

Federal Reserve Bank  
of Minneapolis

Fall 1994

# Quarterly Review

A Progress Report  
on Business Cycle Models (p. 2)

Ellen R. McGrattan

Another Attempt  
to Quantify the Benefits  
of Reducing Inflation (p. 17)

R. Anton Braun

FEDERAL RESERVE  
BANK OF ATLANTA  
RECEIVED  
'95 JAN 4 PM 1 02  
RESEARCH LIBRARY

Federal Reserve Bank of Minneapolis

# Quarterly Review

Vol. 18, No. 4

ISSN 0271-5287

This publication primarily presents economic research aimed at improving policymaking by the Federal Reserve System and other governmental authorities.

*Any views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.*

---

Editor: Arthur J. Rolnick

Associate Editors: S. Rao Aiyagari, John H. Boyd, Preston J. Miller,  
Warren E. Weber

Economic Advisory Board: R. Anton Braun, Edward C. Prescott, Charles H.  
Whiteman

Managing Editor: Kathleen S. Rolfe

Article Editors: Kathleen A. Mack, Kathleen S. Rolfe, Martha L.  
Starr

Designer: Phil Swenson

Associate Designer: Robyn May

Typesetters: Jody Fahland, Correan M. Hanover

Technical Assistants: Shawn Hewitt, Maureen O'Connor

Circulation Assistant: Cheryl Vukelich

---

The *Quarterly Review* is published by the Research Department of the Federal Reserve Bank of Minneapolis. Subscriptions are available free of charge.

*Quarterly Review* articles that are reprints or revisions of papers published elsewhere may not be reprinted without the written permission of the original publisher. All other *Quarterly Review* articles may be reprinted without charge. If you reprint an article, please fully credit the source—the Minneapolis Federal Reserve Bank as well as the *Quarterly Review*—and include with the reprint a version of the standard Federal Reserve disclaimer (italicized above). Also, please send one copy of any publication that includes a reprint to the Minneapolis Fed Research Department.

Direct all comments and questions to

*Quarterly Review*  
Research Department  
Federal Reserve Bank of Minneapolis  
P.O. Box 291  
Minneapolis, Minnesota 55480-0291  
(612-340-2341 / FAX 612-340-2366).

## Another Attempt to Quantify the Benefits of Reducing Inflation

R. Anton Braun  
Economist  
Research Department  
Federal Reserve Bank of Minneapolis

Over the last several years, the U.S. inflation rate has dropped below 4 percent per year—to levels common in the mid-1960s. This is widely regarded as a good thing. But would further reductions in the inflation rate make a good thing even better? Economists can offer answers to this policy question by analyzing economic models that quantify the welfare benefits of a lower inflation rate.<sup>1</sup> Still, a model's results are critically affected by the assumptions made in constructing it. One type of assumption, which I will focus on here, is how the government replaces the revenue lost when the inflation rate is reduced. Economists view inflation as a tax on activities that use money.<sup>2</sup> When this source of revenue is reduced, a government that wants to maintain its current level of spending must replace the lost revenue from inflation by raising other taxes or creating new ones. The size of the welfare benefits from reducing the current inflation rate will, therefore, depend critically on the model's assumptions about how the government replaces this revenue.

One of the best-known prescriptions for monetary policy, for instance, seems to suggest that the welfare benefits from reducing inflation would be quite large. The *Friedman rule*, proposed by Milton Friedman in 1969, calls for a monetary policy that maintains a zero nominal interest rate. In a setting with no uncertainty, that policy involves a negative inflation rate, or *deflation*. Wouter Den Haan (1990) and Robert Lucas (1993) have quantified the benefits of reducing a moderate inflation rate of about 4 or 5

percent to the rate prescribed by the Friedman rule. They find that the benefits of such a reduction could be substantial, ranging from \$60 billion to over \$200 billion. To get some perspective on the size of those numbers, note that the U.S. economy produces about \$6 trillion worth of goods and services each year. Since 1 percent of this gross domestic product (GDP) is around \$60 billion, Den Haan and Lucas' results suggest that the welfare benefits from adopting the Friedman rule would range from 1 percent to 3 percent of GDP.

To get these results, however, Den Haan and Lucas follow Friedman and assume that the government can replace the revenue lost by adopting the Friedman rule with a *lump-sum tax*, that is, a tax independent of an individual's income, wealth, or consumption patterns. Such an assumption may well be unrealistic.

Great Britain's recent experience with the *community charge*—popularly referred to as the *poll tax*—illustrates

---

<sup>1</sup>*Welfare* measures a household's satisfaction level. A household's satisfaction increases with the amount of goods and services it consumes and the time it spends in leisure pursuits. To compare the effects of two different tax policies, economists attempt to ascertain how welfare changes under them. A dollar figure can be assigned to these welfare comparisons by determining how much income would be needed to provide the household with the same level of welfare under the two policies. For another description of this common way to evaluate policy options, see the work of Rao Aiyagari (1990, pp. 2–3).

<sup>2</sup>For a good description of why this is so—and good background reading for my article—see Aiyagari 1990.

the potential problems associated with lump-sum taxation. In 1989, Britain began assessing a poll tax in Scotland. In 1990, the tax was extended to England and Wales. Public reaction to this tax was overwhelmingly negative; the tax provoked street protests throughout Britain and riots in London. At least partly as a result, in November 1990, the Conservative party revolted and put the prime minister, Margaret Thatcher, out of office. The new prime minister, John Major, revoked major provisions of the poll tax in his first budget.

