Optimal Outlooks

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Disclaimer and Acknowledgements

Disclaimer: I am not speaking for others in the Federal Reserve System.

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Need for Outlooks

• A policymaker needs to make a decision today.

• The current decision results in random future net benefits to society.

• Hence, the policymaker’s decision depends on the outlook about those net benefits.
Question

What’s the appropriate notion of an outlook for this policymaker?
Answer

- The needed outlook is not a statistically motivated *predictive density* ...

- But rather an asset-price-based *risk-neutral probability density* (RNPD).
Main Result

• A policymaker reaches the same ex-ante decision by:
  – maximizing social welfare
  – maximizing risk-neutral expected benefits

• Maximizing statistical expectation of benefits is typically different.
Intuition

- To make an ex-ante decision, the policymaker weighs social benefits in different future states against each other.

- To maximize social welfare: relevant weights are households’ ex-ante relative marginal valuations of resources in those states.
• RNPDs are derived from financial market *prices*.

• Those prices reflect households’ ex-ante relative marginal valuations of resources in different future states.

• Hence: the risk-neutral expectation also weighs benefits in different states according to households’ ex-ante relative marginal values of resources.
Outline

1. General Policy Problem

2. Risk-Neutral Probabilities

3. Equivalence

4. Possible Concerns

5. Conclusions
GENERAL POLICY PROBLEM
Random Outcomes

• Policymaker chooses an action $a$ today.

• The result of the action next period depends on the realization of $x$.
  – The random variable $x$ has realizations $\{x_n\}_{n=1}^{N}$.

• The outcome $(a, x)$ results in a benefit of $B(a, x)$.
  – The benefit $B(a, x)$ may be positive or negative.
Examples of $B$

- **Inflation targeting**: $B(a, x) = -(a + x - \pi^*)^2$
  
  - $a$ is accommodation
  
  - $x$ is inflation shock

- **Financial instability**: $B(a, x)$

  - $a$ is bank dividends
  
  - $x$ is financial stress
Social Welfare

- If realization $x_n$ occurs, households consume $(y(x_n) + B(a, x_n))$.

- Households’ ex-ante (subjective) expected utility is:

  $$\sum_{n=1}^{N} \pi_n U(y(x_n) + B(a, x_n), x_n)$$

- The households’ utility function $U$ is possibly state-dependent.

- Also: $\pi_n$ are subjective probabilities, not "true" probabilities.
• Chain rule: optimal choice of $a$ satisfies FOC:

$$\sum_{n=1}^{N} \pi_n MUC_n(a^\ast) \frac{\partial B}{\partial a}(a^\ast, x_n) = 0$$

where $MUC_n(a^\ast)$ is the marginal utility of consumption in state $n$:

$$MUC_n(a^\ast) \equiv U_c(y(x_n) + B(a^\ast, x_n), x_n)$$
Missing Information

- Policymaker needs to know:
  - State-dependent marginal utility: $MUC_n(a^*)$
  - Household subjective probabilities: $\pi_n$.

- No good data on these!

- But we will see:

  Relevant information is encoded in risk-neutral probability density.
RISK-NEUTRAL PROBABILITIES
RNPD

- Suppose households trade assets *before* policymaker chooses $a$.

- Let $q_n$ represent the (implied) price of goods in state $n$.

- Define $q^* = (q^*_n)_{n=1}^{N}$ to be:
  \[
  q^*_n = \frac{q_n}{\sum_{n=1}^{N} q_n}
  \]

- $q^*$ is called the *risk-neutral probability density (RNPD)*.
  - Probability means: $q^*_n$ is positive and $q^*_n$'s sum to 1.
RNPD in Equilibrium

- Households treat $a^*$ as given when trading assets.

- In equilibrium, there is a constant $\xi > 0$ such that:

$$q_n = \xi \pi_n MUC_n(a^*)$$

- Hence:

$$q^*_n = \frac{\pi_n MUC_n(a^*)}{\sum_{m=1}^N \pi_m MUC_m(a^*)}$$
Risk-Neutral and "True" Probabilities

- The RNPD $q^*$ is not the same as the "true" probability density of $x$.

- $q^*$ reflects households’ marginal utilities.

- And $q^*$ reflects households’ subjective probabilities.
• For any random variable $\phi$, define:

$$E^*(\phi) = \sum_{n=1}^{N} q_n^* \phi_n$$

• Define risk-neutral expected benefits:

$$E^*(B(a, x)) = \sum_{n=1}^{N} q_n^* B(a, x_n)$$
EQUIVALENCE
Maximizing $E^*(\text{Benefits})$

- Suppose policymaker chooses $a$ so as to maximize $E^*(\text{Benefits})$.

