Welfare Evaluation of Monetary Policy Rules in a Model with Nominal Rigidities

Jangryoul Kim*
Federal Reserve Bank of Minneapolis

April, 2003

Abstract

This paper examines the welfare implications of monetary policy rules in a business cycle model with nominal rigidities. Rules are compared in terms of a utility-based welfare metric, and the welfare effects of the non-linear dynamics of the model are captured by a quadratic approximate solution method. Rules for fixed paths of nominal income and money outperform the fixed inflation rule, and the former rules, in turn, rank below variants of the Taylor rule. Long run deflationary rules increase welfare. The welfare maximizing rule among a class of Taylor-style rules is characterized by *i*) super-inertial adjustments in interest rates; *ii*) strong short run anti-inflation coupled with long run deflation, and *iii*) increasing interest rates in response to higher real output level and growth.

Keywords: Monetary Policy rules, Nominal Rigidites, Welfare Evaluations, Quadratic Approximate Solutions.

JEL Classification: E3, E5

*Banking and Policy Studies, Banking Supervision Department, Federal Reserve Bank of Minneapolis, 90 Hennepin avenue, Minneapolis, MN 55480-0291; phone (612) 204-5146; fax (612) 204-5114; e-mail: `jangryoul.kim@mpls.frb.org`. I thank Christopher Sims for advice and guidance. My thanks also go to William Brainard, George Hall, and Preston Miller for helpful comments and discussions. All errors are mine.

1. Introduction

Since the work of Taylor (1993), the effects of the monetary regimes on welfare and business cycles have been a key issue in monetary economics. The interest in this topic is best reflected in the plethora of research over the past decade: in the quest for "good" monetary policy rules, researchers have either proposed optimal policy rules (e.g., Clarida et al., 1999, 2001; Ireland, 1996; Khan et al., 2002; King and Wolman, 1999; Rotemberg and Woodford, 1999; Woodford, 1999) in the context of specific models, or compared the performances of alternative rules under different scenarios of the aggregate economy to provide guidelines for desirable policy rules (e.g., Henderson and Kim, 1998; McCallum and Nelson, 1999; Rudebusch and Svensson, 1999). Focusing on the overall performances of alternative rules, others (e.g., Levin et al., 1999) have examined how robust the performances of popular rules are across a wide range of structural models.

A common, if not omnipresent, limitation in the existing literature is that monetary policy rules are evaluated in terms of *ad hoc* or non-structural loss functions usually constructed from variabilities in output gap and inflation (or price level). For example, Levin et al. (1999) use the weighted average of the variances in quarterly output gap and inflation rate, and Rudebusch and Svensson (1999) in addition consider the variabilities of the changes in nominal interest rates. To address this arbitrariness, some in the literature opt to use the utility of agents as a natural metric. To name a few, Rotemberg and Woodford (1997) derive an economically interpretable loss function from the deep structure of the model economy. More specifically, in a model with price stickiness à la Calvo (1983), the authors approximate the unconditional expectation of the households' utility by a weighted average of the variabilities in aggregate inflation and output gap. Amato and Laubach (2003) and Erceg et al. (2000) extend the work of Rotemberg and Woodford (1997) in a model with both price and wage stickiness, and develop welfare measures in terms of the variabilities in aggregate price inflation, aggregate wage inflation, and output gap. Instead of approximating the expected utility with aggregate volatilities, Ghironi (2000) directly evaluates the exact unconditional expectation of the households' utility under the assumption that the system variables have lognormal distributions.

An addition to the recent literature on normative monetary economics, this paper examines the welfare implications of monetary policy rules. To that aim, this paper posits and estimates a monetary business cycle model with price and wage rigidities, subjects the estimated model to alternative monetary policy rules, and compares them in terms of a utility-based welfare metric. For correct welfare evaluations, I construct the welfare metric using a quadratic approximate solution of the model instead of the conventional log-linear approximate solutions. Despite their widespread use and good performance in fitting the model to data, log-linear approximate solution methods have been criticized by recent papers (e.g., Kim and Kim, 2002), which point out the pitfalls in measuring welfare level correctly. To capture the welfare effects of nonlinear dynamics, I use a solution method based on a quadratic approximation of the equilibrium conditions, developed by Sims (2000) and applied to a small open economy model by Kollmann (2002).

The estimated model features a higher degree of nominal rigidities in the labor market than in the goods market, and a reasonable amount of utility (or equivalently, transaction costs saved) from holding money. Given the estimated snapshot of the economy, comparison of welfare levels under different rules suggests that adopting strict inflation targeting would hardly be optimal because that rule puts the whole burden of adjustments to shocks on the wrong (i.e., stickier) variable. Fixed nominal income targeting and money growth targeting show comparable welfare performance, and are superior to fixed inflation targeting. This suggests that, given the trade-off in stabilizing price inflation and wage inflation, targeting neutral nominal anchors is a good firsthand strategy. I also find that variants of Taylor rules perform better than rules targeting fixed paths of nominal anchors, which confirms the desirability of gradual rather than instantaneous adjustments of target variables.

Based on the insights from the comparison results, I find the welfare maximizing rule among a class of Taylor-type rules for endogenous responses of nominal interest rate. The optimized rule requires *i*) super-inertial adjustments of nominal rate, *ii*) a strict anti-inflation reflected in aggressive responses to the increases in nominal anchors such as inflation, wage inflation, and money growth rate, *iii*) long-run deflation in the spirit of Friedman (1969), and *iv*) increasing interest rate in response to higher real output, both in levels and growth rates.

The paper is organized as follows. Section 2 presents and estimates the model with nominal rigidities. In section 3, I describe how to construct a natural welfare measure for policy rules evaluation. Section 4 considers the welfare features of the economy estimated in section 2. Section 5 compares the performance of three fixed targeting rules and two variants of the Taylor rule. Section 6 examines the welfare maximizing rule among a class of Taylor-style rules. Section 7 concludes the paper.

2. The Model

The economy consists of four types of agents: households, firms, government, and the aggregator. Firms produce differentiated products using capital and labor supplied by households and the aggregator, respectively. Households purchase output from the aggregator for consumption and investment purposes, and supply capital and differentiated labor to firms and the aggregator, respectively. Households and firms are obliged to satisfy demands at the wages and prices they set. Nominal rigidities in the labor and goods market take the form of adjustment costs. The aggregator uses a CRS technology of Dixit and Stiglitz to combine the differentiated goods (and labor service) into a homogenized output (and labor), and sells the resulting output (and labor) to households (and firms). Subject to its own period-by-period budget constraint, the government manages monetary policy.

2.1 Households and Wage Setting

An individual household $i \in [0,1]$ carries $M_{i,t-1}$units of nominal money, $B_{i,t-1}$ units of government bond, and K_{it} units of physical capital from the previous period. In period t, the household i earns factor income $W_{it}L_{it} + Q_t K_{it}$ from renting capital K_{it} and labor service L_{it}, where W_{it} and Q_t denote the nominal wage rate and nominal rental rate for capital, respectively. The interest income from government bond holding is $(R_{t-1} - 1)B_{i,t-1}$ where $R_{t-1}$is the gross nominal interest rate between period t-1 and t, and the dividend income from firms is$\int s_{ij}\Gamma_{ijt}dj$ where s_{ij} and Γ_{ijt} are household i's fixed share of firm j and the profit of firm j, respectively. The household also receives a lump-sum nominal transfer payment T_{it} from the government.

The household uses its funds to purchase the final good from the aggregator at the price of P_t, and divide its purchase into consumption C_{it} and investment I_{it}. In order to make new capital operational, the household needs to purchase additional materials in the amount

$$AC_{it}^{k} = \frac{\phi_K}{2}\left[\frac{I_{it}}{K_{it}} - \frac{\overline{I}}{\overline{K}}\right]^2 K_{it} \tag{1}$$

where $I_{it} = K_{i,t+1} - (1-\delta_t)K_{it}$ is the real investment spending, $\phi_K > 0$ is the scale parameter for the capital adjustment costs, and $\frac{\overline{I}}{\overline{K}}$ is the steady state ratio of investment to existing capital stock. Dubbed the *depreciation shock,* δ_t denotes the stochastic decay rate of capital stock. Its stochastic properties are specified later. The household then carries M_{it} units of nominal money, B_{it} units of government bond, and $K_{i,t+1}$ units of capital into period $t+1$. Therefore, the household is subject to the budget constraint

$$\begin{aligned} & C_{it} + K_{i,t+1} - (1-\delta_t)K_{it} + \frac{M_{it}}{P_t} - \frac{M_{i,t-1}}{P_t} + \frac{B_{it}}{P_{it}} - \frac{B_{i,t-1}}{P_t} + AC_{it}^{k} \\ \leq \;& \frac{W_{it}L_{it}}{P_t} + \frac{Q_t K_{it}}{P_t} + T_{it} + \frac{\int s_{ij}\Gamma_{ijt}dj}{P_t} + \frac{(R_{t-1}-1)B_{i,t-1}}{P_t}, \quad t \geq 0. \end{aligned} \tag{2}$$

The household maximizes lifetime utility

$$E_0[\sum_{t=0}^{\infty} \beta^t U(C_{it}^*, L_{it}, M_{it}/P_t)], \quad 0 < \beta < 1 \tag{3}$$

where the instantaneous utility function $U(\cdot)$ has the form

$$U(C_{it}^*, L_{it}, \frac{M_{it}}{P_t}) = \frac{1}{1-\sigma}\left[\left\{C_{it}^{*a}(1-L_{it})^{1-a_t}\right\}^{1-\sigma} - 1\right], \qquad 0 < \sigma, \quad 0 < a < 1, \quad \nu < 0. \tag{4}$$

In equation (3) and (4), $C_{it}^* = (C_{it}^{\nu} + b_t\,(M_{it}/P_t)^{\nu})^{\frac{1}{\nu}}$ is the CES bundle of consumption C_{it} and real money balance M_{it}/P_t. The stochastic properties of the *money demand shock* b_t and the *labor supply shock* a_t will be specified later.

Each household sells its differentiated labor service to the aggregator, who in turn uses a CRS technology

$$L_t = (\int L_{it}^{\theta_L} di)^{\frac{1}{\theta_L}} \quad , \quad \theta_L \in [0,1] \tag{5}$$

to transform the differentiated labor service into a single labor index L_t. The implied demand for household i's labor service and the aggregate wage rate W_t are

$$L_{it}^d = (\frac{W_{it}}{W_t})^{\frac{1}{\theta_L - 1}} L_t \quad , \quad W_t = (\int W_{it}^{\frac{\theta_L}{\theta_L - 1}} di)^{\frac{\theta_L - 1}{\theta_L}} \quad . \tag{6}$$

Each household is subject to quadratic costs of adjusting its nominal wage

$$AC_{it}^w = \frac{\Phi_w}{2}\left(\frac{W_{it}}{W_{i,t-1}} - \Pi_{t-1}^w\right)^2 \frac{W_t}{P_t} \tag{7}$$

where $\Pi_{t-1}^w = W_{t-1}/W_{t-2}$ is the gross wage inflation rate at period $t-1$, and $\Phi_w > 0$ is the scale parameter for the degree of nominal rigidity in the labor market. Note that the presence of lagged wage inflation in equation (7) renders sticky both aggregate wage rate and wage inflation.[1]

The first order conditions for $(C_{it}, M_{ti}, K_{i,t+1}, B_{it}, W_{it})$ are given by

$$\frac{\partial U_{it}}{\partial C_{it}} = \Lambda_{it} \tag{8}$$

[1]This specification generalizes Rotemberg (1982), and addresses the claim of Fuhrer and Moore (1995) that sticky price models should be able to generate sufficient inertia in inflation as well as in price level.

$$1 - R_t{}^{-1} = b_t(C_{it}P_t/M_{it})^{1-\nu} \tag{9}$$

$$\Lambda_{it}\left[1 + \frac{\partial AC^k_{it}}{\partial K_{i,t+1}}\right] = \beta E_t\left[\Lambda_{i,t+1}\left(Q_{t+1}/P_{t+1} + 1 - \delta_{t+1} - \frac{\partial AC^k_{i,t+1}}{\partial K_{i,t+1}}\right)\right] \tag{10}$$

$$\Lambda_{it} = \beta R_t E_t\left[\Lambda_{i,t+1}\frac{P_t}{P_{t+1}}\right] \tag{11}$$

$$\frac{\partial U_{it}}{\partial L_{it}}\frac{dL_{it}}{dW_{it}} = \left\{\Lambda_{it}\frac{\partial AC^w_{it}}{\partial W_{it}} + \beta\left[E_t\Lambda_{i,t+1}\frac{\partial AC^w_{i,t+1}}{\partial W_{it}}\right]\right\} - \Lambda_{it}\frac{1}{P_t}\frac{\partial\left(W_{it}L_{it}\right)}{\partial W_{it}} \tag{12}$$

where Λ_{it} the Lagrangian multiplier on the household i's budget constraint, interpretable as the shadow value of an additional unit of consumption.

Using equations (4), (5), and (7), I rewrite equation (12) as

$$\begin{aligned}
\left[\frac{W_{it}}{W_t}\right]^{\frac{1}{1-\theta_L}-1} MRS_{it} \quad &= \quad \theta_L\left[\frac{W_{it}}{W_t}\right]^{\frac{1}{1-\theta_L}}\frac{W_t}{P_t} + \frac{1}{L_t}(1-\theta_L)\phi_W\left[\frac{W_{it}}{W_{i,t-1}} - \Pi^w_{t-1}\right]\frac{W_t}{P_t} \\
&\quad + \frac{\beta(1-\theta_L)\phi_W}{L_t}E_t\left[\frac{\Lambda_{i,t+1}}{\Lambda_{it}}\left(\frac{W_{i,t+1}}{W_{it}}\right)^2\frac{W_{t+1}}{P_{t+1}}\frac{W_t}{W_{it}}\right] \\
&\quad - \frac{\beta(1-\theta_L)\phi_W}{L_t}E_t\left[\frac{\Lambda_{i,t+1}}{\Lambda_{it}}\Pi^w_t\frac{W_{t+1}}{P_{t+1}}\frac{W_{i,t+1}}{W_{it}}\frac{W_t}{W_{it}}\right]
\end{aligned} \tag{12'a}$$

where

$$MRS_{it} = (1-a_t)C^{*a(1-\sigma)}_{it}(1-L_{it})^{(1-a_t)(1-\sigma)-1}\Lambda^{-1}_{it} \tag{12'b}$$

is the household i's marginal rate of substitution between leisure and consumption. In the analysis that follows, equations (12'a) and (12'b) will be substituted for equation (12).

