TIME CONSISTENCY AND POLICY*

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ABSTRACT

In this paper we review the implications of the time consistency re-
quirement for economic policy. Particular attention is devoted to the
sustainable plan construct. Allocations and policies are defined as
functions of the history of past policies. A sequence of history-
contingent allocation and policy functions is sustainable if it satis-
fies certain sequential rationality conditions. We illustrate these
ideas in a capital taxation model and in a model of default on govern-
ment debt.

The design of fiscal and monetary policy is a central concern in aggregative economics. A useful framework for thinking about optimal policy design is provided by the public finance tradition stemming from Ramsey (1927). Ramsey studied a static, representative consumer economy with many goods. A government requires fixed amounts of these goods, which are purchased at market prices and financed by proportional excise taxes. Given the excise taxes, prices and quantities are determined in a competitive equilibrium. The government's problem is to choose tax rates to maximize the welfare of the representative consumer. It is straightforward to extend Ramsey's formulation to study fiscal policy in dynamic models with uncertainty by reinterpreting the goods in the static problem as state-contingent commodities. In this context a policy for government is a rule specifying state-contingent tax rates. Given a policy, competitive equilibrium prices and allocations are defined as functions of the state of the economy. The design problem is to choose a policy that maximizes a social welfare function defined over the resulting competitive allocations. An optimal policy together with the resulting competitive equilibrium is called a Ramsey equilibrium.

In a Ramsey equilibrium, consumers make decisions once and for all at the beginning of time. But this equilibrium can also be interpreted as a one-time choice of government policy with consumers making decisions sequentially, given the policy. The Ramsey policies then solve the design problem in environments where societies have access to a commitment technology to bind the actions of future governments. In many situations, however, it is

more appropriate to think of policies as being chosen at each date, with society having no ability to commit to future policies. In such environments, one is tempted to conclude that the resulting policy choices coincide with the Ramsey policies. This is not the case. Consider the policy choice problem at some date t, assuming that policies and allocations coincide with the Ramsey equilibrium until that date. The solution to the new policy choice problem typically does not coincide with the Ramsey policies from that date onward. Kydland and Prescott (1977), Prescott (1977), Calvo (1978), and Fischer (1980) have shown that this dynamic inconsistency of the Ramsey policies is pervasive in models of fiscal and monetary policy. This dynamic inconsistency means that the Ramsey policies are simply irrelevant in a world without commitment. Clearly, rational individuals will not base their decisions on the Ramsey policies if a different set of policies will be chosen in the future.

A solution to the design problem without commitment must therefore require that policies be _sequentially rational_. That is, the policy rules must maximize the social welfare function at each date given that private agents behave optimally. Likewise, optimality on the part of private agents requires that they forecast future policies as being sequentially rational for society. A sequence of policy rules, allocations, and prices satisfying these conditions is a _time-consistent equilibrium._ We say there is a time consistency problem if the Ramsey equilibria differ from the time-consistent equilibria. It is worth remarking that time consistency problems can arise in individual decision problems

when preferences change over time (see Strotz 1955). We therefore restrict our focus to situations where preferences are time-consistent in Strotz's sense. The source of time consistency problems cannot then lie in such preferences.

In Section 1, we argue that the source of time consistency problems lies in conflict among agents. We show that in a team environment where all agents share a common objective function, there can be no time consistency problems. Most models of fiscal and monetary policy use a representative agent construct and a social welfare function that coincides with the representative agent's utility function. The representative agent formulation should not mislead us into thinking that the individuals in this economy form a team. The objectives of individuals do not coincide, because in such models individuals care only about their own consumption. Consequently, even in representative agent models there are generally time consistency problems.

We explore the precise nature of the time consistency problem by using two classic illustrations of the problem: capital taxation and default on government debt. In Section 2 we analyze a variant of Fischer's (1980) model of capital taxation; and in Section 3 we analyze Chari and Kehoe's (1987a) debt-default model. We define and characterize time-consistent equilibria for each of these examples.

Our formulation of time-consistent equilibria (based on Chari and Kehoe 1987a,b) allows allocations and policies to depend on the entire history of past decisions by governments as well as on past aggregate (or per capita) allocations. Thus, policies and

allocations are defined as <u>history-contingent</u> functions. This break from the general equilibrium tradition of considering equilibria that are state-contingent functions is essential in imposing the requirement of sequential rationality. Both governments and consumers must forecast how current decisions affect future outcomes. Allowing for history-contingent functions solves this forecasting problem.

For finite-horizon models, sequential rationality implies that this problem is solved by backward induction. For infinite-horizon models such a procedure is no longer available. Indeed, for infinite-horizon models there is typically a large set of time-consistent equilibria that are quite difficult to characterize. It turns out, however, that the set of allocations realized in time-consistent equilibria is fairly easy to characterize. In Section 2 we provide a simple set of inequalities that can be used to characterize such allocations for the capital taxation model. We show that with sufficiently little discounting, even the Ramsey allocations can be supported by some time-consistent equilibrium.

The policy plans and allocations rules used to support the large set of time-consistent allocations for the capital taxation model are closely related to "trigger" strategies of repeated games. (See, for example, Friedman 1971.) Loosely speaking, for any given pair of policy and allocation sequences, the history-contingent policy and allocation rules used to support them specify continuation with these sequences as long as there has been no deviation. If there has been a deviation, the rules

specify reversion to the single-period time-consistent equilibrium forever. Even though such rules resemble the trigger strategies of repeated games, it is important to point out that in our models, private agents behave competitively--not strategically. In particular, private agents do not collude to "punish" the government. Rather, after a deviation, private agents choose the single-period time-consistent allocations because they forecast that the government will choose the single-period time-consistent policy. The government, in turn, takes the aggregate allocation rule for private agents as given and optimally chooses this policy. Because of this feature, some of our results differ from the related results in repeated games with only "large" players.

In Section 3 we extend the debt model of Lucas and Stokey (1983) to allow for default.[1] We model default by the government as a tax on debt. We consider a finite-horizon version of the model. As might be expected, the Ramsey allocations are not, in general, outcomes of a sustainable equilibrium, because the debt issues associated with a Ramsey allocation are positive at some dates. When the inherited debt is positive, the government has an incentive to default. Recognizing this, private agents will not buy such debt in previous periods. This result does not imply, however, that a sustainable equilibrium must have a continuously balanced budget. The government can smooth tax distortions over time by issuing negative debt, that is, by purchasing claims on private agents. Using a backward induction argument, Chari and Kehoe (1987a) show that time-consistent allocations solve a programming problem called the <u>constrained Ramsey</u>

<u>problem</u>. The time-consistent allocations maximize the welfare of the representative consumer at date zero, subject to the budget constraint and a sequence of constraints that require that the present value of the government's surplus be nonpositive at all future dates.

We analyze the time-consistent equilibria in a series of examples. As we show, the Ramsey allocations are typically not time-consistent, and hence there is a value to having a commitment technology. Under plausible assumptions, this value, as measured by the normalized difference between utility in a sustainable equilibrium and the Ramsey utility, can be made arbitrarily small by making the horizon sufficiently long and the discount factor sufficiently close to unity. Note that this result holds in a finite-horizon model and thus does not rely upon "trigger" strategies. Rather, the result holds because the ability to issue negative debt allows for almost as much tax smoothing as occurs in a Ramsey equilibrium.[2]

1. An Overview of the Time Consistency Problem

In this section we formulate policy design as a simple social choice problem and use this framework to provide an overview of the time consistency problem. We compare the equilibria of an environment with commitment to those of an environment without commitment. We formalize commitment as a particular timing scheme for decision making. Society first chooses a policy once and for all, and then private agents choose their actions. In the environment without commitment, decisions are made sequentially. Such a timing scheme in a multiperiod economy occurs when

private agents first choose their first-period actions, then the government chooses its first-period policy, then private agents choose their second-period actions, and so on. For ease of exposition, we consider a one-period economy. In this case the two timing schemes are particularly simple. With commitment, the government first sets policy and then private agents make their decisions. Without commitment, private agents first make their decisions and then the government sets policy. It will be clear that all of the results extend to multiperiod economies.

Throughout this section, we consider special cases of the following environment. There is a society consisting of n private agents. Each agent i (i = 1,...,n) chooses an action x_i from a set of actions X_i. The vector of actions $x = (x_1,...,x_n)$ is called an <u>allocation</u>. Society chooses a policy π from a set of policies Π. The preferences of each private agent are given by a utility function $U^i(x,\pi)$, and society's preferences are given by $S(x,\pi)$. Initially we model allocations as the outcome of a Nash equilibrium and later as the outcome of a competitive equilibrium.

It turns out that the preferences of private agents and society together play a critical role in determining whether the allocations with and without commitment coincide--that is, in determining whether there is a time consistency problem. We first show that if all agents' preferences coincide with those of society, then there can be no time consistency problem. We then give necessary conditions for there to be a time consistency problem, and we illustrate these conditions in a simple model of inflation and unemployment.

1.1 Team Environments

A _team_ is defined as a group of individuals who share a common objective. We show that in a team environment there can be no time consistency problem. Specifically, suppose that each agent's preferences coincide with those of society. Let the utility function of each agent i over the vector x and the policy π be given by some strictly concave, twice-differentiable function $U(x,\pi)$. Let this function also be the social objective function. Notice that together the private agents and society form a team. Even though team members control only their own actions, they all choose these actions to achieve a common goal.

