Transaction Demand for Money and Moral Hazard

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1. Introduction

The only existing tractable coherent model of fiat money is Samuelson's (1958)[1] pure consumption loan model (p.c.l.m.). However, most economists (including Samuelson himself?) do not take the p.c.l.m. as a serious model of money. This paper presents a case in favor of the p.c.l.m. as a serious model of fiat money. Indeed, it is argued that any model which implies fiat money must be essentially similar to the p.c.l.m. That is to say, the p.c.l.m. parsimoniously satisfies conditions which are necessary for any model of valued fiat money. The p.c.l.m. is, then, the preferred model of fiat money and should be the source of the null hypotheses for monetary economics.

Doubtless many will not find the above parsimony argument convincing, and for good reason. To say that the p.c.l.m. satisfies the necessary conditions for a model of fiat money does not say that the p.c.l.m. captures the important attributes of fiat money as it exists in the economy. Certainly the proposition that any model which satisfies the necessary conditions for fiat money will behave in all important respects like the p.c.l.m. is a nonprovable proposition. However, the number of possible modifications and embellishments of the p.c.l.m. is limitless, and to aimlessly produce them is an unproductive exercise.

A common specific criticism of the p.c.l.m. is that it involves only the store-of-value, not the transaction, motive for money holding. This criticism has force only when coupled with the presumption that including a transaction motive substantially alters the implications of the p.c.l.m. This paper answers this particular criticism in two ways. First, it is not at all obvious that the p.c.l.m. fails to include the transaction motive. Second, two models are presented which by anyone's definition should be considered transaction models but which are modifications of the p.c.l.m. and which have the same implications for at least some important questions.

The new models presented in this paper belong to the class of "informa-

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[1]Author names and years refer to the works listed at the end of this book.

tion" models of money (see Brunner and Meltzer 1971). Fiat money is used in transactions because the information costs of other means of transacting are too high. In the models presented here, borrowing and lending is the alternative means of transacting and the cost arises because of moral hazard. Moreover, it is argued that in an important sense these models exhaust the possible "information" models of fiat money.

Three problems which any model of valued fiat money must solve are the possible dominance of money by contracts, seigniorage, and the assignment of a terminal value to money. In the next section we describe the first problem and discuss its relation to the transaction motive of holding money. Then we turn to the p.c.l.m. and show how it solves all three problems. Finally, we describe two new models which solve the first problem in a manner different from that of the p.c.l.m.

2. Dominance by Contracts

Fiat money is money which is intrinsically worthless; it does not enter utility or production functions and is not automatically convertible into something which does. In the usual (Arrow-Debreu) exchange problem, individuals are endowed with varying quantities of several goods which they then trade to reach a Pareto optimal set of consumption bundles. The transformation from endowments into consumption bundles is assumed costless (reversible). There is said to be a complete market if all possible exchanges of goods can be made. In such complete markets there is no role for fiat money.

To introduce fiat money the market must be made incomplete; frictions must be added to the model. Certain sequences of bilateral trades are made impossible or costly, as in Shapley and Shubik 1977. In general, an object is being used as money for transaction purposes if it is used in a trade but rather than consume it the recipient trades it again. Fiat money can be introduced into these transaction models of money by making all goods which enter utility functions costly to transfer and by introducing an object, fiat money, which is not costly to transfer.

Typically, transaction costs are introduced into the technology of the physical exchange of goods, for example, the "trading post" models in Shapley and Shubik 1977. There are innumerable possible nonlinearities in the technology of exchange that can yield the result that all goods are not exchanged at a single geographic place and point in time. Moreover, in a model with costs of exchange (linear or otherwise) a sequence of bilateral exchanges is often more costly than a single multilateral exchange.

Such costly physical exchange of goods is not, however, necessary or sufficient for the existence of money. First, let us consider sufficiency. The individuals involved in a sequence of exchanges can sign a contract for the delivery of goods at the efficient place and time. If such contracts are costless, the transactions made feasible by them include as a subset those made feasible by money (see Brunner and Meltzer 1971, p. 785). The outcome of any set of exchanges involving money can be achieved through contracts for multilateral exchange without the use of money. Therefore, for money to have value it is necessary that such contracts be costly or infeasible. That costly physical exchange is not necessary for a fiat money equilibrium is demonstrated by the p.c.l.m. which does not involve such costs. The introduction of frictions is correct, but the crucial frictions for valued fiat money involve contracts, not the technology of the physical exchange of goods.

