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Postwar U.S. Business Cycles: An Empirical Investigation

We propose a procedure for representing a time series as the sum of a smoothly varying trend component and a cyclical component. We document the nature of the comovements of the cyclical components of a variety of macroeconomic time series. We find that these comovements are very different than the corresponding comovements of the slowly varying trend components.

THE PURPOSE OF THIS ARTICLE is to document some features of aggregate economic fluctuations sometimes referred to as business cycles. The investigation uses quarterly data from the postwar U.S. economy. The fluctuations studied are those that are too rapid to be accounted for by slowly changing demographic and technological factors and changes in the stocks of capital that produce secular growth in output per capita.

As Lucas (1981) has emphasized, aggregate economic variables in capitalist economies experience repeated fluctuations about their long-term growth paths. Prior to Keynes' *General Theory*, the study of these rapid fluctuations, combined with the attempt to reconcile the observations with an equilibrium theory, was regarded as the main outstanding challenge of economic research. Although the Keynesian Rev-

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This paper is substantially the same as our 1981 working paper. The only major change to the paper is the addition of an Appendix of Tables that mirror our originals and contain data ending in 1993. Since we did not update the citations, we apologize to the many authors who have used the Hodrick-Prescott filter and studied its properties in the intervening eighteen years since its original development.

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olution redirected effort away from this question to the one of determining the level of output at a point in time in disequilibrium, the failure of the Keynesian Theory in the 1970s has caused many economists to want to return to the study of business cycles as equilibrium phenomena. In their search for an equilibrium model of the business cycle, modern economists have been guided by the insights of Mitchell (1913) and others who have used techniques of analysis that were developed prior to the development of modern computers. The thesis of this paper is that the search for an equilibrium model of the business cycle is only beginning and that studying the comovements of aggregate economic variables using an efficient, easily replicable technique that incorporates our prior knowledge about the economy will provide insights into the features of the economy that an equilibrium theory should incorporate.

This study should be viewed as documenting some systematic deviations from the restrictions upon observations implied by neoclassical growth theory.[1] Our statistical approach does not utilize standard time series analysis. Our prior knowledge concerning the processes generating the data is not of the variety that permits us to specify a probability model as required for application of that analysis. We proceed in a more cautious manner that requires only prior knowledge that can be supported by economic theory. The maintained hypothesis, based upon growth theory considerations, is that the growth component of aggregate economic time series varies smoothly over time. The sense in which it varies smoothly is made explicit in section 1.

We find that the nature of the comovements of the cyclical components of macroeconomic time series are very different from the comovements of the slowly varying components of the corresponding variables. Growth is characterized by roughly proportional growth in (per capita) output, investment, consumption, capital stock and productivity (output per hour), and little change in the hours of employment per capita or household. In contrast, the cyclical variations in output arise principally as the result of changes in cyclical hours of employment and not as the result of changes in cyclical productivity or capital stocks. In the case of the cyclical capital stocks in both durable and nondurable manufacturing industries, the correlation with cyclical output is even negative. Another difference is in the variability of components of aggregate demand. Cyclical consumption varies only one-half and investment three times as much as does cyclical output.

Section 2 presents our findings regarding the comovements of these series with the cyclical component of real GNP, as well as an examination of the cyclical components of prices, interest rates, and nominal and real money balances. Section 3 examines the serial correlation properties of a number of the series.

Several researchers, using alternative methods, have added and are adding to our knowledge of aggregate economic fluctuations.[2] Our view is that no one approach dominates all the others and that it is best to examine the data from a number of different perspectives. We do think our approach documents some interesting regularities.

1. Lucas (1980) interprets the work of Mitchell (1913) in a similar light.

2. Examples include Litterman and Sargent (1979), Nelson and Plosser (1980), Neftci (1978), Sargent and Sims (1977), Sims (1980, a, b), and Singleton (1980).

1. DECOMPOSITION PROCEDURE

The observed time series are viewed as the sum of cyclical and growth components. Actually, there is also a seasonal component, but as the data are seasonally adjusted, this component has already been removed by those preparing the data series. If growth accounting provided estimates of the growth component with errors that were small relative to the cyclical component, computing the cyclical component would be just a matter of calculating the difference between the observed value and the growth component. Growth theory accounting (cf. Denison 1974), in spite of its considerable success, is far from adequate for providing such numbers. If our prior knowledge were sufficiently strong so that we could model the growth component as a deterministic component, possibly conditional on exogenous data, plus a stochastic process and the cyclical component as some other stochastic process, estimating the cyclical component would be an exercise in modern time series analysis. Our prior knowledge is not of this variety, so these powerful methods are not applicable. Our prior knowledge is that the growth component varies "smoothly" over time.

