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Taxing Capital Income: A Bad Idea

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Corporate profits. Capital gains. Dividend and interest income. These are just a few of the types of capital income that are taxed in the United States—and, some would say, taxed heavily. This situation is quite different from what recent economic theory says is the optimal way to tax capital income: Not at all.

The optimality of a zero capital income tax was first established by Chamley (1986).¹ His result contradicts the conventional view in the public finance literature that capital income should be taxed heavily. The conventional view is based on a model in which the saving rate is assumed to be a fixed fraction of income. In that model, therefore, capital income taxes do not distort economic decisions and, hence, are desirable. More recent economic theory uses models in which the saving rate is not fixed, but is rather chosen by consumers, to maximize their utility from consumption over time. Using such a model, Chamley shows that in the steady state, the optimal tax rate on capital income is zero. This makes sense if you realize that a constant tax rate on capital income is equivalent to an ever-increasing tax rate on consumption. Under a wide variety of assumptions, such a tax on consumption cannot be optimal.

Chamley's (1986) result has not been universally accepted because it is based on a narrow set of assumptions: identical and infinitely lived consumers, steady-state growth not affected by taxes, and a closed economy. Here we lay out a simple framework in which we

describe Chamley's result and then relax his assumptions, one by one, to see if the zero capital income tax result still holds. It does.

That result is not exactly new. Several other researchers have independently extended Chamley's (1986) study in various ways and gotten a similar result for the parts they examined, using various types of models and approaches. (See Judd 1985, Razin and Sadka 1995, and Jones, Manuelli, and Rossi 1997.)

What is new here is our attempt to unify that work. We relax all Chamley's assumptions in just one type of model—a *discrete time* model—using just one approach—the *primal* approach. In the primal approach, the consumer and firm first-order conditions are used to eliminate prices and tax rates, and the problem of determining optimal policy reduces to a simple programming problem in which the choice variables are the allocations. We refer to this programming problem as the *Ramsey problem* and to the associated allocations and policies as the *Ramsey allocations* and the *Ramsey plan*. Our unification of the work on Chamley's result allows a reliable comparison of the results for the various assumptions.

Note that our work does not lead to quite as drastic a policy recommendation as it may seem to. We do not

¹Judd (1985) proves a related result in an economy with different types of consumers.

conclude that capital income taxes should simply be set to zero immediately.

The basic Chamley result is that in a steady state, the optimal capital income tax rate is zero. In practice, we think that this should be interpreted as saying that over the long term, capital income tax rates should be driven to zero. However, with slightly stronger assumptions, the basic Chamley result can be extended to say that it is optimal to have an initial phase of positive capital income tax rates that is soon followed by a tax rate of zero. In practice, even if policymakers decide to move to a system of zero capital income taxes, it will take a while to actually implement the new rules. Perhaps this implementation lag corresponds roughly to the initial phase of positive capital income taxes in the model. If so, the best way to implement the Chamley result is to start the process of dispensing with capital income taxes right away.

Our study, of course, has its own assumptions, which some might see as limitations. Primarily, we assume that the government can commit to follow a long-term program for taxing capital income. Without a technology to make such a commitment, there are *time inconsistency* problems; equilibrium outcomes with government commitment are not necessarily sustainable without it.² The U.S. government has not yet made such an explicit commitment to follow its announced policies. But certainly it does have considerable constitutional and other legal means to do so. Therefore, we do not think that our government commitment assumption should blunt our bottom-line message to U.S. policymakers. Those responsible for shaping the best possible tax system for the nation would be wise to give serious attention to the relatively new principle of public finance demonstrated here: taxing capital income is a bad idea.

The Economy

We start by setting up an economy in which to analyze Chamley's zero capital income tax result.

The framework we use combines two traditions in economics: the *public finance* tradition and the *general equilibrium* tradition. The public finance tradition we follow stems from the work of Ramsey (1927), who considers the problem of choosing an optimal tax structure in an economy with a representative agent when only distorting taxes are available. The general equilibrium tradition we follow models growth as arising from consumers' optimal choices of consumption and investment. This tradition stems from the work of Cass (1965), Koop-

mans (1965), Kydland and Prescott (1982), and Lucas and Stokey (1983).

Consider a production economy populated by a large number of identical, infinitely lived consumers. In each period of time $t = 0, 1, \dots$, the economy has two goods: a consumption-capital good and labor. A constant returns to scale technology which satisfies the standard Inada conditions is available to transform capital k_t and labor l_t into output via the production function $F(k_t, l_t)$. The output can be used for private consumption c_t , government consumption g_t , and new capital k_{t+1} . Government consumption is exogenously specified and constant, so $g_t = g$.

In such an economy, feasibility requires that the resource constraint be satisfied:

$$(1) \quad c_t + g + k_{t+1} = F(k_t, l_t) + (1-\delta)k_t$$

where δ is the depreciation rate on capital. The preferences of each consumer are given by

$$(2) \quad \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

where the discount factor $0 < \beta < 1$ and utility U is strictly increasing in consumption, is strictly decreasing in labor, is strictly concave, and satisfies the standard Inada conditions.

In this economy, consumers own capital and rent it to firms. Government consumption is financed by proportional taxes on the income from capital and labor. Let θ_t and τ_t denote the tax rates on the income from capital and labor. The consumer's budget constraint is

$$(3) \quad \sum_{t=0}^{\infty} p_t (c_t + k_{t+1}) = \sum_{t=0}^{\infty} p_t [(1-\tau_t)w_t l_t + R_{kt} k_t]$$

where

$$(4) \quad R_{kt} = 1 + (1-\theta_t)(r_t - \delta)$$

is the gross return on capital after taxes and depreciation, r_t and w_t are the before-tax returns on capital and labor, p_t is the price of consumption in period t , p_0 is

²Economies with government commitment technologies can be interpreted in at least two ways. One is that the government can simply commit to its future actions by, say, restrictions in its constitution. The other is that the government has no access to such a commitment technology, but the commitment outcomes are sustained by reputational mechanisms. For analyses of optimal policy in environments without commitment, see, for example, Chari, Kehoe, and Prescott 1989; Chari and Kehoe 1990, 1993; and Stokey 1991.

normalized to 1, and the initial capital stock k_0 is given. The first-order conditions for the consumer are

$$(5) \quad \beta' U_{c_t} = \lambda p_t$$

$$(6) \quad \beta' U_{l_t} = -\lambda p_t (1 - \tau_t) w_t$$

$$(7) \quad p_t = R_{k_{t+1}} p_{t+1}$$

where λ is the Lagrange multiplier on the consumer's budget constraint. Here U_{c_t} and U_{l_t} are the partial derivatives of $U(c_t, l_t)$ with respect to c_t and l_t . (We use similar notation throughout our analysis.)

