Staggered Contracts Models of Business Cycle: How Much Nominal Rigidity Do We Have?

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Abstract

This paper presents a monetary business cycle model embodying arbitrary degrees of nominal rigidities in goods and labor markets. Nominal rigidities are introduced in the form of staggered contracts. The structural parameters of the model go through formal reconciliation with data series via maximum likelihood estimation. The estimation results stand in favor of wage stickiness, in the sense that *i*) average duration of contracts is longer in labor market; and *ii*) nominal wage rigidities are crucial for the model's performance in fitting actual U.S. data.

1 Introduction

Over the recent years, there has been a multitude of attempts to construct monetary business cycle models with persistent real effects of monetary disturbances. In particular, since the work of Chari et al. (1998, 2000) questioned the candidacy of staggered price contracts for an effective propagation mechanism, many authors have tried to reinstate the *contract multiplier* as a promising propagation channel generating real persistency of nominal shocks. Gust (1997) shows that imperfect inter-sector capital mobility can increase persistence in the model of Chari et al. (2000). Sharing the insight of Ball and Romer (1990), Kiley (1997) and Jeanne (1997) show that increasing the degree of *real* rigidities also can increase persistence due to the nominal rigidities in the model. Moving away from complete wage flexibility assumed in Chari et al. (2000), Erceg (1997) shows how staggered wage setting jointly with staggered price setting can generate the persistence observed in the data. Another possible resolution of the persistence puzzle suggested

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by Kim (2000 [15]): embedded in a dynamic business cycle model, Taylor style deterministic price contracts adopted in Chari at al. (2000) exhibits an inherent nuisance feature (i.e., the initial overshooting of re-optimized prices following monetary shocks), while deterministic wage contracts or Calvo (1983) style stochastic price/wage contracts are free of such anomalous implications. To my best knowledge, the general picture around the staggered contracts is that the interplay of price and wage contracts is a promising propagation channel for monetary disturbances, wage stickiness being possibly the mightier.[1]

Developing a model which is *in principle* capable of generating realistic contract multiplier is, however, only one half of the story, and the other half in order is the empirical validation of the candidate model by examining how well its implications reconcile with actual data series. This task has both positive and normative importance: being critical to whether real aggregates display non-trivial and realistic responses to nominal shocks, the degree and sources of nominal rigidities are needed to pin down to get a realistic snapshot of the economy. Furthermore, given empirically supported degree of nominal rigidities and implied behavior of hours in the labor market, a vital component in welfare analysis, i.e., the welfare of households, can be explicitly considered under alternative of monetary policies.

This paper seeks to establish the positive results of the empirical validation. For that aim, I formulate an otherwise standard monetary business cycle model, patch it up with arbitrary degree of price and wage rigidities via staggered contracts, and estimate the structural parameters of the model using maximum likelihood estimation method. The estimated model is used to investigate *i*) whether reasonable degree of nominal rigidities are compatible with the observed behavior of key U.S. macro aggregates, and *ii*) what mixture of nominal frictions are supported by the empirical evidence from actual data series.[2]

Estimation results support higher degree of nominal rigidities in the labor market than in the goods market. More specifically,the average durations of price and wage contracts are estimated roughly as two and four quarters, respectively. Consistently with the existing results in the

[1] For example, Kim (2000 [15]) shows that Calvo style staggered price contracts with average price fixity of one year generate only modest degree of real persistency, while wage contracts with the same degree of wage fixity are capable of hump-shaped persistent responses of output following monetary expansions.

[2] The usage of maximum likelihood method gives this paper an advantage over a few recent works such as Christiano et al. (2001) and Rotemberg and Woodford (1997, 1999). In those papers, parameters are esimated by matching the model generated impulse responses with those from strucutural VARs, which are critically dependent upon the validity of identification assumptions.

literature, I also find that critical nominal friction in the estimated model is wage stickiness. In fact, the constrained estimates without price rigidity generates the same qualitative features of key variables as in the model fleshed out with the unconstrained estimates.

The paper is organized as follows. Section 2 presents the model where the impediments to the adjustment of nominal prices and wages are specified in the form of staggered contracts. In section 3, I discuss the method and data used to estimate the deep parameters of the model. Section 4 is devoted to reporting the empirical results. Section 5 concludes the paper.

2 The Model

The economy constructed in this section is a variant of Yun (1998), Kim ([15], 2000) and Erceg at al. (2000). The economy consists of three kinds of agents: households, firms, and government. Firms are monopolistic competitors producing differentiated goods using capital and labor as inputs. Households purchase output for consumption and investment purposes, and supply capital and differentiated labor. The government manages monetary policy by adjusting monetary instruments in response to the aggregate economic conditions, subject to its own period-by-period budget constraint. In the spirit of Calvo (1983), nominal prices and wages are set according to a stochastic staggering mechanism.

2.1 Households

I assume there are infinitely many immortal households indexed by $i \in [0, 1]$, endowed with a fixed amount of time usable for leisure or work each period. They hold two types of nominal assets, money M and interest bearing government bond B, and one real asset, capital K. Income is earned from the capital and labor service they sell to firms and from the interest payment for government debt holding. As shareholders of firms in the economy, households also receive dividend income. When accumulating capital stock, households are subject to quadratic adjustment costs: in order to make new capital operational, the households need to purchase additional materials in the amount

$$AC_{it}^{k} = \frac{\phi_K}{2}\left[\frac{I_{it}}{K_{it}} - \frac{\overline{I}}{\overline{K}}\right]^2 K_{it}, \quad \phi_K > 0 \tag{1}$$

where $I_{it} = K_{i,t+1} - (1-\delta_t)K_{it}$ is the real investment spending, ϕ_K is the scale parameter for the capital adjustment costs, and $\frac{\overline{I}}{\overline{K}}$ is the ratio of investment to existing capital stock in the steady state. The term δ_t denotes the stochastic decay rate of capital stock, which I call *depreciation shock*. Its stochastic properties are specified later.

It is facilitating to interpret the household i's decision making as a sequential one: household i enters period τ with capital, money stock, and nominal government bond holdings carried over form the previous period. In period τ, the household chooses its optimal wage rate to charge, thereby determining its current labor income and working hours. Finally, the household determines how to allocate its total disposable income to consumption, investment, and acquisition of other nominal assets.

Household i derives utility by maximizing

$$E_t[\sum_{\tau=t}^{\infty} \beta^\tau U(C_{i\tau}^*, L_{i\tau}, M_{i\tau}/P_\tau)] \ , \ 0 < \beta < 1$$

where the instantaneous utility function is given by the following form:

$$U(C_{it}^*, L_{it}, \frac{M_{it}}{P_t}) = \left(\left[C_{it}^*(1-L_{it})^{1-a_t}\right]^{1-\sigma} - 1\right)/(1-\sigma), \qquad 0 < a < 1, \nu < 0 \tag{2}$$

In equation (2), $C_t^* = (C_t^\nu + b_t\,(M_t/P_t)^\nu)^{\frac{a}{\nu}}$ is the CES bundle of consumption C_t and real money balance M_t/P_t. The stochastic properties of the *money demand shock* b_t and the *labor supply shock* a_t will be specified later.

The household's budget constraint each period is

$$\begin{aligned} & C_{i\tau} + K_{i,\tau+1} - (1-\delta_\tau)K_{i\tau} + \frac{M_{i\tau}}{P_\tau} - \frac{M_{i,\tau-1}}{P_\tau} + \frac{B_{i\tau}}{P_\tau} - \frac{B_{i,\tau-1}}{P_\tau} + AC_{i\tau}^k \\ \leq \ & \frac{W_{i\tau}L_{i\tau}}{P_\tau} + \frac{Z_\tau K_{i\tau}}{P_\tau} + T_{i\tau} + \frac{\int s_{ij}\Gamma_{j\tau}dj}{P_\tau} + \frac{(R_{\tau-1}-1)B_{i,\tau-1}}{P_\tau}, \quad \tau \geq t \end{aligned} \tag{3}$$

where s_{ij} is household i's share of the firm j which is assumed to be fixed, and Γ_j is the profit of firm j. The LHS of equation (3) denotes the usage of disposable income, comprising *i)* consumption $C_{i\tau}$; *ii)* investment $K_{i,\tau+1} - (1-\delta_\tau)K_{i\tau}$, *iii)* acquisition of new real money balance $\frac{M_{i\tau}-M_{i,\tau-1}}{P_\tau}$ and nominal bonds $\frac{B_{i\tau}-B_{i,\tau-1}}{P_\tau}$; and *iv)* capital adjustment cost $AC_{i\tau}^k$. The RHS denotes the sources of disposable income, such as *i)* labor income $W_{i\tau}L_{i\tau}$; *ii)* capital rental income $Z_\tau K_{i\tau}$, *iii)* the lump-sum government transfers $T_{i\tau}$; *iv)* profit dividend $\int s_{ij}\Gamma_{j\tau}dj$; and *v)* the interest income from previous period's bond holding $B_{i,\tau-1}(R_{\tau-1}-1)$. Note that $R_{\tau-1}$ is the gross nominal interest rate between period $\tau-1$ and τ.

Suppose the current period's individual wage rate $W_{i\tau}$ has already been determined. Then first order conditions for $(C_{it}, M_{ti}, K_{i,t+1}, B_{it})$ are given by

$$\frac{\partial U_{it}}{\partial C_{it}} = \Lambda_{it} \tag{4}$$

$$1 - R_t{}^{-1} = b(C_{it}/RM_{it})^{1-\nu} \tag{5}$$

$$\Lambda_{it}\left[1 + \frac{\partial AC_{it}^k}{\partial K_{i,t+1}}\right] = \beta E_t\left[\Lambda_{i,t+1}(RZ_{t+1} + 1 - \delta_{t+1})\right] - \beta E_t\left[\Lambda_{i,t+1}\frac{\partial AC_{i,t+1}^k}{\partial K_{i,t+1}}\right] \tag{6}$$

$$0 = \frac{\Lambda_{it}}{P_t} - \beta R_t E_t\left[\frac{\Lambda_{i,t+1}}{P_{t+1}}\right] \tag{7}$$

where RM_{it} is the real money balance held by household i, RZ_t is the real rental price, and Λ_{it} the Lagrangian multiplier on the household i's budget constraint, interpretable as the value of one unit of consumption good.

