Disclaimer and Acknowledgements

**Disclaimer:** I am not speaking for others in the Federal Reserve System.

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Need for Outlooks

- A policymaker needs to make a decision today.

- The *current* decision results in random *future* net losses to society.

- Hence, the policymaker’s decision depends on his or her outlook about those net losses.
Question

What’s the appropriate notion of an outlook for this policymaker?
Answer

- The needed outlook is not a statistically motivated predictive density ...

- But rather an asset-price-based risk-neutral probability density (RNPD).
Intuition

• From an ex ante perspective, resources may be more valuable in one state than in another state.

• Optimal decisions should reflect these relative resource valuations.

• RNPDs are derived from financial market prices.

• Hence, an outlook based on an RNPD does reflect the relative values of resources in different states.

• But an outlook based on a statistical forecast does not.
Outline

1. General Policy Problem
2. Risk-Neutral Probabilities
3. Example: Macro-Prudential Supervision
4. Conclusions
GENERAL POLICY PROBLEM
Choice Problem

- Policymaker ($P$) chooses an action $a$.

- The result of the action next period depends on the realization of $x$.
  - The random variable $x$ has realizations $\{x_n\}_{n=1}^N$.

- The outcome $(a, x)$ results in a welfare loss of $L(a, x)$ dollars.
  - The loss $L(a, x)$ may be positive or negative.
Possible Losses

- When \( P \) chooses an action \( a \), there is a vector of possible social losses:
  \[
  (L(a, x_n))_{n=1}^N
  \]

- Dollars in different states are really different goods.

- Hence, each choice of \( a \) results in a distinct bundle of different goods.

- How should \( P \) compare these bundles?
Simple Fruit Analogy

• I face a choice between giving up two baskets of fruit:
  – A apples and B bananas
  – OR A’ apples and B’ bananas

• I need a way to combine apples and bananas together.
  – Should I just add the number of apples and bananas?
  – Should I estimate CES preferences over apples/bananas?
Using Prices

• Right approach: How much will it cost me to replace the lost fruit?

• Hence, I need to compare:

\[ p_A A + p_B B \]

vs. \[ p_A A' + p_B B' \]

• This comparison requires the use of appropriate market prices.
Replacement Cost Approach

- If $P$ chooses $a$, then society suffers a random loss $L(a, x)$.

- By buying a portfolio with random payoff $L(a, x)$, $P$ can replace the losses incurred by the action $a$.

- Hence, the value of that portfolio is the current (replacement) cost of taking action $a$.

- $P$ should choose $a$ so as to minimize this cost.

- This comparison requires the use of appropriate market prices.
RISK-NEUTRAL PROBABILITIES
State Prices

- If $P$ chooses $a$, then society loses $L(a, x_n)$ if $x = x_n$.

- How much would it cost today to reimburse society for the loss in that state?

- To answer this question, we need to know $q_n$ - the current price of a dollar received in the event that $x = x_n$.
  - The vector $(q_n)_{n=1}^N$ is the vector of state prices.
• Given \( q \), it would cost:
\[
\sum_{n=1}^{N} q_n L(a, x_n)
\]
to reimburse society for the losses incurred with action \( a \).

• \( P \) should choose \( a \) so as to minimize \( \sum_{n=1}^{N} q_n L(a, x_n) \).
Risk-Neutral Probabilities

• We don’t affect decisions if we divide $q_n$ by a constant.

• Define:

$$q^*_n = \frac{q_n}{\sum_{m=1}^{N} q_m}$$

• $q^*_n$ is called the risk-neutral probability density (RNPD) of $x$.

  – Probability means: $q^*_n$ sums to one and $q^*_n$ is nonnegative for all $n$. 
Risk-Neutral and "True" Probabilities

- The RNPD $q^*$ of $x$ is not the same as the "true" probability density of $x$.
  - And what exactly is the "true" probability density of $x$?

- $q^*$ reflects asset traders’ aversion to risk.

- And $q^*$ reflects asset traders’ assessments of the likelihood of $x$. 
• For any function $\phi$ of $x$, define:

$$E^*(\phi(x)) = \sum_{n=1}^{N} q_n^* \phi(x_n)$$

• $P$ can optimally choose $a$ by minimizing:

$$E^*(L(a, x))$$

• If $L$ is differentiable with respect to $a$:

$$E^*\left\{ \frac{\partial L}{\partial a}(a^*, x) \right\} = 0$$
Verbal Summary

- Standard: Policymaker’s optimal choice sets the outlook for $L_a$ equal to zero.