Firms in this economy maximize profits:

$$(8) \quad \max F(k_t, l_t) - w_t l_t - r_t k_t.$$

The firm's first-order conditions imply that before-tax returns on capital and labor equal their marginal products, namely, that

$$(9) \quad r_t = F_k(k_t, l_t)$$

$$(10) \quad w_t = F_l(k_t, l_t).$$

The government sets tax rates on capital and labor income to finance the exogenous sequence of government consumption. The government's budget constraint is

$$(11) \quad \sum_{t=0}^{\infty} p_t g = \sum_{t=0}^{\infty} p_t [\tau_t w_t l_t + \theta_t (r_t - \delta) k_t].$$

Let $\pi_t = (\tau_t, \theta_t)$ denote the government policy at t , and let π denote the policies for all t . Let $x_t = (c_t, l_t, k_{t+1})$ denote an allocation for consumers at t , and let x denote an allocation for all t . Let (w, r, p) denote a price system for all t .

A *competitive equilibrium* for this economy is a policy π , an allocation x , and a price system (w, r, p) such that given the policy and the price system, the resulting allocation maximizes the representative consumer's utility, expression (2), subject to the consumer's budget constraint, (3); the price system satisfies equations (9) and (10); and both the government's budget constraint (11) and the economy's resource constraint (1) are satisfied.

Consider now the policy problem faced by the government. Suppose that in the economy an institution, or *commitment technology*, exists through which the government, in period 0, can bind itself to a particular sequence of policies once and for all. We model this by

having the government choose a policy π at the beginning of time, after which consumers choose their allocations. Formally, *allocation rules* are sequences of functions $x(\pi) = (x_t(\pi))$ that map policies π into allocations $x(\pi)$. *Price rules* are sequences of functions $w(\pi) = (w_t(\pi))$, $r(\pi) = (r_t(\pi))$, and $p(\pi) = (p_t(\pi))$ that map policies π into price systems.

Since the government needs to predict how consumer allocations and prices will respond to its policies, consumer allocations and prices must be described by rules that associate government policies with allocations. We impose two restrictions on the set of policies that the government can choose. The government must choose policies for which a competitive equilibrium exists; hence, the allocation rules are defined only over such policies. Also, since the capital stock in period 0 is inelastically supplied, the government has an incentive to set the initial capital tax rate as high as possible. To make the problem interesting, we require that the initial capital income tax rate, θ_0 , be fixed.

A *Ramsey equilibrium* in this economy is a policy π , an allocation rule $x(\cdot)$, and price rules $w(\cdot)$ and $r(\cdot)$ that satisfy these two conditions:

- The policy π maximizes

$$(12) \quad \sum_{t=0}^{\infty} \beta^t U(c_t(\pi), l_t(\pi))$$

subject to the government's budget constraint (11), with allocations and prices given by $x(\pi)$, $w(\pi)$, and $r(\pi)$.

- For every π' , the allocation $x(\pi')$; the price system $w(\pi')$, $r(\pi')$, and $p(\pi')$; and the policy π' constitute a competitive equilibrium.

If multiple competitive equilibria are associated with some policies, our definition of a Ramsey equilibrium requires that a selection be made from the set of competitive equilibria. We focus on the Ramsey equilibrium that yields the highest utility.

Now consider the equilibrium allocations and policies in this economy. For convenience in terms of notation, let U_{c_t} and U_{l_t} denote the marginal utilities of consumption and leisure in period t , and let F_{k_t} and F_{l_t} denote the marginal products of capital and labor in period t . A competitive equilibrium allocation is characterized by two fairly simple conditions: the resource constraint (1) and the implementability constraint

$$(13) \quad \sum_{t=0}^{\infty} \beta^t (U_{ct} c_t + U_{lt} l_t) = U_{c0} R_{k0} k_0$$

where

$$(14) \quad R_{k0} = 1 + (1 - \theta_0)(F_{k0} - \delta).$$

To see that the competitive equilibrium allocations satisfy (13), observe that this implementability constraint is the consumer's budget constraint with the prices and policies substituted out by the consumer and firm first-order conditions.

To see that any allocation which satisfies (1) and (13) is a competitive equilibrium allocation, use these allocations together with the first-order conditions of the consumer and the firm to construct the corresponding equilibrium prices and policies. The prices r_t and w_t are determined by (9) and (10). From (5), the price p_t is given by

$$(15) \quad p_t = \beta^t U_{ct} / U_{c0}.$$

The labor income tax rate τ_t is determined from (5), (6), and (10) and is given by

$$(16) \quad -U_{lt} / U_{ct} = (1 - \tau_t) F_{lt}.$$

The capital income tax rate θ_{t+1} for $t \geq 0$ is determined from (5), (7), and (9) and is implicitly defined by

$$(17) \quad U_{ct} = \beta U_{ct+1} R_{kt+1}$$

where

$$(18) \quad R_{kt+1} = 1 + (1 - \theta_{t+1})(F_{kt+1} - \delta)$$

and the capital income tax rate θ_0 is given.

From our characterization of a competitive equilibrium, we can see immediately that the allocations in a Ramsey equilibrium solve the Ramsey allocation problem of maximizing consumers' utility (2) subject to the constraints (1) and (13). For convenience, write the Ramsey allocation problem in Lagrangian form:

$$(19) \quad \max \sum_{t=0}^{\infty} \beta^t [W(c_t, l_t, \lambda)] - \lambda U_{c0} R_{k0} k_0$$

subject to (1). The function W simply incorporates the implementability constraint into the maximand and is given by

$$(20) \quad W(c_t, l_t, \lambda) = U(c_t, l_t) + \lambda (U_{ct} c_t + U_{lt} l_t)$$

where λ is the Lagrange multiplier on the implementability constraint, (13). The first-order conditions for this problem imply that, for $t \geq 1$,

$$(21) \quad -W_{lt} / W_{ct} = F_{lt}$$

and, for $t = 1, 2, \dots$,

$$(22) \quad W_{ct} = \beta W_{ct+1} (1 - \delta + F_{kt+1})$$

while

$$(23) \quad W_{c0} = \beta W_{c1} (1 - \delta + F_{k1}) + \lambda U_{c0} R_{k0} k_0.$$

In the following results, we will repeatedly use the observation that if the term

$$(24) \quad W_{ct} / U_{ct} = 1 + \lambda \{ [U_{cct} c_t + U_{clt} l_t] / U_{ct} \} + 1 \}$$

has the same value in periods t and $t + 1$, then the capital income tax in period $t + 1$ is zero. To see this, note that if

$$(25) \quad W_{ct} / U_{ct} = W_{ct+1} / U_{ct+1}$$

then (22) can be written as

$$(26) \quad U_{ct} = \beta U_{ct+1} (1 - \delta + F_{kt+1})$$

which from (17) implies that the capital income tax rate $\theta_{t+1} = 0$. Notice from (23) that the first-order condition for consumption in period 0 includes extra terms. Thus, even if

$$(27) \quad W_{c0} / U_{c0} = W_{c1} / U_{c1}$$

the capital income tax in period 1 is not necessarily equal to zero.

