The Friedman Rule Meets the Zero Interest Rate Bound

Jan Kim[*], Preston Miller[†]
Federal Reserve Bank of Minneapolis

Abstract

This paper is an attempt to determine the relative importance of the efficiency and stability effects of monetary policy. The method is to find the policy that maximizes welfare in a general equilibrium model that generates both effects. It is found that the steady-state inflation rate under the optimal policy is significantly above the rate required for maximal efficiency and significantly below that required for maximal stability. Thus, both effects play important roles in determining the optimal rate of inflation. In addition, it is found that if a typical macroeconomic objective function is maximized as a substitute for welfare-maximization, the resultant policy rule puts too much weight on stability. It generates too much inflation and causes the policy instrument to respond too much to new information.

Keywords: Monetary Policy, Welfare, Efficiency, Stability, Optimal Rules.

JEL Classification: E32, E52

[*]Banking and Policy Studies, Division of Supervision, Regulation, and Credit, Federal Reserve Bank of Minneapolis, 90 Hennepin avenue, Minneapolis, MN 55480-0291; phone (612) 204-5146; fax (612) 204-5114; e-mail: `jangryoul.kim@mpls.frb.org`.

[†]Banking and Policy Studies, Division of Supervision, Regulation, and Credit, Federal Reserve Bank of Minneapolis, 90 Hennepin avenue, Minneapolis, MN 55480-0291; phone (612) 204-5144; fax (612) 204-5114; e-mail: `preston.miller@mpls.frb.org`.

1. Introduction

Suppose that the FOMC were to adopt an inflation-targeting framework, as many monetary economists recommend. What inflation rate should it target? One common policy prescription is that it should follow a Friedman Rule; that is, it should seek a rate of deflation equal to the real rate on safe assets in order to make their nominal rate zero. However, another prescription is that it should seek a positive rate of inflation to keep the nominal rate on safe assets comfortably above zero. Obviously, these two prescriptions are in conflict.[1]

The Friedman Rule is motivated by long-run efficiency considerations and relates to the mean of output. The basic argument goes like this: the marginal benefit of holding additional money is the decrease in transaction costs represented by, say, shopping time, going-to-the-bank time, or other costs associated with the purchase of consumption goods. With a positive nominal interest rate, people economize on their cash balances to the point that the marginal benefit, social and private, is equal to the marginal private cost, i.e., the nominal interest rate. However, this is socially suboptimal, because the government can costlessly produce the cash until people are satiated with it. A social optimum occurs when the nominal rate is zero (or deflation is at a rate equal to the real interest rate), so that the marginal social benefit and marginal social cost of holding money are equalized at zero. Thus, the Friedman Rule is designed to remove an inefficiency, and by doing so, raise the mean of output.

The prescription for significantly positive nominal rates is motivated by business cycle considerations and relates to the variability of some key variables, such as output. The basic argument is: at a low inflation target, monetary policy is significantly constrained by the zero bound on the nominal rate, in the sense that the zero bound is encountered frequently. If the federal funds rate were zero and the economy then were hit with a negative demand shock, monetary policy could not

[1] These two conficiting views are best represented by Friedman (1969) and Summers (1991).

respond by lowering the funds rate further, and consequently output would be more variable than at a modestly high inflation target. It does not follow that nominal rates would be expected to be zero when inflation was zero, because nominal rates are the sum of inflation and a positive real rate. However, the outcome for interest rates is a distribution, so that the probability rises of zero nominal rate realizations as the rate of inflation falls. In order to guarantee a small probability of zero nominal rates, a positive rate of inflation is required, usually estimated to be around 2 percent per year.

The arguments underlying both policy prescriptions seem valid, and together they suggest a trade-off with respect to the target rate of inflation. Over a range, extending into negative territory, a lower rate of price change will move the economy to greater efficiency all the time, while making it more likely that monetary stabilization policy will be foregone on occasion. Or, more simply, the trade-off is between the mean and variance of output.

This paper is an attempt to determine the relative importance of the efficiency and stability effects of monetary policy. Our method is to find the policy that maximizes welfare in a model that generates both effects. The rule by its nature determines the weight given to each effect and thus determines the weight given to the Friedman Rule as compared to the positive nominal interest rate rule.

In contrast to finding the rule that maximizes welfare, many studies in the literature instead assume a macroeconomic objective, such as minimizing a weighted sum of the variances of output and inflation.[2] A second aim of this paper is to determine where macro-based rules go wrong. We accomplish this by comparing those rules and associated equilibria to their welfare-maximizing counterparts.

To examine the quantitative welfare implications of the Friedman and positive nominal rate

[2] See, for example, Bernanke (2004) and Levine et al. (1999) who use the weighted average of the variances in quarterly output gap and inflation rate. Rudebusch and Svensson (1999) consider in addition the variabilities of the changes in nominal interest rates.

polices, one needs an appropriate criterion that weighs the alternative outcomes and a model that is capable of generating the trade-off between efficiency and stability. If we assume that tax/transfer policies address distributional concerns, we can take the criterion function to be the expected discounted utility of a representative agent. Because efficiency and stability in reference to monetary policy effects are inherently dynamic and stochastic concepts and because welfare functions are based on households' consumption and leisure paths, a DSGE model is required. Furthermore, in order for the estimated model trade-off to approximate the true trade-off, structural parameters must be fit to the data.

In this paper, we carry out such an analysis with an estimated DSGE model in which monetary policy has both efficiency and stability effects. Our model has three important features tailored for the aims posited above. First, to take into account the welfare costs of a positive nominal rate, we explicitly include nominal money balances in the model. Second, for the correct evaluations of monetary policy's effects on means and variances of relevant variables, we employ a quadratic approximate solution method developed and extended in Sims (2000) and Kim et al. (2002), respectively. Third, we account for the zero bound by assuming that the variability of the nominal rate relative to its average level cannot be greater than the magnitude implied by the data. Thus, given the historical variability of nominal interest rates, a long-run inflation target that is too low will cause nominal rates to hit the zero bound an unacceptably frequent number of times.

The analysis finds that, in the estimated model, both the efficiency effect and the stabilization effect matter in determining the optimal rate of inflation. That is, the policy that maximizes the expected utility of a representative household in the estimated model implies a slightly positive inflation rate of 0.12% per year, which is significantly above the rate required for maximal efficiency and significantly below that required for maximal stability.

The weighting of the efficiency and stability effects that we find is dependent on the model we use. It is conceivable that different models, or the same model estimated over a different period, could generate different policy implications. Nevertheless, comparison of welfare-based and macro-based rules reveals two suboptimal properties of the latter. First, macro-based rules imply too high a rate of inflation. The macro objective recognizes that policy affects the variance of output, but not the mean. Our results suggest that under such objective functions, a policy with zero or very low rate of inflation is hardly optimal: since policy is concerned with economic stabilization but not efficiency, positive inflation is required to keep nominal interest rates above the zero bound. Putting too much emphasis on short-run stabilization, such rules basically shut down a possibly more important long-run efficiency channel for monetary policy to enhance welfare. Second, macro-based rules may stabilize output, but they can actually fail to stabilize the utility of agents, because output is not a good proxy for the utility of consumption and leisure. Our results show that wrong proxy selection can actually de-stabilize utility.

Proponents of macro-based policy rules argue that the efficiency effect of monetary policy can be ignored because empirical studies indicate it is small. But what matters is its contribution to welfare relative to that of monetary stabilization policy. In our model, the efficiency effect cannot be ignored, because it implies at least as large a utility gain as stabilization policy does. As Kiley (2003) showed, the cost from business fluctuations can be large. Yet, for models in which fluctuations are caused by fundamental shocks to the economy and in which individuals respond optimally to the shocks given the constraints that they face, the welfare gains from monetary stabilization policy will be small.

In the text that follows, we first describe general features of our model. We next discuss our methodology and results. Finally, in an appendix, we formally describe the model's construction, solution, and estimation.

2. General Features of the Model

The model, borrowed form Kim (2003), is composed of a private sector, a public sector, and markets for commodities, physical capital, labor, money, and bonds. The private sector includes households and firms. Households purchase commodities to consume and to save in the form of physical capital. They also save by holding money, bonds, and shares of firms. They purchase the assets using the returns on their investments and the income they receive from supplying labor services. Firms purchase physical capital and labor services in order to produce commodities.

The public sector consists of a budget authority and a monetary authority. The budget authority makes transfers to households financed by issuing bonds. The monetary authority performs open-market operations between money and bonds. The monetary authority is assumed to follow a policy feedback rule that determines the nominal interest rate as a simple function of current information. A solution path for inflation and interest rates implies paths of money and bonds. Thus, given an inflation rate and a monetary policy feedback rule, budget balance for the consolidated budget and monetary authorities is assured period-by-period by adjusting the level of nominal transfers to the revenue from money and bond issue.

Commodities and labor are produced as differentiated goods, giving the supplier market power to determine prices. It is assumed that in each market the commodities (or labor) are aggregated into a single composite good (or labor) before being sold at a single price (or wage rate).

We introduce nominal rigidities in the model by assuming that individual firms and households pay adjustment costs each period when setting optimal prices and wages, respectively. Also, we explicitly take money balances into account by following a money-in-utility function approach. As discussed in Feenstra (1986), our formulation is equivalent to viewing money as a means to economize on the transaction costs for the purchase of consumption goods.

An equilibrium in the model is a stochastic process driven by six fundamental disturbances: shocks to aggregate productivity and capital share in the production function, depreciation of capital, money demand, labor supply, and monetary policy.

The model's parameters are determined by jointly applying calibration and estimation to quarterly US data spanning 1959:Q1 - 1999:Q4. Prior to estimation, we fix some parameters at values frequently calibrated in the literature: especially, the inflation rate in the deterministic steady state of the model is fixed at the sample average implying the annual inflation of 4.05%. The remaining parameters are estimated by maximum likelihood applied to the log-linearized solution of the model, which is obtained by the method of Sims (2002). Given the parameter estimates and alternative monetary policy rules, the model is solved again by a second order approximate solution method for the sake of correct welfare statistics.[3] The details on the model, dataset, estimation results, and welfare calculations are given in the appendix.