- Novel: The appropriate notion of the outlook is given by $E^*$.

- Intuitively, policymaker makes choices so as to balance losses across states of the world.

- The relevant trade-offs are governed by state prices, not statistical forecasts.
Aside: Endogeneity of State Prices

- Above: I’ve treated $q^*$ as exogenous to $P$.

- More realistic: Risk-neutral probability density $q^*$ depends on $a$.

- Then, $P$’s problem is to choose $a$ to minimize:

$$\sum_{n=1}^{N} q_n^*(a)L(a, x_n)$$
• Suppose \( P \) ignores endogeneity and chooses \( a^* \) so that:

\[
E^*\left[ \frac{\partial L}{\partial a}(a^*, x_n) \right] = 0
\]

• Result: This choice is nearly optimal as long as this second moment:

\[
Cov^*(L(a^*, x), \frac{\partial \ln q^*(a^*)}{\partial a})
\]

is sufficiently small.

• Note: This second moment is calculated using the RNPD \( q^*(a^*) \).
EXAMPLE:

MACRO-PRUDENTIAL SUPERVISION
Dividend Payouts

- Regulatory question: Large banks want to pay dividends.
- How large a dividend payment should they be allowed to make?
- A low dividend payment today allows banks to have more capital in the future ...
- Which will prove valuable if financial markets are strained in the future.
Model

- Let $S$ be the level of financial market stress next period.

- Let $L(a, S)$ be the net social loss (next period) of a current dividend payment $a$.

- We know that the optimal $a^*$ satisfies:

$$E^* \left\{ \frac{\partial L(a^*, S)}{\partial a} \right\} = 0$$
A Comparative Statics Result

• Intuitively: The approved level of current bank dividends should depend on the outlook for future financial market strains.

• To see how: Consider two different RNPDs for $S$ denoted by $q^*$ and $q^{**}$.

• Assume $q^*$ puts more weight on high realizations of $S$ than $q^{**}$.
  – Formally: $q^*$ dominates $q^{**}$ in a first-order sense.
• Suppose $L$ is supermodular in $(a, S)$.

  – Increasing dividends raises social loss by more when financial markets are strained.

• Then:

  $$a^*(q^*) < a^*(q^{**})$$

• Summary: A regulator should approve lower levels of bank dividends when the RNPD of $S'$ puts more weight on high realizations.
Implementation Challenges

- We need an appropriate proxy $S'$ for $S$.
  - $S'$ must be highly correlated with $S$.
  - There are enough options on $S'$ so that we can construct $q^*$.

- One possibility: treat (the negative of the) logged S&P 500 index as $S'$.

- With options on the S&P 500, we can estimate an RNPD for $S'$.

- Then, if the S&P 500 RNPD has a longer left tail, bank dividends should be lower.
CONCLUSIONS
RNPDs and Predictions

• RNPDs are an ex ante measure of the relative *values* of resources in future states of the world.

• Resources are, all else equal, more valuable in states that are more likely to occur.

• But all else is never equal: RNPDs are shaped by factors other than relative likelihoods.

• So, an RNPD is not the same as a predictive density.
Financial Market Data and Decisions

- BUT, this distinction between RNPDs and predictive densities is exactly what makes RNPDs more useful for policymakers.

- Policymakers form future outlooks so as to make current decisions with future outcomes.

- Optimal decisions trade off future benefits/costs in future states of the world.

- That trade-off should be based on the relative *values* of resources in those states, not their relative likelihods.

For a decision-maker, the relevant outlook is given by an RNPD.
Implementation Challenges

• Decision-making using RNPDs is not necessarily easy.
  – Need to determine appropriate financial proxy.
  – Even then: Available options may not cover longer horizons or extreme tail events.

• Nothing new: Good decisions are always based on a mix of good judgment, good data, and good modeling choices.

BUT:

The right goal is to model/estimate RNPDs, not statistical forecasts.
Ninth District Activities

- Minneapolis Fed’s Banking Group uses options data to compute RNPDs.

- They report the results on the public website for a wide range of assets.
  - Gold, silver, wheat, S&P 500, exchange rates, etc.

- They report and archive the results on a biweekly basis.