Models of Government Expenditure Multipliers

State-of-the-art modifications to the standard neoclassical model increase predictions of the fiscal multiplier, but still not close to what economists think is the actual value

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Abstract
In this note, we demonstrate and analyze the inability of standard neoclassical models to generate accurate estimates of the fiscal multiplier (that is, the macroeconomic response to increased government spending). We then examine whether estimates can be improved by incorporating recently developed theory on demand-induced productivity increases into neoclassical models. We find that neoclassical models modified in this fashion produce considerably better estimates, but they remain unable to generate anything close to an accurate value of the fiscal multiplier.
Introduction

There is a vast empirical literature studying the “fiscal multiplier,” the response of macroeconomic variables to an increase in government spending. A summary of this literature is that the multiplier—defined as the response of gross domestic product to an exogenous change in government expenditures—is somewhere in the range of 0.7 to 1.0 (see Hall 2009 for a discussion of the empirical findings) and perhaps even higher in extraordinary times like ours with a very low nominal interest rate and a very high unemployment rate. Put more concretely, if the federal government were to increase spending by $1 billion, GDP would increase by between $700 million and $1 billion.

Government expenditure provides a variety of goods and services (public roads, national representation and the like), many of which benefit a wide range of people regardless of who provided the revenue to pay for them and thus are unlikely to be privately provided. Policymakers also view public expenditures as a fiscal tool to affect macroeconomic variables such as output and employment. This function has particular relevance during recessions, when policymakers seek mechanisms that can restore employment and economic growth to healthy levels.

To assess the usefulness of government expenditure as a fiscal tool, we need to weigh the costs of fiscal expansion that arise because of the higher taxes that are eventually required against the benefits generated from higher government spending in terms of higher output. This requires explicit models, suitable for policy analysis, that faithfully capture the relevant features of the economy, and this realism can be judged by seeing whether a given model is capable of generating the fiscal multiplier seen in the data.

At least until recently, the workhorse of both macroeconomics and public finance has been the real business cycle model, also referred to as the standard neoclassical model, or SNM. However,
this model of the economy is not able to deliver realistic results for the fiscal multiplier—that is to say, the multiplier values the SNM predicts are close to 0, nowhere near the estimates provided by actual data (again, between 0.7 and 1). In other words, according to the SNM, an increase in government spending would have no discernible effect on economic output, as it will be accompanied by sharp reductions in consumption and investment. Clearly, something is missing from the SNM.

Another type of macroeconomic model, referred to as New Keynesian, is able to yield a multiplier in the range of the empirical literature. But at the same time, these New Keynesian models generate other unrealistic results, such as a decline in the markup ratio of price over cost when output rises, and a dramatically procyclical labor share. The data show the opposite: Markups tend to rise during economic booms and decline in recessions (again, Hall 2009), while labor share tends to shrink in expansions. Unfortunately, then, these models also fail to faithfully reproduce the macroeconomy and so are of limited use to economists or policymakers in designing fiscal policies.

Other models that may be helpful in generating a realistic multiplier use frictions in labor markets to generate involuntary unemployment, but these have yet to be fully developed.

**Seeking better results**

In this note, we briefly explore the implications for the fiscal multiplier of a family of models developed in some of our recent research (Bai, Rios-Rull and Storesletten 2011, Dyrda, Kaplan and Ríos-Rull 2011 and Dyrda and Ríos-Rull 2012) that, while squarely in the neoclassical tradition, imply changes in some of the main properties of neoclassical models.

The impact of an increase in government expenditure on output depends on several factors, and key among these is the extent to which government spending replaces or “crowds out” private consumption and investment. If government spending simply takes the place of private spending that would have occurred in its absence, then there will be no net effect on total output, and the multiplier is 0.