3. Methodology

Our primary aim is to determine the relative importance of the efficiency and stability effects of monetary policy under a utility-based objective function of the monetary authority. A secondary aim is to determine how the conclusion is altered under macro-based objective functions. For these aims, our methodology is to i) choose an objective function the monetary authority seeks to optimize, ii) solve for the optimal rule given the model and that objective function, and iii) compare the implications for macro outcomes and for welfare of different optimized rules.

3.1 Objective Functions

We specify two types of objective functions that the monetary authority is assumed to minimize. The first one is utility-based: the minus of the expected discounted stream of the representative

[3] Kim and Kim (2001) illustrates the necessity of higher order approximate solution methods in measuring the welfare level correctly.

household's lifetime utility. This objective function is represented as

$$\begin{aligned} OF_1 &= -E_0\left[\sum_{t=0}^{\infty}\beta^t U(\zeta_t)\right] \\ &\simeq -\frac{1}{1-\beta_*}U(\overline{\zeta}) - \left[\frac{dU_t(\overline{\zeta})}{d\zeta_t}\otimes\overline{\zeta}\right]'\sum_{t=0}^{\infty}\left(\beta^t E_0\left[d\log\zeta_t\right]\right) \\ &\quad -\frac{1}{2}tr\sum_{t=0}^{\infty}\left(\beta^t Var_0\left[d\log\zeta_t\right]\left[\frac{d^2U_t(\overline{\zeta})}{d\zeta_t^2}\otimes(\overline{\zeta\zeta'})\right]\right) \end{aligned} \tag{1}$$

where β is the subjective discount factor of households, and $E_0(\cdot)$ and $Var_0(\cdot)$ denote the expectation and variance operators, respectively, conditional on some initial state Ω_0. The term ζ_t in the utility function $U(\cdot)$ is the vector of consumption (C_t), real balances (M_t/P_t), labor (L_t), money demand shock (b_t), and labor supply shock (a_t). The term $d\log\zeta_t$ is log-deviation of ζ_t from its deterministic steady state level, and $tr(\cdot)$ is the trace of a square matrix. The symbol $\otimes$ denotes the element-by-element multiplication operator. It is worth noting that, as long as the second derivative of U with respect to consumption is negative, the second order term in (1) penalizes variability in consumption.

The other objective function is macro-based and a variant of the objective function frequently used in the inflation targeting literature: a weighted average of the variances of inflation and output. This objective function has the form

$$OF_2 = \sum_{t=0}^{\infty}\beta_{\Pi}^t\left\{E_0\left[\log\Pi_t - \log\Pi^*\right]^2 + \lambda_y E_0\left[\log y_t - \log y_t^*\right]^2\right\} \tag{2}$$

where β_{Π} is the subjective discount factor for the monetary authority, Π_t and y_t are the gross inflation rate and real output at period t, respectively, Π^* is the target rate of long-run inflation, and y_t^* is the target rate of real output at period t. In the following analysis, we set β_{Π} equal to the household's discount factor, and Π^* to 1.01005, the historical average of the inflation rate over the sample period. In the macro-based objective functions found in the literature, the target rate for inflation is usually taken to be something between what is implied by the Friedman Rule and the positive nominal interest rate constraint. Our choice of the historical average, i.e., 4.05 % per year, is higher than those levels and allows more aggressive adjustment of the nominal interest rate in the context of OF_2. As in Bernanke (2004) and other papers, we take the potential GDP y_t^* to be the deterministic future output implied by the estimated model. In order to see how the policy rule is affected by different relative preferences for output and inflation stability, three different values of the weight on output variability are considered: $\lambda_y = (1, 0.5, 0.1)$.

3.2 Policy Indicators and Zero Bound

In order to solve for optimal policy rules, we need to specify a small set of policy indicators to serve as arguments in the rules.[4] For this purpose, we rely on a general result in the literature adopting linear-quadratic decision making frameworks: for a given objective function, the corresponding optimal rule can be expressed as a function of the current, lagged, and expected future values of the goal variables, i.e., the variables in the objective function. However, although there may be few goal variables, their expected values depend in general on the histories of all the variables in the model. Since the coefficients of optimal policy rules in our model are found using numerical searches, feasibility requires that the arguments in the rules be severely limited. To do so, we further resort to a common finding in the time series literature that the forecast accuracy for any given variable is improved significantly by the addition of very few variables in addition to the lags of the variable itself.[5]

We opt to find the policy indicators for the second objective function OF_2 and use them as the policy indicators for OF_1 as well. We then are able to draw on a large literature for forecasting output and prices, the goal variables for OF_2, and by making the arguments the same in all rules, we can directly compare coefficients to measure differences in responses to new information. Our selection strategy is i) to construct a quarterly VAR(5) in terms of inflation and output, and

[4] As in Kind and Wolman (1999) and Khan et al. (2002), a formal quest for optimal rules associated with objective fucntions like OF_1 requires us to characterize equilibrium allocations under the Ramsey policy, without restricting the form of the policy function to the arguments of a pre-specified interest rate rule. We plan to extend the current exercise toward this direction in future research.

[5] Williams (2003) makes this conclusion in a context very similar to ours.

ii) find a small set of other data series that, if augmented to the bivariate VAR, helps better forecast these goal variables, and iii) proxy the expected future values of the goal variables with the augmented variables.[6] After a few trials, the strategy led to a VAR comprising data series on $(\log R_t, \log MG_t, \log \Pi_t^w)$ in addition to those on $(\log Y_t, \log \Pi_t)$.[7] From this finding, we specify the monetary policy rule of the form

$$\boxed{\begin{aligned} d\log R_t &= \rho_R * d\log R_{t-1} \\ &+ \ a_1 * d\log \Pi_t + a_2 * d\log Y_t + a_3 * d\log Y_{t-1} + a_4 * d\log MG_t + a_5 * d\log \Pi_t^w, \quad |\rho_R| < 1 \end{aligned}} \tag{3}$$

which is a generalization of Taylor (1993) in that it involves money growth and wage inflation as additional nominal anchors.

For each objective function, we proceed to search for optimal values of ρ_R and $a_i's$ that minimize the alternative objectives evaluated at the model solution by the quadratic approximation method described above. Having obtained the best rules for respective objective functions, we compute some key macroeconomic and welfare statistics implied by the optimized rules.

In the search for optimal rules, we need to address the zero bound restriction on nominal interest rates. Generally, two alternative devices are employed in the literature. One approach is to solve the model nonlinearly directly imposing the zero bound, as in Reifschneider and Williams (2000), and Fuhrer and Madigan (1997). The other, as in Rotemberg and Woodford (1999) and Williams (2003), assumes that the variability of the nominal rate is constrained by the average level of the interest rate (or the inflation target). For example, Rotemberg and Woodford assume that the ratio of the standard deviation of the nominal rate to its average level can be no greater than the ratio that describes the '80-'95 sample.

We use a strategy similar to Rotemberg and Woodford: when searching for the coefficients and target inflation rates for an objective function, we require that the k-standard deviation confidence

[6] More specifically, we search for a set of the augmented variables that minimizes the mean squared errors of the one-step-ahead forecasts of the goal variables from the expanded VAR.

[7] The variables in the policy reaction function are as follows: nominal interest rate (R), money growth rate (MG), wage inflation (Π^w), price inflation (Π), and real output (Y).

interval of the nominal interest rate around its deterministic steady state level should not contain zero. Intuitively, the higher the k is, the more concerned the monetary authority is about the occurrence of a zero nominal interest rate. Unless explicitly noted, the level of k in the following analysis is 2.0468, which is equal to the empirical mean-standard deviation ratio from the sample used for estimation.

4. Results

4.1 Trade-off : Efficiency and Stability

As stated in the introduction, the primary goal of this paper is to find an optimal long-run rate of inflation that balances the long-run efficiency and short-run stabilization effects of monetary policy. We start our analysis by examining the trade-off between the two effects in the context of the estimated model.

Figure 1 illustrates the costs and benefits from changing the target rate of inflation, assuming that the estimated policy rule is otherwise maintained. The upper panel plots the costs from higher inflation target, measured by $\frac{1}{1-\beta_*}U(\overline{\zeta}) + \left[\frac{dU_t(\overline{\zeta})}{d\zeta_t} \otimes \overline{\zeta}\right]' \sum_{t=0}^{\infty} \left(\beta^t E_0 \left[d \log \zeta_t\right]\right)$, for the net long-run inflation rate ranging from zero to 5.1% per year.

The plot shows that mean utility decreases monotonically as the inflation rate increases over this range. For example, the household welfare level is 611.83 when the net annual rate of inflation is zero (i.e., $\Pi^* = 1$), and it decreases to 611.09 when the inflation rate is 4% (i.e., $\Pi^* = 1.0099$). In terms of steady-state consumption for the estimated model, this welfare difference amounts to 0.029 units of consumption every quarter for life.

The reason that efficiency is reduced as inflation increases (above the rate associated with a zero real interest rate) seems clear. Higher inflation causes households to hold less real balances. More costs then are incurred to purchase commodities. This acts like a price increase for commodities relative to leisure. In response, the household consumes less and enjoys more leisure (works less).

The main cost of higher inflation thus relates to the higher transaction costs incurred.

Unlike the costs of high inflation rates shown in the upper panel, the benefits from inflating the economy are not straightforward to quantify. We measure the degree of policy "leeway" the monetary authority has by running positive inflation rates. More specifically, assuming that the nominal rate is normally distributed around its mean (dependent upon the steady state inflation rate), we calculate the average duration of non-negative interest rates.

The lower panel of Figure 1 plots how often the zero bound is violated under the estimated rule: with the benchmark level of $\overline{\Pi} = 1.01005$ as the target inflation rate, the nominal rate becomes negative roughly every 47 years. When the target inflation is lowered to 2% per annum, however, the frequency of violation is about nine years, which is unacceptably often.

