Federal Reserve Bank of Minneapolis
Research Department Staff Report 51

Revised April 1983

A PRICE DISCRIMINATION ANALYSIS OF MONETARY POLICY*

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ABSTRACT

Monetary policy is analyzed within a model that ignores transaction costs and appeals solely to legal restrictions on private intermediation to explain the coexistence of currency and interest-bearing default-free bonds. The interaction between such legal restrictions and monetary policy is illustrated in versions of overlapping generations models that contain three assets: government-issued currency and bonds and real capital. It is shown that legal restrictions and the use of both currency and bonds permit the government to levy a discriminatory inflation tax and that such a tax may be better in terms of the Pareto criterion than a uniform inflation tax.

This paper studies the relationship between the amount of seignorage a government can raise and its monetary policy; that is, the composition of its liabilities as between currency and default-free bonds. The approach used differs from most others in that asset demands are generated without appealing to transaction costs. Instead, legal restrictions on private intermediation are used to explain asset demands and rate-of-return dominance--namely, the coexistence of non-interest-bearing currency and interest-bearing default-free bonds. An example of a legal restriction is a prohibition against private currency or private bank note issue.

We analyze monetary policy using price discrimination analysis. On our view, currency and default-free nominal bonds are essentially similar. The currency consists of small-denomination default-free titles to currency in the future, whereas the bonds consist of large-denomination or legally nonmarketable (as with U.S. savings bonds) default-free titles to currency in the future. Legal restrictions create separate markets for these different-sized packages, and the composition of government debt as between currency and bonds determines relative sales by the government to the different markets.

Basing the approach to asset demands on legal restrictions rather than transaction costs is, of course, quite different from the standard view.[1/] It is customary to invoke a Baumol (1952) or Tobin (1956) transaction cost, inventory model or to use a Clower-type, cash-in-advance constraint model to explain desired "cash" holdings in the presence of higher return assets. Those models, however, do not specify the cash that people need to make (consumption good) expenditures. It may, indeed, be plausible to identify this cash with small-denomination bearer notes and to explain the need for such notes by transaction costs. However, this does not imply that the government is the only potential supplier of such notes. In the absence of legal restrictions, general equilibrium models ought to consider the consequences of free entry into the provision of such notes, accomplished, for example, by banks issuing their own notes backed by holdings of Treasury bills. With free entry into such intermediation, the spread between the yield on intermediary assets--Treasury bills--and the zero yield notes is determined by intermediation costs. The nature of these costs can be inferred from the spreads charged by intermediaries for kinds of intermediation (common stock mutual funds, money market funds) for which there is free entry. Those spreads, which are small and constant, suggest that free entry would limit yields on Treasury bills to 1/2 or 1 percent per year.[2/] Because observed yields on Treasury bills are much higher than this, we think they must be explained by legal restrictions on private intermediation. The widespread prohibition against private bank note issue is an example of such a restriction.[3/]

We illustrate the interaction between legal restrictions and the composition of government indebtedness in very simple versions of overlapping generations models. In a sense, our analysis is a rather naive, public finance one. Given a time path of the real government deficit (net of interest payments), we consider the effects of various ways of financing that deficit.

If all taxes are distorting, then the best taxation package for financing a given government expenditure may include an inflation tax. We take that result as given and investigate alternatives to a uniform inflation tax. In some circumstances, the imposition of legal restrictions on private intermediation allows the government to levy discriminatory inflation taxes. In a "second-best" world, we would expect that in some circumstances discriminatory taxes would be more beneficial than a uniform tax. We give examples of such circumstances.

We hope that readers are not put off by our use of an overlapping generations model. As will be seen, the use of that model does not prejudge any of the issues under consideration and does no more than provide a convenient and simple intertemporal background for our analysis. The version we use includes a storage technology for goods, which is a real investment opportunity. Thus, when there are both government bonds and currency in the system, the model allows substitution among three assets: currency, bonds, and a form of real capital. In some circumstances, the model implies high substitution between real capital and bonds and even produces an Irving Fisher-type prediction about inflation and nominal interest rates. In other circumstances, it does not. The use of an overlapping generations model does not even prejudge such questions as whether, in the absence of all "frictions," nominal interest rates are zero or whether "money" disappears.[4/] In our models, the only frictions are legal restrictions. Remove them and one of two results follows: either there exists an equilibrium with valued fiat currency and zero nominal interest on default-free, nominal government bonds; or there does not exist such an equilibrium, and the only equilibria are those that ought to be interpreted as commodity money equilibria. We will provide examples of both situations.

