have using the discounted utility model.

Although the hyperbolic discounting function may be a better model for human behavior, the function is difficult to estimate empirically because of the two parameters, with $\alpha$ occurring twice. Without extremely well-constructed and multidimensional data, the hyperbolic function is an inaccessible model for much research. Several models have been proposed to address this. According to Frederick, Lowenstein and O’Donoghue (2002), the Harvey model constrains $\alpha$ to one and lets $\beta$ vary. Conversely, the Herrnstein and Mazur model fixes the ratio $\beta/\alpha$ to one, and lets the other $\alpha$ vary, although it is referred to as $\omega$. Thus, the two functions are as follows:

$$Harvey: P = F \times (1 + t)^{-\beta}$$

$$Herrnstein and Mazur: P = F \times (1 + \omega t)^{-1}$$

One other function that mimics the effects of the hyperbolic function is the quasi-hyperbolic function. This function is
\[ P = \begin{cases} F & \text{for } t = 0, \\ F \beta \delta^t & \text{for } t > 0. \end{cases} \]  

(7)

where \( \beta \) is the immediate discounting factor, and \( \delta \) is the component of discounting that is affect by time, as evident by being raised to the \( t \) power. \( \delta \) is shorthand for the discounting factor \( \frac{1}{1+r} \), but is abbreviated for clarity’s sake. In the quasi-hyperbolic discounting function, the future reward is always discounted by two factors. The \( \beta \) parameter represents an immediate drop in the first period, but never changes based on time (Meyer, 2012). The \( \delta^t \) component does change with the delay of the reward, and thus it is more influential relative to \( \beta \) as \( t \) grows. Like the hyperbolic function, the quasi-hyperbolic function addresses the problem of dynamic inconsistencies by allowing for short and long run effects in the function but is an easier equation with which to work for estimations, as shown in the Results section.

This data analysis of this paper strictly utilizes the discounted utility (exponential) model and the quasi-hyperbolic. To summarize the various parameters used, see Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model</th>
<th>Increasing the parameter value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>Discounted Utility</td>
<td>Increases discounting of the future</td>
</tr>
<tr>
<td>( \rho ) (long-run portion)</td>
<td>Quasi-Hyperbolic</td>
<td>Increases discounting of the future</td>
</tr>
<tr>
<td>( \beta ) (short-run portion)</td>
<td>Quasi-Hyperbolic</td>
<td>Decreases discounting of the future</td>
</tr>
</tbody>
</table>

**Proposed model incorporating happiness.** In addition to examining the discounted utility model and quasi-hyperbolic model, I propose a model incorporating happiness into the models. As hypothesized, the models integrate the idea of a negative correlation between happiness and discounting rates. Although the research does not
create a causal link between happiness and discounting models, this model assumes that increased levels of happiness decreases one’s discount rates. For the discounted utility model functional form, I propose Equation 8.

\[ P = F \left( \frac{1}{1 + [\mu + \alpha \ln(H)]} \right)^t \]  

(8)

In Equation 10, as before \( P \) is the present value of the reward in time period \( t \), \( F \) is the nominal value of the future reward, \( H \) is the individual’s level of happiness, as measured by the Subjective Happiness Scale, and \( \alpha \) is the coefficient of happiness, and \( \mu \) is the exogenous component of discounting. The behavior of this function is explored further in the discussion section.

For the function incorporating happiness analogous to the quasi-hyperbolic function, I propose Equation 11.

\[
P = \begin{cases} 
F & \text{for } t = 0, \\
F \left( \varphi + \kappa \ln(H) \right) * \left( \frac{1}{1 + \rho} \right)^t & \text{for } t > 0.
\end{cases}
\]

(9)

The only addition this function makes to the quasi-hyperbolic function is changing \( \beta \) to \( \varphi + \kappa \ln(H) \), where \( H \) is again the Subjective Happiness Score, and \( \varphi \) and \( \kappa \) are the analogous components to \( \mu \) and \( \alpha \). Again, as \( H \) increases, what would have been the \( \beta \) term increases, thereby decreasing the amount of discounting. Thus, in this function as well, there is a negative correlation between happiness and the discount rate.

I made the decision to include the happiness term in the formerly \( \beta \) parameter as opposed to the \( \rho \) term because although people tend to one level of happiness, it is felt in the short-run, and thus one’s level of happiness at any given moment is more likely to affect the short-run component of discounting than the long-run parameter. People who are less happy are likely more inclined to boost short term morale rather than consider
the difference between two future selves. Additionally, as the time delay increases, uncertainty increases. People who are happy now can fairly readily predict they will be happy in the immediate future and vice versa for those who are unhappy. However, predicting future happiness is more difficult, simply because people do not know their future circumstances. This uncertainty makes current happy less likely to affect future discounting and more likely to affect the immediate discount because happiness is felt in the present.

Given my research questions, I developed a methodology to study the dynamics described above. I now present the survey and experimental procedure used to elicit discount functions and allow for further analysis of my research questions.

**Method**

**Participants**

A total of 74 respondents participated in the study via an online survey provider. Students in an Introductory to Psychology course at a small, liberal arts college enrolled as participants in the study. Participants took part in the study in exchange for credit for their class. No demographic information was collected.

**Measures**

**Happiness.** To assess an individual’s level of happiness, the experiment has the participants answer the Subjective Happiness Scale. This seven point scale measures an individual’s “set-point” of happiness, or how happy an individual tends to be (Lyubomirsky & Lepper, 1999) in four questions answered on a Likert scale. Because this study is more concerned with overall happiness, the measure of happiness of interest is the Subjective Happiness Scale, because it measures an individual’s tendencies rather than the current state of happiness. Other measures of happiness tend to focus on an
individual’s current state, rather than the long-run characteristics of individuals. This study is concerned with how patterns of temporal discounting and overall happiness are related, rather than how short-run happiness affects short-run decision making. See Appendix A for the full Subjective Happiness Scale.

**Discounting.** Because this study measures periods of happiness, a previously unexplored incentive, I created a new measure to assess discounting for periods of happiness. In order to keep the experiment consistent, I developed an analogous measure for money. I developed a list of fifteen dollar amounts, ranging from $10 to $1000, fifteen periods of happiness, ranging from five days to one year, and fifteen time periods ranging from three days to two years. Using a random number generator, I assigned the fifteen dollar amounts to the fifteen time periods twice, so as to create two different surveys. I also randomly assigned the fifteen periods of happiness to the fifteen time delays, again twice to create two surveys for happiness. Thus, to assess discounting, I had four different surveys, two to measure periods of happiness and two to measure money.

In the survey, the study posed hypothetical questions using the values obtained from the method described above for which the participant would indicate his or her indifference point for a given amount of time in the future in exchange for a given value of a reward today¹. Two examples are below:

---

¹ Research surrounding discounting has found that there is no significant or systematic difference between offering participants real or hypothetical rewards in temporal discounting studies (Johnson & Bickel, 2002; Locey, Jones & Rachlin, 2011). From small to moderately large sums ($0.05 to $250), participants in within-subjects designs picked corresponding payout schedules for both real and hypothetical rewards. In an extension, Ifcher and Zarghami used a method in which participants were told one of their responses would be randomly selected. The participant would then receive their selected preference for that item, and thus the participant should treat all items like they would be selected. A similar methodology which has been used by Albrecht and Weber (1995), is suggested by Frederick, Loewenstein and O’Donohue (2002) and fully explored by Hardisty, Thompson, Krantz, and Weber (2013).
How much is the minimum you would accept in ten weeks instead of $45 today?

How much is the minimum number of days of happiness you would accept in seven months instead of 8 days of happiness starting today?

Although 52% of studies use choice responses to elicit a participant’s discount rate, 31% use a matching, or fill-in-the-blank procedure similar to the one described above (Hardisty, Thompson, Krantz, & Weber, 2013). In this instance, there are two main reasons to go with the matching design. First, because there is a manipulation, having participants give an exact response allows for a greater opportunity to exhibit changes in thought patterns. When confined to choices, participants may not be able to show they have changed, simply because they so heavily preferred one option to the other that even with the manipulation, their preference does not change (e.g. if asked to choose between $100 today and $200 in a year, and the person strongly prefers the $200 in a year option, a manipulation will not likely change that). Secondly, because there are not well-established guidelines for periods of happiness discounting, having participants give their own indifference points rather than trying to impose them on the subjects is more likely to give meaningful results. Matching has also been found to give fewer demand characteristics, as well as fit the hyperbolic discounting model better (Hardisty et al., 2013). This established a baseline for comparing discount rates.

Procedure

For all hypotheses, the set of participants went through a series of tests and manipulations. The sample participants came from Psychology 111: Introduction to Psychology courses at the College of St. Benedict and St. John’s University. The participants received course credit for their participation. Each participant completed several components in order to address the research questions via an online survey.
First, the participants took the Subjective Happiness Scale. Then, the participants read a brief explanation of temporal discounting so that they understood the discounting exercise. The participants responded to thirty-four of these questions in the first portion of the study. The directions informed participants that the first four items would not be used for the study. These initial questions allowed the participant to develop some stabilized thought process for the study. Then, the participants took either of the two survey elements for money, followed by either of the two survey elements for happiness. Because the software used could not randomize the survey order given other randomization constraints, which survey the participants took was assigned based on course instructor. See Table 2 for a further illustration of survey ordering.