Although this is just one experience, a lump-sum tax seems unlikely to be any better received in the United States. But ruling out lump-sum taxes can have a fundamental impact on the desirability of reducing the inflation rate. Most other types of taxes distort households' incentives to save and to supply labor. Consequently, when evaluating the benefits from reducing inflation, researchers must also consider the costs that increasing other taxes, such as the income tax, have on households' incentives. Edmund Phelps (1973) argues that when a benevolent government has only distortionary taxes at its disposal, it will generally choose to raise some revenue from inflation. However, Phelps' analysis is silent on the question of how much inflation is desirable.

Here I investigate whether Phelps' argument is quantitatively important. To do so, I first describe a simple model economy and calibrate it to match some of the main features of the modern U.S. economy. I then use this model to quantify the benefits from reducing inflation under the assumption that the revenue lost from that reduction is replaced with a higher tax on labor income. Two specific findings emerge. I find that the optimal rate of inflation is higher than the rate prescribed by the Friedman rule, but still negative. I also find that the welfare gains from reducing inflation are smaller than \$17 billion, while small mistakes in setting monetary policy could produce welfare losses larger than \$37 billion. Thus, my model suggests that small benefits—in the range of from one-third to one-half of 1 percent of GDP—are all that could be expected from further reductions in the U.S. inflation rate.<sup>3</sup>

### The Model

To quantify the effects described by Phelps (1973), I must model how inflation and the income tax affect households' incentives to save and to supply labor. I do that by making assumptions about households' preferences for goods and leisure, describing how goods are produced, and explaining how wages and interest rates are determined.

### *Firms, Households, and the Government*

To keep things simple, I will consider an economy with no capital, where competitive firms choose labor input to maximize profits. The production technology is assumed to be linear in labor input:

$$(1) \quad y = eh.$$

I will find it convenient to express variables in per capita terms. So, in this expression,  $e$  is the fraction of the population that works,  $h$  is the average number of hours worked per worker in a period, and  $y$  is per capita output. All households have identical preferences, which are defined over consumption and leisure:

$$(2) \quad U = u(c^i) - v(T-l^i).$$

Here  $c^i$  denotes consumption by the  $i$ th household,  $l^i$  is the household's leisure, and  $T$  is the total endowment of time. Household utility  $U$  is assumed to be strictly increasing in consumption and leisure, with diminishing marginal returns. Throughout this article, household utility is the criterion used to make welfare comparisons of alternative government policies.

The way leisure enters the utility function will greatly influence what the model says about the welfare gains from reducing inflation. For instance, if labor supply is totally inelastic to changes in the after-tax wage rate, then following the Friedman rule and raising all government revenue from a tax on labor income is an optimal government policy. More generally, if labor supply is highly inelastic, then a welfare-maximizing tax policy will call for a high tax on labor income and a low inflation rate.

Empirical evidence on labor supply elasticities is mixed. Evidence from international empirical studies using microeconomic data suggests that hours worked by men in their prime working years show little response to changes in after-tax wage rates. [For a survey of this literature, see the work of John Pencavel (1986).] In addition, the preponderance of evidence indicates an inelastic labor supply for married women. [See the work of Thomas Mroz (1987).] But these studies abstract from the workers' decision on whether or not to participate in the labor market,

---

<sup>3</sup>Recently, Robert Lucas (1994) has extended his 1993 analysis to consider cases where the revenue lost from reduced inflation is replaced with a tax on labor income. He argues that the gains from adopting the Friedman rule remain large in that scenario. The difference between Lucas' conclusions and mine is likely due to differences in the way we model the labor supply decision.

and evidence from aggregate data suggests that this decision should not be ignored. While the average number of hours worked weekly per worker is only about one-fourth as variable as gross domestic product (GDP), the aggregate number of hours worked varies by about the same amount as GDP. These facts have led Gary Hansen (1985) and Richard Rogerson (1988) to propose preference specifications in which all of the variation in aggregate labor input is due to variations in *employment*, or the number of workers employed. More recently, Finn Kydland and Edward Prescott (1991), Andreas Hornstein and Edward Prescott (1993), Jang-Ok Cho and Thomas Cooley (1994), and Ellen McGrattan and I (1994) have considered specifications that allow for variation both in the number of days worked in a given period and in the length of the workday.

I consider one such extension here. Suppose, as do Cho and Cooley (1994), that the fraction of days worked by members of household  $i$  in a period is represented by  $e^i$ . If the utility function (2) measures total daily utility, then average daily utility over the period is

$$(3) \quad u(c^i) - v(T-l^i)e^i.$$

Assume also that working more days in a given period produces direct utility costs for a household.<sup>4</sup> This could be true for a variety of reasons. Increasing the number of days worked in a period means less time available for family activities and household chores as well as higher costs from dividing up household responsibilities, such as picking up the kids from day care. These considerations suggest that utility is decreasing in  $e^i$ .<sup>5</sup> Under this additional assumption, average daily utility during the period can be represented as

$$(4) \quad u(c^i) - v(t-l^i)e^i - q(e^i)e^i.$$

Cho and Cooley (1994) have shown that an equilibrium framework with preferences of this form can explain the labor market facts that average weekly hours are smooth although employment variations are large.