The message from Figure 1 is now clear: when choosing a target rate of inflation, the monetary authority should weigh the efficiency gains in the spirit of Friedman against the costs from having its hand tied down due to the zero bound on the nominal rate. This insight is corroborated in a numerical exercise that follows.

4.2 Inflation and Welfare-maximizing Monetary Policy

We reasoned in the previous subsection that the welfare maximizing monetary policy rule balances the objectives of long-term efficiency and short-term stability. In this subsection, we construct formal measures of efficiency and stability from the expected utility of a representative household. We then discuss how a change in the inflation rate affects these two welfare components in terms of the responses of household decision variables. We find the inflation rate that generates the optimal mix of the two, and re-examine the trade-off between efficiency and stability when the long-run inflation rate is varied.

The properties of the welfare-maximizing rule, dubbed (U1), are reported in the upper panel of Table 1. The most conspicuous feature of (U1) is that it requires as optimal a very low level of long-run inflation, i.e., 0.12 % per annum, or equivalently, 2.9% of steady state annual nominal rate. The rule also features i) considerable degree of policy inertia (i.e., $\rho = 0.8844$) in the terminology of Woodford (1999); ii) leaning against the wind in responses to real output levels and changes (i.e., $a_2 = 0.4822$, $a_3 = -0.4806$); iii) more aggressive responses to wage inflation than price inflation (i.e., $a_1 = 0.4964$, $a_5 = 1.4068$); and iv) modest responses to money growth (i.e., $a_4 = 0.8844$). Relative to the estimated policy rule (reported in the appendix), the standard deviations of (Y_t, Π_t, R_t) under (U1) are (1.0912,1.0945, and 0.4446). The relatively low variability in nominal rate seems mainly due to the higher degree of policy inertia under (U1)[8].

In order to see the effects of changes in target inflation on the two welfare components, we find a constrained welfare-maximzing rule, dubbed (U2), for which the long-run inflation rate $\overline{\Pi}$ is fixed at the sample average of 1.01005. This constrained rule (U2) is reported in the bottom panel of Table 1, where we observe i) lower degree of policy inertia; ii) more aggressive responses with respect to nominal anchors; and iii) less aggressive counteraction against real output and its rate of change under (U2).

Our conjecture is that the higher welfare under the unconstrained rule (U1) comes from efficiency gains, while the constrained rule (U2) does better job in terms of stabilization. We now define both efficiency and stability with respect to components of expected utility in (1): we decompose expected utility into that associated with the first moments of the arguments (the sum of the steady-state and conditional mean values) and that associated with the conditional second-moment values of the arguments (variances and covariances). We then associate "efficiency" with the contribution of first moments to expected utility and "stability" with the contribution of second moments to expected utility. It directly follows that expected utility corresponding to any monetary policy rule

[8] According to Woodford (1999), the virtue of inertial adjustments in the nominal rate is that they signal how serious monetary authority is about stabilizing its goal variables even in the distant future, exploiting the forward looking behavior of the private sector in forming their expectations. In fact, the optimal interest rate rules in Rotemberg and Woodford (1997,1999) exhibit super-inertia: for example, in Rotemberg and Woodford (2003), the largest root 1.33 of the autoregressive polynomial for the nominal rate is greater than one.

is the sum of efficiency and stability.

In the upper panel of Table 2, we compare contributions of first-moments of utility function arguments for two steady-state inflation rates, $\overline{\Pi} = 1.0003$ (low) and $\overline{\Pi} = 1.01005$ (high). The utility function is specified so that utility increases with respect to consumption and real balances and decreases with respect to labor. Thus, the decomposition shows that an increase in inflation has the effects described above: at the higher rate of inflation target, consumption falls, leisure increases (labor falls), and real balances fall in the steady state.[9]

With respect to stability, higher inflation has been shown to increase the duration of non-negative nominal interest rate. We maintain that the higher duration enhances stability by increasing the number of times that the monetary authority is able to offset bad preference shocks. To describe how monetary policy in this model can increase stability, we consider two cases. In both cases, the economy is at equilibrium and is then subject to a negative money demand shock. In the first case, the equilibrium nominal interest rate is zero, while in the second case, it is positive. The negative money demand shock corresponds to a fall in the household's desired holdings of real balances. A decline in the demand for real balances in the first case will lead to an actual decline in real money holdings of the same amount because the interest rate is zero and cannot fall. This decline in money holdings leads to an increase in transaction costs, which then causes effects similar to the case examined for higher inflation. However, in the second case, the monetary authority can cushion the fall in real money balances by reducing the nominal interest rate (as would occur in the market if the monetary authority fixed the money stock). In this case, the monetary authority can lessen the welfare cost of an adverse demand shock, as the fall in the interest rate induces larger money holdings. And the higher the inflation rate, the fewer times the monetary authority will be facing the zero nominal interest rate constraint.

[9] The same pattern is also observed for the unconditional expectation of those variables.

In the bottom panel of Table 2, we decompose the contributions of the second moment terms to expected utility for the low and high inflation rates. Overall, we see that higher inflation increases stability, as we have argued. The largest contributor to the gain in stability is a decrease in the covariance of consumption and real balances. This is explained as a drop in the severity of effects of negative demand shocks. That is, while negative demand shocks occur with the same frequency, their effects are dampened more frequently when inflation is high.

Finally, we examine the characteristics of the optimal inflation rate. We first summarize what we already know. Expected utility is the sum of efficiency and stability. As the inflation rate rises, efficiency decreases and stability increases. At the optimal rate of inflation, the marginal loss in efficiency must equal the marginal gain in stability. Further increases in inflation will decrease efficiency and raise stability, and their sum must decline, because the initial inflation rate is optimal. Similarly, a significant decrease in the inflation rate below the optimum will raise efficiency and decrease stability, and their sum must decline.

We have found that the optimal inflation rate given our assumed parameters is the "low" inflation rate above. Thus, the model is giving significant weight to both efficiency and stability, in the sense that the optimal inflation rate falls between the deflation rate required by the Friedman Rule and the positive inflation rate needed to generate a low probability of a zero nominal interest rate realization. Although the optimal inflation rate is small, this is one of the few general equilibrium models to find that is positive.

4.3 How Macro Objective Functions Lead to Flawed Policies

It is natural in a micro-based model to associate optimal policy with the one that maximizes the expected utility of a representative agent. That is what we did in the previous section. However, it also is possible to compute rules that maximize macro-based objective functions, such as those that often appear in the literature. By comparing the rules and associated equilibria for the macro-based objective function with those for the utility-based objective, we are able to determine in what ways the macro-based rules go astray. So, in this section, we compute the rule that minimizes a weighted sum of the variances of output and inflation and then compare the equilibrium under this rule to that under the optimal rule.

Table 3 reports the optimized rules (S1)-(S3) and corresponding welfare statistics for different weight on output variability, with Π fixed at the sample average. As the inflation variability gets higher relative weights, we observe i) increases in the required degree of policy inertia and responses to price inflation, ii) decreases in responses to real output and other nominal anchors such as money growth and wage inflation, iii) decreases in standard deviation of inflation and increases in that of output, and iv) increases in welfare. It is worth noting that, compared with the utility-based rules in Table 1, the macro-based rules for OF in general show i) considerably lower degree of policy inertia, ii) more aggressive responses to price inflation, iii) less aggressive responses to wage inflation and money growth, and iv) lower welfare level.

One problem with the macro-based objective function is that it will encourage too high an inflation rate. The welfare-maximizing rule leads to an optimal inflation rate as low as it is because it recognizes that higher inflation reduces efficiency. The macro-based objective function does not recognize this channel. It assumes that with respect to output, the objective is to minimize deviations around a fixed target, which is usually taken to be its historical trend. Thus, policies that lead to even higher growth will not be rewarded.

We check the inflation bias of macro-based rules by computing a (quasi) partial derivative. We first compute the equilibrium under the macro-based rule when the target rate of inflation is set at the welfare-maximizing rate. We then increase the target inflation rate to its sample average. We find that the value of the macro-based objective function is higher at the higher inflation rate. Thus, a policy maker using the macro-based objective function would opt for an inflation rate above the welfare-maximizing rate.

A second problem with the macro-based objective function is that it encourages too much policy activism; that is, the instrument will respond too much to incoming information. One way to see this is to first imagine the welfare-maximizing rule assuming that there is complete information.[10] In this case, the policy would be a feedback rule with realizations of the fundamental shocks as arguments. Moreover, policy would respond little, if at all, to technology shocks for which the efficacy of monetary policy is questionable. It would respond more to preference shocks for which it could potentially improve welfare. Now suppose that there is incomplete information and that policy must be a feedback rule on the observed variables we specified: past interest rate, wage and price inflation rates, and current and past output. The optimal rule under incomplete information will try to come as close as possible to replicate the rule under complete information. That means that it will try to distill the nature of the shocks from the observed variables in the feedback rule. For instance, the nominal interest rate should be adjusted little to movements in output and prices in the opposite directions since that likely reflects a technology shock.

In contrast, a macro objective function only responds to the variations in the variables it includes. It does not distinguish among the sources of variation. Thus, a macro objective function will lead to too much response in the interest rate to output and price variation caused by technology shocks.

We support our contentions by simple simulations. We begin by identifying the historical value of the shocks based on the estimated model and the historical realizations of the observed variables. In Figure 2, we compute the deviations of key variables from their steady state values. The deviations use the history of values for all the shocks and assume either our computed welfare-maximizing monetary policy rule or the macro-based rule with the weight on output variation

[10] Another way to see this is the lowere degree of policy inertia under the macro-based rules.

$\lambda_y = 0.1$. The figures reveal that relative to the welfare maximizing rule, the macro-based rule:

- Leads to much less variation in the inflation rate,
- Leads to much more variation in the nominal interest rate (too much policy activism),

and

- Leads to somewhat more variation in real balances and labor supply.

We next compute in Figure 3 the responses of variables for shocks to technology, (A, α) only.[11] Two results are stark. One is that the interest rate under the welfare-maximizing rule barely responds to these shocks, while the rate under the macro-based rule responds a lot. The other is that inflation under the welfare-maximizing rule is allowed to vary a lot in response to technology shocks, while inflation under the macro-based rule is essentially smoothed.