Table 2

*Possible Survey Orders*

<table>
<thead>
<tr>
<th>Survey Orders by Instructor</th>
<th>Received Before Treatment</th>
<th>Received After Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Instructor 1</td>
<td>Money Survey A</td>
<td>Money Survey B</td>
</tr>
<tr>
<td></td>
<td>Happiness Survey A</td>
<td>Happiness Survey B</td>
</tr>
<tr>
<td>Instructor 2</td>
<td>Money Survey A</td>
<td>Money Survey B</td>
</tr>
<tr>
<td></td>
<td>Happiness Survey B</td>
<td>Happiness Survey A</td>
</tr>
<tr>
<td>Instructor 3</td>
<td>Money Survey B</td>
<td>Money Survey A</td>
</tr>
<tr>
<td></td>
<td>Happiness Survey A</td>
<td>Happiness Survey B</td>
</tr>
<tr>
<td>Instructor 4</td>
<td>Money Survey B</td>
<td>Money Survey A</td>
</tr>
<tr>
<td></td>
<td>Happiness Survey B</td>
<td>Happiness Survey A</td>
</tr>
</tbody>
</table>

The participants then engaged in either the control portion of the survey or the manipulation portion, decided via random assignment of the survey program used. In the manipulation, participants read a short article on the importance of delaying gratification. As described in the literature, being able to delay gratification is beneficial to one’s health, relationships, as well as happiness (Ifcher & Zarghamee, 2011). This manipulation was used to determine if people who are instructed to discount the future
less in fact discounted the future less. The control group read an article of similar length and reading level, but of an irrelevant topic (classical conditioning). See Appendix B for full readings.

To ensure they read their article, the instructions warned the participants that they will have to answer questions after reading the article. After reading the article, they filled out a question box asking them to essentially summarize the article.

Following the quiz, the participants then completed the two remaining survey elements they had yet to complete, again first for money and secondly for periods of happiness.

Finally, similar to Hardisty et al. (2013), the participants briefly described their decision making process in the previous exercise to allow for some qualitative analysis. The insight provided in this response could play a critical role in the interpretation of the results. Appendix B provides one example of a survey participants could receive, although as Table 2 indicates, the question set order for different participants would vary.

**Results**

**Data Overview**

Standard outlier analysis dictated which results I eliminated from the data set. The method, described by Zijlstra, Ark, & Sijtsma (2007) takes the interquartile range (the 25th percentile score subtracted from the 75th percentile score) and multiplies it by 1.5. This number is added to the third quartile (75th percentile score) to find the upper fence and subtracted from the first quartile (25th percentile score) to find the lower fence. Responses outside the fence are considered outliers, and are thus deleted from the dataset and excluded from analysis.
After the outlier analysis was complete, several participants had portions of the survey with very few or zero responses for analysis. I deleted any section (i.e., one of the four elements of the survey) with fewer than five responses. Five is an arbitrary value selected to avoid biasing any individual survey element score toward any one response. This led to twelve participants being entirely removed from the study, leaving 62 participants. Some participants had all their responses deleted for either money or happiness questions but not both. I kept those individuals in the survey but did not analyze their data across variables. A closer look at the individual data points indicates that these individuals either did not understand the task at hand, or more likely, did not take the task seriously and provided meaningless responses.

A summary of the measures can be found below. The discount rates elicited are daily discount rates. The quasi-hyperbolic metrics fit the formula $P = F \times \beta \times \delta^t$, where $\delta^t \equiv \left[ \frac{1}{1+\alpha} \right]^t$. A standard $\alpha$ of .05 provided the benchmark for statistical significance. To obtain $\beta$ and $\delta$, I ran a linear regression using the below function.

$$P = F \times \beta \times \delta^t$$

$$\ln(P) = \ln(F) + \ln(\beta) + \ln(\delta^t)$$

$$\ln(P) - \ln(F) = \ln(\beta) + \ln(\delta^t)$$

$$\ln\left(\frac{P}{F}\right) = \ln(\beta) + t \times \ln(\delta)$$
From this point, linear regression can be used to approximate estimates for $\beta$ and $\delta$, given one already has $P$, $F$, and $t$.

The regression results produced a result like the following:

Table 3

*Sample Participant Regression Results*

<table>
<thead>
<tr>
<th>Variable</th>
<th>B</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.570</td>
<td>[-1.422,.282]</td>
</tr>
<tr>
<td>Time (d)</td>
<td>-0.00264</td>
<td>[-.00559,.000]</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.239</td>
<td></td>
</tr>
</tbody>
</table>

Table 3 is the regression output for a participant for Money Survey A. This particular participant had fourteen of the fifteen possible responses included in the data analysis.

The time coefficient of -0.0026359 indicates that the natural logarithm of $\delta$ is estimated to be -0.0026359. Thus, the following method solves for $\rho$, where $x$ is the coefficient of time given by the regression output.

\[
\ln(\delta) = \ln\left(\frac{1}{1 + \rho}\right) = x
\]

\[
e^x = e^\ln\left(\frac{1}{1 + \rho}\right)
\]

\[
e^x = \frac{1}{1 + \rho}
\]

\[
1 + \rho = \frac{1}{e^x}
\]

\[
\rho = \frac{1}{e^x} - 1
\]

Using Equation 14, the $\rho$-value for this participant for Money Survey A is 0.0026393.

For the $\beta$ term, the regression output gives the natural logarithm of $\beta$, shown in the _cons value in the regression. Thus, simply finding $e$ raised to the power of the _cons value of the regression output gives the $\beta$ term for an individual for a specific survey element.
I found the exponential discount for an individual for a survey element by finding the mean of the exponential discount rates for each survey element. This data give the descriptive statistics found below in Table 4. The descriptive statistics are for the first set of money and periods of happiness to which the participants responded.

Table 4

Descriptive Statistics

<table>
<thead>
<tr>
<th>Measure</th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective Happiness</td>
<td>61</td>
<td>2.25</td>
<td>6.75</td>
<td>5.05</td>
<td>1.03</td>
</tr>
<tr>
<td>Exponential Discount Rate, Money</td>
<td>57</td>
<td>.0068</td>
<td>.059</td>
<td>.023</td>
<td>.014</td>
</tr>
<tr>
<td>Quasi Hyperbolic $\rho$, Money</td>
<td>58</td>
<td>-.0017</td>
<td>.012</td>
<td>.0028</td>
<td>.0024</td>
</tr>
<tr>
<td>Quasi Hyperbolic $\beta$, Money</td>
<td>58</td>
<td>.00</td>
<td>.90</td>
<td>.54</td>
<td>.20</td>
</tr>
<tr>
<td>Exponential Discount Rate, Happiness</td>
<td>57</td>
<td>.0019</td>
<td>.052</td>
<td>.016</td>
<td>.010</td>
</tr>
<tr>
<td>Quasi Hyperbolic $\rho$, Happiness</td>
<td>53</td>
<td>-.00085</td>
<td>.014</td>
<td>.0024</td>
<td>.0024</td>
</tr>
<tr>
<td>Quasi Hyperbolic $\beta$, Happiness</td>
<td>53</td>
<td>.33</td>
<td>.96</td>
<td>.60</td>
<td>.168</td>
</tr>
</tbody>
</table>

For subjective happiness scores, participants tested between scores of 2.25 and a maximum of 6.75, with a mean happiness score of 5.053. A histogram of the Subjective Happiness Scores is in Figure 1.

![Subjective Happiness Histogram](image)
In the first set of items for money the participants received, they indicated daily exponential discount rates ranging from 0.00681 to 0.05997, with a mean discounting rate of 0.02317. From the regression results assuming the quasi-hyperbolic function for the same set of money items, participants indicated $\rho$ values between -0.00170 and 0.01245, with a mean of 0.00284, and $\beta$ values between 0.0000 and 0.89792, with a mean of 0.54408. In the first set of items for periods of happiness the participants received, they indicated daily exponential discount rates ranging from 0.00191 to 0.05212, with a mean discounting rate of 0.1613. From the regression results assuming the quasi-hyperbolic function for the same set of periods of happiness items, participants indicated $\rho$ values between -0.00085 and 0.01408, with a mean of 0.00244, and $\beta$ values between 0.33230 and 0.96321, with a mean of 0.60420.

Given these overall results, I can test the specific hypotheses outlined in the Introduction.

**Research Question 1**

The first test, paired t-test for differences of means, determined if a difference exists between the extent to which people discount for periods of happiness and money. Table 5 shows the following results. A statistically significant difference was found between an individual’s exponential discount rate for money and for happiness, $t (54) = -3.41, p = .001$. The mean difference between the discount rates ($M = 0.00587, SD = 0.0125$) is statistically greater than zero.

No statistically significant difference was found between an individual’s $\rho$ in the quasi-hyperbolic function for money and for happiness, $t (54) = 1.24, p = .220$. The mean difference between the discount rates ($M = 0.00285, SD = 0.00167$) is not statistically different from zero.
No significant difference was found between an individual’s $\beta$ in the quasi-hyperbolic function for money and for happiness, $t(54) = -1.88$, $p = .066$. The mean difference between the discount rates ($M = -0.0453$, $SD = 0.176$) is not statistically different from zero.

Table 5

<table>
<thead>
<tr>
<th>Paired Samples Test</th>
<th>Paired Differences</th>
<th>t</th>
<th>Sig. (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money Exponential Rate $r$ - Happiness Exponential Rate $r$</td>
<td>0.00586</td>
<td>0.0125</td>
<td>3.40</td>
</tr>
<tr>
<td>Money $\rho$ - Happiness $\rho$</td>
<td>0.00028</td>
<td>0.0016</td>
<td>1.24</td>
</tr>
<tr>
<td>Money $\beta$ - Happiness $\beta$</td>
<td>-0.04529</td>
<td>0.1757</td>
<td>-1.88</td>
</tr>
</tbody>
</table>

An analysis of the effect size confirms the findings above. The Cohen’s $d$ for the exponential rate from the discounted utility model is .47, which is a medium effect size, which the statistical significance of the test picked up. However, for the $\rho$ and $\beta$ parameters from the quasi-hyperbolic discounting function, the effect sizes are .18 and .26, which are both considered small effect sizes and would take a large degree of power to provide statistically significant results.