We label the term in (24) the *general equilibrium expenditure elasticity*. This elasticity captures the distortions relevant for setting taxes on capital income in general equilibrium. Atkinson and Stiglitz (1980) show that for special forms of utility, an elasticity similar to this one reduces to either the price elasticity or the income elasticity of demand.

Throughout, we assume that the solution to the Ramsey problem occurs at an interior point. Note that since

the set of allocations which satisfy the implementability constraint is not necessarily convex, the first-order conditions for the Ramsey problem are necessary but not sufficient. (For a discussion of nonconvexity, see Lucas and Stokey 1983.)

Chamley's Result

Chamley (1986) shows, for a model economy similar to the one just described, that the optimal capital income tax is zero in a steady state. Here we demonstrate that result in our model. Then we restrict attention to a commonly used class of utility functions and analyze optimal capital income taxes in the transition to the steady state as well. The result: With no upper bound on capital income taxation, capital income taxes are zero starting in period 2. And with an upper bound, capital income taxes are zero after a finite number of periods.

To establish Chamley's result in a steady state, suppose that under the Ramsey plan, the allocations converge to a steady state. In our model in such a steady state, W_c and U_c are constant; hence, the general equilibrium expenditure elasticity is constant. Thus, (22) reduces to (26), and steady-state capital income taxes are zero. In sum:

PROPOSITION 1. *If the solution to the Ramsey problem converges to a steady state, then in the steady state, the tax rate on capital income is zero.*

(Note that here—and in the following steady-state results—we prove that if there exists a steady state of the type considered, then the optimal capital income taxes are zero. We do not prove that the solution to the Ramsey problem necessarily converges to the type of steady state considered. Proving this stronger result may require additional assumptions.)

One way to get intuition for Proposition 1 is to note that taxing capital income in period $t + 1$ is equivalent to taxing consumption at a higher rate in period $t + 1$ than in period t . Thus, a positive tax on capital income in a steady state is equivalent to an ever-increasing tax on consumption. Such an increasing tax cannot be optimal in a steady state because all of the relevant general equilibrium expenditure elasticities are constant over time.

For certain utility functions, we can establish a much stronger result, namely, that optimal capital income taxes are zero after only a few periods. (See Chamley 1986, for a related analysis in continuous time.) Here we show that for a commonly used class of utility functions, dis-

torting the capital accumulation decision in period 1 or thereafter is not optimal.

The class of utility functions we consider are of the form either

$$(28) \quad U(c, l) = [c^{1-\sigma}/(1-\sigma)] + V(l)$$

or

$$(29) \quad U(c, l) = (cl^{-\gamma})^{1-\sigma}/(1-\sigma)$$

where $\sigma \leq 1$ and $0 < \gamma < 1$. These utility functions are commonly used in the literature on economic growth because they are consistent with the type of balanced growth observed in the U.S. economy. (Note that in (28), balanced growth occurs only if $\sigma = 1$.) For any utility function of the form (28) or (29), we can easily show that for all periods $t \geq 1$,

$$(30) \quad W_{ct+1}/W_{ct} = U_{ct+1}/U_{ct}$$

Thus, for all periods $t \geq 1$, (22) reduces to (26); hence, the optimal capital income taxes are zero for all periods $t \geq 2$. In sum:

PROPOSITION 2. *For utility functions of the form (28) or (29), it is not optimal to distort the capital accumulation decision in period 1 or thereafter. Therefore, the optimal tax rate on capital income received in period t is zero for $t \geq 2$.*

Note that under the Ramsey plan, the government optimally distorts only the first decision to accumulate capital, which occurs in period 1. The government distorts that decision by levying a positive capital income tax in period 2 on the resulting income. In period 0, of course, the tax rate is fixed by assumption. Intuitively, we can see that for utility functions of the form considered here, the general equilibrium expenditure elasticity is constant even out of steady state, so that except for period 1, the capital income tax should always be zero. This result is much stronger than the standard Chamley result, which refers to steady states.

In a continuous time version of the model with instantaneous preferences given by (28), Chamley (1986) shows that the tax rate on capital income is constant for a finite length of time and is zero thereafter. The reason for Chamley's different result is that he imposes an exogenous upper bound on the tax rate on capital income.

We now impose such an upper bound and prove a discrete time analog of Chamley's result.

In particular, we assume that agents have the option to hold their capital without renting it to firms at a rate of return $1 - \delta$. Under this assumption, the after-tax rate of return on capital is bounded below in equilibrium by $1 - \delta$. The Ramsey equilibrium in this case, in addition to satisfying the analogs of (1) and (13) (the resource and implementability constraints), must satisfy an extra condition derived from (17) to be part of a competitive equilibrium:

$$(31) \quad U_{ct} \geq \beta U_{c,t+1}(1-\delta).$$

Considering the Ramsey problem with (31) as an additional constraint, we have

PROPOSITION 3. *Under an optimal policy, for utility functions of the form (28) and (29) and with a production function in which $F(0,l) = 0$, the constraint (31) on the capital income tax rate is binding for a finite number of periods. After that, the tax takes on an intermediate value for one period and is zero thereafter.*

Proof. We prove this proposition by establishing three claims. First, we claim that the constraint (31) cannot be slack in some period t , bind in periods later than t , and then be slack in some period $t + n$. Second, we claim that the constraint (31) cannot bind in every period. These two arguments together imply that the constraint (31) holds for at most some finite number of periods initially and then does not bind again. Finally, we claim that if t is the last period in which the constraint (31) binds, then the optimal capital income tax is zero in all periods s with $s \geq t + 2$. (In period $t + 1$, the capital income tax may be at some intermediate value.)