The way money is modeled can also have an important effect on the welfare gains from reducing inflation. I will follow the transaction demand literature and assume that conducting transactions has a time cost. [See the work of Bennett McCallum (1983).] This cost, which is increasing in the amount consumed and decreasing in real balances, is given by

$$(5) \quad \phi(c_t^i, m_t^i)$$

where  $m_t^i$  is period  $t$  real balances. The most frequently cited rationale for (5) is the inventory model of cash management. In that model, households carry an inventory of cash to make purchases. Each time this inventory is depleted, households incur a cost to replenish it. These costs might include forgone leisure time, shoe-leather costs, or the fee for using an automatic teller machine.<sup>6</sup>

I will assume that households discount future utility and have infinite planning horizons. Robert Barro (1974), for example, has shown that an infinite planning horizon can be derived from an arrangement in which households have finite planning horizons and value the utility of their children. A typical household's present value utility is given by

$$(6) \quad \sum_{t=0}^{\infty} \beta^t [u(c_t^i) - v(h_t^i + \phi_t^i) - q(e_t^i)e_t^i]$$

where  $\beta$  is the preference discount rate and

$$(7) \quad T - l_t^i = h_t^i + \phi_t^i.$$

The household's budget constraint is

$$(8) \quad P_t c_t^i + M_t^i + B_t^i \leq (1-\tau_t)W_t h_t^i e_t^i + M_{t-1}^i + (1+R_{t-1})B_{t-1}^i + S_t^i.$$

Here  $P_t$  is the price of consumption in period  $t$ ,  $M_t^i \equiv P_t m_t^i$  is the holdings of money at the end of period  $t$ , and  $B_t^i$  is new acquisitions of bonds in period  $t$ . These bonds cost \$1 today and pay their holder  $\$(1+R_t)$  next period. Also,  $\tau_t$  is a proportional tax on labor income,  $W_t$  is the nominal wage rate, and  $S_t^i$  is a lump-sum transfer to the household. Note that negative values of  $S_t^i$  correspond to a lump-sum tax.

Given these assumptions, the household's problem is to maximize (6) subject to (8). The first-order necessary

<sup>4</sup>I will assume that the fraction of days worked in a period is perfectly divisible. For an analysis that assumes this fraction is indivisible, see Hornstein and Prescott 1993.

<sup>5</sup>For a more detailed justification for utility to be decreasing in  $e^i$ , see the work of Cho and Cooley (1994); they explicitly model a home production sector.

<sup>6</sup>I will assume further that  $\phi$  satisfies  $\phi(0, m) = 0$ ,  $\phi_{11} \geq 0$ ,  $\phi_{22} \geq 0$ ,  $\phi_{12} \leq 0$ , and  $\phi_{11}\phi_{22} - \phi_{12}^2 \geq 0$ . The first condition says that transaction costs are zero if no goods are purchased. The remaining assumptions ensure that  $\phi$  is convex.

conditions for this problem include the following equations:

$$(9) \quad [u'(c_t^i) - v'(h_t^i + \phi_t^i)]e_t^i \phi_{1t}^i / v'(h_t^i + \phi_t^i) = P_t / [(1 - \tau_t)W_t]$$

$$(10) \quad e_t^i [v(h_t^i + \phi_t^i) + q(e_t^i) + q'(e_t^i)e_t^i] / v'(h_t^i + \phi_t^i) h_t^i e_t^i = 1$$

$$(11) \quad 1 + (1 - \tau_t)(W_t/P_t)e_t^i \phi_{2t}^i = 1/(1 + R_t)$$

Equations (9) and (10) both equate the marginal rate of substitution between two goods to their effective relative price. Thus, in equation (9), the rate at which households want to exchange consumption for leisure is related to the relative price of the two goods. Similarly, in equation (10), the rate at which households want to exchange a longer workday for fewer days of work is related to the effective relative price of the two goods.

Equation (11) summarizes a portfolio balance restriction. A utility-maximizing household will choose its holdings of money and bonds so that it is indifferent on the margin between saving with bonds or with money. Households are indifferent between saving with bonds or money when the effective return from holding one additional dollar of money for a period equals the return from buying one additional dollar's worth of one-period bonds. That restriction is expressed in equation (11).<sup>7</sup>

To complete my specification of the economy, I need to describe the government's sources and uses of funds. The government raises revenue by taxing labor income and by simply printing money (collecting what is known as *seigniorage*, the difference between how much is printed and how much printing costs). The government's revenue is used to purchase goods from the private sector and to move resources among households (or make *transfer payments*). These assumptions imply the following period  $t$  budget constraint for the government:

$$(12) \quad P_t g_t - \tau_t W_t h_t e_t + S_t = M_t - M_{t-1}$$

Here,  $g_t$  represents government purchases of goods and quantity variables without superscripts are aggregate per capita values.

Finally, the economy's aggregate resource constraint is given by

$$(13) \quad g_t + c_t \leq e_t h_t$$

### The Steady State and Alternative Policies

Suppose that markets are competitive and that the govern-

ment chooses to hold government purchases and transfer payments constant and to conduct its monetary policy so as to maintain a constant growth rate of money.<sup>8</sup> Under these additional assumptions, the economy will have a steady state in which the market-clearing allocations of consumption, real balances, hours, and employment are constant. These allocations can be supported by a constant nominal interest rate and by a price level and a wage rate that grow at a constant rate.