Research Question 2

A Pearson correlation is the best tool to determine the relationship between a subject’s Subjective Happiness Score and the participant’s various components of discounting. Both base score and the natural logarithm of the subjective happiness score were calculated against the base value of the different components of discounting. Table 6 shows the results.
None of the results presented in the table are statistically significant at any significance level. However, all of the correlations for money, the $r$ for periods of happiness in both the base value and natural logarithm of happiness, and the $\rho$ for the head

Table 6

**Correlation Table**

<table>
<thead>
<tr>
<th></th>
<th>Money DUM Discount Rate</th>
<th>Money Quasi Hyperbolic $\rho$</th>
<th>Money Quasi Hyperbolic $\beta$</th>
<th>Happiness DUM Discount Rate</th>
<th>Happiness Quasi Hyperbolic $\rho$</th>
<th>Happiness Quasi Hyperbolic $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subjective Happiness</td>
<td>-.095(^1)</td>
<td>-.008(^1)</td>
<td>.096(^1)</td>
<td>-.043(^1)</td>
<td>.039</td>
<td>-.105</td>
</tr>
<tr>
<td>Natural Log of</td>
<td>(-.096(^1))</td>
<td>(-.031(^1))</td>
<td>.103(^1)</td>
<td>(-.076(^1))</td>
<td>(-.007(^1))</td>
<td>-.105</td>
</tr>
</tbody>
</table>

\(^1\) Results are consistent in direction with hypotheses

natural logarithm for happiness are consistent with the predicted theory. The relatively small correlations (ranging from -.007 to -1.05) indicate the effect size for these correlations are quite small, and thus need a test with extremely high power to statistically prove their correlations.

I also tested the natural logarithm and base value of happiness against the natural logarithm of the discounting parameters but obtained smaller correlations with smaller p-values and fewer results consistent with my hypotheses.

**Research Question 3**

Six independent t-tests compared the group reading the article on delay of gratification (the treatment) and the group reading the article on classical conditioning (the control). A significant difference between the treatment and control group was found for the difference in $\beta$ for money, $t(55) = -2.40$, $p = .020$. The reduction in
discount rate for the treatment group (M = 0.032, SD = 0.146) than for the control group (M = -0.0617, SD = 0.138). The rest of the results are not statistically significant, and thus I cannot conclude the intervention impacted the participants’ discounting. Table 7 shows these results.

Table 7

<table>
<thead>
<tr>
<th>Differences between Treatment and Control Groups</th>
<th>Mean Difference</th>
<th>t</th>
<th>p-value (2-tailed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DUM r, Money</td>
<td>-0.00372</td>
<td>-1.21</td>
<td>.231*</td>
</tr>
<tr>
<td>ρ, Money</td>
<td>0.00017</td>
<td>0.41</td>
<td>.686</td>
</tr>
<tr>
<td>β Money</td>
<td>0.0934</td>
<td>2.40</td>
<td>.02</td>
</tr>
<tr>
<td>DUM r, Happiness</td>
<td>0.00155</td>
<td>0.72</td>
<td>.474</td>
</tr>
<tr>
<td>ρ, Happiness</td>
<td>-0.00053</td>
<td>-0.99</td>
<td>.328*</td>
</tr>
<tr>
<td>β, Happiness</td>
<td>0.0220</td>
<td>0.57</td>
<td>.575</td>
</tr>
</tbody>
</table>

* Result is consistent with predicted direction of change

As indicated in Table 7, four of the six tests aligned with the predicted results, including the statistically significant result for the β for money. The effect sizes for the effects that are consistent with the predicted results are consistent with the statistical significance of the results as well. For r for money, ρ for periods of happiness, and β for periods of happiness, the Cohen’s d values are .33, .28, and .15, respectively. These again are all relatively small effect sizes. These can be compared to the .66 effect size of the β for money, which is a moderate effect size.

For the exploratory question of how one’s happiness affects how being told about the effects of discounting influences one’s change in discount rates, the study used a three-way ANOVA to determine the interaction effect. A summary of the results for each of the six parameters is found below in Table 8. A median split of the data categorized each participant’s happiness into either the first half of happiness score or the second half of happiness scores. The study also tested for interactions with happiness split into
tritiles and quartiles, but the results were less cohesive and had greater p-values than for the median split. None of the results are statistically significant.

Table 8
Interaction of Happiness and Pre- and Post-Test Results

<table>
<thead>
<tr>
<th>Category</th>
<th>F</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential Rate, r, Money</td>
<td>0.985</td>
<td>0.325</td>
</tr>
<tr>
<td>Exponential Rate, r, Happiness</td>
<td>0.171</td>
<td>0.681</td>
</tr>
<tr>
<td>Rho, Money</td>
<td>0.258</td>
<td>0.614</td>
</tr>
<tr>
<td>Rho, Happiness</td>
<td>0.339</td>
<td>0.563</td>
</tr>
<tr>
<td>Beta, Money</td>
<td>2.056</td>
<td>0.157</td>
</tr>
<tr>
<td>Beta, Happiness</td>
<td>2.567</td>
<td>0.116</td>
</tr>
</tbody>
</table>

These results suggest that happiness does not impact the extent to which information changes one’s time preference.

Research Question 4

Most of the quantitative results for Research Question 4 are incorporated into the other three research questions via the differences between the discounted utility model and the quasi-hyperbolic model. The additional component for Research Question 4 is for the estimated properties of the proposed discounting functions which incorporate happiness. For the function analogous to the discounted utility model,

$$P = F \left( \frac{1}{1 + [\mu + \alpha \cdot \ln(H)]} \right)^t$$  \hspace{1cm} (8)

the estimated regression results are

Table 9
Regression Results Predicting r from Happiness

<table>
<thead>
<tr>
<th>Variable</th>
<th>B</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant ((\mu))</td>
<td>.033</td>
<td>[.00698,.0585]</td>
</tr>
<tr>
<td>Natural Logarithm of Happiness</td>
<td>-0.0061</td>
<td>[-.0222,.0099]</td>
</tr>
<tr>
<td>R-Squared</td>
<td></td>
<td>.011</td>
</tr>
</tbody>
</table>
For the function analogous to the quasi-hyperbolic function,

\[
P = \begin{cases} 
  F & \text{for } t = 0, \\
  F * [\varphi + k * \ln(H)] * \left(\frac{1}{1 + \rho}\right)^t & \text{for } t > 0,
\end{cases}
\]  

(9)

the estimated regression results are

Table 10

<table>
<thead>
<tr>
<th>Variable</th>
<th>B</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant ((\varphi))</td>
<td>.483</td>
<td>[.145, .820]</td>
</tr>
<tr>
<td>Natural Logarithm of Happiness ((\kappa))</td>
<td>.0447</td>
<td>[-.165, .255]</td>
</tr>
<tr>
<td>R-Squared</td>
<td>.00329</td>
<td></td>
</tr>
</tbody>
</table>

A further analysis of these regression results is found in the discussion section.

**Discussion**

**Research Question One: Is there a connection between the extent to which someone discounts the future and their happiness?**

I first hypothesized there exists a negative correlation between one’s happiness and the individual’s discount rate. Overall, the mean level of subjective happiness in the study was 5.05, which is slightly higher than the average for college students found in Lyubomirsky (1999). I did not confirm the hypothesis of a negative correlation between discounting and happiness; none of the hypothesis tests showed statistical significance. However, by-and-large, the theory and hypotheses predicted the signs of the correlations. As shown in Table 2, decreasing \(r\) and \(\rho\) decreases the amount of discounting, whereas increasing \(\beta\) decreases the amount of discounting. Thus, negative correlations between happiness and \(r\) and \(\rho\), and positive correlations between happiness and \(\beta\) indicate a negative correlation between discounting and happiness.
When using the straight value of subjective happiness, four of the six parameters had signs consistent with the theory, with the quasi-hyperbolic parameters for happiness being the opposite of the expected sign. When I considered the natural logarithm of subjective happiness, five of the six parameters had signs consistent with theory, as the sign for the $\rho$-value of happiness for the quasi-hyperbolic function switched signs when I took the natural logarithm of subjective happiness and ran that correlation with the discounting parameters. The $\beta$ value for periods of happiness in the quasi-hyperbolic function still showed a negative correlation, inconsistent with the theory. However, as none of these results were statistically significant, I cannot conclude there exists a relationship between happiness and the various parameters measured in relation to discounting. Nonetheless, the consistency of the signs indicate that with a better measure and more participants, the results may fall in accordance with the predictions.

As discussed, according to Icher and Zargham (2011), there is reason to believe in a negative correlation between happiness and discount rates. The lack of such findings here could be explained by several possibilities. First, the preliminary findings of Icher and Zargham were from a series of mid-1970’s surveys of the general population. The two demographics groups differed drastically; the mean age in Icher and Zargham was 44, compared to the college student in this study. As mentioned, a plethora of different studies indicated a link between low discount rates and positive life outcomes. It is possible that college students with low discount rates have not realized the potential of their work; in essence, they have yet to receive the larger, latter reward. College student with low discount rates may earn higher GPAs, save more money and live healthier lifestyles, but to a college student, those increased outcomes may not mean
much during college, but will pay off in the future. In contrast, those in college who are engaging in behaviors demonstrating high discount rates, such as partying, procrastinating on homework, etc., have not yet fully experienced the consequences of their actions.

In the above discussion, I assumed one’s discount rate changes one’s happiness. However, this causal link has not been established in the previous research, and only that there exists a negative correlation between the two. One could also attempt to explain the findings of this survey assuming happiness affects one’s discount rate, i.e., people who are happier tend to discount the future less. It is more difficult to see how the demographic difference between this study and the previous research accounts for the different results. When discounting explains happiness, it is intuitive to say that college students are not yet reaping the rewards, whereas it is less intuitive to claim that after college, people’s behaviors change, even if their happiness remains the same.

However, in other cited studies, researchers found a causal link between affect and discount rates (Ifcher & Zarghamee, 2011; Baumeister, Bratslavsky, & Tice, 1998; Isen & Erez, 2002). Although mood and happiness are related, they are not the same construct. Thus, although the research may suggest that happiness and discounting could be negatively correlated, it does not undermine the findings of the current research.