Finally, in Figure 4, we compute responses of variables to shocks to money demand, b, only. While the paths of variables under the two rules are closer for the money demand shock than they were for the technology shock, it is still apparent that the welfare-maximizing rule allows more inflation variability than does the macro-based rule. However, for the money demand shock, the interest rate under the welfare maximizing rule responds more than it does for the macro-based rule.

In sum, it is clear from these figures that the welfare-maximizing rule attempts to distinguish among the sources of shocks and act accordingly. It responds more to preference shocks than to technology shocks. And it allows considerable variation in inflation. In contrast, the macro-based rule allows too strong a response of the interest rate to technology shocks, and it does too much smoothing of inflation.

A third problem with the macro-based rule is that it stabilizes the wrong thing. In Table 4, we decompose expected utility (as in Table 3) for equilibria under the welfare-maximizing rule and the

[11] We consider these two shock jointly, because they are allowed to be correlated at the estimation stage.

macro-based rule with $\lambda_y = 0.1$ and the target rate of inflation at its historical average. The top panel indicates that the macro-based rule leads to less efficiency. This is expected because it leads to higher inflation. This channel is evidenced by real balances being lower under the macro-based rule. But surprisingly, the macro-based rule also leads to less stability. Apparently, stabilizing output and inflation does not directly translate into utility stabilization, which mainly depends on the second moments of consumption, leisure, and the real balance.

4. Conclusion

Although some of the results in this paper are specific to our model, we believe that there are two general ones that have relevance for current monetary policy. First, a target of price stability is probably not far from optimal. It comes close to balancing the efficiency of a Friedman Rule with the stability of a positive nominal interest rate bound. Second, if policy attempts to stabilize output and/or inflation, it will be too active. The stability of output is not a good proxy for the stability of utility, a function of consumption, leisure. Moreover, fluctuations in inflation are ddesirable when they reflect technology shock, the main force driving the economy. A policy that attempts to neutralize fluctuations in output caused by technology shocks will interfere with optimal individual adjustments to consumption and leisure.

5. References

[1] Bernanke, B.S., 2004, The great moderation, Speech at the meetings of the Eastern Economic Association, Washington, DC, February 20, 2004.

[2] Feenstra, R.C, 1986, Functional equivalence between liquidity costs and the utility of money, Journal of Monetary Economics 17, 271-291.

[3] Friedman, M., 1969, The optimum quantity of money and other essays, (University of Chicago Press, Chicago, IL).

[4] Fuhrer, J.C. and B.F. Madigan, 1997. Monetary policy when interest rates are bounded at zero, The Review of Economics & Statistics 79, 573-585.

[5] Khan, A., R. King, and A. Wolman, 2003, Optimal monetary policy, Review of Economic Studies 60 (4), 825-860.

[6] Kiley, M. T., 2003, An analytical approach to the welfare cost of business cycles and the benefit from activist monetary policy, Contributions to Macroeconomics (3), Article 4.

[7] Kim, J., 2003, Welfare evaluation of monetary policy rules in a model with nominal rigidities, Banking and Policy Studies Working Paper 3-03, Federal Reserve Bank of Minneapolis.

[8] Kim, J. and S. Kim, 2003, Spurious welfare reversals in international business cycle models, Journal of International Economics 60, 471-500.

[9] Kim, J., S. Kim, E. Schaumberg, and C. Sims, 2003, Calculating and using second order accurate solutions of discrete time dynamic equilibrium models, Finance and Economics Discussion Series 2003-61, Board of Governors of the Federal Reserve System.

[10] King, R. and A. Wolman, 1999,What should the monetary authority do when prices are sticky?, in: John B. Taylor, ed., Monetary Policy Rules, (University of Chicago Press, Chicago) 349-398.

[11] Williams, J.C., 2003, Simple rules for monetary policy, FRB of San Francisco Economic Review, 1-12.

[12] Reifschneider, D. and J.C. Williams, 2000, Three lessons for monetary policy in a low inflation era, Journal of Money, Credit, and Banking, 936-966.

[13] Rotemberg, J. and M. Woodford, 1997, An optimization-based econometric framework for the evaluation of monetary policy, in: Ben Bernanke and J. Rotemberg, ed., NBER Macroeconomics Annual 12 (MIT Press, Cambridge, MA) 297-346.

[14] Rotemberg, Julio and Michael Woodford, 1999, Interest rate rules in an estimated sticky price model, in: John B. Taylor, ed., Monetary Policy Rules, (University of Chicago Press, Chicago) 57-119.

[15] Rudebusch, G. and L.E.O. Svensson, 1999, Policy rules for inflation targeting, in: John B. Taylor, ed., Monetary Policy Rules, (University of Chicago Press, Chicago) 203-246.

[16] Sims, C.A., 2000, Second order accurate solution of discrete time series dynamic equilibrium models, mimeo, Princeton University.

[17] Sims, C.A., 2002, Solving linear rational expectations models, Computational Economics 20 (1-2), 1-20.

[18] Summers, L., 1991, How should long-term monetary policy be determined, Journal of Money, Credit and Banking , 23, 625-631.

[19] Taylor, J.B., 1993. Discretion versus policy rules in practice, Carnegie-Rochester Conference Series on Public Policy 39, 195-214.

[20] Woodford, M., 2003, Optimal interest-rate smoothing, Review of Economic Studies 70, 861-886.

6. Appendix (I) : Model and Estimation.

A1. Firms and Price Setting

During period t, an individual firm $j \in [0,1]$ hires K_{jt} units of physical capital (from households) and L_{jt} units of aggregate labor service (from the aggregator), and produce Y_{jt} units of its own product. All firms have the identical CRS production technology

$$Y_{jt} = A_t K_{jt}^{\alpha_t} (g^t L_{jt})^{1-\alpha_t}, \; g \geq 1 \tag{A1}$$

where g is the rate of deterministic labor-augmenting technological growth, and A_t and α_t are the *aggregate productivity shock* and the *capital share shock* α_t, respectively.

Each firm sells its differentiated output to the aggregator, who uses a CRS technology

$$Y_t = (\int Y_{jt}^{\theta_Y} dj)^{\frac{1}{\theta_Y}} \quad , \quad \theta_Y \in [0,1] \tag{A2}$$

to transform the differentiated products into a single output Y_t. The implied demand function for the firm j's output Y_{jt} is

$$Y_{jt}^d = (\frac{P_{jt}}{P_t})^{\frac{1}{\theta_Y - 1}} Y_t \tag{A3}$$

where the aggregate price level P_t is defined as

$$P_t = (\int P_{jt}^{\frac{\theta_Y}{\theta_Y - 1}} dj)^{\frac{\theta_Y - 1}{\theta_Y}} \quad . \tag{A4}$$

Nominal rigidities in the goods market take the form of price adjustment costs

$$AC_{it}^p = \frac{\Phi_p}{2} \left(\frac{P_{jt}}{P_{j,t-1}} - \Pi_{t-1} \right)^2 Y_t \tag{A5}$$

where P_{jt} is the price of the firm j set in period t, and $\Pi_{t-1} = P_{t-1}/P_{t-2}$ is the inflation rate prevailing in period $t-1$. Equation (A5) implies that both the price level and the inflation rate are sticky.

The firm j is assumed to solve its profit maximization problem through two steps. First, given aggregate price level and factor prices, the firm solves the cost minimization problem. Second, given the cost function thus derived, it determines the optimal price P_{jt} to charge by solving the following profit maximization problem

$$\max E_0 \left[\sum_{t=0}^{\infty} \frac{\beta^t \Lambda_t}{\Lambda_0} \left(\frac{P_{jt} Y_{jt}}{P_t} - Y_{jt} \frac{MC_t}{P_t} - AC_{jt}^p \right) \right] \tag{A6}$$

where $\frac{\beta^t \Lambda_t}{\Lambda_0}$ is the discount factor for its real profit between period 0 and t, and $\Lambda_t = \int_{[0,1]} \Lambda_{it} di$ is the average marginal utility of consumption across all households.[12] The marginal cost MC_t, common to all firms, is independent of the output level due to the CRS production function.

The FOCs for $(K_{jt}, L_{jt,} P_{jt})$ require

$$\frac{L_{jt}}{K_{jt}} = \frac{Q_t/P_t}{W_t/P_t} \frac{1-\alpha_t}{\alpha_t} \tag{A7}$$

$$\frac{MC_t}{P_t} = \frac{W_t/P_t}{MPL_t} \tag{A8}$$

$$\frac{1}{P_t} \frac{\partial (P_{jt} Y_{jt})}{\partial P_{jt}} = \frac{MC_t}{P_t} \frac{\partial Y_{jt}}{\partial P_{jt}} + \left\{ \frac{\partial AC_{jt}^p}{\partial P_{jt}} + E_t \left[\frac{\beta \Lambda_{t+1}}{\Lambda_t} \frac{\partial AC_{jt+1}^p}{\partial P_{jt}} \right] \right\}. \tag{A9}$$

where MPL_t is the marginal productivity of labor, and Q_t is the rental price of capital.

A2. Households and Wage Setting

From the previous period $t-1$, an individual household $i \in [0,1]$ carries $M_{i,t-1}$ units of nominal money, $B_{i,t-1}$ units of government bond, and K_{it} units of physical capital. In the current period t, the household earns factor income $W_{it}L_{it} + Q_t K_{it}$ from renting capital K_{it} and labor service L_{it}, where W_{it} and Q_t denote the nominal wage rate and nominal rental rate for capital, respectively. The interest income from government bond holding is $(R_{t-1}-1)B_{i,t-1}$ with R_{t-1} being the gross nominal interest rate between period t-1 and t. The household receives dividend income

[12] If all households are identical and have the same shares Γ_{ijt} of firm $j \in [0,1]$, the assumption of complete markets establishes the unique market discount factor $\frac{\beta^t \Lambda_t}{\Lambda_0}$ between period 0 and t.

