Monetary Unions in a Political Economy Model of Fiscal and Monetary Policy Interaction

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Abstract This paper explores the consequences of a monetary union that pairs multiple independent fiscal authorities microfounded by the political economy model of **?** with a single monetary authority. Membership in a monetary union enables the political coalition in charge of a country's fiscal authority to issue debt that is beneficial to the coalition but detrimental to the country. The incentive to join a monetary union is both the ability to issue nominal debt at a lower interest rate and an initial burst of revenue that rewards the coalition at the time of joining. Without perfect monitoring and reporting between the monetary authority and the politically controlled fiscal authority, the coalition will issue excess debt to reward itself. Repaying the excess debt will lead to adverse welfare consequences when the country experiences a significant negative productivity shock. The model explains Greece's experience with the Eurozone.

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1 Introduction

A monetary union is composed of a single monetary authority and multiple countries each with their own independent fiscal authority. The standard free rider problem of a monetary union in papers such as **?** and **?** is that countries take advantage of a monetary union by issuing debt that is beneficial to the country but detrimental to the union as a whole. This paper highlights a new layer to the free rider problem: membership in a monetary union enables the political coalition in charge of a country's fiscal authority to issue debt that is beneficial to the coalition but detrimental to the country.

The fiscal authority is microfounded by the political economy model of **?** and **?**. Each country's fiscal authority attempts to maximize the utility of a subset of the country's citizens instead of maximizing the welfare of the country or monetary union as a whole. I show that a politically controlled fiscal authority benefits from the discipline an external monetary authority provides. Namely, the fiscal authority will face lower interest rates on its nominal debt if the monetary authority is external rather than domestic. Without sufficient monitoring by the monetary authority nor fiscal rules to constrain it, the coalition in charge of the fiscal authority will issue debt solely to reward itself, creating a situation of unsustainably high debt similar to what has befallen Greece. Fiscal rules as in **?** can be used to mitigate this possibility, but require their own monitoring.

Joining a monetary union is a political decision by the coalition in charge of the fiscal authority to replace the domestic monetary authority with the monetary authority of the monetary union. There is a benefit to joining the monetary union if the domestic monetary authority has been captured by the coalition in charge of the fiscal authority. A captured monetary authority is unable to raise revenue from nominal debt due to the time inconsistency problem of nominal debt: a captured monetary authority will inflate away the real value of nominal debt at the start of every period in order to free revenue for the political coalition to use to reward itself. Consumers anticipate this inflation and will not hold nominal bonds whose real value will disappear.

If the politically distorted fiscal authority decides to join a monetary union, it outsources its monetary authority. The monetary authority is now independent from the coalition in power in the country. An independent monetary authority knows that inflating away the entire real value of nominal bonds will give the politically distorted fiscal authority budgetary freedom to spend revenue on wasteful transfers rather than on public goods. Maintaining a positive level of nominal debt will constrain wasteful spending, but if debt is too high it will require high distortionary taxes to pay off. Thus the split between the aims of monetary union's independent monetary authority and the politically distorted aims of the country's fiscal authority anchors inflation expectations: the independent monetary authority will inflate away some of the debt, so that taxes will be lower, but not all of the debt, so the fiscal authority is still constrained in its spending decisions. The remaining debt will be enough to prevent the country's fiscal authority from spending revenue wastefully but enough to provide some tax smoothing against a productivity shock.

Ideally, the independent monetary authority has full knowledge of the fiscal policies of the countries in the union and will use the threat of inflation to keep the revenue raised from nominal bonds in check. Perfect knowledge may not be possible: a politically distorted fiscal authority may misreport the amount of nominal debt it has issued, or the monetary authority may not pay proper attention to the behavior of the fiscal authority. If so, the monetary authority won't properly use inflation to control the country's nominal debt. The fiscal authority will be able to issue debt that appears nominal but the monetary authority won't inflate away. Effectively, the debt is indexed. A politically distorted fiscal authority with the ability to issue indexed debt will issue it beyond the amount that is optimal for the country. It will issue indexed debt equal to the larger amount that is optimal for the coalition.

The revenue from the excess indexed debt will be used by the coalition to fund transfers to itself now at the cost of higher taxes in the future. The productivity shock will hide the excessive debt as long as the realization of the shock is high enough. With a high shock, a low tax rate will raise sufficient revenue to repay the indexed bonds. If the productivity shock is low, the true nature of the debt will be revealed. The country will have to endure a punishingly high tax rate in order to raise sufficient revenue to repay the indexed bonds without being able to use inflation to lessen its burden.

I illustrate the relevance of the model by analyzing Greece's experience with the Eurozone. Prior to joining the Eurozone Greece faced high interest rates on its debt. Joining the Eurozone enabled Greece to issue debt at lower interest rates because consumers were reassured that the monetary authority would not inflate away the nominal debt. The party in power in Greece, through a combination of misreporting debt, obstinacy by the monetary authority, and a lack of enforcement of fiscal rules was able to issue excess debt whose revenue was used wastefully. While Greece was able to repay the debt during years of expansion, the Great Recession ended its ability to do so.

2 The Model

I first describe the set up of a single country in the model. A monetary union is composed of two such countries, denoted by variables by subscripts a, b respectively. The behavior of the monetary authority in a monetary union comes after the set up for each single country.

Nominal government debt, when sustainable, links periods. Each country operates independently and will only interact through a common price level for their nominal bonds. Fiscal policy consists of setting taxes, expenditure on a public good, direct transfers to citizens, and nominal bond issuance. The timing in a period is as follows: a real shock determines wages (and the distortion due to taxes) at the beginning of every period. After the shock, the monetary authority sets the price level then the fiscal authority chooses its policy. Figure 1 illustrates the sequence of decisions in a period. Timing will be discussed at more length below.

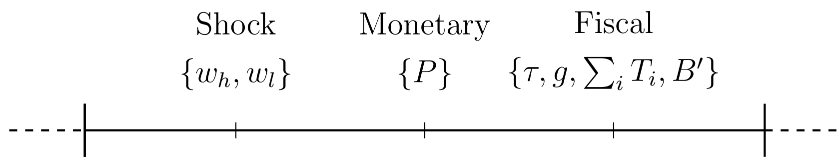

Figure 1: Timing of Monetary and Fiscal Decisions for a Country

2.1 Consumers

Each country has n identical consumers, indexed by i when necessary. A consumer's per period utility function is

$$u(c, g, l) = c + A\log(g) - \frac{l^{1+1/\epsilon}}{\epsilon + 1}$$

and an individual seeks to maximize $U = \sum_t \beta^t u(c_t, g_t, l_t)$ where c is a consumption good, g is government spending on a public good, l is labor, and β the discount rate. The parameter $\epsilon > 0$ is the Frisch elasticity of labor supply. A is a parameter allowing adjustment of the utility of government spending on the public good.

A representative consumer i in period t faces the budget constraint

$$c + q\left(\frac{B'}{n}\right) \leq w_\theta l(1-\tau) + \frac{\left(\frac{B}{n}\right)}{P(B)} + T_i$$

Variables without a prime refer to variables in period t while variables with a prime refer to variables in period $t+1$. The consumer can consume c or purchase nominal bonds $\frac{B'}{n}$ at a price q where each bond pays a nominal unit of income in the next period. Every consumer holds an identical number of bonds so the total number of bonds is B'. The consumer's income consists of labor income at wage w_θ that is taxed by the government at the distortionary tax rate $0 \leq \tau \leq 1$ together with direct transfers $T_i > 0$ from the government. $P(B)$ is the price level determined by the monetary authority at the start of the period as a function of the number of bonds.[1]

The price level in the current period will generally be abbreviated as $P = P(B)$ and the price level in the next period as $P' = P(B')$. The price level is not an intertemporal variable. The ratio of current to next period price level $\frac{P}{P'}$ determines the real return on bonds. For simplicity I normalize this ratio by making bonds pay 1 nominal unit of income in the next period. Thus the real value of bonds will only depend on P' which is set independently every period.