Finally, there is the possibility that the measures in this study were a poor measure of either happiness or discount rates. Because the Subjective Happiness Scale is widely cited and used, compared to the elicitation method created for this survey, it is unlikely the Subjective Happiness Scale caused the lack of significant correlations. A poor elicitation method more likely produced much variation in the data, making it
difficult to obtain statistically significant results. Thus, tests should be run with proven, more reliable measures in order to potentially gauge the parameters of discounting in a more accurate way. Using an open-response survey allowed individuals too much room to interpret the question without truly thinking about the implications of their responses. For example, when posed the question, “What is the minimum amount in dollars you would accept in three days instead of $275 today?” five participants responded with $400 and six responded with $500. At initial glance, these responses seem unreasonable. Because I did not provide any incentive for people to provide accurate responses, participants may have not put the mental effort into the question required to find a true indifference point.

**Research Question Two: Do individuals discount periods of happiness differently than money?**

The second set of hypotheses addressed the different discount rates between money and happiness. When using the exponential functional form, I found a statistically significant difference between the discount rate of happiness and the discount rate of money. Unlike hypothesized, people tended to discount happiness more than money. This is contradictory to previous research indicating consumables are discounted more heavily than money (Doyle, 2013, Estle et al., 2007).

However, according to the quasi-hyperbolic data, no significant difference exists between the discounting parameters, \( \rho \) and \( \beta \), for neither happiness nor discounting. According to the \( \beta \) term, individuals actually had a steeper constant discounting factor for money than happiness, the opposite of expected. The mean difference of \(-0.0453\) indicates that for any given amount of money, they will discount the future value of it by 4.53 percent less than if the reward were in terms of length of happiness, all else being
equal. Although neither result showed statistically significance, the relatively low p-values suggest with more refined measures and greater sample sizes, the effect could become more apparent.

Based on the assumption that happiness is more like a consumable than money, this seems counterintuitive. One may expect consumables to have a high initial discounting constant in order to overcome natural desires such as hunger, pleasure, etc. In this way, happiness may fundamentally differ from consumables in that way.

Another possible explanation for the contradiction to the predicted results is the magnitude effect explained previously. People tend to discount small amounts more heavily than large amounts (Prelec & Loewenstein, 1991). For example, a person may prefer $1 today to $2 in a week, but would prefer $200 in a week to $100 today. This reversal of preference based on magnitude is not predicted by the models used, and could have affect the results. The monetary values used were between $10 and $1000, compared to the periods of happiness of three days to two years. These ranges may not translate; participants perceived they would get a greater magnitude of utility from the values used for days of happiness than the values of money. This discrepancy between the values may have led to the appearance that people were discounting money more heavily than periods of happiness. If the magnitudes of the utilities between money and periods of happiness were more comparable, the results may have been different and confirmed theory.

Because of the abstract nature of comparing periods of happiness to money, it is difficult to interpret what differences in discounting may truly mean beyond a simple analysis of the data. The disparity between money and happiness for the exponential function is easily mirrored to the consumable goods mentioned before; the more direct
the route to pleasure, the greater a reward is discounted in the future. Thus, people want
the immediate gratification the more easily the good is transferable into pleasure.
However, with the quasi-hyperbolic data, this analysis appears flawed, as the immediate
discounting constant is greater for periods of happiness, meaning it is discounted less.

However, recognizing there are different discounting parameters for different
types of goods has market implications. For example, consider ordering a good from
online retailers. Often, people can find better deals online, even when shipping is
included. Despite the better price, many people purchase goods they could buy cheaper
online in stores. Part of this is likely explained by different types of discounting. Some
goods are discounted more heavily than others, and some may have greater initial
discounting constants, whereas others may have greater time dependent parameters.
Based on the variation of these parameters, how a consumer decides to purchase a good
may shift with different shipping times and prices.

Furthermore, knowing how people discount different types of goods is an
important component of setting prices. There are often a couple options when choosing
shipping methods. A typical array would be one day shipping, two to three business
days, or five to seven business days. The more quickly one receives their order (or
reward) the more the customer will have to pay. Each of these scenarios provides a
present value based on the discounting function, and thus the type of function and the
parameters within the function matter tremendously as far as to whether or not a person
purchases a good in store or via the internet. Even beyond the decisions college students
make about when to do various activities such as saving money (Khwaja, Silverman,
Sloan, & Wang, 2007) or exercising (Moreland, 2013) people make decisions based on
temporal discounting. An extreme example is winning the lottery; winners of large
jackpots often get to choose between a lump sum immediately and a larger amount paid out over time. The research (Ifcher & Zarghamee, 2011) indicates that not only is it smarter fiscally to make the decision to receive the money over time, but it will allow for greater happiness through more aggregate consumption. In a more accessible realm, employees of a company often receive equity in the company, sometimes in the form of restricted stock: a certain number of shares that after a certain period of time and or conditions are met, vests into full value shares. In this case, the employee is receiving compensation at a delayed rate. The extent to which the employee (or company) would trade off the shares for an increase in salary is partially an exercise in temporal discounting. Thus, how readily money and other goods or experiences transfer into happiness affect the extent to which they are discounted, which may help explain cross-domain inconsistencies.

**Research Question Three: Does being told about the positive effects of low discount rates and the adverse effects of high discount rates change individuals’ indicated temporal preferences?**

I also hypothesized that reading a brief article highlighting the importance of delaying gratification (i.e., having a low discount rate) would decrease people’s discount parameters. In all but one of the t-tests, the statistics did not confirm this hypothesis. For the $\beta$ value for money, there existed a statistically significant difference between the treatment and control group, indicating that reading the article on the importance of delay of gratification decreased one’s discount rate more than reading a neutral article.

However, none of the other t-tests were statistically significant. Nonetheless, theory predicted the direction of change for all but one test. The hypothesis predicted the difference in exponential rates, $r$, to be greater in the treatment group than in the
control group. The analysis shows the opposite; those in the treatment group increased their discounting rates while those in the control decreased theirs. Only one of the six tests, though, had that result.

This has encouraging implications for policy makers and influencers who hope to educate people to make more long-run oriented decisions. The results for the \( \beta \) value for money suggest that when people are educated about the consequences of having a high discount rate and the benefits to having a lower discount rate, they are able to discount less, specifically in the short-term. This indicates that the education effect has an immediate impact and can bolster one’s short-term will to delay gratification. However, the education does not necessarily impact the long run decisions people make. Therefore, someone may change their behavior for choices between today and tomorrow than two future dates or today and a date far off in the future. People may be more easily swayed to not spend in the short-run, but an individual’s long run planning appears less susceptible to education. For example, education could play impact someone’s decision-making when shopping for luxury items such as clothes or entertainment devices; they can be persuaded to delay gratification and save that money for a future date. However, according to the results, education is less likely to impact schedules of payment for loans. Because all the dates of payment are in the future, the \( \beta \) term does not affect the differences in those time horizons. Thus, education on delaying gratification would not impact the types of payment programs people would choose for loans.

Similarly, the tests for interactions did not yield any statistically significant results. Two of the six cases changed in a direction that indicated happy people are more likely to incorporate new information into their decision process (the \( \rho \) for periods of happiness and amounts of money), whereas the other four cases indicated that unhappy
people are more likely to incorporate new information into their decision process.
Because of the ambiguity of these results, I am unable to derive any meaningful theory
from them, aside from one’s level of happiness does not interact with how well one
incorporates potentially consequential evidence into one’s life.

Again, because of the lack of statistically significant results, there are several
possibilities. As discussed previously, the measure may not have been sensitive enough
to detect changes in small variables. Increased sensitivity and improvement of the
elicitation method could help reduce this. Additionally, the information may not have
been strong or persuasive enough to impact an individual’s time preference, or at least
enough to register as statistically significant.

However, it is also possible that the results are true; informing someone of the
importance of delaying gratification does not affect their discount rate. If this is the case,
then people are not adopting behaviors that potentially could have large positive
repercussions. As discussed previously, several studies and meta-analyses show
scenarios in which people do not adopt new behaviors or attitudes based on warning or
education.

Knowing this allows for the inclusion of the dual-self concept. According to
Thaler and Shefrin (1981), people have a “planner” and “doer” self. The planner tries to
account for the myopic tendencies of the doer, who ultimately makes decisions in the
moment. Knowing the doer tends to be short-sighted, the planner may set up rules or
contingencies to help the doer make more long-term oriented decisions. For example,
economics professors at Yale started a business called “stickK.” This website and app
helps own lose weight by making the person commit to giving money to a charity or
organization (sometimes one to which they would particularly hate donating money) if
the person does not attain their weight loss goals. Thus, the planner is making the active decision to put in place a contingency the doer self must consider when thinking about working out or eating unhealthy food. The planner knows the myopic doer will tend to opt for immediate gratification, but that immediate gratification is not in line with the planner’s long term goals. In this scenario, clearly the planner knows the tendencies of the doer, and that left to the doers’ own choosing, the person will suffer in the long run, so by putting in an enforcing mechanism, the doer chooses the route the planner intends. Therefore, it follows that informing people about the consequences of high discount rates will not change the doers’ time preferences; both the planner and the doer already know the benefits of being able to delay gratification. That leaves the planner to create contingencies like stickK to moderate the behavior of the doer.

According to Laibson (1997), this sort of analysis can be solved mathematically, given the dual-self model and the ability to follow commitment mechanisms to give more predictable results than strictly the discounting models, specifically the quasi-hyperbolic model, alone.

As individuals and as a society people need to decide if this is acceptable or not. Many behavioral economists have studied the effects of “nudges” that help improve individual decision making by providing a framework in which people tend to follow the decision framework a “choice architect” sets up (Thaler & Sunstein, 2008). For important life decisions, such as savings, health choices, and other questions of delaying gratification, there may be significant room for improvement based on the principles of the nudge. Some may impose such commitment plans upon themselves, and others may choose not to do so. Some feel policy makers should utilize such concepts to guide individuals toward socially optimal outcomes. Thus, it is up for society and individuals
to decide if they want to subject themselves to a system where the individual’s actions are guided toward what some would consider a desirable result or if choice should remain uninhibited. Research by Choi (2010) lauds the success of automatic enrollment for 401k retirement plans, which has led to higher enrollments in savings plans. However, some consider mechanisms like default enrollment and other covert nudges manipulative and are opposed to their implementation (Felsen, Castelo, & Reiner, 2013). The discussion of this notion is beyond the scope of the paper, but the results do seem to indicate that there is room for improvement beyond simple education.