Under commitment, society chooses a policy π; and then, given this policy, private agents simultaneously choose their actions. First consider the choice of private agents, given some policy π. Each agent i, faced with a policy π and taking as given the decisions $x_{-i} = (x_1,\ldots,x_{i-1},x_{i+1},\ldots,x_n)$ of all other agents, solves

$$(1.1) \qquad \max_{x_i} U(x,\pi).$$

If we assume an interior solution, the first-order condition is

$$(1.2) \qquad \frac{\partial U}{\partial x_i} = 0.$$

For a given policy π, an equilibrium for private agents is a vector x such that for each i, x_i solves (1.1), given x_{-i}. For any such policy π, denote the resulting equilibrium allocations by $X(\pi)$. For simplicity, assume that for each π there is a unique equilibrium and that the resulting function $X(\cdot)$ is differentiable. This function $X(\cdot)$ is called the _outcome function_.

Society's problem, then, is to choose a policy π to maximize its objective function taking the outcome function $X(\cdot)$ as given. That is, society solves

(1.3) $\max_{\pi} U\big(X(\pi),\pi\big).$

If we assume an interior solution, the first-order condition for society is

(1.4) $\sum_{i=1}^{n} \frac{\partial U}{\partial x_i} \frac{\partial X_i}{\partial \pi} + \frac{\partial U}{\partial \pi} = 0.$

We can now define an equilibrium with commitment.

Definition. An equilibrium with commitment is a policy π^* and an outcome function $X(\cdot)$ that satisfy

(i)   Maximization for society. Given $X(\cdot)$, the policy π^* solves society's problem (1.3).

(ii)   Private equilibrium. For each π, the outcome $X(\pi)$ is an equilibrium for private agents.

Notice that the private decisions actually taken in such an equilibrium are given by $x^* = X(\pi^*)$.

Without a commitment technology, the equilibrium is somewhat different. In particular, private agents first choose a vector x and then society chooses a policy π. Given some allocation x, the problem faced by society is

(1.5) $\max_{\pi} U(x,\pi).$

The first-order condition for society is

$$(1.6) \qquad \frac{\partial U}{\partial \pi} = 0.$$

Assume that for each vector x, the policy π defined by (1.6) is unique and that the resulting policy rule $\pi(\cdot)$ is differentiable. In this equilibrium each private agent i takes the policy rule $\pi(\cdot)$ and the decisions of other private agents x_{-i} as given and solves

$$(1.7) \qquad \max_{x_i} U\big(x, \pi(x)\big).$$

The first-order condition for each agent i is

$$(1.8) \qquad \frac{\partial U}{\partial x_i} + \frac{\partial U}{\partial \pi} \frac{\partial \pi}{\partial x_i} = 0.$$

(Notice that we let each private agent take account of the effect his action has on the policy chosen by society. When the number of private agents is large, this effect will be small; in the limit, it will be zero.) We can now define an equilibrium without commitment.

Definition. An equilibrium without commitment is a vector of private decisions x* and a policy rule $\pi(\cdot)$ that satisfy

(i) Private equilibrium. Given x^*_{-i} and $\pi(\cdot)$, x^*_i solves (1.7).

(ii) Maximization for society. For any x, the policy $\pi(x)$ solves (1.5).

We can now compare the equilibrium outcomes with and without commitment. Combining (1.2) and (1.4), we have that the equilibrium outcome with commitment is completely characterized by

(1.9) $\frac{\partial U}{\partial \pi} = 0$ and $\frac{\partial U}{\partial x_i} = 0$, for all i.

Combining (1.6) and (1.8), we have that the equilibrium outcome without commitment is characterized by

(1.10) $\frac{\partial U}{\partial x_i} = 0$, for all i, and $\frac{\partial U}{\partial \pi} = 0$.

Notice that (1.9) and (1.10) are identical and that either set of equations are the first-order conditions to

$$\max_{x, \pi} U(x, \pi).$$

By strict concavity, the solution to this problem is unique, and hence we have established the following proposition.

Proposition 1. No Time Consistency Problem in a Team Environment

If all agents have the same objective function as society has, the equilibrium allocations and policies with and without commitment are identical.

Thus for there to be a time consistency problem, there needs to be some conflict of interests either between society and private agents or among private agents themselves.

1.2 Benevolent Agents and a Self-Interested Society

A variety of papers in the literature have examined situations in which the preferences of society do not coincide

with those of private agents. In our framework we model such a situation by letting each private agent's objective function be $U(x,\pi)$ and letting society's objective function be some other function, say $S(x,\pi)$. For this specification, an equilibrium with commitment is summarized by

$$\frac{\partial U}{\partial x_i} = 0, \text{ for all } i$$

and

$$\sum_{i=1}^{n} \frac{\partial S}{\partial x_i} \frac{\partial X_i}{\partial \pi} + \frac{\partial S}{\partial \pi} = 0.$$

Likewise, an equilibrium without commitment is summarized by

$$\frac{\partial S}{\partial \pi} = 0$$

and

$$\frac{\partial U}{\partial x_i} + \frac{\partial U}{\partial \pi} \frac{\partial \Pi}{\partial x_i} = 0, \text{ for all } i.$$

It is clear that, in general, the solutions to these two sets of equations will be different.

One justification for assuming that the preferences of society do not coincide with those of its constituent agents is that policy choices are made by a self-interested government. As we have seen, the discrepancy in objectives between such a government and the members of society induces a conflict of interests that can lead to a time consistency problem. It is not clear to us why the preferences of society do not reflect the preferences of its constituents. Thus the time consistency problems just described do not seem an interesting way to model social choice in democratic societies.

1.3 Self-Interested Agents and a Benevolent Society

Consider an environment in which the preferences of each private agent can differ. In particular, let the preferences of agent i be given by $U^i(x,\pi)$. Notice that since each agent's utility is affected by other agents' decisions, this economy has external effects. We model society as being benevolent in the sense that it solves a Pareto problem by maximizing

$$\sum_{i=1}^{n} \lambda_i U^i(x,\pi)$$

for some set of welfare weights $\lambda = (\lambda_1,\ldots,\lambda_n)$.

With commitment, the first-order condition for each agent is

$$\frac{\partial U^i}{\partial x_i} = 0$$

and the first-order condition for society is

$$\sum_{i=1}^{n} \lambda_i \left[\sum_{j=1}^{n} \frac{\partial U^i}{\partial x_j} \frac{\partial X_j}{\partial \pi} + \frac{\partial U^i}{\partial \pi} \right] = 0.$$

Likewise, without commitment the first-order condition for society is

$$\sum_{i=1}^{n} \lambda_i \frac{\partial U^i}{\partial \pi} = 0$$

and the first-order condition for an agent i is

$$\frac{\partial U^i}{\partial x_i} + \frac{\partial U^i}{\partial \pi} \frac{\partial \Pi}{\partial x_i} = 0.$$

Notice that, in general, these two solutions will differ; consequently, when there are externalities, typically there is a time consistency problem.

1.4 Representative Agent Models

A particularly interesting class of social choice problems arises in competitive equilibrium models with a representative agent. It turns out that such problems can be represented in our general social choice framework. For this class of problems, the objective functions of agents are

$$U^i(x,\pi) = U(x_i,\bar{x},\pi)$$

where $\bar{x} = \sum_{i=1}^{n} x_i/n$ denotes the aggregate per capita allocations. The objective function of society is

$$S(x,\pi) = \sum_{i=1}^{n} U(x_i,\bar{x},\pi).$$

Modeling private agents as being competitive amounts to assuming that each agent i takes both the aggregate allocation $\bar{x}$ and the policy π as given. Furthermore, the policy rule Π used by society when there is no commitment is constrained to depend only on the aggregate allocation $\bar{x}$. In the case with commitment, the first-order conditions reduce to

$$\frac{\partial U^i}{\partial x_i} = 0, \text{ for all } i$$

and

$$\sum_{i=1}^{n} \frac{\partial U^i}{\partial \bar{x}} \frac{\partial \bar{x}}{\partial \pi} + \frac{\partial U^i}{\partial \pi} = 0.$$

In the case without commitment, the first-order conditions are

$$\sum_{i=1}^{n} \frac{\partial U^i}{\partial \pi} = 0$$

and

$$\frac{\partial U^i}{\partial x_i} = 0, \text{ for all } i.$$

It is clear that, in general, the solutions to these problems are different. Notice that if the utility functions did not depend on the aggregate allocation $\bar{x}$, then the solutions to these problems would be the same. Dependence of the utility functions on aggregate allocations induces a subtle source of conflict among agents. The nature of this conflict can be illustrated in a simple model of inflation. More complicated examples will be considered in Sections 2 and 3.

1.5 Conflict Among Agents in a Simple Model of Inflation

Perhaps the most widely used example in the time consistency literature is a Phillips curve model of inflation and unemployment. Kydland and Prescott (1977) first used this model to illustrate the problem of time consistency. Barro and Gordon (1983) and Rogoff (1987) have elaborated on the basic model. The idea is that unanticipated inflation provides benefits to society, whereas anticipated inflation is costly. Within our social choice framework, we can model these features as follows.

Each private agent chooses the (log of) his nominal wage x_i. Society, which here is identified with the monetary authority, chooses the (log of) the price level π. The aggregate $\bar{x} = \sum_{i=1}^{n} x_i/n$ is the average nominal wage in the economy. The utility function of private agents is given by

$$U^i(x,\pi) = U(x_i - \pi, \bar{x} - \pi, \pi)$$

while the utility function of the monetary authorities is given by

$$S(x,\pi) = \sum_{i=1}^{n} U(x_i - \pi, \bar{x} - \pi, \pi).$$

As usual, we consider two kinds of commitment technologies. The first-order conditions with commitment reduce to

$$U_1(x_i - \pi, \bar{x} - \pi, \pi) = 0, \text{ for all } i$$

and

(1.11) $$\sum_{i=1}^{n} \left[U_2(x_i - \pi, \bar{x} - \pi, \pi)\left(\frac{\partial \bar{X}}{\partial \pi} - 1\right) + U_3(x_i - \pi, \bar{x} - \pi, \pi) \right] = 0.$$

The first-order conditions without commitment reduce to

$$U_1(x_i - \pi, \bar{x} - \pi, \pi) = 0$$

and

(1.12) $$\sum_{i=1}^{n} \left[U_3(x_i - \pi, \bar{x} - \pi, \pi) - U_2(x_i - \pi, \bar{x} - \pi, \pi) \right] = 0.$$

In this type of model it is common to assume that for a fixed price level π and aggregate nominal wage x_i, the utility of agent i is maximized by setting the nominal wage equal to the price level plus a constant k. Thus $U_1(\cdot) = 0$ when the first argument equals k. One rationalization for this is that a real wage higher than the one given by the constant k leads to lower employment and hence lower utility.