The consumer's budget constraint and linear utility imply the equilibrium bond price

$$q = \beta E_{\theta'}\left[\frac{1}{P'}\right]$$

where the expectation is over possible realizations of the wage shock in the next period. A consumer's utility is defined entirely by the government's choices of taxation τ and public good spending g.

Deriving the optimal amount of labor as a function of the tax rate τ shows

$$l^*_\theta(\tau) = (\epsilon w_\theta(1-\tau))^\epsilon$$

Plugging this into the consumer's utility function shows the simplified indirect utility function before transfers is

$$W_\theta(\tau, g) = \frac{\epsilon^\epsilon(w_\theta(1-\tau))^{\epsilon+1}}{\epsilon+1} + A\log(g)$$

2.2 Firms

Each country has a representative firm with a linear production technology

$$z = w_\theta l$$

[1]The price level is described as a function of B rather than $\left(\frac{B}{n}\right)$ because it is a choice of the monetary authority rather than consumer. For a more in-depth explanation of $P(B)$ see Section **??**

used to produce an intermediate good z at wage w_θ with labor l. At the beginning of each period an i.i.d. technology shock hits the economy such that wages $w_\theta \in \{w_l, w_h\}$ where $w_l < w_h$. The probability that $w_\theta = w_h$ is π, the probability that $w_\theta = w_l$ is $1-\pi$. There will be no trading in intermediate or final goods between countries.

The intermediate good z is split costlessly between the consumption good c and the public good g such that

$$c + g = z.$$

This defines the per period resource constraint in a country as

$$c + g = w_\theta l.$$

2.3 Government

A country's government controls fiscal policy. Raising revenue is possible via a distortionary labor tax τ and by selling nominal bonds B'. A positive bond level means the government is in debt hence owes revenue to consumers. The government can spend revenue on a public good g that benefits all n citizens or on non-negative transfer payments T_i that benefit individuals. It must also repay nominal bonds $\frac{B}{P}$.

The government's budget constraint is

$$g + \sum_i T_i + \frac{B}{P} \leq \mathrm{Rev}_\theta(\tau) + qB'$$

where

$$\mathrm{Rev}_\theta(\tau) = n\tau w_\theta(\epsilon w_\theta(1-\tau))^\epsilon$$

is the total tax revenue raised by the distortionary labor tax on all n consumers.

Define the budget surplus before transfers as

$$S_\theta(\tau, g, B'; \frac{B}{P}) = \mathrm{Rev}_\theta(\tau) + qB' - g - \frac{B}{P}.$$

The surplus must be large enough to pay for any transfers hence $S_\theta(\tau, g, B'; \frac{B}{P}) \geq \sum_i T_i$. Transfers themselves must be non negative: $\forall i \; T_i \geq 0$.

2.3.1 Endogenous Bond Limits

There are endogenous limits to the amount of bonds the government can issue. The upper bound on debt is defined as the maximum amount of bonds the government is able to repay in the case of the bad realization of the wage shock w_l if it spends nothing on the public good and transfers. Define the upper bound $\overline{B}$ as $\overline{B} = \frac{\max_\tau \text{Rev}_l(\tau)}{q}$.

The lower bound on debt is the amount of bonds such that revenue from the bonds would be sufficient to fund optimal public good spending without utilizing the distortionary labor tax. The optimal amount of public good spending is g^s such that $\frac{nA}{g^s} = 1$. This equation equates the declining marginal benefit of providing the public good with the opportunity cost to consumers of consuming the revenue directly with linear utility. Define $\underline{B}$ as $\underline{B} = -nA$ the level of bonds where one more unit of government spending has the same marginal utility as individual consumption. This is the level of bonds such that the government can finance g^s directly from the bonds.

2.3.2 Choice Among Revenue Equivalent Bond Levels

There may be a non-singleton set of bond amounts B' that result in the same bond revenue $qB' = E\left[\frac{1}{P(B')}\right] B'$. Issuing bonds above a specific level can result in the expected price level increasing to perfectly offset the amount of revenue issuing the new bonds would raise. For example the pricing function:

$$P(B) = \begin{cases} B, & \text{if } B \geq 1 \\ 1, & \text{if } B < 1 \end{cases}$$

results in identical revenue for all bond levels greater than or equal to 1. This situation is illustrated in Figure 2.

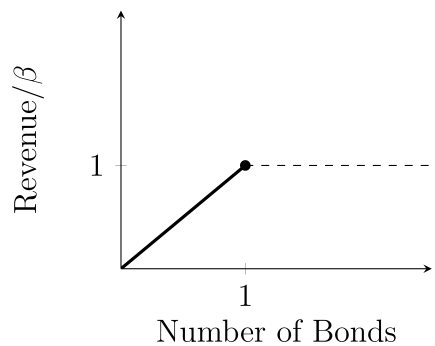

Figure 2: Illustration of Revenue Equivalence for Bonds ≥ 1

Two equilibriums with identical bond revenue will be identical with regard to all real variables. For a revenue level k the set $\{qB = k\}$ may not be a singleton. In the above example, with $k = 1$ this set is equal to $[1, \infty)$. To simplify discussion I assume the government always chooses the minimum element of this set.

2.3.3 Self-Interested Fiscal Policy

A self-interested fiscal authority attempts to maximize the utility of a subgroup of the citizenry. In this section, I provide an overview of the political equilibrium I will be examining in the model. A more precise description is included in the analysis of the behavior of the self-interested fiscal authority in Section **??**.

Following the political system laid out in **?** who extend the political economy model of **?**, citizens vote each period to decide that period's fiscal policy $\{\tau, g, B, T_i\}$. In each period there are T rounds of voting to determine fiscal policy. Each round of voting starts with one citizen being randomly assigned the power to propose a choice of fiscal policy. The proposer puts forward his policy choices of $\{\tau, g, B, T_i\}$. The proposal is enacted if $m < n$ citizens vote for it. If enacted, this ends the voting for that period, a new round will begin next period. If the proposal fails the voting round ends and a new round begins with a new randomly selected proposer. There can be a maximum of

T proposal rounds after which a dictator is appointed. The dictator chooses policies unilaterally with the constraint that all transfers T_i must be equal. A fiscal policy proposal defines the fiscal policy for a single period. The next period a new proposer is randomly selected and the process begins anew. Fiscal commitment across periods possible due to the design of the political system.

I focus on a symmetric Markov-perfect equilibrium. These are proposals that depend only on the current state of the economy $\{w_l, w_h\}$ and debt B. The proposals are independent of both the history of the economy and proposal round. Thus we only need to examine the proposal in the first round.

In order for a proposal to be accepted, the proposal must make the members of the m coalition as well off as the expectation of waiting a round for the next proposal. In practical terms, proposers will propose fiscal instruments to maximize the utility of the m citizens in the coalition without care for non-coalition citizens. This is in contrast to the choices of a benevolent fiscal authority which will maximize the welfare of all n citizens.