The allocations and nominal interest rate that characterize this steady state are found by solving the following set of equations for  $c$ ,  $m$ ,  $h$ ,  $e$ , and  $R$  given values of  $g$ ,  $\pi$ , and  $s_t$ :

$$(14) \quad v'(u' - v'e\phi_1) = (1 - \tau)$$

$$(15) \quad e(v'e + q + q'e)/(v'he) = 1$$

$$(16) \quad (1 - \tau)e\phi_2 = -R/(1 + R)$$

$$(17) \quad g + c = eh$$

$$(18) \quad 1/\beta = (1 + R)/(1 + \pi)$$

where  $\pi_t = P_{t+1}/P_t - 1$  denotes the inflation rate and  $s_t = S_t/P_t$ .<sup>9</sup>

These equations can be used to examine some of the implications of alternative government policies. For instance, if  $R$  is set to zero as the Friedman rule requires, then equation (18) implies that  $1 + \pi = \beta$ . Since  $\beta < 1$ , this expression implies that the Friedman rule will involve deflation:  $\pi < 0$ . The magnitude of deflation will depend on  $\beta$ , the preference discount rate. This line of thought thus explains the well-known characterization of the Friedman rule as a monetary policy that produces deflation at the rate of time preference.

Notice next that the Friedman rule also imposes restrictions on (16). If  $R = 0$ , then so must  $\phi_2$ , if households are

<sup>7</sup>To see this, notice, from equation (8), that the opportunity cost of holding one additional dollar for a period is  $R_t$ , which is the interest rate a bondholder gets from holding a \$1 bond for one period. Holding a dollar for a period, however, reduces the time spent shopping by the amount  $-\phi_{2t}/P_t$ . The dollar value of this leisure time today is  $-(1 - \tau_t)W_t e \phi_{2t}/P_t$ . The value of this leisure time next period is simply its value today multiplied by  $(1 + R_t)$ , or  $-(1 + R_t)(1 - \tau_t)W_t e \phi_{2t}/P_t$ . When this expression is equated to  $R_t$ , the opportunity cost of holding money, the result is equation (11).

<sup>8</sup>Notice that, with equation (12), these government policies also pin down  $\tau$ .

<sup>9</sup>This characterization of the steady-state allocations and prices tacitly assumes that the steady-state growth rate of money equals the inflation rate. This restriction can be derived directly. Suppose that the money supply rule is  $M_t = (1 + \delta)M_{t-1}$ . Then note that in a steady state,  $m_t = m$  implies that  $(1 + \delta)/(1 + \pi) = 1$ . Also, notice that I have imposed the equilibrium restrictions that  $W/P = 1$  and  $\Sigma B_t^i = 0$ .

to be willing to hold both money and bonds. Since  $\phi_2$  is increasing in real balances, the Friedman rule calls for a monetary policy that satiates households with real balances.

These steady-state restrictions can also be used to investigate, more generally, the effects of reducing seigniorage on the government's budget constraint. In a steady state, (12) simplifies to

$$(19) \quad \tau e h - s = g + \pi/(1+\pi)m.$$

Equation (19) demonstrates that if the government decreases seigniorage, then it must increase other taxes, decrease transfers, or decrease government purchases.<sup>10</sup> The work of Friedman (1969) abstracts from taxes on labor income and assumes that budget balance is maintained by imposing a lump-sum tax, which in my framework corresponds to setting  $s$  to be negative. Under these assumptions, the Friedman rule maximizes welfare. Adopting the Friedman rule guarantees households the same real rate of return from holding money that they get from holding bonds, so that households need not waste resources trying to economize on their cash holdings.

Phelps (1973), however, has observed that using lump-sum taxes to offset the revenue lost from reduced seigniorage is not an innocuous assumption. He argues that if a distortionary tax such as  $\tau$  is increased instead, deflating may no longer be a desirable policy. Equation (9) shows that increasing  $\tau$  distorts the relative price of consumption and leisure, making consumption more expensive than leisure. This has a negative effect on the incentive to supply labor. If the government is required to offset any revenue losses from reduced seigniorage with increases in other distortionary taxes, then the welfare gains from reducing inflation must be weighed against the welfare losses that occur when other distortionary taxes are increased. Given these trade-offs, results from public finance suggest that welfare losses increase rapidly as tax rates are increased. Thus, a one percentage point increase in the labor income tax from a base rate of, say, 40 percent induces a much larger welfare loss than a one percentage point increase from a base rate of 10 percent. One implication of this result is that a tax policy that maximizes welfare will often call for some taxation of all goods.

### The Model's Parameters

A question left open by Phelps' (1973) analysis is the magnitude of the welfare-maximizing inflation rate. In order to examine this question, I will choose the model's pa-

rameters to match various features of the U.S. data and then examine how steady-state welfare changes as I alter the inflation rate while holding fixed government purchases and transfers.

The model's parameters can be divided into three types: technology, preference, and government policy. The parameters for government policy I calibrate to match U.S. policy in 1991. To measure the main preference parameters, I rely on the analysis of Cho and Cooley (1994). To measure the parameters of an aggregate transaction cost technology, I use my own strategy—so let's start with them.