Since $U_1(\cdot) = 0$ when the real wage $x_i - \pi$ equals k, then in an equilibrium with commitment, $\partial \bar{X}/\partial \pi = 1$. It follows from equations (1.11) and (1.12) that the solutions to the two problems are different. Notice that it is crucial for society to care

about the second argument $\bar{x} - \pi$, that is, the <u>average</u> real wage in the economy. If the utility functions of private agents or society did not depend upon this argument, then the solutions would be identical. Barro (1985) and Rogoff (1987) recognize the importance of this assumption. They argue that unanticipated inflation provides social benefits if the natural rate of unemployment exceeds the socially optimal level as a result of the presence of externalities or distorting taxation. Since these features are not modeled, it is hard to assess the validity of the argument. Suffice it to say, some such force must be present if this model is to generate a time consistency problem.

1.6 Summary

We have made three main points in this section: First, time consistency problems cannot arise in a team environment. Second, time consistency problems typically arise whenever governments do not maximize the welfare of private agents. Third, even if governments are benevolent, conflicts among private agents can cause time consistency problems. In the sections that follow we consider two examples that investigate how the interactions between external effects and the timing of decisions generate time consistency problems.

2. The Capital Taxation Model

In this section we consider a version of Fischer's (1980) capital taxation model, modified along the lines of Chari and Kehoe (1987b). Initially we consider a one-period version of the model. For this version we define and characterize the equi-

libria with and without commitment. We then show that these results immediately generalize to a finite-period version of the model. In Section 2.4 we discuss the infinite-horizon version of this model.

Consider an economy with a large number of identical consumers and a government. There is a linear production technology for which the marginal product of capital is a constant $R > 1$ and the marginal product of labor is 1. Consumers make decisions at two distinct points in time, the first stage and the second stage. They make consumption-investment decisions at the first stage and consumption-labor supply decisions at the second stage. In particular, at the first stage, consumers are endowed with ω units of the consumption good from which they consume c_1 and save k. At the second stage, they consume c_2 and work ℓ units. Second-stage income, net of taxes, is $(1 - \delta)Rk + (1 - \tau)\ell$, where δ and τ denote the tax rates on capital and labor respectively. For simplicity we assume that first-stage consumption is a perfect substitute for second-stage consumption. A consumer, confronted with tax rates δ and τ, chooses $(c_1,k;c_2,\ell)$ to solve

(2.1) $\max U(c_1+c_2,\ell)$

subject to

$$c_1 + k \leq \omega$$

$$c_2 \leq (1 - \delta)Rk + (1 - \tau)\ell.$$

If the tax rate on capital δ is set so that $(1 - \delta)R = 1$, the consumer is indifferent about the timing of consumption. We assume that in such a case, the consumer saves his entire endowment.

The government sets proportional tax rates on capital and labor income to finance an exogenously given amount of second-stage per capita government spending G. The government's budget constraint is

(2.2) $G \leq \delta RK + \tau L$

where K and L denote the per capita (or aggregate) levels of capital and labor. We assume that $G > R\omega$, so even if consumers save their entire endowments and the tax on capital is set equal to one, the government still needs to tax labor.

In what follows we adopt the notational convention that lowercase letters denote individual variables and uppercase letters denote aggregate variables. This notation is used to emphasize what various agents take as given.

2.1 Capital Taxation With Commitment

In an economy with commitment, the government sets tax rates before private agents make their decisions. In this setup it is straightforward to define an equilibrium. Let $x_1 = (c_1, k)$ and $x_2 = (c_2, \ell)$ denote an individual consumer's first- and second-stage allocations and let $X_1 = (C_1, K)$ and $X_2 = (C_2, L)$ denote the corresponding aggregate allocations. Let $\pi = (\delta, \tau)$ denote government policy. We can now define a competitive equilibrium.

<u>Definition</u>. A competitive equilibrium is an individual allocation (x_1, x_2), an aggregate allocation (X_1, X_2), and a tax policy π that satisfy

(i)   <u>Consumer maximization</u>. Given the tax policy π, the individual allocations solve the consumer's problem (2.1).

(ii)   <u>Government budget constraint</u>. At the aggregate allocation (X_1, X_2), the policy π satisfies the government budget constraint (2.2).

(iii)   <u>Representativeness</u>. The individual and aggregate allocations coincide; that is, $(x_1, x_2) = (X_1, X_2)$.

Given that the individual and aggregate allocations coincide, we can refer to such an equilibrium as a (π, X) pair, where $X = (X_1, X_2)$. Let E denote the set of policies π for which an equilibrium exists. Assume that for each π in E there is a unique equilibrium allocation $X(\pi)$ associated with π. Let $S\big(\pi, X(\pi)\big)$ denote the equilibrium value of utility under the policy π so that

$$S\big(\pi, X(\pi)\big) = U\big(C_1(\pi) + C_2(\pi), L(\pi)\big).$$

We say that a pair (π, X) is a <u>Ramsey equilibrium</u> if π solves

$$\max_{\pi \in E} S\big(\pi, X(\pi)\big)$$

and $X = X(\pi)$. We then have

Proposition 2. The Single-Period Ramsey Equilibrium

The Ramsey equilibrium (π, X) has first-stage allocations $C_1 = 0$ and $K = \omega$ and a capital tax rate $\delta = (R - 1)/R$.

Proof. If the tax on capital is such that $(1 - \delta)R \geq 1$, then consumers save their entire endowments, while if $(1 - \delta)R < 1$, consumers save nothing. Thus the tax on capital acts like a lump-sum tax when it is selected at any level less than or equal to $(R - 1)/R$. Clearly it is optimal to raise as much revenue as possible from this tax. Since $G > R\omega$, government spending is greater than the maximal possible revenues from this capital tax, so it is optimal to set $\delta = (R - 1)/R$. Faced with this tax, consumers save their entire endowments. The tax rate on labor is then set at a level sufficient to raise the rest of the needed revenues. ◊

2.2 Capital Taxation Without Commitment

Formally, the lack of commitment is modeled by assuming that the government does not set policy until after consumers have made their first-stage decisions. The timing is (1) consumers make first-stage decisions, (2) the government sets tax policy, and (3) consumers make second-stage decisions. In this setup the government's tax rates depend on the aggregate first-stage decisions. Thus a government policy is no longer a pair of tax rates $\pi = (\delta, \tau)$ but rather a specification of tax rates for every possible X_1, say, $\sigma(X_1) = \left(\delta(X_1), \tau(X_1)\right)$. To keep the distinction between these clear, we call the function σ a policy plan and we call a specific set of tax rates π simply a policy.

Each consumer's second-stage decisions depend on the first-stage decisions x_1, the aggregate first-stage decisions X_1, and the tax policy selected. Thus, a consumer's second-stage decisions are described by a pair of functions, say $f_2(x_1,X_1,\pi) = [c_2(x_1,X_1,\pi),\ell(x_1,X_1,\pi)]$. We call f_2 a second-stage <u>allocation rule</u> to distinguish it from a particular second-stage <u>allocation</u> x_2. Likewise, the aggregate allocation rule F_2 is defined as a function of the aggregate first-stage decision X_1 and the policy π, and is denoted by $F_2(X_1,\pi)$.

An equilibrium in this environment is defined recursively. First, a second-stage competitive equilibrium is defined, given the history of past decisions by consumers and the government. We then consider symmetric histories (x_1,X_1,π) for which the individual allocation x_1 equals the aggregate allocation X_1. The resulting allocation rules are used to define the problem facing the government. Next, the first-stage competitive equilibrium is defined. Combining all of these gives an equilibrium that we call a <u>time-consistent equilibrium</u>. We define a second-stage competitive equilibrium as follows.

<u>Definition</u>. A competitive equilibrium at the second stage, given the history (x_1,X_1,π), is a set of individual and aggregate allocation rules f_2 and F_2 that satisfy

(i) <u>Consumer maximization</u>. Given the history (x_1,X_1,π), the individual allocation rule $f_2(x_1,X_1,\pi)$ solves

$$\max_{c_2,\ell} \ U(c_1 + c_2,\ell)$$

subject to

$$c_2 \leq (1 - \delta)Rk + (1 - \tau)\ell.$$

(ii) <u>Representativeness</u>. $f_2(X_1, x_1, \pi) = F_2(X_1, \pi)$.

Since this equilibrium is defined for each history, we can summarize it by the function $F_2(X_1, \pi)$.

Next consider the situation of the government. Given the past aggregate decisions X_1 and knowing that future decisions are selected according to the rule $F_2(X_1, \pi)$, the government selects a policy, say, $\pi = \sigma(X_1)$, that maximizes consumer welfare. The government's objective function is

$$(2.3) \qquad S(\sigma, F_2; X_1) = U\big(C_1 + C_2(X_1, \pi), L(X_1, \pi)\big)$$

where $\pi = \sigma(X_1)$. Given X_1 and F_2, the government must select a policy $\sigma(X_1)$ that satisfies its budget constraint:

$$(2.4) \qquad G \leq \delta(X_1)RK + \tau(X_1)L\big(X_1, \sigma(X_1)\big).$$

Let $\Sigma(F_2; X_1)$ denote the set of all policies $\sigma(X_1)$ that satisfy (2.4). The problem of the government is to pick a plan σ such that for every F_1, $\sigma(X_1)$ maximizes utility (2.3) over the set of feasible policies $\Sigma(F_2; X_1)$.