2.4 Monetary Authority

There are two types of monetary authority:

Definition 1 *An independent monetary authority seeks to maximize the welfare of all consumers in all countries for which it is the monetary authority.*

Definition 2 *A captured monetary authority is a monetary authority that is not independent. It is controlled by the same microfounded political process as a self-interested fiscal authority. Namely it tries to maximize the utility of m out of n consumers.*[2]

Both types of monetary authority chooses the price level P to maximize welfare. Inflation is costless in this model. The model utilizes timing akin to a Stackelberg game with the monetary authority as leader and the government

[2]The captured monetary authority's m coalition does not need to be identical to the fiscal authority's m coalition for these results. For simplicity, this is the situation discussed here

as follower. The monetary authority chooses the price level P after the shock in each period. Thus monetary policy controls the real value of government debt which is equivalent to consumer wealth. After the monetary authority moves, the fiscal authorities in both countries choose their fiscal instruments simultaneously. See Figure 1.

The choice of timing is deliberate. The alternative timing is that the fiscal authorities choose their fiscal instruments before the monetary authority chooses the price level. Under this alternative timing the fiscal authority won't raise revenue for bond repayment via distortionary taxes because they knows the monetary authority will inflate away the value of any bonds that are due. The result is that no revenue from nominal bonds will be available to the fiscal authorities.

The monetary authority lacks commitment. Each period the monetary authority chooses the price level for that period only and cannot credibly promise what it will do in the future. Specifically, I constrain the monetary authority to choose the price level solely as a function of its information set $\{B_a, B_b, w_{\theta_a}, w_{\theta_b}\}$ at the beginning of a period. This is a consequence of the monetary authority's lack of commitment. The monetary authority can only use current variables because strategies that threaten non-optimal ex-post actions, such as trigger strategies, require commitment to maintain the threat. The monetary authority is restricted to the current bond levels and shocks because bonds are the only intertemporal good and thus the only variable the monetary authority can observe at the beginning of a period after realizations of the shocks. Predicating monetary policy on fiscal policy decisions that took place in previous periods or will take place in future periods is ruled out because it is equivalent to commitment in another form.

2.4.1 Choice Among Welfare Equivalent Price Levels

There may be a non-singleton set of price levels P that result in identical welfare. I impose two price selection criteria. First, absent welfare gains, the monetary authority will set the price level to 1. This default price level is a normalization brought on by specifying that bonds return 1 unit of nominal income.

Second, the monetary authority will deviate from $P = 1$ only for positive welfare gains. When the monetary authority determines the price level, it will minimize $|P - 1|$ while maximizing welfare. For a welfare level k the set $\{P \text{ s.t. } v(B) = k\}$ where $v(B)$ is the welfare function may not be a singleton. To simplify discussion I assume the government always chooses the element of this set that minimizes $|P - 1|$.

These requirements mimic an aversion to inflation and deflation. Together with the behavior of the fiscal authority described in Section **??**, the price selection criteria allow a simplified description of the price level on the equilibrium path without affecting the actual equilibrium.

2.5 Monetary Unions

A monetary union is composed of two countries each with its consumers, firm and self-interested fiscal authority. Each fiscal authority will issue its own nominal bonds and will be subject to its own shocks. When discussing variables that are unique to a country I will denote the variables by subscripts a, b respectively. For example, B_a will be the amount of nominal bonds issued by country a and B_b will be the amount of nominal bonds issued by country b. Most importantly the shocks w_{θ_a} and w_{θ_b} will be independent of each other though occurring with identical probabilities $\pi, 1 - \pi$ for simplicity.

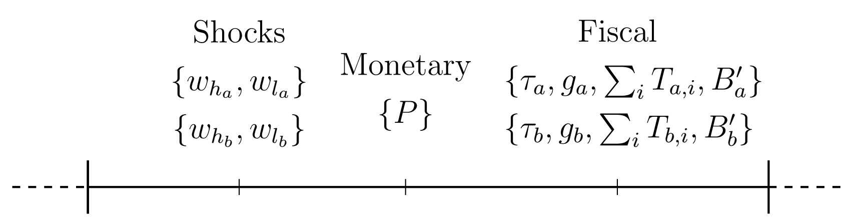

Figure 3: Timing of Monetary and Fiscal Decisions in a Monetary Union

The timing is analogous to the one country case. At the beginning of the period a shock independently occurs in both countries. The monetary authority observes both shocks then chooses the price level P. After that, the fiscal authority in each country chooses its fiscal policies simultaneously. Importantly, neither fiscal authority has information about the decision of the other.

3 Model Analysis

I first describe the problem of a self-interested fiscal authority with a captured monetary authority in country a (without loss of generality). This is the situation of a prospective country that may seek to join a monetary union. The fiscal authority will be unable to issue any nominal bonds due to the time consistency problem of nominal debt. In order to issue nominal debt, the country will need to replace its monetary authority, which it can do by joining a monetary union.

After describing the problem of a self-interested fiscal authority with a captured monetary authority I explain the behavior of a self-interested fiscal authority in country a as part of a monetary union composed of country a and b. The monetary authority's decision will be a function of the amount of nominal bonds B_a issued by both country a and the amount of nominal bonds B_b issued by country b.

3.1 The Self-Interested Fiscal Authority's Problem

3.1.1 Definition of Self-Interested Fiscal Authority's Problem

Following the outline of **?**, I focus on a symmetric Markov-perfect equilibrium. These are proposals that depend solely on the current state of the economy $\{w_l, w_h\}$ and debt B. The proposals are independent of both the history of the economy and proposal round. A citizen will vote for a proposal if it makes him at least as well off as waiting for the next proposal round will. Hence a proposer will propose fiscal instruments that make citizens indifferent between voting for a proposal and waiting for the next round. I choose equilibria where proposals in each round are voted for by the necessary $m-1$ citizens (and the proposer). This means that the equilibrium path consists of a single round with a single proposal that is voted for by the necessary citizens.

The equilibrium is a set of fiscal proposals for each round $r \in \{1, \ldots, T\}$ for the tax rate, public good spending, bond level and transfers $\{\tau^r, g^r, B'^r, T_i^r\}$. The transfers will be used by the proposer to convince a random group of $m-1$ other citizens to support the proposal. Revenue not spent on transfers or public good spending is the effective transfer to the proposer. An equilibirum defines a value function $v_\theta^r(B)$ for each round representing the expected continuation payoff value for a citizen. The last value function $v_\theta^{T+1}(B)$ is the result of the default proposal by the dictator appointed after round T.

Given a set of value functions $\{v_\theta^r(B)\}_{r=1}^{T+1}$ the fiscal proposals must satisfy the proposer's maximization problem. Similarly the fiscal proposals define the optimal value functions. I start with the first relationship. Since the first proposal in the first round is accepted, I drop the r superscripts for simplicity. Formally, given the value functions the optimization problem for the fiscal proposals can be written as

$$\max_{\tau,g,B',T_i} W_\theta(\tau, g) + S_\theta(\tau, g, B'; \frac{B}{P}) - (m-1)T_i + \beta \left[\pi v_H(B') + (1-\pi) v_L(B')\right]$$

$$\text{s.t.} \quad \begin{array}{l} W_\theta(\tau, g) + T_i + \beta \left[\pi v_H(B') + (1-\pi) v_L(B')\right] \geq v_\theta^{r+1}(B) \\ T_i \geq 0 \ \forall i, S_\theta(\tau, g, B'; \frac{B}{P}) \geq (m-1)T_i, B' \in \left[\underline{B}, \overline{B}\right] \end{array}$$

The first constraint is the incentive compatibility constraint that states the proposal must make those citizens receiving a transfer at least as well off in expectation as they would be if they waited for the next proposal round. The other constraints force the proposal to be feasible given government's budget constraint.