Our conceptual framework is that a given time series y_t is the sum of a growth component g_t and a cyclical component c_t:

$$y_t = g_t + c_t \qquad \text{for } t = 1, \ldots, T. \tag{1}$$

Our measure of the smoothness of the $\{g_t\}$ path is the sum of the squares of its second difference. The c_t are deviations from g_t and our conceptual framework is that over long time periods, their average is near zero. These considerations lead to the following programming problem for determining the growth components:

$$\underset{\{g_t\}_{t=-1}^{T}}{\text{Min}} \left\{ \sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T} [(g_t - g_{t-1}) - (g_{t-1} - g_{t-2})]^2 \right\} \tag{2}$$

where $c_t = y_t - g_t$. The parameter λ is a positive number which penalizes variability in the growth component series. The larger the value of λ, the smoother is the solution series. For a sufficiently large λ, at the optimum all the $g_{t+1} - g_t$ must be arbitrarily near some constant β and therefore the g_t arbitrarily near $g_0 + \beta t$. This implies that the limit of solutions to program (2) as λ approaches infinity is the least squares fit of a linear time trend model.

Our method has a long history of use, particularly in the actuarial sciences. There it is called the Whittaker-Henderson Type A method (Whittaker 1923) of graduating or smoothing mortality experiences in constructing mortality tables. The method is still in use.[3] As pointed out in Stigler's (1978) historical review paper, closely related methods were developed by the Italian astronomer Schiaparelli in 1867 and in the ballistic literature in the early forties by, among others, von Neuman.

3. We thank Paul Milgrom for bringing to our attention that the procedure we employed has been long used in actuarial science.

Value of the Smoothness Parameter

The data analyzed, with the exception of the interest rates, are in natural logarithms so the change in the growth component, $g_t - g_{t-1}$, corresponds to a growth rate.

The growth rate of labor's productivity has varied considerably over this period (see McCarthy 1978). In the 1947–53 period, the annual growth rate was 4.20 percent, in the 1953–68 period, 2.61 percent, in the 1968–73 period, only 1.41 percent, and in the subsequent period it was even smaller. Part of these changes can be accounted for by a changing capital-labor ratio and changing composition of the labor force. But, as shown by McCarthy, a sizable and variable unexplained component remains, even after correcting for cyclical factors. The assumptions that the growth rate has been constant over our thirty-year sample period, 1950–79, is not tenable. To proceed as if it were would result in errors in modeling the growth component and these errors are likely to be nontrivial relative to the cyclical component. For this reason, an infinite value for the smoothness parameter was not selected.

The following probability model is useful for bringing to bear prior knowledge in the selection of the smoothing parameter λ. If the cyclical components and the second differences of the growth components were identically and independently distributed, normal variables with means zero and variances σ_1^2 and σ_2^2 (which they are not), the conditional expectation of the g_t, given the observations, would be the solution to program (2) when $\sqrt{\lambda} = \sigma_1/\sigma_2$.

As this probability model has a state space representation, efficient Kalman filtering techniques can be used to compute these g_t.[4] By exploiting the recursive structure, one need not invert a $(T + 2)$ by $(T + 2)$ matrix (T is the number of observations in the sample) as would be necessary if one solved the linear first-order conditions of program (2) to determine the g_t. The largest matrix that is inverted using the Kalman filtering computational approach is 2 by 2. If T is large, this is important because inverting large matrices is costly and there can be numerical rounding problems when implemented on computers. Kalman filtering can be performed with computer packages that are widely available.

Our prior view is that a 5 percent cyclical component is moderately large, as is a one-eighth of 1 percent change in the growth rate in a quarter. This led us to select $\sqrt{\lambda} = 5/(1/8) = 40$ or $\lambda = 1{,}600$ as a value for the smoothing parameter. One issue is, how sensitive are the results to the value of λ that is selected? To explore this issue, various other values of λ were tried. Table 1 contains the (sample) standard deviations and autocorrelations of cyclical real GNP for the selected values of the smoothing parameter as well as statistics to test for the presence of a unit root in the cyclical components.[5] These numbers change little if λ is reduced by a factor of four

4. This minimization has two elements, g_0 and $g_0 - g_{-1}$, which are treated as unknown parameters with diffuse priors. The Kalman smoothing technique (see Pagan 1980) was used to compute efficiently the conditional expectations of the g_t, given the observed y_t. The posterior means of g_0 and $g_0 - g_{-1}$ are the generalized least squares estimates. The conditional expectation of the g_t for $t \geq 1$ are linear functions of these parameters and the observations.

