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Sovereign Default and Maturity Choice^{*}

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Abstract

This paper presents a new quantitative model of endogenous sovereign default, maturity choice, and the term structure of bond yield spreads. Sovereigns usually borrow from international markets at positive term spreads and a duration that exceeds one year. Debt duration and term spreads vary with the business cycle, decreasing during periods of higher credit-market stress. However, when maturity choice is incorporated in models of long-term sovereign debt and default (Chatterjee and Eyigungor, 2012; Hatchondo and Martinez, 2009), countries prefer one-period bonds, largely due to debt dilution. The presence of sudden stops and debt reschedulings can largely account for the maturity choice and term structure of bond yield spreads found in the data. The model also generates the non-monotonic spread curve observed during some debt crises.

JEL Classification: F34, F41, G15.

Keywords: Debt Crises, Restructuring, Yield Curves, Spreads, Bond Duration, Dilution.

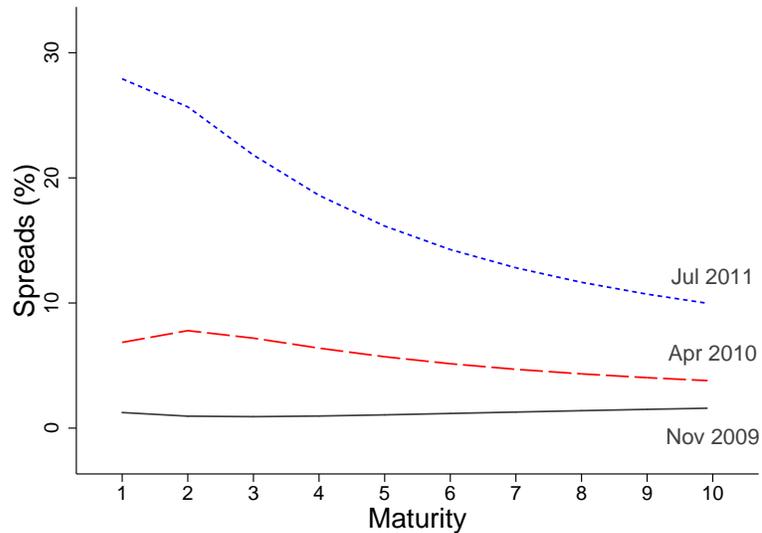
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1 Introduction

Our paper studies the term structure of interest rate spreads and the maturity of sovereign bonds. We develop a framework that helps rationalize the maturity choice for sovereign debt and the pricing of this debt at each maturity. In bad times, sovereign default risk and interest rate spreads increase, while term spreads decline, often resulting in a negative-sloped yield curve for the borrowing country. As we focus on the risk of sovereign default, subsequently we consider sovereign bond yield spreads over a risk-free debt instruments. The spread at each maturity is the difference between the yield on a zero-coupon bond with that maturity and default risk, and the yield on a bond with the same characteristics but with negligible default risk.

Yield spread curves can be non-linear and non-monotonic, as in the case of the Greek spread curve during 2009-2011, shown in Figure 1. The solid black line corresponds to November 2009, when Greece's troubles were mounting, but their full severity had not been yet clearly perceived by many market participants. By the end of October 2009, the credit rating agency Fitch had only downgraded Greece's credit rating from an *A*, which denotes expectations of low default risk, to a slightly worse *A-*. Greek sovereign yield spreads over German bonds were historically low, and the Greek curve was flat. The dashed red line corresponds to April 2010, after Fitch had downgraded Greece's credit rating to *BBB-*. A *BBB* rating means that the capacity for payment of financial commitments is considered adequate but adverse business or economic conditions are more likely to impair this capacity. The April 2010 curve was higher than the November 2009, and was humped, peaking at a 2-year maturity. As explained in Neely (2012), the non-monotonicity can reflect that a debt restructuring is expected, but not immediately, so that bonds maturing within the following year, i.e. before the expected date of the restructuring, were seen as more likely to be served than those maturing in 2 years, i.e., after the anticipated date of the debt operation. Yet, longer maturities carry lower spreads, consistent with lower expected bondholder losses for those tenors. The blue short-dashed line corresponds to July 2011, near the peak of the Greek debt crisis; the Fitch rating for Greece's credit was *CCC*, just one notch above default. The yield spread curve at that time was higher than ever and downward sloping.

Figure 1: Spread curves, Greece

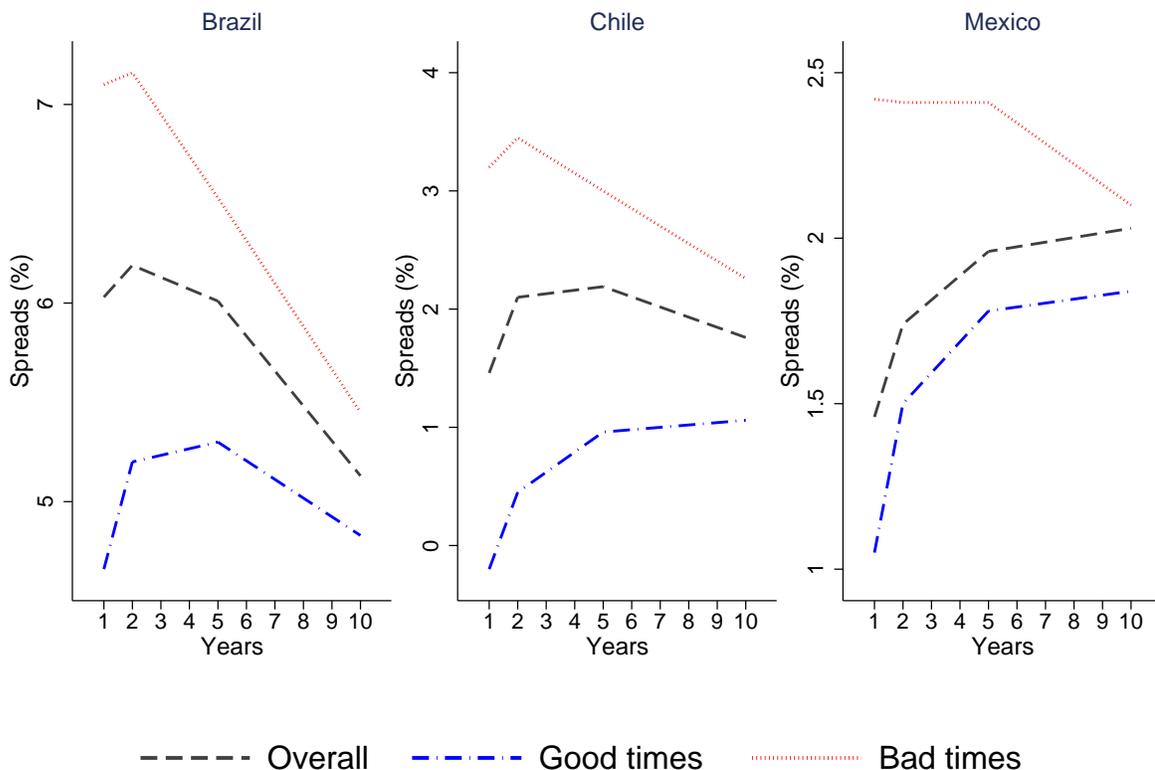


Note: See Appendix A.1 for data sources.

In general, as an economy approaches a period of relatively high financial stress, as measured by higher interest rates and lower output growth, the sovereign spread curve flattens and may invert. Figure 2 shows this pricing pattern for three Latin American countries. The figure also illustrates the strong non-linearity and non-monotonicity of the spread curves during good and bad times, suggesting that analyzing only the 1-year vs 10-year yield term spread, which is the focus of the current models of sovereign debt maturity and default, can leave out pricing information that is relevant to understand the dynamics of debt maturity, and may even be misleading. The overall spread curve for Chile serves to illustrate this simple point. Depending on whether one uses a term spread between 10 and 1 or 2 years, the slope will have a very different magnitude and even a different sign, potentially leading to opposite readings regarding the prospects of the economy. The example highlights that to accurately reflect the pricing information from sovereign bonds, it may not be sufficient to focus on a single term spread, and it may be necessary to instead look at the full spread curve. The non-linearity and non-monotonicity of the yield curve spreads had not been addressed before in the sovereign default literature, as existing debt maturity models analyze spreads for one or at most two different

maturities.

Figure 2: Spread curves, several countries



Note: See Appendix A.1 for data sources. Good (bad) times are observations with 1-year spread lower (higher) than the median. Given numbers are the median spreads for each country.

To understand why default risk may explain changes in the yield spread curve, notice that the yield of a zero-coupon bond that pays in j period is,

$$i_t^0(j) = \frac{1+r}{(\prod_{i=1}^j P_{t+i})^{1/j}},$$

where $P_{t+1}, P_{t+2}, \dots, P_{t+j}$ denote the repayment probabilities in 1, 2, ..., j periods ahead and r is the risk-free interest rate.¹ As a result, depending on the future repayment probabilities P_{t+i} , the

¹The derivation of this formula is presented in Appendix A.3.

shape of the yield curve may be increasing, decreasing, or non-monotone, following the patterns observed in the data, as illustrated in Figure 2 for Latin American economies. Of course, in our model these probabilities will be endogenous and they will depend on the country’s income, as well as the level and the maturity of its debt.