2.2 Firms and Price Setting

During period t, an individual firm $j \in [0, 1]$ hires K_{jt} units of physical capital (from households) and L_{jt} units of aggregate labor service (from the aggregator), and produce Y_{jt} units of its own product. All firms have the identical CRS production technology

$$Y_{jt} = A_t K^{\alpha_t}_{jt}(g^t L_{jt})^{1-\alpha_t},\ g \geq 1\ . \tag{13}$$

The stochastic properties of the *aggregate productivity shock* A_t and the *capital share shock* α_t are detailed later.

As in the labor market, the demand function for the firm j's output Y_{jt} is

$$Y_{jt}^{d} = (\frac{P_{jt}}{P_t})^{\frac{1}{\theta_Y - 1}} Y_t \quad , \theta_Y \in [0,1] \tag{14}$$

where the aggregate demand Y_t and the aggregate price level P_t are defined as

$$Y_t = (\int Y_{jt}^{\theta_Y} dj)^{\frac{1}{\theta_Y}} \quad , \; P_t = (\int P_{jt}^{\frac{\theta_Y}{\theta_Y - 1}} di)^{\frac{\theta_Y - 1}{\theta_Y}} \quad . \tag{15}$$

Nominal rigidities in the goods market take the form of price adjustment costs

$$AC_{it}^{p} = \frac{\Phi_p}{2} \left(\frac{P_{jt}}{P_{j,t-1}} - \Pi_{t-1} \right)^2 Y_t \tag{16}$$

where P_{jt} is the price of the firm j set in period t, and $\Pi_{t-1} = P_{t-1}/P_{t-2}$ is the inflation rate prevailing in period $t-1$. Equation (16) implies that both the price level and the inflation rate are sticky.

The firm j is assumed to solve its profit maximization problem through two steps. First, given aggregate price level and factor prices, the firm solves the cost minimization problem. Second, given the cost function thus derived, it determines the optimal price P_{jt} to charge by solving the following profit maximization problem

$$\max E_0 \left[\sum_{t=0}^{\infty} \frac{\beta^t \Lambda_t}{\Lambda_0} \left(\frac{P_{jt} Y_{jt}}{P_t} - Y_{jt} \frac{MC_t}{P_t} - AC_{jt}^{p} \right) \right] \tag{17}$$

where $\frac{\beta^t \Lambda_t}{\Lambda_0}$ is the discount factor for its real profit between period 0 and t, and $\Lambda_t = \int_{[0,1]} \Lambda_{it} di$ is the average marginal utility of consumption across all households.[2] The marginal cost MC_t, common to all firms, is independent of the output level due to the CRS production function.

The FOCs for $(K_{jt}, L_{jt,} P_{jt})$ require

$$\frac{L_{jt}}{K_{jt}} = \frac{Q_t/P_t}{W_t/P_t} \frac{1-\alpha_t}{\alpha_t} \tag{18}$$

[2] If all households are identical and have the same shares Γ_{ijt} of firm $j \in [0,1]$, the assumption of complete markets establishes the unique market discount factor $\frac{\beta^t \Lambda_t}{\Lambda_0}$ between period 0 and t.

$$\frac{MC_t}{P_t} = \frac{W_t/P_t}{MPL_t} \tag{19}$$

$$\frac{1}{P_t}\frac{\partial\left(P_{jt}Y_{jt}\right)}{\partial P_{jt}} = \frac{MC_t}{P_t}\frac{\partial Y_{jt}}{\partial P_{jt}} + \left\{\frac{\partial AC^p_{jt}}{\partial P_{jt}} + E_t\left[\frac{\beta\Lambda_{t+1}}{\Lambda_t}\frac{\partial AC^p_{jt+1}}{\partial P_{jt}}\right]\right\}. \tag{20}$$

where MPL_t is the marginal productivity of labor.

Using equations (14), (16), and (19), I rewrite the equation (20) as

$$\begin{aligned}
& \theta_Y\left[\frac{P_{jt}}{P_t}\right]^{\frac{\theta_Y}{\theta_Y-1}} - \left[\frac{P_{jt}}{P_t}\right]^{\frac{1}{\theta_Y-1}}\frac{W_t/P_t}{MPL_t} + (1-\theta_Y)\Phi_P\left[\frac{P_{jt}}{P_{j,t-1}} - \Pi_{t-1}\right]\frac{P_{jt}}{P_{j,t-1}} \\
= \quad & \beta(1-\theta_Y)\Phi_P E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\left(\frac{P_{j,t+1}}{P_{jt}}\right)^2\frac{Y_{t+1}}{Y_t}\right] \\
& -\beta(1-\theta_Y)\Phi_P E_t\left[\frac{\Lambda_{t+1}}{\Lambda_t}\Pi_t\frac{Y_{t+1}}{Y_t}\frac{P_{j,t+1}}{P_{jt}}\right]
\end{aligned} \tag{20'a}$$

$$MPL_t = (1-\alpha_t)A_t K_{jt}^{\alpha_t} L_{jt}{}^{-\alpha_t} g^{t(1-\alpha_t)} \tag{20'b}$$

In what follows, I replace equations (19) -(20) with equations (20'a) and (20'b).

2.3 Government

The government maintains a balanced budget every period by equating the total lump-sum payment to households with the sum of seigniorage gain and the increase in net debt

$$T_t = M_t - M_{t-1} + B_t - R_{t-1}B_{t-1} \tag{21}$$

where $T_t = \int_0^1 T_{it}di$, $M_t = \int_0^1 M_{it}di$, $B_t = \int_0^1 B_{it}di$. [3,4] Subject to the condition (21), the government conducts monetary policy by adjusting the short-term nominal interest rate R_t according to the

[3] To offset the effects of monoploistic distortions on the steady state output and/or labor , Rotemberg and Woodford (1997, 1999) and other researchers (e.g., Amato and Laubach, 2003; Erceg et al., 2000) assume government subsidies on sales revenue and/or labor income. I consider the "as-is" economy without such subsidies, because I believe such schemes belong in principle to the realm of fiscal policies.

[4] Although the model exhibits Ricardian Equivalence, fiscal considerations are in order for the equilibrium to exist and be unique. For example, if the growth rate of nominal bonds is higher than the inflation rate, which is possible under a deflationary policy, real government debts will explode. Implicitly, I assume fiscal policy is specified as

$$T_t = g^t T - \tau\frac{B_{t-1}}{P_t}$$

where T and τ are some constants, so that increasing government debts can be financed by negative transfer payments.

rule

$$\log\frac{R_t}{\overline{R}} = \rho_R \log\frac{R_{t-1}}{\overline{R}} + (1-\rho_M)\left[\gamma_\pi \log\frac{\Pi_t}{\overline{\Pi}} + \gamma_y \log\frac{Y_t}{\overline{Y}_t} + \gamma_m \log\frac{MG_t}{\overline{MG}}\right] + \varepsilon_{Mt},\ 0 < \rho_R < 1 \quad (22)$$

where $\overline{R}$ is the gross nominal interest rate, $\overline{MG}$ is the growth rate of nominal money, $\overline{R}$ is the steady state gross nominal interest rate, all in the steady state. Π_t is the gross inflation rate between period $t-1$ and t, and $\overline{Y}_t$ is the deterministic level of output in period t, respectively. $\overline{\Pi}$ is the long-run "reference" level of inflation rate.[5] The monetary policy disturbance ε_{Mt} is a white noise with mean 0 and variance σ_ε^2, independent of all other random shocks in the model. The rule (22) is a generalization of Taylor (1993) in that it allows policy to respond to the variations in money growth in addition to output and inflation.

2.4 Equilibrium

I take the economy to be subject to six structural disturbances. In addition to the monetary policy disturbance ε_{Rt}, the model is driven by stochastic evolution of five structural disturbances $(A_t, \alpha_t, \delta_t, b_t,\ a_t)$, each of which follows a logarithmic AR(1) of the form

$$\log\frac{\chi_t}{\chi} = \rho_\chi \log\frac{\chi_{t-1}}{\chi} + \varepsilon_{\chi t} \quad (23)$$

where χ is the steady state level of χ_t, and $\varepsilon_{\chi t}$ is a white noise with mean 0 and variance σ_χ^2. The autoregressive coefficients are constrained within the stationary region. Innovations in the disturbances are not correlated with one another, except that the two errors $(\varepsilon_{At},\ \varepsilon_{\alpha t})$ in the production function are allowed to be correlated. [6]

[5] I use the term "reference" rate of inflation to denote the long-run inflationary stance of a rule. The term "target" rate probably is used more frequently in the literature. The reason for the unfamiliar nomenclature is to prevent possible confusions around the usage of "targets" or "targeting" observed in the literature.

[6] Christiano et al. (2001) assume variable capital utilization, under which positive productivity shock will increase the effective marginal productivity of labor, and leads to a higher amount of labor employed relative to capital. Allowing for the correlation between A_t and α_t in the present model is intended as a reduced form to capture such dynamics.

An equilibrium of the economy (under the benchmark monetary policy rule) is given by a set of decision rules $\{C_{it}, K_{i,t+1}, M_{it}, B_{it}\}$ and a wage rule W_{it} of household i; a capital demand rule K_{jt}, a labor demand rule L_{jt} and a price rule P_{jt} of firm j, the monetary policy rule of government; and a price vector $\{P_{jt}, W_{it}, Q_t, R_t\}$ such that: *i)* $\{C_{it}, K_{i,t+1}, M_{it}, B_{it}\}$ maximizes lifetime utility (3) subject to the budget constraint (2), adjustment costs (1) and (7), and labor demand in (6); *ii)* K_{jt}, L_{jt}, and P_{jt} maximizes profit stream (17) subject to the production technology (13) and adjustment cost (16); *iii)* $\{R_t, M_t\}$ evolve according to (22) subject to the government's budget constraint (21); *iv)* $\{P_{jt}, W_{it}, Q_t, R_t\}$ clear the goods market, the labor market, capital market, and the money market.

In what follows, I focus on a particular *symmetric* equilibrium in which all firms and households make identical decisions. Since most of the real and nominal variables in the model exhibit deterministic trends due to the constant rate of labor-augmenting technical progress (g) and the reference inflation rate, I deflate variables by their deterministic trends to transform the system into a stationary one.

2.5 Estimation[7]

The transformed system, described in more detail in the appendix, is cast into the form

$$G_1(z_t, z_{t-1}, \varepsilon_t) = 0_{N_1 \times 1} \tag{24a}$$

$$G_2(z_t, z_{t-1}, \varepsilon_t) + \eta_t = 0_{N_2 \times 1} \tag{24b}$$

where ε_t is the vector of the six innovations, and η_t is a vector of endogenous errors satisfying $E_{t-1}\eta_t = 0$, for all t. The N-dimensional system vector z_t is decomposed as $z_t = (z_{1t}', z_{2t}')'$, where z_{2t} denotes the N_2-dimensional auxiliary variables used to denote conditional expectation terms

[7] The estimation results are mostly taken from Kim (2003), who estimates a model with identical first order hahavior.

in equations (10), (11), (12'a) and (20'a), and z_{1t} is the $(N - N_2)$ dimensional vector of all other variables including all exogenous and endogenous state variables.[8] When the system (24) is log-linearized around its steady state, the method of Sims (2002) can be applied to obtain a solution of the form

$$d \log z_t = g_1 d \log z_{t-1} + g_2 \varepsilon_t \tag{25}$$

where g_1 and g_2 are complicated matrix functions of the model parameters. Since the solution (25) takes the form of a state-space model driven by innovations ε_t, maximum likelihood estimates of the parameters can be obtained by an application of Kalman filtering, using data on the observables in z_t.[9]

The raw data used in this study are extracted from DRI BASIC economic series for the sample period 1959:Q1-1999:Q4.[10] Since two main features of the model are *i*) the nominal rigidities in goods and labor markets; and *ii*) the interest rate feedback rule for monetary policy, it is imperative to use the data on monetary aggregates as well as prices and quantities in goods and labor markets. Therefore, the following six series are used for the actual estimation purpose: per capita output (Y), per capita labor hours (L), rate of price inflation (Π), the growth rate of per

[8] In the conventional rubric, z_{1t} and z_{2t} correspond to the "state" and "jump" variables, repectively. As summarized in the appendix, the number of conditional expectations (or endogenous errors) in the present model is 6: one in the capital adjustment equation (10), one in the Fisherian equation (11), two in wage equation (12'a), and two in price equation (20'a).

[9] Maximization of the likelihood function over the parameter set requires one to cope with the parameter regions in which *i)* the candidates of estimates yield nonsensical (mainly negative) steady state values of the variables; and *ii)* the model does not have a unique equilibrium. As in Leeper and Sims (1994), I assign an arbitrary very low likelihood value to parameters in such bad regions, and the resulting discontinuity in the likelihood function is addressed by a "cliff-robust" optimization routine **csminwel.m** written by the latter author.

[10] All raw series, except for interest rate and wage, are seasonally adjusted.

output :	gross domestic products, billions of 1992 dollars.
employment :	average weekly hours of production workers in manufacturing sector.
price :	implicit price deflator for gross national products.
money :	M2 stock, billions of current dollars.
interest rate :	federal funds rate, per annum.
wage :	index of compensation per hour in nonfarm business sector, 1982=100.
population	civilian population, in thousands.

capita money balance (MG), interest rates (R), and wage inflation rates (Π^w). To express the data series conformable with the theoretic counterparts in the model, per capita output and money balance series are obtained by dividing GDP and M2 balance, respectively, by population size. Per capita labor hours are obtained by dividing weekly working hours by 120, under the assumption that each worker is endowed with 5 working days per week. The resulting series imply households devote 33.8% of their time endowment to working. Since federal funds rates are measured in annual percentage rates, I transform them into quarterly rates by dividing by 400 and adding one. Price and wage inflations are obtained by log-differencing the price and wage series.

Since the dataset bears little information about some structural parameters, a few parameters are fixed before estimation: steady state values of capital share $\overline{\alpha}$ and depreciation $\overline{\delta}$ are fixed at 1/3 and 0.025, respectively. The market power θ_Y in the goods market is fixed at the conventionally calibrated value of 0.9, because only two of $(\overline{A}, \theta_Y, \theta_L)$ are identified from the series on output and labor. Assuming the Fed has successfully managed the inflation rate around the intended reference level, I fix the steady state inflation rate $\overline{\Pi}$ at its actual average 1.01005 over the sample period. The CRRA parameter σ is fixed at 1, which amounts to the logarithmic instantaneous utility function.

Two parameters (ν, b), crucial to the form of money demand and welfare calculations, are estimated by running calibration and estimation jointly. More specifically, at each step of maximizing the likelihood function I determine b as

$$b = (1 - \overline{R}^{-1})[V_d]^{\nu-1} \tag{9'}$$

given all other candidate parameters, where the empirical velocity V_d is set equal to the actual average over the sample period.[11]

The estimated parameters are reported in Table 1 along with corresponding functional forms of the structural equations. Asymptotic standard errors are in the parentheses, computed from the

[11] Therefore, the estimates are obtained under the constraint that the estimated velocity is equal to the actual.