Given the fiscal proposals, the value functions are determined by[3]

$$v_\theta(B) = W_\theta(\tau, g) + \frac{S_\theta(\tau, g, B'; \frac{B}{P})}{n} + \beta\left[\pi v_H\left(B'\right) + (1-\pi)v_L\left(B'\right)\right]$$

This expression comes from the three possibilities for a citizen in a proposal round. With probability $1/n$ a citizen is the proposer and thus receives the surplus after transfers. With probability $\frac{m-1}{n}$ a citizen is not the proposer but is a member of the randomly selected coalition that votes for the proposal and thus receives the transfer T_i. With probability $\frac{n-m}{n}$ a citizen is not in the proposer's coalition and receives no transfer. Since utility is quasilinear, the expected utility in a round is the payoff multiplied by the probability.

Because the first proposal is accepted in each round and all proposals will be identical, the value functions will be identical for every proposal round $r \in \{1, \ldots, T\}$. If the round T proposal is rejected, the round $T+1$ dictator's proposal will result in the value function

$$v_\theta^{T+1}(B) = \max_{\tau, g, B'} W_\theta(\tau, g) + \frac{S_\theta(\tau, g, B'; \frac{B}{P})}{n} + \beta\left[\pi v_H\left(B'\right) + (1-\pi)v_L\left(B'\right)\right]$$

subject to uniform transfers and the same feasibility constraints as before.

Definition 3 *An equilibrium is well-behaved if the value function v_θ is continuous and concave on the domain $\left[\underline{B}, \overline{B}\right]$.*

I characterize this equilibrium in the next section and prove its existence in the appendix.

3.1.2 Self-Interested Fiscal Authority with a Captured Monetary Authority

A proposer of fiscal policies has to get m votes for the proposal. Counting the proposer's own vote, $m-1$ other votes are needed. In order to get those votes,

[3] It's helpful to remember that $P = P(B)$ is a function of the current amount of bonds as are all other fiscal choices. Specifically, $S_\theta(\tau, g, B'; \frac{B}{P})$ can be written as $S_\theta(\tau, g, B'; P(B))$ to highlight that B is the only state variable.

the proposer's policies attempt to maximize the utility of the $m-1$ randomly chosen citizens as well as his own. The self-interested fiscal authority's problem is

$$\max_{\tau,g,B',\{T_i\}_1^n} W_{\theta_a}(\tau_a,g_a) + \frac{\sum_i T_{i_a}}{m} + \beta \begin{bmatrix} \pi^2 v_{H,H}\left(B'_a,B'_b\right) + \pi(1-\pi)v_{H,L}\left(B'_a,B'_b\right) + \\ \pi(1-\pi)v_{L,H}\left(B'_a,B'_b\right) + (1-\pi)^2 v_{L,L}\left(B'_a,B'_b\right) \end{bmatrix}$$

$$\text{s.t. } T_{i_a} \geq 0\ \forall i, S_{\theta_a}(\tau_a,g_a,B'_a;\frac{B_a}{P}) \geq \sum_i^n T_{i_a}, B'_a \in \left[\underline{B},\overline{B}\right]$$

There are two possibilities: either transfers are 0 or transfers are positive. If there are no transfers the first order conditions are

$$\frac{1-\tau_a}{1-\tau_a(1+\epsilon)} = \frac{nA}{g_a}$$

$$\frac{1-\tau_a}{1-\tau_a(1+\epsilon)} = -n\beta \begin{bmatrix} \pi^2 v_{H,H}\left(B'_a,B'_b\right) + \pi(1-\pi)v_{H,L}\left(B'_a,B'_b\right) + \\ \pi(1-\pi)v_{L,H}\left(B'_a,B'_b\right) + (1-\pi)^2 v_{L,L}\left(B'_a,B'_b\right) \end{bmatrix}$$

The expression $\frac{1-\tau_a}{1-\tau_a(1+\epsilon)}$ is the marginal distortionary cost of taxation. The first equation equates the marginal cost of raising an additional unit of revenue via taxation with the marginal benefit of spending that revenue on public goods. The second equation equates the marginal cost of raising an additional unit of revenue via taxation with the expected marginal cost of raising the revenue by issuing bonds (and thus smoothing taxation by pushing this cost into the future).

If there are transfers the optimal choices are

$$\frac{n}{m} = \frac{1-\tau_a^*}{1-\tau_a^*(1+\epsilon)}$$

$$\frac{n}{m} = \frac{nA}{g_a^*}$$

$$B_a'^* = \arg\max_{B'_a} \left[\frac{qB'_a}{m} + \beta \begin{bmatrix} \pi^2 v_{H,H}\left(B'_a,B'_b\right) + \pi(1-\pi)v_{H,L}\left(B'_a,B'_b\right) + \\ \pi(1-\pi)v_{L,H}\left(B'_a,B'_b\right) + (1-\pi)^2 v_{L,L}\left(B'_a,B'_b\right) \end{bmatrix}\right]$$

The left hand side $\frac{n}{m}$ term represents the amount each individual in the governing coalition will receive as a transfer if an additional unit of revenue is raised from every consumer. The first equation shows the marginal benefit to coalition members from additional revenue is equal to the marginal cost of raising that additional unit by distortionary taxation. The second equation displays the choice of the government to spend revenue: the marginal benefit of transfers to the governing coalition is equal to the marginal benefit from using that revenue on public good spending. The third equation balances the optimal amount of bonds to issue to fund increased transfers versus the cost of increased bonds in the next period.

When there are transfers the tax rate, government spending, and level of bonds $\{\tau_a^*, g_a^*, B_a^{'*}\}$ will be constant. The government will raise revenue from taxes τ_a^* and bonds $B_a^{'*}$. It will spend g_a^* on the public good. Whatever revenue is left over after that spending will be used to fund transfers.

We can determine when there will be transfers. If the revenue from taxes τ_a^* and bonds $B_a^{'*}$ is sufficient to cover spending g_a^* on the public good, there will be transfers. Thus there is a cutoff

$$C_{\theta_a} = \text{Rev}_{\theta_a}(\tau_a^*) + qB_a^{\prime *} - g^*$$

which is the amount of bonds such that $S_{\theta_a}(\tau_a^*, g_a^*, B_a^{'*}; C_{\theta_a}) = 0$. If the current level of bonds in the period is above C_{θ_a} there will be no revenue for transfers. Thus the optimization problem will be identical to that of the benevolent fiscal authority. If the current level of bonds is below C_{θ_a} there will be transfers while taxes, public good spending and bond issuance are $\{\tau_a^*, g_a^*, B_a^{'*}\}$ respectively.

The problem can be simplified to

$$\max_{\tau_a, g_a, B_a'} W_{\theta_a}(\tau_a, g_a) + \frac{S_{\theta_a}(\tau_a, g_a, B_a'; \frac{B_a}{P})}{n} + \beta \begin{bmatrix} \pi^2 v_{H,H}\left(B_a', B_b'\right) + \pi(1-\pi) v_{H,L}\left(B_a', B_b'\right) + \\ \pi(1-\pi) v_{L,H}\left(B_a', B_b'\right) + (1-\pi)^2 v_{L,L}\left(B_a', B_b'\right) \end{bmatrix}$$

$$\text{s.t.} \quad \begin{array}{l} \tau_a \geq \tau_a^*, g_a \leq g_a^*, B_a' \in \left[B_a'^*, \overline{B_a}\right] \\ S_{\theta_a}(\tau_a, g_a, B_a'; \dfrac{B_a}{P}) \geq 0 \end{array}$$

The new constraints are limits on the lowest taxes, highest government spending and least bonds. If there are transfers, these three variables will equal their starred values.