Finally, consider the consumer's problem at the first stage. Each consumer chooses an individual allocation for the first stage, $x_1 = (c_1, k)$, together with an allocation rule f_2 for taking actions at the second stage. Each consumer takes as given that the current aggregate allocation is some X_1, that future policy is set according to the plan σ, and that future aggregate

allocations are set according to some rule F_2. Given these assumptions, the definition of the first-stage competitive equilibrium is analogous to that of the second-stage competitive equilibrium, so the first-stage equilibrium is summarized by (σ, X_1, F_2).

We have recursively defined the consumer's and the government's problems. Combining these gives an equilibrium with sequential rationality built in for both the private agents and the government. Because of this, we say the equilibrium is time-consistent. Formally, we have

Definition. A time-consistent equilibrium is a triple (σ, X_1, F_2) that satisfies

(i) Sequential rationality by consumers. The triple (σ, X_1, F_2) is a first-stage competitive equilibrium and, for every history (π', X_1'), the allocation rule $F_2(\pi', X_1')$ is a second-stage competitive equilibrium.

(ii) Sequential rationality by the government. Given F_2, the policy plan σ solves the government's problem for every history X_1'.

We then have

Proposition 3. The Single-Period Time-Consistent Equilibrium

The single-period time-consistent equilibrium has first-stage allocations $C_1 = \omega$ and $K = 0$ and a capital tax plan $\delta(X_1) \equiv 1$.

<u>Proof</u>.   Consider first the policy plan σ.   For any given first-
stage aggregate allocation $X_1 = (C_1, K)$, it is clearly optimal for
the government to raise as much revenue as possible from taxing
the given amount of capital.   By assumption, $G > R\omega$, so even if
all the endowments are saved and the resulting capital is fully
taxed, the revenues fall short of government spending.   Thus,
$\delta(X_1) \equiv 1$.   Faced with such a tax, it is optimal for consumers to
save nothing and consume all of their endowments. ◊

It is easy to verify that the utility level of each
consumer in the time-consistent equilibrium is strictly lower than
the level in the Ramsey equilibrium.

But an important question still remains: What is the
source of the conflict in this example?   To investigate this
question we cast our model in the general social choice framework
considered in Section 1.   To accomplish this we need to embed the
budget constraints of consumers and the government into prefer-
ences.   Let the preferences of each private agent be given by

$$U\left(\omega - k + (1 - \delta)Rk + (1 - \tau)\ell, \ell\right) + W(K, L, \delta, \tau)$$

where the function W equals zero if its arguments (K, L, δ, τ) sat-
isfy the government's budget constraint, $G \leq \delta RK + \tau L$, and W
equals some large negative number otherwise.   Let the government's
preferences be

$$U\left(\omega - K + (1 - \delta)RK + (1 - \tau)L, L\right) + W(K, L, \delta, \tau).$$

Since we have assumed that consumers are price-takers, in the sense that they regard aggregates as being unaffected by their decisions, this model is a special case of the representative agent model considered in Section 1.4. Thus the source of conflict is that each private agent cares more about his own allocation than about other private agents' allocations.

2.3 A Finite-Horizon Model of Capital Taxation

Consider a finite repetition of the capital taxation model. To keep things simple, we assume that capital cannot be stored between periods, that there is no borrowing and lending across periods, and that government spending is constant. With commitment, the government chooses a sequence of tax rates once and for all at the beginning of time. A competitive equilibrium is a sequence of individual and aggregate allocations that maximize consumer welfare and that satisfy the government budget constraint and representativeness. The Ramsey equilibrium in this multiperiod model is simply the one-period Ramsey equilibrium repeated finitely many times.

Without commitment, the problem is more complicated because all decisions must be sequentially rational. Consumers must forecast how future tax rates will be chosen, and the government must forecast how its current choices influence future decisions of consumers. Following Chari and Kehoe (1987a,b), we resolve this forecasting problem by making allocations and policies functions of the history of past decisions. Formally, the history of an individual consumer at the first stage of period t is

$$h_{1t} = (x_s, X_s, \pi_s \mid s = 0, \ldots, t - 1)$$

and the aggregate history at the first stage is

$$H_{1t} = (X_s, \pi_s \mid s = 0, \ldots, t - 1).$$

Likewise, the aggregate history confronting the government after consumers have made their first-stage decisions in period t is

$$H_t = (X_s, \pi_s \mid s = 0, \ldots, t - 1) \cup X_{1t}.$$

At the second stage, an individual consumer's history is given by

$$h_{2t} = (h_{1t}, x_{1t}, X_{1t}, \pi_t)$$

and the aggregate history by

$$H_{2t} = (H_{1t}, X_{1t}, \pi_t).$$

In keeping with the assumption that tax rates cannot be altered by the decisions of any single consumer, aggregate histories do not include individual allocations.

Allocations and policies are defined as functions of the histories. Let $f_t = (f_{1t}, f_{2t})$ denote individual allocation functions that map first- and second-stage individual histories into decisions at the respective stages. Let $F_t = (F_{1t}, F_{2t})$ denote the corresponding aggregate allocation function that maps aggregate histories into aggregate allocations. Let σ_t denote the government's policy function that maps histories H_t into decisions at t.

Now in order to define a time-consistent equilibrium, we need to explain how allocation and policy functions induce future histories. In what follows, we consider only symmetric histo-

ries. Let $f^t = (f_t, f_{t+1}, \ldots)$ denote a sequence of individual allocation rules from time t onward. Let F^t and σ^t denote the corresponding objects for the aggregate allocation rules and policy plans. Given a history h_{1t}, the functions f^t, F^t, and σ^t induce individual histories

$$h_{2t} = \{h_{1t}, f_{1t}(h_{1t}), F_{1t}(H_{1t}), \sigma_t(H_{1t}, F_{1t}(H_{1t}))\}$$

$$h_{1t+1} = \{h_{2t}, f_{2t}(h_{2t}), F_{2t}(H_{2t})\}$$

and so on. Likewise, from any initial aggregate history, say, H_{1t}, the functions F^t and σ^t induce future histories $(H_t, H_{2t}, H_{1t+1}, \ldots)$ in a similar fashion.

Consider the first stage of period t. Given some history h_{1t}, an individual consumer chooses a contingency plan f^t. Each consumer takes it as given that future aggregate allocations and policies will evolve according to the histories induced by F^t and σ^t. Recalling that we only consider symmetric histories, we have

Definition. A competitive equilibrium at the first stage of t, given a history H_{1t}, is a set of contingency plans f^t, F^t, and σ^t that satisfy

(i) Consumer maximization. Given H_{1t}, F^t, and σ^t, the individual allocation rules f^t maximize

$$\sum_{s=t}^{T} \beta^{s-t} U\big(c_{1s}(h_{1s}) + c_{2s}(h_{2s}), \ell_s(h_{2s})\big)$$

subject to

$$c_{1s}(h_{1s}) \leq \omega - k_s(h_{1s})$$

$$c_{2s}(h_{2s}) \leq \left[1 - \delta_s(H_s)\right]Rk_s(h_{1s}) + \left[1 - \tau_s(H_s)\right]\ell_s(h_{2s})$$

where for all $s \geq t$, the future histories are induced by f^t, F^t, and σ^t.

(ii) <u>Representativeness</u>. $f^t = F^t$.

We can refer to this equilibrium as a pair (σ^t, F^t). Likewise, a competitive equilibrium at the second stage of t, given a history H_{2t}, is a set of contingency plans (f_{2t}, f^{t+1}), (F_{2t}, F^{t+1}), and σ^{t+1} that satisfy conditions similar to those above. We refer to this equilibrium as $(\sigma^{t+1}, F_{2t}, F^{t+1})$.

Next consider the situation of the government in period t. Given some history H_t and taking as given that future aggregate allocations evolve according to (F_{2t}, F^{t+1}), the government selects a policy plan σ^t that maximizes consumer welfare. The government's objective function is

$$(2.5) \qquad S_t(\sigma^t, F_{2t}, F^{t+1}; H_t) = U\left(C_{1t} + C_{2t}(H_{2t}), L_t(H_{2t})\right)$$

$$+ \sum_{s=t+1}^{T} \beta^{s-t} U\left(C_{1s}(H_{1s}) + C_{2s}(H_{2s}), L_s(H_{2s})\right).$$

Given the history H_t and the allocation rules (F_{2t}, F^{t+1}), the government must select a policy plan that not only satisfies its current budget constraint

$$G \leq \delta_t(H_t)RK_t + \tau_t(H_t)L_t(H_{2t})$$

but that also satisfies its future budget constraints

$$G \leq \delta_s(H_s)RK_s(H_{1s}) + \tau_s(H_s)L_s(H_{2s})$$

for all aggregate histories induced by (F_{2t}, F^{t+1}) and σ^t. Let $\Sigma_t(F_{2t}, F^{t+1}; H_t)$ be the set of all policy plans σ^t that satisfy these budget constraints. The problem of the government at t, then, is to pick a plan σ^t that maximizes consumer welfare (2.5) over the set of all feasible policies $\Sigma_t(F_{2t}, F^{t+1}; H_t)$.