The inversion of the sovereign yield spread curve during bad times is usually accompanied by the shortening of the average debt duration and maturity. Table 1 shows such a pattern for the maturity and duration of sovereign debt in various countries. The table also indicates that even during bad times, countries tend to borrow at maturities and durations that significantly exceed one year.

Table 1: Emerging markets’ choice of maturity (years)

	Argentina	Brazil	Chile	Colombia	Mexico
Maturity					
Overall	9.25	11.32	9.29	6.07	11.91
Good times	10.36	16.12	12.00	6.20	12.03
Bad times	7.42	9.97	8.90	5.95	12.94
Duration					
Overall	6.11	6.47	7.11	3.96	6.69
Good times	6.65	8.23	9.12	4.08	7.45
Bad times	4.86	5.82	6.87	3.85	6.79

Note: See Appendix A.1 for data sources. Good (bad) times are observations with a 1-year spread lower (higher) than the median.

We identify debt dilution as a first crucial factor affecting debt maturity; it generates an increasing yield curve and, as a consequence, it reduces the maturity of debt. Two other factors are important to understand why countries prefer longer maturity, despite the increasing yield curve. First, the maturity of sovereign debt generally increases as a result of debt restructurings. While many restructuring episodes imply reductions in the face value of debt, almost all involve debt maturity extensions. Cruces and Trebesch (2013) document that 123 out of 180 sovereign

debt restructurings from 1970 to 2010 were pure debt reschedulings, i.e., debt events that solely involved lengthening the maturity of old debt instruments. Second, sudden stops in international credit markets can also significantly shape the maturity profile of sovereign debt. To the extent that default is costly, the possibility of a sudden withdrawal of funding that makes the sovereign unable to repay its immediate debt obligations creates a strong incentive for the sovereign to borrow at longer maturities. The presence of sudden stops has been well documented in the literature (see for instance [Edwards, 2007](#)).

The likelihood of sudden stops and debt reprofiling is also relevant to explain the recent debt dynamics of peripheral euro-area economies. A case in this point is Ireland, which exited its 2010 international bailout at the end of 2013 without any European Union or International Monetary Fund precautionary credit line as insurance policy. The sole reliance on market funding without such a safety net, together with firm but tight budget conditions and an upward sloped yield curve consistent with the pricing of the risk of debt dilution, provided the Irish government with strong incentives for short term borrowing. Yet, policymakers opted to issue long term debt. Despite the issuance of long term bonds, 10-2 years Irish term spreads declined about 600 basis points to 350 basis points from 2014Q1 to 2015Q1. During this period, the higher risk of a credit event in Greece increased the likelihood of a sharp fall in short term funding to Ireland. Issuing more long maturity debt helped avoid such rollover risk. Moreover, at the same time, the higher likelihood of a new debt restructuring of Greek debt increased the likelihood that Ireland demanded similar delays in debt payments. Such possibility also contributed to make long term debt relatively more attractive, helping to make the Irish yield spread curve flatter.

Related literature Our analysis borrows from different strands of the literature on sovereign debt, default, and debt maturity. Following the seminal work in international sovereign debt by [Eaton and Gersovitz \(1981\)](#), a large portion of the literature on quantitative models of sovereign debt default has used only one-period debt ([Arellano, 2008](#); [Aguar and Gopinath, 2006](#); [D’Erasmus, 2008](#); [Yue, 2010](#); [Mendoza and Yue, 2012](#), among others). Models of long debt duration, such as [Chatterjee and Eyigungor \(2012\)](#) and [Hatchondo and Martinez \(2009\)](#), feature exogenous maturity. In contrast, our quantitative model features endogenous sovereign debt

maturity and repayment under debt dilution. The work of [Arellano and Ramanarayanan \(2012\)](#) also includes the choice of maturity, but there are several important differences.² First, debt in that model consists of a one-period bond and a long-term bond. The latter is a perpetuity bond with a duration defined by an exogenous parameter of coupon payment decay. The equilibrium duration in these models depends by construction on this exogenous parameter of the long-term bond, which should not be the case if duration were endogenous. For instance, [Hatchondo, Martinez, and Sosa-Padilla \(2014\)](#) calibrate the decay parameter to be 3.4%, so that debt has an average duration of 4.2 years in their model simulations. In addition, as pointed out in [Bai, Kim, and Mihalache \(2014\)](#), the perpetuity bond is restricted to have a fixed, front-loaded repayment schedule, which is at odds with the data but technically required to keep the discounted present value of repayments bounded. While the work by [Bai, Kim, and Mihalache \(2014\)](#) tackles the problem of a front-loaded repayment schedule, its framework does not consider debt dilution, i.e., that additional borrowing decreases the price of outstanding debt. We argue that debt dilution is key to understand maturity choice and the term structure of interest rate spreads.

[Cole and Kehoe \(2000\)](#) study self-fulfilling debt crisis equilibria considering a different timing from [Eaton and Gersovitz \(1981\)](#), where the government issues new bonds before it decides to default within the period. Cole and Kehoe characterize the crisis zones; that is, the values and maturity structure of sovereign debt under which a financial crisis can arise due to a loss of confidence in the government. Interestingly, they find that while lengthening the maturity of debt can shrink the crisis zone. We model sudden stops as exogenous shocks and focus in the country's choice of maturity. In our framework, countries choose longer maturity to prevent the cost of sudden stops.

[Aguiar and Amador \(2013\)](#) also point to the optimality of strategies involving only short-term debt in the context of sovereign default risk models. Their study finds that during periods of deleveraging it is optimal for sovereigns to engage in strategies that use only one-period bonds. Aguiar and Amador argue that active shifts in the maturity structure by the sovereign may affect deleveraging incentives, and hence change the equilibrium price of long-term bonds. The price movements of long-term bonds will shrink the budget set of the sovereign, creating the incentive

²See also [Hatchondo and Martinez \(2013\)](#) and [Hatchondo, Martinez, and Sosa-Padilla \(2014\)](#).

to use only one-period bonds during a period of deleveraging. Our result for the benchmark model, for which we find that the optimal maturity is one, resembles their mechanism.

Debortoli, Nunes, and Yared (2014) incorporate fiscal policy with and without commitment into a framework similar to Buera and Nicolini (2004), in which the government chooses debt maturity and cannot issue state-contingent bonds. Debortoli, Nunes, and Yared (2014) find that while optimal debt maturity is tilted long under fiscal policy commitment, it is optimal to keep a nearly flat maturity structure when the government cannot commit to fiscal policy, as large and tilted positions are too expensive to finance ex-ante because they increase the problem of lack of fiscal commitment ex-post. If households are primarily buying long-term bonds ex-ante, they anticipate that the government lacking commitment will pursue future policies which increase future short-term interest rates, thus diluting their claims. In this case, households require a higher ex-ante interest rate relative to commitment, to induce them to lend long-term to the government. A similar reasoning holds if households are primarily buying short-term bonds ex-ante. Our study also explores the effects of sovereign debt dilution, but via a different dilution mechanism, the lack of government commitment to repay its debt. Our channel is absent in their study because their model does not consider sovereign credit risk.

Recently, Fernandez and Martin (2014) analyze the effect of debt restructurings and reprofiling on maturity in a three-period model. Their main finding is that these policies will increase welfare if they raise total expected payments to creditors in times of crisis. Our analysis of debt restructurings resembles their analysis but in a richer quantitative model.

Our work makes three contributions to the literature: First, we document that debt dilution is key to understand the optimal choice of maturity. Consistently with Hatchondo, Martinez, and Sosa-Padilla (2014), we find that dilution is also largely responsible for sovereign interest rate spreads. Additionally, we measure the effect of dilution on maturity and spreads at different horizons. We find that models of endogenous debt maturity that consider dilution fall short in explaining the maturity observed in the data, delivering corner solutions where agents pick the shortest debt maturity that is available. A remaining key question is what factors may account for the long duration in sovereign debt markets.

Our second contribution is the identification of the factors that explain the maturity and

duration of sovereign debt observed in the data. For this purpose we develop a quantitative model of endogenous sovereign debt maturity with dilution. We find that (i) a high degree of risk aversion by the borrower, (ii) the possibility of sudden stops, and (iii) debt reschedulings help account for the duration of debt documented in the data. We also evaluate quantitatively the role of each factor in explaining the optimal maturity structure over the business cycle. In doing so, our framework yields novel testable implications. By including debt reprofiling, the most common resolution mechanism observed in default events, we can better assess the incentives and effects associated with default decisions and their interaction with optimal maturity choices. Moreover, our model generates the full-term structure of sovereign yield spreads, providing insights not only into the slope of the spread curve but also its curvature. This additional dimension makes our model a suitable framework to explore issues such as the comovement and the spread between government and private sector bond yields. [Cuadra, Sanchez, and Sapriza \(2010\)](#) and [Mendoza and Yue \(2012\)](#) discuss some stylized pricing facts of private and public sector debt in quantitative sovereign default models, but only with one-period bonds.