Let $\beta'\phi_t$ be the Lagrange multiplier on the constraint (31) and $\beta'\gamma_t$ be the Lagrange multiplier on the resource constraint (1). Then the first-order conditions of the Ramsey problem are, with respect to capital,

$$(32) \quad \gamma_t = \beta\gamma_{t+1}[(1-\delta) + F_{k,t+1}]$$

and with respect to consumption,

$$(33) \quad \gamma_t = W_{ct} + [\phi_t - (1-\delta)\phi_{t-1}]U_{cct}.$$

With utility of the form (28) or (29),

$$(34) \quad W_{ct}/W_{c,t+1} = U_{ct}/U_{c,t+1}.$$

To prove our first claim, suppose by way of contradiction that in two periods, t and $t + n$, $\phi_t = \phi_{t+n} = 0$ and $\phi_{t+1}, \phi_{t+2}, \dots, \phi_{t+n-1}$ are all greater than zero. Equations (32) and (33) imply that

$$(35) \quad W_{c,t+1} + \phi_{t+1}U_{cct+1} \geq \beta^{n-1}(1-\delta)^{n-1}[W_{c,t+n} - (1-\delta)\phi_{t+n-1}U_{cct+n}].$$

Equation (34), together with the assumption that constraint (31) is binding in periods $t + 1, t + 2, \dots, t + n - 1$, implies that

$$(36) \quad W_{c,t+1} = \beta^{n-1}(1-\delta)^{n-1}W_{c,t+n}.$$

Plugging this into (35) then gives

$$(37) \quad \phi_{t+1}U_{cct+1} \geq -\beta^{n-1}(1-\delta)^{n-1}(1-\delta)\phi_{t+n-1}U_{cct+n}$$

which is a contradiction since $U_{cc} < 0$.

To prove the second claim, note that if the constraint (31) binds in every period, then the capital stock rented to firms goes to zero at a rate determined by

$$(38) \quad k_{t+1} = (1-\delta)k_t$$

and given the assumption $F(0,l) = 0$, the resource constraint (1) is violated. Thus, the constraint (31) cannot bind in every period.

To prove the third claim, observe that if t is the last period in which the constraint (31) binds, then (32)–(34) imply that

$$(39) \quad U_{cs} = \beta U_{c,s+1}[(1-\delta) + F_{k,s+1}]$$

for periods $s \geq t + 2$, which implies that the capital income tax is zero. Q.E.D.

Extending Chamley's Result

Now we examine whether the zero capital income tax result extends to other economic environments. We consider an economy which has agents not identical, but rather heterogeneous; an economy which grows at a rate determined not exogenously, but rather endogenously; an economy which is not closed, but open; and an economy with agents not infinitely lived, but rather born into overlapping generations. We find that Chamley's basic result extends to all of these environments. The overlapping-

generations economy alone requires somewhat stricter conditions for a zero capital income tax to be optimal.³

Heterogeneous Consumers

We begin by switching from identical to heterogeneous agents. We examine the natural conjecture that, with more than one type of consumer, a nonzero tax on capital income is optimal to redistribute income from one type to another. We study first an environment in which the different types of consumers can be taxed at different rates and then environments in which all consumers have to be taxed at the same rate. We find, with some caveats, that with heterogeneous agents, taxing capital income in a steady state is not optimal.

Assume our economy now has two types of consumers, indexed $i = 1, 2$. Their preferences are given by

$$(40) \quad \sum_{t=0}^{\infty} \beta^t U^i(c_{it}, l_{it})$$

where c_{it} and l_{it} denote the consumption and the labor supply of a consumer of type i . Assume that the discount factors are the same for both types of consumers. The resource constraint for this economy is then given by

$$(41) \quad c_{1t} + c_{2t} + g + k_{t+1} = F(k_t, l_{1t}, l_{2t}) + (1-\delta)k_t$$

where the production function F has constant returns to scale.

Notice that the production function allows for imperfect substitutability between the two types of capital and labor. For this economy, the implementability constraints for the two types of consumers $i = 1, 2$ are given by

$$(42) \quad \sum_{t=0}^{\infty} \beta^t (U_{ct}^i c_{it} + U_{lt}^i l_{it}) = U_{c0}^i R_{k0} k_0^i$$

where k_0^i denotes the initial ownership of capital by consumers of type i . The initial stock of capital $k_0 = k_0^1 + k_0^2$. If the tax system allows tax rates on capital income and labor income to differ across consumer types, then it is straightforward to establish that the resource constraint (41) and the two implementability constraints (42) completely characterize a competitive equilibrium.

For a Ramsey equilibrium, suppose that the government maximizes a weighted sum of consumers' utilities of the form

$$(43) \quad \omega_1 \sum_{t=0}^{\infty} \beta^t U^1(c_{1t}, l_{1t}) + \omega_2 \sum_{t=0}^{\infty} \beta^t U^2(c_{2t}, l_{2t})$$

where the welfare weights $\omega_i \in [0, 1]$ satisfy $\omega_1 + \omega_2 = 1$. The Ramsey problem is to maximize (43) subject to the resource constraint (41) and the implementability constraints (42). Define

$$(44) \quad W(c_{1t}, c_{2t}, l_{1t}, l_{2t}, \lambda_1, \lambda_2) \\ = \sum_{i=1,2} [\omega_i U^i(c_{it}, l_{it}) + \lambda_i (U_{ct}^i c_{it} + U_{lt}^i l_{it})]$$

for $t \geq 0$. Here λ_i is the Lagrange multiplier on the implementability constraint for the consumer of type i . The Ramsey problem is, then, to maximize

$$(45) \quad \sum_{t=0}^{\infty} \beta^t W(c_{1t}, c_{2t}, l_{1t}, l_{2t}, \lambda_1, \lambda_2) - \sum_{i=1,2} \lambda_i U_{c0}^i R_{k0} k_0^i$$

subject to the resource constraint (41). The first-order conditions for capital for this problem imply that for $i = 1, 2$ and for $t = 1, 2, \dots$,

$$(46) \quad W_{cit} = \beta W_{cit+1} (1 - \delta + F_{kt+1}).$$

Here the general equilibrium expenditure elasticity for the consumer of type i is

$$(47) \quad W_{cit}/U_{ct}^i = \omega_i + \lambda_i \{ [(U_{cct}^i c_{it} + U_{clt}^i l_{it})/U_{ct}^i] + 1 \}.$$

In a steady state, W_{cit}/U_{ct}^i is constant over time for $i = 1, 2$, so the steady-state tax on capital income is zero for both types of consumers. Notice that this result is true regardless of the weights ω_i the government places on the two types of consumers. In sum:

PROPOSITION 4. *In an economy with heterogeneous consumers, the steady-state tax rate on capital income is zero for all consumers, regardless of the government's welfare weights ω_i . Furthermore, if utility is of the form (28) or (29), then the optimal capital income tax is zero in periods $t \geq 2$ as well.*

Judd (1985) shows that this result holds when type 1 consumers are workers who supply labor, cannot save or borrow, and hold no initial capital, while type 2 consumers are capitalists who own all the capital but supply no labor. We replace (42) for type 1 consumers with the static constraint

³Throughout, we consider deterministic models. In a stochastic version of the model with identical, infinitely lived consumers, Zhu (1992) and Chari, Christiano, and Kehoe (1994) show that while capital income taxes may be positive sometimes, they are zero on average.

$$(48) \quad U_{c_t}^1 c_{1t} + U_{l_t}^1 l_{1t} = 0$$

for all t . With this constraint, in the solution to the Ramsey problem, (46) for the capitalists continues to hold; thus, the steady-state tax on capital income is zero. This result shows that even if the government puts zero weight on the capitalists, taxing capital in the long run is not optimal.