The captured monetary authority will attempt to maximize the utility of the m out of n consumers who also control the self-interested fiscal authority by setting the price level P

$$P(B) = \begin{cases} \infty, & \text{if } B_a > 0 \\ 1, & \text{if } B_a = 0 \\ 0, & \text{if } B_a < 0 \end{cases}$$

Claim 1 *The self-interested fiscal authority's solution with a captured monetary authority is to issue* 0 *bonds that raise* 0 *revenue in every period.*

An economy with a captured monetary authority will be unable to raise any revenue from nominal bonds. The captured monetary authority will set the price level to infinity for any positive level of bonds. If the amount of bonds is above the cutoff C_{θ_a}, increasing the price level will result in decreased taxes and therefore increased welfare. If the amount of bonds is below the cutoff, a captured monetary authority will derive a benefit from increasing the price level until the entire real value of the nominal bonds is eliminated.

The benefit exists because a captured monetary authority only cares about the welfare of its coalition. Increasing the price level so the real value of bonds declines below C_{θ_a} decreases the real value of bonds and thus the wealth of all citizens while increasing transfers to the coalition. This is a net gain in utility for members of the coalition. Effectively for each unit of revenue freed by raising the price level members of the coalition will lose $\frac{1}{n}$ units of wealth and gain $\frac{n}{m}$ units of transfers. Thus a captured monetary authority will eliminate the real value of any positive amount of nominal bonds. For more details see the proof of Claim **??** in the Appendix.

3.2 Perfect Monetary Union

A perfect monetary union combines two self-interested fiscal authorities with a single independent monetary authority. Relying on the model derivation above, the problem of the monetary authority in a monetary union is

$$v_{\theta_a,\theta_b}(B_a,B_b) = \max_P \left(\begin{bmatrix} \max\limits_{\tau_a,g_a,B'_a} W_{\theta_a}(\tau_a,g_a) + \dfrac{S_{\theta_a}(\tau_a,g_a,B'_a;\frac{B_a}{P})}{n} + \\ \beta \begin{bmatrix} \pi^2 v_{H,H}(B'_a,B'_b) + \pi(1-\pi)v_{H,L}(B'_a,B'_b) + \\ \pi(1-\pi)v_{L,H}(B'_a,B'_b) + (1-\pi)^2 v_{L,L}(B'_a,B'_b) \end{bmatrix} \\ \text{s.t.} \begin{array}{l} \tau_a \geq \tau_a^*, g_a \leq g_a^*, B'_a \in \left[B_a'^*, \overline{B_a}\right] \\ S_{\theta_a}(\tau_a,g_a,B'_a;\dfrac{B_a}{P}) \geq 0 \end{array} \end{bmatrix} + \begin{bmatrix} \max\limits_{\tau_b,g_b,B'_b} W_{\theta_b}(\tau_b,g_b) + \dfrac{S_{\theta_b}(\tau_b,g_b,B'_b;\frac{B_b}{P})}{n} + \\ \beta \begin{bmatrix} \pi^2 v_{H,H}(B'_a,B'_b) + \pi(1-\pi)v_{H,L}(B'_a,B'_b) + \\ \pi(1-\pi)v_{L,H}(B'_a,B'_b) + (1-\pi)^2 v_{L,L}(B'_a,B'_b) \end{bmatrix} \\ \text{s.t.} \begin{array}{l} \tau_b \geq \tau_b^*, g_b \leq g_b^*, B'_b \in \left[B_b'^*, \overline{B_b}\right] \\ S_{\theta_b}(\tau_b,g_b,B'_b;\dfrac{B_b}{P}) \geq 0 \end{array} \end{bmatrix} \right)$$

The pricing function of the monetary authority will depend on which country deviates more from its respective bond cutoff value C_θ

$$P(B_a,B_b) = \begin{cases} \max\left(\frac{B_a}{C_{\theta_a}}, \frac{B_b}{C_{\theta_b}}\right), & \text{if } B_a > C_{\theta_a} \text{ or } B_b > C_{\theta_b} \\ 1, & \text{if } B_a \leq C_{\theta_a} \text{ and } B_b \leq C_{\theta_b} \end{cases}$$

If the amount of nominal bonds in both countries is less than their respective bond cutoffs, there will be transfers. If the monetary authority increases the price level, the amount of debt both fiscal authorities must repay goes down. Thus revenue the fiscal authorities had originally directed to bond repayment will instead go to transfers while taxes and government spending will remain constant at their starred values. The total decrease in the real value of nominal bond holdings is equal to the total increase in transfers in both countries. Because utility is quasilinear it does not matter who gains and who loses: increasing the price level will not change welfare hence the monetary authority will keep the price level constant.

If the amount of nominal bonds in at least one of the countries is greater than or equal to its bond cutoff there will be no transfers in that country. The monetary authority will erase the real value of bonds by increasing the price level as long as at least one fiscal authority will use the newly freed revenue to decrease taxes rather than increase transfers. By definition this is the case when the amount of nominal bonds in a country is greater than or equal to the bond cutoff in that country. The max operator is necessary because the monetary authority will increase the price level until the real value of bonds is at or below each country's respective C_θ cutoff.

Claim 2 *The self-interested fiscal authority in country a in a monetary union will issue C_{h_a} bonds that will raise*

$$\beta\left[\pi^2 C_{h_a} + \pi(1-\pi)C_{h_a}\left(\frac{C_{l_b}}{C_{h_b}}\right) + (1-\pi)\pi C_{l_a} + (1-\pi)^2 C_{h_a}\left(\min\left[\frac{C_{l_a}}{C_{h_a}}, \frac{C_{l_b}}{C_{h_b}}\right]\right)\right]$$

revenue in every period. Country b is identical, mutatis mutandis.

The revenue equation can be decomposed into four parts:

- $\pi^2 C_{h_a}$: Both countries have a high shock. There is no inflation
- $\pi(1-\pi)C_{h_a}\left(\frac{C_{l_b}}{C_{h_b}}\right)$: Country a has a high shock while country b has a low shock. The monetary authority increases the price level to bring the real value of bonds in country b from C_{h_b} down to C_{l_b}. This decreases the real value of bonds in country a from C_{h_a} to $C_{h_a}\left(\frac{C_{l_b}}{C_{h_b}}\right)$.
- $(1-\pi)\pi C_{l_a}$: Country a has a low shock while country b has a high shock. The monetary authority increases the price level to bring the real value of bonds in country a from C_{h_a} to C_{l_a}.
- $(1-\pi)^2 C_{h_a}\left(\min\left[\frac{C_{l_a}}{C_{h_a}}, \frac{C_{l_b}}{C_{h_b}}\right]\right)$: Both countries have a low shock. The monetary authority increases the price level so the real value of bonds in both countries is at or below C_{l_a} and C_{l_b}, respectively. The min operator is necessary because the change in the price level necessary to reduce the

real value of bonds may be different for countries a and b. The max operator in the pricing function for the monetary authority becomes min because I invert the price P in the formula for the bond price q

Consider a one step deviation where country a issues $B_a > C_{h_a}$ bonds and country b issues $B_b = C_{h_b}$. From the point of view of the monetary authority, increasing the price level will decrease taxes and therefore increase welfare in country a. Since country b issued C_{h_b} bonds increasing the price level will increase transfers while taxes and public good spending are constant. This means increasing the price level will leave welfare in country b unchanged while total welfare will be increased. Similarly if each country issues nominal bonds above their respective bond cutoff, the monetary authority will increase the price level until the real value of both country's nominal bonds is at or below their respective cutoffs.