$\int s_{ij}\Gamma_{ijt}dj$ from firms, where s_{ij} and Γ_{ijt} are household i's fixed share of firm j and the profit of firm j, respectively. The household also receives a lump-sum nominal transfer payment T_{it} from the government.

The household i uses its funds to purchase the final good from the aggregator at the price of P_t, and divide its purchase into consumption C_{it} and investment I_{it}. In order to make new capital operational, the household needs to purchase additional materials in the amount

$$AC_{it}^k = \frac{\phi_K}{2}\left[\frac{I_{it}}{K_{it}} - \frac{\overline{I}}{\overline{K}}\right]^2 K_{it} \tag{A10}$$

where $I_{it} = K_{i,t+1} - (1-\delta_t)K_{it}$ is the real investment spending, $\phi_K > 0$ is the scale parameter for the capital adjustment costs, and $\frac{\overline{I}}{\overline{K}}$ is the steady state ratio of investment to existing capital stock. Dubbed the *depreciation shock*, δ_t denotes the stochastic decay rate of capital stock. The household then carries M_{it} units of nominal money, B_{it} units of government bond, and $K_{i,t+1}$ units of capital into period $t+1$.

Therefore, the household i maximizes its lifetime utility

$$E_0[\sum_{t=0}^{\infty}\beta^t U(C_{it}, L_{it}, \frac{M_{it}}{P_t})], \quad 0 < \beta < 1 \tag{A11}$$

subject to the budget constraint

$$\begin{aligned} & C_{it} + K_{i,t+1} - (1-\delta_t)K_{it} + \frac{M_{it}}{P_t} - \frac{M_{i,t-1}}{P_t} + \frac{B_{it}}{P_{it}} - \frac{B_{i,t-1}}{P_t} + AC_{it}^k \\ \leq \quad & \frac{W_{it}L_{it}}{P_t} + \frac{Q_tK_{it}}{P_t} + \frac{T_{it}}{P_t} + \frac{\int s_{ij}\Gamma_{ijt}dj}{P_t} + \frac{(R_{t-1}-1)B_{i,t-1}}{P_t}, \quad t \geq 0 \end{aligned} \tag{A12}$$

where the instantaneous utility function $U(\cdot)$ in (A11) has the form[13]

$$U(C_{it}, L_{it}, M_{it}/P_t) = \frac{a}{\nu}\log(C_{it}^{\nu} + b_t\,(M_{it}/P_t)^{\nu}) + (1-a_t)\log(1-L_{it}), \quad 0<a<1,\ \nu<0 \quad \text{(A13)}$$

with b_t and a_t being the *money demand shock* and the *labor supply shock,* respectively.[14]

As in the goods market, the demand for household i's individual labor service is

$$L_{it}^{d} = (\frac{W_{it}}{W_t})^{\frac{1}{\theta_L - 1}} L_t \quad , \quad \theta_L \in [0,1] \quad \text{(A14)}$$

where the aggregate labor supply L_t and aggregate wage are defined as

$$L_t = (\int L_{it}^{\theta_L}\, di)^{\frac{1}{\theta_L}} \quad , \quad W_t = (\int W_{it}^{\frac{\theta_L}{\theta_L - 1}}\, di)^{\frac{\theta_L - 1}{\theta_L}}. \quad \text{(A15)}$$

Each household, when changing its nominal wage, pays the quadratic costs of the form

$$AC_{it}^{w} = \frac{\Phi_w}{2}\left(\frac{W_{it}}{W_{i,t-1}} - \Pi_{t-1}^{w}\right)^{2}\frac{W_t}{P_t} \quad \text{(A16)}$$

where $\Pi_{t-1}^{w} = W_{t-1}/W_{t-2}$ is the gross wage inflation rate at period $t-1$, and $\Phi_w > 0$ is the scale parameter for the degree of nominal rigidity in the labor market.[15]

Utility maximization requires the following first order conditions:

$$\frac{\partial U_{it}}{\partial C_{it}} = \Lambda_{it} \quad \text{(A17)}$$

$$1 - {R_t}^{-1} = b_t(C_{it}P_t/M_{it})^{1-\nu} \quad \text{(A18)}$$

[13] As discussed in Feenstra (1986), one can construct an isomorphic model in terms of transaction costs, by redefining C^* in the utility function (A13) as the usual consumption and replacing C in the budget constraint (12) with

$$C^{**} = C^* \times \left[1 - b\left(\frac{M/P}{C^*}\right)^{\nu}\right]^{\frac{1}{\nu}}$$

where C^{**} represents the gross spending on consumption inclusive of (multiplicative) transaction costs. The ratio $C^{**}/\,C^*$ (evaluated at the estimated steady state) is 1.0006, implying transaction costs are a reasonably small fraction of consumption C^*.

[14] The steady state level of a_t is equal to a.

[15] Nominal rigidites can alternatively be specified via Calvo-style contracts, under which the model has the same first order properties. In that case, however, the way labor service enters $U(\cdot)$ in (13) prevents the equlibrium conditions from being cast into what is suitable for the second order solution method used later for welfare calculations. Therefore, the welfare imlications in the present work are to be interpreted on the caveat that the inefficient distribution of work hours across households is ignored.

$$\Lambda_{it}\left[1+\frac{\partial AC_{it}^{k}}{\partial K_{i,t+1}}\right]=\beta E_t\left[\Lambda_{i,t+1}\left(Q_{t+1}/P_{t+1}+1-\delta_{t+1}-\frac{\partial AC_{i,t+1}^{k}}{\partial K_{i,t+1}}\right)\right] \tag{A19}$$

$$\Lambda_{it}=\beta R_t E_t\left[\Lambda_{i,t+1}\frac{P_t}{P_{t+1}}\right] \tag{A20}$$

$$\begin{aligned}\left[\frac{W_{it}}{W_t}\right]^{\frac{1}{1-\theta_L}-1} MRS_{it} \quad = \quad & \theta_L\left[\frac{W_{it}}{W_t}\right]^{\frac{1}{1-\theta_L}}\frac{W_t}{P_t}+\frac{1}{L_t}(1-\theta_L)\phi_W\left[\frac{W_{it}}{W_{i,t-1}}-\Pi_{t-1}^{w}\right]\frac{W_t}{P_t}\\ & +\frac{\beta(1-\theta_L)\phi_W}{L_t}E_t\left[\frac{\Lambda_{i,t+1}}{\Lambda_{it}}\left(\frac{W_{i,t+1}}{W_{it}}\right)^2\frac{W_{t+1}}{P_{t+1}}\frac{W_t}{W_{it}}\right]\\ & -\frac{\beta(1-\theta_L)\phi_W}{L_t}E_t\left[\frac{\Lambda_{i,t+1}}{\Lambda_{it}}\Pi_t^{w}\frac{W_{t+1}}{P_{t+1}}\frac{W_{i,t+1}}{W_{it}}\frac{W_t}{W_{it}}\right]\end{aligned} \tag{A21}$$

$$MRS_{it}=(1-a_t)C_{it}^{*a(1-\sigma)}(1-L_{it})^{(1-a_t)(1-\sigma)-1}\Lambda_{it}^{-1} \tag{A22}$$

where MRS_{it} is the household i's marginal rate of substitution between leisure and consumption.

A3. Closing the Model

The government is assumed to maintain balanced budget every period by financing the total lump-sum payment to households with the seigniorage gain and issuance of net debt:

$$T_t=M_t-M_{t-1}+B_t-R_{t-1}B_{t-1} \tag{A23}$$

where $T_t=\int_0^1 T_{it}di$, $M_t=\int_0^1 M_{it}di$, and $B_t=\int_0^1 B_{it}di$.

In the benchmark economy to be estimated, monetary policy is specified as a generalized feedback rule of Taylor (1993)

$$\log\frac{R_t}{\overline{R}}=\rho_R\log\frac{R_{t-1}}{\overline{R}}+(1-\rho_R)\left[\gamma_\pi\log\frac{\Pi_t}{\overline{\Pi}}+\gamma_{y_1}\log\frac{Y_t}{\overline{Y}_t}+\gamma_m\log\frac{MG_t}{\overline{MG}}\right]+\varepsilon_{Rt},\ 0<\rho_R<1 \tag{A24}$$

where $\overline{R}$ is the gross nominal interest rate, $\overline{MG}$ is the growth rate of nominal money, $\overline{R}$ is the steady state gross nominal interest rate, all in the steady state. Π_t and MG_t are the rates of gross inflation and money growth, respectively, between period $t-1$ and t, and $\overline{Y}_t$ is the deterministic level of output at period t. $\overline{\Pi}$ is the long-run level of inflation rate the monetary authority maintains. The monetary policy disturbance ε_{Mt} is a white noise with mean 0 and variance σ_ε^2 and independent of all other disturbances in the model.

Beside the monetary policy disturbance ε_{Rt}, the model is driven by five structural shocks $(A_t, \alpha_t, \delta_t, b_t, h_t)$, each of which follows a stationary AR(1) in logarithmic form

$$\log \frac{\chi_t}{\overline{\chi}} = \rho_1 \log \frac{\chi_{t-1}}{\overline{\chi}} + \varepsilon_{\chi t} \quad \text{(A25)}$$

where $\overline{\chi}$ is the steady state level of χ_t, and $\varepsilon_{\chi t}$ is a white noise with mean 0 and variance σ_χ^2. The six innovations are uncorrelated with one another, except that those in A_t and α_t are correlated because they appear jointly in the production function.

Most of the model's real and nominal variables inherit deterministic trends due to the constant rate of labor-augmenting technical progress (g) and the long-run rate of inflation the monetary authority maintains. I deflate those variables by their respective deterministic growth rates, and use lowercase letters for the resulting stationary-transformed variables. In what follows, I focus on a particular *symmetric* equilibrium in which all firms and households make identical decisions. Equations describing this stationary-transformed symmetric equilibrium of the model are given in the appendix.