However, a multiplier of 1 results when increased government spending does not induce a reduction in private consumption and investment, but rather adds to it. But, of course, increases in government expenditure have to be paid for eventually, and the effects of this burden depend on the duration of the increase in expenditures and on the form of taxation. Throughout this note, we will explore the effects of short-lived additional public expenditures that are paid for in a very efficient manner, with lump-sum taxes.

Our research suggests that models in the neoclassical tradition can produce higher estimates of the fiscal multiplier via two channels. Dyrda, Kaplan and Ríos-Rull (2011) demonstrate the
importance of adjusting neoclassical models to increase the response of hours worked to temporary changes in wages. Model parameters of this responsiveness—that is, the change in labor supplied by workers when wage rates rise or drop temporarily—should be consistent with new evidence about how household size changes in response to macroeconomic change. In contrast to the traditional notion that people can be identified with households, our data analysis reveals that people’s living arrangements change often in ways that are partly synchronized with the business cycle: During recessions, households take in more members as young adults move into (or delay departure from) their parents’ home. Seniors may also move in with middle-aged relatives, and unrelated singles may form group households.

More importantly, in terms of the fiscal multiplier, Bai, Ríos-Rull and Storesletten (2011) articulate a neoclassical model in which movements in productivity are not the result of technological shocks, but of greater willingness to spend or higher demand. As we will see, this explanation gives solid theoretical rationale for macroeconomic expansion when government spending increases.

In the next section, we describe reasons behind the inability of the SNM to generate realistic values of the fiscal multiplier. We then move to a brief discussion of our findings about fluctuations in household size that affect the responsiveness of hours worked to changes in wages (the “Frisch elasticity”) and how that affects the fiscal multiplier. Finally, we discuss neoclassical models with “demand-enhanced” productivity based on our earlier and ongoing work (Bai, Ríos-Rull and Storesletten 2011 and Dyrda and Ríos-Rull 2012) and the extent to which these modifications to the SNM can deliver values of the fiscal multiplier closer to the empirically documented range. Along the way, we make some observations about what could be capable of generating fiscal multiplier values in the actual empirical range without drastic departures from the neoclassical tradition.

The failure of the standard neoclassical model

Two features of the SNM are central to its estimates of the fiscal multiplier; both concern household decisions about how many hours they’ll work. And both ultimately lead to the model’s inability to realistically predict the fiscal multiplier.

The first is that the number of hours worked is the outcome of choices that households make in response to wages and interest rates. That hours worked respond to wage rates is rather intuitive, but households also adjust labor in response to interest rates because, among other things, interest rates affect the value of the present relative to the future. Through so-called substitution effects, hours worked are then affected.

The model’s second key feature: The two inputs essential to producing economic output—capital and labor—are subject to diminishing returns, and each is paid its marginal product (the market
value of the last unit of output it contributes). So, the more units of labor or capital that are added to the economy, the less additional output will be obtained. Therefore, as the supply of capital increases, its marginal product (the interest rate) decreases. The same applies to hours worked and the wage.

These features, and their complex interaction with other economic factors, mean that increasing the number of labor hours in order to increase economic output—the mechanism through which a fiscal multiplier works in the neoclassical world—ultimately requires that both consumption and investment decrease in the short term.  

But to estimate the net effect of a fiscal multiplier, the question is, How large are these drops? That is, to what degree will the positive impact of government expenditures on economic output—through increased labor supply—be negated by the short-term decreases in consumption and investment?

To answer this question quantitatively, economists feed specific parameter values into the model’s equations. The values are selected to match key statistics in the U.S. economy. So, government expenditure is usually set at 16 percent of GDP, a typical figure for the United States. Investment and private consumption are about 19 percent and 65 percent of GDP, respectively. Labor income is roughly two-thirds of total GDP. The nominal interest rate is 4 percent. Another important statistic is the “intertemporal rate of substitution,” a term economists use for people’s willingness to forgo consumption now in order to consume in the future; this is usually set at 0.5. And people are assumed to work 30 percent of their nonsleeping time. All of these parameter values are widely agreed upon by macroeconomists.