Hessian of the maximized likelihood function. The estimate of growth rate g is 1.0056, which is higher than the actual average growth rate 1.0052 of per capita GDP over the sample period. The estimate of discount factor β is 0.9986, falling between the estimate 0.9974 for post-79 era in Ireland (2001) and 0.9999 in Kim (2000) for 59:Q1- 95:Q1, although higher than the usually calibrated value of 0.99. The share a of consumption bundle C_t^* in the instantaneous utility function is 0.4681, which is higher than the usually calibrated value of 0.4. The estimates of (ν, b) are (-22.7561, 0.0008), which shows that the indifference curves on the (C, RM) plain are highly convex to origin in the estimated steady state: one percent increase in $\frac{C}{M/P}$ ratio results in 23.7561 percent decrease in the marginal rate of substitutions between C and RM.[12] These estimates also imply that an interest-semi-elasticity of money demand is about 0.04, which is well below the usual empirical estimates.[13]

The estimate of θ_L is 0.6888, lower than the calibrated value 0.75 in Huang and Liu (1999). The real rigidity parameter for capital adjustment cost (ϕ_K =16.8456) shows a considerable degree of real rigidity in the economy: when the economy is initially at the estimated steady state, transforming one unit of consumption good into the same unit of operational capital involves an additional 0.0668 units of output as adjustment costs.

The parameters for the monetary policy rule, used as the benchmark rule in the following analysis, show the systematic evolution of nominal interest rates in response to inflation and money growth, but none to output over the sample period.[14] The estimate $\rho_M = 0.1395$ implies a modest

[12] As discussed in Feenstra (1986), one can construct an isomorphic model in terms of transaction costs, by redefining C^* in the utility function (4) as the usual consumption and replacing C in the budget constraint (2) with

$$C^{**} = C^* \times \left[1 - b\left(\frac{M/P}{C^*}\right)^{\nu}\right]^{\frac{1}{\nu}}$$

where C^{**} represents the gross spending on consumption inclusive of (multiplicative) transaction costs. The ratio $C^{**}/\ C^*$ (evaluated at the estimated steady state) is 1.0006, implying transaction costs are a reasonably small fraction of consumption C^*.

[13] This seems to arise because the model makes the demand for money adjust instantaneously, whereas empirical work usually allows lags or uses longer frequency data.

[14] Also observed in Ireland (1999) are the small responses of nominal interest rate to output in the presence of

degree of policy inertia.

Regarding the structural disturbances, the estimated AR(1) coefficients show the economy has been subject to highly persistent structural shocks. Except for the labor supply shock, the half-lives of the aggregate shocks are around 6 years. The labor supply shock a_t exhibits negative serial correlations. Finally, the innovations in the shocks A_t and α_t are negatively correlated with correlation coefficient -0.9755.

The estimates of (Φ_w, Φ_p) =(20.0341,10.0970) show the degree of nominal rigidities is higher in the labor market than in the goods market. Those parameters are precisely estimated with respective standard errors 0.7025 and 0.7393.

3. Welfare Metric

I now construct a natural utility-based metric, to be used for welfare evaluation of alternative monetary policy rules. Since the instantaneous utility function $U(\cdot)$ has a deterministic trend due to the labor-augmenting technological growth, I first transform $U_t = U(C_t^*, L_t, M_t/P_t)$ to achieve stationarity:

$$\begin{aligned} U_t &= g^{a(1-\sigma)t} \times u_t \\ &= g^{a(1-\sigma)t} \left(\frac{1}{1-\sigma} \left[(c_t^\nu + b_t r m_t{}^\nu)^{\frac{a}{\nu}} (1 - L_t)^{(1-a_t)} \right]^{1-\sigma} - 1 \right) \end{aligned} \tag{26}$$

where c_t and rm_t are the stationary transformed consumption and real balance, respectively. Using the second order Taylor expansion of u_t around the deterministic steady state, I get the present discount value of a representative household's expected utility conditional on an initial condition

inflation and money growth as policy indicators.

Ω_0:

$$
\begin{aligned}
EW &= E\left[\sum\nolimits_{t=0}^{\infty}\beta_*^t u_t \mid \Omega_0\right] \\
&\simeq \frac{1}{1-\beta_*}u(\overline{\zeta})+\left[\frac{du_t(\overline{\zeta})}{d\zeta_t}\otimes\overline{\zeta}\right]'\sum\nolimits_{t=0}^{\infty}\left(\beta_*^t E\left[\widehat{\zeta}_t \mid \Omega_0\right]\right) \\
&\quad +\frac{1}{2}tr\sum\nolimits_{t=0}^{\infty}\left(\beta_*^t Var\left[\widehat{\zeta}_t \mid \Omega_0\right]\left[\frac{d^2u_t(\overline{\zeta})}{d\zeta_t^2}\otimes(\overline{\zeta\zeta'})\right]\right) \qquad (27)
\end{aligned}
$$

where $\beta_* = \beta g^{a(1-\sigma)}$, and $tr(\cdot)$ is the trace of a square matrix.[15]The symbol $\otimes$ denotes the matrix operator of element by element multiplication.

The presence of conditional expectation $E\left[\widehat{\zeta}_t \mid \Omega_0\right]$ in equation (27) highlights the need to use higher order approximate solution methods for correct welfare evaluation. For a simple illustration, suppose that the exact solution of ζ_t can be represented as a function $\zeta(\cdot)$ of ε_t, so that the second order Taylor expansion of ζ_t around the steady state is $d\zeta_t = \overline{\zeta} + \zeta'\varepsilon_t + \frac{1}{2}\zeta''\varepsilon_t^2$. If a first order approximate solution of ζ_t is inserted into EW, the expectation of the third term in $d\zeta_t$ is ignored, which is in general the same size as the other second order terms that appear in the last term in EW.

To get a solution of the system (24) with the accuracy of up to the second order, I use the method by Sims (2000) based on a quadratic Taylor expansion of (24) around the deterministic steady state $\overline{z}$. Under a set of regularity conditions, a unique and stationary second order accurate solution to (24) is obtained of the form

$$
\begin{aligned}
\widehat{z}_{1it} &= F_{1ij}\widehat{z}_{1j,t-1} + F_{2ij}\varepsilon_{jt} + F_{3i} \qquad (28a) \\
&\quad +0.5\left(F_{11ijk}\widehat{z}_{1j,t-1}\widehat{z}_{1k,t-1} + 2F_{12ijk}\widehat{z}_{1j,t-1}\varepsilon_{kt} + F_{22ijk}\varepsilon_{jt}\varepsilon_{kt}\right), \\
\widehat{z}_{2it} &= S_i\widehat{z}_{1it} + T_iM_{11ijk}\widehat{z}_{1jt}\widehat{z}_{1kt} + T_iM_{2i}\ . \qquad (28b)
\end{aligned}
$$

where $\widehat{z}_t = \log z_t - \log\overline{z}$ denotes the % deviation of z_t from its deterministic steady state, and

[15] Many researchers (e.g., Clarida et al.,1999; Rotemberg and Woodford, 1997,1999; and Erceg et al., 2000; Kollmann, 2002) have used unconditional expectation of utility, which corresponds to using $E[u_t \mid \Omega_0]$ with $t = \infty$. Acknowledging the importance of transitional dynamics, this paper uses the discounted stream of utility.

S, T, F's, and M's are matrix functions of the deep parameter of the model.[16] In particular, the terms F_3 and M_2 represent the degree of certainty non-equivalence. Note that equations in (28) utilize the tensor notation for the simplicity of exposition. For example, the term $F_{11ijk}\widehat{z}_{1j,t-1}\widehat{z}_{1k,t-1}$ can be interpreted as the quadratic form in terms of lagged $\widehat{z}_t$ for the i^{th} equation, constructed by the lag of $\widehat{z}_{1t}$. By using equation (28a) recursively, I can compute $\{\mu_{1t}, \Sigma_{1t} : t \geq 0\}$, the conditional first and second moments of $\widehat{z}_{1t}$, from which the welfare measure EW is constructed. The details involved are given in the appendix.

When the metric EW is applied, it is essential to use the same initial condition Ω_0 for all rules being compared. This in turn requires one to use the same pair of (μ_0, Σ_0) or the same distribution of the initial state for every policy rule.[17] In the following analysis, I set both $\mu_0$and Σ_0 to be 0, under the assumption that the economy has been at the estimated steady state until the initial period.[18]

For interpretational convenience, the relative performances of alternative rules are measured by *consumption compensations* defined as follows: suppose that the benchmark rule (22) (say, rule 0) and another rule (say, rule 1) deliver EW^0 and EW^1 level of expected welfare, respectively. The *consumption compensation* for the rule 1 (relative to rule 0) is defined as

$$dc = \frac{1}{2}\frac{\left[EW^0 - EW^1\right]}{\lambda^0} + \frac{1}{2}\frac{\left[EW^0 - EW^1\right]}{\lambda^1} \tag{29}$$

where λ^0 and λ^1 are the marginal utility of consumption evaluated at steady state under the rule 0 and rule 1, respectively.[19]The interpretation of the relative measure dc is straightforward: if EW^0

[16]The original codes of Sims (2000) give a slightly different (but essentially equivalent) solution, in which the solutions for the "jump" variables z_{2t} are indirectly given by linear combinations of the whole system vector in log-deviations. The routine for transforming the original solutions into those in (28) is available upon request.

[17]If each policy rule's own implied unconditional mean and variance are used, the ranking is identical to what results when the non-discounted unconditional expectation of u_t is used.

[18]Since welfare calculations vary depending on the initial conditions, I also use the conditioning information set which comprises the unconditional mean and variance under the benchmark rule (22). The qualitative results, which remain the same, are available upon request.

[19]Since the monetary authority manipulates the "reference" rate of inflation, the steady state level of marginal

is higher than EW^1, the representative household under the rule 1 should be compensated with dc amount of one time consumption to have (approximately) the same level of lifetime expected welfare as under the benchmark rule.

Before further analysis, it is meaningful to gauge the accuracy gains from using a higher order approximate solution method. Equation (28b) shows a key difference between the linear and the quadratic approximate solution methods: the latter method gives a quadratic parametrization of the conditional expectations as on the RHS of (28b), while the former only gives the first term for linear parametrization. Therefore, I compare the accuracy of the two solution methods in the spirit of parametrized expectation approach (PEA) as follows: *i)* setting off from the estimated deterministic steady state, I generate a very long path of exogenous disturbances of the sample size 100,000; *ii)* the linear and quadratic parametrizations of the conditional expectations z_{2t} are combined with the original subsystem (24a) for z_{1t}; *iii)* for each parametrized version of the whole system thus constructed, I solve forwardly for the whole system variable z_t given the path of exogenous disturbances, again starting from the deterministic steady state; *iv)* the solved paths of z_t are substituted in (24b) to generate the simulated paths of endogenous errors η_t under the two solution methods; and *iv)* I check the accuracy of the solutions by regressing the simulated errors on state variables: the R^2s should come out very small for both solution methods, and even smaller for the quadratic approximation method.[20]

utility of consumption is also policy-dependent. Taking arithmetic average is analogous to the same convention in calculating arc elasticities.

[20] The state variables are categorized into

$$x_{1t} = (mg_t, k_{t+1}, \Pi_t, rw_t, rm_t, R_t, \Pi_t^w),\ x_{2t} = (A_t, \alpha_t, \delta_t, b_t, a_t)$$

where the first and second sets denote endogenous and exogenous state variables, respectively. The resulting simulated endogenous errors are regressed on the following 25 "explanatory" variables for corresponding solution method:

i) constant	(1 term)
ii) $(\log x_{1t} - \log \overline{x}_1)$, and $(\log x_{2t} - \log \overline{x}_2)$	(12 terms)
iii) square terms in $(\log x_{1t} - \log \overline{x}_1)$ and $(\log x_{2t} - \log \overline{x}_2)$	(12 terms)

Table 2 reports the R^2s and F statistics for the null of "all-zero coefficients" from the two regressions. The left panel is for the conventional first order solution, and the right panel is for the second order solution. The F statistics show that the null is rejected for all endogenous errors regardless of the solution methods. The null is eventually rejected with as large a simulation sample size as 100,000, however, because both solution methods are only approximate. The results in Table 2 also indicate that the inaccuracy in the approximate solutions will not be detected with sample size as large as the historical data, because the rejection of null for the sample size of 160 requires R^2 be 0.242 or higher. At any rate, the results in Table 2 demonstrate higher accuracy of the second order approximate solution: for all of the six endogenous errors, the quadratic approximate solution generates smaller R^2s and F statistics, and the improvement is most conspicuous for the errors in the pricing equation.

4. Insights from the Benchmark Rule

Table 3 reports the performance of **BM**, the estimated benchmark rule (22) implemented without ε_{Mt}.[21] The first column of the upper panel shows that, at the current stance $\overline{\Pi} = 1.01005$ of long-run inflation, the welfare measure EW amounts to 607.8661. Evaluated at the estimated steady state, this level of welfare translates into 17.5563 units of consumption each period for an eternal life.

As shown in (27), EW comprises three terms on the RHS representing *i*) steady state utility; *ii*) utility from the first order deviations from steady state; and *iii*) utility from the second order deviations from steady state. Had the welfare metric EW been constructed naively from the conventional first order approximate solution, the second term would not have come into play. In that case, the naive welfare calculation under **BM** would have been lower by 5.1690 than the correct

[21] The omission of policy mistakes ε_{Mt} is for fair comparison with other rules, which are assumed to be implemented exactly in the later section.

level. In terms of the consumption compensation measure, this "measurement error" amounts to 107.4934 units of one-time consumption, or equivalently 0.8370 units each period for life which is 6.22% of output in the deterministic steady state.[22]

Two features of the present model are suggestive of ways to improve upon the benchmark rule in terms of welfare. The first one is the nature of nominal rigidities in the model: both price and wage are sticky, not only in *levels* but also in the *rates of changes*. One striking and frequently criticized implication of many New Keynesian models is that, when there is a single nominal variable whose *level* is sticky, Pareto optimum is attainable in the absence of other distortions because there is no trade-off between stabilization of inflation and stabilization of the output gap. This rosy implication is an inherent artifact of price adjustment processes that involve only forward looking expectations of private sector.[23] Roughly speaking, purely forward looking expectations implies a Phillips curve in which inflation depends only on the current and expected future output gap, without any lagged dependence on past inflation. This being the case, the monetary authority manipulates the parameters of the policy rule to stabilize output gap, achieving inflation stabilization as a serendipity.

In a recent paper with both price and wage *level* rigidities, however, Erceg et al. (2000) establish that even if expectations are purely forward looking, monetary authorities *cannot* achieve Pareto optimum unless either prices or wages are perfectly flexible. In the present model, the trade-off that the monetary authority faces is all the more serious because the aggregate price and wage

[22] For **ABM**, the underprediction of welfare due to using the first order approximate solution amounts to 110.3507 units of one-time consumption.

