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Demand Management: An Illustrative Example

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This paper presents a simple coherent general equilibrium example in which optimal provision of a public good implies counter-cyclical government expenditure.

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An Illustrative Example

by John Bryant*

Demand management has recently come under attack. In particular, varying the government's financing mix between taxing and money and bond creation with the business cycle has been questioned. This paper illustrates through simple example that, in contrast, expenditure demand management may be justified. In the example, optimal provision of a public good implies countercyclical government expenditure.

To analyze demand management, one needs a model with fluctuating employment. One widely accepted attribute of the economy is its inherent stability. Therefore the first task is to determine the source of shocks that generate fluctuation.

There are three possible sources of shock. First, there are shocks to technologies. Second, there are shocks to preferences. Third, there are stochastic nonneutral government policies. We discuss the second two, reject them, and use the first source of shocks.

A frequently used source of shocks is random preferences. One major problem with this source is it's lack of credibility. Are employment fluctuations really explained by sudden massive changes in taste? Secondly, the purpose of economics is to explain economic behavior in a given environment. The subject of the study, economic behavior, should not be the primitive of the model!

A second source of shock is stochastic nonneutral government policy. The problem with this explanation is the suboptimality of such policies. It seems unreasonable to analyze the optimal expenditure policy response to suboptimal policy. If the government can follow optimal expenditure policy, then

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why cannot the government cease the fluctuation generating suboptimal policy? If the expenditure policy itself generates shocks, then our model of optimal expenditure policy is one without fluctuation.

The last possibility is shocks to technology. This explanation must address the observation that demand shocks frequently generate fluctuations. This fact explains why shocks to preferences are a popular assumption. Our model employs the obvious answer to this problem. Anticipated future shocks to technology change demand.

A model with employment fluctuation need not only specify the source of shocks. One wants the model to generate certain behavior. Our model is designed to exhibit three characteristics. First, the economy generally moves along a full employment path. Secondly, employment occasionally falls rapidly and then converges back to the full employment path. Lastly, after a large decline in employment, the economy does not converge back to the full employment path.

In this model with employment fluctuation, we consider government expenditure demand management only. That is to say, we study balanced budget fiscal policy. Demand management through financing policies has elsewhere been found to be unjustified. See, for example, Bryant and Wallace (8), Prescott and Kydland (10), and Sargent and Wallace (11).

Expenditure demand management is justified in our model by a market failure. However, our approach differs from recent Keynesian attempts to resurrect demand management (see Azariadis (2)). Those models assume an incompleteness in markets which restricts risk sharing by individuals. Demand management then redistributes risk in a way unavailable to the private market. However, there are several problems with this approach. The implementation of such policies seems to require much sophisticated knowledge of the financial markets, and of individuals' preferences and behavior. Moreover, such policies seem unlikely to "look" anything like simple demand management policies. Also, demand management cannot "bridge" very many incompletenesses. Lastly, direct interference in an incomplete market may be a better solution.

We assume that government expenditure is for a public good, not for goods producible by the private market. Otherwise, without market failure, we just reject government expenditure. The individual's utility depends upon the total amount of the public good produced. As each individual is infinitesimally small, none is produced privately. In a sense the model really addresses the optimal provision of public goods, not demand management. However, the crucial observation here is that the optimal expenditure policies "look" like demand management.

The Example

The example is a simple overlapping generations model. Time is discrete and is without beginning or end. Each period an equal number of individuals are born. They live two periods. There are three goods in the model, leisure time, a transferable but nonstorable consumption good, and the nonstorable public good. The individual is endowed with leisure only in his first period. He can use his leisure time to work, up to a fixed constraint $\overline{W}$. The model has constant returns to scale. Therefore separate firm entities are superfluous. Working in the private good technology for W_p hours yields the individual w_pW_p goods next period. Working in the public good technology for W_g hours yields w_gW_g public goods to the government. w_p and w_g are independent of the number of hours worked by the individual or all individuals. These technologies are known to the individual when he makes his decisions in his first period.

There also is a futures market. The individual when young can buy goods in his youth with promises to deliver goods next period. Similarly, when this generation meets its obligations next period, the following generation buys them with promises. We do not worry about how this market got here, it always existed. Nor do we worry about individuals meeting their contracts, they just do (but see Bryant (3)).

The individual maximizes a strictly concave, two-smooth utility function of a particular form. Let the individual work $W \leq \overline{W}$ hours, and purchase goods on the futures market with a promise of $P\ell$ goods tomorrow. Let per capita government expenditures be G. Let the individuals' second-period consumption be C_2. Then the individuals' utility function for $W \leq \overline{W}$ is

$$U_1(-W+\ell) + U_2(C_2) + U_3(G)$$

where $U_1'(-\overline{W}) = U_2'(0) = \infty$. The utility function is additive except that leisure and first-period consumption goods are perfect substitutes for $W < \overline{W}$. One interpretation is that leisure does not enter the utility function for $W < \overline{W}$, and the individual can produce current consumption goods one for one "at home" (out of the economy) in normal working time. In any case, this assumption generates the stable full employment path.