5. The tests for the presence of a unit root are augmented Dickey-Fuller tests in which the change in the cyclical component is regressed on a constant, the level of the cyclical component, and six lags of the

TABLE 1

STANDARD DEVIATION AND SERIAL CORRELATIONS OF CYCLICAL GNP FOR DIFFERENT VALUES OF THE SMOOTHING PARAMETER; SAMPLE PERIOD: 1950.1–1979.2

	λ = 400	λ = 1600	λ = 6400	λ = infinity
Standard Deviations	1.56%	1.80%	2.03%	3.12%
Autocorrelations				
Order 1	.80	.84	.87	.94
Order 2	.48	.57	.65	.84
Order 3	.15	.27	.41	.73
Order 4	−.14	−.01	.17	.61
Order 5	−.32	−.20	.00	.52
Order 6	−.39	−.30	−.11	.44
Order 7	−.42	−.38	−.20	.38
Order 8	−.44	−.44	−.27	.31
Order 9	−.41	−.44	−.31	.25
Order 10	−.36	−.41	−.32	.20
Unit-Root Test	−5.02	−4.47	−3.57	−1.15

to 400 or increased by a factor of four to 6,400. As λ increases, the standard deviation increases and there is greater persistence, with the results being very different for $\lambda = \infty$. It is noteworthy that only the results for the linear detrending violate the assumption that no unit root is giving rise to nonstationarity in the cyclical component.

With our procedure for identifying the growth component ($\lambda = 1{,}600$), the annual rate of change of the growth component varied between 2.3 and 4.9 percent over the sample period, with the minima occurring in 1957 and in 1974. The maximum growth rate occurred in 1964, with another peak of 4.4 percent in 1950. The average growth rate over the period was 3.4 percent. The differences between our cyclical components and those obtained with perfect smoothing ($\lambda = \infty$) are depicted in Figure 1, along with the cyclical component. The smoothness of the variation in this difference, relative to the variation in the cyclical component, indicates that the smoothing parameter chosen is reasonable. We caution against interpreting the cyclical characteristic of the difference as a cycle of long duration. Such patterns can appear as artifacts of the data analysis procedure.

The same transformation was used for all series: that is, for each series j

$$g_{jt} = \sum_{i=1}^{T} w_{it}^{T} y_{ji}, \tag{3}$$

where T is the length of the sample period. If the sample size were infinite, it would not be necessary to index these coefficients by t and

$$g_{jt} = \sum_{i=-\infty}^{\infty} w_i^{\infty} y_{j,t+i} \tag{4}$$

change in the cyclical component. One rejects the presence of a unit root in the cyclical component if the t-statistic for the coefficient on the level of the cyclical component is more negative than the critical value of −2.89 (5 percent) or −3.50 (1 percent).

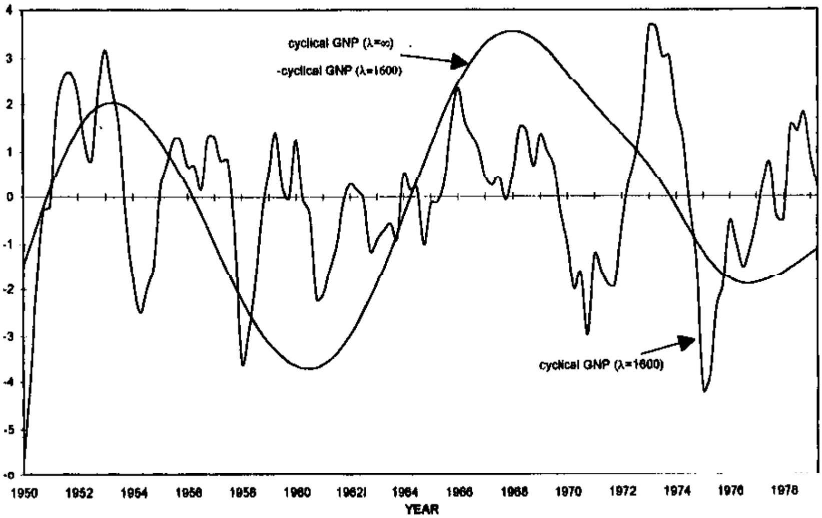

FIG. 1.

where

$$w_i^{\infty} = 0.8941^i\,[0.056168\cos(0.11168\,i) + 0.055833\sin(0.11168\,i)] \qquad (5)$$

for $i \geq 0$ and $w_i = w_{-i}$ for $I < 0$.[6] For t, far from either the end or the beginning of the sample, the w_{it}^T are near w_{t-i}^{∞}, so our method is approximately a two-way moving average with weights subject to a damped harmonic. The advantage of using the exact solution is that observations near the beginning and the end of the sample period are not lost.