[23] If the price adjustment cost is specified as

$$AC_{it}^{P} = \frac{\Phi_P}{2}\left(\frac{P_{jt}}{P_{j,t-1}} - \overline{\Pi}\right)^2 Y_t$$

where $\overline{\Pi}$ is the steady state rate of inflation, then inflation is driven by purely forward-looking expectations. Similar results hold for the wage adjustment costs.

are dependent upon both their past history as well as expectations on future levels. Therefore, the monetary authority in the present model is required to find the best compromise between the stabilization of prices and wages, while keeping low volatility of output gap as well. This in turn requires wage inflation to be another nominal anchor to which monetary instruments are adjusted.

The second one, closely related to Friedman (1969), is the explicit consideration of money in the model. The dictum of Friedman (1969) is that the optimal inflation policy is what makes the private cost of holding money equal to the social cost, or a policy achieving zero nominal interest rate via long-run deflation. Equipped with an optimization based money demand function invariant to the changes in policies, the present model can be used to measure the costs/benefits in terms of welfare of changing long-run inflation, so long as the money demand shock b_t is not abstracted away.[24]

At this point, the two possibilities of welfare improvement are addressed informally in the framework of the benchmark policy rule by considering how much welfare metric varies *i*) if the reference inflation rate varies, and *ii*) if wage inflation is used as another indicator variable in the benchmark rule **BM**.

The lower panel of Table 3 summarizes the changes in welfare under **ABM**, the benchmark rule augmented with wage inflation. Evidently, the welfare gains from using another indicator are uniformly positive for all three reference inflation rates. At the current rate of reference inflation, the augmented benchmark rule **ABM** yields slightly higher welfare level equivalent to 5.4643 units of one-time consumption. For higher and lower reference rates, the consumption gains over the

[24] Rotemberg and Woodford (1997,1999) and Erceg et al. (2000) exclude money from the models on the implicit assumption that money is additively separable in the period utility function so that the behavior of the model will be invariant to adding money. However, the welfare implications of different monetary policies are not invariant, because the welfare costs of positive nominal interest rates are ignored under that assumption. The sensitivity of welfare measure EW with respect to the inclusion/exclusion of money will be examined in more detail later in section 5.

benchmark rule amount to 2.6183 and 8.3205 units, respectively.

The last two columns of Table 3 report the performances of **BM** and **ABM** for roughly symmetrically higher ($\overline{\Pi}$ =1.011) and lower ($\overline{\Pi}$ =1.0091) levels of reference inflation rates. As expected, more inflationary policy yields lower welfare level. For example, when the reference inflation rate is increased to 4.473% per annum, households demand 2.9042 additional units of one-time consumption to be as happy as ever, which amounts to 21.6% of steady state output. When the reference inflation rate is lowered, households are willing to forego 2.9019 units of one-time consumption.

As will be shown later in section 6, the welfare maximizing rate of long-run inflation is negative at least among a class of simple endogenous rules adjusting short term rates. The legacy of Friedman (1969) in the present model is in fact in contrast with a few papers in the literature (e.g., King and Wolman, 1999; Wolman, 2001) which favor zero or slightly positive long-run inflation depending on whether the policy objective is present value of welfare or steady state. It is therefore worthwhile to briefly discuss why I reach the opposite end of the policy spectrum. In models with staggered contracts and transaction costs, steady long run inflation has two offsetting welfare effects. On the one hand, positive long run inflation affects the distribution of relative prices, decreasing the markups of firms whose prices were set in the previous periods. This erosion of relative prices (and increased output) of those firms can be welfare improving, because monopolistically competitive firms are obliged to satisfy all demands at their individual prices posted. On the other hand, it is desirable to have long run deflation so that the nominal interest rate is zero, as long as there exists deadweight losses (i.e., the "shoe leather costs" of inflation) under the money demand curve. In the present model where nominal rigidities are imposed via adjustment costs, there are no adjustment costs under steady inflation and therefore only the second negative effect of long run inflation comes into play. Equipped with a simple quantity equation and staggered prices à la Taylor, however, the models in King and Wolman (1999) and Wolman (2001) allow only the first welfare channel of long run inflation to work, deviating from the dictum of Friedman.

At any rate, the findings from Tables 3 are supportive of a welfare-improving deflationary policy rule equipped with wage inflation as another policy indicator. The analyses in the two subsequent sections, where I examine alternative policy rules and an optimized rule among a restrictive class of rules, are based upon this insight.

5. Welfare Analysis: a Performance Derby

In this section, I consider two types of alternative monetary policy rules in which nominal interest rate is used as the policy instrument. The first type of rules, dubbed "*targeting*" rules, postulate that the monetary authority maintains pre-specified deterministic paths of three variables: price, money stock, and nominal income.[25] The second type of rules are basically generalizations of the Taylor rule, in which monetary instruments are adjusted in response to the endogenous indicator variables such as output gap and inflation rate. The insight developed in the previous section is further assayed by considering versions of each alternative rule with lower reference inflation rates or endogenous responses to wage inflation.

An important yet frequently ignored issue in the literature is that a candidate rule should be supported by the economy. In particular, the zero bound on the nominal interest rate should be accounted for: given that a solution method involves approximations around a deterministic steady state with inflation rate close to 0, the nominal interest rate would be negative a nonnegligible portion of time under a rule implying highly volatile nominal rates. Therefore, I focus on the set

[25] Usually, as in McCallum and Nelson (1998), the expression "X-targeting" or "having a target level X^* for variable X" describes a regime in which the monetary authority sets its instrument according to a rule involving responses to deviation of X from its desired path. Alternatively, in a series of papers, Svensson (1999) and Rudebusch and Svensson (1999) identify X-targeting as a regime in which the monetary authority sets a level for the variable X and use all available information to bring X in line with that level. According to the terminology of Svensson, the Taylor rule is a rule "responding" to inflation and output gap, while according to McCallum and Nelson (1998) it is a rule that "targets" both variables. In this paper, the term "targeting" is used to denote the usage advocated by Svensson.

of *feasible* monetary rules, i.e., rules under which the 2.55 times standard deviation confidence intervals for the nominal interest rate, constructed around its unconditional expectation, do not contain zero.[26] Crude as it is, this apparatus does impose a condition that too aggressive an activist rule yielding too volatile nominal interest rate is not compatible with low reference inflation rate, a generic implication of models such as the present one.

5.1 Alternative Policy Rules

5.1.1 Inflation Targeting (PHIT)

If the monetary authority sets the nominal interest rate to keep inflation rate Π_t at a fixed level, the corresponding path of nominal interest rate (up to the first order accuracy) can be found from the money demand function (9). Taking the first difference of the log-linearized version of (9) and using $\Delta\widehat{rm}_t = \widehat{MG}_t - \widehat{\Pi}_t$, one gets

$$\frac{1}{R-1}\widehat{R}_t = \frac{1}{R-1}\widehat{R}_{t-1} + \Delta\widehat{b}_t + (1-\nu)\Delta\widehat{c}_t - (1-\nu)\widehat{MG}_t \quad . \tag{30}$$

Therefore, the strict inflation targeting dictates the change in interest rate be positively related with consumption growth, and negatively related with nominal money growth. This rule will be labelled **PHIT** for future references.[27] If the rule (30) is implemented with $\overline{\Pi} = 1$, it is equivalent to strict price targeting.

It is worth noting that, again, up to the first order, fixed inflation targeting is equivalent to targeting a constant markup in the goods market that would result if nominal prices were fully

[26] In a quarterly model like the present one, if nominal interest rate is normally distributed, such a restriction alows zero interest rate once every $\frac{1}{4(1-0.9946)} = 46.3$ years. The threshold of 2.55 is higher than the empirical mean-standard deviation ratio 2.0248 during the etimation period.

[27] Note that all three targeting rules considered in this section are not policy *recipes* in a strict sense. Instead of giving a functional form for a policy instrument to follow, it describes how the instrument evolves with the other variables on the RHS if inflation is *somehow* kept constant.

flexible. This can be seen from the log-linearized version of the pricing equation:

$$\widehat{\Pi}_t = \widehat{\Pi}_{t-1} + \frac{\theta_Y}{(1-\theta_Y)\Phi_p \overline{\Pi}^2}\left[\widehat{rw}_t - \widehat{mpl}_t\right] + \beta g^{a(1-\sigma)} E_t\left[\widehat{\Pi}_{t+1} - \widehat{\Pi}_t\right] \tag{31}$$

where keeping $\widehat{\Pi}_t$ at zero implies $\widehat{rw}_t = \widehat{mpl}_t$ for all t.

5.1.2 Nominal Income Targeting (NIT)

Taking the first difference of the log-linearized version of (9) and using $\Delta\widehat{p_t y}_t = \widehat{\Pi}_t + \Delta\widehat{y}_t$, I get the nominal income targeting rule of the form

$$\frac{1}{R-1}\widehat{R}_t = \frac{1}{R-1}\widehat{R}_{t-1} + \Delta\widehat{b}_t + (1-\nu)\Delta\widehat{c}_t - (1-\nu)\widehat{MG}_t - (1-\nu)\Delta\widehat{y}_t \ . \tag{32}$$

This rule requires the current nominal rate increase over the previous level in response to positive consumption growth and negative growth in nominal money stock and real output. This rule is labelled as **NIT** for future reference

5.1.3 Money Growth Targeting (MGT) When the central bank stabilizes the aggregate money growth rate, (9) implies

$$\frac{1}{R-1}\widehat{R}_t = \frac{1}{R-1}\widehat{R}_{t-1} + \Delta\widehat{b}_t + (1-\nu)\Delta\widehat{c}_t + (1-\nu)\widehat{\Pi}_t \tag{33}$$

Under the strict money growth targeting, therefore, the current interest rate relative to previous rate is positively related with consumption growth and inflation rate. This rule will be labelled as **MGT** for future reference.

5.1.4 Variants of Taylor Rules (TR, ATR) I consider two versions of rules that share the spirit of Taylor (1993). The first one is an endogenous rule by which the central bank adjusts the nominal rate gradually in response to price inflation and GDP gap:

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + \gamma_1\widehat{\Pi}_t + \gamma_2\widehat{y}_t \ . \tag{34}$$

Clarida et al. (1999) estimate the rule (34) over the tenure of Volker and Greenspan 1979:Q3-1996:Q4, and obtain the estimates $(\rho, \gamma_1, \gamma_2)$=(0.79,0.4915,0.1953). When the rule (34) is implemented with those estimates, however, the feasibility condition is violated. To obtain a feasible solution of the model, I put $(\rho, \gamma_1, \gamma_2)$=(0.8,1.7,0.01) implying very small responses to current output gap and aggressive response to inflation rate. This feasible rule is labelled **TR** for future reference.

Also considered is another version of (34), augmented with wage inflation as another indicator:

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + \beta_1 \widehat{\Pi}_t + \beta_2 \widehat{y}_t + \beta_3 \widehat{\Pi}_t^w \tag{35}$$

labelled as **ATR** for future reference. I put $(\rho, \beta_1, \beta_2, \beta_3)$=(0.8,1.7,0.01,0.17) in the following analysis.

5.2 Performances of Alternative Rules

Table 4 summarizes how the five alternative rules compare with regard to the current (i.e., $\overline{\Pi} = 1.01005$), modest (i.e., $\overline{\Pi} = 1.0063$), and low (i.e., $\overline{\Pi} = 1.0025$) rates of reference inflation. **NIT** and **MGT** achieve comparably higher welfare than **PHIT** does. In fact, at the current stance of long-run inflation, adopting **PHIT** instead of the **BM** actually lowers welfare, while gains from moving toward **NIT** or **MGT** is tantamount to about 2.2 units of additional one-time consumption.

The two versions of Taylor rules, **TR** and **ATR**, outperform all of the three fixed targeting rules at all rates of reference inflation. This finding reflects the emphasis in the nearly entire literature that a policy should bring target variables to designated levels only gradually. Because monetary policy effects on nominal anchors are usually smooth and delayed, strict targeting rules like the ones examined here necessarily lead to instability, which is penalized by the third term in the welfare measure *EW*. Also observed in Table 4 is that augmenting **TR** with wage inflation leads to additional welfare gains amounting to 2.14 units of consumption compensation for both current and modest inflation stance.

In addition to the instability due to targeting a fixed path of aggregate price, there is another reason for the poor performance of **PHIT** in the present model: the estimated structure of the economy imposes a much higher degree of nominal rigidities in the labor market than in the goods market. Intuitively, when monetary authority faces a trade-off in stabilizing output, price inflation, and wage inflation, it is a better strategy to put more weight on stabilizing the more rigid nominal variable (i.e., wage inflation in the present setup), thus letting the other more flexible one (i.e., price inflation in this model) account for a larger share of the adjustment process.[28] Simply put, if wages are not fully flexible, holding prices stable prevents the real wage from adjusting as it should. Hence, assuming the estimated structure of the economy as granted, **PHIT** is chasing the wrong variable. In view of this intuition, the good performance of **MGT** (relative to **PHIT**) is explained as follows: in an economy where output is demand determined, monetary authority can stabilize the economy by controlling a neutral (i.e., money growth) nominal anchor. In doing so, monetary policy implicitly allows the more flexible one (i.e., price) between the two sticky nominal variables to adjust more to the changes in economic environment.

5.3 Sensitivity Analysis

The results in the previous subsection hinge on two aspects of substitutions in the model: First, in the intra-temporal context, the welfare measure EW is dependent upon the elasticity of substitution between consumption and real balance (or, equivalently, the transaction technology) implied by the utility function (4). Second, in the inter-temporal context, the attitude of households toward cyclical variations also affects the welfare measure. In terms of a sensitivity checkup, the former is related to the pair of parameters (ν, b) governing transaction technology, and the latter to the

[28] This insight is provided by Aizenman and Frenkel (1986) in a static model, and by Erceg et al. (2000) in a dynamic setting.

parameter σ measuring the degree of risk aversion. In particular, the money demand parameter b is central to measuring the welfare costs incurred by the need to economize on non-interest-bearing money.

Tables 5A-5E report how the initial ranking **ATR**$\gg$**TR**$\gg$**NIT**$\gg$**MGT**$\gg$**PHIT** observed in Table 4 changes for different values of the three parameters (σ, ν, b). In those tables, the reference inflation rate is fixed at the benchmark level $\overline{\Pi} = 1.01005$.

Table 5A compares the alternative rules for higher degrees of risk aversion, where the overall ranking is quite different from that under $\sigma = 1$. In particular, the two versions of Taylor rules show less impressive performance: when σ is raised to 1.3, for example, the once second best performer **TR** finishes barely ahead of **PHIT**, and **ATR** steps down from the top to third place. The relative performance **MGT** shows the most conspicuous improvement with higher degrees of risk aversion, finishing first for higher degrees of risk aversion. **PHIT** is consistently the worst performer.

