

# Incomplete Labor Markets\*

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## Abstract

In this paper, I explore the properties of *incomplete labor markets models*. These models drop the usual equilibrium restriction that households optimally choose their level of labor supply, and treat the real interest rate as exogenous. I show that in incomplete labor markets models with overlapping generations or credit constraints, a fall in the price of land generates an inefficient decline in employment if the real interest rate remains constant. The low employment means that, in these kinds of models, declines in the real interest rate or debt-financed increases in government spending are Pareto improving.

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# 1 Introduction

Many, if not most, macroeconomic models make three assumptions about labor markets: agents are homogeneous, agents behave competitively in making labor supply decisions, and the aggregate labor market clears instantaneously. Any model that includes these three assumptions is sharply inconsistent with the U.S. data from the past five years.<sup>1</sup> From June 2006 to June 2011, real hourly earnings have risen slightly (around 3%), and per capita real consumption has fallen slightly (less than 1%). The fall in consumption indicates that people are less wealthy. If leisure is a normal good, then people should buy less of it, and so work more. The rise in real wages means that leisure has become more expensive to buy. Again, people should buy less leisure, and so work more. Yet, over this same five-year period, hours worked per person have fallen sharply.

Motivated by this observation, I explore the theoretical properties of what I term *incomplete labor markets* models. I use this terminology because, as in (now conventional) incomplete markets models of financial markets, I restrict the ability of agents within the model to engage in all mutually beneficial exchanges. In incomplete financial markets models, agents cannot make all possible trades of state-contingent and date-contingent claims to

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<sup>1</sup>The inconsistency is also a product of a fourth assumption about preferences: household preferences are such that the marginal rate of substitution between current consumption and current leisure is independent of consumption and leisure in other dates and states. Virtually all macroeconomic models make this assumption - but see Kydland and Prescott (1982) for an exception.

consumption. In incomplete labor markets models, households are not able to offer to supply more labor for firms at a lower real wage. More specifically, when I define the equilibrium conditions of an incomplete labor markets model, I drop the usual equilibrium condition that households choose their labor supply so as to maximize utility. I replace this condition with the assumption that the real interest rate is exogenous.

Treating the real interest rate as exogenous in a closed economy is unconventional. The idea here is that the central bank can control the nominal interest rate. Hence, the different assumptions about the exogeneity (endogeneity) of the real interest rate in the two different notions of equilibrium are really different assumptions about the exogeneity (endogeneity) of the expected inflation rate. In a complete labor markets equilibrium, the expected inflation rate is an equilibrium object that adjusts to clear the labor market. In an incomplete labor markets equilibrium, the expected inflation rate is independent of market forces.

I consider the impact of a permanent fall in land prices in incomplete and complete labor markets models with overlapping generations and/or credit constraints. (As will become clear, the fall in land prices can be generated by either fundamental or non-fundamental shocks.) To heighten the contrast between the two kinds of models, I assume that households are willing to supply labor inelastically, up to some maximal amount.

The fall in land prices serves to cut back on the outside supply of risk-free assets. In a complete labor markets model, the reduction in the supply of

risk-free assets, combined with the households' willingness to supply labor inelastically, drives down the real interest rate while leaving employment unchanged. In an incomplete labor markets model, the impact of a fall in land prices depends on how policymakers move the exogenous real interest rate. If the real interest rate is not changed, then the fall in the supply of risk-free assets results in a fall in consumption. The permanent fall in land prices leads to a permanent fall in employment.

Interventionist policies can be beneficial in an incomplete labor markets model. For example, a short-term debt-financed increase in government spending raises employment and output permanently and is Pareto improving. Lowering the real interest rate sufficiently can also lead to a Pareto-improving increase in employment and output. Perhaps more surprisingly, within these models, any government intervention that reduces the private demand for saving can generate an increase in employment. Thus, in these models, the government can raise current employment by committing to raise future (lump-sum) taxes so as to provide Social Security and Medicare payments for the currently young.

In incomplete labor markets models, equilibrium employment falls in response to an adverse demand shock (here, a fall in consumption demand triggered by a fall in land prices). In this sense, my paper is highly related to recent work by Hall (2011a, 2011b). My contribution over Hall's is that my analysis provides a tighter connection between changes in asset values and the ultimate impact of those changes in labor markets. In this sense, my

paper tries to answer Ohanian's (2010) call for further research that builds connections between financial market shocks and labor market distortions. See Farmer (2011) for an alternative approach.

Like incomplete labor markets models, New Keynesian models also have the property that equilibrium employment falls in response to an adverse demand shock. Nonetheless, incomplete labor markets models are distinct from New Keynesian models in a number of ways. Incomplete labor markets models provide a more explicit model of the change in the demand for consumption, which is typically attributed to an increase in patience in New Keynesian models. More importantly, product prices are flexible in an incomplete labor markets model, whereas price stickiness is the hallmark of New Keynesian modeling. Simple New Keynesian models are consistent with all three of the assumptions described in the first paragraph (homogeneity, competitive workers, and labor market clearing). As I've discussed there, such models are inconsistent with macroeconomic data from the past five years. Richer New Keynesian models allow workers to have market power in the setting of nominal wages, and these models can qualitatively rationalize the data pattern described in the first paragraph through an increase in workers' labor market power. But these explanations are not wholly compelling; at a minimum, they leave open the desirability of exploring alternative modeling approaches that do not rely on this kind of change in the economy.