To understand the Euler equation (6) for capital stock, consider the following experiment: suppose that, at period t, the household i decreases its consumption by one unit, thereby increasing capital holding in the next period, and sells the remaining capital at the end of period $t+1$. Then, the LHS of equation (6) measures i) the utility value the consumption foregone, *plus* ii) that of the increase in the capital adjustment cost paid to increase capital stock in the next period. In the RHS, the first term denotes the utility value of i) the remaining capital after depreciation, *plus* ii) that of the decrease in the capital adjustment cost at period $t+1$ due to higher level of period $t+1$ capital stock. So the equation (6) is interpreted as an arbitrage condition between the current and future consumption.

2.1.1 Wage Setting

As a supplier of differentiated labor service, each household is a wage setter and satisfies the quantity of its own labor demanded at its individual wage rate posted. As is standard in the literature, we assume that the demand function for L_i, the labor service supplied by household i, is of the Dixit-Stiglitz form

$$L_{it} = (\frac{W_{it}}{W_t})^{\frac{1}{\theta_L - 1}} L_t \tag{8}$$

where $\theta_L \in [0, 1]$ measures the degree of household i's market power over its own labor service, W_{it} is the wage set by the household i, L_t is the aggregate labor demand. In the labor market equilibrium, the total labor demand L_t is in turn equal to the aggregate labor supply $(\int L_{it}^{\theta_L} di)^{\frac{1}{\theta_L}}$, the CES index constructed from differentiated labor service. The aggregate wage rate W_t is in turn defined as

$$W_t = (\int W_{it}^{\frac{\theta_L}{\theta_L - 1}} di)^{\frac{\theta_L - 1}{\theta_L}}. \tag{9}$$

Households set wages according to a variation of the mechanism in Calvo (1983). In each period t, a randomly chosen $1-\phi_L$ fraction of households, denoted by *opt(t)*, are able to reoptimize their individual nominal wages. The other ϕ_L fraction of households, denoted by *rev(t)*, are assumed to reset their wages according to an index rule

$$W_{it} = \Pi_{t-1}^{\omega} W_{i,t-1}, \qquad i \in rev(t) \tag{10}$$

where Π_{t-1}^{ω} is the actual wage inflation rate in the last period.[3] Therefore, even if a household is able to reoptimize in the current period t, current wage decision continues to affect its utility in future period $\tau \geq t$ with geometrically decaying probability $\phi_L^{\tau-t}$.

The First order condition for optimal wage setting for a household $i \in opt(t)$ is:

$$-E_t \left[\sum_{\tau=t}^{\infty} (\beta\phi_L)^{\tau-t} \frac{\partial U_{i\tau}}{\partial L_{i\tau}} \frac{dL_{i\tau}}{dW_{it}} \right] = E_t \left[\sum_{\tau=t}^{\infty} (\beta\phi_L)^{\tau-t} \frac{\Lambda_{i\tau}}{P_\tau} \left(L_{i\tau} \frac{dW_{i\tau}}{dW_{it}} + W_{i\tau} \frac{dL_{i\tau}}{dW_{it}} \right) \right] \tag{11}$$

where $U_{i\tau}, L_{i\tau}, W_{i\tau}$, and $\Lambda_{i\tau}$ all depend on W_{it} via equations (2) and (8). Equation (11) requires the optimal wage equate the present discounted value of marginal disutility from work and that of real wage income measured in utility terms through the Lagrangian multiplier $\Lambda_{i\tau}, \forall \tau \geq t$.

Under the assumption that all households in *opt*(t) set the same wage rate W_t^*, equation (4) and equation (11) yield the following wage setting condition:

$$\begin{aligned} & a\theta_L E_t \left[\sum_{\tau=t}^{\infty} (\beta\phi_L)^{\tau-t} C_{i\tau}^{*a(1-\sigma)-1} C_{i\tau}^{\nu-1} \left(1 - (\frac{\varpi_t \Pi_t^{\omega}}{\Pi_\tau^{\omega}})^{\frac{1}{\theta_L - 1}} L_\tau \right)^{(1-a_t)(1-\sigma)} RW_\tau (\frac{\varpi_t \Pi_t^{\omega}}{\Pi_\tau^{\omega}})^{\frac{\theta_L}{\theta_L - 1}} L_\tau \right] \\ = \; & E_t \left[\sum_{\tau=t}^{\infty} (\beta\phi_L)^{\tau-t} (1-a_\tau) C_{i\tau}^{*a(1-\sigma)} \left(1 - (\frac{\varpi_t \Pi_t^{\omega}}{\Pi_\tau^{\omega}})^{\frac{1}{\theta_L - 1}} L_\tau \right)^{(1-a_t)(1-\sigma)-1} (\frac{\varpi_t \Pi_t^{\omega}}{\Pi_\tau^{\omega}})^{\frac{1}{\theta_L - 1}} L_\tau \right] \end{aligned} \tag{12}$$

where $\varpi_t = W_t^*/W_t$ is the optimal wage rate relative to the aggregate, and $RW_t = W_t/P_t$ is the aggregate real wage.[4]

[3] This "backard-looking" indexation scheme is also used in Christiano et al. (2001) to achieve higher persistence in both price level and inflation rate.

[4] See Kim (2000) for the conditions under which the optimal wage rate for optimizing households are identical.

2.2 Firms

There are infinitely many monopolistically competitive firms indexed by $j \in [0, 1]$, having access to the identical CRS production technology

$$Y_{jt} = A_t K_{jt}^{\alpha_t} (g^t L_{jt})^{1-\alpha_t} \tag{13}$$

where K_{jt} and L_{jt} are the quantity of capital composite labor input, respectively, used by firm j. By assuming $g \geq 1$, we allow all real variables of the model(except L) to have deterministic balanced growth paths. The stochastic properties of the *aggregate productivity shock* A_t and the *capital share shock* α_t are detailed later.

A firm j is assumed to solve its profit maximization problem through two steps. First, given its desired output level(determined by its relative price and aggregate output) and factor prices, the firm solves a cost minimization problem. Second, given the cost function thus derived, it determines optimal price by solving the following profit maximization problem:

$$\max E_t \left[\sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{\Lambda_\tau}{\Lambda_t} \left(\frac{P_{j\tau} Y_{j\tau}}{P_\tau} - \frac{MC_\tau}{P_\tau} Y_{j\tau} \right) \right] \tag{14}$$

where $MC_t = A_t^{-1} Z_t^{\alpha_t} W_t^{1-\alpha_t} (1-\alpha_t)^{\alpha_t - 1} \alpha_t^{-\alpha_t}$ is the marginal cost, and $\beta^{\tau-t}\frac{\Lambda_\tau}{\Lambda_t}$ is the common discount factor for real profit between period τ and t, with $\Lambda_t = \int \Lambda_{it} di$ is the average marginal utility of consumption for all households.[5]

The FOCs for cost minimization are summarized by the following two equations:

$$\frac{L_{jt}}{K_{jt}} = \frac{Z_t}{W_t} \frac{1-\alpha_t}{\alpha_t}, \qquad j \in [0, 1] \tag{15}$$

$$\frac{MC_\tau}{P_\tau} = \frac{RW_t}{MPL_t} \tag{16}$$

where MPL_t is the marginal productivity of labor, which is identical for all firms due to the CRS production function.

2.2.1 Price Setting

Analogously in the labor market, I assume that the demand function for firm j's output Y_{jt} is of the Dixit-Stiglitz form, and that the firm is obliged to set its product supply equal to demand:

$$Y_{jt} = \left(\frac{P_{jt}}{P_t}\right)^{\frac{1}{\theta_Y - 1}} Y_t \tag{17}$$

[5] See also Kim (2000) for the discussion on the common discount factor across firms.

where $\theta_Y \in [0,1]$ measures the degree of firm j's market power over its own product, P_{jt} is the price of the individual firm j's product, Y_t is the aggregate demand by households. In goods market equilibrium, the total output demand Y_t is in turn equal to the aggregate supply $(\int Y_{jt}^{\theta_Y} dj)^{\frac{1}{\theta_Y}}$, the CES index constructed from differentiated products. The aggregate price level P_t is in turn defined as

$$P_t = (\int P_{jt}^{\frac{\theta_Y}{\theta_Y - 1}} di)^{\frac{\theta_Y - 1}{\theta_Y}} \tag{18}$$

Nominal rigidity in the goods market is formulated as in the labor market: In each period t, a randomly chosen $1-\phi_Y$ fraction of firms, denoted by *opt(t)*, are able to reoptimize their individual nominal prices. The other ϕ_Y fraction of firms, denoted by *rev(t)*, are assumed to reset their prices according to an index rule

$$P_{jt} = \Pi_{t-1} P_{j,t-1}, \qquad j \in rev(t) \tag{19}$$

where Π_{t-1} is the actual aggregate inflation rate in the last period. Therefore, even if a firm is able to reoptimize in period t, its current price decision continues to affect profit in period $\tau \geq t$ with probability $\phi_Y^{\tau - t}$.

Using equation (17) and (19), I derive the following FOC for a firm $j \in opt(t)$ to set its optimal price:

$$\psi_t E_t \left[\sum_{\tau=t}^{\infty} (\beta\phi_Y)^{\tau-t} \frac{\Lambda_\tau}{\Lambda_t} \left(\frac{\Pi_t}{\Pi_\tau} \right)^{\frac{\theta_Y}{\theta_Y - 1}} Y_\tau \right] = \frac{1}{\theta_Y} E_t \left[\sum_{\tau=t}^{\infty} (\beta\phi_Y)^{\tau-t} \frac{\Lambda_\tau}{\Lambda_t} \left(\frac{\Pi_t}{\Pi_\tau} \right)^{\frac{1}{\theta_Y - 1}} \frac{RW_\tau}{MPL_t} Y_\tau \right] \tag{20}$$

where $\psi_t = P_t^* / P_t$ is the optimal price relative to the aggregate.[6]

2.3 Government

The behavior of government is described by its own budget constraint and monetary policy equations. The government is assumed to maintain balanced budget every period by financing the total lump-sum payment to households with the seigniorage gain and issuance of net debt:

$$T_t = M_t - M_{t-1} + B_t - R_{t-1} B_{t-1} \tag{21}$$

where $T_t = \int_0^1 T_{it} di$, $M_t = \int_0^1 M_{it} di$, and $B_t = \int_0^1 B_{it} di$.