Claim 3 *Unexpected inflation driven by one country results in increased transfers in the other country.*

There are two reasons for the monetary authority to raise the price level: a low realization of the wage shock in a country or an off equilibrium decision by a country to issue more than C_{h_*} bonds. To explain the first, imagine both countries issue C_h bonds and country a's shock is w_{l_a} while country b's is w_{h_b}. The monetary authority will increase the price level to drive the real value of country a's bonds to C_{l_a}. Country a's bonds will be at the appropriate low shock bond cutoff C_{l_a} hence there won't be transfers in country a. By raising the price level, the real value of country b's bonds will be driven below the high shock cutoff C_{h_b} resulting in positive transfers in country b.

Similarly imagine country a issues too many bonds so $C_{b_a} > C_{h_a}$ while country b stayed on the equilibrium path by issuing C_{h_b} bonds. Assume both countries experience their respective high wage shock w_{h_a}, w_{h_b}. The monetary authority will raise the price level to eliminate the real value of country a's excess bonds driving the amount country b must repay below the cutoff C_{h_b} value. Country a which originally deviated will have no transfers, country b which stayed on path will have positive transfers.

3.3 Imperfect Monetary Union Results

An imperfect monetary union is a monetary union where the monetary authority ignores fiscal decisions of one country. Mathematically without loss of generality this means $P(B_a, B_b) = P(B_a)$. I will say country b is *ignored*. Here are three possible explanations for why a country may be ignored:

- **Pareto Weights** The monetary authority may have Pareto weights on the welfare of the countries in the monetary union. For example, fitting with the simplicity of the model, the monetary authority may put 0 weight on the welfare of country b. Clearly if the monetary authority doesn't care about the welfare of country b it won't change the price level in response to fiscal decisions by country b.

- **Unobservable Debt** The budget decisions of country b could be unobservable by the monetary authority. For example, the monetary authority could always perceive the amount of debt issued by country b to be C_{l_b}. This constant level of debt would never provoke the monetary authority to change the price level. The misperception could exist due to deception by country b in reporting either its debt or the other characteristics that define C_{θ_b}.

- **Judgmental Decisions** The monetary authority could be judgmental in its welfare consideration of transfers by assigning a negative utility value to them. The monetary authority would then try to avoid transfers. Since some price level changes will lead to transfers (e.g. Claim **??**), this would change the reaction function of the monetary authority. For example, interacting with the *Pareto Weights* explanation, the monetary authority may try to avoid transfers altogether in country a due to the combination of a low Pareto weight on country b and a high aversion to transfers in country a.

There's a difference between a country deceiving the monetary authority (the unobservable debt explanation) and one that the monetary authority is judging (the Pareto weights and judgmental decisions explanations). In the former situation, the country knows it is being ignored (it is causing the ignoring) while the monetary authority does not. In the latter situation the monetary authority knows the country is ignored while the country does not.

Since country b is ignored, we no longer need to consider the behavior of the monetary authority when deciding its fiscal behavior. The problem of the self-interested fiscal authority in an ignored country is as before

$$\max_{\tau_b, g_b, B'_b} W_{\theta_b}(\tau_b, g_b) + \frac{S_{\theta_b}(\tau_b, g_b, B'_b; \frac{B_b}{P})}{n} + \beta \left[\pi v_H\left(B'\right) + (1-\pi) v_L\left(B'\right)\right]$$

$$\text{s.t.} \quad \begin{array}{l} \tau \geq \tau^*, g \leq g^*, B' \in \left[B'^*, \overline{B}\right] \\ S_\theta(\tau, g, B'; \dfrac{B}{P}) \geq 0 \end{array}$$

Claim 4 *The self-interested fiscal authority in an ignored country b in a monetary union will issue an amount of bonds above C_{l_b}.*

Without the option to use inflation caused by its own bond issuance is to reduce the real value of nominal bonds, the fiscal authority will have to use tax revenue rather than inflation to smooth taxes between periods. Since taxes are distortionary this will lead to lower welfare. When ignored the self-interested fiscal authority will issue (what will be ex-post) too many bonds because it doesn't know the realization of tomorrow's productivity shock.

The benefit of bonds is that they increase transfers to the coalition today, the cost is the possibility that repaying the bonds will require higher taxes tomorrow. The shock in country b determines the consequences of issuing bonds. If the shock is w_{h_b} the tax rate τ_b^* will raise sufficient revenue to repay the bonds. If the shock is w_{l_b}, the tax rate τ_b^* won't raise sufficient revenue to repay the bonds thus necessitating higher taxes. But there is only a $1-\pi$ probability of the bad shock w_l so the fiscal authority issues bonds hoping it does not happen.

The shock in country a will determine whether or not the price level changes. The change in the price level is independent from the shock in country b since country b is ignored and the shocks are independent. Inflation may help repay bonds issued by country b, but it also may not. Thus country b can't rely on inflation to help repay bonds.

4 Results

The main result is that while a country as a whole may not benefit from joining a monetary union, the political coalition in charge of the fiscal authority will always benefit.

Proposition 1 *The welfare of a country with a captured monetary authority will be higher if it joins a perfect monetary union or if it joins an imperfect monetary union but is not the ignored country.*

The comparison of welfare for the country as a whole is clear from comparing Claim **??** and Claim **??**. A country with a captured monetary authority will be unable to raise any revenue from nominal bonds. Joining a monetary union allows the fiscal authority to issue nominal bonds that can be used to raise revenue to smooth taxes. If the country is in an imperfect monetary union but not ignored the fiscal authority behaves identically to how it would behave in a perfect monetary union.

Proposition 2 *The welfare of the political coalition in charge of the fiscal authority in a country with a captured monetary authority will be higher if it joins either a perfect or a imperfect monetary union.*

The political coalition always benefits when the country as a whole benefits. I need to show that the political coalition benefits if the country is ignored in an imperfect monetary union. If it is ignored, the monetary authority does not respond to the bond decisions of the country. The country is effectively issuing indexed debt. Debt that won't be inflated away must be repaid with tax revenue. Claim **??** shows that the country as a whole will suffer if the fiscal authority issues indexed debt. However, in expectation the governing coalition will benefit from issuing indexed debt because it will use the revenue to fund transfers in the current period.

Whether a country is ignored or not has a significant welfare impact on its citizens (not just the coalition). It's important that a country that is ignored is revealed as such.

Proposition 3 *A country in a monetary union is revealed to be ignored only after a bad realization w_l of the ignored country's wage shock.*

Assume country b is the ignored country. If there is a good shock w_{h_b} productivity will be high enough that the lowest possible tax rate τ_b^* will raise sufficient revenue to repay bonds for both an ignored and a non-ignored country. If there is a bad shock w_{l_b} inflation by the monetary authority will eliminate some of the real value of bonds in a non-ignored country so that the tax rate τ^* will raise enough revenue to repay bonds. In an ignored country this inflation will not occur. Taxes will have to be higher than τ_b^* in order to repay bonds.

One way to constrain an ignored fiscal authority is through the use of *fiscal rules*. In this model a fiscal rule would be an exogenous rule that prevents a country from issuing debt beyond a certain limit. Optimally, a perfect monetary union provides a limit on the amount of nominal debt (to C_{θ_a}). A fiscal rule is another method of constraining debt with the power of the monetary union rather than the threat of inflation. An effective fiscal rule would constrain an ignored country from issuing excess debt.