Incomplete labor markets models rule out the possibility that workers can offer to supply labor at a below-equilibrium real wage. Nonetheless, incom-

plete labor markets models are entirely distinct from models with real wage rigidities (such as those described in Shimer (2012) and Michaillat (2012)). A fall in consumption demand will typically not generate a fall in employment in a model with real wage rigidities. In such a model, employment changes take place because of shocks to labor demand - that is, changes in the willingness of firms to hire workers at a given real wage. These changes in labor demand are driven by changes in technology and factor endowments. In Section 3.1.2, I discuss the impact of introducing real wage rigidities into an incomplete labor markets model.

## 2 A Benchmark Model

Consider an overlapping generations economy, in which all agents live two periods. Each cohort has a unit measure of agents. The initial old agents are each endowed with one unit of land. A unit of land pays off 1 unit of land services in every period. An agent born in period 1 and thereafter is endowed with one unit of time. He can produce  $An$  units of consumption with  $n$  units of time. Old agents are unproductive.

An agent born in period 1, or thereafter, has a utility function of the form:

$$u(c_y) + \beta u(c_o + \theta d_o)$$

Here,  $c_y$  represents consumption of goods when young,  $c_o$  is consumption of goods when old, and  $d_o$  is land services consumed when old. I assume

that  $u', -u''$  are both positive. The agents supply labor inelastically. The parameter  $\theta$  is common across all agents and represents how much they value land. The initial old prefer more consumption to less.

The initial old each own real government debt that pays off  $B$  units of consumption. In period 1 and thereafter, the government pays off its obligation by selling one-period bonds that promise  $B$  units of consumption and levying lump-sum taxes on young agents.

Financial markets are complete, so that agents are able to trade consumption for land and bonds.

At this point, I will define two notions of steady-state equilibrium. The first is based on the assumption that labor markets are complete, in the sense that it includes a labor market in which households trade consumption for labor with firms. The second features incomplete labor markets, because households cannot trade labor with firms. In this second notion of equilibrium, the price of real bonds is exogenous.

Treating the real interest rate as exogenous in a closed economy is unconventional. The idea here is that the central bank has control over the nominal interest rate. Hence, the different assumptions about the exogeneity (endogeneity) of the real interest rate in the two different notions of equilibrium are really different assumptions about the exogeneity (endogeneity) of the expected inflation rate.<sup>2</sup> In a complete labor markets equilibrium, the

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<sup>2</sup>As is typical, this nominal interest rate peg by the central bank is consistent with any equilibrium stochastic process for inflation that has the same (constant) expected inflation rate. However, because there is no outside supply of nominal assets, all of these

expected inflation rate is an equilibrium object that adjusts to clear the labor market. In an incomplete labor markets equilibrium, the expected inflation rate is independent of market forces.

## 2.1 Complete Labor Markets Equilibrium

Given the level of government debt  $B$ , I define a (steady-state) complete labor markets equilibrium in this economy to be a specification of a land price  $p_L$ , a bond price  $q$ , a wage  $w$ , and quantities  $(c_y^*, c_o^*, n^*, \tau^*)$  such that:

1. Households optimize:

$$\begin{aligned}
 (c_y^*, c_o^*, n^*, L^*, b^*) &\in \arg \max_{(c_y, c_o, L, n, b)} u(c_y) + \beta u(c_o + \theta L) \\
 &s.t. \ c_y + p_L L + qb = wn - \tau^* \\
 &s.t. \ c_o = p_L L + b \\
 &s.t. \ c_y, c_o, L, b, n, 1 - n \geq 0
 \end{aligned}$$

2. Firms optimize:

$$n^* \in \arg \max_{n \geq 0} An - wn$$

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equilibrium processes for inflation are consistent with the same equilibrium real allocation.



3. Markets clear:

$$c_y^* + c_o^* = An^*$$

$$L^* = 1$$

$$b^* = B$$

$$\tau^* = B - qB$$

This is a rather vanilla overlapping generations model. Given  $B$ , the complete labor markets equilibrium  $(c_y^*, c_o^*, n^*, w, \tau^*, p_L, q)$  is characterized by the following conditions:

$$w = A \tag{1}$$

$$n^* = 1 \tag{2}$$

$$qu'(c_y^*) = \beta u'(c_o^* + \theta) \tag{3}$$

$$\tau^* = B(1 - q) \tag{4}$$

$$c_y^* = wn^* - p_L - B \tag{5}$$

$$c_o^* = p_L + B \tag{6}$$

$$p_L = q\theta/(1 - q) \tag{7}$$

Since the real wage  $w > 0$ , households find it optimal to set  $n^* = 1$ . The bond price  $q$  is determined so that households are marginally indifferent between consumption when young and old. The land price is simply the present value

of the perpetual stream of services generated by land.