The only controversial number in this model is the one that describes the willingness of people to work longer than usual if the wage is temporarily high—again, the Frisch elasticity. (See Chetty, Guren, Manoli and Weber 2011 for a discussion of the debate among economists as to its magnitude.) Although disputable, a very conventional value used in such models is 0.7. The value is calculated using a married couple as the notion of household (see Heathcote, Storesletten and Violante 2010).

With these parameter values, the model predicts that a temporary increase in government expenditures of 1 percent of GDP will have very little net impact. It results in just a 0.023 percent increase in GDP. Why so little effect? The primary cause is a dramatic drop in investment. A 1 percent increase in government expenditures leads (according to this model) to investment levels falling from 19 percent of GDP to 18.04 percent, wiping out 96 percent of the 1 percent boost in government spending.

Consumption also drops, but just slightly, falling from 65 percent to 64.98 percent of GDP. And, indeed, the increase in government spending barely increases the number of labor hours supplied
by households, from 30 percent of total available time to 30.01 percent. Put in more practical terms, the neoclassical model predicts that if government boosts public spending by 1 percent of GDP for three months, the average adult would work no more than 10 minutes longer.

**First candidate for a bigger multiplier: A higher Frisch elasticity**

How can the model achieve better results, closer to actual estimates of the impact of a fiscal multiplier (again, in the range of 0.7 to 1, rather than the 0.023 just obtained)? The lynchpin appears to be labor’s response to an increase in wages, the Frisch elasticity. A higher figure would have dramatic influence over the model’s estimate of the fiscal multiplier.

As mentioned above, a conventional if debatable value used for Frisch elasticity is 0.7. But our recent work, Dyrda, Kaplan and Ríos-Rull (2011), makes the case for a higher estimate. The gist of our argument is based on the fact that standard measurements of the Frisch elasticity of labor assume that households are stable in that they keep the same characteristics over time (for instance, that married people stay married). But, of course, many people are not “stable” in this sense; young people move in and out of their parents’ home, some people divorce and become two households and the like.

In our work, we show that those “unstable people” display a higher Frisch elasticity than the 0.7 figure based on a stable population. In addition, we argue that the concept of the household itself is not set in stone, and we document that the size of households is larger in recessions as many people move in with family or friends to bear the harder times. When they do so, they work even fewer hours than they would normally. According to our calculations, properly accounting for unstable people and their movements in and out of households changes the Frisch elasticity of the economy as a whole from 0.7 to 1.1.

When we apply this higher value of the Frisch elasticity of labor to the standard neoclassical model, we obtain a higher multiplier. Indeed, the multiplier goes up by 31 percent. Unfortunately, this improvement is less impressive than it sounds: Given the low initial value of 0.023, a 31 percent increase yields very little: a prediction of just 0.029—still nowhere close to empirical measurements in the range of 0.7 to 1.

**Why standard neoclassical models fail**

Neoclassical models contain two “first-order conditions” (or mathematical requirements) that together determine the response of workers to increases in government expenditures. Understanding these conditions helps explain why neoclassical models generate such low estimates of the fiscal multiplier.
One of these conditions—the “intra-temporal”—links variables such as working hours, consumption levels and wage rates in the here-and-now: “today.” So, the “intra-temporal first-order condition” says what all hourly workers know: Consumption by a household is partially determined by the prevailing wage rate times the number of hours that household members work at that wage.

But because labor in neoclassical models is paid its marginal product and is also subject to diminishing returns, prevailing wages in the economy as a whole will decline as the labor force grows, discouraging households from offering more labor hours. To increase working hours (in order to generate more output) as wages decline, a fall in consumption is necessary.

But to increase output in a neoclassical model, a fall in consumption isn’t enough. The other first-order condition—the “inter-temporal”—must also be satisfied. This condition refers to the way variables interact over time—that is, between the here-and-now and the future: “today” and “tomorrow.” So in this context, this condition mathematically links the ratio of hours worked “today” and “tomorrow,” on one hand, with the ratio of wages today and wages tomorrow, multiplied by tomorrow’s interest rate, on the other hand.