The above makes it clear that the data are being filtered. As any filter alters the serial correlation properties of the data, the reported serial correlations should be interpreted with caution. The results do indicate that there is considerable persistence in the rapidly varying component of output. When using the statistics reported here to examine the validity of a model of the cyclical fluctuations of an artificial economy, the serial correlation of the rapidly varying component of the model's aggregate output series should be compared to these numbers. That is, the model's output series should be decomposed precisely as was the data for the U.S.

6. See Miller (1946) for a derivation. There are certain implicit restrictions on the y_t sequence when the sample is infinite. Otherwise, the g_{jt} may not exist. We require that the $\{y_t\}$ sequence belongs to the space for which

$$\sum_{t=-\infty}^{\infty} .8941^{|t|}|y_{jt}| < \infty .$$

economy. Only then, would the model's statistics and those reported here be comparable.

As the comovement results were not particularly sensitive to the value of the smoothing parameter λ selected, in the subsequent analysis only the statistics for $\lambda = 1{,}600$ are reported. With a larger λ, the amplitudes of fluctuations are larger, but the relative magnitudes of fluctuations of the series change little. We do think it is important that all series be filtered using the same parameter λ.

2. VARIABILITY AND COVARIABILITY OF THE SERIES

The components being studied are the cyclical components and subsequently all references to a series relate to its cyclical component. The sample standard deviations of a series is our measure of a series's variability, and the correlation of a series with real GNP is our measure of a series's covariability. These measures are computed for the first half and the second half of the sample, as well as for the entire sample. This is a check for the stability of the measures over time.

A variable might be strongly associated with real output, but lead or lag real output. Therefore, as a second measure of the strength of association with real output, the R-squared for the regression

$$c_{jt} = \alpha_j + \sum_{i=-2}^{2} \beta_{ji} GNP_{t-i} \tag{6}$$

for each series j was computed.

The ratio of the explained sum of the squares for this regression to the explained sum of squares for the regression when the coefficients are not constrained to be equal in the first and the second halves of the sample is our measure of stability. It is a number between zero and one, with one indicating that the best-fit equation is precisely the same in the first and second halves of the sample.

We chose this measure rather than applying some F-test for two reasons. First, we do not think the assumption of uncorrelated residuals is maintainable. Second, even if it were, it is very difficult to deduce the magnitude of the instability from the reported test statistic.

Aggregate Demand Components

The first set of variables studied are the real aggregate demand components. The results are summarized in Tables 2 and 3. The series that vary the least are consumption of services, consumption of nondurables and state and local government purchases of goods and services. Each of these has standard deviation less than the 1.8 percent value for real output. The investment components, including consumer durable expenditures, are about three times as variable as output. Covariabilities of consumption and investment with output are much stronger than the covariability of government expenditures with output.

TABLE 2

AGGREGATE DEMAND COMPONENTS: STANDARD DEVIATIONS AND CORRELATIONS WITH GNP
SAMPLE PERIOD: 1950.1-1979.2

	Standard Deviations in Percents			Correlations with Real Output			Average Percent of Real GNP
	Whole	First Half	Second Half	Whole	First Half	Second Half	
Real GNP	1.8	1.7	1.9	—	—	—	—
Total Consumption	1.3	1.2	1.4	.739	.503	.917	61.7
Services	.7	.7	.6	.615	.441	.781	26.8
Nondurables	1.2	1.0	1.3	.714	.575	.808	26.5
Durables	5.6	6.1	5.0	.574	.298	.884	8.4
Total Invest. Fixed	5.1	4.2	5.9	.714	.454	.884	14.2
Residential	10.7	8.5	12.4	.436	.123	.637	4.4
Nonresidential	4.9	4.4	5.3	.684	.554	.777	9.7
Equipment	5.8	5.6	5.9	.707	.642	.760	6.0
Structures	4.5	3.8	5.1	.512	.225	.698	3.7
Total Government	4.8	6.5	2.2	.258	.353	.152	22.6
Federal	8.7	11.6	4.2	.266	.377	.125	10.8
State and Local	1.3	1.6	1.0	−.170	−.408	.131	11.8