Third, we develop a model of endogenous maturity is simpler than [Arellano and Ramanarayanan \(2012\)](#) and less computationally demanding, as it replaces a continuous state variable with a discrete one that takes a few values. The savings in terms of runtime and computer resources and the simpler implementation make it easier to apply our methodology to other economic environments.

In Section 2, we present the economic environment and the benchmark model and define the equilibrium. In Section 3, we give a quantitative analysis of the benchmark model. In Section 4, we study the quantitative implications of introducing different features to the economic environment in the benchmark. In Section 5, we quantitatively assess a preferred economy embedding sudden stop shocks and debt reschedulings into the benchmark setup. In Section 6, we present the conclusions.

2 Benchmark model

We consider a small open economy model with households, a benevolent government, and foreign lenders. The government trades in bonds of different maturities with risk-neutral foreign lenders. Debt contracts are not enforceable, as the government has the option to default on them. Default is costly for the country, and foreign lenders charge a premium to account for the probability of not being paid back by the government.

2.1 Preferences and endowments

Time is discrete and denoted by $t \in \{0, 1, 2, \dots\}$. Each period the small open economy has a stochastic endowment y_t that follows a finite-state first-order Markov chain with state space $Y \subset \mathbb{R}_{++}$ and transition probability $\Pr\{y_{t+1} = y' \mid y_t = y\}$.

The benevolent government in the country maximizes the expected utility over consumption sequences of the representative household. The discount factor is $\beta \in (0, 1)$ and the risk aversion coefficient is $\gamma \geq 1$. The momentary utility function is

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}. \quad (1)$$

2.2 Bond prices

A country's bond portfolio is described by a fixed coupon payment, b , and its maturity, m . That portfolio may consist of different bonds.³ In this section, we introduce the notation that we use to write the price of those bonds.

Given a country's portfolio, characterized by (b, m) , the market value of a bond that pays \tilde{b} for n periods is

$$\text{bond value} = \tilde{b}q(y, b, m; n).$$

Notice that since default occurs on the entire portfolio, this bond price, q , depends on the portfolio characteristics $\{b, m\}$. When the portfolio maturity, m , coincides with the bond maturity, n , the

³If the portfolio is composed of one bond or several bonds is irrelevant in this framework.

price q represents the unit price of the portfolio. The market value of the portfolio can be written as

$$\text{portfolio value} = bq(y, b, m; n = m).$$

To illustrate the notation further, consider the following example. Suppose that the country has the portfolio:

$$\text{portfolio} = \{b, b, b, b\}.$$

Then,

$$\text{portfolio value} = b \times q(y, b, 4; 4).$$

Given that the country is holding that portfolio, we can also compute the value of a promise to pay only a part of the coupon, \underline{b} , for the following four periods.⁴ That is,

$$\text{holding part of the coupon} = \{\underline{b}, \underline{b}, \underline{b}, \underline{b}\} \rightarrow \text{value} = \underline{b} \times q(y, b, 4; 4).$$

This pricing is possible because q is the price per unit of the coupon. More importantly, this notation also allows us to price a subset of the portfolio that consists of payments of \underline{b} for only the first three periods.⁵ That is,

$$\text{subset of portfolio} = \{\underline{b}, \underline{b}, \underline{b}, 0\} \rightarrow \text{value} = \underline{b} \times q(y, b, 4; 3).$$

Similarly, we can obtain the market value of a (zero coupon) bond that pays \underline{b} in the fourth period given that the country's portfolio is the same than above. That is,

$$\text{four-period zero coupon bond} = \{0, 0, 0, \underline{b}\} \rightarrow \text{value} = \underline{b} \times [q(y, b, 4; 4) - q(y, b, 4; 3)].$$

These prices will be very useful for providing notation for the country's choice of maturity.

⁴For instance, this may be the case because the country has two outstanding bonds, one paying \underline{b} and other paying $b - \underline{b}$.

⁵For instance, this may be the case because the country has three bonds outstanding, one for $\{\underline{b}, \underline{b}, \underline{b}, 0\}$, other for $\{b - \underline{b}, b - \underline{b}, b - \underline{b}, 0\}$, and a zero-coupon bond $\{0, 0, 0, b\}$.

2.3 Decision problem

A country with an outstanding amount of assets, b (debt if $b < 0$), has two actions to choose from. The first option is to make the payment (G , for good credit status), which enables the country to issue new debt. If it issues new debt, the country may change the size of the coupon and the maturity. The second option for the country is to default (D , for default). The country's choice to either pay or default is represented by

$$V(y, b, m) = \max [V^G(y, b, m), V^D(y)]. \quad (2)$$

The policy function describing this decision, $D(y, b, m)$, is 1 if default is preferred and 0 otherwise. If the country chooses to default, lifetime utility is represented by

$$V^D(y) = \frac{(y - \Phi(y))^{1-\gamma}}{1-\gamma} + \beta E_{y'|y} [(1-\lambda)V^D(y') + \lambda V(y', 0, 0)],$$

where λ is the probability that the country gains access to credit markets in the next period, and $\Phi(y)$ represents the income losses to the country while it is excluded from credit markets.

Not defaulting involves paying and potentially changing both the coupon b' and the maturity m' of the debt:

$$V^G(y, b, m) = \max_{b', m'} \frac{c^{1-\gamma}}{1-\gamma} + \beta E_{y'|y} V(y', b', m')$$

subject to

$$c = y + b - \underbrace{q(y, b', m'; m')b'}_{\text{issue new debt}} + \underbrace{q(y, b', m'; m-1)b}_{\text{retire old debt}}$$

$$b' \in \mathbb{R}_-, m' \in \mathbf{M}(m),$$

where \mathbf{M} is the set maturities that the country can choose from.⁶ Notice that while the term $q(\cdot)b'$ represents the value of the portfolio issued this period, the last term can be interpreted

⁶For instance, the set \mathbf{M} could contain any maturity between 1 and 30 years.

as the cost of retiring the old debt at the market price. This is just one possible interpretation, however. The country does not need to take this specific action. Rewriting the budget constraint, we can see that it is equivalent to the country changing the maturity by simply issuing (or buying back) zero coupon bonds:

$$c = y + b - \underbrace{q(y, b', m'; m')(b' - b)}_{\text{change coupon, given } m'} - \underbrace{[q(y, b', m'; m') - q(y, b', m'; m - 1)] b}_{\text{change maturity, given } b}.$$

The term $q(\cdot)(b' - b)$, which represents a change in the coupon for a given maturity, captures the debt dynamics described in the literature on long-duration bonds as in [Hatchondo and Martinez \(2009\)](#). The last term captures the choice of maturity, what the country achieves by buying/selling zero-coupon bonds.

The policy functions for choosing debt and maturity are $B(y, b, m)$ and $M(y, b, m)$. Notice that this representation is general enough to allow the country to make only the debt payment. In that case, the country would choose $b' = b$ and $m' = m - 1$.

2.4 Equilibrium

Given the world interest rate r , the price of the country's debt must be consistent with zero expected discounted profits. The price of a bond of maturity $n > 0$ in a country with income y , new debt $-b'$, and maturity $m' > 0$ can be represented by

$$q(y, b', m'; n) = \frac{E_{y'|y}}{1+r} \left\{ (1 - D(y', b', m')) \underbrace{[1 + q(y', B(y', b', m'), M(y', b', m'); n - 1)]}_{\text{value of remaining debt}} \right\}.$$

The value of debt captures the expected payment in the next period and the value of the remaining debt once that payment is made. Importantly, the value of those remaining obligations depend on the country's next-period choices for the level and the maturity of debt, $B(\cdot)$ and $M(\cdot)$, respectively. Thus, the lender is taking into account the possibility of *debt dilution* arising from the borrower's future behavior.

3 Quantitative analysis

We solve the model numerically, calibrating the parameters based on the literature and available data. This section describes the calibration strategy and compares the results with the data. Details on the computation are presented in Appendix A.4.

3.1 Calibration

We set the maximum possible maturity to 15 years, which is significantly larger than the maturity observed for emerging markets.⁷ In addition, we assume that each year the country can change the maturity by at most one year: i.e., $\mathbf{M}(m) = \{m - 1, m, m + 1\}$. This seems to be a reasonable assumption given the observed variations in maturity and it reduces the time of computation significantly.⁸

We use the standard function for the income loss in case of default:

$$\Phi(y) = \begin{cases} 0, & \text{if } y < \phi; \\ \phi, & \text{o/w.} \end{cases}$$

Table 2 summarizes the model parameters. We follow [Arellano and Ramanarayanan \(2012\)](#) in setting most of the model parameters. In particular, the yearly risk-free interest rate is set to 0.032 and the probability of redemption if the country is excluded from the financial markets is set to 0.17. The standard deviation of the income shock is set to 0.017, and the persistence is set to 0.9. In the benchmark calibration, we assume a value of 2 for the risk aversion parameter.

⁷Our results are robust to allowing for longer maximum maturities.

⁸In addition, our results change only slightly if we assume $\mathbf{M}(n) = \{m - 2, m - 1, m, m + 1, m + 2\}$.