Because the intra-temporal condition has just indicated that wage rates are declining, the only way the inter-temporal condition can generate an increase in the first ratio (working hours today over working hours tomorrow) is to significantly increase interest rates tomorrow. Because interest rates are the marginal productivity of capital (just as labor is paid its marginal product) and capital’s marginal productivity is higher when there’s less of it, then the only way to get a high interest rate is to decrease the amount of capital. This means that investment “today” has to be low.

So, the bottom line (of neoclassical models with these two conditions) is that to obtain the increase in working hours necessary for an increase in GDP, both consumption and investment must fall. But as we’ve seen in our model simulations, only investment fell significantly; consumption barely budged. Therefore, the models fail to generate a realistic increase in working hours or—thereby—a rise in GDP.

Other economic models exist in which the intra-temporal first-order condition is not necessary. New Keynesian models with rigid wages, for example, operate without it. In this class of models, firms are unwilling to hire more workers at the prevailing wage. Increased government expenditures increase profits for firms, and that induces them to hire more workers at the current fixed wage. The number of hours worked then rises.

Another type of model holds that firms and workers expend considerable and costly effort in seeking good fits. These search models of unemployment hold that there are always willing workers, but they are costly to find. In this class of models, the intra-temporal first-order condition is absent, but a zero-profit condition on firms applies. Increases in government
expenditures may provide firms with incentives to look for more workers, and this might provide a satisfactory mechanism for generating a realistic multiplier.

**Can government expenditures increase wages?**
As we’ve seen, the neoclassical model is unable to generate realistic estimates for the fiscal multiplier, in large part because decreasing returns to labor—the diminishing additional output from each additional hour of work—mean that workers face diminished wages as government expenditures increase. Is there a different way to design a model so that government expenditures could increase wages instead of reducing them? If so, that would certainly help to increase hours worked.

In fact, this is what New Keynesian models with rigid prices do. As government expenditures increase, firms in these models are required to deliver the goods or increase prices or both. Since some firms cannot adjust prices, they need to hire more labor to deliver the goods. To induce households to provide more labor hours, wages have to go up. However, fixed prices and higher wages imply that profits and markups fall as a result of the increase in government expenditures.

Again, Hall (2009) provides a very nice description of the issue and of the lack of evidence for a drop in markups when government expenditures rise. Let us add that the behavior of labor share, which is slightly countercyclical in the data, is also at odds with the prediction of New Keynesian models with fixed prices.

In fact, greater potential lies in a model in the neoclassical tradition, with a twist.

**A second candidate: The shopping model**
In a recent paper, Bai, Ríos-Rull and Storesletten (2011) built a modified neoclassical model that goes a significant way toward generating a realistic multiplier. This model allows any increase in expenditures (which we refer to as demand) to increase productivity and, with it, to increase wages. The innovation in this model is that generating output requires not only inputs of production, but also the active participation of the purchasers of goods and services.

A few examples can help to illustrate this mechanism, and perhaps the best illustrations are from service industries. The tourism industry needs tourists, restaurants need diners, hotels need guests, hospitals need patients, movies have to be seen, advice has to be heard, pedicures need toes, and so on. In our model, consumption and investment require the active participation of consumers and firms. Without spenders, there is no output. Productivity goes up when the contribution of buyers (consumers, firms and the government) increases. It is the buyers—not producers—who are ultimately responsible for increased productivity by exerting more—but unmeasured—effort to use the economy’s productive capacity more intensely.
In the model, these ideas are implemented by requiring consumers not only to pay for the goods, but also to find them, a disagreeable task that is costly in terms of utility (just as is work in the standard model for those who prefer leisure). An increase in consumption can be achieved through both an increase in labor that raises the potential output of the economy, and an increase in search effort that allows households to find more output, thus making the economy operate at higher capacity.