## 2.2 Incomplete Labor Markets Equilibrium

Given the level of government debt  $B$  and a real bond price  $q$ , I define an incomplete labor markets (steady-state) equilibrium in this economy to be a land price  $p_L$ , a wage  $w$ , and quantities  $(c_y^*, c_o^*, n^*, \tau^*)$  such that:

1. Households optimize with respect to consumption, land, and bonds:

$$\begin{aligned} (c_y^*, c_o^*, L^*, b^*) &\in \arg \max_{(c_y, c_o, L, b, a)} u(c_y) + \beta u(c_o + \theta L) \\ &s.t. \ c_y + p_L L + qb = wn^* - \tau^* \\ &s.t. \ c_o = p_L L + b \\ &s.t. \ c_y, c_o, L, b \geq 0 \end{aligned}$$

2. Firms maximize profits:

$$n^* \in \arg \max_{n \geq 0} An - wn$$

3. Markets clear:

$$\begin{aligned} c_y^* + c_o^* &= An^* \\ L^* &= 1 \\ b^* &= B \\ \tau^* &= B - qB \end{aligned}$$

In this incomplete labor markets equilibrium, households do not optimize with respect to  $n$ , because they cannot offer to work at a real wage less than  $w$ . In this version of equilibrium,  $1 - n^*$  units of time do not get used.

Given  $q$  and  $B$ , an incomplete labor markets equilibrium  $(c_y^*, c_o^*, n^*, w, \tau^*, p_L)$  is defined by the equations:

$$w = A \tag{8}$$

$$qu'(c_y^*) = \beta u'(c_o^* + \theta) \tag{9}$$

$$\tau^* = B(1 - q) \tag{10}$$

$$c_y^* = wn^* - p_L - B \tag{11}$$

$$c_o^* = p_L + B \tag{12}$$

$$p_L = q\theta/(1 - q) \tag{13}$$

These equations are the same as the ones defining complete labor markets equilibrium, except that I've dropped the household's labor supply decision  $n^* = 1$  and replaced it with an exogenous specification of  $q$ .

I think of the economics of an incomplete labor markets equilibrium in the following way. Firms treat total product demand as given, and they engage in price competition over market share. (Unlike in New Keynesian models, prices are flexible.) This kind of competition drives down product prices to the point that the real wage equates the marginal product of labor  $A$ . However, labor markets are dysfunctional: firms will not accept an offer from a worker to work more hours at a lower wage.

## 2.3 Comparative Statics

In this subsection, I analyze the impact of changes in exogenous variables within the two different notions of equilibrium. It is useful to note that because land is an asset that pays off  $\theta$  in every period,  $q$  must be less than one (positive real interest rates) in either notion of equilibrium. It follows that:

$$u'(c_y^*) > \beta u'(c_o^* + \theta)$$

Both notions of equilibrium are dynamically efficient, and that means that young households are “over-saving” relative to the natural rate of interest (equal to the population growth rate of zero). In the next section, I expand the analysis to include models with dynamically inefficient equilibria.

### 2.3.1 Complete Labor Markets Equilibrium

In a complete labor markets equilibrium with a lower value of  $\theta$ , the price of bonds  $q$  has to rise so as to satisfy the young household’s intertemporal Euler equation:

$$qu'(A - \frac{q\theta}{1-q} - B) = \beta u'(\frac{\theta}{1-q} + B)$$

Thus, if people don’t like land services as much, their demand for bonds rises (so as to better fund their retirements). If the supply of bonds is fixed at  $B$ , then bond prices rise and the real interest rate falls. In equilibrium, the

young end up consuming more.

In a complete labor markets equilibrium with a higher value of  $B$ , the price of bonds  $q$  has to fall so as to satisfy the young household's intertemporal Euler equation:

$$qu'(A - \frac{q\theta}{1-q} - B) = \beta u'(\frac{\theta}{1-q} + B)$$

With more bonds available to save for retirement, the real interest rate rises.

In a complete labor markets equilibrium with a higher value of  $A$ , the consumption of both young and old households rises. There is a fall in the real interest rate, as young households demand more bonds.

Because of the simplicity of the model, these steady-state calculations are actually informative about transitions. Suppose there is an unanticipated permanent fall in  $\theta$  in period  $t$ . That fall in  $\theta$  will make the price of land fall immediately. The old agents are worse off.

Should the government make up for the losses of the old by selling more bonds in that period and then transferring the proceeds to the old? The old in period  $t$  will be made better off by the government's doing so. However, the government needs to pay off its obligations by rolling over this higher level of debt. Because the complete labor markets equilibrium is dynamically efficient, all future generations are made worse off with the higher level of government debt.

### 2.3.2 Incomplete Labor Markets Equilibrium

Given a fixed value of  $q$ , in an incomplete labor markets equilibrium with a lower value of  $\theta$ , the value of  $n^*$  has to fall so as to satisfy the household's intertemporal Euler equation:

$$qu'(An^* - \frac{q\theta}{1-q} - B) = \beta u'(\frac{\theta}{1-q} + B)$$

Intuitively, when  $\theta$  falls, the price of land falls, and old households have to consume less. With a fixed real interest rate, the young households also consume less. Total output falls, and all households are worse off in steady-state.

