Optimal Outlooks

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Disclaimer and Acknowledgements

Disclaimer: I am not speaking for others in the Federal Reserve System.

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Need for Outlooks

• A policymaker needs to make a decision today.

• The *current* decision results in random *future* net losses to society.

• Hence, the policymaker’s decision depends on his or her outlook about those net losses.
What’s the appropriate notion of an outlook for this policymaker?
Answer

- The needed outlook is not a statistically motivated *predictive density* ...

- But rather an asset-price-based *risk-neutral probability density* (RNPD).
Intuition

• From an ex ante perspective, resources may be more valuable in one state than in another state.

• Optimal decisions should reflect these relative resource valuations.

• RNPDs are derived from financial market prices.

• Hence, an outlook based on an RNPD does reflect the relative values of resources in different states.

• But an outlook based on a statistical forecast does not.
Outline

1. General Policy Problem

2. Risk-Neutral Probabilities

3. Example: Inflation-Targeting

4. Conclusions
GENERAL POLICY PROBLEM
Choice Problem

- Policymaker (P) chooses an action $a$.

- The result of the action next period depends on the realization of $x$.
  - The random variable $x$ has realizations $\{x_n\}_{n=1}^N$.

- The outcome $(a, x)$ results in a welfare loss of $L(a, x)$ dollars.
  - The loss $L(a, x)$ may be positive or negative.
Possible Losses

• When $P$ chooses an action $a$, there is a vector of possible social losses:

$$ (L(a, x_n))_{n=1}^N $$

• Dollars in different states are really different goods.

• Hence, each choice of $a$ results in a distinct \textit{bundle} of different goods.

• How should $P$ compare these bundles?
Simple Fruit Analogy

• I face a choice between giving up two baskets of fruit:
  – A apples and B bananas
  – OR A’ apples and B’ bananas

• I need a way to combine apples and bananas together.
  – Should I just add the number of apples and bananas?
  – Should I estimate CES preferences over apples/bananas?
Using Prices

- Right approach: How much will it cost me to replace the lost fruit?

- Hence, I need to compare:
  \[
  p_A A + p_B B \\
  \text{vs. } p_A A' + p_B B'
  \]

- This comparison requires the use of appropriate market prices.
Replacement Cost Approach

- If $P$ chooses $a$, then society suffers a random loss $L(a, x)$.

- By buying a portfolio with random payoff $L(a, x)$, $P$ can replace the losses incurred by the action $a$.

- Hence, the value of that portfolio is the current (replacement) cost of taking action $a$.

- $P$ should choose $a$ so as to minimize this cost.

- This comparison requires the use of appropriate market prices.
RISK-NEUTRAL PROBABILITIES
State Prices

• If $P$ chooses $a$, then society loses $L(a, x_n)$ if $x = x_n$.

• How much would it cost today to reimburse society for the loss in that state?

• To answer this question, we need to know $q_n$ - the current price of a dollar received in the event that $x = x_n$.

  – The vector $(q_n)_{n=1}^N$ is the vector of state prices.
• Given $q$, it would cost:

$$\sum_{n=1}^{N} q_n L(a, x_n)$$

to reimburse society for the losses incurred with action $a$.

• $P$ should choose $a$ so as to minimize $\sum_{n=1}^{N} q_n L(a, x_n)$. 
Risk-Neutral Probabilities

• We don’t affect decisions if we divide $q_n$ by a constant.

• Define:

$$q^*_n = \frac{q_n}{\sum_{m=1}^{N} q_m}$$

• $q^*$ is called the risk-neutral probability density (RNPD) of $x$.

  – Probability means: $q^*$ sums to one and $q^*_n$ is nonnegative for all $n$. 
Risk-Neutral and "True" Probabilities

- The RNPD $q^*$ of $x$ is not the same as the "true" probability density of $x$.
  - And what exactly is the "true" probability density of $x$?

- $q^*$ reflects asset traders’ aversion to risk.

- And $q^*$ reflects asset traders’ assessments of the likelihood of $x$. 
For any function $\phi$ of $x$, define:

$$E^*(\phi(x)) = \sum_{n=1}^{N} q_n^* \phi(x_n)$$

- $P$ can optimally choose $a$ by minimizing:

$$E^*(L(a, x))$$

- If $L$ is differentiable with respect to $a$:

$$E^* \left\{ \frac{\partial L}{\partial a} (a^*, x) \right\} = 0$$
Verbal Summary

- **Standard:** Policymaker’s optimal choice sets the *outlook* for \( L_a \) equal to zero.