The behavior of the government must be described. The government hires individuals to work in the public goods industry at the private industry wage rate, w_p. To avoid financing issues, we assume a balanced budget each period. The government imposes equal, costless lump-sum taxes in the second period of an individual's life. It uses the proceeds to pay workers in the second period of their life with the private consumption good. Moreover, only the public goods produced by a given generation enter that generation's utility function. One interpretation is that only the old consume the public good, and it takes a period to make the public good just as it takes a period to make the private good. We conclude that $C_2 = w_pW - T - P\ell = w_p(W-W_g) - P\ell$ and $G = w_gW_g$.

"Unemployment" in this model consists of all individuals working part-time, not a mix of fully employed and unemployed workers. In addition, unemployment is "voluntary" not "involuntary." The author views these points as minor technicalities which have been adequately treated in the "new-new" labor economics (see, for example, Azariadis (1) and Bryant (4), (5)). It is useful to abstract away from such complications.

Now we verify that the model exhibits the desired characteristics. Time subscripts are used only when necessary. Time subscripts refer to the birthdate of the generation affected, not the time that the subscripted variable is observed. The individual of generation t takes the "wage rates," w_p and w_g, government expenditure and tax, and the value of futures contracts, P, as given.

His problem is:

$$\max_{\ell_t, W_t} \; U_1(-W_t+\ell_t) + U_2[(w_p)_t W_t - T_t - P_t\ell_t] + U_3(G_t).$$

The first-order necessary conditions are

(1) $$-U_1'(-W+\ell) + w_p U_2'(w_p W - T - P\ell) \geq 0, \; = \text{ if } W < \overline{W}$$

(2) $$U_1'(-W+\ell) - PU_2'(w_p W - T - P\ell) \leq 0, \; = \text{ if } \ell > 0.$$

The first-order necessary conditions imply the desired characteristics of the model. These inequalities imply $P \leq w_p$, = if $W < \overline{W}$. While the individual chooses ℓ, in the aggregate ℓ is determined by the previous generation's decision. P is determined in equilibrium by $\ell_t = P_{t-1}\ell_{t-1}$. This says that the goods purchased in the futures market by generation t equals the goods supplied to the futures market by generation t-1. Suppose $P_{t-1}\ell_{t-1} > 0$ and $w_p > 1$. Suppose the individual's decision is $W < \overline{W}$. Then $P = w_p > 1$. Therefore, $\{\ell_t\}$ approaches infinity at the rate w_p. We conclude that $W = \overline{W}$ after a finite number of periods. At the point where W just equals $\overline{W}$, $P = w_p > 1$, so $\{\ell_t\}$ must continue to grow from this point until $U_1' - U_2' = 0$, or $P = 1$. From this position only a large change in w_p reduces W below $\overline{W}$ as ℓ provides a "cushion."

Let us be more precise about shifts to the private technology. Suppose $w_p = (1+\gamma)\omega_p$. γ is a random variable bounded below by -1. In each period γ is an independent drawing with the same probability distribution. The realization of its and the next generation's γ is known to a generation at birth. Once again we consider the individual's problem. Let "~" mean the solution value. Assume $\tilde{W} < \overline{W}$. Then differentiating (1) we have:

$$(U_1''+w_p^2U_2'')dW + \omega_p[U_2'+(1+\gamma)\omega_p(W-W_g-\ell)U_2'']d\gamma = U_1''dW + \omega_p[U_2'+C_2U_2'']d\gamma = 0.$$

If second-period consumption is a gross substitute for the other goods (implying $U_2' + C_2U_2'' > 0$), then $\partial \tilde{W}/\partial\gamma > 0$. If second-period consumption is a gross compliment, $\partial \tilde{W}/\partial\gamma < 0$.

With gross substitutes the model generates asymmetric employment and output behavior with positive serial correlation. Assume second-period consumption is a gross substitute for the other goods. Further assume that $\omega_p > 1$. We start at the solution $\tilde{W} = \overline{W}$, $P = 1$. Realizations of γ greater than zero, and realizations of γ not too far below zero do not move the economy away from full employment, $\tilde{W} = \overline{W}$. Such realizations affect real output. However, the effect on real output is muted by the fact that employment is not influenced. Only large negative deviations in γ have the output effect magnified by employment fluctuations. Suppose such a large negative realization occurs to generation t, with $\gamma_{t+1} > -1$. Then $W_t < \overline{W}$. Moreover, as $P_t = (w_p)_t = (1+\gamma_t)\omega_p$, $\ell_{t+1} = P_t\ell_t$ is small. If ℓ_{t+1} is small enough, then $W_{t+1} < \overline{W}$ as $U_1'(-\overline{W}) = \infty$. Moreover, we see from $P = W_p$ and (1) that $d\tilde{W}/d\ell = 1$ for $\tilde{W} < \overline{W}$. The model generates positive serial correlation in employment and output.