#### Transaction Technology

I will assume that the transaction technology is of the following general form:

$$(20) \quad \phi(c_t, m_t) = k c_t \{m_t/c_t\}^{1-\theta}$$

where  $\theta$  is assumed to be greater than one and  $\mu$  and  $k$  are nonnegative. [This form of the transaction technology is similar to that used by David Marshall (1992).] Note that if (20) is substituted into (11), the resulting expression can be manipulated to produce

$$(21) \quad \log(m_t) = v_0 - v_1 \log[R_t/(1+R_t)] + v_2 \log(c_t) + v_3 \log(1-\tau_t) e_t$$

where

$$(22) \quad \theta = 1/v_1 = 1/v_3$$

$$(23) \quad \mu = (\theta v_2 - 1)/(\theta - 1).$$

In order to derive a relationship that can be estimated, I will assume that  $\log(m_t)$ ,  $\log[R_t/(1+R_t)]$ , and  $\log(c_t)$  have unit roots and that a linear combination of these three variables is stationary or that these variables *cointegrate*. Under this assumption, I can apply results from the econometric literature on cointegration to consistently estimate the coefficients  $v_1$  and  $v_2$  from the following empirical specification:

$$(24) \quad \log(m_t) = v_0 - v_1 \log[R_t/(1+R_t)] + v_2 \log(c_t) + \varepsilon_t$$

<sup>10</sup>I am tacitly assuming here that a decrease in  $\pi$  decreases government revenue. At very high inflation rates, of course, a reduction in inflation might actually increase government revenue.

where  $\varepsilon_t$  is a stationary random variable.<sup>11</sup> In practice, I estimate (24) using the canonical cointegration regression estimator proposed by Joon Park (1990).

#### □ *A Note on the Data*

Before discussing estimates of the parameters in (24), let me discuss the data I use.

U.S. consumption data only extend back to 1929, but Milton Friedman and Anna Schwartz (1982) have constructed a much longer time series for the net national product (NNP). Using longer data sets is desirable when estimating cointegration relations because these estimators identify the parameters from trends in the data. So I substitute NNP for consumption in equation (24) and use data from Friedman and Schwartz 1982 to extend the sample period back to 1900. As long as the trend in NNP is the same as the trend in consumption, this substitution is innocuous.

As my measure of money, I use the monetary base. This monetary aggregate is the appropriate one for calculating seigniorage, but it overstates the amount of cash used by U.S. households to conduct transactions.<sup>12</sup> I construct real balances by dividing the monetary base by the NNP deflator and measure the nominal interest rate using data on commercial paper rates.

#### □ *The Estimates*

Using annual data on commercial paper rates, real balances, and NNP expressed in constant 1982 dollars and a sample period extending from 1900 to 1986, I estimate the coefficients in equation (24) to be

$$(25) \quad \log(m_t) = 2.55 - 0.55 \log[R_t/(1+R_t)] + 0.98 \log(y_t),$$

(0.400) (0.036) (0.053)

The numbers in parentheses are standard errors. These estimates are quite similar to previous estimates by Lucas (1993), who uses M1 as the monetary aggregate and an interest rate for long-term securities. Lucas estimates the interest elasticity to be  $-0.5$  and imposes a unit income elasticity.

#### *Preferences*

Now let's turn to the preference parameters.

I assume that households discount future utility at the rate of 2 percent per year. This implies a value of 0.98 for  $\beta$ , the preference discount rate. Preferences are assumed to be of this general form:

$$(26) \quad u(c_t) - v(h_t + \phi_t)e_t - g(e_t)e_t \\ = \log(c_t) - [\alpha_1/(\gamma_1+1)(h_t + \phi_t)^{\gamma_1+1}] \\ - [\alpha_2/(\gamma_2+1)e_t^{\gamma_2+1}].$$

I use Cho and Cooley's (1994) parameterization of (26). They calibrate the  $\alpha$ 's so that one-third of the daily time endowment is spent in market activities and the employment rate is 65 percent, and they set  $\gamma_1 = 1$  and  $\gamma_2 = 0.62$ . These choices allow them to replicate the fraction of variation in labor input due to variations in hours per worker and employment in U.S. data as well as some of the other main features of the U.S. business cycle.

#### *Government Policy*

The government policy parameters are calibrated to match U.S. data for 1991.<sup>13</sup>

In that year, the GDP deflator grew at an annual rate of 4.2 percent, while the consumer price index (CPI) for urban consumers grew at a rate of 3.9 percent. So I have chosen an inflation rate of 4 percent. In conjunction with my assumption that  $\beta = 0.98$ , this rate implies a nominal interest rate of 6.12 percent.

To link data from the national income and product accounts to my model, I use a narrower measure of output than GDP. My measure consists of total consumption expenditures, total government purchases, and one-third of the value added from the depository institutions sector. According to this measure of output, in 1991 government purchases were 22 percent of output, and transfers plus interest payments on the debt were 17 percent. These ratios are used to pin down  $g/y$  and  $s/y$  in the model. The value added by depository institutions was about 3 percent of my output measure in 1991. I assume that one-third of this value added is directly related to activities that help households economize on their cash balances. Therefore, I calibrate the scale of the transaction function so that transaction costs are 1 percent of output.

<sup>11</sup>In this expression,  $\varepsilon_t$  includes employment and taxes. Thus, the classical assumption of independence of the disturbance and other right-side variables is violated. The cointegration literature has established conditions under which the sample covariances of the right-side variables and  $\varepsilon_t$  converge in probability to zero, and estimators have been proposed that allow the use of standard distributional assumptions to perform statistical inference. Note also that Lawrence Christiano and I (1994) find that the biases from estimating (25) in samples of length 85 are small even when the interest rate is assumed to be stationary in levels.

<sup>12</sup>If the fraction of currency that has been used for, say, black market activities have been stable over time, then my estimates should still be reliable.