Money makes certain transactions feasible or less costly. If the set of possible transformations of endowments without money is binding in the direction affected by money, money has value.

The problem of dominance of money by contracts and the question of transaction versus store-of-value motives for holding money are closely related. To illustrate this we consider Hahn's (1973a) distinction between models in which money is "essential" and those in which it is "inessential." Goods can differ both in their physical attributes and in their time dimension. That is to say, two goods which are identical except in their date of production can be treated as separate goods. Naturally, goods which do not overlap in time cannot be physically exchanged one for the other. If transactions are limited to such exchanges, then money can be introduced into the market to allow transactions between goods which do not overlap in time. Hahn objects to such store-of-value models in that in them money is not "essential." That is to say, if one treats time like any other attribute and assumes that trade occurs in a single market, then the role of money disappears in such models. Hahn's objection to the store-of-value models is a special case of the observation that it is frictions in contract writing that are crucial to the existence of money. If contract writing is impossible or costly, then the inability to physically exchange goods that do not overlap in time is a constraint on the possible transformations of initial endowments, a constraint which can generate money holding. If contracts are costless, then the constraint on physical exchanges is irrelevant and money is "inessential."

3. The Pure Consumption Loan Model (p.c.l.m.)

Now let us consider a version of the Samuelson pure consumption loan model. First we show how it solves the dominance by contracts problem, and then we consider the seigniorage and terminal value problems.

Time is discrete and is a double-countable infinity of periods; time has no beginning or end. Each period, N identical individuals are born, and they live two periods. In their first period of life they are endowed with an amount K of a transferable but nonstorable consumption good, that good being the same in every period. In their second period of life they are endowed with nothing. Individuals maximize $U(C_1,C_2)$ where $U(\cdot)$ is a utility function with the usual properties, and C_1 and C_2 are the individual's consumption of the consumption good in the first and second periods of life, respectively.

In this model no trades are possible without money because *contracts* with the yet to be born are impossible. The time sequence imposed on the goods does matter in an essential way. The model can be viewed as a single market in which certain contracts, the only desirable contracts, cannot be written. There is no need to interpret the index as time, although doing so provides a good explanation of why the contracts cannot be written. If fiat money is imposed on the model, then individuals can trade first-period endowment for second-period consumption without ever contracting with the unborn, and with simple restrictions on the utility functions, for example, $U_2(C_1,0) = \infty$, there is a valued fiat money equilibrium.

One crucial element of the p.c.l.m. is the fact that all generations cannot get together and write contracts. However, the impossibility of writing contracts is not the only attribute of the p.c.l.m. which makes it a viable model of fiat money. Shell (1971) has noted that the *seigniorage* problem is central to the inefficiency of competitive equilibrium in the p.c.l.m.[2] This is a problem which must be addressed by any model of valued fiat money. Generally it is assumed that fiat money is costless to produce. Therefore, the only market solution is for fiat money to be without value. Valued fiat money is not a market solution, so it must be imposed from outside the market and the market participants must not be allowed to produce it. In the p.c.l.m. this problem is avoided by each generation being unable to produce fiat money but getting fiat money from the previous generation in exchange for goods.

There is a third problem which must be addressed by any model of fiat money. When the market closes someone must be holding the money. Therefore, that person must have been willing to make a transaction of goods for money or must have foregone a transaction of money for goods when the market was operating. Since fiat money by definition has no intrinsic value, this behavior requires explanation. In the p.c.l.m. a generation sells its money holdings for goods to the next generation next period; in other words, the market never closes. This problem we refer to as the *terminal value problem*.

Thus, the p.c.l.m. very parsimoniously handles necessary problems for a model of money: money is given an advantage over contracts (the latter are infeasible), and the seigniorage and terminal value problems are solved. One difficulty with the model is the interpretation of it as a store-of-value not a transaction model of money. We have already seen that by Hahn's definition the p.c.l.m. is not a store-of-value model. The reason the p.c.l.m. is given this store-of-value interpretation is that in the model people hold money as a means of saving for their retirement. This criticism of the p.c.l.m. does have appeal. After all, the reason many transactions are handled in money rather than by contract is not that some of the potential contractors are not yet born. And if an implication of the model is taken to be that transactions in money will only be of that sort, the model is rejected. In other words, this is an objection to the way the p.c.l.m. handles the dominance by contracts problem.