Table 2: Calibrated parameters, benchmark

Parameter	Value	Basis
Interest rate, r	0.032	Arellano and Ramanarayanan (2012)
Redemption prob, λ	0.170	”
Income shock std, σ_y	0.017	”
Income autocorrelation, ρ_y	0.900	”
Risk aversion, γ	2.000	”
Discount factor, β	0.750	Debt-to-output ratio of 25%
Cost of default, ϕ	0.900	Default rate of 2%

We calibrate the discount factor, β , and the threshold of income in the income loss function, ϕ , to match two key moments in the data: the debt-to-output ratio and the default rate. In what follows, whenever we modify the framework, we adjust these two parameters such that indebtedness and default are constant. This strategy facilitates comparison between alternative specifications.⁹

3.2 Comparison with the data

The benchmark model is able to mimic some of the features of the data. The second and third columns of Table 3 present the key statistics from the data and from the baseline specification, respectively. In both the model and the data, income and consumption are highly positively correlated and consumption is more volatile. As in the data, 10-year spreads tend to be above one-year spreads, and spreads are higher in bad times at both the short and long ends of the spread curve. The model cannot reproduce the duration and maturity observed in the data, however. Table 3 shows that the model generates debt duration of one year, significantly shorter than that in the data, which is about 5.5 years on average. Similarly, the model is unable to generate any of the debt maturity observed in the data. Finally, the cyclical nature of maturity and duration observed in the data, shown by the correlation of 0.24 and 0.22 between maturity and

⁹In Appendix A.6 we show the calibrated values of β and ϕ for each setting.

duration with income, cannot be reproduced by the model, for which duration and maturity are basically acyclical.

Finally, the last column of Table 3 highlights the dramatic effects of sovereign debt dilution on debt duration and maturity. The next subsection analyzes in detail the role of debt dilution in generating the short debt maturity and duration in the benchmark economy.

Table 3: Key statistics, data, benchmark, and no dilution

	Data	Benchmark	No dilution
Duration	5.52	1.00	3.50
Maturity	8.78	1.00	6.34
Duration (good times)	6.99	1.00	4.07
Duration (bad times)	5.97	1.00	3.08
Maturity (good times)	10.00	1.00	7.32
Maturity (bad times)	8.71	1.00	5.48
$\rho(mat, \log(y))$	0.24	0.01	0.43
$\rho(dur, \log(y))$	0.22	0.02	0.49
1-year spread	3.17	2.36	1.55
1-year spread (good times)	1.70	1.43	0.69
1-year spread (bad times)	4.59	3.64	3.23
10-year spread	3.58	2.76	0.46
10-year spread (good times)	3.35	2.59	0.32
10-year spread (bad times)	4.33	3.11	0.65
$\sigma(\log(c))/\sigma(\log(y))$	1.32	1.39	1.32
$\rho(\log(c), \log(y))$	0.80	0.73	0.76
Default (%)	2.00	2.29	2.27
Value of debt / Income	0.25	0.24	0.24

3.3 Debt dilution

Our benchmark model explicitly allows for the possibility of debt dilution, i.e., that the price of outstanding bonds decreases when new bonds are issued. This feature increases the spreads of long-maturity bonds and, as a direct consequence, reduces the optimal maturity. The work of

Hatchondo and Martinez (2009) already highlighted that spreads are higher for longer maturity bonds because of debt dilution. The difference with their study is that here, we endogeneize the choice of maturity, so countries choose to avoid paying that higher spread by reducing the maturity of their debt.

To analyze the effect of debt dilution, we recompute the model but without debt dilution. For this purpose, we assume that to change the maturity of outstanding obligations, those obligations must be retired at the net present value discounted at the risk-free rate, and only then new bonds can be issued. Hence, the country's problem is the same as before except for the value of issuing new debt, which is

$$V^G(y, b, m) = \max_{b', m'} \frac{c^{1-\gamma}}{1-\gamma} + \beta E_{y'|y} V(y', b', m')$$

subject to

$$c = y + b - \mathbb{1}(b' \neq b \vee m' \neq m - 1) \underbrace{(q(y, b', m'; m')b')}_{\text{issue new debt}} + \underbrace{q^F(m-1)b}_{\text{retire old debt}}$$

$$b' \in \mathbb{R}_-, m' \in \mathbf{M}(m),$$

where for $\tilde{m} > 0$

$$q^F(\tilde{m}) \equiv \sum_{s=1}^{\tilde{m}} \left(\frac{1}{1+r} \right)^s,$$

and $q^F(0) = 0$. The indicator function $\mathbb{1}(B(y', b', m') \neq b' \vee M(y', b', m') \neq m' - 1)$ reflects the fact that the country must retire the old debt and issue new debt only if it changes either the level or the maturity of the debt.

The pricing scheme also has to be modified accordingly,

$$q(y, b', m'; n) = \frac{E_{y'|y}}{1+r} \{ (1 - D(y', b', m')) ([1 + \mathbb{1}(B(y', b', m') \neq b' \vee M(y', b', m') \neq m' - 1) q^F(m' - 1) + \mathbb{1}(B(y', b', m') = b' \wedge M(y', b', m') = m' - 1) q(y', b', m' - 1; n - 1)] \}.$$

This specification resembles that in [Bai, Kim, and Mihalache \(2014\)](#). The last two columns of [Table 3](#) show the results of the model with debt dilution (benchmark) and without debt dilution. Eliminating dilution increases the optimal maturity of debt. With dilution, the optimal maturity is 1, while without dilution it is 3.5, more than three-fold. Our calibration is very similar to [Chatterjee and Eyigungor \(2012\)](#) and [Hatchondo and Martinez \(2009\)](#). Therefore, the takeaway from this exercise is that allowing for endogenous maturity choice in a model of long-term bonds eliminates long bonds from the equilibrium.

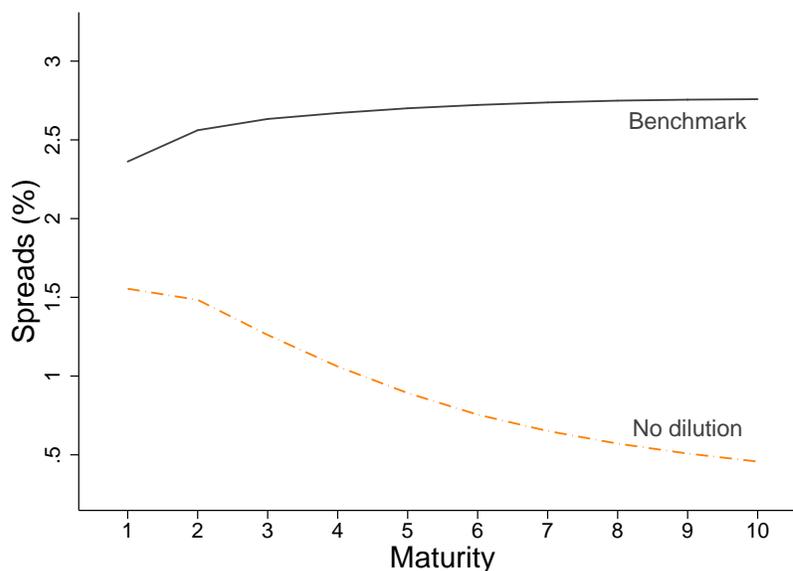
Eliminating dilution increases the optimal maturity in a counterfactual manner, however. [Figure 3](#) shows the average spread curve with and without dilution. The spread curve with dilution is slightly increasing, while it is strongly decreasing once dilution is eliminated. An upward-sloping spread curve means that it is cheaper for the country to fund its financing needs with shorter-maturity debt, which makes the country optimally choose such maturity. The model without debt dilution can generate longer duration but with counterfactual spread curves and equilibrium spreads. [Table 3](#) shows, for instance, that in the case of no dilution, the median 10-year yield spread is 0.46% while the median one-year yield spread is 1.55%. Consistent with these findings, our next step is to determine which country or market features can explain both the observed duration and maturity, while also matching the shape of the spread curve and equilibrium spreads found in the data.

4 Key features to account for long debt duration

The model described before fails to generate the long debt duration observed in the data. In this section, we highlight three features that help the benchmark model replicate the observed duration without abstracting away from debt dilution. In particular, we consider risk aversion, sudden stops, and distressed debt reschedulings. We discuss the role of each of these factors within the baseline framework presented above.¹⁰

¹⁰Other features were considered but they have little impact on the choice of maturity. Among others, we considered i.i.d. expenditure shocks, fixed costs of issuing new bonds, and shocks to the risk free interest rate.

Figure 3: Spread curve with and without dilution



Note: Given numbers are the averages across the median yields of each sample.

4.1 Risk aversion

One advantage of long-maturity debt is that the country is insured (by risk neutral lenders) by fixing the terms of the debt contract. As a result, the optimal maturity of debt should increase with the borrower's degree of risk aversion. Table 4 compares the data with the model results at different levels of risk aversion for the borrowing country. A risk aversion parameter of 2 has an optimal duration of one year, whereas risk aversion parameters of 5 and 7.5 have durations of 2.53 and 4.21 years, respectively. The model also captures the cyclical nature of most macroeconomic variables of interest in the analysis, although increasing risk aversion reduces the volatility of consumption relative to that of output. Overall, higher risk aversion seems reasonable to bring the model closer to the data.