Now suppose that the tax system does not allow tax rates on either capital income or labor income to differ across consumer types. These restrictions on the tax system imply extra constraints on the allocations that can be achieved in a competitive equilibrium.

Consider first the restriction that tax rates on capital income do not differ across consumers. To derive the restrictions that this adds to the Ramsey problem, consider the consumers' intertemporal first-order conditions, which can be written as

$$(49) \quad U_{c_t}^i / U_{c_{t+1}}^i = \beta[1 + (1 - \theta_{t+1})(F_{k_{t+1}} - \delta)].$$

Since the right side of (49) does not vary with i , the restriction

$$(50) \quad U_{c_t}^1 / U_{c_{t+1}}^1 = U_{c_t}^2 / U_{c_{t+1}}^2$$

holds in any competitive equilibrium. Thus, (50) is an extra restriction that must be added to the Ramsey problem. Note that (46) is still the first-order condition with respect to capital of the Ramsey problem with the additional constraint (50). Thus, we conclude that the steady-state tax on capital income is zero.

Consider next the restriction that tax rates on labor income do not differ across consumers. The consumers' first-order conditions for labor supply can be written as

$$(51) \quad -U_{l_t}^i / U_{c_t}^i F_{l_{1t}} = 1 - \tau_t.$$

Since the right side of (51) does not vary with i , the restriction

$$(52) \quad U_{l_t}^1 U_{c_t}^2 / U_{c_t}^1 U_{l_t}^2 = F_{l_{1t}} / F_{l_{2t}}$$

holds in any competitive equilibrium and thus must be added to the Ramsey problem. Note that this additional constraint does, in general, depend on the level of capital k if and only if the ratio $F_{l_{1t}} / F_{l_{2t}}$ depends on k . Recall that the production function is separable between k

and (l_1, l_2) if $F_{l_{1t}} / F_{l_{2t}}$ does not depend on k . Such separable production functions can be written in the form

$$(53) \quad F(k, l_1, l_2) = F(k, H(l_1, l_2))$$

for some function H . In this case, it is straightforward to show, again, that the steady-state tax on capital income is zero. (For some related discussion, see Stiglitz 1987.)

The discussion of the extra constraints on the Ramsey problem implied by restrictions on the tax system suggests this observation: Zero capital income taxation in the steady state is optimal if the extra constraints do not depend on the capital stock and is not optimal if these constraints depend on the capital stock (and, of course, are binding).

Endogenous Growth

Now we return to a version of Chamley's original model, but relax his exogenously determined growth assumption. We consider a model in which the long-run growth rate of the economy is not simply given, but rather is determined by agents' decisions to accumulate both physical and human capital. Analysis of optimal policy in this endogenous growth model leads to a remarkable result: Along a balanced growth path, all taxes are zero.

Our discussion is restricted to a version of the model with both physical and human capital described by Lucas (1990). In this model, the long-run growth rate is endogenously determined by agents' decisions to accumulate these two forms of capital. (Bull 1992 and Jones, Manuelli, and Rossi 1997 discuss extensions of the result that the optimal capital income tax is zero to a larger class of endogenous growth models.)

Consider an infinite-horizon model in which the technology for producing goods is given by a constant returns to scale production function $F(k_t, h_t, l_{1t})$, where k_t denotes the physical capital stock in period t , h_t denotes the human capital stock in period t , and l_{1t} denotes labor input to goods production in period t . Human capital investment in period t is given by $h_t G(l_{2t})$, where l_{2t} denotes labor input into human capital accumulation and G is an increasing concave function. The resource constraints for this economy are

$$(54) \quad c_t + g + k_{t+1} = F(k_t, h_t, l_{1t}) + (1 - \delta_k)k_t$$

$$(55) \quad h_{t+1} = h_t G(l_{2t}) + (1 - \delta_h)h_t$$

where c_t is private consumption, g is exogenously given government consumption, and δ_k and δ_h are depreciation rates on physical and human capital, respectively.

The consumer's preferences are given by

$$(56) \quad \sum_{t=0}^{\infty} \beta^t [c_t^{1-\sigma} / (1-\sigma)] \nu(l_{1t} + l_{2t})$$

where ν is a decreasing convex function. Government consumption is financed by proportional taxes on the income from capital and labor in the goods production sector. Let θ_t and τ_t again denote the tax rates on the income from capital and labor. The consumer's budget constraint is

$$(57) \quad \sum_{t=0}^{\infty} p_t (c_t + k_{t+1}) = \sum_{t=0}^{\infty} p_t [(1-\tau_t) w_t h_t l_{1t} + R_{kt} k_t]$$

where

$$(58) \quad R_{kt} = 1 + (1-\theta_t)(r_t - \delta)$$

is the gross return on capital after taxes and depreciation and r_t and w_t are, again, the before-tax returns on capital and labor. Note that human capital accumulation is a nonmarket activity.

The consumer's problem in this economy is to choose sequences of consumption, labor, and physical and human capital to maximize utility subject to (55) and (57). The firms maximize these profits:

$$(59) \quad F_k(k_t, h_t l_{1t}) - r_t k_t - w_t h_t l_{1t}$$

The government's budget constraint is

$$(60) \quad \sum_{t=0}^{\infty} p_t g = \sum_{t=0}^{\infty} p_t [\tau_t w_t h_t l_{1t} + \theta_t (r_t - \delta) k_t]$$

Along a balanced growth path for this economy, l_1 and l_2 are constant, and consumption, output, and both types of capital all grow at rate $G(l_2) + 1 - \delta$.