Combining these various definitions gives a type of equilibrium that will not break down as time evolves, since by construction the various contingency plans will be carried out for any possible set of histories. We then have

<u>Definition</u>. A time-consistent equilibrium is a pair (σ, F) that satisfies

(i) <u>Sequential rationality by consumers</u>. For every history H_{1t}, (σ^t, F^t) is a first-stage competitive equilibrium, and for every history H_{2t}, the triple $(\sigma^{t+1}, F_{2t}, F^{t+1})$ is a second-stage competitive equilibrium.

(ii) <u>Sequential rationality by the government</u>. For every history H_t, the plan σ^t maximizes consumer welfare over the set of feasible plans $\Sigma_t(F_{2t}, F^{t+1}; H_t)$.

We abbreviate notation and let $S_0(\sigma, F)$ denote the value of utility at time zero in a time-consistent equilibrium.

It is easy to characterize the time-consistent equilibria by using backward induction. At the second stage of the last period, the consumer's decision problem depends only upon current tax rates and the current capital stock; it is independent of the rest of the history. Consequently, the government's decision problem depends only upon the current capital stock. It follows

that the equilibrium in the last period is identical with the single-period equilibrium and is independent of the history. Next consider the problem in period T - 1. Clearly, neither the government's decisions nor private agents' decisions have any effect on outcomes in period T. Hence, the period T - 1 problem is also static and the outcomes are identical to those in the single-period case. It follows by repetition of this argument that for the finite-horizon case, the time-consistent equilibrium is unique and is simply the sequence of the single-period time-consistent equilibria.

2.4 An Infinite-Horizon Model of Capital Taxation

With commitment, the characterization of equilibrium is straightforward. The infinite-horizon Ramsey equilibrium is simply the one-period Ramsey equilibrium of Proposition 2, repeated forever. Notice that this equilibrium is the limit of a sequence of finite-horizon Ramsey equilibria.

Without commitment, the way to characterize the set of equilibria is not obvious. One way to proceed is simply to take the limits of a sequence of finite-horizon time-consistent equilibria. This technique will indeed yield a time-consistent equilibrium. However, there are many other time-consistent equilibria that are not the limits of any sequence of finite-horizon equilibria. In fact, the set of time-consistent equilibria is very large and difficult to characterize. However, it is relatively easy to characterize the policies and allocations induced by time-consistent equilibria.

Recall that, in general, a time-consistent equilibrium (σ, F) is a sequence of functions that specify policies and allocations for all possible histories. Starting from the null history at date 0, a time-consistent equilibrium induces a particular sequence of policies and allocations, say, (π, X). We call this the outcome induced by the time-consistent equilibrium.

The technique for characterizing the set of such outcomes builds on Abreu's (1984) seminal work on repeated games. In our models, however, agents behave competitively rather than strategically, and thus we need to reformulate Abreu's arguments. We prove that a sequence of policies and allocations can be induced by some time-consistent equilibrium if and only if the sequence can be induced by a particular time-consistent equilibrium called the revert-to-autarky equilibrium. We then use this result to show that an arbitrary sequence is an outcome of a time-consistent equilibrium if and only if it satisfies two conditions: first, the sequence is a competitive equilibrium at date 0; second, the sequence must satisfy some simple inequalities.

We proceed in three steps. First, we define the autarky equilibrium and the revert-to-autarky equilibrium. Second, we prove that an arbitrary sequence of policies and allocations can be induced by some time-consistent equilibrium if and only if it can be induced by the revert-to-autarky equilibrium. Third, we use this result to provide a simple characterization of time-consistent policies and allocations.

The autarky equilibrium (σ^a, F^a) is defined as follows. For the government, the plan $\sigma_t^a(H_t)$ specifies the single-period

time-consistent plan of Proposition 3 regardless of the history up until time t. For private agents, the allocation rules $F_{1t}^a(H_{1t})$ and $F_{2t}^a(H_{2t})$ specify the single-period time-consistent allocation rules regardless of the history up until time t. It is easy to verify that these policy plans and allocation rules constitute a time-consistent equilibrium. Chari and Kehoe (1987b) prove the following proposition.

Proposition 4. Autarky Is the Worst Time-Consistent Equilibrium

Any time-consistent equilibrium (σ,F) must have a utility level $S(\sigma,F)$ greater than or equal to the utility level $S(\sigma^a,F^a)$ of the autarky equilibrium.

Proof. We sketch the proof here. To establish the proposition, we show that for an arbitrary equilibrium (σ,F), the following inequalities hold:

$$S(\sigma,F) \geq S(\sigma^a,F) \geq S(\sigma^a,F^a).$$

Both inequalities rely on a fact about competitive equilibria. That is, for any period t, the second-stage labor supply and consumption decisions solve the same static problem. From this fact it follows that a deviation by the government from σ to σ^a is feasible in that σ^a satisfies the government budget constraint for any equilibrium allocation rule F. Sequential rationality by the government then yields the first inequality.

Next, if the allocation rule F specifies positive savings at some date t, then the second equality holds, because distorting taxes are being replaced by lump-sum taxes. If the

rule F specifies zero savings for all dates, then the resulting allocations under F and F^a are identical. ◊

The next proposition uses a modified version of the autarky plans called the revert-to-autarky plans. For an arbitrary sequence of policies and allocations (π, X), the revert-to-autarky plans (σ^r, F^r) specify continuation with the candidate sequences (π, X) as long as they have been chosen in the past; otherwise, they specify revert to the autarky plans (σ^a, F^a). Thus, for example, at time t given a history H_t, this policy plan specifies: Choose the tax rates π_t specified by π if the tax rates $(\pi_0, \ldots, \pi_{t-1})$ have been chosen according to π and the allocations $(X_0, X_1, \ldots, X_{t-1})$ and X_{1t} have been chosen according to X. If they have not, then revert to the autarky tax rule σ^a. The revert-to-autarky allocation rules F^r are similarly defined.

Consider, then, some arbitrary sequences (π, X) and the associated revert-to-autarky plans. It will be useful to define the single-period utility when the government reverts to autarky. Given that the first-stage allocations X_{1t} at time t have been chosen according to X, let $U^d(X_{1t})$ be the maximized value of utility under the autarky rule. It is easy to show that

$$U^d(X_{1t}) = \max_{(\tau, C_2, L)} U(C_{1t} + C_2, L)$$

subject to

$$C_2 \leq (1 - \tau)L$$

$$\frac{U_\ell}{U_c} = (1 - \tau)$$

$$G \le RK_t + \tau L.$$

We then have

Proposition 5.   Time-Consistent Equilibrium Outcomes

An arbitrary pair of sequences (π, X) is the outcome of a time-consistent equilibrium if and only if

(i)   The pair (π, X) is a competitive equilibrium at date 0.

(ii)   For every t, the following inequality holds:

(2.6) $$\sum_{s=t}^{\infty} \beta^{s-t} U(X_s) \ge U^d(X_{1t}) + \frac{\beta}{1 - \beta} U(X^a)$$

where X^a denotes the autarky equilibrium allocation.

Proof.   Suppose first that (π, X) is the outcome of a time-consistent equilibrium (σ, F).   Sequential rationality by consumers requires that (π, X) be a competitive equilibrium at date 0.   By an argument similar to the one in Proposition 4, a deviation by the government to the autarky plan σ^a is feasible.   Also, by Proposition 4, the autarky equilibrium is the worst equilibrium. Clearly, then, the utility of the government must be at least as large as the right-hand side of (2.6) for every period t.

Next, suppose some arbitrary pair of sequences (π, X) satisfies (i) and (ii) of Proposition 5.   We need to show that the associated revert-to-autarky plans (σ^r, F^r) constitute a time-consistent equilibrium.   Consider histories under which there have been no deviations from (π, X) up until t.   Since (π, X) is a competitive equilibrium at date 0, it is clear that its continuation

from date t is also a competitive equilibrium. Thus sequential rationality by consumers is satisfied for such histories. Now consider the situation of the government when it is confronted with allocation rules F^r. The highest utility it can obtain from any deviation is given by the right-hand side of (2.6). We have proved that σ^r is sequentially rational for the government for such histories.

Next, consider histories for which there has been a deviation before time t. The revert-to-autarky rules (σ^r, F^r) specify autarky from then onward. Clearly, the autarky policies and allocations constitute a competitive equilibrium at t. Finally, faced with the autarky allocation rule, the government finds it optimal to choose the autarky policy. Thus (σ^r, F^r) is a time-consistent equilibrium. ◊

An immediate corollary to this proposition is a result that resembles the folk theorem for repeated games (see, for example, Fudenberg and Maskin 1986).

Proposition 6. Supporting Ramsey Allocations

There is some discount factor $\underline{\beta} \in (0,1)$ such that for all $\beta \in (\underline{\beta}, 1)$ the Ramsey allocations can be supported by a time-consistent equilibrium.

Proof. Recall that the Ramsey allocations are the same in all periods. Denote the Ramsey allocation of any period by X^*. By Proposition 5 we need only verify that inequality (2.6) is satisfied by X^*. Rearranging terms, we need to show

(2.7) $\qquad \dfrac{\beta}{1-\beta}\left[U(X^*) - U(X^a)\right] \geq \left[U^d(X_1^*) - U(X^*)\right].$

Since the Ramsey allocations yield a strictly higher level of utility than the autarky allocations do, the left-hand side of (2.7) is positive. Since the left-hand side of (2.7) increases monotonically to infinity as the discount factor approaches one, the proposition follows. ◊

Propositions 5 and 6 have shown that the set of time-consistent equilibria for the infinite-horizon is much larger than the limit of the finite-horizon equilibria. The result depends critically on the fact that both policy and allocation rules were allowed to depend on histories. If either of these rules were restricted so as not to depend on the history prior to the current period, then the unique time-consistent equilibrium would be the limit of the finite-horizon equilibrium. We see no compelling reason to restrict attention to such rules.