[6] Unlike in equation (12), the two expectational terms in equation (20) do not have the firm index j. Therefore, the optimal price set each period is common to all firms.

Monetary policy is specified as a generalized feedback rule of Taylor (1993)

$$\log \frac{R_t}{\overline{R}} = \rho_R \log \frac{R_{t-1}}{\overline{R}} + (1-\rho_R)\left[\gamma_\pi \log \frac{\Pi_t}{\overline{\Pi}} + \gamma_y \log \frac{Y_t}{\overline{Y}_t} + \gamma_m \log \frac{MG_t}{\overline{MG}}\right] + \varepsilon_{Rt},\ 0 < \rho_R < 1 \quad (21)$$

where $\overline{R}$ is the gross nominal interest rate, $\overline{MG}$ is the growth rate of nominal money, $\overline{R}$ is the steady state gross nominal interest rate, all in the steady state. Π_t is the rate of gross inflation rate between period $t-1$ and t, and $\overline{Y}_t$ is the deterministic level of output at period t, respectively. $\overline{\Pi}$ is the long-run "target" level of inflation rate.[7] The monetary policy disturbance ε_{Mt} is a white noise with mean 0 and variance σ_ε^2 and independent of all other disturbances in the model.

I now discuss the stochastic structure of the model. I put the economy subject to six structural disturbances. More specifically, beside the monetary policy disturbance ε_{Rt}, the model is driven by stochastic evolution of five shocks $(A_t, \alpha_t, \delta_t, b_t, a_t)$,each of which follows a stationary AR(1) in logarithmic form

$$\log \frac{\chi_t}{\overline{\chi}} = \rho_1 \log \frac{\chi_{t-1}}{\overline{\chi}} + +\varepsilon_{\chi t} \quad (22)$$

where $\overline{\chi}$ is the steady state level of χ_t, and $\varepsilon_{\chi t}$ is a white noise with mean 0 and variance σ_χ^2. I allow the innovations in A_t and α_t, the two productivity shocks, are correlated with each other but uncorrelated with those in (δ_t, b_t, a_t). Innovations is the latter three shocks are uncorrelated with one another.

The final equations completing the model are those for the evolution of aggregate price and wage rate. In a symmetric equilibrium, they evolve as

$$P_t^{\frac{\theta_Y}{\theta_Y - 1}} = (1-\phi_Y) P_t^{*\frac{\theta_Y}{\theta_Y - 1}} + \phi_Y \Pi_{t-1} P_{t-1}^{\frac{\theta_Y}{\theta_Y - 1}} \quad (23)$$

and

$$W_t^{\frac{\theta_L}{\theta_L - 1}} = (1-\phi_L) W_t^{*\frac{\theta_L}{\theta_L - 1}} + \phi_L \Pi_{t-1}^{\omega} W_{t-1}^{\frac{\theta_L}{\theta_L - 1}}\ . \quad (24)$$

In what follows is focused on a particular *symmetric* equilibrium in which i) all firms (or households) in $opt(t)$ set the same optimal price (or wage); and ii) all households make identical decision on (C, K, M, B). In such a equilibrium, most of the model's real and nominal variables inherit deterministic trends due to the constant rate of labor -augmenting technical progress (g) and the

[7]Since output and price have deteministic trends, their steady state values grow at constant rates. See footnote (12) and Appendix for further discussion.

"target" inflation rate of the monetary authority.[8] By dividing each variable by its deterministic gross rate of growth, I make the system stationary. We use lowercase letters for stationary transformed variables. For example, the output, price, and wage rate are transformed as:

$$y_t = Y_t/g^t, p_t = P_t/\overline{\Pi}^t, w_t = W_t/\left(\overline{\Pi}^{\omega}\right)^t \ .$$

Equations describing the stationary-transformed symmetric equilibrium in the model are given in the Appendix.

3 Methodology and Data

3.1 Maximum Likelihood Estimation

If log-linearized around the deterministic steady state, the stationary-transformed system is cast into the form

$$G_0 d\log x_t = G_1 d\log x_{t-1} + G_2\varepsilon_t + \Psi\eta_t \tag{25}$$

where $d\text{log}x_t = \log\ x_t - \log\overline{x}$, and x_t is the n-dimensional vector of starionary-transformed system variables. ε_t is the vector of innovations in exogenous disturbances, and η_t is a vector of endogenous errors satisfying $E_{t-1}\eta_t = 0$. The matrices (G_0, G_1, G_2) of the derivatives of $G(\cdot,\cdot,\cdot)$ are with respect to $\log x_t$, $\log x_{t-1}$, and ε_t, respectively, evaluated at the steady state.

The log-linearized system (25) is solved using the method by Sims (2000). If there exists a unique equilibrium, the solution takes the form

$$d\log x_t = F_1 d\log x_{t-1} + F_2\varepsilon_t \tag{26}$$

where F_1 and F_2 are complicated matrix functions of the model parameters.

If equation (26) is seen as a transition equation for the "state" variable x_t, constructing likelihood functions is a straightforward application of Kalman filtering. With a selection matrix H that singles out of the observables out of the state vector x_t, I have the following state-space representation:

$$\boxed{\begin{array}{lc} \text{transition equation :} & d\log x_t = F_1 d\log x_{t-1} + F_2\varepsilon_t, \quad \varepsilon_t \backsim iiN(0,\Sigma_\varepsilon) \\ \text{observation equation :} & d\log z_t = H d\log x_t \end{array}} \tag{27}$$

[8] In more detail, in the steady state *i)* Y, K, and C grow at a rate of g; *ii)* P and Z grow at a rate of $\overline{\Pi}$; *iii)* M and W grow at a rate of $\overline{\Pi}g$; and *iv)* Λ grows at a rate of $g^{-1+a(1-\sigma)}$. All other variables, including R and L, are constant in the steady state.

where z_t denotes the variables corresponding to the observable data series and Σ_ε is the (block diagonal) covariance matrix of the innovations. The Gaussian likelihood function for the entire parameter vector Θ is constructed as

$$L_T(\Theta \mid z_1, ..., z_T) = -\frac{1}{2}\sum_{t=1}^{T} \log |{}_{t-1}\Sigma_t^z(\Theta)| \\ -\frac{1}{2}\sum_{t=1}^{T} [d\log z_t - {}_{t-1}\, d\log z_t(\Theta)]' [{}_{t-1}\Sigma_t^z(\Theta)]^{-1} [d\log z_t - {}_{t-1}\, d\log z_t(\Theta)] \qquad (28)$$

where ${}_{t-1}d\log z_t(\Theta)$ and ${}_{t-1}\Sigma_t^z\ (\Theta)$ are one step ahead forecasts of mean and variance of $d\log z_t$, respectively.[9] The likelihood function in equation (29) is maximized over the whole parameter set Θ. The asymptotic standard errors of the estimates are computed from the Hessian matrix of the log-likelihood function evaluated at the estimates.

Other than the economic restrictions on a few parameters imposed above, maximization of (29) over the parameter set Θ requires one to cope with the parameter regions in which *i)* the candidate value of $\theta \in \Theta$ yield nonsensical (mainly negative) steady state values of the variables; and *ii)* the model does not have a unique equilibrium. The usual constrained optimization routines cannot be used to deal with such nonstandard situations, because the implied restrictions cannot be written in analytic forms. As in Leeper and Sims (1994), I assign an arbitrary low likelihood value to parameters in such bad regions, and the resulting discontinuity in the likelihood function is addressed by a "cliff-robust" optimization routine written by Sims.[10]

Another issue around constructing the likelihood function is how to initialize the Kalman filter. I assume that the initial information set is null, and therefore the initial one step ahead forecasts of mean and variance of $d\log z_1$ are equal to the unconditional expectation and variance:

$${}_0d\log z_1 = E[d\log z_1] = 0, \quad {}_{t-1}\Sigma_t^z = var\,[d\log z_1] = H\Sigma^x H'$$

where Σ^x is the unconditional variance of $d\log x_t$ determined by the Lyapunov equation

$$\Sigma_x = F_1\Sigma_x F_1' + F_2\Sigma_\varepsilon F_2' \ .$$

[9] More specifically, the evolution of $\left({}_{t-1}\widehat{d\log z}_t, {}_{t-1}\Sigma_t^z\right)$ is summarized as follows:

$$\begin{aligned} {}_td\log x_{t+1} &= F_1\, {}_{t-1}d\log x_t + F_1K_t(d\log z_t - H\ {}_{t-1}d\log x_t)\ , \\ {}_td\log z_{t+1} &= H\ {}_td\log x_{t+1}, \\ {}_t\Sigma_{t+1}^x &= F_1(I-K_t)\,{}_{t-1}\Sigma_t^x F_1' + F_2\Sigma_\varepsilon F_2', \\ {}_t\Sigma_{t+1}^z &= H_t\Sigma_{t+1}^x H', \end{aligned}$$

where $K_t = {}_{t-1}\Sigma_t^x H'(H_{t-1}\Sigma_t^x H')^{-1}$ is the time-varying Kalman gain matrix.

[10] The routine is **csmiwel.m** written in a Matlab M-file.

3.2 Preliminary Calibrations and Data

Since the data series bears little information about some structural parameters, they are fixed before estimation: steady state values of capital share $\overline{\alpha}$ and depreciation $\overline{\delta}$ are fixed at 1/3 and 0.025, respectively. The market power θ_Y in the goods market is fixed at the conventionally calibrated value of 0.9, because only two of $(\overline{A}, \theta_Y, \theta_L)$ are identified from the series on output and labor. Assuming the Fed has been successfully managed the inflation rate around its "target" level, I fix the steady state inflation rate $\overline{\Pi}$ at its actual average 1.01005 over the sample period. The CRRA parameter σ is fixed at 1, which amounts to a logarithmic instantaneous utility function.