When it comes to the transaction technology, Tables 5B and 5C show that the initial ranking is almost entirely preserved for higher values of ν and b alike. The only exceptions are when either parameter is much higher than the benchmark value, in which cases **NIT** and **MGT** trade their places.

The cases with b very close to 0 are worthy of consideration, since these cases correspond to the abstraction of money. It is evident from Table 5C that ignoring money leads to the overprediction of welfare level. In fact, higher values of b are associated with lower welfare levels for each rule considered. Observing the near-invariance with respect to b of the ranking in Table 5C, however, I interpret that the relative performances of rules are not affected by the size of welfare gains from deflating the economy.

Noting that the benchmark value of $\nu = -22.7561$ implies that the transaction technology is highly insensitive to b, I also report the sensitivity with respect b for two higher values of ν =-12 and

-5 in Tables 5D and 5E, respectively. Generally, the initial ranking for the benchmark parameters is preserved.

The analyses thus far have been in the context of the "structural" metric EW. Here, I consider briefly how robust the previous results are when a conventional *ad-hoc* loss function evaluated at the first order approximate solutions is employed as a performance criterion. More specifically, I consider the following expected loss function constructed from the conditional variances:

$$EL = E\left[\sum_{t=0}^{\infty} \beta_*^t L_t \mid \Omega_0\right] \tag{36a}$$

where the period loss function L_t is constructed as

$$L_t = var(\widehat{\Pi}_t) + \lambda_1 var(\widehat{y}_t) + \lambda_2 var(\widehat{R}_t - \widehat{R}_{t-1}) \tag{36b}$$

The expected loss function above is similar to that used in Rudebusch and Svensson (1999). Tables 6A-C repeat the sensitivity analyses with $\overline{\Pi} = 1.01005$ for different combinations of (λ_1, λ_2). Since the actual values of the expected loss in (36) are hard to interpret in economic terms, I only report the ranking of the five alternative rules. The initial "non-structural" ranking evaluated at the estimates in Table 1 are shown in the square brackets, displaying **NIT**$\gg$**MGT**$\gg$**ATR**$\gg$**TR**$\gg$**PHIT** for all different weights (λ_1, λ_2).

It is striking that the performance of alternative rules is highly sensitive to welfare measures employed. In Tables 6A-6C, the new ranking under EL shows the two variants of Taylor rules are now running behind the two fixed targeting rules **NIT** and **MGT**. In fact, as shown in Table 6C, **ATR** and **TR** reclaim their thrones only when the non-structural measure EL is so constructed that *i)* interest rate changes are severely penalized, and *ii)* transaction technology is more sensitive to b due to higher values of ν.

PHIT, however, is the poorest performer under the new measure as well despite the fact that the measure in (36) explicitly incorporates price inflation (not wage inflation) as an argument. It ranks fourth under the current inflationary regime of $\Pi = 1.01005$. This finding corroborates the non-optimality of strict price inflation targeting in an economy with a higher degree of nominal rigidities in the labor market, as discussed in the previous subsection.

In section 4, it was demonstrated that the naive welfare measure based on first order approximate solutions would have resulted in considerable distortions in welfare calculations. What directly follows is whether such distortions are serious enough to reverse the ranking of policy rules and result in the wrong policy implications for the monetary authority. Table 7 reports how the five rules would compare with one another if they were evaluated in terms of the first order solutions. What is striking is that the two decent fixed targeting rules, **NIT** and **MGT**, are now running ahead of the two versions of Taylor rules. In fact, **TR** and **ATR** are worse than the benchmark rule, requiring positive consumption compensation!

The intuition behind the (spurious) dominance of the two fixed targeting rules is as follows. As was discussed above, fixed targeting rules incur inherent instabilities trying to force target variables on track every period. This "behind the scene" welfare diminishing feature of fixed targeting rules is not fully captured unless the first order bias terms $E\left[\widehat{\zeta}_t \mid \Omega_0\right]$ in (27) are taken into account by using second order approximate solutions in the construction of the welfare measure.

The fact that even the relative ranking of alternative rules is highly dependent upon the performance measures suggests that the monetary authority should be cautious in choosing a proper metric. Formulating loss function grounded upon the utility function of households is a justifiable way to resolve this criterion-dependency. Of course, this is the case only insofar as the assumed structure of the economy is free of controversy.

6. Welfare Analysis: Toward the Optimal Policy Rule

In this section, I construct an endogenous interest rate rule of the form

$$\widehat{R}_t = \rho \widehat{R}_{t-1} + \beta_1 \widehat{\Pi}_t + \beta_2 \widehat{MG}_t + \beta_3 \widehat{\Pi}_t^w + \gamma_1 \widehat{y}_t + \gamma_2 \widehat{y}_{t-1} \tag{37}$$

which maximizes the welfare metric EW. On the RHS of (37), the money growth rate and lagged real output are included as policy indicators in order to exploit the good performance of two fixed targeting rules **MGT** and **NIT** shown in the preceding analysis.[29] Having observing the performance of **ABM** and **ATR**, I also include wage inflation as an essential indicator. In additions to the coefficients on the RHS of (37), the reference inflation rate is also another, possibly the most important, policy variable for monetary authority.

To find the optimized coefficients for the class of the rules in (37), one has to contend with complicated boundaries defined by *i*) the need for the solution of the model to exist and be unique, and *ii*) the need for policy rules to be feasible. At each maximization step, I check whether the candidate parameters fall outside such boundaries, and assign an artificially large negative value of EW to those cases. The resulting discontinuity at such boundaries is addressed again by the optimization routine used for estimating the model.

Given the estimates of non-policy parameters in Table I, the optimized rule is

$$\widehat{R}_t = 2.0156\widehat{R}_{t-1} + 0.8429\widehat{y}_t - 0.8463\widehat{y}_{t-1} + 0.8134\widehat{\Pi}_t + 0.6584\widehat{\Pi}_t^w + 0.1791\widehat{MG}_t \tag{38}$$

with the optimal reference inflation rate of $\overline{\Pi} = 0.9952$.

The optimized rule (38), dubbed **OPT**, exhibits many features advocated in the literature as what good monetary rules should have. First, it is a deflationary policy rule: the corresponding annual rate of deflation is 1.91% which falls between the 2.93% of Friedman and 0.76% of Khan et al. (2002). Second, nominal interest rate is adjusted with "super-inertia" in the terminology of Woodford (1999). The coefficient 2.0156 on the lagged nominal rate is much higher than the estimate 0.79 of Clarida et al. (1999) over the tenure of Volker and Greenspan 1979:Q3- 1996:Q4, and 0.795 in Levin et al. (1999). According to Woodford (1999), the virtue of inertial adjustments

[29] The estimated policy rule in Levin et al. (1999) shows that historical US monetary policy over 1980:Q1- 1996:Q4 responded to not only the level but also the recent growth rate of output. In their work, the estimated coefficients on current and lagged real output are around 1 and -1, respectively.

in nominal rate is that they signal how serious monetary authority is about stabilizing its goal variables even in the distant future, exploiting the forward looking behavior of the private sector in forming their expectations. In fact, the optimal interest rate rules in Rotemberg and Woodford (1997,1999) also exhibit super-inertia: for example, in Rotemberg and Woodford (1997), the largest root 1.33 of the autoregressive polynomial for nominal rate is greater than one.

Third, **OPT** exhibits strong anti-inflationary adjustments of nominal rate. The coefficients on the nominal anchors sum up to 1.6509, which is *per se* higher than the 1.5 in the simple rule of Taylor (1993). With super-inertia, the long run degree of aggressiveness under the optimized rule is literally explosive. It is particularly noteworthy that **OPT** embodies "targeting" for wage inflation and money growth in the sense of McCallum and Nelson (1998), with coefficients 0.6584 and 0. 1791, respectively.

Fourth, the coefficients on current and real output in **OPT** show the monetary authority needs to "lean against the wind", by increasing the nominal rate in response to the increase in *growth rate* of real output as well as its current *levels*. Coupled with the inflation coefficient of comparable magnitude, those coefficients translates into a rule responding to the nominal income growth. Hence, the optimized rule (38) has the feature of nominal income "targeting," again in the usual sense.

Table 8A reports the performance of the optimized rule. Implemented with the optimal degree of deflation, the rule (38) yields considerable welfare gain over the benchmark rule with $\overline{\Pi} = 1.01005$: the households are willing to make 50.9776 units of one time sacrifice in consumption, which is more than five times the steady state consumption.

It would be interesting to see if the welfare dominance of **OPT** is still preserved in economies with higher long-run inflationary stance. Furthermore, it is in order to net out the effect of deflationary stance to see how the other features of **OPT** contribute to welfare improvement. Therefore, I compare in Table 8B the performance of **OPT** and the other five rules for different reference rates of inflation, which shows an almost consistent dominance of **OPT** for both inflationary and deflationary economies. The only exception is when $\overline{\Pi} = 1.01005$, under which **TR** outperforms **OPT** by an equivalent of 0.5593 units of one-time additional consumption.

Since the rule (37) is designed to mimic the two good targeting rules (**NIT, MGT**), it is worth asking what common features of those rules contribute to the welfare improvement by **OPT**. To get some insight, I plot the impulse responses of some key variables under four rules **PHIT, NIT, MGT,** and **OPT** toward one unit of favorable technology shock in Figure 1 to 4, respectively.[30]

It is evident that even a casual eyeball test for choosing stabilizing policies would reject **PHIT**: as displayed in Figure I, the costs of targeting the wrong variable appear as "boom-bust" responses in money growth, real money, and output, and higher volatilities in wage inflation and nominal interest rate.

Compared with **PHIT**, **NIT** generates much smoother or dampened responses (except for inflation) as displayed in Figure II: real output shows monotone initial increases, and subsequent adjustments are distributed over a very long horizon. Money growth also shows deviations of considerably smaller magnitudes and shorter length. The initial below-zero responses of inflation show that the goal of maintaining constant (up to a deterministic trend) nominal income is achieved by suppressing price level. It is worth noting that, unlike under **PHIT,** almost all the burden of nominal adjustments falls on the right variable (i.e., wage rate).

Figure III displays impulse responses under **MGT**. In comparison with **NIT**, **MGT** shows a similar degree and longevity of responses in output and inflation, coupled with slightly smaller volatilities in monetary variables such as money growth and interest rate. These findings strongly support that controlling volatilities in the monetary sector also contributes to welfare improvements,

[30] In those figures, the reference rate of inflation is set at 1.0.

which is evident from the fact that, as long as other things are held constant, the welfare metric EW decreases in the volatility of real balance (or nominal interest rate in view of money demand function (8)). Furthermore, as under **NIT**, the nearly constant responses of wage inflation suggest that chasing the "neutral" nominal variable (i.e., money growth) in effect prevents the problem in targeting the "wrong" variable (i.e., price).

The responses of the economy under **OPT** are displayed in Figure IV, where the impulse responses under **OPT** show striking resemblance to those under **NIT** and **MGT.** In particular, the two latter rules share with **OPT** *i*) the hump-shaped responses of output and real money; *ii*) very small and smooth responses of money growth and interest rate; and, most of all, *iii*) near constancy of wage inflation.

As alluded by the coefficients in **OPT**, the resemblance of impulse responses under **OPT** and (**NIT, MGT**) is not a coincident: **OPT** inherits the virtues of **MGT** and **NIT**, by incorporating money growth as an indicator and by effectively "targeting" the growth in nominal income, respectively. Also, by not putting the whole weight on price inflation in adjusting nominal rates to the short run inflationary pressure, the optimized rule shuns the undesirable feature in **PHIT** of favoring the wrong variable.

The responses of money growth and nominal rates suggest that **OPT** improves upon **MGT** by mimicking **NIT**. As shown in Figure 2, **NIT** procyclically accommodates the initial increase in real output by lowering nominal rates when a positive technology shock occurs.[31] This feature is present under **OPT** as well, although with very small magnitude. The cost of accommodation is higher volatility in money growth under **OPT** than under **MGT**.

7. Conclusion

In this paper, I have applied an estimated monetary business cycle model with nominal rigidities

[31] Ireland (1996) also argues optimal policy should be procyclical to supply shock, in the sense that positive technology shock is followed by an increase in money growth.

to evaluating performances of monetary policy rules. Performances are measured in terms of a natural metric based on the utility function of agents, and the task of accurate welfare evaluation is achieved owing to the second order approximate solution method.

The results suggest that in the presence of a higher degree of nominal rigidities in the labor market than in the goods market, strict inflation targeting cannot be an optimal policy. The optimized rule has a strict anti-inflation stance requiring that the nominal interest rate be aggressively adjusted to the increases in nominal anchors such as inflation, wage inflation, and money growth rate. Furthermore, the optimized rule is deflationary in the spirit of Friedman, with long-run deflation of 1.51% per annum. It also features a high degree of inertia, and the countercyclical adjustment of nominal rates with respect to higher real output growth.

All of these results are highly dependent upon the estimated model and the performance measure constructed from it. The price and wage adjustment scheme, degree of nominal rigidities in markets, and the way nominal money enters the present model are particularly critical features that limit the sense in which the optimized rule is really welfare improving. In particular, the instability of money demand coupled with the advent of the evermore increasing interest bearing monetary assets is an important issue to be addressed before taking the results in this paper as warranted. One suggestion by Lucas (2000) of applying Divisia monetary index is a promising way for further research to resolve this difficulty. I hope in the future work to extend the methodological framework used in this paper to examine how robust the findings in this paper are in light of such critical model aspects.