Table 4: Risk aversion

	Data	Benchmark, $\gamma = 2$	Risk aversion, γ	
			5	7.5
Duration	5.52	1.00	2.53	4.21
Maturity	8.78	1.00	4.21	8.34
Duration (good times)	6.99	1.00	2.50	4.42
Duration (bad times)	5.97	1.00	2.59	4.09
Maturity (good times)	10.00	1.00	4.11	8.39
Maturity (bad times)	8.71	1.00	4.38	8.24
$\rho(mat, \log(y))$	0.24	0.01	-0.11	0.29
$\rho(dur, \log(y))$	0.22	0.02	-0.09	0.41
1-year spread	3.17	2.36	0.47	0.32
1-year spread (good times)	1.70	1.43	0.07	0.03
1-year spread (bad times)	4.59	3.64	1.86	2.08
10-year spread	3.58	2.76	1.64	2.52
10-year spread (good times)	3.35	2.59	1.08	1.60
10-year spread (bad times)	4.33	3.11	2.45	3.92
$\sigma(\log(c))/\sigma(\log(y))$	1.32	1.39	1.11	1.03
$\rho(\log(c), \log(y))$	0.80	0.73	0.91	0.91
Default (%)	2.00	2.29	1.64	2.26
Value of debt / Income	0.25	0.24	0.25	0.25

4.2 Sudden stops

Financial globalization has exposed countries to the risk of funding shocks (see, for instance, [Fratzscher, 2012](#)). The likelihood of experiencing a sudden stop, i.e., an abrupt reduction in net capital inflows to a country, affects the optimal maturity choice of the sovereign. In our baseline model, we introduce the possibility of a sudden stop as an exogenous shock that occurs with probability p_s and prevents the country from issuing new debt.

This possibility adds an additional state variable $a \in \{0, 1\}$ that takes value 0 if the country has no access to credit markets, i.e., if it is experiencing a sudden stop. The value function before

the choice of repayment is

$$V(y, a, b, m) = \max [V^G(y, a, b, m), V^D(y)] , \quad (3)$$

and the policy function $D(y, a, b, m)$ is 1 if default is preferred and 0 otherwise. For a country with access to credit markets ($a = 1$), the value of having a good credit standing is very similar to the benchmark:

$$V^G(y, 1, b, m) = \max_{b', m'} \frac{c^{1-\gamma}}{1-\gamma} + \beta E_{y', a' | y} V(y', a', b', m')$$

subject to

$$\begin{aligned} c &= y + b - q(y, b', m'; m')b' + q(y, b', m'; m - 1)b \\ b' &\in \mathbb{R}_-, m' \in \mathbf{M}(m). \end{aligned}$$

In contrast, a country that receives a sudden stop shock and as a consequence has no access to credit markets ($a = 0$) can make its payment but cannot issue new debt:

$$V^G(y, 0, b, m) = \frac{(y+b)^{1-\gamma}}{1-\gamma} + \beta E_{y', a' | y} V(y', a', b, m - 1).$$

The policy functions for choosing debt and maturity are $B(y, a, b, m)$ and $M(y, a, b, m)$. Notice that when a country makes only its debt payment, the policies are $B(y, a, b, m) = b$ and $M(y, a, b, m) = m - 1$. If $a = 0$, then we must have $B(y, 0, b, m) = b$ and $M(y, 0, b, m) = m - 1$.

Finally, the value function of defaulting is also similar to the benchmark:

$$V^D(y) = \frac{(y - \Phi(y))^{1-\gamma}}{1-\gamma} + \beta E_{y', a' | y} [(1 - \lambda)V^D(y') + \lambda V(y', a', 0, 0)] .$$

As sudden stops influence the choices of borrowing and defaulting, the bond prices must

contemplate the probability of receiving this shock:

$$q(y, b', m'; n) = \frac{E_{y', a' | y}}{1 + r} \{(1 - D(y', a', b', m'))[1 + q(y', B(y', a', b', m'), M(y', a', b', m'); n - 1)]\}.$$

In the calibration, we first set the probability of a sudden stop at 10%, consistent with the estimate for developing countries presented by [Edwards \(2007\)](#) and [Bianchi, Hatchondo, and Martinez \(2013\)](#). We also conduct a sensitivity analysis using a more conservative value of 5%. The advantage of long-maturity debt is that the country does not need to roll over all its obligations every period. This is particularly important if there exists the possibility of a sudden stop. Thus, the optimal maturity of sovereign debt increases with the probability of a sudden stop. The last two columns in [Table 5](#) illustrate quantitatively how changing the probability of a sudden stop affects the key statistics. The maturities across good and bad times are substantially higher with 10% probability of a sudden stop than with 5%. The model also exhibits a larger pro-cyclicality of both debt duration and maturity, as observed in the data. In addition, the model captures the pro-cyclicality of the term spread generally found in the data, as well as the cyclicity of most macroeconomic variables described in the table, especially with 10% probability of a sudden stop. Indeed, incorporating sudden stops helps reducing the gap between model and data.

Table 5: Sudden stops

	Data	Benchmark, $p_s = 0\%$	Probability of sudden stops, p_s	
			5%	10%
Duration	5.52	1.00	1.97	2.48
Maturity	8.78	1.00	3.00	4.22
Duration (good times)	6.99	1.00	2.00	2.49
Duration (bad times)	5.97	1.00	1.96	2.50
Maturity (good times)	10.00	1.00	3.03	4.21
Maturity (bad times)	8.71	1.00	3.00	4.29
$\rho(mat, \log(y))$	0.24	0.01	0.42	0.34
$\rho(dur, \log(y))$	0.22	0.02	0.44	0.36
1-year spread	3.17	2.36	1.41	1.27
1-year spread (good times)	1.70	1.43	0.51	0.32
1-year spread (bad times)	4.59	3.64	3.03	2.91
10-year spread	3.58	2.76	2.30	2.43
10-year spread (good times)	3.35	2.59	1.95	2.07
10-year spread (bad times)	4.33	3.11	2.59	2.89
$\sigma(\log(c))/\sigma(\log(y))$	1.32	1.39	1.35	1.32
$\rho(\log(c), \log(y))$	0.80	0.73	0.75	0.74
Default (%)	2.00	2.29	1.89	1.94
Value of debt / Income	0.25	0.24	0.27	0.29

4.3 Restructuring debt in default

One salient feature of sovereign debt restructurings is that in most cases they involve extending the maturities of the old debt instruments, which is sometimes accompanied by a reduction in their face values. Debt restructurings involving maturity extensions are known as reprofiling or reschedulings. Pure debt reschedulings were particularly prevalent in the debt crises of the 1980s, many of which affected the Latin American economies. While the 1990s saw a shift toward restructurings involving larger face value haircuts, the extension of debt maturities continues to be a very heavily used mechanism in for debt crisis resolution. Moreover, a recent International

Monetary Fund (IMF) proposal has brought renewed attention to debt rescheduling policies as a key debt crisis resolution mechanism. The IMF proposal aims to introduce greater flexibility to its 2002 lending framework by providing the IMF with a broader range of policy responses during sovereign debt distress episodes, including using debt reprofiling operations as a partial substitute for bailouts.

A sovereign debt restructuring affects the maturity and duration of sovereign debt through two channels: first, the direct extension of maturity increases debt maturity and duration. Second, a restructuring encourages long term borrowing by reducing the relative prices of short term bonds. As holders of short term debt incur in larger losses than holders of longer maturity debt during a restructuring, the yield spread curve becomes flatter relative to an economy where restructurings are not an option, providing stronger incentives to the sovereign to borrow long term.

In this section, we quantitatively assess the role of distressed debt reschedulings on the maturity, yield spread, and term structure of sovereign debt. In cases of default, the sovereign faces two possible outcomes: (i) a disorderly default, which occurs with probability $1 - \epsilon$ and brings, as in the benchmark, immediate financial autarky for a stochastic number of periods and a direct output loss to the defaulting country, or (ii) an orderly default, or debt rescheduling, which occurs with probability ϵ and implies that the sovereign suffers a direct output loss, restructures its outstanding obligations without making the current payment, and stays in credit markets. In the latter case, the maturity of the debt automatically increases by η years and the debt amount is changed so that the new debt level together with the new maturity give a face value equal to the previous value with a haircut Ψ . The country learns about the type of default once the default decision is made.