Two parameters (ν, b) are of particular interest, because they jointly determine the form of money demand or equivalently the transaction technology.[11] Figure 1 illustrates how. In the upper panel are drawn the isovelocity curves, on which two parameters (ν, b) yield constant consumption velocities of money $V = PC/M$ in steady state. The isovelocity curves are defined by

$$\overline{V} = b^{\frac{1}{\nu-1}}(1 - \overline{R}^{-1})^{\frac{1}{1-\nu}} \tag{8'}$$

given the level of nominal rate $\overline{R}$. Among the three isovelocity curves in Figure 1, the one with V =1.1321 is drawn for the actual average velocity over the sample period. The arrow denotes the particular point on that isovelocity curve corresponding to the estimates of (ν, b) reported in Table 1.

The positive slopes of the isovelocity curves are interpreted as follows: suppose ν increases form a point on an isovelocity curve. Higher ν makes consumption and real balance more substitutable, and leads to lower money demand and higher velocity. Such increase in velocity should be counteracted by higher value of money demand parameter b.

The lower panel plots transaction costs $TC = \left[1 - \frac{b}{b+\overline{V}}\right]^{\frac{1}{\nu}}$, evaluated along the isovelocity curves in the upper panel.[12] Note that in the neighborhood of the estimated (ν, b), transaction costs are close to 1, implying that transaction costs, or the components of them that can be

[11] As discussed in Feenstra (1986), exactly the same model can be described in terms of transaction costs approach: one can redefine C^* in the utility function (2) as the usual consumption and replacing C in the budeget constraint (3) with

$$C^{**} = C^* \times \left[1 - b\left(\frac{M/P}{C^*}\right)^{\nu}\right]^{\frac{1}{\nu}}$$

where C^{**} represents the gross spending on consumption inclusive of (multiplicative) transcation costs. Note that the tranasaction costs are increasing in consumption velocity.

[12] More specifically, TC represents C^{**}/C^* in footnote 10.

eliminated by saturating the economy with nominal money is a very small fraction of total output. The positive slopes of transaction cost curves reflect higher money demand needed to maintain constant velocity for higher levels of ν. It is also displayed that, for a fixed ν, higher velocity leads to higher transaction cost or equivalently, greater level of money demand parameter b.

The monotonicity of the relation between (ν, b) along an isovelocity curve is utilized to implement estimation and calibration jointly. More specifically, at each step of maximizing the likelihood function, b is determined as

$$b = (1 - \overline{R}^{-1})V_d^{\nu-1} \tag{8"}$$

given all other candidate parameters, where V_d is set equal to the actual average velocity over the sample period.

The raw data used in this study are extracted from DRI BASIC economic series for the sample period 1959:Q1-1999:Q3.[13] Since two main feature of the model are *i*) the nominal rigidities in goods and labor markets; and *ii*) interest rate feedback rule for monetary policy, it is imperative to examine the data on monetary aggregates as well as prices and quantities in goods and labor markets. Therefore, the following six series are used for the actual estimation purpose: per capita output (Y), per capita labor hours(L), rate of price inflation (Π), the growth rate of per capita money balance (MG), interest rates (R), and wage inflation rates (Π^w). To express the data series conformable with the theoretic counterparts in the model, per capita output and money balance are obtained by dividing GDP and M2 balance, respectively, by population size. Per capita labor hours are obtained by dividing weekly working hours by 120, under the assumption that each worker is endowed with 5 working days per week. The resulting series imply households devote 33.8% of their time endowment to working. Since federal funds rates are measured in annual percentage rates, I transform them into quarterly rates by dividing by 400 and adding one. Price and wage inflations are obtained by log-differencing the price and wage series. The resulting six

[13] All series, except for interest rate and wage, are seasonally adjusted.

output :	gross domestic products, billions of 1992 dollars.
employment :	average weekly hours of production workers in manufacturing sector.
price :	implicit price deflator for gross national products.
money :	M2 stock, billions of current dollars.
interest rate :	federal funds rate, per annum.
wage :	index of compensation per hour in nonfarm business sector, 1982=100.
population	civilian population, in thousands.

series are plotted in Figure 2, against their estimated steady state levels.

4 Empirical Results

4.1 Benchmark Estimates

Table I reports the functional forms of the model equations and the maximum likelihood estimates of parameters, whose standard errors are given in parentheses.[14]The growth rate g of the real output is 1.0056, which is higher than the actual average growth rate of output 1.0050 over the sample period. The estimate 0.9986 of discount factor β is sharply estimated with standard deviation 0.0003. The estimate of β is higher than the calibrated value 0.9898 of Chari et al.(2000), but comparable to the estimate 0.999 of Kim ([16], 2000) for 1959:Q1-1995:Q1 period. The share a of consumption bundle C_t^* in the instantaneous utility function is 0.4681, higher than the conventionally calibrated value of 0.4. The estimate -22.7561 of ν is much higher than the estimate -7.4459 of Ireland (2001) for 1979:Q3-1998:Q4. The estimate of b is 0.0008, giving a very small weight on real balance in the utility function.

The market power θ_L of households in the labor market is 0.6888. It is lower than the estimate 0.8821 of Amato and Laubach (1999) and the calibrated value 0.75 of Huang and Liu (1999). The estimate 16.8465 of capital adjustment cost ϕ_K shows a considerable degree of real rigidity in the economy. More specifically, setting off the estimated steady state, the transformation of one unit of consumption good into the same unit of working capital requires additional 0.0680 units of additional output.

The parameters for the monetary policy rule show the systematic manipulation of monetary aggregates in response to inflation, but not to output over the sample period. The estimate $\rho_R = 0.1395$ implies a small degree of policy inertia, compared with Rotemberg and Woodford (1999) and Woodford (1999).[15]

I now turn to the estimates of nominal rigidity parameters. The degree of nominal rigidities in the labor market ($\phi_L = 0.7333$ is higher than that in the goods market ($\phi_Y = 0.4052$) : the resulting expected duration of wage and price fixities are 3.75 quarters and 1.68 quarters,

[14]Standrad erors are computed from the Hessian of the likelihood function, evaluated at the maximum likelihood estimates.

[15]In the context of generalized Taylor rule in which interest rate depends upon its lags as well, Woodford (2000) shows that small but persistenct changes in short-term interest rates in response to shocks constitute an optimal policy, because such inertial responses allow a larger effect of monetary policy on long rates and aggreagte demand, for a given degree of overall interest rate variability.

respectively. Both parameters are precisely estimated with respective standard deviations 0.0297 and 0.0234.

Regarding the structural disturbances, the estimated AR(1) coefficients show the economy has been subject to highly persistent structural shocks. Except for the labor supply shock, the half-lives of the aggregate shocks are around 6 years. The labor supply shock a_t exhibits negative serial correlations. Finally, the innovations in the shocks A_t and a_t are negatively correlated with correlation coefficient -0.9775. One "structural" interpretation for this negative correlation can be drawn from Christiano et al. (2001) who assume variable capital utilization: in the presence of cyclical capital utilization, positive productivity shock will increase the effective marginal productivity of labor, and leads to higher amount of labor employed.

In view of the VAR structure of the linearized solution of the model in (26), the likelihood function has a straightforward interpretation as a measure of the mean squared errors of forecasts. Therefore, by comparing the likelihood values, one can assess the fit of the estimated model relative to unrestricted VARs fitted to the same data. In Table 2A, the likelihood value of the estimated model is compared with those of two VARs with constants of order 1, one with time trend and the other without it.[16]

The likelihood values of the model at the best fit is 4328.2055, quite a bit lower than the 4749.3752 and 4724.6128 of the VARs with and without trend, respectively. Since the two VARs in this case have 69 and 63 free parameters compared with 25 for the model, it would be worthwhile to see how the model's fit compares to those of the VARs against the Schwarz criterion

$$BIC = L_T(\widehat{\Theta} \mid z_1, ..., z_T) - \frac{p}{2} \log T \tag{29}$$

where p denotes the number of estimated parameters. As shown in the lower panel of Table 2A, however, the fit of the model leaves plenty of room of improvement even in this criterion. For example, despite being penalized for the additional 44 parameters, the VAR with trend yields the BIC of 4574.4980, much higher than the 4264.8442 of the model.

Another way to examine the estimation results is to compare the means and standard deviations for the observed series with those predicted by the model, as reported in Table 2B. Setting aside the inflation rate whose steady state level is preset equal to the data average, the estimated

[16] To allow a more direct comparison, I calculate the unconditional likelihood values of the two VARs, using the impled unconditional distributions for the initial observations.

steady states of the other five variables closely match their sample averages.[17] The main differences are in the second moments: for examples, the model overpredicts the standard deviations of output and labor hours by factors of 1.81 and 1.93, respectively.[18]

To illustrate how the estimated model works, Figure 2 displays the impulse response functions of the estimated model toward unit increase in monetary policy shock. Following such expansionary nominal shock, money stock increases and nominal interest rate decreases initially as displayed in panel (g) and (h). The real economy is boosted up, yielding about 2% initial increase in output in panel (a) beyond steady state level. The "built-in" stabilization tendency of the monetary policy rule, however, pushes the real output back toward its steady state level 4 quarters after the shock. This feature is also reflected in the responses of interest rate in panel (h), where the initial hike in the interest rate is almost completely dampened out in less than one year.

In Panel (c) and (e), price and wage display gradual adjustments to their new steady state levels, even if the money stock shows considerable initial hike followed by fast adjustments to steady state level. The nominal rigidities in goods and labor markets exhibit themselves in initial increases followed by slow dampening in the price and wage inflation rates in panel (d) and (f), respectively. As expected from the higher degree of nominal rigidity in the labor market, nominal wage shows more sluggish adjustment after the shock, both in level and growth rate, than does nominal price. Such a naive interpretation may be misleading, however, because the monetary feedback rule has price inflation as an indicator, forcing faster adjustment of price inflation. Therefore, as in Chari et al. (2000), I also consider the effects of unit increase in money growth shock.[19] The dashed lines in panel (d) and (f) represent the responses of price and wage inflation toward such shock: the hump-shaped responses of wage inflation is more pronounced under this shock. I hereby conclude more sluggish adjustments in wage rate are not merely due to the specific form of monetary policy rule considered.[20]

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[17] In calculating data counterparts, output series is stationary transformed via its average growth rate.

[18] In a calibration exercise, Ellison and Scott (2000) critcize this "overvolatility of output" as a failure of staggered price contracts in replicating typical business cycle fatures.