Firms stand ready to produce, with capital and labor, but output occurs only when consumers find the firms and generate demand for that output. The search efforts of consumers are not measured in GDP, and the higher output is imputed to higher productivity.

While the spirit of this “shopping economy” is neoclassical—prices are flexible, people and firms are restricted only by technology, markets clear—the aggregate production function with constant productivity (a traditional workhorse of the SNM) does not hold.

In the shopping economy, as in all models, there is a budget constraint that requires households to pay for what they (and the government) purchase. An additional constraint, unique to this economy, requires households to search in order to find consumption goods. The more search effort consumers expend, the more goods will be found and, thus, the higher output will be. Consequently, output can increase even if there are no changes in measured inputs (since search effort itself is unmeasured).

In our paper, we use the modern concept of competitive search that achieves an optimal allocation and thus guarantees that the model has a unique prediction. However, we include all forms of hassle associated with searching for consumption goods, such as receiving worse service in restaurants at capacity, a lengthy wait in emergency rooms on Saturday nights, not getting the right options or color when buying a car and so on.

All of these hassles are greater when the economy is in an expansion, generating higher productivity as a result of higher demand. During recessions, hassles diminish—parking spaces, shopping lines, waiting times all decline; clerks and salespeople stand idle as they wait for customers. This dynamic applies to firms, as well, since they have to search for investment goods. Purchasing departments and shopping professionals need to find the right type of capital goods, a task that is clearly less costly during recessions.

The shopping economy model holds potential for generating a fiscal multiplier more in line with the empirical estimates. This is because increased government spending in this model generates higher productivity, and that may generate higher wages, in contrast to the SNM. To gauge this potential, we added a government sector financed with lump-sum taxation to the model in Bai, Rios-Rull and Storesletten (2011).
As in the SNM, an increase in government expenditures induces an increase in hours worked. But unlike in the SNM, productivity goes up, which can potentially increase wages, or at least slow down their reduction. Productivity increases because now people and firms have to look harder to find goods since higher government spending has raised effective demand.

When the relevant parameters are applied to the shopping model, it generates much larger effects than did the SNM. An increase in government expenditures of 1 percent of GDP (from 16 percent of the average value of GDP to 17 percent) now yields a fiscal multiplier of 0.172, over seven times that generated by the SNM. This results from both a 0.07 percent increase in productivity and a 0.09 percent increase in hours worked. By comparison, the SNM generates no increase in productivity and a 0.034 percent increase in hours worked.

The main difference between the shopping economy and the SNM is in investment, with the reduction being much smaller in the shopping economy. Interestingly, consumption in the shopping economy falls by more than in the SNM—from 65 percent to 64.94 percent—and investment falls from 19 percent to 18.22 percent. As in the SNM, an increase in the Frisch elasticity to 1.1 also increases the multiplier, albeit by a small amount (from 0.172 to 0.187).

These fiscal multiplier estimates are still a far cry from the values between 0.7 and 1 found in the empirical literature. One way for models of the neoclassical tradition to produce a higher value would be to have “time to build” features to avoid wild oscillations in investment. For example, we could pose a requirement that investment projects take a long time to both plan and implement. Therefore, in any period, only a fraction of the current investment is chosen, the rest being the outcome of previous decisions. Similarly, in the current period, decisions are made about future investment. With this formulation, we would obtain some rigidity in investment, not because markets do not work well, but because of technological—indeed, engineering—reasons. Short-lived increases in government expenditures would not in this case disrupt investment very much, and a swift increase in government expenditures would imply a larger multiplier. The exact calculations would require a specification of how to absorb the losses of firms that are required to secure the investment goods. In this scenario, a much larger drop in consumption is likely. This is a topic that deserves more attention.