4. Two Alternative Models

To respond to this criticism we turn to some alternative "information" models of fiat money. In these models we use the method of the p.c.l.m. to solve the seigniorage and terminal value problems. That is to say, the models are overlapping generations models. However, the models solve the dominance by contracts problem in a different manner. People live many periods, and the reason that money exists is not because they cannot contract with the unborn, but because contract writing is costly or impossible.

4.1. *Model I*

This first model is structured so that contracts are impossible, yielding a valued fiat money equilibrium. Except as noted below, the model is identical to the p.c.l.m. Suppose N individuals are born each period and live $2n$ periods. $N/2$ have endowment stream $\Omega^1_{1\times 2n} = k, k, \ldots, k$, and $N/2$ have endowment stream $\Omega^2_{1\times 2n} = K, k, K, k, \ldots, K, k$, where $K > k$. All individuals

[2] In a model with costless contracts this seigniorage problem can arise if there are a countably infinite number of individuals. There is no Nash equilibrium set of contracts in such a model. Perhaps this could be used as an explanation of why a fiat money solution is imposed by a government, but it does seem a farfetched explanation for the existence of valued fiat money.

have the identical concave utility function $W(C_{1\times 2n}) = W(C_1, \ldots, C_{2n}) = U(C_1) + \ldots + U(C_{2n})$.

To consider the possibility of contracts, we must introduce more structure. Our bankruptcy law is that no one can be reduced below k. Moreover, people are unidentifiable, except as to having made a commitment last period to give or receive goods this period. The only possible contracts are, then, one-period contracts. But there is no Nash equilibrium with one-period contracts either. For suppose there were. The optimal strategy for a deviant poor person is to offer to supply an infinite amount of goods tomorrow for goods today. If this is an equilibrium strategy, then the probability of a rich person getting a legitimate contract is zero, so the demand for contracts is zero. As another way to see this, consider a futures market. Suppose the promise of 1 unit tomorrow has price P in terms of goods today. For $P > 0$, all the poor will offer an infinite amount, so $P = 0$. We have a moral hazard explanation for the impossibility of contracts.

The stationary fiat money solution is that the $N/2$ poor consume $C^1_{1\times 2n} = k, \ldots, k$ and don't use money, and the $N/2$ rich consume $C^2_{1\times 2n} = (K+k)/2, \ldots, (K+k)/2$ and do use money. The nonmonetary equilibrium is $C^1 = \Omega^1$, $C^2 = \Omega^2$. Without moral hazard and with a futures market, the nonmonetary equilibrium is $C^1_{1\times 2n} = k, \ldots, k$ and $C^2_{1\times 2n} = (K+k)/2, \ldots, (K+k)/2$, and there is no valued fiat money equilibrium.

In this model there is valued fiat money and contracts cannot exist. We now modify the model to have borrowing and lending be costly and to allow the coexistence of valued fiat money and borrowing and lending.

4.2. *Model 2*

N individuals are born each period, they live $2n \geq 4$ periods, and all have endowment stream $\Omega_{1\times 2n} = k, K, \ldots, k, K$. Because of the way money is introduced into this market—purchase from the previous generation—there is no way money alone can yield an optimal distribution of consumption for individuals. Only by borrowing can they increase first-period consumption.

Assume that U is unbounded above. Unless it is possible to verify that individuals can make good on their promises, no contracts are possible for the same reason as in Model 1. Instead of assuming that no such verification is possible, as before, let us assume a verification technology of a particular simple and unrealistic form. For every unit of good promised tomorrow it costs v units of goods today to verify that the borrower will not be bankrupt next period. Since offering to borrow is costless, the only solution is for all borrowers to have their contracts verified. Clearly, if $v = 1$, there will be no borrowing because the equilibrium rate of interest is nonnegative (zero actually). However, there is a stationary valued money equilibrium. If $v = 0$, a futures market yields the optimal consumption stream and there is no demand for money. Is there an intermediate value of v such that borrowing and lending and valued fiat money can coexist? While the proof is rather tedious (see the Appendix), the answer is that the model can be rigged so that this is so. If v and $U(\cdot)$ satisfy

$$\frac{U'(K)}{U'[(k+K)/2]} > (1-v) > \frac{U'[(k+K)/2]}{U'(k)}$$

then there is an integer I such that if $n \geq I$, there is such an equilibrium.

Borrowing and lending and money can coexist in Model 2 because individuals are poorly endowed at birth and because people live long enough that desired lending exceeds borrowing. However, the p.c.l.m. can be similarly modified as shown in Wallace (this volume). If some individuals are endowed in their second period of life but not their first, yet aggregate endowment is still larger in the first period, borrowing and lending can coexist with money holding in the p.c.l.m. The advantage of Model 2 in this regard is only that it assumes identical endowments and has costly contracts rather than a mix of costless (within-generation) and impossible (between-generation) contracts.