[19] More specifically, I consider the following monetary policy

$$\widehat{MG}_t = 0.57\widehat{MG}_{t-1} + \varepsilon_t \quad .$$

[20] It is in order to note that higher degree of wage rigidity than of price rigidity yields weakly contercyclical repsonses of real wage, as criticized by those who favor staggered price contracts to wage contracts.

4.2 Which Rigidity?

The general picture that emerges from Table 1 and Figure 2 is consistent with the consensus that wage stickiness is crucial in generating real persistence of monetary disturbances. To assess how strongly the "dominance" of wage rigidity is supported by the data, two exercises are done below: First, I set $\phi_Y = 0$ in the estimated version of the benchmark model and see if there are significant qualitative difference in the impulse response functions. Second, I re-estimated model parameters subject to the constraint of $\phi_Y = 0$, and examine the point estimates and implied impulse responses.

Figure 3 displays the impulse responses where ϕ_Y is set to be 0 and other parameters are held at their benchmark estimates. As expected, the qualitative features of the benchmark model is preserved, while *i*) the initial hikes in output and money stock are less conspicuous; and *ii*) inflation and price level shows higher initial hikes and faster adjustment due to complete price flexibility. Interestingly, the responses of nominal wage and its growth rate are not sensitive to such change in ϕ_Y. As shown in panel (b), however, the countercyclicality of real wage is more conspicuous because wage stickiness is now the only source of nominal rigidity.

Table 3 reports the parameters re-estimated under the restriction $\phi_Y = 0$. In fact, the estimation results in Table 1 and Table 3 shows that the price rigidity parameter ϕ_Y is statistically significant: it has a significant $t-$value in Table 1, and the likelihood ratio test statistic is $2\times(L_u - L_c)$ =2×(4328.2005-4296.5478)=63.3054> $\chi^2(1, 0.01) = 6.63$. It is worth noting, however, that the estimates in Table 3 are as a whole robust to the alternative specification of the goods market nominal rigidity, with some increase in the estimate of ϕ_L. The now higher estimate of ϕ_L is intuitively sensible, because the restriction will shift part of the goods market rigidity to labor market. It is worth noting that the increase in ϕ_L is less than half of the decrease (to 0) in ϕ_Y : as shown in Kim (2000 [15]), given the same degree of average durations, wage contracts yield higher degree of nominal rigidities than price contracts, because the former directly dampens the incentive to raise individual prices set by firms while the latter controls only the degree by which the increase in marginal costs are passed on to the re-optimized prices.

The impulse responses in Figure 4 reflects the robustness of the estimates with respect to the restriction: as a whole, the qualitative features in Figure 2 and 3 are preserved. Taken as a whole, the statistical significance of ϕ_Y does not imply its economic significance of equal magnitude.

5 Conclusion

In this paper, I address the question of how much and what kind of nominal rigidities are supported by the U.S. aggregate economy. For that purpose, I present and estimate a general equilibrium model embodying nominal rigidities in both price and labor markets in the form of staggered contracts. The estimation results suggest that reasonable amounts of nominal rigidities are compatible with real economy, and that stickiness in nominal wages is crucial for the model's performance. Stickiness in prices plays a relatively minor role in fitting the model with actual data.

The formally estimated snapshot of the aggregate economy is indispensable for further research. For one thing, welfare levels under alternative monetary policy rules can be evaluated at the formally estimated structural parameters. In particular, the estimated functional form of households' utility function can be used as a natural welfare metric. This research topic is pursued in a sequel of this paper.

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7 Appendix

7.1 Steady States

The deterministic steady states of the stationary transformed model are given below. To save space, we define $\beta_g = \beta g^{a(1-\sigma)-1}$ and $\delta_g = g - 1 + \delta$.

$$R = \overline{\Pi}\beta_g^{-1} \tag{30}$$

$$rz = \beta_g^{-1} - 1 + \delta \tag{31}$$

$$\frac{k}{y} = \theta_y \alpha \left[\frac{z}{p}\right]^{-1} \tag{32}$$

$$\frac{c}{y} = 1 - \frac{k}{y}\left[g - 1 + \delta\right] \tag{33}$$

$$k/L = \left[A\frac{k}{y}\right]^{\frac{1}{1-\alpha}} \tag{34}$$

$$\left[\frac{c}{c^*}\right]^{\nu} = \frac{c^{\nu}}{c^{\nu} + b \times rm^{\nu}} = \frac{1}{1 + b^{\frac{1}{1-\nu}}(1 - \frac{1}{R})^{\frac{\nu}{\nu-1}}} \tag{35}$$

$$\frac{L}{1-L} = \theta_Y \theta_L (1-\alpha)\frac{a}{1-a} D \left[\frac{c}{y}\right]^{-1} \tag{36}$$

$$\frac{w}{p} = \frac{1-a}{a}\frac{c}{\theta_Y}\frac{c}{1-L}\left[\frac{c}{c^*}\right]^{-\nu} \tag{37}$$

$$y = L \times \overline{A}^{\frac{1}{1-\alpha}} \left[\frac{k}{y}\right]^{\frac{\alpha}{1-\alpha}} \tag{38}$$

$$c = \frac{c}{y}y, \quad k = \frac{k}{y}k, \quad rm = b^{\frac{1}{1-\nu}}(1 - \frac{1}{R})^{\frac{1}{\nu-1}} \times c \tag{39}$$

$$mrs = \frac{1-a}{a}\frac{1}{1-L}C^{\nu-1}(C^{\nu} + bRM^{\nu}) \tag{40}$$

Note that three parameters $(\theta_Y, \theta_L, \overline{A})$ in equations (32), (36), and (38) are underidentified because, given other parameters, they jointly determine the steady state level output and labor.

7.2 Equations Describing an Equilibrium

The stationary-transformed version of the households' block of the system is given below. For notational simplicity, we define two variables : $x_t = \left[\frac{gk_t}{k_{t-1}} - 1 + \delta_{t-1} - \delta_g\right]$ and $\beta_L = \beta\phi_L g^{a(1-\sigma)}$

$$\lambda_t := \int_{[0,1]} \lambda_{it} di = a[c_t^{\nu} + b_t (m_t/p_t)^{\nu}]^{\frac{a-a\sigma-\nu}{\nu}} c_t^{\nu-1} \int_{[0,1]} (1 - L_{it})^{(1-a_t)(1-\sigma)} di \tag{41}$$

$$1 - (1 + R_t)^{-1} = b_t (c_t/rm_t)^{1-\nu} \tag{42}$$

$$\lambda_t [1 + \phi_K x_{t+1}] = \beta_g E_t \left\{\lambda_{t+1}\left[1 - \delta_{t+1} + rz_{t+1} - \frac{\phi_K}{2} x_{t+2}^2 + \phi_K \frac{gk_{t+2}}{k_{t+1}} x_{t+2}\right]\right\} \tag{43}$$

$$\frac{\lambda_t}{p_t} = \frac{\beta_g}{\overline{\Pi}} R_t E_t \frac{\lambda_{t+1}}{p_{t+1}} \tag{44}$$

$$\begin{aligned} & a\theta_L E_t \left[\sum_{\tau=t}^{\infty} (\beta_L)^{\tau-t} c_{i\tau}^{*a(1-\sigma)-1} c_{i\tau}^{\nu-1} \left(1 - (\frac{\varpi_t \Pi_t^{\omega}}{\Pi_\tau^{\omega}})^{\frac{1}{\theta_L - 1}} L_\tau\right)^{(1-a_t)(1-\sigma)} rw_\tau (\frac{\varpi_t \Pi_t^{\omega}}{\Pi_\tau^{\omega}})^{\frac{\theta_L}{\theta_L - 1}} L_\tau\right] \\ = \; & E_t \left[\sum_{\tau=t}^{\infty} (\beta_L)^{\tau-t} (1 - a_\tau) c_{i\tau}^{*a(1-\sigma)} \left(1 - (\frac{\varpi_t \Pi_t^{\omega}}{\Pi_\tau^{\omega}})^{\frac{1}{\theta_L - 1}} L_\tau\right)^{(1-a_t)(1-\sigma)-1} (\frac{\varpi_t \Pi_t^{\omega}}{\Pi_\tau^{\omega}})^{\frac{1}{\theta_L - 1}} L_\tau\right] \end{aligned} \tag{45}$$

The corresponding firms' block of the system is reported below. For notational simplicity, I define $\beta_Y = \beta\phi_Y g^{a(1-\sigma)}$:

$$y_t = A_t k_t^{\alpha_t} L_t^{1-\alpha_t} \tag{46}$$

$$L_t = \frac{1-\alpha_t}{\alpha_t} \frac{z_t}{w_t} k_t \tag{47}$$

$$\psi_t E_t \left[\sum_{\tau=t}^{\infty} (\beta\phi_Y)^{\tau-t} \frac{\lambda_\tau}{\lambda_t} \left(\frac{\Pi_t}{\Pi_\tau}\right)^{\frac{\theta_Y}{\theta_Y - 1}} y_t\right] = \frac{1}{\theta_Y} E_t \left[\sum_{\tau=t}^{\infty} (\beta_Y)^{\tau-t} \frac{\lambda_\tau}{\lambda_t} \left(\frac{\Pi_t}{\Pi_\tau}\right)^{\frac{1}{\theta_Y - 1}} \frac{rw_\tau}{mpl_t} y_\tau\right] \quad . \tag{48}$$

After combining the budget constraint of households, aggregate profit of firms, and the government budget constraint, we get the following resource constraint :

$$c_t + gk_{t+1} - (1 - \delta_t)k_t + \frac{\phi_K}{2} x_{t+1}^2 = y_t \tag{49}$$

Stationary transformation of the monetary policy rule equation is done as follows: I define the gross rate of growth in money stock $MG_t = M_t/M_{t-1}$, which is in itself stationary. Then assuming the hypothetical zero-th period observations on output is equal to its steady state level of the stationary transformed output, the stationary transform of the monetary policy rule is given by

$$\log\frac{R_t}{R} = \rho_M \log\frac{R_{t-1}}{R} + (1-\rho_M)\left[\gamma_\Pi \log\frac{\Pi_t}{\overline{\Pi}} + \gamma_m \log\frac{MG_t}{\overline{MG}} + \gamma_y \log\frac{y_t}{\overline{y}}\right] + \varepsilon_{Mt}\ . \tag{50}$$

Equations (24) and (25) for the evolution of aggregate price and wage are transformed into

$$1 = \phi_Y \left[\frac{\Pi_{t-1}}{\Pi_t}\right]^{\frac{\theta_Y}{\theta_Y - 1}} + (1-\phi_Y)\left[\psi_t\right]^{\frac{\theta_Y}{\theta_Y-1}}, \qquad 1 = \phi_L\left[\frac{\Pi^\omega_{t-1}}{\Pi^\omega_t}\right]^{\frac{\theta_L}{\theta_L-1}} + (1-\phi_L)\left[\varpi_t\right]^{\frac{\theta_L}{\theta_L-1}}\quad . \tag{51}$$

The aggregate real wage evolves as

$$g\frac{rw_t}{rw_{t-1}} = \frac{\Pi^\omega_t}{\Pi_t}\quad . \tag{52}$$

Equations for stochastic disturbances, stationary in themselves, complete the system.