The multiplier in a recession
So far we have looked at the multiplier in average times, when the economy is operating normally. But “normal times” are not when policymakers consider using government expenditures as a tool to expand output. Leading researchers (Christiano, Eichenbaum and Rebelo 2010, for instance) postulate that in certain circumstances, such as when the nominal interest rate is 0 percent, New Keynesian models with fixed prices give rise to large fiscal multipliers, suggesting that such a policy tool would be particularly potent under such conditions.
To analyze the multiplier in recessions, we need to “engineer” a recession in our models. In neoclassical contexts, recessions occur because of poorer-than-average technology or because of preference changes that induce people to work less. In this exercise, we first consider shocks to preferences in both SNMs and shopping models that make work less agreeable. We set the sizes of the shocks so that GDP shrinks by 1 percent, and we use a high Frisch elasticity of 1.1.9

In the SNM, a 1 percent drop in output is generated via a 1.5 percent reduction in hours worked. However, consumption barely changes (it goes down just 0.02 percent), while the bulk of the reduction in output is accommodated by lower investment. In actual recessions, investment typically falls by more than consumption, but in a less extreme manner. In the shopping economy, a similar 1 percent output drop comes from a drop not only in labor, but also in productivity, which falls by 0.4 percent.

Now that we’ve engineered a recession in the model economies, we can consider an expansionary policy that increases government expenditures the same amount in both. We find that the fiscal multipliers are higher than in normal times, as theory has suggested, but barely so. In the SNM, the multiplier is up to 0.0298 relative to 0.0296 in normal times. In the shopping economy, it is 0.1916 in a recession, up from 0.1871 in normal times. The gains in the shopping economy come from productivity (0.07 percent) and from labor (0.12 percent); in the SNM, the gains are all from labor.10

The shopping economy allows for an additional type of recession: a shock to the cost of search—the willingness of people to put up with the hassle needed to find goods and services. An increase in search costs thus acts as a shock to demand and generates a recession. Such a recession compounded with the expansionary policy generates the largest multiplier found in these exercises, a value of 0.221.

This quick review of multipliers during recessions generated by neoclassical models arrives at the same destination: The fiscal multiplier is almost 0 for the SNM and about 0.2 for the shopping economy. The reason these models predict such similar multipliers, whether the economy is in a recession and not, is clear. Neoclassical models assume that there are no market frictions, such as wages that don’t adjust quickly or prices that don’t change right away. Therefore, in these sorts of models, recessions are generated because people simply choose to work less. Therefore, the models respond in recessions very much the same as they do in normal times: The model economy operates optimally.

**Conclusions**
This note shows that neoclassical models of the business cycle have serious shortcomings that limit their ability to evaluate the effect of changes in government expenditures. State-of-the-art extensions can increase the predicted values for the fiscal multiplier, but these estimates are still much smaller than those measured empirically.
Many researchers have embraced New Keynesian models in which the central mechanism generating large fluctuations is the assumption that prices are fixed, so firms and workers are unable to adjust (nominally denominated) contracts. While this may be fruitful, we think that some of its counterfactual implications (specifically, on price-to-cost markups and labor share) and the lack of solid theoretical explanation for why prices are fixed justify the exploration of models with neoclassical flavor, but with enough frictions so as to allow for recessions as situations with idle resources. We think that such models should include frictions in labor markets that make many households work fewer hours than they would like and consume less than desired because of difficulties in borrowing.

Models with these features in addition to those provided by our shopping economy, in which increases in government expenditures boost productivity, may be capable of yielding multiplier values consistent with those in the empirical literature. These models are the subject of our ongoing work.
Endnotes

1 The authors thank Doug Clement and Carolyn Wilkins for their editorial comments and Fabrizio Perri for discussions of theory. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.


3 Unlike income taxes, for example, that not only make people poorer but also provide an incentive to substitute away from consumption goods and into leisure, which is not taxed, lump-sum taxes are not distortionary and do not provide such a disincentive to work.