The debt value of a country with debt b with maturity m and income y is,¹¹

$$V(y, b, m) = \max [V^G(y, b, m), V^D(y, b, m)]. \quad (4)$$

The value function V^G is specified as in the benchmark, but now the expected value of

¹¹Here, we shut down the sudden stop shocks to isolate the effects of debt crisis resolutions. We will allow for the interaction of these shocks with debt reschedulings in the next section when we introduce our preferred economy.

defaulting, V^D , takes into account the possibility of orderly and disorderly default. Hence, V^D depends on the value of debt and its maturity:

$$V^D(y, b, m) = \epsilon V^{D,o}(y, b, m) + (1 - \epsilon) V^{D,u}(y).$$

The value of orderly default is

$$V^{D,o}(y, b, m) = \frac{(y - \Phi(y))^{1-\gamma}}{1-\gamma} + \beta E_{y'|y} V(y', b^R, n^R),$$

where

$$n^R = \min(n + \eta, N)$$

$$b^R = (1 - \Psi) b \frac{m}{n^R}.$$

The value of disorderly default resembles the value of default in the benchmark case,

$$V^{D,u}(y) = \frac{(y - \Phi(y))^{1-\gamma}}{1-\gamma} + \beta E_{y'|y} [(1 - \lambda) V^D(y', 0, 0) + \lambda V(y', 0, 0)],$$

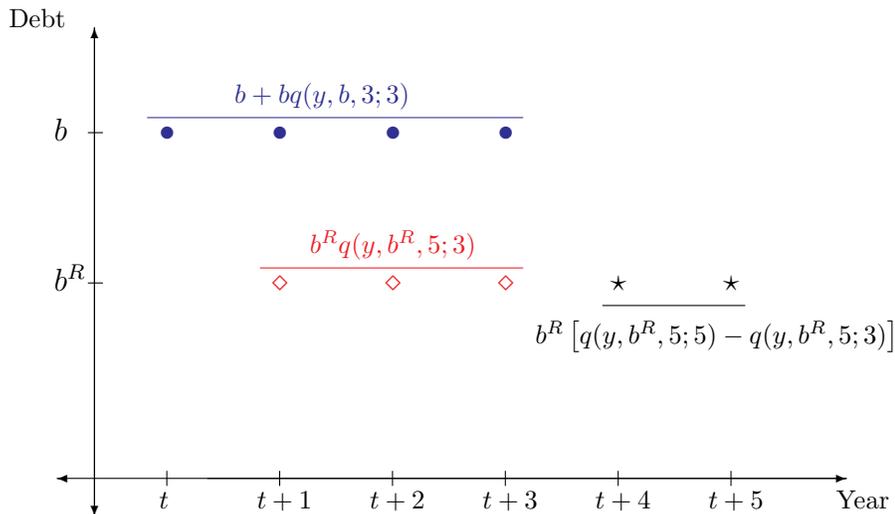
with the decision to default denoted by $D(y, b, m)$, as before. The equilibrium prices with the possibility of orderly default are captured by the following equation:

$$\begin{aligned} q(y, b', m'; n) = & \\ & \frac{E_{y'|y}}{1+r} \{ (1 - D(y', b', m')) [1 + q(B(y', b', m'), y', M(y', b', m'); n - 1)] \\ & + \epsilon D(y', b', m') \frac{b^R}{b'} (q(y', b^R, n^R; n - 1) + \frac{n-1}{m'-1} [q(y', b^R, n^R; n^R) - q(y', b^R, n^R; m' - 1)]) \}, \end{aligned}$$

where the last term reflects the possibility of restructuring debt. The ratio $\frac{b^R}{b'}$ captures the change in the value of the coupon and is typically smaller than 1. The term $q(y', b^R, n^R; n - 1)$ reflects the value of the coupons up to the maturity of the bond that is being priced, $n - 1$ years. The value of the years added to the previous maturity is captured by the term $[q(y', b^R, n^R; n^R) - q(y', b^R, n^R; m' - 1)]$, which is the unit price of bonds that start to pay in m' periods until n^R

periods from now. The added debt is distributed among debtholders. How much do the creditors receive? That amount is given by $\frac{n-1}{m'-1}$, which captures the sum of the discounted payments that the holder of this bond was promised relative to all the country's obligations.

Figure 4: Debt rescheduling



To illustrate how distressed debt rescheduling works, consider a simple example in which the maturity extension parameter η is set to 2. Restructuring implies that bondholders' losses are not uniform across different maturities: holders of long-maturity bonds suffer smaller losses. Consider an economy entering distressed debt rescheduling at time t with a debt portfolio maturity of 4 years and an annual coupon payment of b . The value of the debt entering the rescheduling is the sum of b , the current coupon payment, and the discounted value of the stream of coupon payments of size b to be disbursed over the next three years, priced at $q(y, b, 3; 3)$. Figure 4 illustrates that value with blue circles. Under the terms of the rescheduling, the face value of the coupon payments is reduced from b to $b_R < b$. In addition, the country postpones the first disbursement for one year, so it starts paying b_R in period $t + 1$. In addition, the payments are now extended

over the next five years. This means that the country will make payments for two additional years in the future compared with its original payment scheme. The value of the new five-year maturity portfolio can be separated in two subsets: first, the value of the stream of coupons up to the third year, i.e., the same maturity as the original portfolio, $b_R q(y, b_R, 5; 3)$; second, the value of the coupons b_R paid over the two years that are added to the original maturity, i.e., the pricing of the stream of payments $\{0, 0, 0, b_R, b_R\} \rightarrow q(y, b_R, 5; 5) - q(y, b_R, 5; 3)$. The last term, representing the bonds at new maturity, will be distributed among bondholders in proportion to the face value of the debt they hold, and represented by $\frac{n-1}{m'-1}$.

When we allow for orderly defaults, the model-specific calibrated parameters are the probability that a rescheduling be implemented, ϵ , which takes the values of 25% and 50% (see [Cruces and Trebesch, 2013](#)), and the minimum number of years that the debt maturity could be extended under a rescheduling, η , which is set to two years, a reasonable value for a lower threshold (see, for instance, [IMF, 2014](#)). In this exercise, we assume no haircut: $\Psi = 0$. The results are presented in [Table 6](#).

Table 6: Orderly default

	Data	Benchmark, $\epsilon = 0\%$	Probability of orderly default, ϵ	
			25%*	50%
Duration	5.52	1.00	1.47	1.67
Maturity	8.78	1.00	2.00	2.35
Duration (good times)	6.99	1.00	1.33	1.50
Duration (bad times)	5.97	1.00	1.47	1.89
Maturity (good times)	10.00	1.00	1.27	2.03
Maturity (bad times)	8.71	1.00	2.00	2.77
$\rho(mat, \log(y))$	0.24	0.01	-0.17	-0.27
$\rho(dur, \log(y))$	0.22	0.02	-0.20	-0.34
1-year spread	3.17	2.36	4.61	2.18
1-year spread (good times)	1.70	1.43	2.21	1.01
1-year spread (bad times)	4.59	3.64	7.96	5.11
10-year spread	3.58	2.76	3.89	1.34
10-year spread (good times)	3.35	2.59	3.39	1.19
10-year spread (bad times)	4.33	3.11	4.31	1.59
$\sigma(\log(c))/\sigma(\log(y))$	1.32	1.39	1.28	1.25
$\rho(\log(c), \log(y))$	0.80	0.73	0.77	0.79
Reprofiling (%)	—	0.00	1.33	1.97
Default (%)	2.00	2.29	4.05	2.01
Value of debt / Income	0.25	0.24	0.18	0.34

(*) For this case, the calibration is exactly the same as that for the benchmark.

Focus on the last column of Table 6, which presents the results for the probability of orderly defaults $\epsilon = 50\%$. We find that including the possibility of debt rescheduling increases the optimal duration and maturity. Because rescheduling is associated with bad times, duration and maturity are countercyclical. Moreover, the reschedulings make the yield spread curves inversions in bad times become more pronounced, because holding short-term debt at times when the government is close to default is riskier than in the absence of the rescheduling option. Short-term debt is now riskier because in a rescheduling, the short-term bondholder will lose the current coupon payment and receive the future coupon payments under a delayed schedule. Therefore, such a

bondholder loses both from the loss of any immediate payment due and from the delay in receipt of future payments.

The fourth column of Table 6 presents the results for the probability of orderly defaults $\epsilon = 25\%$. One caveat is in order for this case. To show the effect of introducing orderly defaults on debt and default, this case is computed with the same parameters that the benchmark. We find that as reschedulings are less costly than disorderly defaults, they lower the expected cost of experiencing a default, and increase the optimal frequency of debt crisis episodes (disorderly default plus reprofiling). As a result of the higher default frequency due to a larger number of reschedulings, the volatility of term spreads is higher and increasing in the conditional rescheduling probability.

5 Preferred economy

The results in the previous section show that three features that we study can separately help the model generate maturity patterns that are closer to those observed in the data. We now consider the three features together in a single framework, our preferred economy, and show that they can jointly account for the observed duration without sacrificing other aspects of the data. In particular, we solve a model that exhibits high risk aversion, sudden stop, and debt rescheduling as highlighted above, where the arrival of a sudden stop is independent from that of debt rescheduling. Since the formal description of the value functions and equilibrium prices of each feature were discussed in the previous section, we defer the formulations corresponding to this economy to Appendix A.5.

The third column of Table 7, referred to as “preferred,” presents the moments from a model that includes a risk aversion coefficient of 5, a 10% probability of a sudden stop, and a 50% probability of the country entering debt rescheduling conditional on experiencing a default. The model generates a debt duration of 4.8 years, almost five times that of the benchmark specification and close to that observed in the data, 5.5 years. The model result for sovereign debt maturity, which is almost 9.7 years, is only slightly higher than in the data, 8.8 years. The model also captures the difference in optimal maturity and duration between good and bad times found in

the data. If measured by the median value in good and bad times, defined by the one-year spread, duration and maturity are less cyclical in the model than in the data. However, if we focus on the correlation of duration and maturity with income, we find that they are more procyclical in the model than in the data. When looking at spreads, the model replicates the generally positive observed term premium and the countercyclical, although the spread levels are lower in the model than in the data. Finally, the model also captures the high correlation between output and consumption, and the higher volatility of consumption relative to output observed in the data.

