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Real Business Cycles*

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Abstract: Real business cycles are recurrent fluctuations in an economy's incomes, products, and factor inputs—especially labor—that are due to nonmonetary sources. These sources include changes in technology, tax rates and government spending, tastes, government regulation, terms of trade, and energy prices. Most real business cycle (RBC) models are variants or extensions of a neoclassical growth model. One such prototype is introduced. It is then shown how RBC theorists, applying the methodology of Kydland and Prescott (*Econometrica* 1982), use theory to make predictions about actual time series. Extensions of the prototype model, current issues, and open questions are also discussed.

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Real business cycles.

Real business cycles are recurrent fluctuations in an economy's incomes, products, and factor inputs—especially labor—that are due to nonmonetary sources. Long and Plosser (1983) coined the term *real business cycles* and used it to describe cycles generated by random changes in technology. Other real sources of fluctuations that have been studied include changes in tax rates and government spending, tastes, government regulation, terms of trade, and energy prices.

Kydland and Prescott (1982), who studied the *quantitative* predictions of a stochastic growth model with shocks to technology, found that covariances between model series and autocorrelations of model output were consistent with corresponding statistics for U.S. data. These findings were viewed as surprising for two reasons. First, the findings were counter to the idea that monetary shocks are the driving force behind business cycle fluctuations. Second, the policy implication for Kydland and Prescott's model was that stabilization policies are counterproductive. Fluctuations arise when households optimally respond to changes in technology.

The methodology that Kydland and Prescott (1982) used in their study of business cycles transformed the way in which applied research in macroeconomics is done. For this reason, the term 'real business cycles' is often associated with a methodology rather than Kydland and Prescott's original findings. Indeed, the methods of their 1982 paper have been used to study many different sources of business cycles, including monetary shocks.

Most real business cycle (RBC) models are variants or extensions of a neoclassical growth model. One such prototype is introduced. It is then shown how RBC theorists, following Kydland and Prescott (1982), use theory to make predictions about actual time series. Extensions of the prototype model are discussed. Current issues and open questions follow.

Prototype real business cycle model

Households choose sequences of consumption and leisure to maximize expected discounted utility. When aggregated, preferences are defined for a *stand-in* household that maximizes the expected value of

$$\sum \beta^t u(c_t, 1 - h_t) N_t, \tag{1}$$

where u is the utility function, c_t is per capita consumption at date t, $1 - h_t$ is per capita leisure at date t, N_t is the population at date t which grows at rate η, and β is a discount factor.

The technology available in period t is $z_t F_t(K_t, H_t)$, where $z_t F_t$ is the output produced at date t with K_t units of capital and H_t hours. The function F_t has constant returns to scale so that doubling the inputs doubles the output. The variable z_t is a stochastic technology shock assumed to follow a Markov process. The variation in z modeled here is variation in the effectiveness of factor inputs, capital and labor, to produce final goods and services or *total factor productivity* (TFP). Fluctuations in TFP arise from many possible sources. For example, improvements in TFP can arise from new inventions or innovations in existing production processes. Reductions in TFP can arise from increased regulation on producers.

Households are endowed with time each period, normalized without loss of generality to 1, which they can allocate to work or to leisure. They can invest x_t (per capita) in new capital goods. Doing so yields

$$N_{t+1} k_{t+1} = N_t[(1 - \delta)k_t + x_t], \tag{2}$$

where k_t is per capita beginning-of-period t capital, k_{t+1} is per capita end-of-period t capital, and δ is the rate of per period depreciation.

Households face taxes on purchases of consumption and investment and on incomes to capital and labor. With taxation, the household budget constraint in period t is

$$(1+\tau_{ct})c_t + (1+\tau_{xt})x_t = r_t k_t - \tau_{kt}(r_t - \delta)k_t + (1-\tau_{ht})w_t h_t + \psi_t. \tag{3}$$

Variables r_t and w_t are pre-tax payments to capital and labor, respectively. Variables τ_{ct}, τ_{xt}, τ_{kt}, and τ_{ht} are tax rates on consumption, investment, capital, and labor, respectively. These tax rates are assumed to be stochastic and follow a Markov process. Variable ψ_t is the per capita transfer payment at date t made by the government to each household. Total transfer payments are equal to tax revenues less total spending by the government. The per capita spending of the government at date t is g_t.