4 See the appendix for a technical explanation of this conclusion.

5 Doing so requires an adjustment in the value of parameter $\theta$ (see appendix) to ensure that the amount of hours worked on average is 30 percent.

6 For simplicity, we have assumed that the government has a pile of projects in a drawer and is exempted from having to search for goods (a trivial simplification). Also, it is very easy to model similar search frictions for the government by being explicit about the part of measured government expenditures that are used for search purposes rather than providing strictly useful goods.

7 Both because people are poorer and because the reduction in investment increases the rate of return, propelling a wealth effect and a (intertemporal) substitution effect.

8“Time to build” is an expression used by Kydland and Prescott (1982), referring to multiperiod construction as a fundamental characteristic of most economies and one that helps explain macroeconomic fluctuations. As they put it: “That wine is not made in a day has long been recognized by economists.”

9 Let’s for now ignore the fact that in this type of recession, a benevolent government should do nothing: The economy is optimal on its own.

10 Another type of recession, one due to a short-lived shock that reduces the household’s relative preference for current consumption. It turns out that a recession of 1 percent in output via a reduction in the willingness to consume generates wild oscillations in consumption and investment. In the SNM with the high Frisch elasticity of 1.1, consumption drops by half(!) and investment goes up two and a half times. The behavior of the shopping economy is more subdued: Consumption falls from 65 percent of GDP to 59.9 percent, and investment goes from 19 percent to 23.1 percent. These recessions are clearly uninteresting, and for what it is worth, the multipliers are not very different from those in normal times: 0.030 in the SNM and 0.195 in the shopping economy.
References


Appendix

Models of Government Expenditure Multipliers

In the neoclassical world, labor is paid whatever the last unit contributes (its marginal product), which means that an increase in the quantity of labor lowers wages. In the SNM, people care about two things: consumption, which they like, and hours worked, which they do not like. People make choices based on consumption and hours worked, and they evaluate their levels of consumption and work both currently and in the future (although the further into the future, the less these things are of concern).

The choices that people make in this case have to satisfy a relation called the intratemporal first-order condition, which can often be described mathematically with the following equation: where $w$ is the wage, $c$ is consumption and $h$ is hours worked.

$$w\ c^{-\sigma} = \theta \ h^{\frac{1}{\psi}}$$

The Greek letters are positive parameters that describe the details of people’s preferences:

- $\sigma$ tells us how unwilling people are to tolerate fluctuation over time in consumption.
- $\theta$ tells us how much they dislike work relative to leisure.
- $\psi$ expresses the responsiveness of hours worked to small changes in the wage rate.

We see from this expression that a simultaneous decrease in the wage rate and increase in hours worked requires a drop in consumption. If this intratemporal condition applies, this result is inevitable.

Still, that is not enough to ensure an increase in hours, and a reduction of the wage could very well be accommodated by a reduction both in hours worked and in consumption.

For hours worked to increase, something else is also necessary: an increase in the interest rate. This can be seen from the other relation that people’s choices have to satisfy, the intertemporal first-order condition that is described in this equation:

$$\left( \frac{h}{h'} \right)^{\frac{1}{\psi}} = \beta \left( 1 + r' \right) \frac{w}{w'}$$

Variables displayed with a “prime” (as in $h'$ instead of a simple $h$) denote values of the variable in the following period.
The letter $r$ is the interest rate.

The Greek letter $\beta$, a positive parameter between 0 and 1, indicates patience—the willingness to delay consumption.

We can see from this equation that in order for hours worked today to be larger than hours worked tomorrow, the interest rate has to be large enough so that $\beta \times (1+r')\left[\frac{w}{w'}\right] > 1$, even when wages today are lower than wages tomorrow, $\left[\frac{w}{w'}\right] > 1$.

In normal times, $\beta \times (1+r) = 1$, so this requires that the interest rate next period is much larger than it is normally. The problem is that interest rates are the marginal productivity of capital, so high interest rates require capital to be lower than usual, and this means that investment today has to be low.