A4. Estimation

The system of equations describing the stationary symmetric equilibrium of the model can be cast into the form

$$\Psi(z_t, z_{t-1}, \varepsilon_t) + \Xi\eta_t = 0 \quad \text{(A26)}$$

where ε_t is the vector of the six innovations, and η_t is a vector of endogenous errors satisfying $E_{t-1}\eta_t = 0$, for all t. The N-dimensional system vector z_t is decomposed as $z_t = (z'_{1t}, z'_{2t})'$, where z_{2t} denotes the N_2-dimensional auxiliary variables used to denote conditional expectation terms, and z_{1t} is the $(N - N_2)$ dimensional vector of all other variables including all exogenous and endogenous state variables

A log-linear approximate solution of the model can be obtained using the method by Sims (2002), and the resulting solution takes the form

$$d \log z_t = g_1 d \log z_{t-1} + g_2 \varepsilon_t \tag{A27}$$

where g_1 and g_2 are complicated matrix functions of the model parameters, and $d \log z_t$ is the log-deviation of the system variables from steady state. With a selection matrix H that singles out the observables from the state vector z_t, I have the following state-space representation:

$$\boxed{\begin{array}{ll} \text{transition equation :} & d \log z_t = g_1 d \log z_{t-1} + g_2 \varepsilon_t, \quad \varepsilon_t \backsim iiN(0, \Sigma_\varepsilon) \\ \text{observation equation :} & \qquad d \log \omega_t = H d \log z_t \end{array}} \tag{A28}$$

where ω_t denotes the variables corresponding to the observable data series and Σ_ε is the covariance matrix of the innovations ε_t. From (A28), it is straightforward to construct a Gaussian likelihood function for the entire parameter vector θ:

$$\begin{aligned} L_T(\theta \quad | \quad z_1, ..., z_T) = & -\frac{1}{2} \sum_{t=1}^{T} \log |_{t-1}\Sigma_t^z(\theta)| \\ & -\frac{1}{2} \sum_{t=1}^{T} [d \log z_t -_{t-1} d \log z_t(\theta)]' \left[_{t-1}\Sigma_t^z(\theta)\right]^{-1} [d \log z_t -_{t-1} d \log z_t(\theta)] \end{aligned} \tag{A29}$$

where $_{t-1}d \log z_t(\theta)$ and $_{t-1}\Sigma_t^z \ (\theta)$ are one-step ahead forecasts of mean and variance of $d \log z_t$, respectively, and T is the sample size. For $t = 1$, the unconditional mean (which is zero) and variance of $d \log z_t$ implied by the first order solution (27) are used for $_{t-1}d \log z_t(\theta)$ and $_{t-1}\Sigma_t^z(\theta)$, respectively.

The raw data used for estimation summarized, in Table A1, are extracted from DRI BASIC economic series for the sample period 1959:Q1-1999:Q3. The following six series are used for the actual estimation purpose: per capita output (Y), per capita labor hours (L), rate of price inflation

(Π), the growth rate of per capita money balance (MG), interest rates (R), and wage inflation rates (Π^w). To express the data series conformable to their model counterparts, output and money growth are suitably transformed via population size. Per capita labor hours are obtained by dividing weekly working hours by 120, under the assumption that each worker is endowed with 5 working days per week. The resulting series imply households devote 33.8% of their time endowment to working. Since federal funds rates are measured in annual percentage rates, I transform them into quarterly rates by dividing by 400 and adding one. Price and wage inflations are obtained by log-differencing the price and wage series.

Some structural parameters are fixed before estimation: steady state values of capital share $\overline{\alpha}$ and depreciation $\overline{\delta}$ are fixed at 1/3 and 0.025, respectively. The market power θ_Y in the goods market is fixed at the conventionally calibrated value of 0.9, because only two of $(\overline{A}, \theta_Y, \theta_L)$ are identified from the series on output and labor. Assuming the Fed has been successfully managed the inflation rate around its "long-run target" level, I fix the steady state inflation rate $\overline{\Pi}$ at its actual average 1.01005 over the sample period. Two parameters (ν, b), crucial to the form of money demand and welfare calculations, are estimated by running calibration and estimation jointly. I first fix the steady state consumption velocity $V = PC/M$ in equation (18) at its actual average over the sample period. Then, at each step of maximizing the likelihood function, I use the resulting relation to determine b given all other candidate parameters. Table A2 summarizes the functional forms of the model equations and the estimates of parameters, with standard errors in parentheses.[16]

A5. Quadratic Approximate Solution

As discussed in Sims (2000) and Kim et al. (2003), correct welfare evaluations in terms of utility levels require higher order accuracy of the model solutions beyond those by the conventional log-

[16] The standard errors are constructed from the numerically computed Hessian at the optimum.

linear methods. I use the second order approximate solution method developed and extended by those authors in order to compute the paths of the first and second moments of $d\log z_t$ conditional on an state $\Omega_0 : \{E[d\log z_t \mid \Omega_0], Var[d\log z_t \mid \Omega_0], t = 1, 2, ...\}$. In the following analysis, the initial state Ω_0 comprises the unconditional first and second moments of $d\log z_t$, calculated by the same solution method given the estimated monetary policy rule (A27).

The method used here is the second-order accurate solution developed by Sims (2000). Under a set of regularity conditions, a unique and stationary second order accurate solution to (A26) is given by

$$\begin{aligned}\widehat{z}_{1it} &= F_{1ij}\widehat{z}_{1j,t-1} + F_{2ij}\varepsilon_{jt} + F_{3i} \qquad \text{(A30a)}\\ &\quad +0.5\left(F_{11ijk}\widehat{z}_{1j,t-1}\widehat{z}_{1k,t-1} + 2F_{12ijk}\widehat{z}_{1j,t-1}\varepsilon_{kt} + F_{22ijk}\varepsilon_{jt}\varepsilon_{kt}\right),\\ \widehat{z}_{2it} &= S_i\widehat{z}_{1it} + T_iM_{11ijk}\widehat{z}_{1jt}\widehat{z}_{1kt} + T_iM_{2i}\ . \qquad \text{(A30b)}\end{aligned}$$

where $\widehat{z}_t = \log z_t - \log \overline{z}$ denotes the % deviation of z_t from its deterministic steady state, and S, T, F's, and M's are matrix functions of the deep parameter of the model. In particular, the terms F_3 and M_2 represent the degree of certainty non-equivalence. Note that equations in (A30) utilize the tensor notation for the simplicity of exposition. For example, the term $F_{11ijk}\widehat{z}_{1j,t-1}\widehat{z}_{1k,t-1}$ can be interpreted as the quadratic form in terms of lagged $\widehat{z}_t$ for the i^{th} equation, constructed by the lag of $\widehat{z}_{1t}$.

Once the solution of the form in (A30) is obtained, we can recursively calculate the first and second conditional moments $\{E_0[d\log z_t], Var_0[d\log z_t], t \geq 0\}$ which will be used evaluate the welfare implications of minimizing alternative objective functions of the monetary authority.

A6. Calculating conditional first and second moments By using equation (A30a) recursively, we can compute $\{\mu_{1t}, \Sigma_{1t}, t = 1, 2, ...\}$, the conditional first and second moments of $\widehat{z}_1$, from which the welfare measure OF_1 is constructed. For the sake of second order accuracy, all terms of orders higher than two may be dropped out: accordingly, only the first two terms in equation (A30a) describe the evolution of the conditional variances of $\widehat{z}_{1t}$:

$$\Sigma_{1t} = F_1 \Sigma_{1t} F_1' + F_2 \Sigma_\varepsilon F_2' \tag{A31}$$

where F_1 and F_2 are the matrices of the coefficients on $\widehat{z}_{1,t-1}$ and ε_t, respectively, representing the first order parts of the solution.

Recursive calculations of μ_{1t} are more involved. The subsystem (A30a) is first re-written in an expanded form as

$$\begin{aligned} \widehat{z}_{1t} &= F_1 \widehat{z}_{1,t-1} + F_2 \varepsilon_t + F_3 \\ &+ \frac{1}{2} \begin{bmatrix} \widehat{z}_{1,t-1}' F_{11}^{(1)} \widehat{z}_{1,t-1}' \\ \vdots \\ \widehat{z}_{1,t-1}' F_{11}^{(N_1)} \widehat{z}_{1,t-1}' \end{bmatrix} + \begin{bmatrix} \widehat{z}_{1,t-1}' F_{12}^{(1)} \varepsilon_t \\ \vdots \\ \widehat{z}_{1,t-1}' F_{12}^{(N_1)} \varepsilon_t \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \varepsilon_t F_{22}^{(1)} \varepsilon_t \\ \vdots \\ \varepsilon_t F_{22}^{(N_1)} \varepsilon_t \end{bmatrix} \end{aligned} \tag{A32}$$

where F_3 is a $N_1 \times 1$ column vector, and $F_{jk}^{(i)\prime}$s are the matrices constructing quadratic terms for the i^{th} equation in the second order solution (A30a).

Taking expectation of (A32) conditional on Ω_0, I get

$$\begin{aligned} \mu_{1t} &= F_1 \mu_{1,t-1} + F_3 \\ &+ \frac{1}{2} \begin{bmatrix} tr\left(\Sigma_{1,t-1} F_{11}^{(1)}\right) \\ \vdots \\ tr\left(\Sigma_{1,t-1} F_{11}^{(N_1)}\right) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} tr\left(\Sigma_\varepsilon F_{22}^{(1)}\right) \\ \vdots \\ tr\left(\Sigma_\varepsilon F_{22}^{(N_1)}\right) \end{bmatrix} \end{aligned} \tag{A33}$$

where $tr(\cdot)$ is the trace of a square matrix. One can calculate $\{\mu_{1t}, \Sigma_{1t} : t \geq 1\}$ recursively by using (A20) and (A22) jointly given some initial condition $\Omega_0 = (\mu_{10}, \Sigma_{10})$.