Factors of Production

The second set of variables considered are the factors of production and productivity which is output per hour. These results are summarized in Tables 4 and 5. There is a strong and stable positive relationship between hours and output. In addition, the variability in hours is comparable to the variability in output. The contemporaneous association between productivity and output is weak and unstable with the standard deviation of productivity being much smaller than the standard deviation of output. It is interesting to note that when lead and lag GNPs are included, the

TABLE 3

AGGREGATE DEMAND COMPONENTS: STRENGTH OF ASSOCIATION WITH GNP AND MEASURE OF STABILITY
SAMPLE PERIOD: 1950.1-1979.2

	Correlation with Real Output Squared	R^2 for Regression $c_{jt} = \alpha_j + \sum_{i=-2}^{2} \beta_{ji} GNP_{t+i}$	Stability Measure
Total Consumption	.546	.620	.922
Services	.378	.424	.877
Nondurables	.510	.589	.968
Durables	.329	.415	.829
Total Invest. Fixed	.509	.552	.785
Residential	.190	.441	.809
Nonresidential	.468	.602	.831
Equipment	.500	.631	.908
Structures	.262	.367	.834
Total Government	.067	.119	.509
Federal	.071	.129	.482
State and Local	.029	.095	.298

TABLE 4

FACTORS OF PRODUCTION: STANDARD DEVIATIONS AND CORRELATIONS WITH GNP
SAMPLE PERIOD: 1950.1-1979.2

	Standard Deviations in Percents			Correlations with Real Output		
	Whole	First Half	Second Half	Whole	First Half	Second Half
Real GNP	1.8	1.7	1.9	—	—	—
Capital Stocks						
Inventory	1.7	2.0	1.4	.507	.686	.309
Capital Stock Durables	1.2	1.4	1.0	−.210	−.178	−.274
Capital Stock Nondurables	.7	.7	.7	−.236	−.185	−.297
Hours	2.0	2.1	1.8	.853	.896	.824
Work Week	.5	.6	.5	.820	.854	.800
Employees	1.4	1.6	1.2	.773	.831	.732
Productivity	1.0	1.0	1.1	.100	−.231	.361

association between GNP and productivity increases dramatically with the R-squared increasing from .010 to .453.

Capital stocks, both in durable goods and nondurable goods industries, are less variable than real output and negatively associated with output. Inventory stocks, on the other hand, have a variability comparable to output, and their correlations with output are positive. Further, the strength of association of inventories with GNP increases when lag and lead GNPs are included in the regression. This is indicated by the increase in the R-squared from .257 to .622.

Monetary Variables

Results for the final set of variables are presented in Tables 6 and 7. Correlations between nominal money, velocity, and real money with GNP are all positive. The differences in the correlations in the first and second halves of the sample, with the exception of nominal M1, suggest considerable instability over time in these relationships. A similar conclusion holds for the short-term interest rate. The correlations of GNP with the price variables are positive in the first half of the sample and

TABLE 5

FACTORS OF PRODUCTION: STRENGTH OF ASSOCIATION WITH GNP AND MEASURE OF STABILITY
SAMPLE PERIOD: 1950.1-1979.2

	Correlation with Real Output Squared	R^2 for Regression $c_{ji} = \alpha_j + \sum_{i=-2}^{2} \beta_{ji} GNP_{t+1}$	Stability Measure
Capital Stocks			
Inventory	.257	.622	.828
Capital Stock Durables	.044	.235	.782
Capital Stock Nondurables	.056	.129	.740
Hours	.728	.838	.954
Work Week	.672	.700	.513
Employees	.600	.801	.935
Average Product of Labor	.010	.453	.773

TABLE 6

MONETARY AND PRICE VARIABLES: STANDARD DEVIATIONS AND CORRELATIONS WITH GNP
SAMPLE PERIOD: 1950.1-1979.2

	Standard Deviations in Percents			Correlations with Real Output		
	Whole	First Half	Second Half	Whole	First Half	Second Half
Real GNP	1.8	1.7	1.9	—	—	—
M1						
Nominal Value	.9	.8	1.0	.661	.675	.649
Velocity	1.6	2.0	1.0	.614	.801	.415
Real Value	1.5	1.2	1.7	.565	.079	.865
M2						
Nominal	1.1	.9	1.3	.480	.175	.665
Velocity	1.9	2.4	1.2	.529	.818	.131
Real Value	1.8	1.4	2.1	.432	−.221	.828
Interest Rates						
Short	.24	.27	.19	.510	.738	.255
Long	.06	.06	.06	.193	.640	−.175
Price Indexes						
GNP Deflator	1.0	1.0	1.1	−.239	.490	−.814
CPI	1.3	1.3	1.3	−.316	.223	−.799

negative in the second half with the correlation for the entire period being small and negative.