<sup>13</sup>This is the most recent year for which I can get a complete set of data. The binding constraint is the value added by depository institutions.



Given this parameterization, I use equations (14)–(18) to numerically calculate what I will refer to as the *baseline steady state* of the model. Doing so yields implications for two other variables: the amount of revenue raised by seigniorage and the rate at which labor income is taxed. The baseline steady state predicts that seigniorage revenue is 0.22 percent of output. This value is somewhat less than that in the data. Seigniorage revenue in 1991 was about 0.47 percent of my measure of output. The model also predicts a labor tax rate of 39 percent.

### The Results

Now that my model is calibrated, I can use it to quantify the welfare benefits from reducing inflation.

The accompanying chart shows how the model says welfare changes with the inflation rate. Welfare is expressed as a percentage increase or decrease relative to consumption under the baseline parameterization described in the preceding section. The values of the chart are calculated by solving the steady state of the economy described earlier at alternative values of the inflation rate while holding fixed government purchases and transfers. The government's budget constraint is balanced by adjusting the labor tax rate. Given a steady state indexed by some  $\pi_j$ , and the baseline steady state indexed by  $\pi_b = 0.04$ , welfare is calculated by finding the value of  $x$  that satisfies this equation:

$$(27) \quad u(c(\pi_b)(1+x)) - v(h(\pi_b) + \phi(\pi_b))e(\pi_b) - g(e(\pi_b))e(\pi_b) - u(c(\pi_j)) + v(h(\pi_j) + \phi(\pi_j))e(\pi_j) + g(e(\pi_j))e(\pi_j) = 0.$$

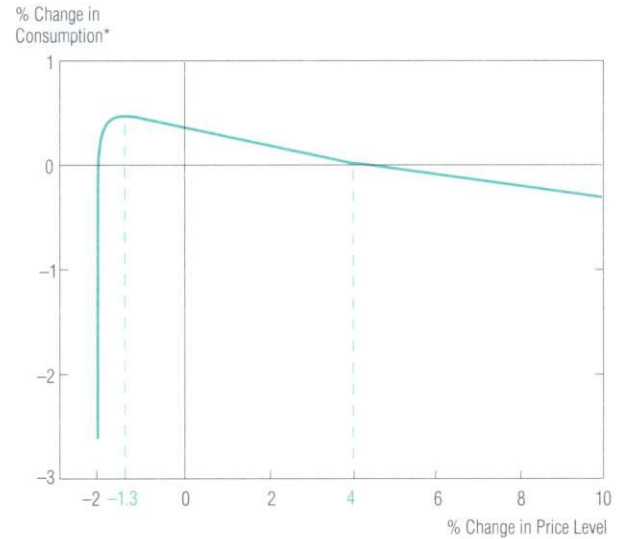
In words,  $x$  indexes the increment to baseline consumption that would make households indifferent to the steady state with an inflation rate of  $\pi_j$ . Thus, at the baseline inflation rate of 4 percent, welfare is zero.

The chart has several notable features.

One is that the highest welfare gain occurs when the inflation rate is –1.3 percent. This inflation rate is only slightly higher than the inflation rate of –2 percent prescribed by the Friedman rule. These results show that Phelps' (1973) argument that the optimal inflation rate should lie above the rate prescribed by the Friedman rule when lump-sum taxes are ruled out is not quantitatively important: deflation is still optimal.

Another notable feature of the chart is that in general the welfare gains from reducing inflation are small. Reducing inflation from its baseline value of 4 percent to the op-

### The Model's View of How Welfare Varies With Inflation



\*Change in baseline level of consumption at each inflation rate that makes households indifferent between that rate and the baseline rate of 4%.

timal rate of –1.3 percent would produce a welfare gain of 0.43 percent of baseline consumption, or about \$16.8 billion.<sup>14</sup> For comparison, suppose instead that I adopt the approach of Den Haan (1990) and Lucas (1993) and offset the revenue lost when reducing the inflation rate by increasing lump-sum taxes. Under this assumption, my model says, the maximum welfare increase occurs when the Friedman rule is adopted. That is, for my parameterization, welfare is maximized when the inflation rate is –2 percent. Redoing the calculations reported in the chart under these alternative assumptions yields a welfare gain of 0.95 percent of baseline consumption, or \$37 billion, when the inflation rate is –2 percent.<sup>15</sup> Thus, adopting the more plausible assumption that revenue losses are offset by increases in taxes on labor income has a significant effect on the potential gains from deflation; they are more than cut in half.

<sup>14</sup>This welfare gain is converted into a dollar figure by multiplying 0.0043 by aggregate consumption in 1991, which was \$3,906 billion.

<sup>15</sup>This number is smaller than the one reported by Lucas (1993). The difference can be explained by differences in how we measure output. In Lucas' framework, output is divided between consumption and transaction costs. Thus, a welfare gain of 1 percent of consumption is also about 1 percent of output. Recall that GDP was about \$6 trillion in 1992, so 1 percent of GDP is \$60 billion. I assume that the welfare gain of 1 percent of consumption is only 0.68 percent of GDP. So to compare Lucas' number with mine, the \$60 billion must be multiplied by 0.68, which yields \$40.8 billion.

Finally, note that the chart has an asymmetry. As the inflation rate falls from its baseline level, welfare first increases gradually, then decreases sharply. However, as the inflation rate rises from its baseline level, welfare decreases only gradually. While an inflation rate of  $-1.3$  percent increases welfare  $0.43$  percent, an inflation rate of  $-1.91$  percent decreases it by more than  $1$  percent. A welfare loss of  $1$  percent is quite large. For instance, it is three times larger than the loss associated with an inflation rate of  $10$  percent.