These results suggest that the level and the business cycle dynamics of sovereign debt maturity and yield spreads are highly affected by the borrower's risk aversion, the likelihood of events such as sudden stops that may unexpectedly limit the ability of the country to roll over its short-term obligations, and the type of debt restructuring that the country may face during a debt crisis. Once we account for such factors, our sovereign debt default framework captures most moments in the data.

We evaluate the sensitivity of our results to the specifications of parameters governing the rescheduling by running two other exercises in addition to our preferred economy. First, we allow for a 20% haircut when debt is rescheduled: $\Psi = 0.20$. Second, we set the maturity extension to five years instead of two: $\eta = 5$. The last two columns of Table 7, referred to as "haircut" and "longer extension," show that those alternative parameterizations actually improve the model's fit of some moments. Hence, the conclusions in this section with our preferred economy do not depend on the assumptions regarding the rescheduling of debt in the case of orderly default.

Table 7: Alternative economies

	Data	Benchmark	Preferred	Haircut*	Longer extension*
Duration	5.52	1.00	4.82	5.18	5.24
Maturity	8.78	1.00	9.72	10.36	10.72
Duration (good times)	6.99	1.00	4.98	5.17	5.07
Duration (bad times)	5.97	1.00	4.73	5.19	5.34
Maturity (good times)	10.00	1.00	9.72	10.19	10.04
Maturity (bad times)	8.71	1.00	9.67	10.66	11.17
$\rho(mat, \log(y))$	0.24	0.01	0.26	0.18	0.15
$\rho(dur, \log(y))$	0.22	0.02	0.42	0.30	0.33
1-year spread	3.17	2.36	0.40	0.62	0.64
1-year spread (good times)	1.70	1.43	0.03	0.06	0.06
1-year spread (bad times)	4.59	3.64	2.85	3.79	3.67
10-year spread	3.58	2.76	1.74	2.00	2.02
10-year spread (good times)	3.35	2.59	1.14	1.31	1.28
10-year spread (bad times)	4.33	3.11	2.68	3.03	3.14
$\sigma(\log(c))/\sigma(\log(y))$	1.32	1.39	1.11	1.07	1.09
$\rho(\log(c), \log(y))$	0.80	0.73	0.84	0.87	0.85
Reprofiling (%)		0.00	1.79	1.77	1.83
Default (%)		2.29	1.76	1.78	1.88
Value of debt / Income		0.24	0.36	0.32	0.35

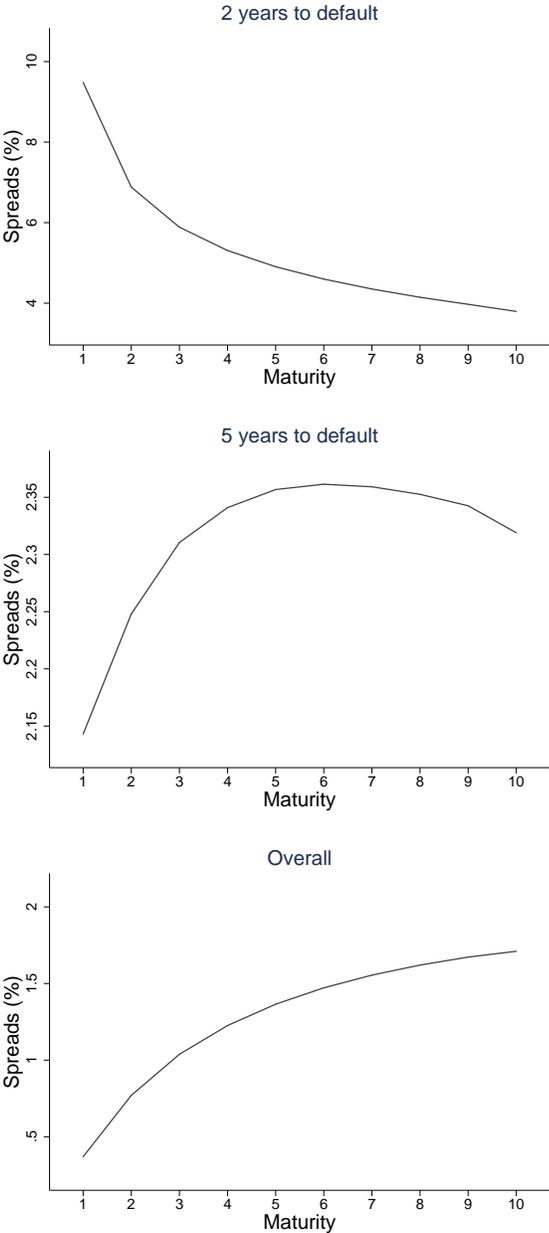
(*) For this case the calibration is exactly the same that for the preferred economy.

Our preferred economy also shows interesting changes in the yield spread curve as countries approach default, as shown in Figure 5.¹² The top panel shows that yields are high and decreasing with maturity in periods near default episodes. The middle panel shows that a few years before default the yield spread curve is non-monotone, peaking at the period in which default is most likely, as we described for the Greek yield spread curve during 2011. Finally, the bottom panel shows the overall curve, which is increasing and concave. Notice that the concavity and non-monotonicity of the yield spread curve cannot be generated, by construction, in the model of

¹²To compute the yield spread curve described in Figure 5 we computed the yields of zero-coupon bonds of different maturities in every period. The values presented are the median values for each maturity across bad and good times.

endogenous maturity of [Arellano and Ramanarayanan \(2012\)](#), where only two points of the spread curve are analyzed.

Figure 5: Spread curves, preferred economy



Note: Spreads are the averages across the median yields of each sample.

To better demonstrate the implications of an orderly default, we now analyze the lenders' losses in such an event. In particular, we derive a measure of losses accrued by debtholders if a country enters an orderly default. The measure compares the value of debt in the aftermath of the orderly default with the same value had the country not defaulted, where the two values are discounted at the median market price for a portfolio with a given maturity.¹³ In particular, consider a country with a portfolio with level of debt b and maturity m . Losses of a lender that holds a bond of maturity n are:

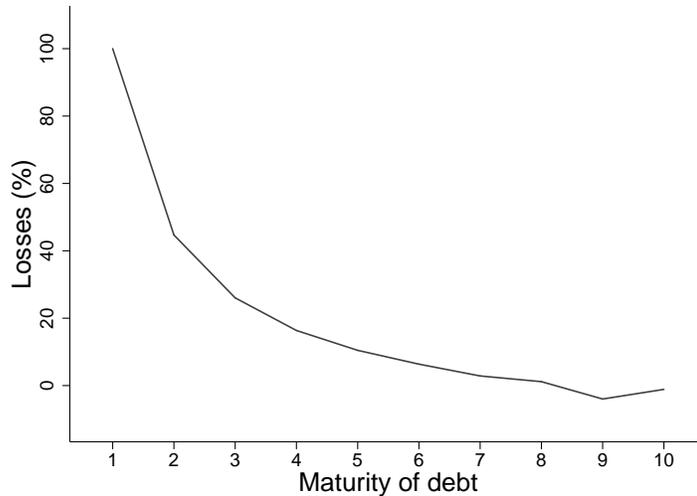
$$Losses(n) = - \left(\frac{\frac{b^R}{b} \{ \tilde{q}(n-1) + \frac{n-1}{m-1} [\tilde{q}(n^R) - \tilde{q}(m-1)] \}}{\tilde{q}(n)} - 1 \right),$$

where b^R is the new level of debt, n^R is the new maturity after rescheduling, and $\tilde{q}(n)$ is the median price of portfolios with maturity n from the simulations.

To compute the losses implied by our model, we consider the orderly default observations that occur with portfolios of maturity 10 in the simulated samples used in Table 7. Figure 6 shows the average of those losses for bonds with 1 to 10 years of maturity. Two main results stand out. First, notice that investors' losses are positive. This occurs because the rescheduling increases the maturity and therefore it reduces the market value of the debt. Second, the distribution of losses is not uniform across investors, with holders of shorter-maturity instruments accruing larger losses. This happens because they receive a smaller share of the rescheduled debt portfolio.

¹³This measure follows [Sturzenegger and Zettelmeyer \(2007\)](#).