3. SERIAL CORRELATION PROPERTIES OF DATA SERIES

A sixth-order autoregressive process was fit to a number of the series which displayed reasonable stable comovements with real output. Figure 2 presents plots of the unit impulse response functions for GNP and nine other series for the estimated

TABLE 7

MONEY AND PRICE VARIABLES: STRENGTH OF ASSOCIATION WITH GNP AND MEASURE OF STABILITY
SAMPLE PERIOD: 1950.1-1979.2

	Correlation with Real Output Squared	R^2 for Regression $c_{jt} = \alpha_j + \sum_{i=-2}^{2} \beta_{ji} GNP_{t+i}$	Stability Measure
M1			
Nominal Value	.437	.445	.378
Velocity	.378	.408	.281
Real Value	.319	.495	.678
M2			
Nominal Value	.230	.371	.749
Velocity	.280	.376	.650
Real Value	.187	.428	.684
Interest Rates			
Short	.260	.506	.748
Long	.037	.381	.724
Price Index			
GNP Deflator	.057	.261	.567
CPI	.010	.330	.481

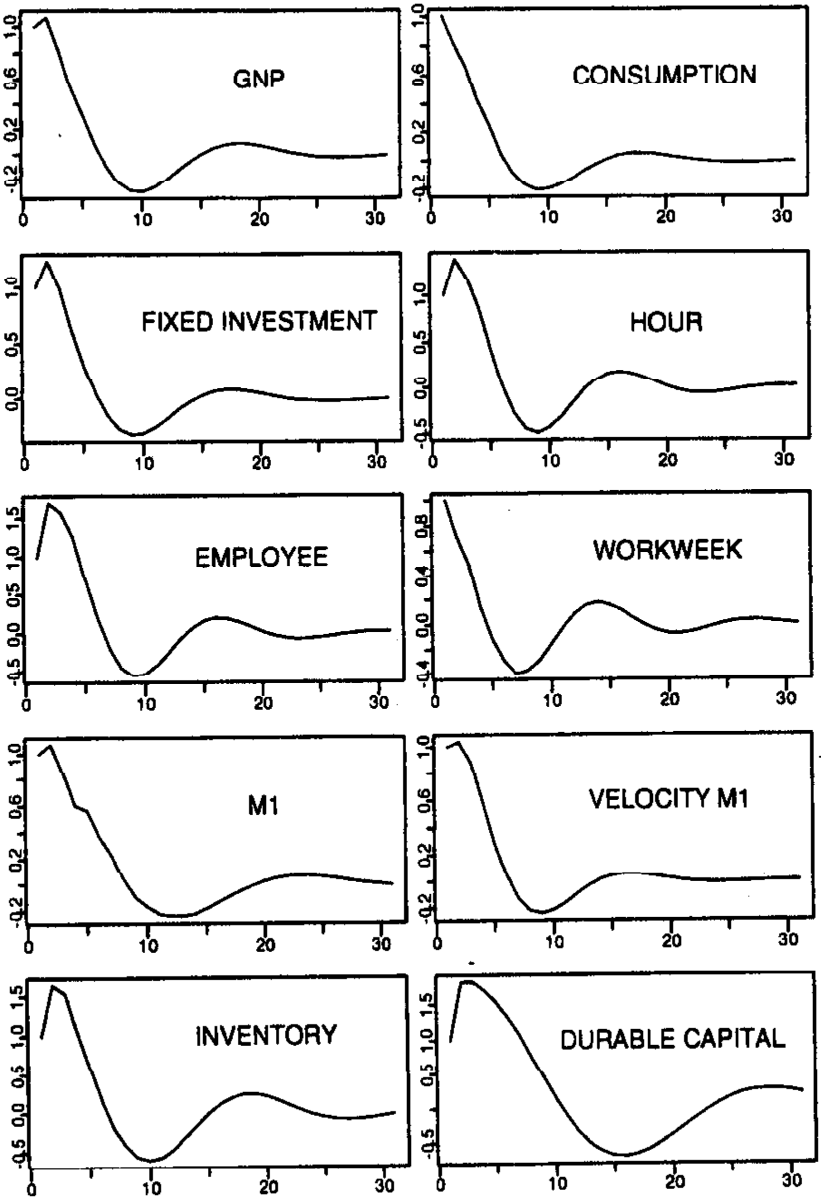

FIG. 2. Unit Impulse Response Function

autoregressive function.[7] The function for GNP increases initially to a peak of 1.15 in period one and has a minimum of −.39 in period eight. The patterns for consumption and investment are similar except that the peak for consumption is in the initial period. The function for consumption and each of its three components (not pictured) are similar to the one for the aggregate.

