

Exercise 1

Present Value of a Person's Wealth

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You are multi millionaires and may not know it. What is the value of your wealth? To answer this question we must define what we mean by wealth. Wealth is the value of your endowments. If you don't currently know the meaning of endowment and value, this definition is not of much use. In what follows these terms are defined using the language of economic theory.

First, there are *commodities* that can be traded, $\{x_i\}$. In macroeconomics the number of commodities is big since a commodity, such as apples or labor, at different dates are different commodities and there is an infinity of dates. But, we will get to this later and now will deal with the case that there are only a finite number of commodities. If you understand the present value concept for economies that last only a finite amount of time, you will understand the concept for an economy that lasts forever.

Second, each commodity has a price. The *price of commodity i* is p_i . The value v of commodity bundle $\{x_i\}$ is

$$v(x) = \sum_i p_i x_i .$$

Third, an *endowment* is a commodity bundle $\{e_i\}$ owned by the individual. Its value is

$$v(e) = \sum_i p_i e_i .$$

A person's *wealth* is the value of that person's endowment.

For this exercise the commodities will be labor services for $t = 0, 1, 2, \dots$ and the composite output good for $t = 0, 1, 2, \dots$. As the endowments of the composite output goods are zero, they will not enter into your calculation. I assume that you will be

receiving no inheritance and that currently you own no stocks, own no debt assets, and have no liabilities. Many of you will have a student loan, but you will see the present value of your wealth is big relative to this number. That is why I said ignore it in the exercise.

Your endowment is 3500 hours of productive time for each year $t = 0, 1, \dots, 45$. This assumes that you can work for 45 years. This means that you are 22 years old now and will retire and begin receiving social security benefits when you are 67. After retirement your productive time endowment is zero. The price of year t labor services is 25×1.02^t \$ per hour. These wage rates are corrected for inflation, that is changes in the price level. The assumption is that the general wage level will continue to increase by on average 2 percent a year as it has for the last 100 years.

Note that a unit of consumption today has a different value than a unit of consumption eighteen years hence in precisely the same way that a new car has a different value today than a new loaf of bread today. The relative value of two goods is the relative price.

The value of a bundle of period t goods is $p_{1t}x_{1t} + p_{2t}x_{2t} + \dots + p_{nt}x_{nt}$, where x_{it} is the quantity of the i^{th} good at date t in the bundle and p_{it} is the corresponding price. In macro economics we deal with a single composite consumption good at each date, c_t . A bundle of different date consumption goods is $\{c_t\}_{t=0}^T$. The value of this bundle, called its present value, is

$$\sum_{t=0}^T p_t c_t,$$

where the $\{p_t\}_{t=0}^T$ are the inter temporal prices. Notice that value is price time quantities summed over goods. Note also that c_t can be negative. This corresponds to delivery of $|c_t|$ units of the consumption good at date t .

The units in which the consumption good is measured is selected so that $p_0 = \$1$. Assume that the price of c_t is given by:

$$p_t = 1.04^{-t}.$$

This price is in terms of date 0 dollars. This set of prices corresponds to a constant real interest rate of 4 percent. If you don't understand this, don't worry. The relation between interest rates and inter temporal prices will be developed later.

You will be asked below to find the wealth of a person in terms of period zero \$.

Question 1: Calculate the wealth of this person.

Question 2: In fact people do not allocate all their productive time to market activities.

People allocate about 2000 hours a year to this activity in the United States and Japan. The number is much smaller in Europe. Calculate the present value of this person's labor income.

The U.S. government will expropriate about 40 percent of this person's labor income. The government does not tax non-market productive activities. If marginal tax rates are high, people allocate more of their productive time to non-market activities and less to market activities.

Question 3: What is the wealth of this American under the assumption that there are lump sum payments of \$5,000 per year to every person including 15 years of retirement?

Include the values of the productive time allocated to non-market activities as well as the values of these lump sum payments and of labor incomes.

Include your name, student ID number, course number, and date at the top. You are limited to a single page for the answers. All that I want to see are the numerical results and the algebraic formulas used to compute them. Points will be taken off for those who do not follow instructions.