8. Appendix

8.1 Stationary Transform of the System Three different transform schemes are used to make the system stationary in a symmetric equilibrium. First, all occurrences of deflated nominal variables $(M_t/P_t, Q_t/P_t, W_t/P_t)$ are re-defined as real variables:

$$RM_t = M_t/P_t,\ RQ_t = Q_t/P_t,\quad RW_t = W_t/P_t \quad .$$

Second, real variables $(Y_t, C_t, K_t, \Lambda_t, MRS_t, RM_t, RQ_t, RW_t)$ are transformed using respective deterministic trend growth rates. For example:

$$y_t = Y_t/g^t,\ c_t = C_t/g^t,\ k_t = K_t/g^t,\quad \lambda_t = \Lambda_t/g^{[-1+a(1-\sigma)]t},\ rm_t = RM_t/g^t \quad .$$

Finally, occurrences of $(P_t/P_{t-1}, W_t/W_{t-1}, M_t/M_{t-1})$ are replaced by growth rates:

$$\Pi_t = P_t/P_{t-1},\quad \Pi_t^w = W_t/W_{t-1},\quad MG_t = M_t/M_{t-1} \quad .$$

For notational simplicity, I define

$$x_t = \left[\frac{gk_t - (1-\delta_{t-1})k_{t-1}}{k_{t-1}} - \delta_g\right],\quad \delta_g = g - 1 + \delta,\ \beta_g = \beta g^{a(1-\sigma)-1} \ .$$

8.1.1 Household Block The stationary-transformed version of the households' block of the system is given below.

$$\lambda_t = a[c_t^\nu + b_t\,(rm_t)^\nu]^{\frac{a-a\sigma-\nu}{\nu}}(1-L_t)^{(1-a_t)(1-\sigma)}c_t^{\nu-1} \tag{A1}$$

$$1 - 1/R_t = b_t(c_t/rm_t)^{1-\nu} \tag{A2}$$

$$\lambda_t\left[1+\phi_K x_{t+1}\right] = \beta_g \mathcal{Z}_{1t} \tag{A3}$$

$$\mathcal{Z}_{1,t-1} = \lambda_t\left[1 - \delta_t + rq_t + \phi_K\frac{gk_{t+1}}{k_t}x_{t+1} - \frac{\phi_K}{2}x_{t+1}^2\right] + \eta_{1t} \tag{A4}$$

$$\lambda_t = \beta_g R_t \mathcal{Z}_{2t} \tag{A5}$$

$$\mathcal{Z}_{2,t-1} = \lambda_t \Pi_t^{-1} + \eta_{2t} \tag{A6}$$

$$\begin{aligned} mrs_t &= \theta_L rw_t + (1-\theta_L)\Phi_w\left[\Pi_t^w - \Pi_{t-1}^w\right]rw_t\Pi_t^w L_t^{-1} \\ &-\beta g^{a(1-\sigma)}\Phi_w(1-\theta_L)\mathcal{Z}_{3t} + \beta g^{a(1-\sigma)}\Phi_w(1-\theta_L)\mathcal{Z}_{4t} \end{aligned} \tag{A7}$$

$$mrs_t = \lambda_t^{-1} c_t^{*a(1-\sigma)} (1-a_t)(1-L_t)^{(1-a)(1-\sigma)-1} \tag{A8}$$

$$\mathcal{Z}_{3,t-1} = \frac{\lambda_t}{\lambda_{t-1}} \left[\Pi_t^w\right]^2 rw_t L_{t-1}^{-1} + \eta_{3t} \tag{A9}$$

$$\mathcal{Z}_{4,t-1} = \frac{\lambda_t}{\lambda_{t-1}} \Pi_t^w \Pi_{t-1}^w rw_t L_{t-1}^{-1} + \eta_{4t} \tag{A10}$$

where η_t's are the martingale difference expectational errors.

8.1.2 Firms Block The equations for the decision problems of firms are given by

$$\frac{L_t}{k_t} = \frac{rq_t}{rw_t} \frac{1-\alpha_t}{\alpha_t} \tag{A11}$$

$$\begin{aligned} & \lambda_t \left[\theta_Y - \frac{rw_t}{mpl_t} + (1-\theta_Y)\Phi_p(\Pi_t - \Pi_{t-1})\Pi_t\right] \\ = \quad & (1-\theta_Y)\beta g^{a(1-\sigma)} \Phi_p \mathcal{Z}_{5t} - (1-\theta_Y)\beta g^{a(1-\sigma)} \Phi_p \mathcal{Z}_{6t} \end{aligned} \tag{A12}$$

$$mpl_t = A_t(1-\alpha_t) k_t^{\alpha_t} L_t^{-\alpha_t} \tag{A13}$$

$$\mathcal{Z}_{5,t-1} = \lambda_t \left[\Pi_t\right]^2 \frac{y_t}{y_{t-1}} + \eta_{5t} \tag{A14}$$

$$\mathcal{Z}_{6,t-1} = \lambda_t \Pi_t \Pi_{t-1} \frac{y_t}{y_{t-1}} + \eta_{6t} \tag{A15}$$

8.1.3 Other Equations Combining the budget constraint of households, aggregate profit of firms, and the government budget constraint, we get the resource constraint :

$$c_t + gk_{t+1} - (1-\delta_t)k_t + \frac{\phi_K}{2}\left[\frac{gk_{t+1} - (1-\delta_t)k_t}{k_t} - \delta_g\right]^2 k_t = y_t \tag{A16}$$

The aggregate real wage and real money stock evolve following

$$g\frac{rw_t}{rw_{t-1}} = \frac{\Pi_t^\omega}{\Pi_t}, \quad g\frac{rm_t}{rm_{t-1}} = \frac{MG_t}{\Pi_t} \tag{A17}$$

For the benchmark economy, the monetary policy rule is transformed into

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1-\rho_R)\left[\gamma_\pi \widehat{\Pi}_t + \gamma_y \widehat{y}_t + \gamma_m \widehat{MG}_t\right] + \varepsilon_{Mt} \tag{A18}$$

The exogenous shocks, stationary themselves, do not need to be transformed.

8.3 Recursive Calculations of (μ_t, Σ_t) For the sake of second order accuracy, all terms of orders higher than two may be dropped out: accordingly, only the first two terms in equation (28a) describe the evolution of the conditional variances of $\widehat{z}_{1t}$:

$$\Sigma_{1t} = F_1 \Sigma_{1,t-1} F_1' + F_2 \Sigma_\varepsilon F_2' \tag{A19}$$

where F_1 and F_2 are the matrices of the coefficients on $\widehat{z}_{1,t-1}$ and ε_t, respectively, representing the first order parts of the solution.

Recursive calculations of μ_t are more involved. The subsystem (28a) may be re-written in an expanded form as

$$\begin{aligned} \widehat{z}_{1t} &= F_1 \widehat{z}_{1,t-1} + F_2 \varepsilon_t + F_3 \\ &+ \frac{1}{2} \begin{bmatrix} \widehat{z}_{1,t-1}' F_{11}^{(1)} \widehat{z}_{1,t-1} \\ \vdots \\ \widehat{z}_{1,t-1}' F_{11}^{(N_1)} \widehat{z}_{1,t-1} \end{bmatrix} + \begin{bmatrix} \widehat{z}_{1,t-1}' F_{12}^{(1)} \varepsilon_t \\ \vdots \\ \widehat{z}_{1,t-1}' F_{12}^{(N_1)} \varepsilon_t \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \varepsilon_t F_{22}^{(1)} \varepsilon_t \\ \vdots \\ \varepsilon_t F_{22}^{(N_1)} \varepsilon_t \end{bmatrix} \end{aligned} \tag{A20}$$

where F_3 is a $N_1 \times 1$ column vector, and $F_{jk}^{(i)\prime}$s are the matrices constructing quadratic terms for the i^{th} equation in the second order solution (28a).

Taking expectation of (A20) conditional on Ω_0, I get

$$\begin{aligned} \mu_{1t} &= F_1 \mu_{1,t-1} + F_3 \\ &+ \frac{1}{2} \begin{bmatrix} tr\left(\Sigma_{1,t-1} F_{11}^{(1)}\right) \\ \vdots \\ tr\left(\Sigma_{1,t-1} F_{11}^{(N_1)}\right) \end{bmatrix} + \frac{1}{2} \begin{bmatrix} tr\left(\Sigma_\varepsilon F_{22}^{(1)}\right) \\ \vdots \\ tr\left(\Sigma_\varepsilon F_{22}^{(N_1)}\right) \end{bmatrix} \end{aligned} \tag{A21}$$

where $tr(\cdot)$ is the trace of a square matrix. One can calculate $\{\mu_{1t}, \Sigma_{1t} : t \geq 1\}$ recursively by using (A19) and (A21) jointly given some initial condition $\Omega_0 = (\mu_{10}, \Sigma_{10})$.

References

[1] Aizenman, Joshua and Jacob. A. Frenkel, 1986, Targeting rules for monetary economy, Economics Letters 21, 183-187.

[2] Amato, Jeffrey D. and Thomas Laubach, 2003, Monetary policy in an estimated optimization-based model with sticky prices and wages, Journal of Economic Dynamics and Control 27, 1181-1215.

[3] Calvo, Guillermo, 1983, Staggered prices in a utility maximizing framework, Journal of Monetary Economics 12, 383-398.

[4] Christiano, Lawrence J., Martin Eichenbaum, and Charles Evans, 2001, Nominal rigidities and the dynamic effects of a shock to monetary policy, NBER Working Paper 8403.

[5] Clarida, Richard, Jordi Gali, and Mark Gertler, 1999, The science of monetary policy: a new Keynesian perspective, Journal of Economic Literature 37, 1661-1707.

[6] Clarida, Richard, Jordi Gali, and Mark Gertler, 2001, Optimal monetary policy in closed versus open economies: an integrated approach, American Economic Review 91, 248-252.

[7] Erceg, Christopher J., Dale W. Henderson, and Andrew Levin, 2000, Optimal monetary policy with staggered wage and price contracts, Journal of Monetary Economics 46, 281-313.

[8] Feenstra, Robert C, 1986, Functional equivalence between liquidity costs and the utility of money, Journal of Monetary Economics 17, 271-291.

[9] Friedman, Milton, 1969, The Optimal quantity of money, in: The optimum quantity of money, and other essays, Aldine Publishing Company, Chicago.

[10] Fuhrer, Jeffrey C. and George R. Moore, 1995, Inflation persistence, Quarterly Journal of Economics 440, 127-159.

[11] Ghironi, Fabio, 2000, Alternative monetary rules for a small open economy: the case of Canada," Economics Department Working Paper 466, Boston College.

[12] Henderson, Dale W. and Jinill Kim, 1998, The choice of monetary policy reaction function in a simple optimizing model, International Finance Discussion Paper Series 601, Federal Reserve Board.

[13] Huang, Kevin X. D. and Zheng Liu, 1999, Staggered contracts and business cycle persistence, Discussion Paper Series 127, Federal Reserve Bank of Minneapolis.

[14] Ireland, Peter N., 1996, The role of countercyclical monetary policy, Journal of Political Economy 104, 704-723.

[15] Ireland, Peter N., 1997, A small, structural, quarterly model for monetary policy evaluation, Carnegie-Rochester Conference Series on Public Policy 47, 83-108.

[16] Ireland, Peter N. 2001. Sticky-price models of the business cycle: specification and stability, Journal of Monetary Economics 47, pp.3-18.

[17] Khan, Aubhik, Robert G. King, and Alexander L. Wolman, 2002, Optimal monetary policy, NBER Working Paper 9402, forthcoming in Review of Economic Studies.

[18] Kim, Jangryoul, 2003, Staggered contracts models of business cycle: how much nominal rigidity do we have?, Banking and Policy Studies Working Paper 2-03, Federal Reserve Bank of Minneapolis.

[19] Kim, Jinill, 2000, Constructing and estimating a realistic optimizing model of monetary policy, Journal of Monetary Economics 45, 329-359.

[20] Kim, Jinill and Sunghyun Henry Kim, 2002, Spurious welfare reversal in international business cycle models, Journal of International Economics, in press.

[21] King, Robert G. and Alexander L. Wolman, 1999, What should monetary authority do when prices are sticky?, in: John B. Taylor, eds., Monetary Policy Rules, University of Chicago Press, Chicago, 349-398.

[22] Kollmann, Robert, 2002, Monetary policy rules in the open economy: effects on welfare and business cycles, Journal of Monetary Economics 49, 989-1015

[23] Leeper, Eric M. and Christopher A. Sims, 1994, Toward a modern macroeconomic model usable for policy analysis, in: S. Fischer and J. Rotemberg, eds., NBER Macroeconomics Annual Vol. 9, The MIT Press, Cambridge, 81-117.

[24] Levin, Andrew, Volker Wieland, and John. C. Williams, 1999, Robustness of simple monetary policy rules under model uncertainty, in: John B. Taylor, eds., Monetary Policy Rules, University of Chicago Press, Chicago, 263-299.

[25] Lucas, Robert E., Jr, 2000, Inflation and welfare, Econometrica 68, 247-274.

[26] McCallum, Bennet T. and Edward Nelson, 1998, Nominal income targeting in an open-economy optimizing market, NBER Working Paper 6675.

[27] McCallum, Bennet T. and Edward Nelson, 1999, Performance of operational policy rules in an estimated semiclassical structural model, in: John B. Taylor, eds., Monetary Policy Rules, University of Chicago Press, Chicago, 15-45.

[28] Rotemberg, Julio, 1982, Monopolistic price adjustment and aggregate output, Review of Economic Studies 49, 517-531.

[29] Rotemberg, Julio and Michael Woodford, 1997, An optimization-based econometric framework for the evaluation of monetary policy, in: Ben Bernanke and J. Rotemberg, eds., NBER Macroeconomics Annual Vol. 12, The MIT Press, Cambridge, 297-346.

[30] Rotemberg, Julio and Michael Woodford, 1999, Interest rate rules in an estimated sticky price model," in: John B. Taylor, eds., Monetary Policy Rules, (University of Chicago Press, Chicago) 57-119.

[31] Rudebusch, Glenn, D. and Lars E. O. Svensson, 1999, Policy rules for inflation targeting, in: John B. Taylor, eds., Monetary Policy Rules, (University of Chicago Press, Chicago) 203-246.

[32] Sims, Christopher A., 2000, Second order accurate solution of discrete time series dynamic equilibrium models, mimeo, Princeton University.

[33] Sims, Christopher A., 2002, Solving linear rational expectation models, Computational Economics 20, 1-20.

[34] Svensson, Lars E. O, 1999, Inflation targeting as a monetary policy rule, Journal of Monetary Economics 43, 607-654.

[35] Taylor, John B, 1993, Discretion versus policy rules in practice, Carnegie-Rochester Conference Series on Public Policy 39, 195-214.

[36] Wolman, Alexander L, 2001, A primer on optimal monetary policy with staggered price-setting, Economic Quarterly 83:Fall, Federal Reserve Bank of Richmond.

[37] Woodford, Michael, 1999, Optimal monetary policy inertia, NBER Working Paper 7261.