- Then, $\hat{a}$ satisfies FOC:

$$E^*\left\{ \frac{\partial B}{\partial a}(\hat{a}, x) \right\} = 0$$
Result - Setup

\[ 0 = \mathbb{E}^* \{ \frac{\partial B}{\partial a}(\hat{a}, x) \} \]
\[ = \sum_{n=1}^{N} q_n^* \{ \frac{\partial B}{\partial a}(\hat{a}, x_n) \} \]

- But we know that for some constant \( \xi > 0 \):

\[ q_n^* = \xi \pi_n MUC_n(\hat{a}) \]
It follows that $\hat{a}$ also satisfies:

$$0 = \sum_{n=1}^{N} \pi_n MUC_n(\hat{a}) \frac{\partial B}{\partial a}(\hat{a}, x_n)$$

This is the same FOC that characterized $a^*$. 

Thus: maximizing $E^*(\text{Benefits})$ is the same as maximizing social welfare.

- But: maximizing $E^*$ only requires knowledge of RNPD.
Verbal Summary

- Standard: Policymaker’s optimal choice sets the outlook for marginal net benefits equal to zero.

- Novel: The appropriate notion of the outlook is given by $E^*$.  

- Policymaker should balance benefits across states of the world using households’ relative marginal valuations of resources in different states.

- The relative marginal valuations are given by RNPD, not statistical density.
CONCERNS
Lack of Predictive Power

**Concern**: RNPDs predict poorly.

**Response**: This is true but irrelevant.

- Policymakers’ decisions should be based on households’ relative valuations of resources in different states.

- These aren’t predictive: they incorporate subjective probabilities and marginal utilities.
Heterogeneity

Concern: Households aren't the same.

Response: The basic equivalence result extends as long as ...

- Redistributions of resources generated by choice of \( a \) can be offset using transfers.

- Similar to: "expanding the pie" argument for free trade.
Costly Information Acquisition

**Concern:** Possible loss of private incentives to acquire information.

- If policy is set so as to keep an asset’s current price constant ...

- Investors have no incentive to get information about its future payoffs.

- Consequence: policy choice does not adequately reflect available information.

- See Bernanke-Woodford (1997) for elegant exposition.
Response

- This concern is mitigated by existence of options with varying strikes.

- With options, investors value information about each outcome of $x$ even if the policymaker ensures that $E^*(\frac{\partial B}{\partial \alpha}(x, a^*))$ always equals zero.

- Note: In constructing RNPDs, we need data on prices from many options with distinct strikes.
Incompleteness of Observed Assets

**Concern:** Given observed assets, there may be multiple RNPDs.

**Response:** The basic equivalence result extends as long as ...

- For any action $a$, the benefit $B(a, x)$ is spanned by the payoffs of observed assets.

- Even without spanning: we can find upper and lower bounds to $B(a, x)$ consistent with absence of arbitrage.
Limited Participation

**Concern**: Few households trade in option mkts used to construct RNPDs.

**Response**: This is a problem if they’re barred from participating.

- However, I find it more plausible that they are *choosing* not to participate.

- That decision suggests that their relative marginal valuations of resources in various states are similar to that implied by option markets.
Illiquidity

**Concern:** Asset prices could differ because of liquidity, not risk, differences.

**Response:** This is a potential issue.

- Specifically: options with similar strikes might have very different prices.

- Right response: appropriate attention to robustness.

- Wrong response: abandon RNPDs completely.
CONCLUSIONS
• Policy decisions often impact the economy a lag.

• Hence, policymakers need some way to gauge the relative likelihoods of future events.

• Monetary: How likely is deflation? How likely is high inflation?

• Financial regulation: How likely is significant financial instability?
• Typical approach: attempt to figure out "true" probability of future events.

• Point of this talk: For policymakers that care about social welfare, the relevant probability is a risk-neutral probability.

• RNPDS encode households’ ex-ante marginal valuations of resources in different states.

• Good policymaking should be based on these relative valuations.
Thus, the risk-neutral probability of deflation could rise because:

- Households view that outcome as more likely
- Households’ marginal utility of resources in that outcome has risen.

Both of these changes should matter for a monetary policymaker who can influence the likelihood of deflation.
Implementation Challenges

• Decision-making using RNPDs is not necessarily easy.
  – Need to determine appropriate financial proxy for relevant event.
  – Even then: Available options may not cover longer horizons or extreme tail events.

• Nothing new: Good decisions are always based on a mix of good judgment, good data, and good modeling choices.

BUT:

The right goal is to model/estimate RNPDs, not statistical forecasts.
Ninth District Activities

• Minneapolis Fed’s Banking Group uses options data to compute RNPDs.

• They report the results on the public website for a wide range of assets.
  – Gold, silver, wheat, S&P 500, exchange rates, etc.

• They report and archive the results on a biweekly basis.

• See http://www.minneapolisisfed.org/banking/rnpd.