Despite the general trend of statistically insignificant results, there is still room to interpret the data under different models. Furthermore, the economics significance of the statistically non-significant results still has relevant applications.

**Research Question Four: How does the type of discounting model affect the interpretations and implications of the results?**

The mean discount rate for money of 0.02318 implies that the average person would be indifferent between $100 today, $107.06 in three days, $117.24 in one week, $197.75 in thirty days, and $773.26 in ninety days. The mean quasi-hyperbolic function for money implies the average person would be indifferent between $100 today, $182.20 in three days, $196.67 in thirty days, and $234.22 in ninety days. For the happiness exponential function, the data suggest the average person would be indifferent between 100 days of happiness starting today and 104.9 days of happiness starting in three days, 111.9 days of happiness starting in seven days, 161.7 days of happiness starting in thirty days, and 422.4 days of happiness starting in ninety days. The quasi-hyperbolic function for happiness suggests that the average person would be indifferent between 100 days of happiness starting today and 168.1 days of happiness starting
starting in three days, 169.8 days of happiness starting in 7 days, 179.7 days of happiness starting in thirty days and 208.4 days of happiness starting in ninety days. Appendix D has a full tabulation of these data.

When one uses the function incorporating one’s level of happiness, some interesting results occur. Although Appendix D has the full data results, the Figure 2 demonstrates the differences between different levels of happiness and indifference points. Using Equation 8 for the discounted utility model and Equation 9 for the quasi-hyperbolic model (reshown below)

\[ P = F \left( 1 + \frac{1}{\mu + \alpha \ln(H)} \right)^t \]  

\[ P = \begin{cases} 
  \text{For } t = 0, & F \\
  \text{For } t > 0, & F \left[ \varphi + \kappa \ln(H) \right] \ast \left( \frac{1}{1 + \rho} \right)^t 
\end{cases} \]

I can model the indifference curves for various time delays for different levels of happiness, given an amount to be received today. The new model allows deviations in happiness to change the amount of discounting that occurs.

At small time frames, the indifference points between levels of happiness are fairly close to each other. A happy person (Subjective Happiness Scale score of 6.5) is indifferent between $100 today and $106.52 in three days, whereas a person at the average level of happiness (Subjective Happiness Scale score of 5.05) is indifferent between $100 today and $107.00 in three days, and an unhappy person (Subjective Happiness Scale Score of 3) is indifferent between $100 today and $108.01 in three days. However, as one extends the time period out to ninety days, the indifference point for $100 today is $664.54 for the happy person, $761.46 for the person with an average
level of happiness, and $1,007.77 for an unhappy person. Thus, as the delay gets larger, the effect of happiness is exacerbated given this functional form.

Simply looking at the graph for the two types of functions, as well as comparatively for the different levels of happiness, illustrates the distinctions between the quasi-hyperbolic and discounted utility model. The discounted utility model starts off increasing slowly, followed by rapidly increasing rates as time increases. The quasi-hyperbolic, however, starts with an initially large leap from the $100 today to $181.15 at day one, but the increase over time is much more tempered than in the discounted utility model. This effect causes the quasi-hyperbolic function to overestimate discounting in the short-run and underestimate it in the long run, whereas the discounted utility model underestimates discounting in the initial periods and grossly overestimates discounting in the further out periods. More mathematically sophisticated models could alleviate this problem. Nevertheless, the two discounting functions provide a solid basis and framework to analyze discounting.

One can compare these effects to what happens in the quasi-hyperbolic model when one incorporates happiness. For $100 today, using the quasi-hyperbolic function taking into account percentile of happiness, the happy person is indifferent for $178.16 in three days, a person with average happiness is indifferent for $181.26, and an unhappy person is indifferent for $189.73. For the same $100 today, but in ninety days, the happy person is indifferent for $229.03, the average person, $233.68, and someone who is unhappy is indifferent for $243.91. As opposed to the discounted utility model, in the quasi-hyperbolic model incorporating happiness, the relative differences between people at different levels of happiness stays the same as time goes on.
Figure 2  Indifference Curves For Various Discounting Functions

- Standard DUM
- DUM Subjective Happiness of 6.5
- DUM Subjective Happiness of 5.05
- DUM Subjective Happiness of 3
- Standard Quasi-Hyperbolic
- QH Subjective Happiness of 6.5
- QH Subjective Happiness of 5.05
- QH Subjective Happiness of 3
This is because the function is constructed in a way where happiness only affects the first time period’s decision process, and all subsequent time periods are discounted the same amount.

Another way to analyze this data is to generate a consumption over time function. In a method outlined by Silberberg (1978), one can determine optimal consumption at a given time $C(t)$ given an initial capital stock $K_0$, income flow $iK$, utility function $U(C)$, a consumption span $T$, and a discounting function. For this analysis, I let the discounting function be $m^t$ where $m = \frac{1}{1+r} \text{ or } \frac{1}{1+[\mu+a+ln(t)]}$ depending on whether or not the analysis is using the function incorporating happiness. In this method, one maximizes the function

$$\int_0^T U(C(t)) \ast m^t \, dt \quad \text{such that } K' = iK - C, \quad K(0) = K_0, \quad K(T) \geq 0 \quad (15)$$

This maximizes utility as a function of consumption with a discounting factor, given that one has an initial capital stock which can receive interest, at the rate of income flow minus consumption. According to Kahneman and Krueger (2006), there are diminishing marginal returns to overall consumption, so I let $U(C) = \ln(C)$, a function which exhibits that behavior. I solve this method using the Hamiltonian maximization technique fully in Appendix C. There, I show the solution to be:

$$C(t) = - \frac{K_0 \ln(m) e^{it} m^t}{1 - m^T} \quad (16)$$

Thus, by imposing parameters into the function, one can map out consumption over time. Although one could easily use this sort of analysis for consumption over one’s life, I restrict the time-frame to a more narrow range. The data collected looks at daily discount rates over a relatively short period of time (two years), and thus if the data
collected is imposed onto an entire life, the results become extreme and difficult to interpret. For exploration, consider this scenario. Two parents go on month long vacation, and leave their 22 year old son home with $1000 to survive, which they put in a bank account that earns 0.1% interest daily. The son wants to spend all the money available by the time his parents return. The son has the discount function of the same form and with the same parameters as elicited through the data collection in the preceding experiment, and wants to maximize utility. The graph on the following page (Figure 3) illustrates the son’s consumption over time given different discounting functions. An exact tabulation of the data can be found in Appendix F.

From the table, one can see the effects of the various factors influencing discount rates. For the elicited discounting function based on the average discount rates of the participants, the average participant in the circumstances described above would consume $41.98 on the first day. The average person would then decrease consumption slightly each day over the month until finishing the month by consuming $35.26 on the final day. My proposed model which integrates happiness into the function aligns very closely with the average elicited discounting function, starting at $41.95 on the first day and ending at $35.29. According to my model, a happy person starts the month consuming $41.45 and finishes at $35.75, whereas an unhappy person starts the month consuming $42.99 and ends the month spending $34.34.

The specific values of this exercise are less important than the behavior of the different functions. As stated, a person with an “average” score on the Subjective Happiness Scale should have a very similar consumption curve to the average elicited discounting function. Because the person discounts the future, the person starts off consuming more at the beginning of the time period and ends the month spending less,
assuming the individual’s discount rate is greater than the interest rate they could receive on the money.

Comparatively, a happier person will have a flatter consumption pattern than the average person, because a happier person values the future more relative to an unhappy person, according to the model. Thus, in order to maximize utility, the person consumes more relative to the unhappy person at the later dates, leading to the more even consumption pattern. Nonetheless, the happy person still does discount the future and therefore consumes more at the beginning than at the end of the time period. Conversely, an unhappy person has a steeper consumption pattern because the unhappy person values the present more than the future, relative to the happy person.

In the above example, the economic significance of the difference between the happy and unhappy person appears negligible. The unhappy person starts the month spending $1.54 more and ends the month spending $1.41 less than the happy person. This spending behavior would not likely lead to noticeably different behavior patterns to a casual observer or even to the consumers themselves. However, as one extends the model further into time, the difference between happy and unhappy consumers expands. The exact point at which these disparities become economically significant is open to interpretation, but knowing the overall trends of the model proves useful for making such determinations.
Consumption Over Time

- Elicited Discounted Utility Model
- Subjective Happiness of 6.5 (Very Happy)
- Subjective Happiness is 5.05 (Average Happiness)
- Subjective Happiness is 3 (Unhappy)

Initial Conditions:
- Initial Capital Stock: $1000
- Time Frame: 30 days
- Daily Interest Rate: 1%

Figure 3
As stated, one could extend this analysis to consumption of money over a lifetime. If one were to do that, however, one would need to make additional assumptions about income not related to initial capital. Because this study solely collects data on college students, making inferences about discounting over one’s life is an extrapolation. One could also apply this analysis with a greater spread of activities, each of which have different weighted utility curves to add complexity and depth to the interpretations. One cannot, however, apply this analysis to the quasi-hyperbolic function. Because the quasi-hyperbolic function is piecewise and therefore not continuous, the model violates the assumptions of the process outlined by Silberberg (1978). If one eliminates the piecewise component to the quasi-hyperbolic function, the consumption pattern is the exact same form as the discounted utility model, as the immediate discounting component disappears. Consequently, to consider the quasi-hyperbolic model results further, I would need to use a dynamic programming approach, which is beyond the scope of this paper.

Limitations and Future Research

There are limitations to this research. First, discount rates may not be a perfect measure of impulsiveness. Although temporal discounting tendencies offer a general model of impulsiveness, an individual will have different discount rates for different goods, suggesting that strict dollar values are not a perfect predictor (Dittmar & Bond, 2010). Thus, making the connection between making impulsive decisions and happiness may be slightly off base.