In an incomplete labor markets equilibrium with a higher value of  $B$ , the value of  $n$  has to rise so as to satisfy the household's intertemporal Euler equation:

$$qu'(An^* - \frac{q\theta}{1-q} - B) = \beta u'(\frac{\theta}{1-q} + B)$$

Intuitively, with a higher value of  $B$ , old households can consume more. Given the fixed value of  $q$ , young households also consume more.

In an incomplete labor markets equilibrium with a higher value of  $A$ , the value of  $n^*$  has to fall to satisfy the household's intertemporal Euler equation:

$$qu'(An^* - \frac{q\theta}{1-q} - B) = \beta u'(\frac{\theta}{1-q} + B)$$

An increase in productivity has no impact on output and lowers employment.

In an incomplete labor markets equilibrium, the value of  $q$  is exogenous. What happens if  $q$  increases? Look at the household's intertemporal Euler equation. We can see that:

$$qu'(An^* - \frac{q\theta}{1-q} - B) = \beta u'(\frac{\theta}{1-q} + B)$$

It follows that  $n^*$  has to rise to satisfy this Euler equation. In other words, lower values of the real interest rate lead to higher steady-state output.<sup>3</sup>

### 3 Other Incomplete Labor Markets Models

The basic logic in the last section was simple. In both models, I considered the impact of a fall in the price of land that shrank the outside supply of assets. In a complete labor markets model, the decline in the supply of outside assets led the real interest rate to fall. People saved less and consumed more when young. However, total output remained unchanged. In an incomplete labor markets model, the real interest rate only falls if the government adjusts it downward. Without that downward adjustment, people view current consumption as being too expensive and demand little of it. Equilibrium consumption and output both fall.

In this section, I discuss how this basic logic can be extended to four other kinds of incomplete labor markets models. First, I allow for the pos-

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<sup>3</sup>Given that  $n^*$  is capped at one in this economy, there is no incomplete markets equilibrium for values of  $q$  larger than the complete markets equilibrium bond price  $q^*$ .

sibility of diminishing returns in labor by endowing the young agents with a (perishable) fixed factor of production. Next, I add capital to the overlapping generations version of the incomplete labor markets model discussed above. Then I consider the case of a bursting bubble in land prices. Finally, I examine the implications of a fall in land prices in an incomplete labor markets model with infinitely lived agents who face borrowing constraints. This last analysis builds on Woodford's (1986) classic equivalence result between overlapping generations economies and economies with infinitely lived finance-constrained agents.

### **3.1 Adding Diminishing Returns to Labor**

In this subsection, I change the earlier model so that the young agents are endowed with one unit of a fixed factor to production that disappears when they are old. Firms produce  $F(n, a)$  units of consumption from  $n$  units of labor and  $a$  units of the fixed factor, where  $F$  is a constant returns-to-scale production function. They rent the fixed factor from the young in a competitive market. I consider two distinct notions of equilibrium. In the first, real wages are free to adjust downward (as described in the benchmark model). In the second, I introduce a simple labor market friction, in that real wages are bounded from below.



### 3.1.1 Flexible Real Wages

Given a real bond price  $q$  and a real debt level  $B$ , I define an incomplete labor markets (steady-state) equilibrium in this economy with a fixed factor to be a land price  $p_L$ , a wage  $w$ , a rental rate  $r$ , and quantities  $(c_y^*, c_o^*, n^*, k^*, \tau^*, b^*, a^*)$  such that:

1. Households optimize with respect to consumption, land, bonds, and factor rental:

$$\begin{aligned} (c_y^*, c_o^*, L^*, b^*, a^*) &\in \arg \max_{(c_y, c_o, L, b, a)} u(c_y) + \beta u(c_o + \theta L) \\ &s.t. \ c_y + p_L L + qb = wn^* - \tau^* + ra \\ &s.t. \ c_o = p_L L + b \\ &s.t. \ c_y, c_o, L, b, a, 1 - a \geq 0 \end{aligned}$$

2. Firms maximize profits:

$$(n^*, a^*) \in \arg \max_{n, a \geq 0} F(n, a) - wn - ra$$

3. Markets clear:

$$c_y^* + c_o^* = F(n^*, 1)$$

$$L^* = 1$$

$$b^* = B$$

$$\tau^* = B - qB$$

Given  $q$  and  $B$ , an incomplete labor markets equilibrium  $(c_y^*, c_o^*, n^*, w, r, \tau^*, p_L, b^*, a^*)$  is defined by the equations:

$$w = F_n(n^*, a^*) \quad (14)$$

$$r = F_a(n^*, a^*) \quad (15)$$

$$a^* = 1 \quad (16)$$

$$qu'(c_y^*) = \beta u'(c_o^* + \theta) \quad (17)$$

$$\tau^* = B(1 - q) \quad (18)$$

$$c_y^* = wn^* - p_L - B + r \quad (19)$$

$$c_o^* = p_L + B \quad (20)$$

$$p_L = q\theta/(1 - q) \quad (21)$$

Here, the notations  $F_n$  and  $F_a$  represent the corresponding partial derivatives.