Table A1: Raw Data Series

output :	gross domestic products, billions of 1992 dollars.
employment :	average weekly hours of production workers in manufacturing sector.
price :	implicit price deflator for gross national products.
money :	M2 stock, billions of current dollars.
interest rate :	federal funds rate, per annum.
wage :	index of compensation per hour in nonfarm business sector, 1982=100.
population	civilian population, in thousands.

Note : All series, except for interest rate and wage, are seasonally adjusted.

Table A2: Functions and Parameter Estimates ($\overline{\Pi} = 1.01005$)

Functional Forms	Estimates and Standard Deviations
$Y_t = A_t K_t^{\alpha_t}(g^t L_t)^{1-\alpha_t}$	A=5.5668(0.0718), g=1.0056(8.8×10^{-5})
$\beta^t U(C_t, L_t, \frac{M_t}{P_t})$ $= \beta^t \log\left[C_t^{*a}(1-L_t)^{1-a_t}\right]$	$\beta = 0.9986(0.0003)$, $a = 0.4681(0.0016)$
$C_t^* = (C_t^\nu + b_t (M_t/P_t)^\nu)^{\frac{1}{\nu}}$	ν=-22.7561(0.4765), b =0.0008(5.5×10^{-5})
$L_{it} = (\frac{W_{it}}{W_t})^{\frac{1}{\theta_L - 1}} L_t$	$\theta_L = 0.6888(0.0087)$
$AC_t^k = \frac{\phi_K}{2}(\frac{I_t}{K_t} - \frac{I}{K})^2 K_t$	$\Phi_k = 16.8456(1.5501)$
$\log\frac{R_t}{R} = \rho_R \log\frac{R_{t-1}}{R} + (1-\rho_M)\times$ $\left[\gamma_\pi \log\frac{\Pi_t}{\Pi} + \gamma_y \log\frac{Y_t}{\bar{Y}_t} + \gamma_m \log\frac{MG_t}{MG}\right]$ $+\varepsilon_{Mt}$	$\rho_R = 0.1395(0.0112), \gamma_\pi$=0.8042(0.0045) γ_y=4.4×10^{-6}(4.5×10^{-5}), γ_m=0.4276(0.0187) $\sigma_M^2 = 4.3 \times 10^{-5}(5.2 \times 10^{-6})$
$AC_{it}^P = \frac{\Phi_P}{2}\left(\frac{P_{jt}}{P_{j,t-1}} - \Pi_{t-1}\right)^2 Y_t$	$\Phi_p = 10.0970(0.7393)$
$AC_{it}^W = \frac{\Phi w}{2}\left(\frac{Wi}{W_{i,t-1}} - \Pi_{t-1}^w\right)^2 \frac{W_t}{P_t}$	$\Phi_w = 22.0341(0.7025)$
$\log\frac{A_t}{A} = \rho_A \log\frac{A_{t-1}}{A} + \varepsilon_{At}$	ρ_A=0.9761(0.0002), σ_A^2=0.0012(9.9×10^{-5})
$\log\frac{\alpha_t}{\alpha} = \rho_\alpha \log\frac{\alpha_{t-1}}{\alpha} + \varepsilon_{\alpha t}$	ρ_α=0.9690(0.0015), σ_α^2=0.0003(2.7×10^{-5})
$\log\frac{\delta_t}{\delta} = \rho_\delta \log\frac{\delta_{t-1}}{\delta} + \varepsilon_{\delta t}$	ρ_δ=0.9563(0.0012), σ_δ^2=0.0129(0.0021)
$\log\frac{b_t}{b} = \rho_b \log\frac{b_{t-1}}{b} + \varepsilon_{bt}$	ρ_b=0.9450(0.0022), σ_b^2=0.0716(0.0078)
$\log\frac{a_t}{a} = \rho_a \log\frac{a_{t-1}}{a} + \varepsilon_{ht}$	ρ_a=-0.4573(0.0482), σ_a^2=0.0319(0.0046)
	$cov(\varepsilon_{At}, \varepsilon_{\alpha t}) = -0.0006$(5.0×10^{-5})

Table A3: Model and Data Moments

Series	Model Steady State	Prediction Std. dev.	U.S. Mean	Data Std. dev.
Output, y	13.3297	0.0846	13.5535	0.0468
Labor, L	0.3310	0.0340	0.3382	0.0176
Inflation, Π	1.01005	0.0083	1.01005	0.0063
Money Growth, MG	1.0155	0.0161	1.0144	0.0097
Interest Rate, R	1.0174	0.0082	1.0164	0.0063
Wage Inflation, Π^w	1.0155	0.0111	1.0142	0.0076

7. Appendix (II) : Tables and Figures

Table 1: Optimized Rules for OF_1

Rule	$\overline{\Pi}$	Coeff.						Welfare	Std. Dev.		
		ρ	a_1	a_2	a_3	a_4	a_5		y	Π	R
U1	1.0003 (free)	0.8844	0.4964	0.4822	-0.4806	0.1885	1.4068	611.2754	1.0912	1.0945	0.4446
U2	1.01005 (fixed)	0.8727	0.5086	0.4683	-0.3159	0.2420	1.8332	611.0778	0.9120	1.2106	0.9297

Table 2: Welfare Decompositions[17]

	U1			U2		
	Weight	Discounted sum	Contribution	Weight	Discounted sum	Contribution
[1] Welfare			**611.2754**			**611.0778**
[2] Efficiency			**612.2771**			**611.9987**
Steady State						
U			606.7934			605.8955
C			9.6441			9.5921
L			0.3363			0.3345
RM			8.8233			8.4705
Mean Effects						
U	-	-	5.4837	-	-	6.0992
C	0.4650	44.8971	20.8772	0.4613	47.1219	21.7352
L	-0.2695	57.6827	-15.5442	-0.2673	59.9627	-16.0921
RM	0.0031	48.8557	0.1515	0.0069	57.3349	0.3931
[3] Stability						
U	-	-	**-1.0017**	-	-	**-0.9209**
C	-0.2674	7.9156	-2.1166	-0.3075	8.2632	-2.5409
L	-0.0683	0.9314	-0.0636	-0.0672	0.9324	-0.0626
RM	-0.0365	9.4695	-0.3455	-0.0803	11.6456	-0.9351
(C, RM)	0.0698	8.1647	0.5699	0.1537	9.4334	1.4503
(C, b)	-0.0031	-0.6018	0.0019	-0.0068	-0.7239	0.0048
$(L.a)$	0.2371	3.7734	0.8947	0.2353	4.4129	1.0382
(RM, b)	0.0031	18.4393	0.0572	0.0068	18.8537	0.1274

[17] Negliegible terms ignored, the sum of individual contributions does not exactly match efficiency or stability.

Table 3: Optimized Rules for $\mathbf{OF}_2$ ($\overline{\Pi} = 1.01005$)

Rule	Coeff.						Welfare	Std. Dev.		
	ρ	a_1	a_2	a_3	a_4	a_5		y	R	Π
S1 (λ_y= 1)	0.0159	3.5883	1.0457	-1.0667	0.0321	0.5152	609.8514	1.0426	0.5734	0.5556
S2 (λ_y= 0.5)	0.1546	44.9160	0.1983	-0.2152	0.0152	0.0335	610.3316	1.1187	1.5726	0.0492
S3 (λ_y= 0.1)	0.1713	46.1230	0.0084	-0.0228	0.0004	0.0289	610.3834	1.1238	1.6660	0.0319

Table 4: Welfare Decompositions[18]

		U1			S3	
	Weight	Discounted sum	Contribution	Weight	Discounted sum	Contribution
[1] Welfare			**611.2754**			**610.3834**
[2] Efficiency			**612.2771**			**611.5154**
Steady State						
U			606.7934			605.8955
C			9.6441			9.5921
L			0.3363			0.3345
RM			8.8233			8.4705
Mean Effects						
U	-	-	5.4837	-	-	5.6199
C	0.4650	44.8971	20.8772	0.4613	44.7015	20.6187
L	-0.2695	57.6827	-15.5442	-0.2673	57.3446	-15.3292
RM	0.0031	48.8557	0.1515	0.0069	48.1931	0.3304
[3] Stability						
U	-	-	**-1.0017**	-	-	**-1.1320**
C	-0.2674	7.9156	-2.1166	-0.3075	8.6625	-2.6637
L	-0.0683	0.9314	-0.0636	-0.0672	1.8273	-0.1228
RM	-0.0365	9.4695	-0.3455	-0.0803	10.7809	-0.8657
(C, RM)	0.0698	8.1647	0.5699	0.1537	9.1633	1.4087
(C, b)	-0.0031	-0.6018	0.0019	-0.0068	-0.1521	0.0010
$(L.a)$	0.2371	3.7734	0.8947	0.2353	4.1365	0.9732
(RM, b)	0.0031	18.4393	0.0572	0.0068	20.1384	0.1360

[18] Nnegliegible terms ignored, the sum of individual contributions does not exactly match efficiency or stability.