7.3 Log-linearization of equation (45) and (48)

I discuss the log-linearized version of equations (45) and (48) only, since other equations are straightforward to lo-linearize.[21]

Log-linearizing equation (45) and arranging terms, I get

$$\Xi\widehat{\varpi}_t = (1-\beta_L)E_t\left\{\textstyle\sum_{\tau=t}^{\infty}\beta_L^{\tau-t}\left[(\widehat{mrs}_t - \widehat{rw}_t) - \Xi\left(\widehat{\Pi}^\omega_t - \widehat{\Pi}^\omega_\tau\right)\right]\right\} \tag{53}$$

where

$$\begin{aligned}\widehat{mrs}_t &= (1-\nu+\nu\frac{c}{c^*})\widehat{c}_t + \nu(1-\frac{c}{c^*})\widehat{rm}_t + (1-\frac{c}{c^*})\widehat{b}_t \\ &\quad + \frac{L}{1-L}\widehat{L}_t - \frac{a}{1-a}\widehat{a}_t\end{aligned} \tag{54}$$

is the log-linearized marginal rate of substitution between consumption and labor$(-\frac{\partial U_t/\partial L_t}{\lambda_t})$, and $\Xi = 1 + \frac{1}{1-\theta_L}\frac{L}{1-L}$.

The law of motion for aggregate wage rate in (51) is log-linearized as

$$\widehat{\varpi}_t = \frac{\phi_L}{1-\phi L}\left(\widehat{\Pi}^\omega_t - \widehat{\Pi}^\omega_{t-1}\right) \tag{55}$$

Combining (53) and (55), I get

$$\Xi(\frac{\phi_L}{1-\phi_L} + \beta_L)\widehat{\Pi}^\omega_t = \Xi(\frac{\phi_L}{1-\phi L})\widehat{\Pi}^\omega_{t-1} + (1-\beta_L)E_t\left\{\textstyle\sum_{\tau=t}^{\infty}\beta_L^{\tau-t}\left[(\widehat{mrs}_t - \widehat{rw}_t)\right]\right\} \tag{56}$$

$$+(1-\beta_L)E_t\left\{\textstyle\sum_{\tau=t+1}^{\infty}\beta_L^{\tau-t}\Xi\widehat{\Pi}^\omega_\tau\right\} \tag{57}$$

[21] Refer to see Kim(2000) for detailed discussion on other equations.

Using the law of iterated projection, it is straightforward to show

$$\Xi\frac{\phi_L+\beta_L}{1-\phi_L}\widehat{\Pi}_t^{\omega}=\Xi\beta_L\frac{1}{1-\phi_L}E_t\widehat{\Pi}_{t+1}^{\omega}+\frac{\phi_L}{1-\phi_L}\Xi\widehat{\Pi}_{t-1}^{\omega}+(1-\beta_L)\left[\widehat{mrs}_t-\widehat{rw}_t\right] \tag{58}$$

which is the final equation to use for solving the linearized model.

Similar steps with equations (45) and (51) yields the following equation describing the evolution of inflation rate :

$$\frac{\phi_Y+\beta_Y}{1-\phi_Y}\widehat{\Pi}_t=\beta_Y\frac{1}{1-\phi_Y}E_t\widehat{\Pi}_{t+1}+\frac{\phi_Y}{1-\phi_Y}\widehat{\Pi}_{t-1}+(1-\beta_Y)\left[\widehat{rw}_t-\widehat{mpl}_t\right] \tag{59}$$

where

$$\widehat{mpl}_t=\widehat{A}_t+\alpha\widehat{k}_t-\alpha\widehat{L}_t+\left[\alpha\log\frac{k}{L}-\frac{\alpha}{1-\alpha}\right]\widehat{\alpha}_t$$

is the log-linearized marginal productivity of labor.

Table 1: Functions and Parameter Estimates

$L_u = 4328.2055$	
$Y_t = A_t K_t^{\alpha_t}(g^t L_t)^{1-\alpha_t}$	$A = 5.5668(0.0718),\ \ g = 1.0056(8.8 \times 10^{-5})$
$\beta^t U(C_t, L_t, \frac{M_t}{P_t}) = \beta^t \log\left[C_t^{*a}(1-L_t)^{1-a_t}\right]$	$\beta = 0.9986(0.0003),\ a = 0.4681(0.0016)$
$C_t^* = (C_t^\nu + b_t\,(M_t/P_t)^\nu)^{\frac{1}{\nu}}$	$\nu = -22.7561(0.4765), b = 0.0008(5.5 \times 10^{-5})$
$L_{it} = (\frac{W_{it}}{W_t})^{\frac{1}{\theta_L - 1}} L_t$	$\theta_L = 0.6888(0.0087)$
$AC_t^k = \frac{\phi_K}{2}(\frac{I_t}{K_t} - \frac{I}{K})^2 K_t$	$\Phi_k = 16.8456(1.5501)$
$\log\frac{R_t}{R} = \rho_R \log\frac{R_{t-1}}{R} + (1-\rho_M)\times$ $\left[\gamma_\pi \log\frac{\Pi_t}{\Pi} + \gamma_y \log\frac{Y_t}{\overline{Y}_t} + \gamma_m \log\frac{MG_t}{MG}\right]$ $+\varepsilon_{Mt}$	$\rho_R = 0.1395(0.0112), \gamma_\pi = 0.8042(0.0045)$ $\gamma_y = 4.4 \times 10^{-6}(4.5 \times 10^{-5}), \gamma_m = 0.4276(0.0187)$ $\sigma_M^2 = 4.3 \times 10^{-5}(5.2 \times 10^{-6})$
$P_t^{\frac{\theta_Y}{\theta_Y - 1}} = (1-\phi_Y) P_t^{*\frac{\theta_Y}{\theta_Y - 1}} + \phi_Y \Pi_{t-1} P_{t-1}^{\frac{\theta_Y}{\theta_Y - 1}}$	$\phi_Y = 0.4052(0.0297)$
$W_t^{\frac{\theta_L}{\theta_L - 1}} = (1-\phi_L) W_t^{*\frac{\theta_L}{\theta_L - 1}} + \phi_L \Pi_{t-1}^w W_{t-1}^{\frac{\theta_L}{\theta_L - 1}}$	$\phi_L = 0.7333(0.0234)$
$\log\frac{A_t}{A} = \rho_A \log\frac{A_{t-1}}{A} + \varepsilon_{At}$ $\log\frac{\alpha_t}{\alpha} = \rho_\alpha \log\frac{\alpha_{t-1}}{\alpha} + \varepsilon_{\alpha t}$ $\log\frac{A_t}{\delta} = \rho_\delta \log\frac{\delta_{t-1}}{\delta} + \varepsilon_{\delta t}$ $\log\frac{A_t}{A} = \rho_b \log\frac{b_{t-1}}{b} + \varepsilon_{bt}$ $\log\frac{a_t}{a} = \rho_a \log\frac{a_{t-1}}{a} + \varepsilon_{at}$	$\rho_A = 0.9761(0.0002),\ \sigma_A^2 = 0.0012(9.9 \times 10^{-5})$ $\rho_\alpha = 0.9690(0.0015),\ \sigma_\alpha^2 = 0.0003(2.7 \times 10^{-5})$ $\rho_\delta = 0.9563(0.0012),\ \sigma_\delta^2 = 0.0129(0.0021)$ $\rho_b = 0.9450(0.0022),\ \sigma_b^2 = 0.0716(0.0078)$ $\rho_a = -0.4573(0.0482),\ \sigma_a^2 = 0.2080(0.0302)$ $cov(\varepsilon_{At}, \varepsilon_{\alpha t}) = -0.0006(5.0 \times 10^{-5})$

Table 2A: Comparison of Fits

	Model P & W Stickiness	Model Wage Stickiness	VAR Trend	VAR No Trend
L_T	4328.2055	4296.5478	4749.3752	4724.6128
BIC	4264.8442	4235.7210	4574.4980	4564.9423