We have, then, models of fiat money in which fiat money exists because individuals cannot costlessly be known to be able to meet their contractual obligations. The transactions which occur do take place through time, so it still may be objected that these are store-of-value, not transaction models. However, the important point here is that money enters not because contracts with the unborn are impossible, but because contracts with anyone are impossible or costly to police.

To make our case that these are not store-of-value models more convincing, some simple further modifications can be sketched. Suppose, for example, that individuals live n periods, not $2n$, but there are two goods in each period, goods 1 and 2. Let there be two kinds of people with endowments $\Omega^1_{2\times n} = (k,K)', (k,K)', \ldots, (k,K)'$, and $\Omega^2_{2\times n} = (K,k)', (K,k)', \ldots, (K,k)'$, respectively. Suppose further that the markets for each of the two goods must be physically separate, there is no direct exchange, one cannot be two places at once, and the markets for the two goods operate in sequence. Then as long as contracts cannot be costlessly verified across markets, money and borrowing and lending can coexist as in Model 2. In this model money is held for between-period transactions by the first group of individuals but is held for within-period transactions by the second group of individuals. Note that without the ordering of markets or some other scheme to limit velocity money dominates borrowing and lending, but money has limiting value zero. The exchange of goods today for goods tomorrow is not desirable, and the terminal value problem reappears. Or consider a model with identical individuals in a world of three goods with endowment path $\Omega_{3\times n} = (k,K,k)', (K,k,K)', (k,K,k)', \ldots, (k,K,k)'$.

Any appearance of a store-of-value nature of the demand for money in these models comes from the overlapping generations structure. One can use some other means of solving the seigniorage and terminal value problems —for example, appropriate government intervention — and have multigood models that remove even this slight appearance of a store-of-value role of money. In such multigood models it is not important to the existence of valued fiat money that markets be ordered, although the coexistence of money and borrowing and lending requires that money not dominate the latter. But efforts in this direction do seem misguided in the face of the empirical fact that all actions take place through time. Moreover, by eliminating the time dimension in the model one removes the possibility of analyzing some problems of interest, for example, the distortions of inflation.

One objection to the above models is that they do not exhaust the set of models which generate valued fiat money by assuming that income streams are not known with certainty and contracts are not enforceable. Specifically, it could be argued that moral hazard is not the explanation for the use of money, but that uncertain income streams and costly information are. This objection does not stand up under scrutiny.

Suppose the problem is not the individual making bad faith contracts but the individual having a random income stream. With complete markets and costless knowledge of the stochastic structure this would not matter; the individual would just trade contingent claims. Suppose learning about the stochastic structure is costly. If transactions are limited to bilateral exchanges, then an individual's promises would not be used for transaction purposes in a series of exchanges. But this is not relevant, for if costless contracts are available, a single multilateral exchange would be negotiated anyway. Also, with scale technologies, or interdependent production costs at a point in geographic location and time, promises are not traded for goods at a single point. But this also does not explain the need for money. Multiple exchange contracts can be negotiated with the intermediary engaged in evaluating such promises. It may be that such contracts are costly to organize, not to verify. However, organization costs can be modeled just as verification costs are.

This is not to say that uncertain endowments are not interesting in their own right. For example, in the above models there is a unique rate of return at which individuals diversify between money and loans, and at this rate the portfolio is indeterminate. With stochastic repayment there may be many rates of return consistent with diversification, each implying a unique portfolio. However, such randomness can easily be included in the p.c.l.m.

The above models in an important sense exhaust the set of "information" models of money, models in which fiat money dominates contracts because of uncertain returns, costly information, and costly organization and enforcement of contracts. Yet these models are simple modifications of the p.c.l.m., and by almost anyone's definition they involve the transaction motive. Does the inclusion of transaction motive affect the important implications of the models? It would seem not. Let us consider the major properties of the p.c.l.m. as treated by Wallace (in this volume). As the reader can easily verify, in the above models, as with the p.c.l.m., dominance in return distribution by another asset drives out fiat money; there is a nonmonetary equilibrium; there is the possibility of multiple monetary equilibria with different rates of inflation; and a costless endowment tax is Pareto superior to printing money to cover deficits. Also like the p.c.l.m., with only a change in some details the models can be included in the Lucas (1972) incomplete information model, the Kareken-Wallace (1978) country-specific fiat money model, the Bryant-Wallace (1979b) model of open market operations, or the Bryant (1978) model of depression.