We have, then, full employment occasionally disrupted by transitory reductions in employment. What about a large enduring reduction in employment? Suppose for generation t $\gamma = -1$. Then $W_t = 0$ and generation t offers nothing in return for goods today, as it will have no goods tomorrow. Moreover, no generation offers a positive amount of goods tomorrow for zero goods today, so $\ell = 0$ for all future generations. The futures market is wiped out. In all future periods, by $U_1'(-\overline{W}) = \infty$, $\tilde{W} < \overline{W}$ with $U_1'(-\tilde{W}) = w_pU_2'[w_p\tilde{W}-T]$ for $W_g < \tilde{W}$.

Demand Management

Having seen that our model has the desired characteristics, we turn to government policy. The government is a dominant player in a game. Its strategy is to announce W_g. We consider two polar cases. First, $w_p = (1+\gamma)\omega_p$ and $w_g = \omega_g$. Second, $w_p = (1+\gamma)\omega_p$ and $w_g = (1+\gamma)\omega_g$. We examine $d\tilde{W}_g/d\gamma$ and $d\tilde{W}_g/d\gamma/dW/d\gamma$. Because of the symmetry between W and ℓ, we do not treat $d\tilde{W}_g/d\ell$ in detail and study the first period of a shock. Similar analysis holds for subsequent periods where the effects come through ℓ.

The objective function of the government is not obvious. We assume that while the government maximizes individual utility, it does not purposefully redistribute income between generations. The government acts as a competitive purchasing agent for the representative individual.[1/] We consider the government decision in the current period given w_p, w_g, and ℓ rather than the government decision functions. The government decision this period influences future generations only by its effect on $P\ell$. The government maximizes the sum of this generation's utility and a valuation function on $P\ell$, $V(P\ell)$. V is a device for generating the government's behavior as competitive purchasing agent. However, this device underlines the fact that the competitive purchasing agent does not advance the interest of the current generation alone. At the moment we only constrain V to be increasing and continuous. However, we take the derivative of V below. While V is a construct, we wish to preserve its interpretation as a valuation function of some dynamic programming problem. Therefore, we do not impose differentiability. The reader can interpret $V'(P\ell)$ as the derivative at

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[1/] The government can further increase prebirth expected utility by forcing each generation to bear some of the following generation's risk (but do the unborn vote?). If, counter to our assumption, each generation does not know the following generation's realization, P is a vector of prices contingent on that realization. The government cannot then play a useful insurer's role.

the points where V is differentiable, almost everywhere, and not worry about the (hopefully) zero probability event of being at a nondifferentiable point. More generally, the reader can interpret $V'(P\ell)$ as a number appropriately bounded by the right- and left-hand derivatives of V, which exist everywhere.

The government's problem can be written:

$$\max_{W_g} U_1(-\tilde{W}+\ell) + U_2[w_p(\tilde{W}-W_G)-\tilde{P}\ell] + U_3[w_g W_g] + V(\tilde{P}\ell).$$

The first-order necessary condition is that

$$-w_p U_2' + w_g U_3' + [-U_1'+w_p U_2']\, d\tilde{W}/dW_g + \ell[-U_2'+V']\, d\tilde{P}/dW_g = 0.$$

Note that if $\tilde{W} = \overline{W}$, then $d\tilde{W}/dW_g = 0$ and if $\tilde{W} < \overline{W}$, $-U_1' + w_p U_2' = 0$ by (1) so the third term on the LHS is zero. We now impose that the government does not desire to redistribute income between generations. This implies that $-U_2' + V' = 0$. The marginal return to redistribute this generation's second-period consumption to next generation's first-period consumption is zero. As a result a necessary condition for the government maximization problem is: $-w_p U_2' + w_g U_3' = 0$. Marginal utilities are equated to relative wages, certainly the intuitive result. From (1) and (2), for $\tilde{W} < \overline{W}$ this can be written as:

$$(3) \qquad -U_1'(-\tilde{W}+\ell) + w_g U_3'(w_g W_g) = 0$$

and for $\tilde{W} = \overline{W}$ as

$$(4) \qquad -\left(\frac{w_p}{\bar{p}}\right)U_1'[-\overline{W}+\ell] + w_g U_3'(w_g W_g) = 0.$$