Table 1: Estimation Results

Functional Forms	Estimates and Standard Deviations
$Y_t = A_t K_t^{\alpha_t} (g^t L_t)^{1-\alpha_t}$	A=5.5668(0.0718), g=1.0056(8.8×10^{-5})
$\beta^t U(C_t, L_t, \frac{M_t}{P_t})$ $= \beta^t \log \left[C_t^{*a}(1-L_t)^{1-a_t}\right]$	$\beta = 0.9986(0.0003)$, $a = 0.4681(0.0016)$
$C_t^* = (C_t^\nu + b_t\,(M_t/P_t)^\nu)^{\frac{1}{\nu}}$	ν=-22.7561(0.4765), b =0.0008(5.5×10^{-5})
$L_{it} = (\frac{W_{it}}{W_t})^{\frac{1}{\theta_L - 1}} L_t$	$\theta_L = 0.6888(0.0087)$
$AC_t^k = \frac{\phi_K}{2}(\frac{I_t}{K_t} - \frac{I}{K})^2 K_t$	$\Phi_k = 16.8456(1.5501)$
$\log \frac{R_t}{R} = \rho_R \log \frac{R_{t-1}}{R} + (1-\rho_M)\times$ $\left[\gamma_\pi \log \frac{\Pi_t}{\Pi} + \gamma_y \log \frac{Y_t}{\overline{Y}_t} + \gamma_m \log \frac{MG_t}{MG}\right]$ $+\varepsilon_{Mt}$	$\rho_R = 0.1395(0.0112), \gamma_\pi$=0.8042(0.0045) γ_y=4.4×10^{-6}(4.5×10^{-5}), γ_m=0.4276(0.0187) $\sigma_M^2 = 4.3 \times 10^{-5}(5.2 \times 10^{-6})$
$AC_{it}^P = \frac{\Phi_P}{2}\left(\frac{P_{jt}}{P_{j,t-1}} - \Pi_{t-1}\right)^2 Y_t$	$\Phi_p = 10.0970(0.7393)$
$AC_{it}^W = \frac{\Phi w}{2}\left(\frac{Wi}{W_{i,t-1}} - \Pi_{t-1}^w\right)^2 \frac{W_t}{P_t}$	$\Phi_w = 22.0341(0.7025)$
$\log \frac{A_t}{A} = \rho_A \log \frac{A_{t-1}}{A} + \varepsilon_{At}$	ρ_A=0.9761(0.0002), σ_A^2=0.0012(9.9×10^{-5})
$\log \frac{\alpha_t}{\alpha} = \rho_\alpha \log \frac{\alpha_{t-1}}{\alpha} + \varepsilon_{\alpha t}$	ρ_α=0.9690(0.0015), σ_α^2=0.0003(2.7×10^{-5})
$\log \frac{A_t}{\delta} = \rho_\delta \log \frac{\delta_{t-1}}{\delta} + \varepsilon_{\delta t}$	ρ_δ=0.9563(0.0012), σ_δ^2=0.0129(0.0021)
$\log \frac{A_t}{A} = \rho_b \log \frac{b_{t-1}}{b} + \varepsilon_{bt}$	ρ_b=0.9450(0.0022), σ_b^2=0.0716(0.0078)
$\log \frac{a_t}{a} = \rho_a \log \frac{a_{t-1}}{a} + \varepsilon_{at}$	ρ_a=-0.4573(0.0482), σ_a^2=0.2080(0.0302)
	$cov(\varepsilon_{At}, \varepsilon_{\alpha t}) = -0.0006(5.0\times 10^{-5})$

Table 2: Accuracy of the Solution

Endogenous Errors in	**First Order** R^2	**Solution** $F-stat.$	**Second Order** R^2	**Solution** $F-stat.$
∂K	0.0649	289.1125	0.0648	288.6361
∂B	0.0092	38.6796	0.0088	36.9829
∂W [1]	0.0046	19.2504	0.0031	12.9536
∂W [2]	0.0037	15.4701	0.0021	8.7662
∂P [1]	0.0138	58.3900	0.0030	12.5345
∂P [2]	0.0272	116.4731	0.0032	13.3728

Table 3: Performance of Benchmark Rule[32]

Ref. Inf.	**1.01005**		**1.011**		**1.0091**
I. BM					
$\widehat{R}_t =$	$0.1395\widehat{R}_{t-1}+$	$0.6920\widehat{\Pi}_t+$	$3.8{\times}10^{-6}\widehat{y}_t$	$+0.3680\widehat{MG}_t$	
EW	607.8661		607.7265		608.0061*
dC	-		2.9042		-2.9109*
II. ABM					
$\widehat{R}_t =$	$0.1395\widehat{R}_{t-1}+$	$0.6920\widehat{\Pi}_t+$	$3.8{\times}10^{-6}\widehat{y}_t$	$+0.3680\widehat{MG}_t$	$+0.3460\widehat{\Pi}^w_t$
EW	608.1288		607.9916		608.2603
dC	-5.4633		-2.6103		-8.3205

[32] Result with asterisk is feasible with a 2-SD bound around the unconditional expectation of nominal interest rate.

TABLE 4: PERFORMANCES OF ALTERNATIVE RULES

Ref. Inf.	**1.01005**		**1.0063**		**1.0025**
I. PHIT	$\frac{1}{\overline{R}-1}\widehat{R}_t=$	$\frac{1}{\overline{R}-1}\widehat{R}_{t-1}+\Delta\widehat{b}_t$	$+(1\text{-}\nu)\ \Delta\widehat{c}_t$	$-(1\text{-}\nu)\ \widehat{MG}_t$	
EW	607.5599		607.8834		607.8954
dC	6.3672		-0.3595		-0.6082
II. NIT	$\frac{1}{\overline{R}-1}\widehat{R}_t=$	$\frac{1}{\overline{R}-1}\widehat{R}_{t-1}+\Delta\widehat{b}_t$	$+(1\text{-}\nu)\ \Delta\widehat{c}_t$		
			$-(1\text{-}\nu)\ \widehat{MG}_t$	$-(1\text{-}\nu)\Delta\widehat{y}_t$	
EW	607.9744		608.5208		609.0646
dC	-2.2527		-13.6072		-24.8974
III. MGT	$\frac{1}{\overline{R}-1}\widehat{R}_t=$	$\frac{1}{\overline{R}-1}\widehat{R}_{t-1}+\Delta\widehat{b}_t$	$+(1\text{-}\nu)\ \Delta\widehat{c}_t$	$+(1\text{-}\nu)\ \widehat{\Pi}_t$	
EW	607.9720		608.5304		609.1142
dC	-2.2014		-13.8818		-25.9267
IV. TR	$\widehat{R}_t=$	$0.8\widehat{R}_{t-1}$	$+\ 1.7\widehat{\Pi}_t$	$+\ 0.01\widehat{y}_t$	
EW	608.0639		608.6387*		*Unfeasible*
dC	-4.1124		-16.0591*		
V. ATR	$\widehat{R}_t =$	$0.8\widehat{R}_{t-1}+$	$1.7\widehat{\Pi}_t+$	$0.01\widehat{y}_t$	$+0.17\widehat{\Pi}^w_t$
EW	608.1667		608.7401*		*Unfeasible*
dC	-6.2504		-18.1836*		

Table 5A: Sensitivity Analysis

σ	1.1	1.2	1.3	1.4	1.5
PHIT	510.5009 [5]	439.8141 [5]	386.0093 [5]	343.6682 [5]	309.4715 [5]
NIT	510.8381 [1]	440.0892 [2]	386.2328 [2]	343.8482 [3]	309.6140 [4]
MGT	510.7469 [3]	440.1430 [1]	386.5505 [1]	343.4870 [1]	310.5966 [1]
TR	510.6630 [4]	439.8604 [4]	386.0635 [4]	343.7981 [4]	309.7150 [3]
ATR	510.7599 [2]	439.9529 [3]	386.1526 [3]	343.8851 [2]	309.7788 [2]

Table 5B: Sensitivity Analysis (Cont.)

ν	-20	-15	-12	-8	-5
PHIT	607.6681 [5]	607.8090 [5]	607.8333 [5]	607.7123 [5]	607.3538 [5]
NIT	608.1090 [3]	608.3410 [3]	608.4654 [3]	608.5986 [3]	608.6517 [4]
MGT	608.1027 [4]	608.3239 [4]	608.4432 [4]	608.5901 [4]	608.7473 [3]
TR	608.2178 [2]	608.4858 [2]	608.6354 [2]	608.8099 [2]	608.9067 [2]
ATR	608.3201 [1]	608.5869 [1]	608.7357 [1]	608.9086 [1]	609.0035 [1]

Table 5C: Sensitivity Analysis (Cont.)

b	10^{-10}	10^{-5}	10^{-4}	10^{-3}	10^{-2}
PHIT	609.0876 [5]	608.0913 [5]	607.8300 [5]	607.5430 [5]	607.2280 [5]
NIT	609.2143 [3]	608.4072 [3]	608.1946 [3]	608.9607 [3]	608.7033 [4]
MGT	609.1882 [4]	608.3967 [4]	608.1881 [4]	608.9584 [4]	608.7059 [3]
TR	609.3986 [2]	608.5312 [2]	608.3018 [2]	608.0490 [2]	608.7703 [2]
ATR	609.4957 [1]	608.6320 [1]	608.4036 [1]	608.1518 [1]	609.8744 [1]

Table 5D: Sensitivity Analysis (Cont.)[33]

b	10^{-10}	10^{-5}	10^{-4}	10^{-3}	10^{-2}
PHIT	609.8504 [4]*	608.6501 [5]	608.2625 [5]	607.8053 [5]	607.2666 [5]
NIT	609.9169 [3]	609.0610 [3]	608.7796 [3]	608.4448 [3]	608.0462 [4]
MGT	609.7902 [5]	608.9970 [4]	608.7357 [4]	608.4240 [4]	608.0524 [3]
TR	610.1601 [2]	609.2627 [2]	608.9666 [2]	608.6136 [2]	608.1926 [2]
ATR	610.2538 [1]	609.3603 [1]	609.0655 [1]	608.7140 [1]	609.2948 [1]

Table 5E: Sensitivity Analysis (Cont.)[34]

b	10^{-10}	10^{-5}	10^{-4}	10^{-3}	10^{-2}
PHIT	610.4774 [5]	609.0797 [5]	608.3393 [5]	607.2833 [5]	605.7927 [5]
NIT	610.3910 [3]	609.6310 [3]	609.2164 [3]	608.6106 [3]	607.7259 [4]
MGT	609.8523 [4]	609.3766 [4]	609.1123 [4]	608.7204 [4]	608.1361 [2]
TR	610.6599 [2]	609.8956 [2]	609.4774 [2]	608.8652 [2]	607.9687 [3]
ATR	610.7512 [1]	609.9893 [1]	609.5725 [1]	608.9621 [1]	609.0684 [1]

[33] ν is set at -12. Results with asterisk are feasible with a 2-SD bound around the unconditional expectation of nominal interest rate.

[34] ν is set at -5.

Table 6A: Non-structural measures: $(\lambda_1, \lambda_2) = (1, 0.5)$

	σ					ν					$b=$ 10^{-x}				
	1.1	1.2	1.3	1.4	1.5	-20	-15	-12	-8	-5	-10	-5	-4	-3	-2
PHIT[5]	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
NIT[1]	1	1	1	2	2	1	1	1	1	1	1	1	1	1	1
NGT[2]	2	2	2	1	1	2	2	2	2	2	2	2	2	2	2
TR[4]	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
ATR[3]	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3

Table 6B: Non-structural measures: $(\lambda_1, \lambda_2) = (0.2, 0.5)$

	σ					ν					$b=$ 10^{-x}				
	1.1	1.2	1.3	1.4	1.5	-20	-15	-12	-8	-5	-10	-5	-4	-3	-2
PHIT [5]	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
NIT [1]	1	1	1	2	2	1	1	1	1	1	1	1	1	1	1
NGT [2]	2	2	2	1	1	2	2	2	2	4	2	2	2	2	2
TR [4]	4	4	4	4	4	4	4	4	4	3	4	4	4	4	4
ATR [3]	3	3	3	3	3	3	3	3	3	2	3	3	3	3	3

Table 6C: Non-structural measures: $(\lambda_1, \lambda_2) = (1, 1)$

	σ					ν					$b=$ 10^{-x}				
	1.1	1.2	1.3	1.4	1.5	-20	-15	-12	-8	-5	-10	-5	-4	-3	-2
PHIT [5]	5	5	5	5	5	5	5	5	5	5	5	5	5	5	5
NIT [1]	1	1	1	2	2	1	1	1	4	4	1	1	1	1	1
NGT [2]	2	2	2	1	1	2	2	2	3	3	2	2	2	2	2
TR [4]	4	4	4	4	4	4	4	4	2	2	4	4	4	4	4
ATR [3]	3	3	3	3	3	3	3	3	1	1	3	3	3	3	3

Table 7: Performances of Alternative Rules:

First Order Approx. Solutions[35]

Ref. Inf.	1.01005	1.0063	1.0025
I. PHIT			
EW	602.4210	603.0270	603.6425
dC	5.7420	-6.8564	-19.6390
II. NIT			
EW	602.9524	603.5398	604.1416
dC	-5.3098	-17.5160	-30.0074
III. MGT			
EW	602.9539	603.5397	604.1408
dC	-5.3397	-17.5145	-29.9919
IV. TR			
EW	602.5488	603.1506*	*Unfeasible*
dC	3.0849	-9.4263*	
V. ATR			
EW	602.5512	603.1521*	*Unfeasible*
dC	3.0339	-9.4564*	

[35] Results with asterisk are feasible with a 2-SD bound around the unconditional expectation of nominal interest rate. *dC* measures are relative to the benchmark rule, which is also evaluated via the first oreder approximate solutions.

Table 8A: Performance of Optimized Rules

Ref. Infla		**0.9952**		**1.01005**	**1.0063**	**1.0025**
OPT						
$\widehat{R}_t =$	$2.0156\widehat{R}_{t-1}+$	$0.8429\widehat{y}_t$-	$0.8463\widehat{y}_{t-1}+$	$0.8139\widehat{\Pi}_t+$	$0.6584\widehat{\Pi}^w_t+$	$0.1791\widehat{MG}_t$
EW		610.3226		608.1398	608.6909	609.2489
dC		-50.9756		-5.6928	-16.7500	-28.6143

Table 8B: Comparison of OPT and Alternative Rules[36]

Ref. Inf.	**0.9952**	**0.9975**	**1**	**1.01005**	**1.025**	**1.05**
OPT	610.3226 [1]	609.9634 [1]	609.5681[1]	608.1398 [2]	606.1331 [1]	603.0531[1]
PHIT	Unfeasible	604.0447 [4]	607.1164[4]	607.5599 [6]	605.7222 [6]	602.5542[6]
NIT	609.5016 [2]	609.6620 [3]	609.3711[3]	607.9744 [4]	605.9756 [4]	602.8985[4]
MGT	Unfeasible	609.9330 [2]	609.4821[2]	607.9720 [5]	605.9584 [5]	602.8871[5]
TR	Unfeasible	Unfeasible	Unfeasible	608.0639 [3]	605.9836 [3]	602.7932[3]
ATR	Unfeasible	Unfeasible	Unfeasible	608.1667 [1]	606.0886 [2]	602.9022[2]

[36] b is set at the benchmark estimate 0.0008.