Results similar to ours are well known for repeated games. Our models, however, differ from repeated games in an important aspect: private agents behave competitively rather than strategically. For example, in the revert-to-autarky equilibrium, consumers do not "punish" the government when it deviates; rather, they choose the autarky allocations, because taking the future aggregate allocations and policies as beyond their control, it is optimal to choose these allocations.

3. A Finite-Horizon Model of Debt and Default

In the multiperiod capital taxation model of Section 2, it was assumed that capital depreciated completely between periods

and that agents could not borrow or lend. Technically, this implied that there were no state variables, like capital or debt, connecting one period to the next. In addition, we assumed a linear production function so that the calculation of equilibrium prices would be trivial. These features helped make our analysis of the model simple, and thus the model served as a useful intro- duction to multiperiod models with time-consistency problems. In most macroeconomic models of interest, however, there are physical state variables and the calculation of equilibrium prices is nontrivial. The main goal of this section is to provide an intro- duction to such models and to highlight some of the issues that arise. We accomplish this by studying a simple model of debt and default. A secondary goal is to show that even in this simple model it is a nontrivial problem to determine whether or not there is a time consistency problem. In particular, we show that while the conflicts among agents of the type considered in Section 1 are necessary for a time consistency problem, they are not sufficient.

We consider a finite-horizon model of debt similar to the models of Prescott (1977), Barro (1979), and Lucas and Stokey (1983). In the model, government consumption fluctuates over time and the revenues to finance this consumption are raised through distortionary taxation of labor. The government is also allowed to tax debt. Any tax on debt is interpreted as a partial default and a 100-percent tax is interpreted as a complete default. For simplicity, we assume there is no capital.

In the commitment equilibrium, the government uses debt to smooth distortions from labor taxation over time. With a

fluctuating stream of government consumption, optimality implies that the Ramsey policy will not have the budget balanced in each period. In the no-commitment equilibrium, it is this lack of budget balance that drives the time consistency problem. In particular, without commitment, whenever the outstanding government debt is positive, the government has an incentive to default on the debt in order to decrease the amount of distortionary labor taxation. When the outstanding debt is negative, however, the government has no incentive to default. Chari and Kehoe (1987a) use this fact to show that the time-consistent equilibria solve a certain programming problem called the constrained Ramsey problem. This problem is to maximize the welfare of a representative consumer at date 0, subject to the budget constraint and a sequence of constraints that require that the present value of the government's surplus be nonpositive at all future dates.

3.1 Debt and Default With Commitment

Consider an economy populated by a large number of identical agents who live for $T + 1$ periods. In each period there are two goods: labor and a consumption good. A constant returns-to-scale technology is available to transform one unit of labor into one unit of output. The output can be used for private consumption or for government consumption. The per capita level of government consumption in each period, denoted G_t, is exogenously specified. Let c_t and ℓ_t denote the individual levels of consumption and labor, and let C_t and L_t denote the aggregate (or per capita) values of these variables. An aggregate allocation $(C,L) = \{C_t, L_t\}_{t=0}^{T}$ is feasible if it satisfies

(3.1) $C_t + G_t = L_t.$

The preferences of each agent are given by

(3.2) $\sum_{t=0}^{T} \beta^t U(c_t, \ell_t)$

where U is increasing, concave, and bounded, and where $0 < \beta < 1$.

Let p_t denote the price of the consumption good at time t in an abstract unit of account, and denote the vector of prices by $p = \{p_t\}_{t=0}^{T}$. Since the constant returns-to-scale technology transforms a unit of labor into one unit of output, the wage rate equals the price of the consumption good. We assume that revenues can be raised only through a proportional tax on labor income. Let τ_t denote the tax rate on the labor income earned in period t, and let $\tau = \{\tau_t\}_{t=0}^{T}$ denote the sequence of such tax rates. The budget constraint of the representative consumer is then

(3.3) $\sum_{t=0}^{T} p_t[c_t - (1 - \tau_t)\ell_t] = 0.$

Notice that we have written the consumer's budget constraint in date-0 or present-value form. Implicit in this constraint is a sequence of government debt held by consumers. One can understand the government's incentives to tax (or to default on the) debt by explicitly writing out this sequence.

Following Lucas and Stokey (1983), we allow for government debt of all maturities. In each period t the government has outstanding net claims denoted $_{t-1}B = \{_{t-1}B_s\}_{s=t}^{T}$ where $_{t-1}B_s$ is a claim to goods at time s. At time t the issue of new debt claims by the government results in a net debt position of $_tB$. (One can

think of $_tB$ as a single bond with time-varying coupon payments.) We model default as a tax on outstanding debt. Let $\delta_t \in [0,1]$ denote the tax on debt outstanding in period t. Let $_tq_s$ be the price at time t of the debt claim maturing in period s. The value of the outstanding debt at time t, prior to the tax $(1 - \delta_t)$ is given by $\sum_{s=t}^{T} {}_tq_s \, {}_{t-1}B_s$. The government's budget constraint at time t is

$$(3.4) \qquad p_t\left[\tau_t L_t - G_t\right] + \sum_{s=t+1}^{T} {}_tq_s \, {}_tB_s = (1 - \delta_t) \sum_{s=t}^{T} {}_tq_s \, {}_{t-1}B_s$$

where $_{-1}B \equiv 0$.

The analogous sequential budget constraints for the aggregate allocations $\{C_t, L_t\}_{t=0}^{T}$ are

$$(3.5) \qquad p_t\left[C_t - (1 - \tau_t)L_t\right] + \sum_{s=t+1}^{T} {}_tq_s \, {}_tB_s$$

$$= (1 - \delta_t) \sum_{s=t}^{T} {}_tq_s \, {}_{t-1}B_s$$

where $_{-1}B \equiv 0$. Obviously, in a competitive equilibrium there is an arbitrage relation between the prices of the consumption goods and the prices of the debt claims, namely, $_tq_t = p_t$, and for all $s \geq t + 1$

$$(3.6) \qquad {}_tq_s = p_s(1 - \delta_{t+1})(1 - \delta_{t+2}) \ \cdots \ (1 - \delta_s)$$

In this economy an individual agent's allocation is a vector of consumption and labor, denoted by $x = \{x_t\}_{t=0}^{T}$, where $x_t = (c_t, \ell_t)$. An aggregate allocation is defined analogously and denoted by $X = \{X_t\}_{t=0}^{T}$, where $X_t = (C_t, L_t)$. A policy for the government is a sequence of tax rates on labor, tax rates on debt, and debt issues, denoted by $\pi = \{\pi_t\}_{t=0}^{T}$, where $\pi_t = (\tau_t, \delta_t, {}_tB)$.

We then have

Definition. A competitive equilibrium is a set of individual allocations x, an aggregate allocation X, price systems p and q, and a policy π that satisfy

(i)   Consumer maximization.   Given π, p, q, and X, the individual allocation x maximizes (3.2) subject to (3.3).

(ii)   Sequential constraints for aggregate allocations. The aggregate allocation X satisfies (3.5) for each t.

(iii) Sequential constraints for government policies. The policy π satisfies (3.4) for each t.

(iv)   No arbitrage. The price systems p and q satisfy (3.6) for all t.

(v)   Representativeness. x = X.

Notice that the sequential constraints (3.4) and (3.5) imply the feasibility condition (3.1).

We comment briefly on the no-arbitrage condition and the sequence of constraints for aggregate allocations. We can derive these conditions from consumer maximization by including the sequence of period budget constraints for each of the consumers. In these period budget constraints are the debt claims held on other consumers as well as on the government. Consumer maximization then implies the no-arbitrage condition. Market clearing in private debt and representativeness then imply the sequence of constraints for aggregate allocations. For notational convenience

we have simply imposed these conditions as part of the definition
of equilibrium.

In any equilibrium the individual and aggregate alloca-
tions coincide, so we refer to such a competitive equilibrium as
(π,X,p,q). Let E denote the set of policies for which an equilib-
rium exists. Assume that for each π in E there is a unique allo-
cation $X(\pi)$. (A sufficient condition for this to be true is that
consumption and leisure are normal goods.) The equilibrium value
of utility under a policy π is given by

$$S\big(\pi,X(\pi)\big) = \sum_{t=0}^{T} \beta^t U\big(C_t(\pi),L_t(\pi)\big).$$

We say (π,X,p,q) is a <u>Ramsey equilibrium</u> if π solves

$$\max_{\pi\in E} S\big(\pi,X(\pi)\big)$$

and $X = X(\pi)$, $p = p(\pi)$, and $q = q(\pi)$.

In this model we have allowed government to tax labor
and debt. As we shall see in the no-commitment equilibrium, the
incentive to use the tax on debt to renege on claims drives the
time inconsistency problem. However, interestingly enough, in the
Ramsey equilibrium the ability to tax debt is irrelevant; and, in
terms of allocations, all that really matters is the tax on la-
bor. Specifically, the Ramsey equilibrium for this economy coin-
cides with the Ramsey equilibrium considered by Lucas and Stokey
(1983) in which governments are not allowed to tax debt. The
reason for this is that letting the government tax debt does not
expand the set of allocations attainable under a government pol-
icy. We then have

Proposition 7. The Ramsey Equilibrium of the Debt Model

The consumption and labor allocations C and L in the Ramsey equilibrium solve

$$\max \sum_{t=0}^{T} \beta^t U(C_t, L_t)$$

subject to

$$(3.7) \qquad C_t + G_t = L_t$$

$$(3.8) \qquad \sum_{t=0}^{T} \beta^t R_t = 0$$

where $R_t \equiv U_c C_t + U_\ell L_t$ is the government surplus in period t in units of marginal utility.