From Euler's theorem, we can conclude that:

$$c_y^* = F(n^*, 1) - p_L - B$$

The level of labor is then determined by the household's Euler equation:

$$qu'(F(n^*, 1) - p_L - B) = \beta u'(\theta/(1 - q) + B) \quad (22)$$

This equation implies that the qualitative comparative statics described in Section 2 apply in this incomplete labor markets model with diminishing

returns.

### 3.1.2 Real Wage Floor

Models of the labor market often emphasize the importance of lower bounds on real wages (due to minimum wage restrictions or collective bargaining considerations). The model with diminishing returns allows us to think about the impact of such a lower bound on the real wage. Define *an incomplete labor markets equilibrium with a real wage floor*  $\bar{w}$  to be a land price  $p_L$ , a wage  $w$ , a rental rate  $r$ , and quantities  $(c_y^*, c_o^*, n^*, k^*, \tau^*, b^*, s^*)$  such that:

$$w = F_n(n^*, a^*) \quad (23)$$

$$r = F_a(n^*, a^*) \quad (24)$$

$$a^* = 1 \quad (25)$$

$$qu'(c_y^*) = \beta u'(c_o^* + \theta) \quad (26)$$

$$\tau^* = B(1 - q) \quad (27)$$

$$c_y^* = wn^* - p_L - B + r \quad (28)$$

$$c_o^* = p_L + B \quad (29)$$

$$p_L = q\theta/(1 - q) \quad (30)$$

$$w \geq \bar{w} \quad (31)$$

The lower bound on the real wage does not affect the comparative statics results sketched in the previous subsection. However, if  $F_{nn} < 0$ , then the

lower bound on the real wage does imply that, in any equilibrium:

$$n^* \leq \min(1, \bar{n})$$

where:

$$F_n(\bar{n}, 1) = \bar{w}$$

In other words, changes in  $\theta$ ,  $B$ , and  $q$  cannot raise  $n$  above  $\bar{n}$ , even if  $\bar{n}$  is less than one. Kocherlakota (2012) emphasizes this kind of upper bound on employment as a restriction that implies that monetary policy, and other kinds of aggregate demand management, cannot necessarily achieve full employment.

## 3.2 Adding Capital

In this subsection, I change the earlier model so that the initial old are endowed with  $k$  units of capital (along with the bond and land endowments mentioned earlier). Capital depreciates at rate  $\delta$  from one period to the next, and firms produce output using capital and labor according to the constant returns-to-scale production function  $F(k, n)$ . Then, given a real bond price  $q$  and a real debt level  $B$ , I define an incomplete labor markets (steady-state) equilibrium in this economy to be a land price  $p_L$ , a wage  $w$ , a capital rental rate  $r$ , and quantities  $(c_y^*, c_o^*, n^*, k^*, \tau^*, b^*)$  such that:

1. Households optimize with respect to consumption, land, bonds, and

capital:

$$\begin{aligned}(c_y^*, c_o^*, L^*, b^*, k^*) \in \arg \max_{(c_y, c_o, L, b, a)} & u(c_y) + \beta u(c_o + \theta L) \\ \text{s.t. } & c_y + p_L L + qb + k = wn^* - \tau^* \\ \text{s.t. } & c_o = p_L L + b + rk + (1 - \delta)k \\ \text{s.t. } & c_y, c_o, L, b, k \geq 0\end{aligned}$$

2. Firms maximize profits:

$$(k^*, n^*) \in \arg \max_{n \geq 0} F(k, n) - wn - rk$$

3. Markets clear:

$$c_y^* + c_o^* + \delta k^* = F(k^*, n^*)$$

$$L^* = 1$$

$$b^* = B$$

$$\tau^* = B - qB$$

Given  $q$  and  $B$ , an incomplete labor markets equilibrium  $(c_y^*, c_o^*, n^*, w, r, \tau^*, p_L, k^*, b^*)$

is defined by the equations:

$$w = F_n(k^*, n^*) \quad (32)$$

$$r = F_k(k^*, n^*) \quad (33)$$

$$qu'(c_y^*) = \beta u'(c_o^* + \theta) \quad (34)$$

$$q(1 - \delta + r) = 1 \quad (35)$$

$$\tau^* = B(1 - q) \quad (36)$$

$$c_y^* = wn^* - p_L - B - k^* \quad (37)$$

$$c_o^* = p_L + B + k^*(1 - \delta + r) \quad (38)$$

$$p_L = q\theta/(1 - q) \quad (39)$$

Here, the notations  $F_n$  and  $F_k$  represent the corresponding partial derivatives.

Given  $q$ , equation (35) determines  $r$ , equation (33) determines the ratio  $\widehat{k} = k^*/n^*$ , and equation (32) determines  $w$ . The level of labor is then determined by the household's Euler equation:

$$qu'(wn^* - p_L - B - \widehat{k}n^*) = \beta u'(\theta/(1 - q) + B + \widehat{k}(1 - \delta + r)n^*) \quad (40)$$

In the appendix, I prove that if  $u$  exhibits non-increasing relative risk aversion and  $q \leq \beta$ , then  $n^*$  is increasing as a function of  $B$  and  $\theta$  in the neighborhood of a steady-state in which  $w > \widehat{k}(2 - \delta + r)$ . Under these same conditions,  $n^*$  is increasing as a function of  $q$  in the neighborhood of a steady-state in which  $-F_{kk}k < 1$ , where  $F_{kk}$  represents the partial second derivative with respect

to  $k$ . I show too that this restriction on  $F$  is satisfied if  $F$  is Cobb-Douglas and  $q$  is sufficiently close to one.