To develop the implementability constraints on the Ramsey problem for this economy, we use the consumer's and firm's first-order conditions to substitute out for prices, policies, and Lagrange multipliers. We obtain the following two constraints:

$$(61) \quad \sum_{t=0}^{\infty} \beta^t U_{c_t} c_t = A_0$$

where

$$(62) \quad A_0 = U_{c_0} [1 + (1-\theta_0)(F_{k_0} - \delta)] k_0 - U_{l_0} (l_{10} + \{[1 - \delta_h + G(l_{20})]/G'(l_{20})\})$$

and

$$(63) \quad U_{l_t} / h_t G'(l_{2t}) = \{[\beta U_{l_{t+1}} / h_{t+1} G'(l_{2t+1})][1 - \delta_h + G(l_{2t+1})]\} + (\beta U_{l_{t+1}} l_{1t+1} / h_{t+1})$$

The first of these constraints (61) is the consumer's budget constraint, and the second (63) is the first-order condition governing the consumer's human capital accumulation. Constraint (63) is required because human capital accumulation occurs outside the market and cannot be taxed. Thus, in any competitive equilibrium, the Euler equation for human capital accumulation is undistorted. Therefore, no tax instrument can be used to make the Euler equation for human capital accumulation hold for arbitrary allocations. In contrast, for arbitrary allocations, the Euler equation for physical capital can be made to hold by choosing the tax on capital income appropriately. This incompleteness of the tax system implies that the undistorted Euler equation for human capital accumulation is a constraint on the set of competitive allocations.

The economy's implementability constraints (61) and (63) together with its resource constraints (54) and (55) characterize competitive equilibrium allocations. The corresponding Ramsey problem for this economy is to maximize utility (56) subject to these constraints.

We prove that along a balanced growth path, the first-order conditions for the Ramsey problem are the same as those for a government which has access to lump-sum taxes. (This, of course, does not mean that the government can achieve the lump-sum tax allocation; there are distortions along the equilibrium path.) Let

$$(64) \quad W(c_t, l_{1t} + l_{2t}; \lambda) = U(c_t, l_{1t} + l_{2t}) + \lambda c_t U_{c_t}$$

where λ is the Lagrange multiplier on (61). For our specified utility function,

$$(65) \quad W(c_t, l_{1t} + l_{2t}; \lambda) = [1 + \lambda(1-\sigma)] U(c_t, l_{1t} + l_{2t})$$

The Ramsey problem, then, is to maximize

$$(66) \quad \sum \beta^t W(c_t, l_{1t} + l_{2t}; \lambda) - \lambda A_0$$

subject to (54), (55), and (63).

Consider a relaxed problem in which we drop (63). Since in this rewritten problem the objective function from period 1 onward is proportional to that of a government which has access to lump-sum taxes, the solutions to the two problems are the same along a balanced growth path. Along such a path, this solution also satisfies (63). Thus, along a balanced growth path, the Ramsey problem has the same solution as the lump-sum tax problem. However, the solutions to these last two problems differ along the transition paths. In sum:

PROPOSITION 5. *In our endogenous growth model, if the Ramsey allocation converges to a balanced growth path, then along such a path, all taxes are zero.*

(Jones, Manuelli, and Rossi (1997) prove a similar result for a more general economy.)

One might be concerned that this result depends on the ratio of government consumption to output going to zero. Concern about that is not warranted. Consider an extension of the model described above, one with an environment in which the government chooses the path of government consumption optimally. Suppose that the period utility function is given by $U(c_1, l_1 + l_2) + V(g)$, where V is some increasing function of government consumption. The government problem in this setup is to choose both tax rates and government consumption to maximize the consumer's utility.

We can solve this problem in two parts. In the first part, government consumption is taken as exogenous and tax rates are chosen optimally. In the second part, government consumption is chosen optimally. The proof described above obviously goes through for extensions of this kind. For

$$(67) \quad V(g) = ag^{1-\sigma}/(1-\sigma)$$

it is easy to show that along a balanced growth path, government consumption is a constant fraction of output.

An Open Economy

Now we consider the optimal capital income tax in a small open-economy model. In so doing, we abstract from the strategic issues that arise when more than one authority sets taxes and from the general equilibrium linkages between an economy's fiscal policy and world prices. We determine that Chamley's zero capital income tax result holds even in an open economy.

When an economy is open, besides taxing its citizens, a government can tax foreign owners of factors that are located in its country. To allow this possibility in our model, we allow the government to use two types of taxes. *Source-based taxes* are taxes that governments levy on income generated in their country at the income's source, regardless of the income's ownership. *Residence-based taxes* are taxes that governments levy on the income of their country's residents regardless of the income's source. We show that the optimal source-based taxes on capital income are zero in all periods and that the optimal residence-based taxes are too, at least when the economy has a steady state. This result is much stronger than the corresponding results for closed economies. (See Razin and Sadka 1995 for some closely related work.)

So, consider an open-economy model with both source-based and residence-based taxation. We model source-based taxes as those levied on a firm and residence-based taxes as those levied on consumers.

Let r_t^* be the world rental rate on capital income when the world has no domestically levied taxes. A firm's problem is to

$$(68) \quad \max F(k_t, l_t) - (1 + \theta_{ft})r_t^*k_t - (1 + \tau_{ft})w_t l_t$$

where θ_{ft} and τ_{ft} are the source-based tax rates on income from capital and labor. The firm first-order conditions are

$$(69) \quad \theta_{ft}r_t^* = F_{kt} - r_t^*$$

$$(70) \quad \tau_{ft}w_t = F_{lt} - w_t$$

Consumers solve this problem:

$$(71) \quad \max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to

$$(72) \quad \sum_{t=0}^{\infty} p_t c_t = \sum_{t=0}^{\infty} p_t w_t (1 - \tau_{ct}) l_t$$

where $p_t = \prod_{s=1}^t (1/R_s)$, $R_s = 1 + (1 - \theta_{cs})(r_s^* - \delta)$, $p_0 = 1$, θ_{ct} and τ_{ct} are residence-based taxes on the income from capital and labor, and initial assets are set to zero for convenience. The consumer first-order conditions are

$$(73) \quad -U_{lt}/U_{ct} = w_t(1 - \tau_{ct})$$

$$(74) \quad \beta U_{ct+1}/U_{ct} = 1/R_{t+1}$$

In the closed-economy models we have studied, the competitive equilibrium has consumer budget constraints, a government budget constraint, and a resource constraint. In this small open economy, there is no resource constraint, and the government budget constraint can be replaced by the economywide budget constraint (which is simply the sum of the consumer and government budget constraints):

$$(75) \quad \sum_{t=0}^{\infty} q_t [c_t + g + k_{t+1} - (1-\delta)k_t] = \sum_{t=0}^{\infty} q_t F(k_t, l_t)$$

where $q_t = \prod_{s=1}^t (1/R_s^*)$ and $R_s^* = r_s^* + 1 - \delta$. Notice that the economy as a whole borrows and lends at the before-tax rate R_s^* , while consumers borrow and lend at the after-tax rate R_s . In this economy, any taxes on borrowing or lending levied on consumers are receipts of the government and cancel out in the combined budget constraint.