To derive explicit predictions about the behavior of these households, it is necessary to first define and then compute an equilibrium for the economy. In doing so, it is convenient to detrend any variables that grow over time and deal only with stationary processes. To be precise, assume that there is a constant rate of improvement in production processes over time so that $F_t(K_t, H_t) \equiv F(K_t, (1+\gamma)^t H_t)$ with F homogeneous of degree 1. If the per capita capital stock grows at rate γ and z_t and h_t are stationary, then output grows at rate γ. Certain assumptions on utility and the process for government spending also ensure that components of output grow at rate γ. Denote by $\tilde{v}_t$ the detrended level of variable v_t, that is, $\tilde{v}_t = v_t/(1+\gamma)^t$.

A competitive equilibrium is defined as household policy functions for consumption $c(\tilde{k}, \tilde{K}, s)$, investment $x(\tilde{k}, \tilde{K}, s)$, and hours $h(\tilde{k}, \tilde{K}, s)$, where $\tilde{k}$ is the (detrended) stock of capital for the household, $\tilde{K}$ is the (detrended) aggregate stock of capital, and $s = (\log z$, τ_c, τ_x, τ_k, τ_h, $\log \tilde{g})$; pricing functions $w(\tilde{K}, s)$ and $r(\tilde{K}, s)$; a function governing the evolution of the aggregate capital stock $\tilde{K}' = \Psi(\tilde{K}, s)$ that maps the current state into the capital stock next period ($\tilde{K}'$), and a function $\Phi(s', s)$ governing the transition of the stochastic shocks from s to s' such that (i) households maximize the expected value of (1) subject to (2) and (3) with the initial capital stock $\tilde{k}_0$ and functions for prices, aggregate capital, and the transition of s taken as given; (*ii*) productive factors are paid their marginal products; (*iii*) expectations are rational so that $\tilde{k} = \tilde{K}$ and

$$\Psi(\tilde{k}, s) = [(1-\delta)\tilde{k} + x(\tilde{k}, s)]/[(1+\eta)(1+\gamma)];$$

and (*iv*) markets clear:

$$c(\tilde{k}, \tilde{k}, s) + x(\tilde{k}, \tilde{k}, s) + g(s) = z(s)F(\tilde{k}, h).$$

Note that in forming expectations about the future, households take processes for prices, tax rates, and transfers as given. If households behave competitively, they assume that their own choice of capital next period does not affect the economy-wide level of capital. Therefore, in computing optimal decision functions for the household, it is necessary to distinguish the household's holdings of capital and the aggregate holdings of capital.

Comparing model predictions to data

Given equilibrium functions, properties of the model time series can be compared to data in a straightforward way. Starting with initial conditions on the state, the evolution of the state is determined by functions Ψ and Φ, resulting in sequences $\{\tilde{k}, s\}_{t=0}^{\infty}$ for the state. Equilibrium price and decision functions are then used with these sequences for the state to determine sequences of all prices and allocations.

A standard assumption for the transition $\Phi(s', s)$ is the vector autoregression

$$s_{t+1} = P_0 + Ps_t + Q\epsilon_{t+1}$$

where each element of ϵ_t is a normally distributed random variable, independent of the other elements of ϵ and across time, with mean equal to zero and variance equal to one. Allowing non-zero off-diagonals in the matrices P and Q allows for correlations in the elements of the vector s. For example, a standard assumption is that tax rates and spending are positively correlated.

If the elements of the matrix QQ' are not large, the equilibrium evolution of the capital stock is well approximated by the following function:

$$\log \tilde{k}_{t+1} = A_0 + A_k \log \tilde{k}_t + B_k s_t,$$

which is linear in the log of the detrended, per capita capital stock and the stochastic states. Similarly, the logarithms of consumption, investment, output, and hours of work can be well approximated as linear functions of $\log \tilde{k}_t$ and s_t. (See Marimon and Scott 1999 for an introduction to log-linear methods and nonlinear methods.)

Stacking the results in matrix form yields a system of equations

$$X_{t+1} = AX_t + B\epsilon_{t+1}$$

$$Y_t = CX_t + \omega_t,$$

where X contains all variables of interest, some of which may not be observable, and Y is a vector of observables. This system can be easily simulated and lends itself nicely to standard methods of estimating model parameters. (See Anderson et al. 1996.)