5 Greece

To illustrate the importance of the results above, I analyze Greece's experience with the Eurozone through the lens of the model. Although the model is simplified for tractability the main results hold. Greece suffered from a captured monetary authority prior to joining the Eurozone. Upon joining, Greece operated as if it had an independent monetary authority. The Eurozone proved to be an imperfect monetary union and Greece suffered the consequences.

Specifically, joining the Eurozone allowed Greece to raise more revenue from debt than it previously did. Greece used some of this increased revenue on increased transfers. While Greece benefitted while times were good, the significant negative shock of the Great Recession revealed that its nominal debt was in fact indexed and would require high taxes to repay.

6 Conclusion

A monetary union is an effective way for a country to gain an independent, and benevolent, monetary authority. A self-interested fiscal authority benefits from an independent monetary authority because the monetary authority's independence allows the fiscal authority to issue nominal debt whose revenue will be used smooth taxes. This relationship keeps the amount of nominal debt under control and allows the country to effectively buffer shocks.

The relationship can be abused if neither party properly monitors the other. The monetary authority may not weigh the well-being of the country when choosing the price level. The self-interested fiscal authority may not accurately report the amount of debt to the monetary authority. If either of these hold true the self-interested fiscal authority can act without restraint. The nominal debt it issues will not be inflated away by the monetary authority. The self-interested fiscal authority will issue too many nominal bonds that are in fact indexed bonds. When a bad productivity shock hits, the fiscal authority will have to raise taxes to an extremely high level to raise the revenue necessary to pay off those bonds.

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A Proof of Claim ??

[These proofs not quite aligned with the paper, yet]

To prove Claim **??** regarding the optimal choices of a self-interested fiscal authority I need to show the pricing function

$$P(B) = \begin{cases} \frac{B}{C_\theta}, & \text{if } B > C_\theta \\ 1, & \text{if } B \leq C_\theta \end{cases}$$

The first order condition for the monetary authority with a self-interested fiscal authority is

$$\frac{\partial v_\theta(B)}{\partial P} = \begin{cases} \left[\frac{\epsilon\tau_\theta(\frac{B}{P})}{1-\tau_\theta(\frac{B}{P})(1+\epsilon)}\right]\frac{B}{nP^2}, & \text{if } B > C_\theta \\ 0, & \text{if } B < C_\theta \end{cases}$$

Concentrating on the case of $B > C_\theta$, I show

$$\frac{\partial v_\theta(B)}{\partial P} = \left[\frac{\epsilon\tau(\frac{B}{P})}{1-\tau(\frac{B}{P})(1+\epsilon)}\right]\frac{B}{nP^2}.$$

This expression is always positive hence there is always a benefit for the monetary authority to increasing the price level.

To find this derivative choose $B_0 > C_\theta$. I will build a non-optimal function $\phi_\theta(B)$ that equals $v_\theta(B)$ at B_0 but is less elsewhere (and strictly concave). This will fulfill the conditions of Theorem 4.10 of **?** stating that derivatives of $v_\theta(B)$ are equal to the derivatives of $\phi_\theta(B)$ at B_0. For clarity and notational simplicity let $b = \frac{B}{P(B)}$ and $b_0 = \frac{B_0}{P(B_0)}$.

Choose B from a neighborhood of B_0. Define

$$g(b) = \text{Rev}\left(\tau(b_0)\right) + qB'(b_0) - b$$

which is a non-optimal amount of government spending while still fulfilling debt repayment obligations. The amount of transfers will be the residual after paying back b bonds

$$S_\theta(\tau_\theta(b_0), g(b), B'(b_0); b) = \text{Rev}\left(\tau(b_0)\right) + qB'(b_0) - g(b) - b$$

Define the non-optimal utility function to be

$$\phi_\theta(B) = W\left(\tau(b_0), g(b)\right) + \frac{S_\theta(\tau_\theta(b_0), g(b), B'(b_0); b)}{n} + \beta\left[\pi v_H\left(B'(b_0)\right) + (1-\pi) v_L\left(B'(b_0)\right)\right]$$

Expand the indirect utility and transfers terms. Note that the terms dependent on P are the direct utility benefit of government spending, the bond holdings of the household in the current period, and transfers. Differentiate, noting that the terms dependent on P in transfers will cancel, and find

$$\begin{aligned}
\frac{\partial \phi(B)}{\partial P} &= -\frac{B}{P^2} + \frac{A}{g}\left(\frac{B}{P^2}\right) \\
&= -\frac{B}{P^2} + \left[\frac{1-\tau\left(\frac{B}{P}\right)}{1-\tau\left(\frac{B}{P}\right)(1+\epsilon)}\right]\left(\frac{B}{nP^2}\right) \\
&= \left[\frac{\epsilon\tau\left(\frac{B}{P}\right)}{1-\tau\left(\frac{B}{P}\right)(1+\epsilon)}\right]\left(\frac{B}{nP^2}\right)
\end{aligned}$$

where I've substituted in the first order condition of the fiscal authority. Taking the second derivative confirms the necessary conditions.

When $\frac{B}{P} < C_\theta$ the derivative can be taken directly from the definition of $v(B)$. Increasing the price level causes no change in taxes or government spending which are pegged at τ^*, g^* respectively. The mechanism behind this result is that increasing the price level decreases the real value of debt and thereby decreases the wealth of all n citizens. Revenue previously used to repay the debt is freed and the self-interested fiscal authority redirects it into transfers to the m citizens in the coalition. Because utility is linear taking 1 unit of wealth from n citizens while giving n/m to m citizens results in identical welfare from the perspective of the benevolent monetary authority.

To establish the full pricing function I need to show that the value function is constant for all $B \leq C_\theta$. The derivative establishes that $v(B)$ is maximized on $B < C_\theta$, I need to show the value is identical on the boundary C_θ. By definition at C_θ and below the tax rate, public good spending and bond issuance are (τ^*, g^*, B'^*). As explained above, a lower amount of bonds means less wealth but more transfers. The two effects offset hence the set of maximizers of $v(B)$ includes the bound C_θ.

Claim **??** says that a self-interested fiscal authority will issue C_h bonds. Revenue from issuing bonds is used to either lower the current tax rate or increase transfers. Both of these result in gains for the self-interested fiscal authority. Hence the self-interested fiscal authority will attempt to maximize bond revenue. Claim **??** is equivalent to stating that revenue is maximized by issuing C_h bonds. The core of the argument is that issuing more bonds than C_h will result in no additional revenue due to an offsetting rise in the price level, issuing fewer bonds than C_h will result in foregone revenue if tomorrow has high productivity.

Assume the self-interested fiscal authority issues $B_0 \in (C_l, C_h)$ bonds. There are two possibilities for the next period. If the shock is w_h, The price level P' will be 1 hence the real value of the bonds will be B_0. If the shock is w_l, the price level will be $P' = \frac{B_0}{C_l}$ hence the real amount of bonds will be C_l. Thus issuing B_0 bonds results in revenue of $\beta(\pi B_0 + (1-\pi)C_l)$.

Compare this revenue level to the revenue level that would result if the self-interested fiscal authority issued C_h bonds. If the shock is w_h, The price level P' will be 1 hence the real value of the bonds will be C_h. If the shock is w_l, the price level will be $P' = \frac{C_h}{C_l}$ hence the real amount of bonds will be C_l. Thus issuing C_h bonds results in revenue of $\beta(\pi C_h + (1-\pi)C_l)$.

Issuing C_h bonds raises the maximum possible amount of revenue. From the perspective of the benevolent monetary authority welfare (i.e. the value function at the beginning of next period) is identical at B_0 and C_h. There is no harm to welfare from issuing C_h bonds and there is a gain to the coalition of the self-interested fiscal authority in doing so.