- **Novel:** The appropriate notion of the outlook is given by \( E^* \).

- Intuitively, policymaker makes choices so as to balance losses across states of the world.

- The relevant trade-offs are governed by state prices, not statistical forecasts.
Aside: Endogeneity of State Prices

- Above: I’ve treated $q^*$ as exogenous to $P$.

- More realistic: Risk-neutral probability density $q^*$ depends on $a$.

- Then, $P$’s problem is to choose $a$ to minimize:

$$
\sum_{n=1}^{N} q_n^*(a) L(a, x_n)
$$
• Suppose $P$ ignores endogeneity and chooses $a^*$ so that:

$$E^*[\frac{\partial L}{\partial a}(a^*, x_n)] = 0$$

• Result: This choice is nearly optimal as long as this second moment:

$$Cov^*(L(a^*, x), \frac{\partial \ln q^*(a^*)}{\partial a})$$

is sufficiently small.

• Note: This second moment is calculated using the RNPD $q^*(a^*)$. 
EXAMPLE:

INFLATION-TARGETING
Model of Inflation-Targeting

• Consider a hypothetical central bank (CB) with a single mandate: inflation target $\pi$.

• CB chooses accommodation $a$ that, next period, results in:
  
  – inflation rate $\pi = (a + x)$
  
  – where $x$ is random
• Sticky prices imply that there is an efficiency loss if $\pi$ differs from the target $\bar{\pi}$.

• The gap $|\pi - \pi^*|$ generates an approximate dollar loss:

$$\kappa(\pi - \bar{\pi})^2$$

• That is, the CB’s loss function is well approximated by:

$$L(a, x) = \kappa(a + x - \bar{\pi})^2$$
First Order Condition

- The CB chooses $a$ to minimize:
  \[ E^*(a + x - \bar{\pi})^2 \]

- This results in the first-order condition:
  \[ E^*(\pi) = \bar{\pi} \]

- The inflation-targeting CB ensures that the outlook for $\pi$ is kept near $\bar{\pi}$.

- Standard result - except the relevant outlook is given by $E^*$, not $E$. 
Intuition

- $E^*(\pi)$ can be measured with inflation break-evens.
  - on TIPS bonds or on zero coupon inflation swaps

- These break-evens imply that $E^*(\pi)$ is generally larger than (usual measures of) $E(\pi)$.

- Keeping $E^*(\pi)$ equal to $\pi$ will result in $E(\pi)$ being less than $\pi$.

- Why is this desirable?
• $E^*(\pi) > E(\pi) \Rightarrow$ state prices tend to be high when inflation is high.

• This means that $\pi > \bar{\pi}$ is more costly to society than $\pi < \bar{\pi}$.

• Hence, optimal monetary policy should lead to $E(\pi)$ being lower than $\bar{\pi}$. 

RNPDs and Predictions

• FAQ: Do RNPDs forecast the future better than statistical models?

• Similar: Did RNPDs in 2006 reveal the coming asset price corrections?

• My point today is that these are the wrong questions for policymakers to ask.
Financial Market Data and Decisions

- Policymakers form future outlooks so as to make current decisions with future outcomes.

- Optimal decisions trade off benefits/costs in future states of the world.

- The trade-off should *not* be based on ex ante (or ex post!) assessments of the states’ probabilities.

- Instead, the trade-off should be based on the ex ante relative *values* of resources in those states.

Hence, the relevant outlook for a policymaker is an RNPD.
Implementation Challenges

• Decision-making using RNPDs is not necessarily easy.
  – Need to determine appropriate financial proxy.
  – Even then: Available options may not cover longer horizons or extreme tail events.

• Nothing new: Good decisions are always based on a mix of good judgment, good data, and good modeling choices.

BUT:

The right goal is to model/estimate RNPDs, not statistical forecasts.
Ninth District Activities

- Minneapolis Fed’s Banking Group uses options data to compute RNPDs.

- They report the results on the public website for a wide range of assets.
  - Gold, silver, wheat, S&P 500, exchange rates, etc.

- They report and archive the results on a biweekly basis.