To derive the constraints for the Ramsey problem in an open economy with both types of taxes available, first substitute the consumer first-order conditions into (72) to get the implementability constraint:

$$(76) \quad \sum_{t=0}^{\infty} \beta^t (U_{ct} c_t + U_{lt} l_t) = 0$$

where we have used the fact that (74) implies that $p_t = \beta^t U_{ct} / U_{c0}$. Next notice that the first-order conditions of the firm and the consumer can be summarized by (69), (74), and

$$(77) \quad -U_{lt} / U_{ct} = F_{lt} (1 - \tau_{ct}) / (1 + \tau_{ft})$$

Thus, for each marginal condition, there is at least one tax rate, and the Ramsey problem has no additional constraints. With both source- and residence-based taxes available, therefore, the Ramsey problem is to maximize (71) subject to (75) and (76).

With either purely source-based taxation or purely residence-based taxation, the Ramsey problem does have additional constraints. With purely source-based taxation, $\tau_{ct} = \theta_{ct} = 0$ for all t , so $R_t = R_t^*$ for all t . For such a tax system, therefore, (74) implies that the Ramsey problem has this additional constraint:

$$(78) \quad \beta U_{ct+1} / U_{ct} = 1/R_{t+1}^*$$

With purely residence-based taxation, $\tau_{ft} = \theta_{ft} = 0$, so

(69) implies that the Ramsey problem has this additional constraint:

$$(79) \quad F_{kt} = r_t^*$$

With both source- and residence-based taxes available, the Ramsey problem can be written as

$$(80) \quad \max \sum_{t=0}^{\infty} \beta^t W(c_t, l_t, \lambda)$$

subject to (75). Here

$$(81) \quad W(c_t, l_t, \lambda) = U(c_t, l_t) + \lambda (U_{ct} c_t + U_{lt} l_t)$$

The first-order condition for capital then implies that

$$(82) \quad F_{kt} = r_t^*$$

while the first-order condition for consumption implies that

$$(83) \quad \beta W_{ct+1} / W_{ct} = 1/R_{t+1}^*$$

Condition (82) implies that setting $\theta_{ft} = 0$ for all t is optimal. We know that this small economy will have a steady state only if

$$(84) \quad \beta R_t^* = 1$$

for all t . Under this parameter restriction, (83) implies that $W_{ct} = W_{ct+1}$; thus, the Ramsey allocations are constant. In particular, $U_{ct} = U_{ct+1}$. Hence, equations (74) and (84) imply that $\theta_{ct} = 0$ for all t .

Under a system with only source-based taxes, the Ramsey problem is to maximize (80) subject to (75) and (83). For a relaxed version of this problem, with constraint (83) dropped, the above analysis makes clear that the solution satisfies the dropped constraint and hence solves the original problem. The first-order condition for capital then implies (82); hence, $\theta_{ft} = 0$ for all t .

Similarly, under a system with only residence-based taxes, the Ramsey problem is to maximize (80) subject to (75) and (79). For a relaxed version of this problem, with constraint (79) dropped, the above analysis makes clear that the solution satisfies the dropped constraint and hence solves the original problem. The first-order condition for consumption in the relaxed problem is (83). Under the parameter restriction (84), $W_{ct} = W_{ct+1}$,

so $U_{c_t} = U_{c_{t+1}}$. Hence, equations (74) and (84) imply that $\theta_{c_t} = 0$ for all t .

In sum:

PROPOSITION 6. *In our open-economy model, either under a system with both source- and residence-based taxes or under a system with only source-based taxes, $\theta_{f_t} = 0$ for all t . Also, in this model, with the additional restriction (84), either under a system with both source- and residence-based taxes or under a system with only residence-based taxes, $\theta_{c_t} = 0$ for all t .*

Notice that the Ramsey allocations from the problem with both source- and residence-based taxes can be achieved with residence-based taxes alone. With the additional restriction (84), these allocations can also be achieved with source-based taxes alone. The intuition for why optimal source-based taxes are zero is that with capital mobility, each government faces a perfectly elastic supply of capital as a factor input and therefore optimally chooses to set capital income taxes on firms to zero. The intuition for why optimal residence-based taxes are zero is that under restriction (84), the small economy instantly jumps to a steady state, so the Chamley-type logic applies for all t .

Overlapping Generations

Finally, we consider optimal capital income taxes in a closed economy with overlapping generations rather than infinitely lived agents.⁴ We show that in this type of economy, tax rates on capital income in a steady state are optimally zero if certain homotheticity and separability conditions are satisfied. This result has been independently derived by Atkeson, Chari, and Kehoe (1999) and Garriga (1999).

We briefly formulate optimal fiscal policy in an overlapping-generations model. Consider a two-period overlapping-generations model with a constant population normalized to 1. The resource constraint for this economy is

$$(85) \quad c_{1t} + c_{2t} + k_{t+1} + g = F(k_t, l_{1t}, l_{2t}) + (1-\delta)k_t$$

where c_{1t} and c_{2t} denote the consumption of a representative young agent and a representative old agent in period t , l_{1t} and l_{2t} denote the corresponding labor inputs, k_t denotes the capital stock in t , δ denotes the depreciation rate on capital, and g denotes government consumption. Each young agent in t solves the problem to

$$(86) \quad \max U(c_{1t}, l_{1t}) + \beta U(c_{2t+1}, l_{2t+1})$$

subject to

$$(87) \quad c_{1t} + k_{t+1} + b_{t+1} = (1-\tau_{1t})w_{1t}l_{1t}$$

$$(88) \quad c_{2t+1} = (1-\tau_{2t+1})w_{2t+1}l_{2t+1} + [1 + (1-\theta_{t+1})(r_{t+1} - \delta)]k_{t+1} + R_{t+1}b_{t+1}$$

where τ_{1t} and τ_{2t} are the tax rates on the two types of labor inputs, θ_t is the tax rate on capital income, b_{t+1} is the government debt held by the young generation at t , and R_t again is the return on capital. The government budget constraint in this economy is

$$(89) \quad \tau_{1t}w_{1t}l_{1t} + \tau_{2t}w_{2t}l_{2t} + \theta_t r_t k_t + b_{t+1} = g + R_t b_t$$

To define an optimal policy here, we must assign weights to the utility of agents in each generation. We assume that the government assigns weight λ^t to generation t with $\lambda < 1$. Then the Ramsey problem can be written as

$$(90) \quad \max [U(c_{20}, l_{20})/\lambda] + \sum_{t=0}^{\infty} \lambda^t [U(c_{1t}, l_{1t}) + \beta U(c_{2t+1}, l_{2t+1})]$$

subject to the resource constraint for each t and the implementability constraint

$$(91) \quad R(c_{1t}, l_{1t}) + \beta R(c_{2t+1}, l_{2t+1}) = 0$$

for each t , where

$$(92) \quad R(c, l) \equiv cU_c(c, l) + IU_l(c, l)$$

and $U(c_{20}, l_{20})$ is the utility of the initial old. Constraint (91) is the implementability constraint associated with each generation except the initial old. (The implementability constraint for the initial old plays no role in our steady-state analysis.) It is straightforward to show that