An important feature of the analysis in Kydland and Prescott (1982) was the construction of the same statistics for the model and for the U.S. data. Employing this methodology requires two necessary steps. The first concerns measurement: data series must be consistent with model series. For example, consumer durable expenditures are investments much like expenditures on new housing. National accountants treat expenditures on durables and housing differently, but the prototype model does not. Thus, revising the national accounts to include services, rents, and depreciation of durables is necessary for data and model series to be consistent. The second step of Kydland and Prescott's (1982) methodology concerns reporting: the same statistics should be computed for the model and the revised data. Such comparisons are useful in highlighting similarities and deviations, which are both necessary ingredients to further the development of good theory.

Applying the two methodological tenets to the prototype model and U.S. data reveals a number of interesting results. Both the theory and the U.S. data display procyclical movements in consumption and investment, with the movements in investment being far greater in percentage terms. With tax rates and government spending fixed at mean U.S. levels, the theory predicts fluctuations in per capita hours that are too smooth relative to U.S. hours and a correlation between hours worked and productivity that is too high relative to the correlation in U.S. data. When fiscal shocks consistent with U.S. policy are introduced, the theory predicts movements in per capita hours and a correlation between hours worked and productivity that are in line with the data.

Extensions of the prototype

During the 1980s and 1990s, business cycle research was exploratory but methodologically rooted. Researchers investigated the effects of many different shocks, the mechanisms that propagate them, and the welfare implications—in a consistent way that made clear what factors were important and why. A brief history is provided here, but interested readers are referred to the volume edited by Cooley (1995) and to a summary of more recent work in King and Rebelo (1999) and Rebelo (2005).

Kydland and Prescott (1982) and Long and Plosser (1983) emphasize technology shocks as an important source of fluctuations. Greenwood et al. (1988) also explore the role of technology shocks for the business cycle but restrict attention to technological changes affecting the productivity of new capital goods and allow for accelerated depreciation of old capital. Mendoza (1995) includes shocks to the terms of trade in an international business cycle model and shows that responses of real exchange rates to productivity shocks and terms-of-trade shocks are quite different, both qualitatively and quantitatively. Braun (1994), Christiano and Eichenbaum (1992), and McGrattan (1994) add fiscal shocks which are important for movement in hours and labor productivity, as noted above. Kim and Loungani (1992) add shocks to energy prices and show that the addition has only a modest impact on the variability of output and hours. Cooley and Hansen (1989) include monetary shocks and a cash-in-advance constraint and show that these additions have negligible effects on business cycle predictions.

The original technology-driven business cycle models underpredicted fluctuations in observed hours and overpredicted the correlation between hours and productivity, leading to further investigations of the model of the labor market and alternative mechanisms for propagating shocks. High—possibly infinite—elasticities were required in the original RBC models to generate fluctuations in aggregate hours comparable to the data. Rogerson (1988) motivates an infinite *aggregate* elasticity of labor supply in a world with variation in the fraction of people working: individuals work a standard workweek or not at all. This idea is implemented in an RBC model by Hansen (1985), who finds a significant increase in hours fluctuations relative to Kydland and Prescott (1982).

Another factor affecting the labor market is explored by Benhabib et al. (1991) and Greenwood and Hercowitz (1991) who introduce home production. These researchers show that business cycle predictions depend crucially on the willingness and opportunity of households to substitute time in home work and market work. Under plausible parameterizations, the models do in fact generate greater variability of hours and lower correlations between hours and productivity.

The empirical performance of the RBC model is also improved when labor-market search frictions are introduced, as in Andolfatto (1996) and Merz (1995). Labor-market search models have also been used to study movements in unemployment and vacancies.

Current research and open questions

RBC research has evolved beyond the study of business cycles. The methodology that

Kydland and Prescott (1982) introduced is now being applied to central questions in labor, finance, public finance, history, industrial organization, international macroeconomics, and trade.

Within business cycle research, some open questions remain. What is the source of large cyclical movements in TFP? This question is especially interesting in the case of the U.S. Great Depression when TFP declined significantly (Cole and Ohanian, 2004). Are movements in TFP primarily due to new inventions and processes that are, by the nature of research and development, stochastically discovered? Or are movements in TFP primarily due to changing government regulations that may alter the efficiency of production? Are they due to unmeasured investments that fluctuate over time? The answers matter for policymakers, and they matter for economists who calculate the welfare costs or gains of changing policies.

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