Figure 6: Losses of lenders in orderly default



6 Conclusions

Our study explains key properties of the maturity and the yield spread curve of sovereign debt found in the data. First, the duration of sovereign debt generally exceeds one year and co-moves positively with the borrowing country's business cycle. Second, yield spread curves show an upward slope in good times and are inverted during periods of credit-market stress. Third, the curve is non-linear and, under certain conditions such as distressed debt reschedulings, can become non-monotonic. This paper presents a novel quantitative model of sovereign default and maturity choice that helps identify key factors that matter for understanding the stylized facts described above. In particular, we identify debt dilution as a first crucial factor reducing the maturity of debt. Actually, we find that in our benchmark model, calibrated as it is standard in the literature of quantitative models of sovereign default (Arellano, 2008; Aguiar and Gopinath, 2006; Chatterjee and Eyigungor, 2012; Hatchondo and Martinez, 2009; Arellano and Ramanarayanan, 2012), countries would prefer to use mostly one-period bonds. Eliminating debt dilution, the model is able to capture the maturity observed in the data, but at the cost of counterfactual implications for spread curves. Instead of ignoring debt dilution, this paper asks the following question: What features of financial markets can help us explain the data? Our

findings indicate that the presence of sudden stops, particularly for more risk averse borrowers, and debt reschedulings, go a long way in reconciling the model with the data.

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A Appendix

A.1 Data

For all statistics presented as “data” in tables 3 to 7, we use the data from Argentina, Brazil, Chile, Colombia, Mexico, Peru and Turkey. We treat the data and the model moments in the same way. In particular, for spread, duration and maturity statistics that we report from the data, we use country-specific medians and then compute the average across countries.

A.2 Notes on variable definitions

Duration For the duration of a bond, we use the Macaulay definition (as in Hatchondo and Martinez (2009)) which is a weighted sum of future coupon payments:

$$\frac{q(y, b', m'; 1) + 2 \times (q(y, b', m'; 2) - q(y, b', m'; 1)) + \dots + n \times (q(y, b', m'; n) - q(y, b', m'; n - 1))}{q(y, b', m'; n)}.$$

Yield to maturity Yield for maturity n for an observation with borrowing level b' , income y and maturity of the held bond m is $\left(\frac{1}{q(b', y, m; n) - q(b', y, m; n - 1)}\right)^{\frac{1}{n}} - 1$.

$$YTM(y, b', m'; n) \equiv \left(\frac{1}{q(y, b', m'; n) - q(y, b', m'; n - 1)}\right)^{\frac{1}{n}} - 1.$$

Then the spread for maturity m is $YTM(y, b', m'; n) - r$.

A.3 Yield curve and repayment probability

Consider bonds that make constant unitary coupon payments for m periods. In this section we explain how the unit bond price depends on the default probabilities. For simplicity, we consider the repayment probabilities in periods $1, 2, \dots, j$ denoted by $P_{t+1}, P_{t+2}, \dots, P_{t+j}$. Then, the price of a bond that pays one unit the next period is

$$q_t(1) = \frac{P_{t+1}}{1 + r},$$

where r is the risk free rate. Similarly, the price for bonds that pay one unit for two periods is

$$q_t(2) = \frac{P_{t+1}}{1+r} + \frac{P_{t+1}P_{t+2}}{(1+r)^2},$$

and more generally, the price for a bonds that pays for j periods is

$$q_t(j) = \frac{P_{t+1}}{1+r} + \frac{P_{t+1}P_{t+2}}{(1+r)^2} + \dots + \frac{\prod_{i=1}^j P_{t+i}}{(1+r)^j}.$$

With these prices at hand, we can write prices of zero-coupon bonds that pay in 1, 2, ..., j periods

$$\begin{aligned} q_t^0(1) &= q_t(1) = \frac{P_{t+1}}{1+r}, \\ q_t^0(2) &= q_t(2) - q_t(1) = \frac{P_{t+1}P_{t+2}}{(1+r)^2}, \\ &\dots \\ q_t^0(j) &= q_t(j) - q_t(j-1) = \frac{\prod_{i=1}^j P_{t+i}}{(1+r)^j}. \end{aligned} \tag{5}$$

Then, the yield of a zero-coupon bond that pays in j period is $i_t^0(j)$,

$$\begin{aligned} i_t^0(1) &= \frac{1+r}{P_{t+1}}, \\ i_t^0(2) &= \frac{1+r}{(P_{t+1}P_{t+2})^{1/2}}, \\ &\dots \\ i_t^0(j) &= \frac{1+r}{(\prod_{i=1}^j P_{t+i})^{1/j}}. \end{aligned} \tag{6}$$

A.4 Computation

Basics

We solve the model numerically using value function iteration on a discrete state space for debt and income shocks. We use 201 points for debt grid and 41 points income. We approximate the process of income using the Rouwenhorst technique described by [Kopecky and Suen \(2010\)](#).

Approximating the solution using shocks on cost of default

In order to achieve a better convergence in the iterations of value and price functions, we introduce uncertainty in the cost of default in a similar fashion with the idiosyncratic income shocks highlighted in [Chatterjee and Eyigungor \(2012\)](#). In particular, we approximate the solution of each model by solving the corresponding models featuring idiosyncratic shocks to the cost of default:

$$\tilde{\Phi}(y, \mu) = \Phi(y) + \mu, \mu \sim N(0, 0.1\sigma_y).$$

Then the value functions for the benchmark are:

$$V(y, b, m, \mu) = \max [V^G(y, b, m), V^D(y, \mu)]. \quad (7)$$

If the country chooses to default it gets:

$$V^D(y, \mu) = \frac{(y - \tilde{\Phi}(y, \mu))^{1-\gamma}}{1-\gamma} + \beta E_{y', \mu' | y} [(1 - \lambda)V^D(y', \mu') + \lambda V(y', 0, 0, \mu')].$$

In case of not defaulting, the budget constraint is the same as before, but the value for next period should take the expectation over the cost of default shocks:

$$V^G(y, b, m) = \max_{b', m'} \frac{c^{1-\gamma}}{1-\gamma} + \beta E_{y', \mu' | y} V(y', b', m', \mu').$$

The policy for default now also depends on the cost of default shock: $D(y, b, m, \mu)$. Notice that we can define the threshold cost shock that leaves the sovereign indifferent between defaulting and paying:

$$\tilde{\mu}(y, b, m) \equiv y - \Phi(y) - [(1 - \gamma) (V^G(y, b, m) - \beta E_{y', \mu' | y} [(1 - \lambda)V^D(y', \mu') + \lambda V(y', 0, 0, \mu')])]^{\frac{1}{1-\gamma}}$$

Above (below) this threshold the sovereign pays (defaults). Then we can write the probability of not defaulting in terms of the c.d.f. of μ , and use it in the price equation:

$$q(y, b', m'; n) = \frac{E_{y' | y}}{1+r} \{Pr(\mu \geq \tilde{\mu}(y', b', m')) [1 + q(y', B(y', b', m'), M(y', b', m'); n - 1)]\}.$$

This way, we mitigate the problem of having big jumps in the price function across similar debt levels, since we replace a binary variable of not defaulting with a continuous c.d.f. within the expectation.

Simulations

Once we solved for the policy and value functions, we simulate 1500 countries (paths) for 500 years, and take out the first 100 periods before calculating the moments. The model counterparts of the spread, duration and maturity statistics are averages across country-specific medians.

A.5 Detailed formulations for the preferred economy

Here we give the value functions and the equilibrium conditions for bond prices for our preferred economy, which exhibits sudden stops and debt reschedulings. In this setting, the value of a sovereign that has debt b with maturity m and income y is:

$$V(y, a, b, m) = \max [V^G(y, a, b, m), V^D(y, b, m)]. \quad (8)$$

The value function V^G is the same that the one in the economy that only features sudden stops, so is omitted here. The value of default captures the probability of orderly default, and the probability of sudden stops in the following period:

$$V^D(y, b, m) = \epsilon V^{D,o}(y, b, m) + (1 - \epsilon) V^{D,u}(y).$$

The value of orderly default is:

$$V^{D,o}(y, b, m) = \frac{(y - \Phi(y))^{1-\gamma}}{1-\gamma} + \beta E_{y', a' | y} V(y', a', b^R, n^R),$$

where

$$n^R = \min(n + \eta, N)$$

$$b^R = (1 - \Psi) b \frac{m}{n^R}.$$

Meanwhile, disorderly default gives:

$$V^{D,u}(y) = \frac{(y - \Phi(y))^{1-\gamma}}{1-\gamma} + \beta E_{y',a'|y} [(1 - \lambda)V^D(y', 0, 0) + \lambda V(y', a', 0, 0)].$$

The equilibrium condition for bond prices in this economy is:

$$\begin{aligned} q(y, b', m'; n) = & \\ & \frac{E_{y',a'|y}}{1+r} \{(1 - D(y', a', b', m'))[1 + q(B(y', a', b', m'), y', M(y', a', b', m'); n - 1)] \\ & + \epsilon D(y', a', b', m') \frac{b^R}{b'} (q(y', b^R, n^R; n - 1) + \frac{n-1}{m'-1} [q(y', b^R, n^R; n^R) - q(y', b^R, n^R; m' - 1)])\}. \end{aligned}$$

which takes into account the probability of sudden stops and orderly default in the next period.

A.6 Calibrated parameters across alternative settings

Setting	β	ϕ
Benchmark	0.75	0.90
No dilution	0.77	0.91
Risk aversion, γ		
	5	0.90
	7.5	0.88
Probability of sudden stops, p_s		
	5%	0.86
	10%	0.87
Probability of orderly default, ϵ		
	25%	0.75
	50%	0.86
Preferred economy	0.90	0.85
Haircut	0.90	0.85
Long extension	0.90	0.85