This asymmetry is important for anyone considering policies that would reduce the inflation rate further. Common measures of inflation have large margins of error. David Lebow, John Roberts, and David Stockton (1992, 1994) estimate, for example, that growth in the CPI, a widely used measure of inflation, may overstate the true inflation rate by between one-half of a percentage point and  $1.5$  percentage points.<sup>16</sup> Suppose that monetary policy were set to target the inflation rate as measured by CPI growth at the optimal rate. With the amount of inherent uncertainty in this measure, a target of, say,  $-1.3$  percent implies a true inflation rate in the region where welfare losses occur.

These measurement problems for the CPI have other implications for my analysis. If CPI growth overstates the true inflation rate by about one percentage point, then an evaluation of the gains from reducing inflation should start from a baseline inflation rate of around  $3$  percent rather than  $4$  percent. If I redo my calculations from a baseline inflation rate of  $3$  percent, the welfare gain from adopting an inflation rate of  $-1.3$  percent shrinks to  $\$14$  billion. The welfare gain from adopting a stable price level is even smaller—only about  $\$10$  billion.

Taken together, these results indicate that the maximum welfare gain from reducing inflation below its current rate is less than one-half of  $1$  percent of consumption. The asymmetry in the chart also shows that reducing the inflation rate enough to achieve a gain of this magnitude is very risky. If the inflation rate is reduced too far, households could be made much worse off than they would be with moderate levels of inflation. Given the large amount of uncertainty in the measured value of inflation, the largest likely achievable gain seems to be  $\$14$  billion. This is only  $0.36$  percent of consumption, which is small relative to the gains that other reforms could achieve. Thomas Cooley and Gary Hansen (1992), for example, estimate the gains from removing the U.S. tax on capital income and replacing it with a higher inflation rate to be much

larger, more than  $2.5$  percent of gross national product, or  $\$150$  billion.

Plausible modifications of the model's specification weaken the case for deflation even more. For instance, the form of the money demand function derived here implies that households' demand for real balances are unbounded as the interest rate on bonds,  $R$ , approaches zero. This is the source of the asymmetry in the chart. As  $R$  approaches zero, the revenue requirements needed to offset the loss to government revenue from deflating get very large. Raising the labor tax to meet these additional revenue requirements imposes a large welfare cost on households. Suppose instead that the transaction technology is specified as

$$(28) \quad \log(m_t) = v_0 - v_1[R_t/(1+R_t)] + v_2\log(c_t) + \varepsilon_t.$$

The difference between (28) and (24) is that the interest rate term in (28) is not logged. This is often referred to as a *semilog* money demand specification. For this specification, a zero nominal interest rate is consistent with finite holdings of real balances.

Thomas Cooley and Gary Hansen (1991) and Lucas (1993) consider the welfare gains from reducing inflation in frameworks that are consistent with a semilog money demand function. Cooley and Hansen find that welfare declines when the inflation rate is reduced from  $5$  percent to zero and the forgone revenue is replaced with a higher tax on either labor or capital income. Lucas (1993) compares the welfare costs of moderate inflation using transaction cost functions that are consistent with each of the two alternate specifications of the money demand function shown in (24) and (28). He finds that shifting to a transaction cost function that implies a semilog money demand function cuts welfare costs by two-thirds.

Adopting such a function can also overturn the conclusion that deflation is optimal. In Braun 1994, I consider a cash-in-advance economy that imposes restrictions on a semilog money demand function. For this economy, when lump-sum taxes are ruled out, the welfare-maximizing inflation rate is positive.

## Conclusion

My model suggests that the maximum welfare gains from reducing U.S. inflation from its current rate are quite small, somewhere in the range of from one-third to one-half of

---

<sup>16</sup>Mark Wynne and Fiona Sigalla (1993) find that these measurement problems are equally severe for other measures of the aggregate price level.

1 percent of annual GDP. These results appear quite reasonable, but they must be viewed cautiously. My analysis has an obvious limitation: it ignores the effects of uncertainty, which can have important welfare implications. Ayşe İmrohoroğlu (1992), for example, finds that if households have limited access to financial markets and experience periodic, idiosyncratic shocks to labor income, then the welfare costs of moderate inflation are substantially larger than my model suggests. Still, I think the most direct way to deal with those costs is not to try to reduce inflation, but rather to introduce regulatory reforms that provide individuals with readier access to financial markets.

Some economists think that the most important costs of inflation stem not from its average value, but from its variability. However, finding a formal model consistent with that idea is difficult. Work by V. V. Chari, Lawrence Christiano, and Patrick Kehoe (1991), for example, shows that a highly variable inflation rate is one component of an optimal government policy in a flexible price model. True, the costs of a variable inflation rate would be higher if prices were not free to adjust due to, say, labor contracts that limit employers' ability to adjust wages. On net, however, I suspect that the welfare gains from adopting a monetary policy aimed at stabilizing cyclical fluctuations are quite small. My basis for this is calculations by Robert Lucas (1987) which suggest that a representative individual would pay only 0.1 percent of consumption, or about \$15 per year, for insurance that would guarantee the person a smooth consumption path that grew at the rate of GDP. This means that even if monetary policy could remove all cyclical fluctuations in consumption, the welfare gains would be only about 0.1 percent of consumption per year. For the U.S. economy in 1991, this would have been about \$3.9 billion. In fact, however, monetary policy cannot smooth consumption completely; thus, the gains are likely to be even smaller.