I. $w_p = (1+\gamma)\omega_p$, $w_g = \omega_g$.

Now we are ready to examine our first polar case. The private technology is subject to random shocks, while the public technology is not. We only treat the case $\tilde{W} < \overline{W}$. The same qualitative results for $\tilde{W} = \overline{W}$ can be derived manipulating (4) instead of (3) (except that $d\tilde{W} = 0$). Totally differentiating (3) we get

$$U_1'' dW + w_g^2 U_3'' dW_g = 0$$

while totally differentiating (1) yields

$$U_1'' dW - (1+\gamma)^2 \omega_p^2 U_2'' dW_g = -\omega_p [U_2' + C_2 U_2''] d\gamma.$$

Solving these two equations simultaneously by Cramers rule we conclude that if second-period consumption is a gross substitute for the other goods (as assumed), $d\tilde{W}/d\gamma > 0$, $d\tilde{W}_g/d\gamma < 0$, and, therefore, $d\tilde{W}_g/d\gamma/d\tilde{W}/d\gamma < 0$. If second-period consumption is a gross compliment, $d\tilde{W}/d\gamma < 0$, $d\tilde{W}_g/d\gamma > 0$, and it still holds that $d\tilde{W}_g/d\gamma/d\tilde{W}d\gamma < 0$.

The government's hiring policy is countercyclical relative to aggregate employment. The government should hire some of the unemployed when the private economy suffers unemployment. Given gross substitutes we have the intuitively obvious result. If the private sector technology becomes less productive, but the public sector technology is unchanged, private sector employment should fall and public sector employment should rise.

II. $w_p = (1+\gamma)\omega_p$, $w_g = (1+\gamma)\omega_g$.

Now we turn to the case where both private and public sector technologies are hit by a real shock. Once again we treat only the case $\tilde{W} < \overline{W}$. Similar manipulation of (4) yields similar results for $\tilde{W} = \overline{W}$ as before.

Totally differentiating (3) we now get

$$U_1'' dW + (1+\gamma)^2 \omega_g^2 U_3'' dW_g = -\omega_g [U_3' + G U_3''] d\Upsilon,$$

and the total derivative of (1) is unaffected at

$$U_1'' dW - (1+\gamma)^2 \omega_p U_2'' dW_g = -\omega_p [U_2' + C_2 U_2''] d\gamma.$$

Solving by Cramers rule again we conclude that if second-period consumption is a gross substitute and the public good a gross compliment, then $d\tilde{W}/d\gamma > 0$, $d\tilde{W}_g/d\gamma < 0$, and $d\tilde{W}_g/d\gamma/d\tilde{W}/d\gamma < 0$. Similarly, for second-period consumption a gross compliment and the public good a gross substitute $d\tilde{W}/d\gamma < 0$, $d\tilde{W}_g/d\gamma > 0$, and $d\tilde{W}_g/d\gamma/d\tilde{W}/d\gamma < 0$ still. Otherwise the signs are ambiguous. The case for countercyclical policy is weaker here, as would be anticipated. For example, if both second-period consumption and the public good are gross substitutes and if both technologies become less productive, then there are offsetting effects and no general results on employment public or private. This is not surprising.

So far we have only examined the first period of a shock. A shock this period influences future periods only by its effects upon $P_t \ell_t = \ell_{t+1}$. From (3) and our previous conclusion that $d\tilde{W}/d\ell = 1$ for $\tilde{W} < \overline{W}$, we conclude that $d\tilde{W}_g/d\ell = 0$ for $\tilde{W} < \overline{W}$. Any reduction in employment caused by a previous shock has the same policy response independent of the size of the reduction in employment. Using analysis similar to that in (I) above, we conclude that for $\tilde{W} = \overline{W}$, $d\tilde{W}_g/d\ell < 0$ if first-period consumption is a gross substitute and $d\tilde{W}_g/d\ell > 0$ if first-period consumption is a gross compliment. With gross substitutes the government follows a counter-cyclical policy. This subsequent counter-cyclical policy is optimal whether or not the public sector technology suffers the shock.

We have examined the optimal government expenditure response to a temporary employment decline. But what if the decline in employment is permanent, if the futures market is wiped out? If this occurs $\ell = 0$ in subsequent periods. This influences optimal government expenditure as discussed above. Note, however, that the government expenditure does not move the economy back to full employment. That is achieved by reinstitution of the futures market.

Concluding Comments

In our simple model with gross substitutes, the government should follow a counter-cyclical policy. Activist expenditure "demand management" is implied by the government acting as a competitive purchasing agent.

In the preceding analysis the government observes the anticipated shocks to technology. In our simple world this is reasonable. In the vastly more complicated real world, this assumption may not be justified. Individuals anticipate the shocks to their own differing technologies, but have reasons to conceal those anticipations. The government has data on stocks and preceding flows, but does not have data on the anticipated future shocks to technology. In our model, but with the government not observing γ, countercyclical policy can be based on employment. The government only loses the ability to adjust to small shocks on the full employment path.

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