### 3.2.1 Dynamic Inefficiency: The Collapse of a Bubble

I return to the overlapping generations model without capital of the last section. However, I change the model of preferences so that agents have a utility function of the form:

$$u(c_y) + \beta u(c_o)$$

so that land has no intrinsic service flow. This modeling approach allows for the possibility that land prices have a bubble. In particular, given a real bond price  $q = 1$  and an outside debt level  $B$ , an incomplete labor markets (steady-state) equilibrium  $(c_y^*, c_o^*, n^*, w, \tau^*, p_L)$  in this economy is defined by the equations:

$$w = A \tag{41}$$

$$u'(c_y^*) = \beta u'(c_o^*) \tag{42}$$

$$\tau^* = 0 \tag{43}$$

$$c_y^* = An^* - p_L - B \tag{44}$$

$$c_o^* = p_L + B \tag{45}$$

There is a continuum of possible steady-state equilibria in this economy, indexed by the price of land. Across these steady-states, output and employment (that is,  $n^*$ ) are increasing functions of  $p_L$  (that is, the bubble).

What if  $q$  is set to a constant value other than one? In that case, the price of land cannot be constant in equilibrium (because it must rise at the rate of interest). If  $q$  is set to a constant that is larger than one, there is a continuum of possible equilibria. In all of these equilibria, the land price eventually converges to zero (because the real interest rate is negative). In contrast, if  $q$  is set equal to a constant less than one, then the unique equilibrium is one in which the price of land equals its fundamental value of zero. In this sense, this model justifies the notion that easy monetary policy generates bubbles.

### 3.2.2 Borrowing Constraints

Consider an economy with a unit measure of *even* agents and a unit measure of *odd* agents. All agents are infinitely lived and have the same utility function:

$$\sum_{t=1}^{\infty} \beta^{t-1} u(c_t + \theta d_t)$$

where  $c_t$  is the agent's consumption in period  $t$ , and  $d_t$  is the land owned by the agent at the beginning of period  $t$ . All agents are endowed with one unit of time in every period. Odd agents can produce  $An$  units of consumption by working  $n$  units of time in odd-dated periods, but they are unproductive



in even-dated periods. The reverse is true of even agents.

The even agents (who are unproductive in period 1) are initially endowed with one unit of land. The even agents are also each endowed with  $B$  units of a one-period bond that pays off one unit of consumption in period 1. The government raises the taxes to pay that bond by taxing the productive agents  $\tau^*$  and issuing  $B$  units of new one-period bonds. Agents cannot short-sell land or bonds.

I consider a steady-state incomplete labor markets equilibrium, in which productive agents always consume the same amount  $c_p^*$ , unproductive agents always consume the same amount  $c_u^*$ , and agents who are productive in a given period begin that period holding no land. Mathematically, given a constant one-period real bond price, I define an incomplete labor markets equilibrium in this economy to be a specification of a land price  $p_L$ , a wage

$w$ , and quantities  $(c_p^*, c_u^*, n^*, \tau^*)$  such that:

$$qu'(c_p^*) = \beta u'(c_u^* + \theta) \quad (46)$$

$$qu'(c_u^* + \theta) \geq \beta u'(c_p^*) \quad (47)$$

$$c_p^* + c_u^* = An^* \quad (48)$$

$$w = A \quad (49)$$

$$\tau^* = B(1 - q) \quad (50)$$

$$p_L = q\theta/(1 - q) \quad (51)$$

$$c_p^* = wn^* - B - p_L \quad (52)$$

$$c_u^* = B + p_L \quad (53)$$

If we translate  $c_y^*$  into  $c_p^*$  and  $c_o^*$  into  $c_u^*$ , these conditions are identical to the ones characterizing incomplete labor markets equilibrium in the original overlapping generations economy. The exception is (47). That condition guarantees that the unproductive agents don't want to buy any land or bonds. It is satisfied if  $q \geq \beta$ .

## 4 Recent Macroeconomic Experience

In this section, I interpret recent macroeconomic data using the benchmark incomplete labor markets model and describe the model's implications for the impact of various policy interventions.

## 4.1 Interpreting Data

From the middle of 2006 through the end of 2010, the price of residential land in the United States fell sharply. I believe that it is not unreasonable to treat this fall as largely unanticipated. Consider the benchmark overlapping generations model with incomplete labor market models discussed in Section 2. Suppose that, at some date  $T$ , the utility  $\theta$  from land services falls permanently to a new value  $\theta' < \theta$ . How does this change affect the path of employment in this benchmark model - assuming, for the moment, that the real bond price  $q$  and the government debt  $B$  do not change?