## References

- Aiyagari, S. Rao. 1990. Deflating the case for zero inflation. *Federal Reserve Bank of Minneapolis Quarterly Review* 14 (Summer): 2–11.
- Barro, Robert J. 1974. Are government bonds net wealth? *Journal of Political Economy* 82 (November–December): 1095–1117.
- Braun, R. Anton. 1994. How large is the optimal inflation tax? *Journal of Monetary Economics* 34 (October): 201–14.
- Braun, R. Anton, and Christiano, Lawrence J. 1994. Is long-run M1 demand stable? Research Department Working Paper 529. Federal Reserve Bank of Minneapolis.
- Braun, R. Anton, and McGrattan, Ellen R. 1994. A model of family labor supply in war and peace. Research Department Working Paper 527. Federal Reserve Bank of Minneapolis.
- Chari, V. V.; Christiano, Lawrence J.; and Kehoe, Patrick J. 1991. Optimal fiscal and monetary policy: Some recent results. *Journal of Money, Credit and Banking* 23, part 2 (August): 519–39.
- Cho, Jang-Ok, and Cooley, Thomas F. 1994. Employment and hours over the business cycle. *Journal of Economic Dynamics and Control* 18 (March): 411–32.
- Cooley, Thomas F., and Hansen, Gary D. 1991. The welfare costs of moderate inflations. *Journal of Money, Credit and Banking* 23, part 2 (August): 483–503.
- \_\_\_\_\_. 1992. Tax distortions in a neoclassical monetary economy. *Journal of Economic Theory* 58 (December): 290–316.
- Den Haan, Wouter J. 1990. The optimal inflation path in a Sidrauski-type model with uncertainty. *Journal of Monetary Economics* 25 (June): 389–409.
- Friedman, Milton. 1969. The optimum quantity of money. In *The optimum quantity of money and other essays*, pp. 1–50. Chicago: Aldine.
- Friedman, Milton, and Schwartz, Anna J. 1982. *Monetary trends in the United States and the United Kingdom: Their relation to income, prices, and interest rates, 1867–1975*. Chicago: University of Chicago Press for the National Bureau of Economic Research.
- Hansen, Gary D. 1985. Indivisible labor and the business cycle. *Journal of Monetary Economics* 16 (November): 309–27.
- Hornstein, Andreas, and Prescott, Edward C. 1993. The firm and the plant in general equilibrium theory. In *General equilibrium, growth, and trade II: The legacy of Lionel McKenzie*, ed. Robert Becker, Michele Boldrin, Ronald Jones, and William Thomson, pp. 393–410. San Diego: Academic Press.
- İmrohoroğlu, Ayşe. 1992. The welfare cost of inflation under imperfect insurance. *Journal of Economic Dynamics and Control* 16 (January): 79–91.
- Kydland, Finn E., and Prescott, Edward C. 1991. Hours and employment variation in business cycle theory. *Economic Theory* 1 (January): 63–81.
- Lebow, David E; Roberts, John M.; and Stockton, David J. 1992. Economic performance under price stability. Economic Activity Section Working Paper 125. Board of Governors of the Federal Reserve System.
- \_\_\_\_\_. 1994. Monetary policy and the price level. Manuscript. Board of Governors of the Federal Reserve System.
- Lucas, Robert E., Jr. 1987. *Models of business cycles*. Yrjö Jahnsson Lectures series, Helsinki, Finland. London: Blackwell.
- \_\_\_\_\_. 1993. On the welfare cost of inflation. Manuscript. University of Chicago.
- \_\_\_\_\_. 1994. On the welfare cost of inflation. CEPR Publication 394. Center for Economic Policy Research, Stanford University.
- Marshall, David A. 1992. Inflation and asset returns in a monetary economy. *Journal of Finance* 47 (September): 1315–42.
- McCallum, Bennett T. 1983. The role of overlapping-generations models in monetary economics. In *Money, monetary policy, and financial institutions*, ed. Karl Brunner and Allan H. Meltzer. Carnegie-Rochester Conference Series on Public Policy 18 (Spring): 9–44. Amsterdam: North-Holland.
- Mroz, Thomas A. 1987. The sensitivity of an empirical model of married women's hours of work to economic and statistical assumptions. *Econometrica* 55 (July): 765–99.
- Park, Joon Y. 1990. Testing for unit roots and cointegration by variable addition. In *Co-integration, spurious regressions, and unit roots*, ed. Thomas B. Fomby and George F. Rhodes, Jr., pp. 107–33. Advances in Econometrics 8. Greenwich, Conn.: JAI Press.
- Pencavel, John H. 1986. Labor supply of men: A survey. In *Handbook of labor economics*, Vol. 1, ed. Orley Ashenfelter and Richard Layard, pp. 3–102. Handbooks in Economics 5. Amsterdam: North-Holland.
- Phelps, Edmund S. 1973. Inflation in the theory of public finance. *Swedish Journal of Economics* 75 (March): 67–82.
- Rogerson, Richard. 1988. Indivisible labor, lotteries and equilibrium. *Journal of Monetary Economics* 21 (January): 3–16.
- Wynne, Mark A., and Sigalla, Fiona. 1993. A survey of measurement biases in price indexes. Research Paper 9340. Federal Reserve Bank of Dallas.