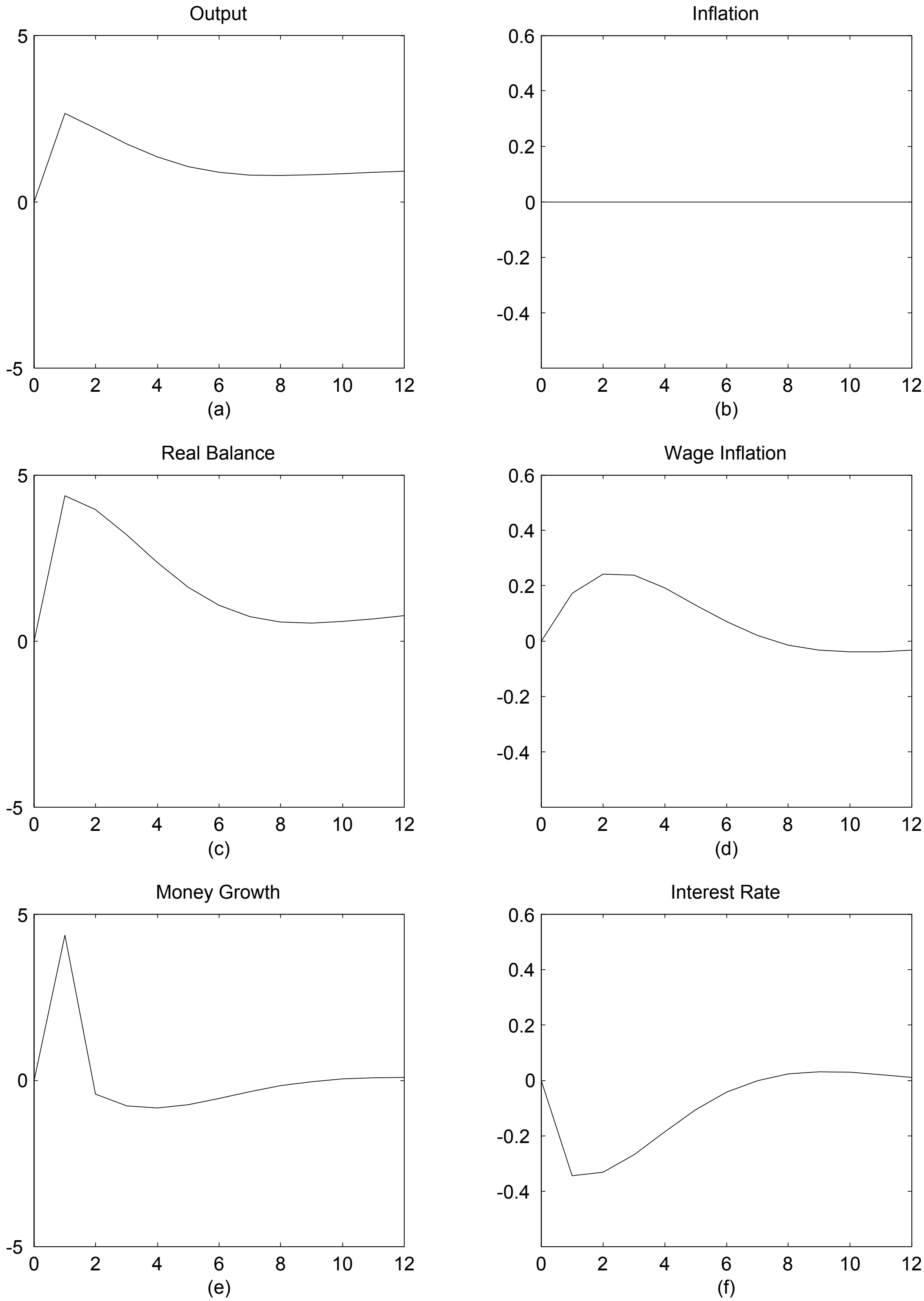

Figure 1: Impulse Responses under **PHIT**

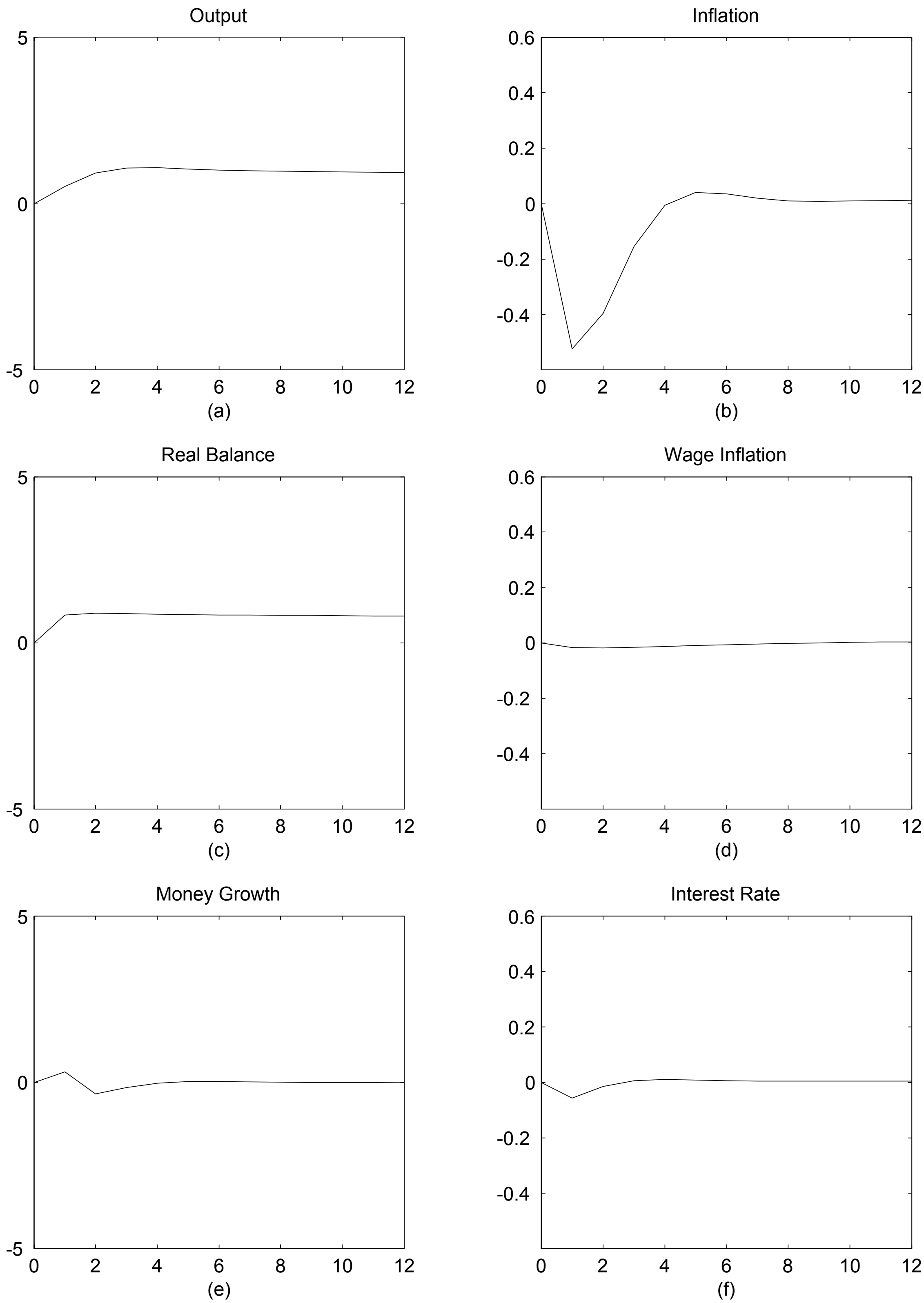

Figure 2: Impulse Responses under **NIT**

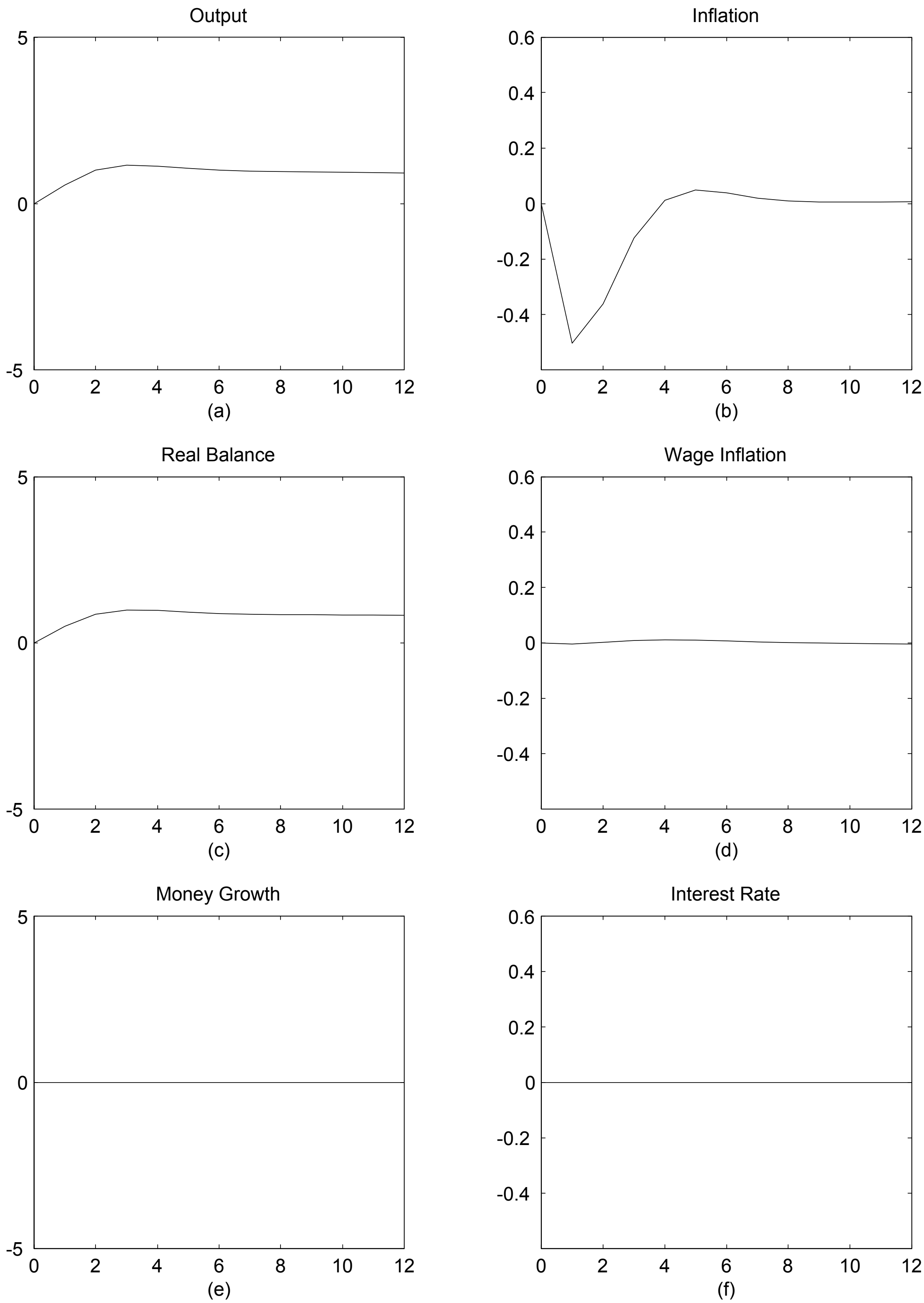

Figure 3: Impulse Responses under **MGT**

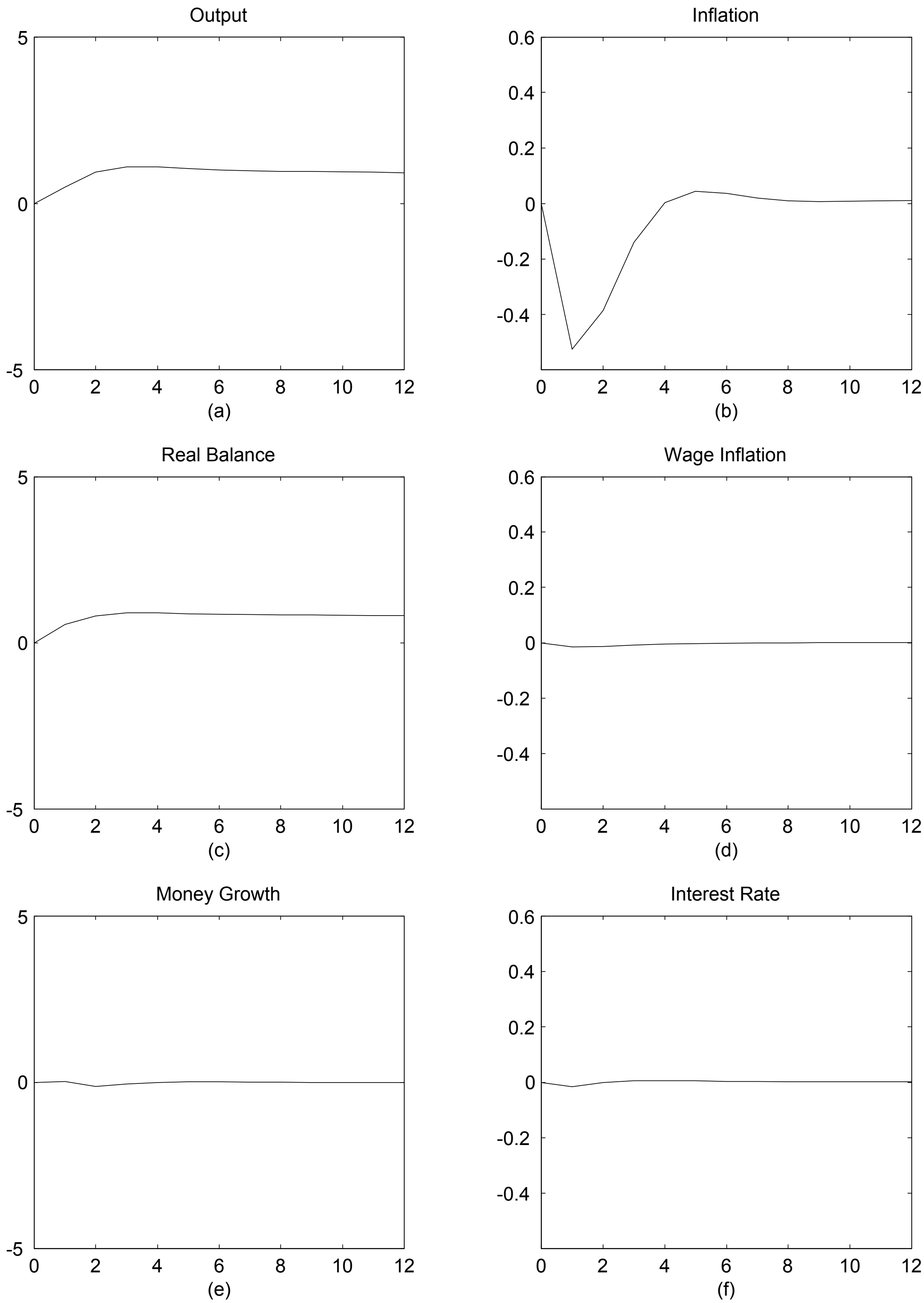

Figure 4: Impulse Responses under **OPT**