In the benchmark model, transitions are simple. The old agents' demand for consumption is always equal to the sum of the real value of land and the real payoff from bonds. Hence, after the unexpected fall in  $\theta$  occurs in period  $T$  and thereafter, employment falls to  $n'$ , which solves:

$$\begin{aligned}qu'(c'_y) &= \beta u'(c'_o + \theta') \\An' &= c'_y + c'_o\end{aligned}$$

where  $c'_o$  is the new consumption of the old agents and  $c'_y$  is the new consumption of the young agents. The fall in  $\theta$  lowers  $c'_o + \theta'$ . Given a fixed  $q$ , the fall in  $c'_o + \theta'$  leads to a fall in  $c'_y$ :

$$c'_y = u'^{-1}(\beta q^{-1} u'(c'_o + \theta'))$$

The impact on  $n'$  depends on the functional form of  $u$  and the impact of the fall in  $\theta'$  on the consumption of the old agents. Suppose, for example, that  $u'(c) = c^{-\gamma}$ . In that case, if  $c'_o + \theta'$  falls by  $\alpha\%$ , so does employment and output, regardless of the size of  $\gamma$ .

Hence, in the benchmark incomplete labor markets model, a fall in  $\theta$  triggers a simultaneous fall in land prices, consumption, and employment even though real wages do not change. This implication is roughly consistent with the observation that land prices, consumption, and employment have fallen in the United States over the past five years, while real wages have remained roughly constant.<sup>4</sup>

## 4.2 Policy Perspectives

In the previous subsection, I considered the impact of an unanticipated fall in  $\theta$  at date  $T$  in the benchmark model. Suppose that the government reacts to that fall in  $\theta$  by raising  $q$  (lowering interest rates) in period  $T$  and thereafter. That increase in  $q$  translates into an increase in  $p_L$ ,  $c'_y$ , and thereby into a permanent increase in  $n'$ . Note that all agents are better off. The young agents are unhappy because they are receiving a lower real interest rate - but that decline is offset by the increase in their labor incomes.

Now suppose that the government sells  $B'$  units of bonds in period  $T$  and

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<sup>4</sup>From 2006:I to 2011:I, non-farm output per hour rose nearly 10%, while non-farm real hourly earnings rose only 3%. The models that I've constructed - with competitive product markets - are inconsistent with these observations. These data on wages and productivity suggest that firms enjoy considerably more product market power in 2011 than in 2006.

thereafter, where  $B' > B$ . It spends the extra resources in period  $T$  on public goods or alternatively distributes them to the old agents in period  $T$ . The additional bonds serve to deliver more consumption to old agents in period  $(T + 1)$  and thereafter. That leads the young to demand more consumption in period  $T$  and thereafter. Again, all agents are better off, *regardless of how the government spends the extra resources in period  $T$* .

More generally, the government could raise employment and welfare by introducing any lump-sum tax/transfer program that drives out private saving. For example, suppose the government were to tax the young and transfer more to the old. This tax/transfer scheme raises the consumption of the old agents. Given a fixed  $q$ , productive agents' demand for consumption rises and that generates more employment.

## 5 Some Conclusions

There is an ongoing debate among policymakers and academics about the appropriate policy response to the recent large and persistent decline in employment. Sometimes, this debate seems to hinge on whether Ricardian equivalence is valid. Sometimes, it seems to focus on whether prices and wages are sticky or not. In this paper, I consider what I term *incomplete labor markets models*. In these models, the real interest rate is exogenous and workers are unable to offer work more hours for a lower real wage. I show that in these models, there is a natural way in which an unanticipated

decline in the price of land results in an inefficient decline in employment. Relatedly, expansionary fiscal and monetary policy are both welfare-improving responses to such a decline in the price of land. The model's analysis suggests that a key issue in the policy debate is the nature of labor market competition - or the lack thereof.

Some might simply discard incomplete labor markets models out of hand based on (at least) two philosophical considerations. The first is Lucas' famous criticism (1980, p. 709) that disequilibrium models inevitably involve more free parameters than equilibrium models, and we should be wary of theorists bearing free parameters. The second is Barro's (1977) equally famous skepticism that much unemployment could result from the failure of firms and workers to reach mutually beneficial agreements.

There was a time, I think, when these observations might well have seemed very compelling. (They certainly did to me for many years.) I have to say, though, that time is past. In terms of Lucas' observation, empirically relevant equilibrium models always come equipped with a host of less-than-fully-motivated shocks to preferences, technology, and market power. (Indeed, I am reasonably confident that the only way that a modern New Keynesian DSGE model can account for the fall in employment that we've seen in the past five years is through changes in (shocks to) preferences or household monopoly power in labor markets.) It is not obvious why this approach is preferable to a disequilibrium approach of the kind that I pursue in this paper.

In terms of Barro’s observation, there is a great deal of cutting-edge modeling in macroeconomics that is based on the assumption that financial markets are incomplete. This modeling approach proceeds by imposing ad hoc restrictions on the ability of model entities to make mutually beneficial exchanges. As my language in the current paper suggests, I see little a priori distinction between this modeling paradigm and the one that I adopt in this paper.

So, given how macroeconomics has developed in the past thirty years, I don’t find the above philosophical reactions all that useful. I would rather proceed empirically. Here, it seems to me that - nearly thirty years after Kydland and Prescott (1982) and Long and Plosser (1983) - there are still important holes in economists’ understanding of the cyclical determinants of employment. (And I mean to include both labor economists and macro-economists in that characterization.) Can models of the kind described in this paper help fill those holes? Answering this question seems like a first-order intellectual issue - and a first-order policy issue.