⁴The literature on optimal policy in overlapping-generations models includes, for example, Atkinson 1971, Diamond 1973, Pestieau 1974, and Atkinson and Sandmo 1980; the surveys Auerbach 1985 and Stiglitz 1987; and the applied works Auerbach and Kotlikoff 1987 and Escolano 1992. Of course, as Barro 1974 demonstrates, if bequests are allowed, then the overlapping-generations model is equivalent to a model with infinitely lived agents, and our earlier analysis applies.

if the solution to the Ramsey problem converges to a steady state with constant allocations

$$(93) \quad (c_{1t}, l_{1t}, c_{2t+1}, l_{2t+1}, k_{t+1}) = (c_1, l_1, c_2, l_2, k)$$

then the Ramsey allocations satisfy

$$(94) \quad \lambda^{-1} = F_k + 1 - \delta.$$

In a steady state in this economy, the first-order condition for capital accumulation is

$$(95) \quad U_{c_1}(c_1, l_1) / \beta U_{c_2}(c_2, l_2) = 1 + (1 - \theta)(F_k - \delta).$$

These equations imply that unless

$$(96) \quad \lambda^{-1} = U_{c_1}(c_1, l_1) / \beta U_{c_2}(c_2, l_2)$$

the tax rate on capital income is not zero in this economy. In general, we would not expect condition (96) to hold. Notice the contrast with infinitely lived representative-consumer models in which, in a steady state, the marginal utility of the representative consumer $U_c(c, l)$ is constant. In an overlapping-generations model, we would not expect the marginal utility of a consumer to be constant over the consumer's lifetime.

In our overlapping-generations model, the first-order conditions for consumption in the Ramsey problem, evaluated at the steady-state allocations, are

$$(97) \quad U_{c_1} + \alpha_t R_{c_1} = \mu_t$$

$$(98) \quad \beta(U_{c_2} + \alpha_t R_{c_2}) = \lambda \mu_t$$

where $\lambda \alpha_t$ is the Lagrange multiplier on the implementability constraint (91) for the generation born in period t and $\lambda \mu_t$ is the Lagrange multiplier on the resource constraint (85) in period t . With a utility function of the form (28), R_c is proportional to U_c , so that (97) and (98) imply (96). In sum:

PROPOSITION 7. *In our overlapping-generations economy, if the utility function is of the form (28), then in a steady state, the optimal tax on capital income is zero.*

When $\lambda = \beta$ and $F(k, l_1, l_2) = F(k, l_1 + l_2)$, we can show that for all strictly concave utility functions, the optimal tax on capital income is zero in a steady state. (See Atkeson, Chari, and Kehoe 1999.)

Theory vs. Practice

By formally describing and extending Chamley's (1986) result, we have demonstrated how the primal approach can be used to answer a fundamental question in public finance: What is the optimal capital income tax? This approach has produced a substantive lesson for policy-makers: In the long run, in a broad class of environments, the optimal tax on capital income is zero. With further restrictions on our model, we have shown that this result applies to the short run as well. Theoretically, that is, our result concurs with that of Chamley (1986): taxing capital income is a bad idea.

We think that this result should be applied in the real world, and we see signs that some U.S. policymakers agree. Currently, of course, U.S. capital income tax rates are far from zero. That is understandable, since until relatively recently, the dominant economic theory supported positive taxes on capital income; policymakers were relying on what has become outdated theory. Recently, however, practice seems to have shifted toward the new theory's result. During the Reagan administration, tax rates on dividends and capital gains began to be lowered and tax exemptions for retirement savings expanded. Recently, too, influential proponents of the supply-side view, like Boskin (1978), Feldstein (1978), Lucas (1990), and Hall and Rabushka (1995), have advocated lowering capital income taxes still further. Hall and Rabushka (1995) have laid out a detailed proposal on how to implement zero capital income taxation.⁵

Some researchers might disagree with this movement. They might argue that the new theory is just too simple to be applicable in the real world. The results of our theory require what might seem to be unrealistic assumptions, especially full commitment of the government to keep to its announced tax policy and perfect markets. Without such assumptions, the doubters might say, this theory does not work.

They're right, and they're wrong. The assumptions are necessary, to some extent, for the optimal capital income tax to be zero. But the assumptions are not necessarily unrealistic barriers that should block the theory's practical application.

⁵In addition to eliminating capital income taxation, Hall and Rabushka's proposal reduces the progressivity of the tax system. Our theory is silent on the optimal progressivity of the tax system. Their proposal can be easily adapted to yield any desired degree of progressivity.

The highest perceived barrier is the difficulty in ensuring that the government keep its promises. If the government cannot commit to follow some prespecified policies, then implementing the solution to the Ramsey problem can be difficult. In any period, the government has an incentive to renege on its past promises, tax the income from existing capital highly, and promise that future capital income will not be taxed. Kydland and Prescott (1977) have shown that this tension could lead to high capital income taxes in every period.

This barrier may be surmountable by one of two means. First, as Chari and Kehoe (1990) have shown, a desire to maintain a good reputation may give the government an incentive to keep its promises, at least if the government is sufficiently patient. Second, if problems with commitment are the reasons for high capital income tax rates, the appropriate policy is to use available constitutional and legal methods to commit to low rates. At an extreme, if the U.S. legal system can guarantee free speech, why can't it guarantee that the government keep its promises on tax policy?

Another perceived barrier is that in the real world, private markets are not perfect, while in our theory, they are. Doubters might argue that if we incorporate into the theory imperfections, like externalities or missing markets, and still allow only income tax policies, then the optimal capital income tax rate may not be zero. For example, Aiyagari (1995) has argued that if the only instrument available to the government is income tax policies, then positive capital income tax rates are desirable because they partially offset the distortions from missing markets. Intuitively, Aiyagari's argument relies on trying to get one policy instrument to achieve two conflicting goals: minimize tax distortions and partially replace the missing markets.

We think this argument is weak. If there are imperfections in markets, the appropriate policy is to use some direct policy instrument to deal with them. For example, the appropriate direct policy in Aiyagari's model is for the government either to provide insurance or, even better, to remove the unmodeled impediments to the private provision of insurance. Once these direct means are used to deal with the market imperfections, tax policy can be left to do what it should be doing: minimizing tax distortions by not taxing capital income.

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