Similarly, there is the possibility that hypothetical rewards do not function as a valid proxy for real rewards. Research by Johnson and Bickel (2002) suggest that people do not differ between real and hypothetical rewards, the research does not have
participant under emotional duress. Furthermore, non-monetary rewards might fluctuate in different ways in real versus hypothetical situations, which could cause an inconsistency between the findings and reality. It may be difficult for participants to consider hypothetical dollar amounts as a coping mechanism for present emotion.

Another limitation of the study is the inability to assess whether the directionality of the causation (if there is one) between happiness and temporal discounting. Because the data are only correlation, a causal claim cannot be made. The only way to truly identify whether or not temporal discounting affects happiness would be to coach someone to manipulate their discount rates and then track their happiness over time and compare that to a control group.

There is also a limitation associated with the sample; using only Intro to Psychology students at one college will certainly lead to a biased sample. People in this demographic may have certain traits that limit the extent to which the findings are true. Any participants may also have a difficult time making temporal discounting decisions about abstract ideas such as happiness. Furthermore, participants may suffer from fatigue in the survey, and produce bias results because of their fatigue. Research by Brazell, Gray-Lee, Louviere, Dallaert, and Pullman (1995) indicated this effect may be minimal, as participants may actually “learn” over the course of the experiment, and become more efficient at the decision making. Furthermore, Louviere (1997) found similar discount rates for respondents answering between 12, 24, 48, or 96 questions. However, the abstract nature of this experiment may cause problems.

A bigger potential problem with the study is the overall lack of unhappy participants. Figure 1 illustrates that the large majority rated themselves as being happy, with very few students on the low end of the spectrum. Without having an
adequate sample of unhappy participants, obtaining an overall trend in discounting as happiness varies proves difficult. This restriction of range bias could be solved by obtaining a larger, more diverse sample of participants to include more unhappy people in the study. Additionally, although I collected no demographic information, the sample was likely very homogenous. Based on the population from which I drew the sample, the participants were aged 18-22, with the majority being at the lower end of that spectrum. Furthermore, they were likely white, middle- to upper-middle class citizens and fairly intelligent. Without the full spectrum of diversity, making broad claims is extrapolating beyond what the sample allows.

Future research could be done in many areas. One could see if one could instill certain mindsets or philosophies that emphasize either present or future-oriented thinking, and the implications on temporal discounting, similar to what Mogilner, Aaker, Kamvar did with tea and excitedness (2012).

A mirror study could be done with punishments as opposed to rewards; instead of posing “Would you rather receive $100 today or $120 in a week?” one could ask “Would you rather pay off a debt with $100 today or $120 in a week?” and “Would you rather be sad for the next 3 days or 5 days next week?” Studies have shown people to have loss aversion, and that may have an effect on discount rates (Tversky & Kahneman, 1991). This could confirm and extend or limit the scope of the current study. Additionally, research could be done exploring a different set of time periods than used in this study. With longer time horizons, one could more accurately assess long term discounting tendencies.

Research could also analyze other intrapersonal characteristics that vary with discount tendencies. There may be personality traits, demographic differences, or other
variations between people that affects how individuals discount the future. If this information is found, then one could potentially control for these factors and determine a more comprehensive model of temporal discounting.

Finally, one could test temporal discount rates at the macro level and the mood of nations. One could analyze different policies that value future generations at different rates and compare those to the political and overall mood of a country or state at a given time to see if the effects of individual temporal discounting persist at higher ordered groups. A country, in theory, should make more future oriented decisions when the mood in the country is high, because the economy is likely doing well and thus could afford to make future-oriented decisions. However, the results of this study may suggest that the citizens may try to maintain the current mood, and have a high discount rate even when morale is high.

Conclusions

Overall, almost all the tested hypotheses gave inconclusive results. The main problem likely is usage of a poor discounting elicitation method. Nonetheless, examining the results, as well as their implications has value. If happiness is negatively correlated with discounting, that means by adding it to previously existing discounting models, researchers can more accurately predict behavior if one’s happiness is known. Furthermore, the connection between happiness and discounting may go a long way in terms of explaining differences in life outcomes between individuals. Although I do not have the evidence to suggest a causal relationship between happiness and discounting, there is an intuitive connection in each direction; happier people do not need the immediate boost in morale as badly as unhappy people, and thus discount less, or the
converse, people who tend to discount less have better life outcomes and are thus happier.

Additionally, how people respond to information greatly impacts policy decisions. If people do not incorporate information about the effects of discounting into their decisions, people may achieve suboptimal outcomes. In those cases, individuals or policy makers can implement systems to shift people’s behavior closer to what is considered ideal. Social workers and well-being assistants could also help their clients more by knowing this research. If a social worker has a depressed client, the social worker may be able to better help the person if they know what types of poor decisions the client is likely to make because the person is unhappy. Thus, beyond strictly mental strategies, caretakers could encourage healthy lifestyle choices, such as eating right and saving, both because the client may be unlikely to engage in those activities autonomously, and that lowering one’s discount rate may actually lead to more happiness down the line. Furthermore, consideration of how those ideals reflect back onto happiness should be a driving force in the discussion.

As stated, many consider happiness to be a premiere force in life, if not the ultimate force. Because so much of what people do is connected to the pursuit of happiness, understanding the connection between well-being and decision making is of the utmost importance: do people make decisions that are likely to make them happy? Do happy people make decisions differently than unhappy people? Are people aware of how to become happier and live better? Because of the prominent role intertemporal choice plays in our lives, the connection between happiness and temporal decision making should be fully understood.
References


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Appendix A

For each of the following statements and/or questions, please circle the point on the scale that you feel is most appropriate in describing you.

1. In general, I consider myself:
   
   not a very happy person 1 2 3 4 5 6 7 a very happy person

2. Compared with most of my peers, I consider myself:

   less happy 1 2 3 4 5 6 7 more happy

3. Some people are generally very happy. They enjoy life regardless of what is going on, getting the most out of everything. To what extent does this characterization describe you?

   not at all 1 2 3 4 5 6 7 a great deal

4. Some people are generally not very happy. Although they are not depressed, they never seem as happy as they might be. To what extent does this characterization describe you?

   not at all 1 2 3 4 5 6 7 a great deal
Appendix B

INTRODUCTION
You are invited to be in a research study about time preferences. This study is being conducted by: Daniel Flesher, under the supervision of Dr. Rodger Narloch and Dr. Parker Wheatley. You were selected as a possible participant because of your expressed interest in the study via the PRIA system. We ask that you read this form and ask any questions you may have before agreeing to be in the study.

BACKGROUND and PROCEDURES
The purpose of this study is explore the relationship between individuals’ preferences for rewards over different periods in time, and the factors that affect an individual’s preferences.
If you agree to be in this study, we would ask you to do the following things.
Take a four question survey regarding your personal happiness. Complete a thirty item exercise indicating your preferences over time. Read one short article. Complete another thirty item exercise indicating time preference. Write a short response regarding the task you performed.

RISKS/BENEFITS
This study has no known risks.
The benefits of participation are a potentially deeper understanding of how you make decisions regarding time.

CONFIDENTIALITY
The specific data connected with this study will be completely anonymous, and your answers will not be able to be tied back to your identity. The study does not require you to provide any identifying information. The records of this study will be kept private.
Research records will be kept in a secure file on a password protected computer. Only the researchers will have access to the records. In any reports or public presentations, no information will be included that would make it possible to identify a participant.

VOLUNTARY NATURE OF THE STUDY
Your participation in this research study is completely voluntary. You may stop participating at any time without penalty or costs of any kind. Your decision whether or not to participate will not affect your current or future relations with the College of Saint Benedict or Saint John’s University.

CONTACTS AND QUESTIONS
The researchers conducting this study is: Daniel Flesher, under supervision of Dr. Rodger Narloch and Dr. Parker Wheatley. You may ask any questions you have now. If you have questions later, you may contact them at: Daniel Flesher – daflesher@csbsju.edu, (952)-250-0744. Dr. Rodger Narloch – rnarloch@csbsju.edu, Dr. Parker Wheatley – pwheatley@csbsju.edu. If you have additional questions you may also contact the CSB/SJU Institutional Review board chair, Dr. Robert Kachelski: irb@csbsju.edu.
You will be given a copy of this form to keep for your records.

STATEMENT OF CONSENT
I have read the above information. I have asked questions and have received answers. I consent to participate in the research.
I attest that:
• I am at least 18 years of age.
For each of the following statements and/or questions, please indicate the number on the scale that you feel is most appropriate in describing you.

1. **In general, I consider myself:**

<table>
<thead>
<tr>
<th>Selection</th>
<th>Not at all (1)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>A very happy (7)</th>
</tr>
</thead>
</table>

2. **Compared with most of my peers, I consider myself:**

<table>
<thead>
<tr>
<th>Selection</th>
<th>Less happy (1)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>More happy (7)</th>
</tr>
</thead>
</table>

3. **Some people are generally very happy. They enjoy life regardless of what is going on, getting the most out of everything. To what extent does this characterization describe you?**

<table>
<thead>
<tr>
<th>Selection</th>
<th>Not at all (1)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>A great deal (7)</th>
</tr>
</thead>
</table>

4. **Some people are generally not very happy. Although they are not depressed, they never seem as happy as they might be. To what extent does this characterization describe you?**

<table>
<thead>
<tr>
<th>Selection</th>
<th>Not at all (1)</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>A great deal (7)</th>
</tr>
</thead>
</table>
The purpose of the following study is to compare your preferences for different amounts of money and periods of happiness over different periods of time. In this experiment, you will be asked to make assessments about the minimum amount of either money or period of happiness you would accept in the future in exchange for the provided amount in the today. For example, if the question prompts, “How much is the minimum you would accept instead of $X today?” you would answer with $Y, where Y is the lowest amount you would accept today instead of taking the money in the future. Similarly, a question may prompt, “How much is the minimum number of days you would accept in 6 months instead of X days starting today?” you would answer with Y days, where Y is the fewest number of days of happiness you would accept starting at the given point in the future instead of the number of days today. Use decimals and non-round numbers when appropriate.