Proof. First, the set of allocations attainable is the same as those in an economy where the government sets the tax on debt identically equal to zero. To see this, note that if (π, X, p, q) is an equilibrium with δ_t possibly positive for some t, then so is $(\hat\pi, X, \hat p, \hat q)$ with $\hat\tau_t = \tau_t$, $\hat\delta_t = 0$, $_tB_s = {}_tB_s(1-\delta_{t+1}) \cdots (1-\delta_s)$, and $_t\hat q_s = p_s$, for all s and t with $s \geq t + 1$. Next, notice that in any competitive equilibrium, the consumer's first-order conditions imply

$$(3.9) \qquad p_t = \beta^t U_c(C_t, L_t)$$

and

$$(3.10) \qquad (1 - \tau_t) = -U_\ell(C_t, L_t)/U_c(C_t, L_t).$$

Substituting (3.9) and (3.10) into the consumer's budget constraint (3.3) gives (3.8). Clearly there are many debt sequences $\{_tB\}_{t=0}^T$ and debt taxes $\{\delta_t\}_{t=0}^T$ that satisfy the sequential budget constraints (3.4) and (3.5). ◊

The first-order conditions for the Ramsey problem are (3.7), (3.8), and for all t,

$$(3.11) \quad (1 + \lambda_0)(U_c + U_\ell)$$

$$+ \lambda_0\left[C_t(G_t)(U_{cc} + U_{c\ell}) + L_t(G_t)(U_{c\ell} + U_{\ell\ell})\right] = 0$$

where λ_0 is the Lagrangian multiplier on (3.8). Clearly the allocations that solve this problem depend on only the current value of government consumption G_t and the multiplier λ_0. Suppressing the multiplier, we let $R(G_t)$ denote the value of the government surplus (in marginal utility units) under the Ramsey allocations.

In the Ramsey plan, the government optimally smooths distortions over time. We now provide a parametric example for which this smoothing of distortions is accomplished by having tax rates constant over time.

Example 1. Let U be given by

$$U(C,L) = \frac{C^{1-\alpha_1}}{1 - \alpha_1} - \alpha_3 \frac{L^{1-\alpha_2}}{1 - \alpha_2}.$$

Note that $\alpha_3 \geq 0$ and that from concavity we have $\alpha_1 \geq 0$ and $\alpha_2 \leq 0$. Given the additive separability of U, we can manipulate the first-order conditions (3.11) to get

$$(1 + \lambda_0)\tau_t + \lambda_0 \left[\frac{C_t U_{cc}}{U_c} - (1 - \tau_t) \frac{L_t U_{\ell\ell}}{U_\ell} \right] = 0$$

where we have suppressed dependence on G_t and have let $\tau_t = 1 + U_\ell/U_c$. Substituting for the derivatives of U in this equation, we see that tax rates are constant over time and independent of the current level of government consumption. Furthermore, it is easy to show that the surplus under the Ramsey plan, $R(G_t)$, will be decreasing in G_t if $\alpha_1(1 - \alpha_1) \geq \alpha_2(1 - \alpha_2)$. We will exploit this feature of the Ramsey plan in some later examples.

3.2 Debt and Default Without Commitment

In an environment without commitment, we can no longer retain the fiction that all agents make decisions once and for all at the beginning of time and then simply execute those decisions at the appropriate time. Indeed, we need to ensure that these decisions are sequentially rational. In terms of the timing of decisions, we model the sequential decision making by assuming that governments in each period choose a policy at the beginning of the period and then consumers choose their consumption and labor supply decisions. As in Section 2, governments choose policies as a function of the aggregate history, which for this model consists of the past aggregate consumption, labor, and debt-holding decisions and the past policies. Thus an aggregate history confronting the government at time t is

$$H_t = (X_s, \pi_s | s = 0, \ldots, t - 1).$$

Consumers make their choices over consumption, labor, and their debt holdings at date t as functions of their individual

histories. Such a history includes the policy choice π_t as well as the past individual decisions, past aggregate decisions, and past policy choices. The individual history is given by

$$h_{1t} = (x_s, X_s, \pi_s | s = 0, \ldots, t - 1) \cup \pi_t$$

and the aggregate history H_{1t} is given by

$$H_{1t} = (X_s, \pi_s | s = 0, \ldots, t - 1) \cup \pi_t.$$

In keeping with the representative agent model used, only symmetric histories are considered.

For this environment, a time-consistent equilibrium consists of an individual allocation rule f, an aggregate allocation rule F, a policy plan σ, and price systems p and q that satisfy certain sequential rationality conditions. An individual allocation rule is a sequence of functions $f = \{f_t\}_{t=0}^{T}$, where f_t maps each individual history h_{1t} into an agent's current choice of consumption and labor. Likewise, an aggregate allocation rule is a sequence of functions $F = \{F_t\}_{t=0}^{T}$, where F_t maps each aggregate history H_{1t} into an aggregate amount of consumption and labor. A policy plan σ is a sequence of functions $\sigma = \{\sigma_t\}_{t=0}^{T}$, where σ_t maps each history H_t into current taxes on labor and debt and new debt issues. Finally, price systems p and q are sequences of functions $p = \{p_t\}_{t=0}^{T}$ and $q = \{_t q\}_{t=0}^{T}$, where p_t maps each history H_{1t} into a price for the consumption good at t and where $_t q$ maps each history H_{1t} into a vector of debt prices $\{_t q_s\}_{s=t}^{T}$. Now, just like the functions in Section 2, given any individual history h_{1t}, the contingency plans f^t, F^t, and σ^t induce future individual histories. For example, an agent's history at time t + 1 is

$$h_{1t+1} = \{h_{1t}, f_t(h_{1t}), F_t(H_{1t}), \sigma_t(H_t), \sigma_{t+1}(H_{1t}, F_t(H_{1t}))\}$$

and so on. In a similar fashion, given any aggregate history, the contingency plans F^t and σ^t induce future aggregate histories in the obvious way.

In a time-consistent equilibrium, sequential rationality by consumers is modeled by assuming that the policy plans, allocation rules, and price functions form a competitive equilibrium for each aggregate history. In this equilibrium each consumer is assumed to act competitively in that he assumes the evolution of policies and prices is not influenced by his actions. In particular, since future policies and prices are determined by aggregate histories, acting competitively implies that each consumer believes his actions have no effect on aggregate histories.

The problem of the consumer at time t, for some given functions F^t, σ^t, p^t and history h_{1t}, is to choose f^t to maximize

$$(3.12) \qquad \sum_{s=t}^{T} \beta^{s-t} U\left(c_s(h_{1s}), \ell_s(h_{1s})\right)$$

subject to the budget constraint

$$(3.13) \qquad \sum_{s=t}^{T} p_s(H_{1s})\{c_s(h_{1s}) - [1 - \tau_s(H_s)]\ell_s(h_{1s})\}$$

$$= [1 - \delta_t(H_t)] \sum_{s=t}^{T} {}_t q_s(H_t)_{t-1} B_s.$$

In such a competitive equilibrium, the allocation rule F^t must satisfy the sequence of constraints: for all $s \geq t$,

$$(3.14) \quad p_s(H_{1s})\{C_s(H_{1s}) - [1 - \tau_s(H_s)]L_s(H_{1s})\}$$

$$+ \sum_{r=s+1}^{T} {}_s q_r(H_{1s})\, {}_s B_r(H_{1s})$$

$$= [1 - \delta_s(H_{1s})] \sum_{r=s}^{T} {}_s q_r(H_{1s})\, {}_{s-1} B_r(H_{1s-1}).$$

We then have

<u>Definition.</u> A sequence of individual and aggregate allocation rules f^t and F^t, price functions p^t and q^t, and policy plans σ^t are sequentially rational for consumers at time t, given a history H_t, if it satisfies

    (i)   <u>Consumer maximization.</u> Taking F^t, p^t, q^t, and σ^t as given, f^t solves consumer's problem of maximizing (3.7) subject to (3.8).

    (ii)   <u>Sequential constraints for aggregate allocations.</u> F^t satisfies (3.9) for all $s \geq t$.

    (iii)   <u>No arbitrage.</u> The price systems p^t and q^t satisfy

$${}_s q_r(H_{1s}) = p_r(H_{1r})[1 - \delta_{s+1}(H_{s+1})] \ \cdots \ [1 - \delta_r(H_r)] \text{ and}$$

$${}_s q_s(H_{1s}) = p_s(H_{1s}) \text{ for all } r, s \text{ with } r \geq s \geq t + 1.$$

    (iv)   <u>Representativeness.</u> $f^t = F^t$.

It is important to note that in this definition the future histories h_{1s}, H_{1s}, and H_s are induced by σ^t, f^t, and F^t. Since representativeness is part of the definition of sequential rationality, we summarize these functions by $(\sigma^t, F^t, p^t, q^t)$.

Next consider the problem of the government. At time t the government, faced with an aggregate history H_t, takes as given that the future aggregate allocations and prices evolve according to the functions F^t, p^t, and q^t. It is important to note that in contrast to individual consumers, the government can influence the future allocations and prices by affecting the aggregate history. The objective function of the government at t is given by the utility of the representative agent from t onward under F^t and σ^t; namely,

$$(3.15) \qquad S_t(\sigma^t, F^t; H_t) = \sum_{s=t}^{T} \beta^s U\big(C_s(H_{1s}), L_s(H_{1s})\big).$$

The government choice set at time t, given a history H_t, is the set of policy plans σ^t from t onward that satisfy the government budget constraints

$$(3.16) \qquad p_s(H_{1s})\big[\tau_s(H_s)L_s(H_s) - G_s\big] + \sum_{r=s+1}^{T} {}_s q_r(H_{1s})_s B_r(H_s)$$

$$= \big[1 - \delta_s(H_s)\big] \sum_{r=s}^{T} {}_s q_r(H_{1s})_{s-1} B_r(H_{s-1})$$

for all $s \geq t$, where the future histories are induced from H_t by σ^t and F^t. We denote this choice set by $\Sigma^t(F^t, p^t, q^t; H_t)$. We then have

Definition. A time-consistent equilibrium is a (σ, F, p, q) that satisfies

(i)   Sequential rationality by consumers. For every history H_{1t}, the sequence of functions $(\sigma^t, F^t, p^t, q^t)$ are sequentially rational for consumers.