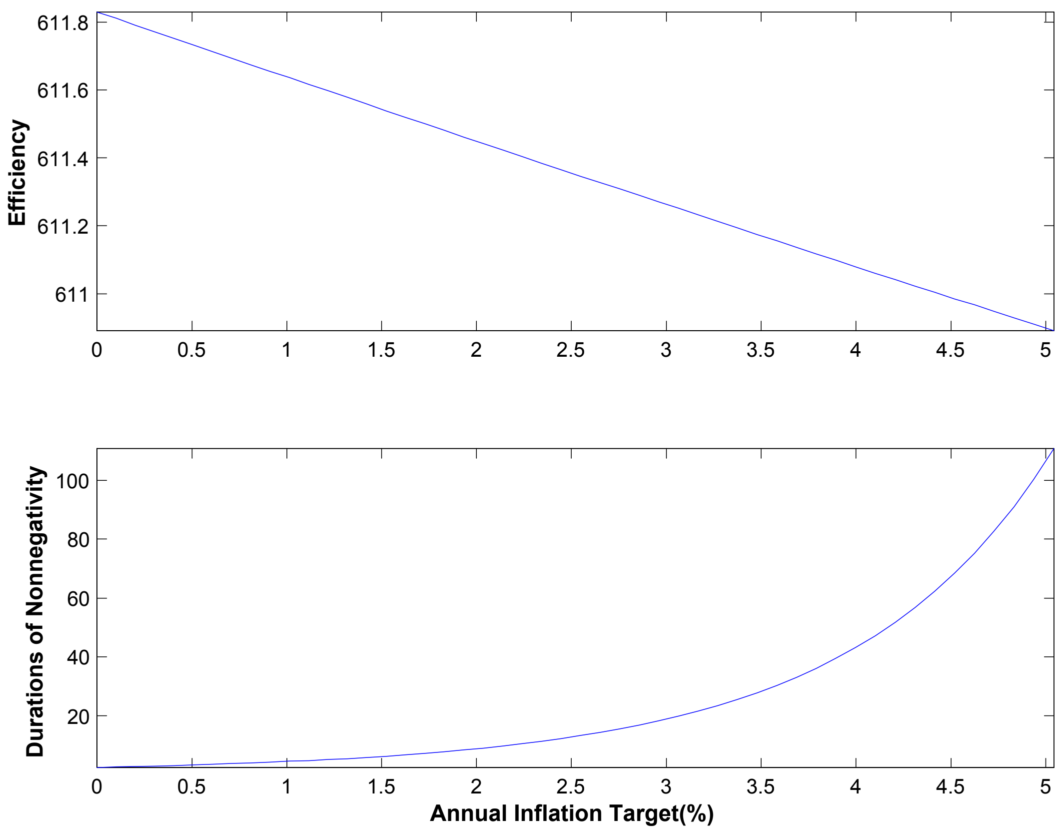

Figure 1: Efficiency and Policy Effectiveness

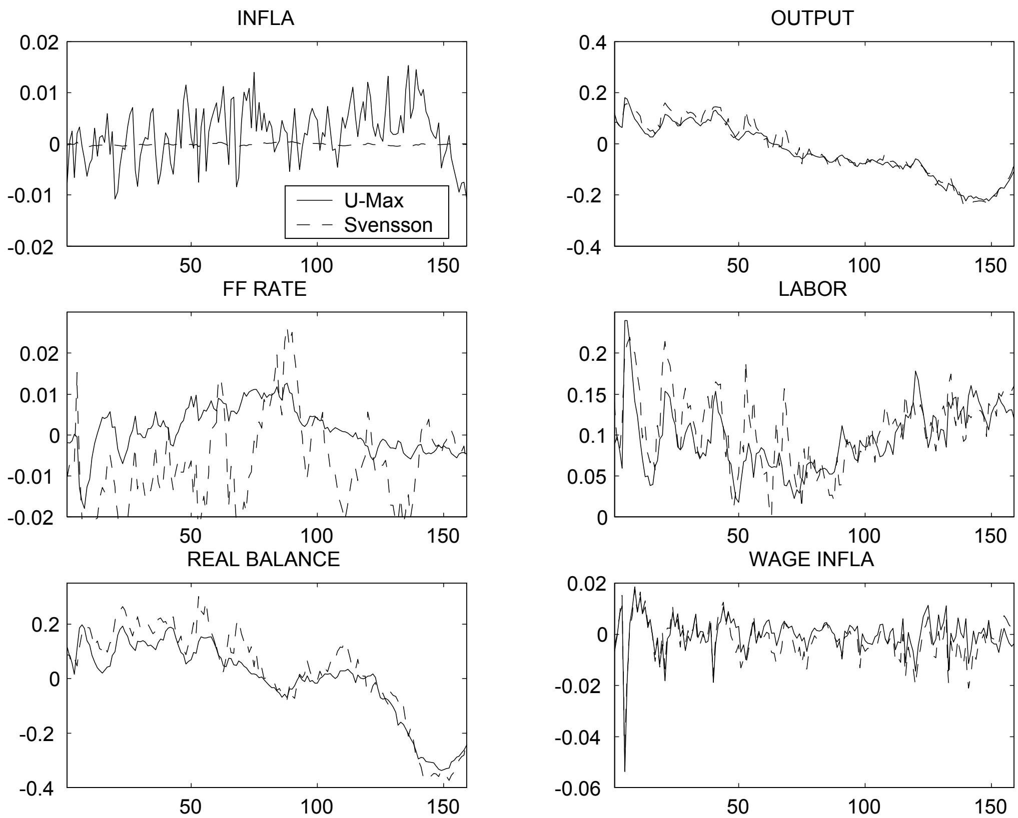

Figure 2: Simulated paths under alternative rules: all shocks.

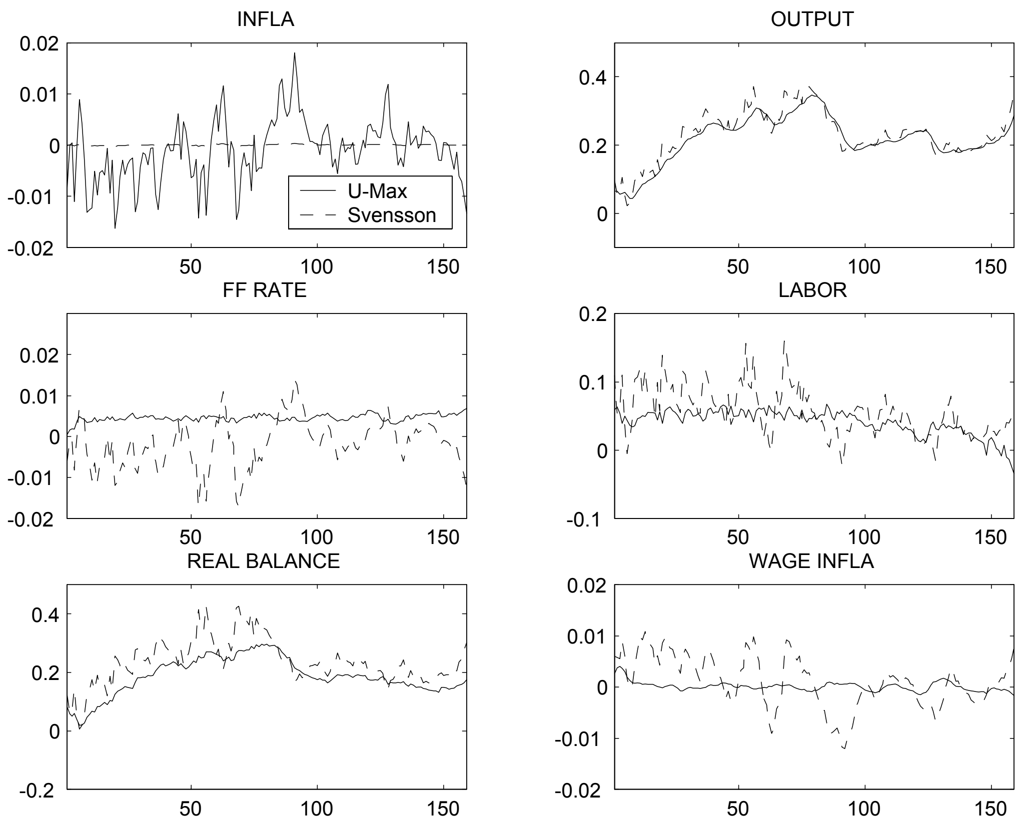

Figure 3: Simulated paths under alternative rules: (A, α) shocks.

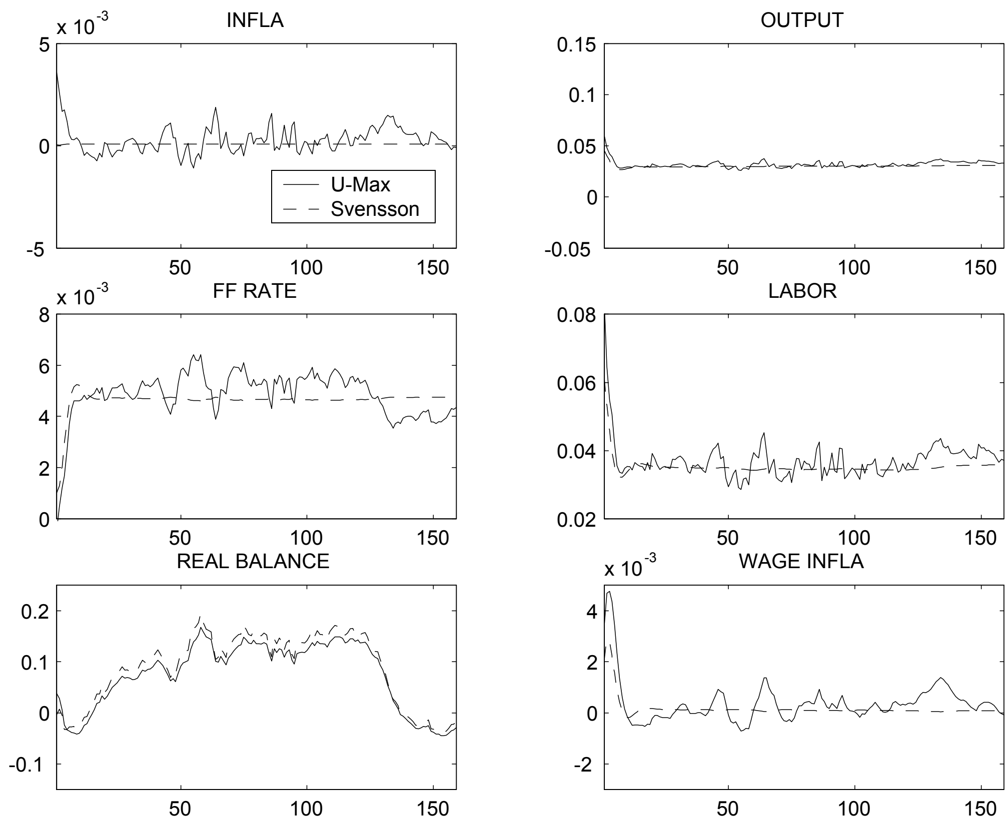

Figure 4: Simulated paths under alternative rules: b shocks.