The pattern for total hours and the number of employees, except for the greater amplitude, is very similar to the pattern for GNP. The average work-week pattern, however, begins to decline immediately and the period of damped oscillation is shorter. The monetary variables have very different response patterns, indicating serial correlation properties very different than those of real output.

There is a dramatic difference in the response pattern for the capital stock in durable goods industries. The maximum amplitude of the response is much greater, being about 3.6, and occurs slightly over a year subsequent to the unit impulse. The pattern for the capital stock in the nondurable goods industries (not pictured) is similar though the maximum amplitude is smaller, being 2.8. For both capital stocks the peaks in the unit response function are in period five.

APPENDIX

All the data from the original paper were obtained from the Wharton Economic Forecasting Association Quarterly Data Bank. The short-term interest rate was the taxable three-month U.S. Treasury bill rate, and the long-term interest rate, the yield on U.S. Government long-term bonds.

Tables A.1–A.7 contain data from 1947.1 to 1993.4. All data for Tables A.1–A.3 come from the National Income and Product Accounts: Historical NIPA Quarterly Data, Survey of Current Business, U.S. Department of Commerce. The capital stock data in Tables A.5 and A.6 come from the Survey of Current Business as annual series. We used quarterly investment series from the NIPA with the annual capital stocks to construct quarterly series. All labor data in Tables A.5 and A.6 come from Citibase. Data for the price series in Tables A.6 and A.7 also come from Citibase. The interest rate series are from the Federal Reserve Bulletin and are constructed from the monthly series in Tables 1.33 and 1.35. Real M1 and Real M2 were obtained from the Business Cycle Indicators Historical Diskette, published by the U.S. Department of Commerce. Nominal series were calculated by multiplying by the GNP deflator.

7. Letting a_t be the innovations and

$$c_t = \sum_{i=0}^{\infty} \theta_i a_{t-i}$$

be the invertible moving average representation, parameter θ_i equals the value of the unit response function in period i. One must take care in interpreting the response pattern. Two moving average processes can be observationally equivalent (same autocovariances function) yet have very different response patterns. We chose the invertible representation because it is unique. It is just one way to represent the serial correlation properties of a covariance stationary stochastic process. Others are the spectrum, the autoregressive representation, and the autocovariance function.

TABLE A1

STANDARD DEVIATION AND SERIAL CORRELATIONS OF CYCLICAL GNP FOR DIFFERENT VALUES OF THE SMOOTHING PARAMETER. SAMPLE PERIOD: 1947.1-1993.4

	$\lambda = 400$	$\lambda = 1600$	$\lambda = 6400$	$\lambda =$ infinity
Standard Deviations	1.47%	1.80%	2.14%	4.94%
Autocorrelations				
Order 1	.81	.86	.90	.96
Order 2	.53	.64	.73	.91
Order 3	.22	.39	.53	.86
Order 4	−.03	.16	.34	.80
Order 5	−.21	−.05	.18	.74
Order 6	−.32	−.27	.02	.69
Order 7	−.39	−.30	−.09	.63
Order 8	−.43	−.37	−.19	.58
Order 9	−.40	−.40	−.26	.52
Order 10	−.35	−.40	−.28	.47
Unit-Root Test	−6.52	−5.91	−4.98	−2.34

TABLE A2

AGGREGATE DEMAND COMPONENTS: STANDARD DEVIATIONS AND CORRELATIONS WITH GNP SAMPLE PERIOD: 1947.1-1993.4

	Standard Deviations in Percents			Correlations with Real Output			Average Percent of Real GNP
	Whole	First Half	Second Half	Whole	First Half	Second Half	
Real GNP	1.8	1.8	1.8	—	—	—	—
Total Consumption	1.2	0.9	1.4	.719	.511	.875	61.7
Services	0.7	0.7	0.8	.685	.544	.810	31.2
Nondurable	1.2	1.0	1.3	.707	.558	.827	24.5
Durables	5.5	5.4	5.6	.457	.112	.787	6.9
Total Invest. Fixed	5.5	4.5	6.4	.732	.470	.927	15.2
Residential	10.9	9.1	12.6	.462	.755	.745	5.1
Nonresidential	5.1	4.6	5.6	.746	.659	.820	10.1
Equipment	6.1	5.8	6.4	.798	.715	.871	6.1
Structures	4.8	3.8	5.6	.469	.397	.528	4.0
Total Government	3.9	5.4	1.2	.350	.515	−.012	21.6
Federal	6.9	9.5	1.9	.348	.540	−.164	10.7
State and Local	1.5	1.9	1.1	−.216	−.453	.015	10.8