For the questions regarding periods happiness, define happiness however you define being happy. In other words, use whatever happiness means to you.

These first four are practice problems, and will not be used in the data, but are simply to give you an opportunity to familiarize yourself with the task.

The following questions measure a personal preference, and therefore there is no incorrect answer.

5. How much is the minimum in dollars you would accept in ten weeks instead of $45 today?

Amount in dollars in ten weeks:

6. How much is the minimum in dollars you would accept in two weeks for $700 today?

Amount in dollars in two weeks:

7. How many days of happiness is the minimum you would accept in 20 days instead of 90 days of happiness starting today?

Amount you would accept in 20 days:

8. How many days of happiness is the minimum you would accept in seven months instead of 8 days of happiness starting today?

Amount you would accept in seven months:
The following responses will be used in data collection. Please consider your answers carefully.

Remember, the following questions measure personal preference. There are no incorrect answers.

9. What is the minimum amount in dollars you would accept in three days instead of $275 today?
   Amount in dollars you would accept in three days:

10. What is the minimum amount in dollars you would accept in one week instead of $820 today?
    Amount in dollars you would accept in one week:

11. What is the minimum amount in dollars you would accept in ten days instead of $10 today?
    Amount in dollars you would accept in ten days:

12. What is the minimum amount in dollars you would accept in two weeks instead of $650 today?
    Amount in dollars you would accept in two weeks:

13. What is the minimum amount in dollars you would accept in one month instead of $350 today?
    Amount in dollars you would accept in one month:

14. What is the minimum amount in dollars you would accept in six weeks instead of $220 today?
    Amount in dollars you would accept in six weeks:

15. What is the minimum amount in dollars you would accept in two months instead of $100 today?
    Amount in dollars you would accept in two months:

16. What is the minimum amount in dollars you would accept in three months instead of $40 today?
    Amount in dollars you would accept in three months:
17. What is the minimum amount in dollars you would accept in four months instead of $1000 today?
Amount in dollars you would accept in four months:

18. What is the minimum amount in dollars you would accept in six months instead of $500 today?
Amount in dollars you would accept in six months:

19. What is the minimum amount in dollars you would accept in eight months instead of $80 today?
Amount in dollars you would accept in eight months:

20. What is the minimum amount in dollars you would accept in nine months instead of $50 today?
Amount in dollars you would accept in nine months:

21. What is the minimum amount in dollars you would accept in twelve months instead of $175 today?
Amount in dollars you would accept in twelve months:

22. What is the minimum amount in dollars you would accept in eighteen months instead of $25 today?
Amount in dollars you would accept in eighteen months:

23. What is the minimum amount in dollars you would accept in twenty-four months instead of $400 today?
Amount in dollars you would accept in twenty-four months:
For the questions regarding periods happiness, define happiness however you define being happy. In other words, use whatever happiness means to you.

Remember, the following questions measure personal preference. There are no incorrect answers.

24. How many days of happiness is the minimum you would accept in three days instead of 40 days starting today?

Number of days of happiness you would accept in three days:

25. How many days of happiness is the minimum you would accept in one week instead of 20 days starting today?

Number of days of happiness you would accept in one week:

26. How many days of happiness is the minimum you would accept in ten days instead of 65 days starting today?

Number of days of happiness you would accept in ten days:

27. How many days of happiness is the minimum you would accept in two weeks instead of 100 days starting today?

Number of days of happiness you would accept in two weeks:

28. How many days of happiness is the minimum you would accept in one month instead of 80 days starting today?

Number of days of happiness you would accept in one month:

29. How many days of happiness is the minimum you would accept in six weeks instead of 25 days starting today?

Number of days of happiness you would accept in six weeks:

30. How many days of happiness is the minimum you would accept in two months instead of 365 days starting today?

Number of days of happiness you would accept in two months:
31. How many days of happiness is the minimum you would accept in three months instead of 7 days starting today?

Number of days of happiness you would accept in three months:

32. How many days of happiness is the minimum you would accept in four months instead of 300 days starting today?

Number of days of happiness you would accept in four months:

33. How many days of happiness is the minimum you would accept in six months instead of 50 days starting today?

Number of days of happiness you would accept in six months:

34. How many days of happiness is the minimum you would accept in eight months instead of 35 days starting today?

Number of days of happiness you would accept in eight months:

35. How many days of happiness is the minimum you would accept in nine months instead of 200 days starting today?

Number of days of happiness you would accept in nine months:

36. How many days of happiness is the minimum you would accept in twelve months instead of 5 days starting today?

Number of days of happiness you would accept in twelve months:

37. How many days of happiness is the minimum you would accept in eighteen months instead of 15 days starting today?

Number of days of happiness you would accept in eighteen months:

38. How many days of happiness is the minimum you would accept in twenty-four months instead of 150 days starting today?

Number of days of happiness you would accept in twenty-four months:
Read the article on the next page. Note: You will be given a brief quiz on the material you read.
The concept of the delay of gratification has been long studied in psychology and economics. Delaying gratification is the process of forgoing small, immediate rewards in exchange for larger rewards in the future. One of the foundational delay of gratification studies was done in the 1960’s and 1970’s by Dr. Walter Mischel. In these studies, children were offered either one marshmallow immediately, or two marshmallows in fifteen minutes if the child could resist the temptation to eat single marshmallow now. Some children were able to wait for the two marshmallows, but many ate the marshmallow presented to them immediately. Longitudinal studies later found that those children who waited for the larger rewards tended to do better in the future – they earned higher SAT scores and received more education.

Similar studies have connected one’s ability to delay gratification a variety of other life outcomes. People who are less able to delay gratification are more likely to drive drunk, be addicted to drugs, gamble, or be obese. These results all suggest that people who delay less are more impulsive, and thus tend to make decisions associated with being impulsive (overeating, engaging in drug use, etc.)

Furthermore, people who are able to delay longer (i.e. wait more for rewards) tend to be better off overall by several objective measures. Those who wait for the larger rewards tend to save more for retirement. Additionally, those that wait for gratification in the future procrastinate less, and thus tend to be more productive. Finally, studies have shown a positive correlation between one’s ability to wait for rewards in the future and overall happiness. This means that people who delay gratification are more likely to be happy. With such high-impact consequences of delaying gratification, understanding the mental processes behind the delay of gratification may be able to significantly benefit one’s life.

**39. In a few sentences, please explain how, in your opinion, being able to delay gratification is important.**
Classical conditioning theory involves learning a new behavior via the process of association. In simple terms two stimuli are linked together to produce a new learned response in a person or animal.

For example, Little Albert was a 9-month-old infant who was tested on his reactions to various stimuli. He was shown a white rat, a rabbit, a monkey and various masks. Albert was described as "on the whole stolid and unemotional," and showed no fear of any of these stimuli. However what did startle him and cause him to be afraid was a hammer being struck against a steel bar behind his head. The sudden loud noise would cause Little Albert to burst into tears.

When Little Albert was just over 11 months old the white rat was presented and seconds later the hammer was struck against the steel bar. This was done 7 times over the next 7 weeks and each time Little Albert burst into tears. Eventually, Little Albert only had to see the rat and he immediately showed every sign of fear. He would cry (whether or not the hammer was hit against the steel bar) and he would attempt to crawl away whenever the rat was presented.

Watson and Rayner had shown that classical conditioning could be used to create a phobia. A phobia is an irrational fear, i.e. a fear that is out of proportion to the danger. Over the next few weeks and months Little Albert was observed and 10 days after conditioning his fear of the rat was much less marked. This dying out of a learned response is called extinction. However even after a full month the phobia was still evident.

40. In a few sentences, please explain how Little Albert developed a phobia to a white rat.
For this portion of experiment, we are going to again give you a series of different rewards, and you will again give the minimum value you would accept today in exchange for the reward in the future. Please consider your answers carefully.

Remember, the following questions measure personal preference. There are no incorrect answers.

41. What is the minimum amount in dollars you would accept in three days instead of $80 today?
   Amount in dollars you would accept in three days:

42. What is the minimum amount in dollars you would accept in one week instead of $275 today?
   Amount in dollars you would accept in one week:

43. What is the minimum amount in dollars you would accept in ten days instead of $820 today?
   Amount in dollars you would accept in ten days:

44. What is the minimum amount in dollars you would accept in two weeks instead of $10 today?
   Amount in dollars you would accept in two weeks:

45. What is the minimum amount in dollars you would accept in one month instead of $650 today?
   Amount in dollars you would accept in one month:

46. What is the minimum amount in dollars you would accept in six weeks instead of $175 today?
   Amount in dollars you would accept in six weeks:

47. What is the minimum amount in dollars you would accept in two months instead of $350 today?
   Amount in dollars you would accept in two months:

48. What is the minimum amount in dollars you would accept in three months instead of $1000 today?
   Amount in dollars you would accept in three months:
49. What is the minimum amount in dollars you would accept in four months instead of $80 today?

Amount in dollars you would accept in four months:

50. What is the minimum amount in dollars you would accept in six months instead of $220 today?

Amount in dollars you would accept in six months:

51. What is the minimum amount in dollars you would accept in eight months instead of $50 today?

Amount in dollars you would accept in eight months:

52. What is the minimum amount in dollars you would accept in nine months instead of $25 today?

Amount in dollars you would accept in nine months:

53. What is the minimum amount in dollars you would accept in twelve months instead of $500 today?

Amount in dollars you would accept in twelve months:

54. What is the minimum amount in dollars you would accept in eighteen months instead of $400 today?

Amount in dollars you would accept in eighteen months:

55. What is the minimum amount in dollars you would accept in twenty-four months instead of $100 today?

Amount in dollars you would accept in twenty-four months:
For the questions regarding periods happiness, define happiness however you define being happy. In other words, use whatever happiness means to you.