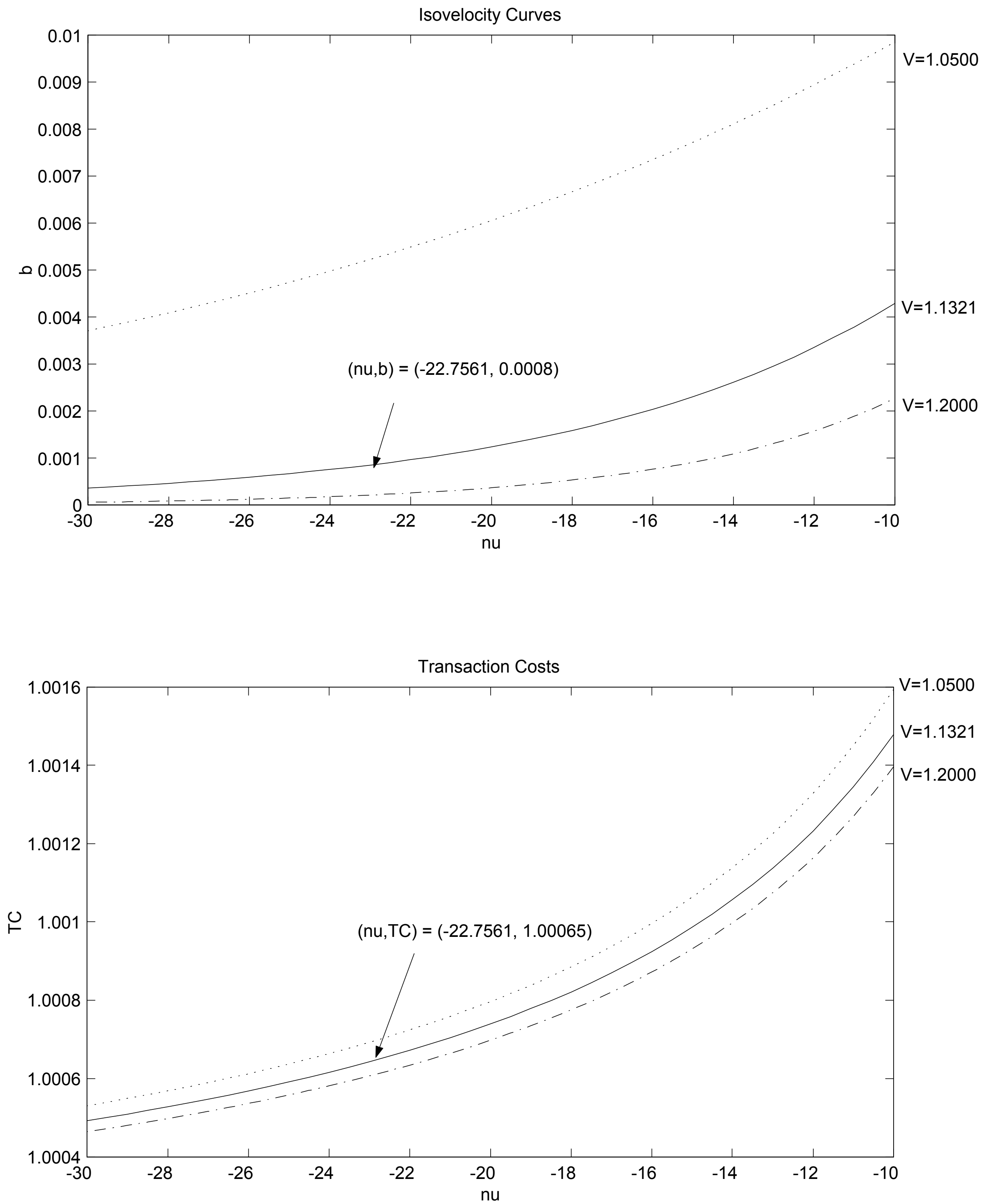
Table 2B: Model and Data Moments[22]

Series	Model Steady State	Prediction Std. dev.	U.S. Mean	Data Std. dev.
Output, y	13.4510	0.0980	13.6851	0.0451
Labor, L	0.3345	0.0338	0.3382	0.0177
Inflation, Π	1.01005	0.0071	1.01005	0.0063
Money Growth, MG	1.0157	0.0162	1.0144	0.0095
Interest Rate, R	1.0171	0.0068	1.0165	0.0080
Wage Inflation, Π^w	1.0157	0.0102	1.0142	0.0077

[22] Standard deviations are for logarithmic deviations from either deterministic steady states or sample means, where sample output series is stationary transformed via its average growth rate over the sample period.

Table 3: Constrained Estimation

$L_c = 4296.5478$		
A	5.6571	(0.0362)
g	1.0057	(1.4×10^{-5})
β	0.9988	(0.0001)
a	0.4704	(0.0013)
ν	-23.5989	(0.6351)
θ_L	0.6758	(0.0078)
b	0.0007	(6.8×10^{-5})
Φ_K	18.4476	(2.8×10^{-7})
ρ_R	0.0770	(0.0072)
γ_π	0.7716	(0.0052)
γ_y	2.3×10^{-5}	(2.3×10^{-5})
ϕ_L	0.7660	(0.0193)
ρ_A	0.9763	(0.0002)
ρ_α	0.9724	(0.0018)
ρ_δ	0.9589	(0.0015)
ρ_b	0.9499	(0.0025)
ρ_a	-0.3623	(0.0379)
σ^2_A	0.0008	(7.1×10^{-5})
σ^2_α	0.0003	(2.3×10^{-5})
σ^2_δ	0.0119	(0.0018)
σ^2_b	0.0653	(0.0032)
σ^2_a	0.2447	(0.0368)
σ^2_M	4.3×10^{-5}	(4.9×10^{-6})
$cov(\varepsilon_A, \varepsilon_\alpha)$	-0.0005	(3.8×10^{-5})



Isovelocity Curves
b
nu
V=1.0500
V=1.1321
V=1.2000
(nu,b) = (-22.7561, 0.0008)
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
-30
-28
-26
-24
-22
-20
-18
-16
-14
-12
-10
Transaction Costs
TC
nu
V=1.0500
V=1.1321
V=1.2000
(nu,TC) = (-22.7561, 1.00065)
1.0016
1.0014
1.0012
1.001
1.0008
1.0006
1.0004

Figure 1:

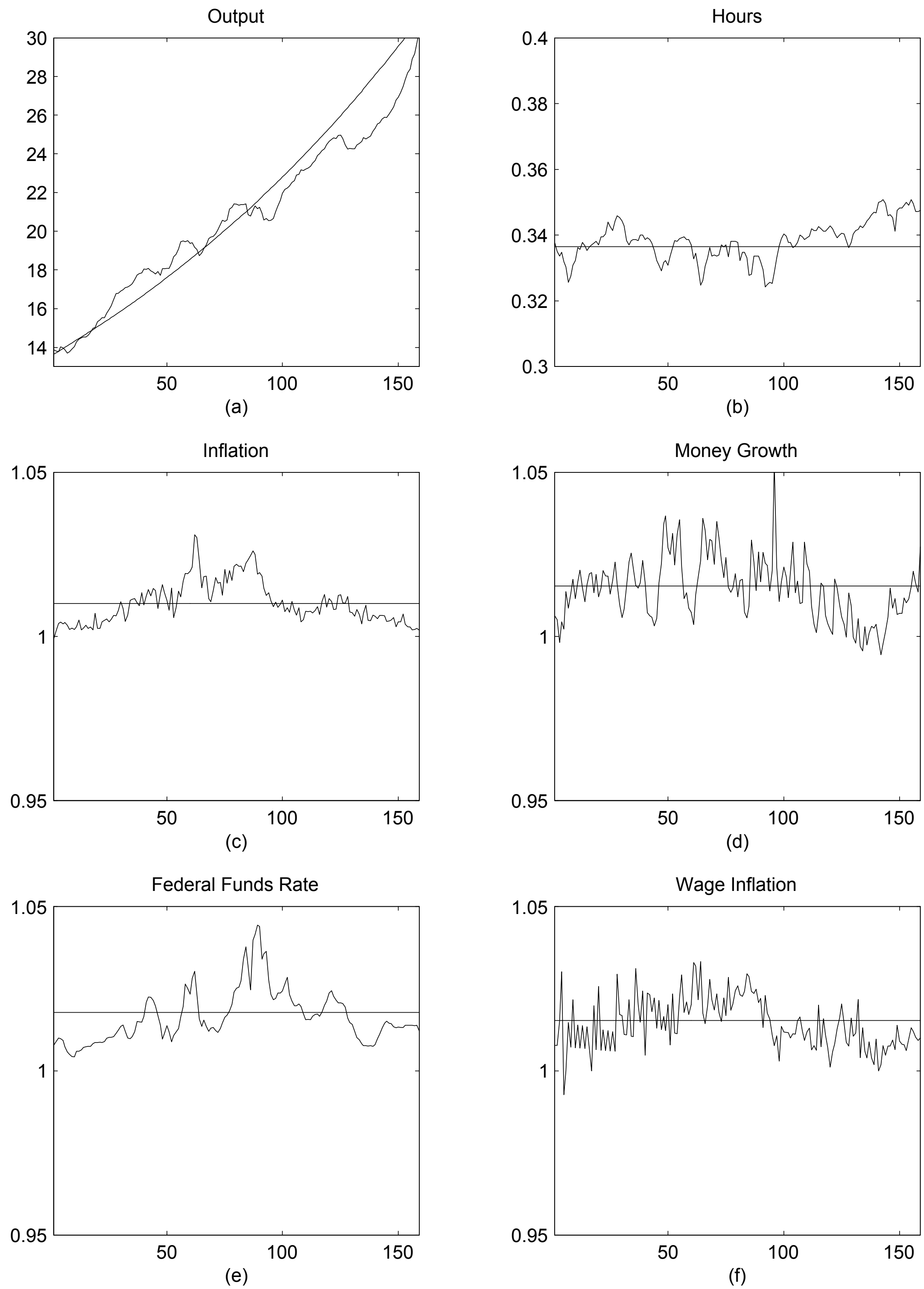


Figure 2: Data Plots

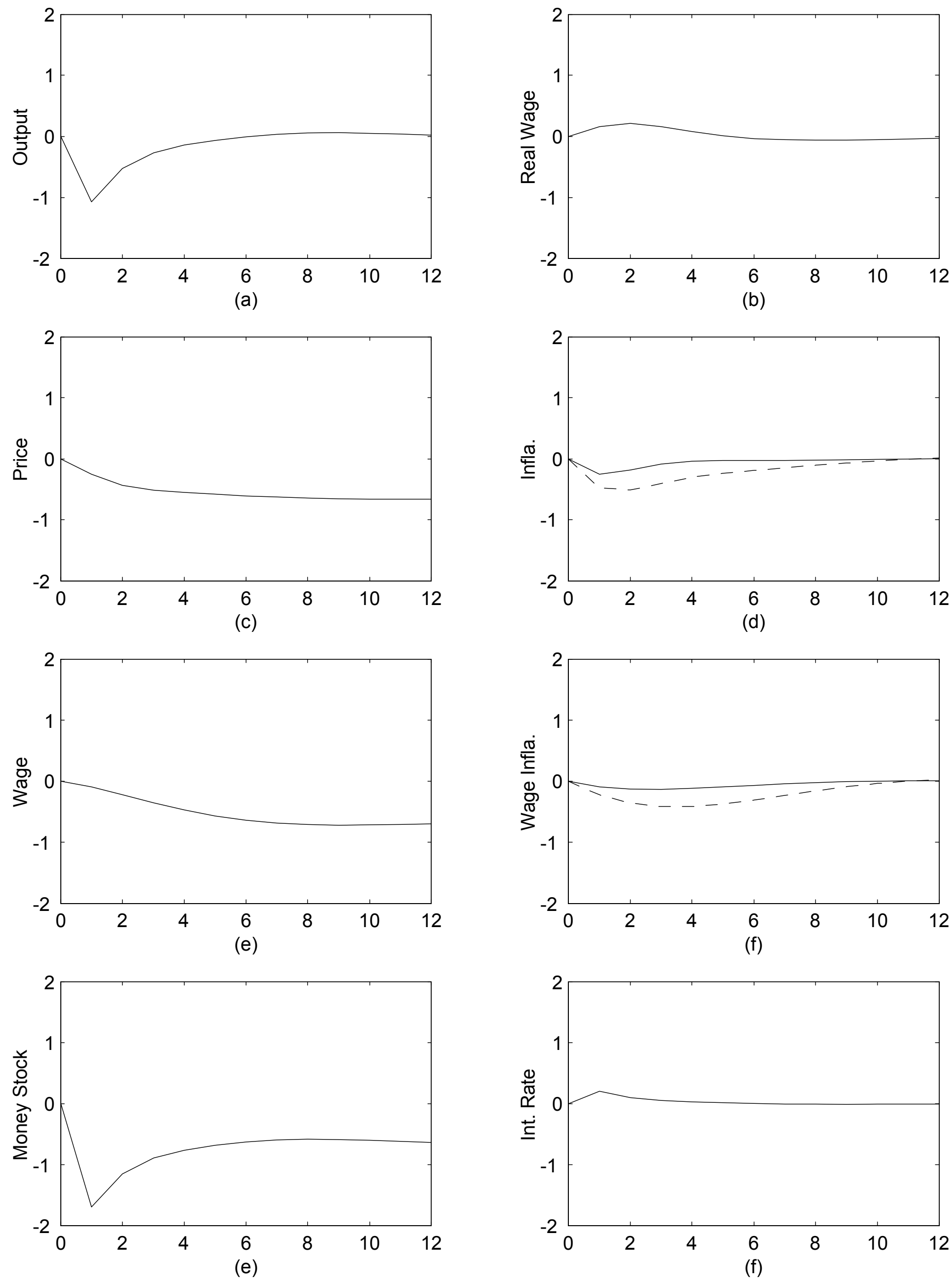


Figure 3: Estimated Impulse Responses

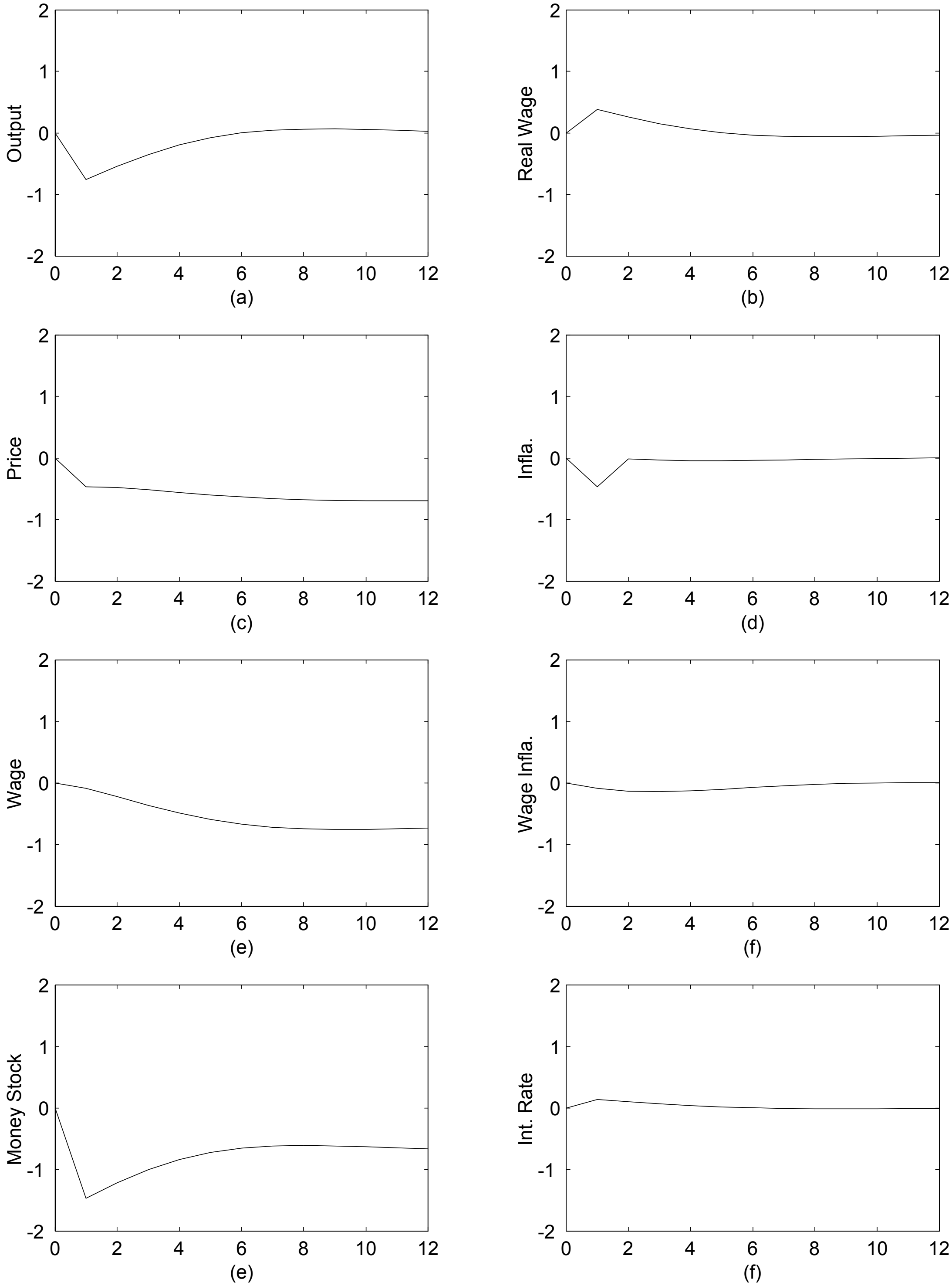


Figure 4: Constrained Impulse Responses

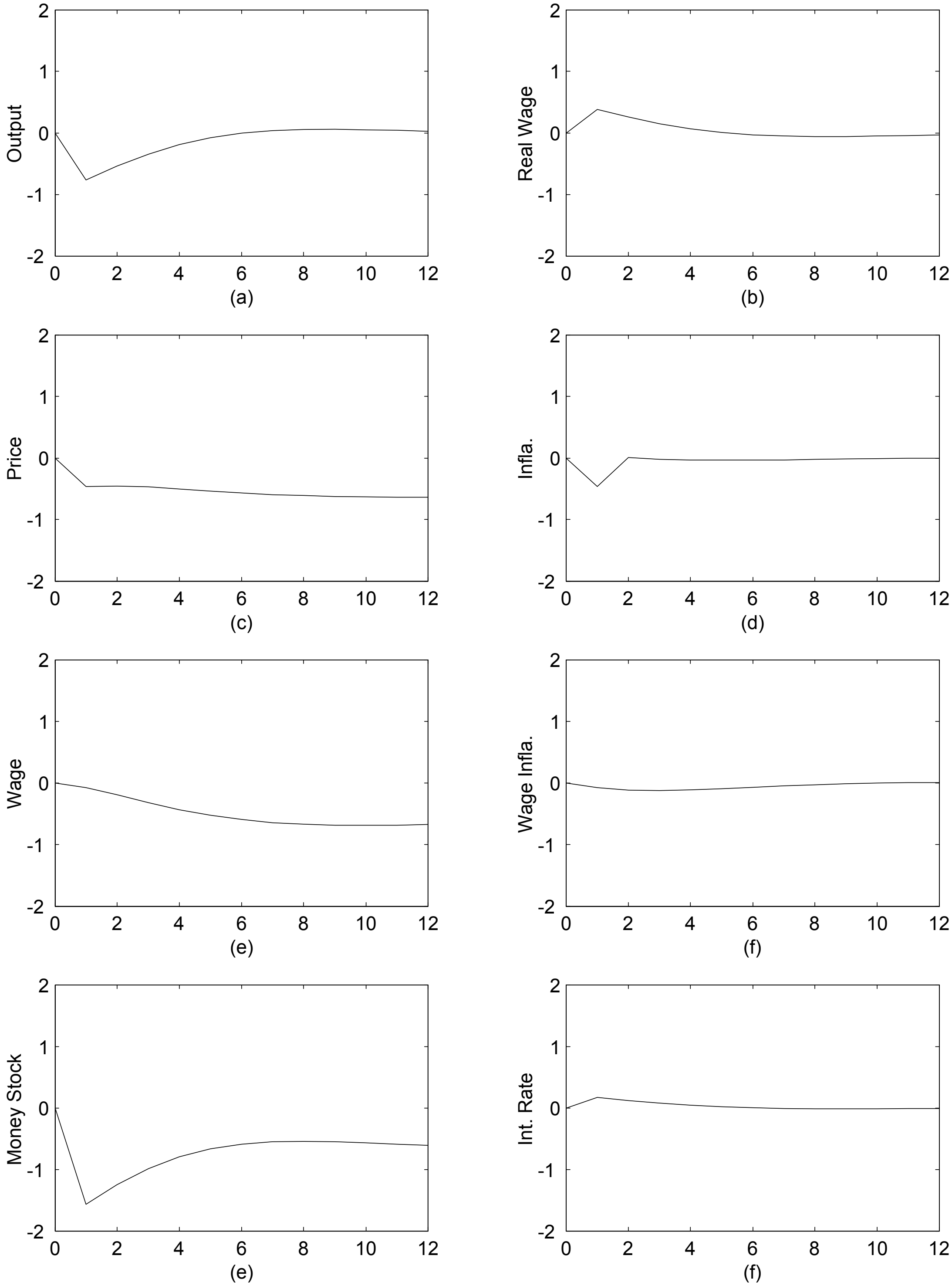


Figure 5: Re-estimated Impulse Responses