## 6 Appendix<sup>5</sup>

In this appendix, I use the equilibrium condition:

$$qu'(wn^* - \frac{q\theta}{1-q} - B - \widehat{kn}^*) - \beta u'(\frac{\theta}{1-q} + B + \widehat{kn}^*(1 - \delta + r)) = 0 \quad (54)$$

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<sup>5</sup>I thank Bernabe Lopez-Martin for his excellent research assistance with this appendix.

from the model with capital to derive local comparative statics of  $n^*$  with respect to  $\theta$ ,  $B$ , and  $q$ . I begin by assuming that:

$$\begin{aligned} u &\text{ is NIARA} \\ q &\leq \beta \\ w &> \widehat{k}(2 - \delta + r) \end{aligned}$$

The derivative of the condition (54) with respect to  $n^*$  is:

$$qu''(c_y^*)(w - \widehat{k}) - \beta u''(c_o^* + \theta)\widehat{k}(1 - \delta + r)$$

Since  $q \leq \beta$ ,  $c_y^* \leq (c_o^* + \theta)$ . Since  $u$  is NIARA, we know that  $u''(c_y^*)/u'(c_y^*) \leq u''(c_o^* + \theta)/u'(c_o^* + \theta)$ . Hence  $-qu''(c_y^*) \geq -\beta u''(c_o^* + \theta)$ , and the derivative:

$$\begin{aligned} &qu''(c_y^*)(w - \widehat{k}) - \beta u''(c_o^* + \theta)\widehat{k}(1 - \delta + r) \\ &\leq qu''(c_y^*)(w - \widehat{k} - \widehat{k}(1 - \delta + r)) \\ &< 0 \end{aligned}$$

is negative. This result immediately implies that, for a fixed  $q$ ,  $n^*$  is an increasing function of  $\theta$  and  $B$  in a neighborhood of the steady-state.

I next add the assumption that:

$$-F_{kk}k < 1 \tag{55}$$



The derivative of the condition (54) with respect to  $q$  is:

$$u'(c_y^*) + qu''(c_y^*)\left(\frac{\partial w}{\partial q}n^* - \frac{\partial \hat{k}}{\partial q}n^* - \frac{\partial p_L}{\partial q}\right) \quad (56)$$

$$-\beta u''(c_o^* + \theta)\left(\frac{\theta}{(1-q)^2} + \frac{\partial \hat{k}}{\partial q}n^*(1-\delta+r) + \hat{k}n^*\frac{\partial r}{\partial q}\right) \quad (57)$$

I want to prove that this derivative is positive (which will imply that  $n^*$  is an increasing function of  $q$  in a neighborhood of the steady-state).

Define  $f(\hat{k}) = F(\hat{k}, 1)$ . The assumption (55) implies that  $-f''(\hat{k})\hat{k} < 1$ .

From the first-order condition defining  $\hat{k}$ , we know that:

$$-q^{-2} = f''(\hat{k})\frac{\partial \hat{k}}{\partial q}$$

and so:

$$\frac{\partial \hat{k}}{\partial q} > 0$$

It is easy to show from the first-order condition for labor, and the homogeneity of  $F$ , that:

$$\frac{\partial w}{\partial q} = -f''(\hat{k})\hat{k}\frac{\partial \hat{k}}{\partial q}$$

Since  $-f''(\hat{k})\hat{k} < 1$ , this ensures that the first term (56):

$$qu''(c_y^*)\left(\frac{\partial w}{\partial q}n^* - \frac{\partial \hat{k}}{\partial q}n^* - \frac{\partial p_L}{\partial q}\right)$$

is positive. As well:

$$\frac{\partial r}{\partial q} = f''(\hat{k}) \frac{\partial \hat{k}}{\partial q}$$

and so:

$$\begin{aligned} & \frac{\partial \hat{k}}{\partial q} n^* (1 - \delta + r) + \hat{k} n^* \frac{\partial r}{\partial q} \\ = & \frac{\partial \hat{k}}{\partial q} n^* (1 - \delta + f'(\hat{k})) - \hat{k} n^* f''(\hat{k}) \frac{\partial \hat{k}}{\partial q} \\ = & \frac{\partial \hat{k}}{\partial q} n^* (q^{-1} - \hat{k} f''(\hat{k})) \end{aligned}$$

which is positive because  $q^{-1} > \beta^{-1} > 1 > -\hat{k} f''(\hat{k})$ . It follows that (57) is also positive.

Note that if  $F(k, n) = k^\alpha n^{1-\alpha}$ , then  $-F_{kk}k = \alpha(1-\alpha)\hat{k}^{\alpha-1}$ . Since  $\alpha\hat{k}^{\alpha-1} = q^{-1} - 1 + \delta$ ,  $-F_{kk}k < 1$  iff  $(1-\alpha)(q^{-1} - 1 + \delta) < 1$ , which is true if:

$$q > \frac{(1-\alpha)}{1 + (1-\alpha)(1-\delta)}$$

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