Remember, the following questions measure personal preference. There are no incorrect answers.

56. How many days of happiness is the minimum you would accept in three days instead of 20 days starting today?  
Number of days of happiness you would accept in three days:

57. How many days of happiness is the minimum you would accept in one week instead of 80 days starting today?  
Number of days of happiness you would accept in one week:

58. How many days of happiness is the minimum you would accept in ten days instead of 300 days starting today?  
Number of days of happiness you would accept in ten days:

59. How many days of happiness is the minimum you would accept in two weeks instead of 7 days starting today?  
Number of days of happiness you would accept in two weeks:

60. How many days of happiness is the minimum you would accept in one month instead of 15 days starting today?  
Number of days of happiness you would accept in one month:

61. How many days of happiness is the minimum you would accept in six weeks instead of 40 days starting today?  
Number of days of happiness you would accept in six weeks:

62. How many days of happiness is the minimum you would accept in two months instead of 200 days starting today?  
Number of days of happiness you would accept in two months:
63. How many days of happiness is the minimum you would accept in three months instead of 50 days starting today?

Number of days of happiness you would accept in three months:

64. How many days of happiness is the minimum you would accept in four months instead of 365 days starting today?

Number of days of happiness you would accept in four months:

65. How many days of happiness is the minimum you would accept in six months instead of 25 days starting today?

Number of days of happiness you would accept starting in six months:

66. How many days of happiness is the minimum you would accept in eight months instead of 35 days starting today?

Number of days of happiness you would accept in eight months:

67. How many days of happiness is the minimum you would accept in nine months instead of 150 days starting today?

Number of days of happiness you would accept in nine months:

68. How many days of happiness is the minimum you would accept in twelve months instead of 5 days starting today?

Number of days of happiness you would accept in twelve months:

69. How many days of happiness is the minimum you would accept in eighteen months instead of 100 days starting today?

Number of days of happiness you would accept in eighteen months:

70. How many days of happiness is the minimum you would accept in twenty-four months instead of 65 days starting today?

Number of days of happiness you would accept in twenty-four months:
71. Briefly describe the decision making process in which you engaged. How did you make your decisions?
72. For this research, your responses will not be connected with your identity. However, the researchers still request permission to use and quote your responses in reports, presentations, and in other academic settings. Again, your responses are completely anonymous.

Please indicate whether or not the researchers have permission to use your responses in future reports, presentations, and other academic settings.

☐ The researchers HAVE MY PERMISSION to use my responses.

☐ The researchers DO NOT HAVE MY PERMISSION to use my responses.
Debriefing

Thank you very much for participating in this study!

The purpose of this study is to measure individuals’ tendencies for preferring different rewards over time. These results will be compared to how participants respond to questions about their happiness, as well as to the article that the participant read.

All participants are completing the same time preference & happiness survey you did. However, the article that you read may be different than other participants. One article, which was read by half the students, focuses on the importance of delaying gratification, and the repercussions delaying gratification has on one’s life. The other article, read by the other half of participants, focuses on an example of Pavlovian conditioning.

We, the researchers, are looking at several different components in this study. We want to see if people’s reported happiness correlates with their time preference tendencies. There is evidence to suggest that happier people will tend to accept less in the future than less happy people. Secondly, we want to test how people prefer money over time compared to periods of happiness over time. Previous research indicates consumable goods, such as candy or vacations, are preferred more immediately than money. Finally, we want to test if knowing the effects of delaying gratification affects how one makes choices over time. There is evidence to indicate that being informed of the importance of delaying gratification does not have much effect.

Because we won’t be done with this experiment for several months, it is very important that you don’t discuss it with any other students. Many of them will be participating, and any information they have about the study might contaminate the results. Therefore, please don’t talk to anyone about this study!

Thank you, and please let us know if you have any questions or concerns about this research.
Appendix C

Maximize \( \int_0^T U(C(t))m^t \, dt \) such that \( iK - C = K' \) where \( T \) is the total number of time periods over which one is maximizing, \( t \) is time, \( C \) is consumption, \( U \) is utility, \( m \) is the discounting factor \( \frac{1}{1+r} \), \( i \) is the market interest rate, \( K \) is the amount of capital and \( K' \) is the rate of change of capital.

Using the Hamiltonian function:

\[
H(t) = U(C(t))m^t + \lambda(iK - C)
\]

Take the derivative of the Hamiltonian with respect to consumption and set equal to zero, and derivative of the Hamiltonian with respect to \( K \) and set equal to \(-\lambda'\).

\[
\frac{\partial H}{\partial C} = U'(C)m^t - \lambda = 0 \rightarrow U'(C)m^t = \lambda \quad \text{Appendix Equation (A.E.) 1}
\]

\[
\frac{\partial H}{\partial K} = \lambda i = -\lambda' \rightarrow -\lambda i = \lambda'
\]

A.E. 2

Take the derivative of A.E. 1 with respect to \( t \).

\[
\frac{\partial \lambda}{\partial t} = U'(C(t))m^t \ln(m) + U''(C(t))C'm^t
\]

A. E. 3

Divide A.E.3 by A.E. 1

\[
\frac{\frac{\partial \lambda}{\partial t}}{\lambda} = \frac{U'(C(t))m^t \ln(m) + U''(C(t))C'm^t}{U'(C)m^t} = \ln(m) + \frac{U''(C(t))C'}{U'(C)} = \frac{\lambda'}{\lambda} \quad \text{A. E. 4}
\]

From A.E. 2, \( \frac{\lambda'}{\lambda} = -i \) and thus the LHS of A.E. 2 and the RHS of A.E. 4 are equivalent, and thus the RHS of A.E. 2 and the LHS of A.E. 4 can be set equal to each other

\[
-i = \ln(m) + \frac{U''(C(t))C'}{U'(C)} \rightarrow -\frac{U''(C(t))C'}{U'(C)} = i + \ln(m)
\]

A.E. 5

From A.E.2, \(-\lambda i = \lambda'\), solving the first order differential equation with respect to \( t \) gives 
\( \lambda(t) = \lambda_0 e^{-it} \). Plugging the RHS of this function into the RHS of A.E. 1 gives:

\[
U'(C)m^t = \lambda_0 e^{-it} \rightarrow U'(C) = \frac{\lambda_0 e^{-it}}{m^t}
\]

A.E. 6

In order to exhibit the laws of diminishing marginal consumption

Let \( U(C) = \ln(C) \). Thus, \( U'(C) = \frac{1}{C}, U''(C) = -\frac{1}{C^2} \)
Plugging the above components into the LHS of A.E. 5 gives

\[-\left(-\frac{1}{C^2}\right) \frac{C'}{C} = \frac{C'}{C} = i + \ln(m)\]  

A.E. 7

Solve the first order differential equation in A.E. 7

\[\frac{C'}{C} = i + \ln(m) \rightarrow C' = C(i + \ln(m))\]  

A.E. 8

\[C(t) = C_0 e^{(i+\ln(m))t} = C_0 e^{it} e^{\ln(m)t} = C_0 e^{it} m^t\]  

A.E. 9

From the initial conditions of the maximization problem,

\[K' - iK = -C\]  

A.E. 10

Plugging the LHS of A.E. 10 into the LHS of A.E. 9 gives

\[K' - iK = -C_0 e^{it} m^t\]  

A.E. 11

Solve the differential equation via the integration factor

\[e^{\int -it \, dt} (K' - iK) = (e^{\int -it \, dt})(-C_0 e^{it} m^t)\]  

\[\frac{d}{dt} (e^{\int -it \, dt} K) = (e^{-it})(-C_0 e^{it} m^t)\]  

\[\int \frac{d}{dt} (e^{\int -it \, dt} K) = \int -C_0 m^t\]  

\[e^{-it} K = -\frac{C_0 m^t}{\ln(m)} + A\]  

A.E. 12

\[K = -\frac{C_0 m^t e^{it}}{\ln(m)} + Ae^{it}\]  

At time t=0, K(0)=K_0

\[K_0 = -\frac{C_0 m^0 e^{i0}}{\ln(m)} + Ae^{i0} = -\frac{C_0}{\ln(m)} + A\]  

A.E. 13

\[A = K_0 + \frac{C_0}{\ln(m)}\]  

A.E. 14

Plugging the RHS of A.E. 14 in for A in Equation L gives:

\[K(t) = -\frac{C_0 m^t e^{it}}{\ln(m)} + \left(K_0 + \frac{C_0}{\ln(m)}\right) e^{it}\]  

At time T, K(T) = 0.

\[K(T) = 0 = -\frac{C_0 m^T e^{iT}}{\ln(m)} + \left(K_0 + \frac{C_0}{\ln(m)}\right) e^{iT} = e^{iT} \left(-\frac{C_0 m^T e^{iT}}{\ln(m)} + K_0 + \frac{C_0}{\ln(m)}\right) = 0\]
Solve for $C_0$.

\[-\frac{C_0 m^\tau e^{i\tau}}{\ln(m)} + K_0 + \frac{C_0}{\ln(m)} = 0\]

\[-K_0 = \frac{-C_0 m^\tau e^{i\tau}}{\ln(m)} + \frac{C_0}{\ln(m)} = \frac{C_0}{\ln(m)} (-m^\tau + 1)\]

\[C_0 = -\frac{K_0 \ln(m)}{1 - m^\tau}\]

**A.E. 15**

Plug the RHS of A.E. 15 into A.E. 9 for $C_0$.

\[C(t) = -\frac{K_0 \ln(m) e^{i\tau} m^t}{1 - m^\tau}\]

**A.E. 16**

$C(t)$ gives the optimal consumption at time $t$, given the parameters in the equation.
### Appendix D

**Indifferences Points for Various Discounting Functions versus $100 today**

<table>
<thead>
<tr>
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<th>Discounted Utility</th>
<th>Money</th>
<th>Quasi-Hyperbolic</th>
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### Indifferences Points for Various Discounting Functions versus 100 days starting today

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Appendix E

Consumption at time $t$

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