5. Summary

The p.c.l.m. parsimoniously handles three problems essential to a model of fiat money. Contracts which can substitute for money are not feasible, and the seigniorage and terminal value problems are solved. The p.c.l.m. should, then, be the source of null hypotheses concerning the behavior of fiat money in the economy.

The objection that the p.c.l.m. is not a proper model of fiat money because it fails to capture the transaction motive of holding money is invalid. We have produced "information" models of fiat money, models which are unambiguously transaction models but which are modifications of the p.c.l.m. with the same important properties. Moreover, these models exhaust the possible "information" models of money, which suggests that searching for further modifications or alternative models of fiat money is not now a productive activity.

Appendix

To prove

(A1) $$\frac{U'[(K+k)/2]}{U'[k]} < (1-v) < \frac{U'[K]}{U'[(K+k)/2]}$$

implies that if individuals live long enough, valued fiat money and borrowing and lending can coexist in Model 2.

Let $\Omega_{1\times 2J} = (k, K, \ldots, k, K)$ where J is a large integer. It is easily shown that interest on loans is zero in a stationary monetary equilibrium and that the borrower pays the verification cost. Also, the individual borrows only in odd periods of life and lends or acquires money only in even periods. The individual's problem can be written as

$$\max \sum_{i=1}^{2J} U(C_i)$$

subject to

$$C_{2j+1} = k + l_{2j} + m_{2j} + (1-v)b_{2j+1} \qquad j = 0, \ldots, J-1$$

$$C_{2j+2} = K - l_{2j+2} - m_{2j+2} - b_{2j+1}$$

$$l_0 = m_0 = b_{2J} = 0$$

where l, m, and b are lending, money holding, and borrowing. Lending and money holding are perfect substitutes to the individual. The first-order necessary conditions are

(A2) $$(1-v)U'(C_j) \leqslant U'(C_{j+1}) \quad = \text{if } b_j > 0 \qquad j=1,\ldots,2J$$

(A3) $$U'(C_j) \geqslant U'(C_{j+1}) \qquad = \text{if } l_j \text{ or } m_j > 0.$$

First we prove that the left-hand side of (A1) implies there is borrowing in the first period. Suppose there is no borrowing in the first period ($C_1=k$). Since (A3) implies that consumption is nondecreasing, and consuming K from period 2 on is not feasible, the individual must lend or hold money in period 2. Then (A3) implies $C_2 = C_3$ or $K - l_2 - m_2 = k + l_2 + m_2 + (1-v)b_3$. This implies $C_2 \geqslant (K+k)/2$. But then

$$(1-v)U'(C_1) = (1-v)U'(k) > U'[(K+k)/2] \geqslant U'(C_2)$$

by the left-hand side of (A1), which violates (A2).

Next we show that there is a first odd-numbered period in which the individual does not borrow. Suppose in odd-numbered periods up to $2j-1$ the individual does borrow. By iterative substitution of (A2) and (A3), $U'(C_{2j-1}) \leq (1-v)^{j-1}U'(C_1)$. As j becomes large, $U'(C_{2j-1}) \rightarrow 0$, but this is impossible as it implies that consumption grows without bound which clearly is nonoptimal (K is an upper bound).

Now we show that if there is no borrowing in period $2j-1$, there is no borrowing in period $2j+1$. The person who did not borrow in period $2j-1$ must lend or hold money in period $2j$. Then (A3) implies $C_{2j} = C_{2j+1}$ or $K - l_{2j} - m_{2j} = k + l_{2j} + m_{2j} + (1-v)b_{2j+1}$. This implies $C_{2j+1} \geq (½)[K+k]$. However, $C_{2j-2} \leq K$. This and the right-hand side of (A1) imply

$$(1-v)U'(C_{2j+1}) \leq (1-v)U'[(K+k)/2] < U'[K] \leq U'[C_{2j+2}]$$

which implies $b_{2j+1} = 0$ by (A2).

So there is a period beyond which the individual does not borrow. But lending or acquiring money in even periods continues. Therefore, if J is large enough, the individual's lending or holding money exceeds her or his borrowing. Because this is a representative individual, aggregate lending and money holding exceeds aggregate borrowing, and there is a valued fiat money equilibrium. Since all individuals borrow in their first period of existence, valued fiat money and borrowing and lending coexist.