B Proof of Claim ??

Assume the coalition controlling the captured monetary authority is the same as the coalition controlling the self-interested fiscal authority. The derivative of the captured monetary authority's value function with respect to P is

$$\frac{\partial v(B)}{\partial P} = \begin{cases} \left[\frac{\epsilon\tau_2(\frac{B}{P})}{1-\tau_2(\frac{B}{P})(1+\epsilon)}\right]\frac{B}{nP^2}, & \text{if } B > C_\theta \\ \left(\frac{n}{m}-1\right)\frac{B}{nP^2}, & \text{if } B < C_\theta \end{cases}$$

The case $B < C_\theta$ arises from the equivalence of government debt and transfers in a consumer's budget constraint: both are income. Receiving a transfer is identical to holding government debt. Increasing the price level decreases the real value of the nominal government bonds every consumer holds. The total decrease in debt will equal the total increase in transfers that benefit the coalition that controls the captured monetary authority.

An independent monetary authority weighs this increase averaged across all n consumers compared to the decrease in debt and sees it had no effect. (The derivative was 0 in this region.) For a captured monetary authority, those transfers aren't averaged. Increasing the price level decreases the amount the government has to repay everyone while increasing the transfers to the coalition controlling the captured captured monetary authority. Inflation is in effect a lump sum tax on all to fund direct transfers to the coalition.

The pricing function is

$$P(B) = \begin{cases} \infty, & \text{if } B > 0 \\ 1, & \text{if } B = 0 \\ 0, & \text{if } B < 0 \end{cases}$$

If the government has a negative amount of bonds the captured monetary authority will deflate in order to maximize the real amount consumers owe the government. Any amount owed to the government will be directed into transfers to the governing coalition. For every unit of revenue the captured monetary authority raises from all n consumers it can increase transfers to its coalition by $\frac{n}{m}$. For a member of the coalition the net effect $\frac{n}{m} - 1$ is strictly positive.

C Proof of Claim ??

To prove Claim **??** I need to show that with price level commitment the tax level will be higher than the minimum τ^* for at least a single period. The model with price level commitment is equivalent to a simplified version of **?**. See the paper for an in-depth description of the dynamics of the model. Without a monetary authority to keep debt at the bond cutoff C_θ, debt will exceed the cutoff. Specifically it will do so in periods of low realizations of the productivity shock. A self-interested fiscal authority will attempt to fund transfers today by counting on a good realization w_h of the shock tomorrow. The good realization would allow bond repayment with the minimum labor tax τ^*. If tomorrow productivity is low at w_l, taxes will exceed the minimum value.

D Proof of Claim ??

$$P(B_a, B_b) = \begin{cases} \max\left(\frac{B_a}{C_{\theta_a}}, \frac{B_b}{C_{\theta_b}}\right), & \text{if } B_a > C_{\theta_a} \text{ or } B_b > C_{\theta_b} \\ 1, & \text{if } B_a \leq C_{\theta_a} \text{ and } B_b \geq C_{\theta_b} \end{cases}$$

This proof rests on two points: the two countries in the monetary union face independent constraints and differentiation is distributes over addition. The shared price level P enters solely (and identically) through each country's budget constraint. This enables the monetary authority to treat each country's problem independently. Mathematically

$$\frac{\partial v_{\theta_a,\theta_b}(B_a, B_b)}{\partial P} = \frac{\partial v_{\theta_a}(B_a)}{\partial P} + \frac{\partial v_{\theta_b}(B_b)}{\partial P}$$

The derivative is the sum of the utility gains from the two countries

$$\frac{\partial v_{\theta_a,\theta_b}(B_a,B_b)}{\partial P}=\begin{cases}\left[\frac{\epsilon\tau_{\theta_a}(\frac{B_a}{P})}{1-\tau_{\theta_a}(\frac{B_a}{P})(1+\epsilon)}\right]\frac{B_a}{nP^2}+\left[\frac{\epsilon\tau_{\theta_b}(\frac{B_b}{P})}{1-\tau_{\theta_b}(\frac{B_b}{P})(1+\epsilon)}\right]\frac{B_b}{nP^2}, & \text{if } B_a>C_{\theta_a} \text{ and } B_b>C_{\theta_b}\\ \left[\frac{\epsilon\tau_{\theta_a}(\frac{B_a}{P})}{1-\tau_{\theta_a}(\frac{B_a}{P})(1+\epsilon)}\right]\frac{B_a}{nP^2}, & \text{if } B_a>C_{\theta_a} \text{ and } B_b<C_{\theta_b}\\ \left[\frac{\epsilon\tau_{\theta_b}(\frac{B_b}{P})}{1-\tau_{\theta_b}(\frac{B_b}{P})(1+\epsilon)}\right]\frac{B_b}{nP^2}, & \text{if } B_a<C_{\theta_a} \text{ and } B_b>C_{\theta_b}\\ 0, & \text{if } B_a<C_{\theta_a} \text{ and } B_b<C_{\theta_b}\end{cases}$$

In the region $B_a > C_{\theta_a}$ and $B_b > C_{\theta_b}$ increasing the price level decreases the real value of nominal bonds in both countries. Since each country is individually above the bond cutoff, decreasing the real value of nominal bonds results in lower taxes hence higher welfare. The two regions ($B_a > C_{\theta_a}$ and $B_b < C_{\theta_b}$) and ($B_a < C_{\theta_a}$ and $B_b > C_{\theta_b}$) are situations when one country, but not the other is above the the bond cutoff. In one country, increasing the price level decreases the real value of bonds and leads to lower taxes and higher welfare. In the other country, increasing the price level decreases the real value of bonds and leads to increased transfers and constant welfare. The last region $B_a < C_{\theta_a}$ and $B_b < C_{\theta_b}$ considers the case that both countries are below their respective bond cutoffs. If this is the case, increasing the price level decreases the real value of bonds and leads to increased transfers and constant welfare in both countries.

All regions except for the last $B_a < C_{\theta_a}$ and $B_b < C_{\theta_b}$ have a positive welfare gain to increasing the price level. Hence the pricing function is

$$P(B_a,B_b)=\begin{cases}\max\left(\frac{B_a}{C_{\theta_a}},\frac{B_b}{C_{\theta_b}}\right), & \text{if } B_a>C_{\theta_a} \text{ or } B_b>C_{\theta_b}\\ 1, & \text{if } B_a\le C_{\theta_a} \text{ and } B_b\ge C_{\theta_b}\end{cases}$$

where the max operator is combining the two cases where either ($B_a > C_{\theta_a}$ and $B_b < C_{\theta_b}$) or ($B_a < C_{\theta_a}$ and $B_b > C_{\theta_b}$) holds.

As before, issuing C_h bonds raises the maximum possible amount of revenue for each fiscal authority. From the perspective of the benevolent monetary authority welfare (i.e. the value function at the beginning of next period) is identical at B_0 and C_h. There is no harm to welfare from issuing C_h bonds and there is a gain to the coalition of the self-interested fiscal authority in doing so. Due to the possibility that increases in the price level will be caused by actions of the other country in the monetary union, issuing C_h bonds raises a different amount of revenue than before

$$\beta\left[\pi^2 C_{h_a} + \pi(1-\pi)C_{h_a}\left(\frac{C_{l_b}}{C_{h_b}}\right) + (1-\pi)\pi C_{l_a} + (1-\pi)^2 C_{h_a}\left(\min\left[\frac{C_{l_a}}{C_{h_a}}, \frac{C_{l_b}}{C_{h_b}}\right]\right)\right]$$