TABLE A3

AGGREGATE DEMAND COMPONENTS: STRENGTH OF ASSOCIATION WITH GNP AND MEASURE OF STABILITY

SAMPLE PERIOD: 1947.1-1993.4

	Correlation with Real Output Squared	R^2 for Regression $c_{jt} = \alpha_j + \sum_{i=-2}^{2} \beta_{ji} GNP_{t+1}$	Stability Measure
Total Consumption	.517	.571	.808
Services	.469	.512	.873
Nondurables	.500	.520	.872
Durables	.209	.324	.669
Total Invest. Fixed	.536	.580	.796
Residential	.213	.482	.731
Nonresidential	.557	.662	.929
Equipment	.637	.702	.955
Structures	.220	.396	.792
Total Government	.123	.229	.500
Federal	.121	.224	.436
State and Local	.047	.080	.200

TABLE A4

FACTORS OF PRODUCTION: STANDARD DEVIATIONS AND CORRELATIONS WITH GNP
SAMPLE PERIOD. 1947.1-1993.4

	Standard Deviations in Percents			Correlations with Real Output		
	Whole	First Half	Second Half	Whole	First Half	Second Half
Real GNP	1.8	1.8	1.8	—	—	—
Capital Stocks						
Inventory	2.1	2.4	1.8	.510	.547	.475
Capital Stock Durables	1.2	1.1	1.2	.510	.387	.619
Capital Stock Nondurables	1.0	1.0	0.9	−.055	−.125	.021
Hours	1.8	1.9	1.7	.883	.860	.911
Work Week	1.1	1.1	1.0	.778	.778	.783
Employees	1.5	1.6	1.5	.828	.808	.850
Productivity	0.9	1.0	0.8	.239	.151	.360

TABLE A5

FACTORS OF PRODUCTION: STRENGTH OF ASSOCIATION WITH GNP AND MEASURE OF STABILITY
SAMPLE PERIOD: 1947.1-1993.4

	Correlation with Real Output Squared	R^2 for Regression $c_{jt} = \alpha_j + \sum_{i=-2}^{2} \beta_{ji} GNP_{t+1}$	Stability Measure
Capital Stocks			
Inventory	.260	.373	.801
Capital Stock Durables	.260	.728	.967
Capital Stock Nondurables	.003	.356	.874
Hours	.779	.869	.992
Work Week	.605	.764	.994
Employees	.685	.858	.989
Average Product of Labor	.057	.465	.933

TABLE A6

MONETARY AND PRICE VARIABLES: STANDARD DEVIATIONS AND CORRELATIONS WITH GNP
SAMPLE PERIOD: 1947.1-1993.4

	Standard Deviations in Percents			Correlations with Real Output		
	Whole	First Half	Second Half	Whole	First Half	Second Half
Real GNP	1.8	1.8	1.8	—	—	—
M1						
Nominal Value	2.1	1.3	2.7	.368	.542	.318
Velocity	2.7	2.1	3.1	.328	.680	.104
Real Value	2.7	1.6	3.4	.347	.219	.436
M2						
Nominal	1.8	1.4	2.2	.337	.324	.357
Velocity	2.5	2.5	2.6	.404	.672	.151
Real Value	2.4	1.8	2.9	.319	.058	.491
Interest Rates						
Short	1.1	0.6	1.5	.324	.335	.358
Long	0.6	0.2	0.8	.032	.228	−.020
Price Indexes						
GNP Deflator	1.0	1.0	1.0	−.156	.327	−.635
CPI	1.6	1.4	1.7	−.222	.247	−.585

TABLE A7

MONEY AND PRICE VARIABLES: STRENGTH OF ASSOCIATION WITH GNP AND MEASURE OF STABILITY
SAMPLE PERIOD: 1947.1-1993.4

	Correlation with Real Output Squared	R^2 for Regression $c_{jt} = \alpha_j + \sum_{i=-2}^{2} \beta_{ji} GNP_{t+1}$	Stability Measure
M1			
Nominal Value	.135	.229	.783
Velocity	.108	.280	.747
Real Value	.120	.270	.738
M2			
Nominal Value	.114	.291	.782
Velocity	.163	.377	.755
Real Value	.102	.321	.707
Interest Rates			
Short	.105	.336	.701
Long	.001	.191	.701
Price Index			
GNP Deflator	.024	.199	.430
CPI	.049	.248	.485

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