(ii) <u>Sequential rationality by the government</u>. For every history H_t, the policy plan σ^t maximizes consumer welfare (3.15) over the set $\Sigma^t(F^t, p^t, q^t; H_t)$.

We can characterize the time-consistent equilibria of this model using a backward induction argument. Recall that for the capital taxation model of Section 2, we used such an argument to reduce the multiperiod time-consistent equilibrium to a sequence of static equilibria. The key to the reduction was that there were no state variables connecting the periods. In this model, government debt is such a state variable. This feature, together with the fact that government consumption fluctuates over time, implies that the time-consistent equilibrium does not reduce so simply. Rather, the backward induction argument can be used to show that the time-consistent equilibrium solves a constrained Ramsey problem in which debt issues are constrained to be nonpositive. Specifically, Chari and Kehoe (1987a) show that if debt of all maturities is allowed, the following proposition holds.

<u>Proposition 8</u>. <u>The Time-Consistent Equilibrium for the Debt Model</u>

The allocations in the unique time-consistent equilibrium solve the constrained Ramsey problem: Choose $\{C_t, L_t\}_{t=0}^{T}$ to solve

$$\max \sum_{t=0}^{T} \beta^t U(C_t, L_t)$$

subject to

$$C_t + G_t = L_t$$

$$\sum_{t=0}^{T} \beta^{t} \left[U_{c} C_{t} + U_{\ell} L_{t} \right] = 0$$

and for all $s = 0, 1, \ldots, T,$

$$(3.17) \qquad \sum_{s=t}^{T} \beta^{s} \left[U_{c} C_{s} + U_{\ell} L_{s} \right] \leq 0.$$

Notice that this problem is simply the Ramsey problem of Proposition 7 with the extra constraints (3.17). These constraints ensure that debt issues at each date are nonpositive.

The reason that the time-consistent equilibrium solves such a problem is fairly intuitive. Consider the last period T. If the government inherits positive debt, it clearly has an incentive to default in order to minimize the amount of revenue it must collect through a distortionary labor tax. However, if the government inherits negative debt, so that the government holds claims on private agents, it has no incentive to tax the debt. By induction it follows that for any period t, regardless of the history, the government will default on positive debt but not tax negative debt. From the no-arbitrage condition, it follows that if the government ever issues positive debt, it will have a zero price. Using this fact, the consumer's first-order conditions, and the government's sequence of budget constraints, we can recursively derive constraints (3.17).

We present some examples that illustrate the nature of the time-consistent equilibrium. For these we let the horizon be infinite. (As Chari and Kehoe 1987a shows, even with an infinite-horizon, the solutions to the constrained Ramsey problem are sustainable outcomes.) In Example 2, the Ramsey allocations are time-consistent.

Example 2. Let $T = \infty$. Let $G_t = 0$ for t even and $G_t = \gamma$ for t odd. Let $R(G_t)$ denote the surplus function for the Ramsey plan. Let U be like U in Example 1 (Section 3.1), so that $R(G_t)$ is decreasing. It immediately follows that under the Ramsey plan, the budget is balanced over each two-period cycle; thus,

$$R(0) + \beta R(\gamma) = 0.$$

Since $R(G_t)$ is decreasing, $R(0)$ is positive and $R(\gamma)$ is negative. For t even, $\sum_{r=t}^{\infty}\beta^r R(G_r) = 0$ and for t odd, $\sum_{r=t}^{\infty}\beta^r R(G_r) = R(\gamma) < 0$. Since the debt issues are negative, the Ramsey allocations solve the constrained Ramsey problem. Thus the Ramsey allocations are time-consistent.

The next example is a slight variant of Example 2. In it there is a time consistency problem; however, the value of a commitment technology is not very large.

Example 3. Let $T = \infty$. Let $G_t = \gamma$ for t even and $G_t = 0$ for t odd. Let U be like U in Example 1. Let $R(G_t)$ denote the surplus function for this pattern of government consumption. Again, under the Ramsey plan, the budget is balanced over each two-period cycle:

$$R(\gamma) + \beta R(0) = 0.$$

For t even, $\sum_{r=t}^{\infty}\beta^r R(G_r) = R(0) > 0$ and for t odd, $\sum_{r=t}^{\infty}\beta^r R(G_r) = 0$. Thus the Ramsey allocations do not solve the constrained Ramsey problem, so there is a time consistency problem. We can compute an upper bound for the welfare loss due to the time consistency problem as follows. Consider the policy of

balancing the budget in period 0 and then following the constrained Ramsey allocations from date 1 on. From Example 2 we know that from date 1 on, this policy gives the Ramsey allocations of that example. The utility difference between this plan and the Ramsey plan is at most the utility lost from balancing the budget in the first period.

In Example 3, the value of a commitment technology is bounded above by the utility lost in a single period. Chari and Kehoe (1987a) examine the value of a commitment technology when government spending is stochastic. They measure this value by the expected utility difference between the Ramsey allocations and the constrained Ramsey allocations, where this difference is normalized by dividing by $\sum_{t=0}^{T} \beta^t$. This normalization converts the measure into a type of average discounted utility.

In the comparison of the Ramsey and the constrained Ramsey allocations, two assumptions are used. The first assumption is on the stochastic process for government consumption.

Assumption 1. Government consumption follows a stationary Markov process with strictly positive elements. Furthermore, it is persistent in that $\text{Prob}\{G_{t+1} \leq \gamma | G_t\}$ is a decreasing function of G_t for all γ.

Note that the persistence condition requires that higher values of government consumption at t give stochastically larger values of government consumption at t + 1.

The second assumption is on the surplus function $R(G_t)$ from the Ramsey plan.

<u>Assumption 2</u>. $R(G_t)$ is decreasing in G_t.

This assumption requires that the value of tax revenues is smoother than the value in government consumption. Recall that the parametric utility function of Example 1 satisfies this assumption. Chari and Kehoe (1987a) prove the following proposition.

<u>Proposition 9</u>. <u>The Value of a Commitment Technology</u>

Given Assumptions 1 and 2, for any $\varepsilon > 0$ there is some horizon length $T < \infty$ and some discount factor $\beta < 1$ such that the difference in the normalized value of utilities under the Ramsey and the constrained Ramsey allocations is at most ε.

This proposition implies that for an interesting class of economies, the value of a commitment technology (measured in units of normalized utility) is not very large.

To sum up, in this section we have shown (1) how the introduction of state variables complicates the computation of time-consistent equilibria, (2) that the presence of conflict among agents does not guarantee there is a time consistency problem, and (3) that the value of a commitment technology may be quite small in the debt-default model.

4. Conclusions

There is a large and growing body of literature on the time consistency problem and its implications for macroeconomic policy. In this paper we have sought to provide a perspective on the issue of time consistency rather than to survey the litera-

ture. In our view, conflict among agents plays a central role in creating time consistency problems. Because much of the literature has used representative agent models, the nature of this conflict has been obscured. We have shown how two representative agent models (of capital taxation and of debt and default) can be cast into a social choice theoretic framework in which the nature of this conflict among agents is made explicit. Optimal taxation models, where revenues are raised through distorting taxes, have the feature that each agent is better off by forcing others to bear a larger share of the burden of providing public goods. This conflict plays an essential role in establishing that the timing of policies matters for allocations.

We have also provided a careful definition of time-consistent equilibria for the capital taxation and debt and default models. Correctly defining a time-consistent equilibrium requires that we consider history-contingent allocation and price functions. These are essential to ensure that the forecasting problems of policymakers and agents are well defined. In particular, this way of defining an equilibrium ensures that we do not fall into the trap of thinking about policy as a sequence of "announcements" of future plans, each of which is fully believed by private agents. The problem with this approach to modeling sequential rationality can be understood by examining the debt and default model. In each period the government would default on the inherited debt and announce it will never do so again in the future. If private agents believe such announcements, they will buy the debt issued by the government and invariably be disap-

pointed in the future. With this approach, therefore, no equilibrium can exist. With history-contingent functions we avoid such problems.

We should reemphasize that in no sense can societies choose between commitment or time-consistent equilibria. Commitment technologies are like technologies for making shoes in an Arrow-Debreu model--they are either available or not. In particular, commitment technologies are not objects of choice. This fact has important implications for the debate over rules versus discretion.

There is a temptation to view rules as describing policies chosen under commitment and discretion as describing policies chosen without commitment. Under our interpretation, society cannot choose between commitment and no commitment. Consequently, society cannot choose between rules and discretion. However, we think there are deeper issues in this debate. We have described policies here as being chosen by society, but actual policy choices must necessarily be delegated to specific institutions or individuals. Society's problem, then, is more than choosing from alternative policy rules; rather, the problem is designing the process by which policies are chosen. Formally, this is a problem in mechanism design. (See, for example, Hurwicz 1973, Myerson 1979, and Harris and Townsend 1981.) From this perspective, the debate over rules versus discretion is actually a debate about how much authority should be delegated to policymakers. Research directed at integrating the issues of mechanism design into aggregative models is essential if we are to progress further in this debate.

Footnotes

[1]For some further work using the model of Lucas and Stokey, see Alesina and Tabellini (1987); Persson and Svensson (1984); Persson, Persson, and Svensson (1987); and Rogers (1987).

[2]In an interesting paper, Bulow and Rogoff (1988) have investigated the implications of allowing for negative debt in an open economy setting.

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