The rest of the paper proceeds as follows. We first describe a somewhat general setting, an overlapping generations model, and the kind of legal restriction we study. We then describe stationary policies, a way of studying stationary equilibria, and a proposition that relates inflation rates and initial price levels. Then we provide two kinds of examples, one involving two-part pricing and the other involving price discrimination between groups.

THE MODEL

The model is peopled by overlapping generations living for two periods. We will describe equilibrium conditions for the model under laissez-faire (LF) and under portfolio restrictions that preclude all within-generation intertemporal trades, a regime that is labeled portfolio autarky (PA). We assume that portfolio autarky is enforced without cost. Portfolio autarky is, of course, a very stringent form of legal restriction. We study it primarily because it is relatively easy to work with and because it allows us to illustrate some general principles. Moreover, in our examples, portfolio autarky is equivalent to a government monopoly on the making of small change, a situation that has often prevailed (see, for example, Timberlake 1978, chap. 6).

Endowments, Preferences, and the Technology

The model is a stationary economy operating in discrete time.[5/] We let t, an integer, denote the date and let t = 1 be the current or initial date. A new generation, generation t, appears at each date t and is present in the economy at t and t+1. There is a single consumption good at each date t, and member h of generation t is endowed with some time t good, $w_t^h(t) \geq 0$, and some time t+1 good, $w_t^h(t+1) \geq 0$.

As for preferences, each member of generation 0 (those who at t = 1 are in the second and last period of their lives) maximizes consumption of time 1 good, while each member h of generation t, t > 0, has preferences that are represented by a twice differentiable, increasing, and strictly concave utility function, $u_t^h[c_t^h(t), c_t^h(t+1)]$, where $c_t^h(t+i)$ is consumption of time t+i good by member h of generation t. Under uncertainty, expected utility is maximized.[6/]

We assume that different generations are identical with regard to number and patterns of endowments and preferences, but we allow and make some use of intrageneration diversity.

There is also a technology for converting time t good into time t+1 good. The input is time t good and the output is, in general, a probability distribution of time t+1 good. Thus, if k is the input of time t good, the output of time t+1 good is zero if k < K and is x(t+1)k if $k \geq K$, where $x(t+1) = x_j > 0$ with probability θ_j; j = 1, 2, . . ., J. It is assumed that x(t+1) is observed after the input decision at t is made but before generation t+1 appears. Note that $K \geq 0$ is the minimum scale on which this technology can be operated. For inputs greater than this minimum, the technology is a stochastic (storage) technology with constant returns to scale. We assume that the minimum scale is such that it plays a role only under PA.

Government

We assume that the government attempts to consume $G(t) > 0$ units of time t good and that its only method of financing this expenditure is by a deficit. It can issue fiat currency, and it can issue one-period, default-free discount bonds. Each bond issued at t is a title to a known amount of currency at t+1. Thus, the cash flow constraint of the government is

$$G(t) = p(t)[M(t) - M(t-1)] + p(t)P_b(t)B(t) - p(t)B(t-1) \tag{1}$$

where $p(t)$ is the time t price of a unit of currency in terms of time t good (the inverse of the price level), $M(t-1)$ is the stock of currency held by the public from t-1 to t, $B(t-1)$ is the total face value in units of time t currency of the government bonds issued at t-1, and $P_b(t)$ is the price at t in terms of currency of an amount of bonds that pay one unit of currency at t+1. [$1/P_b(t)$ is unity plus the nominal interest rate on bonds issued at t.][7]

We describe the government financing scheme in terms of the ratio $B(t)/[B(t) + M(t)] \equiv \gamma(t) \in [0,1]$. The government also specifies a minimum size per bond. As with the minimum scale for storage, we assume that this minimum size plays a role only under PA. It is a minimum expenditure on bonds in terms of time t good, $F(t)$; that is, the minimum nominal face value at t, $\underline{b}(t)$, say, satisfies $p(t)P_b(t)\underline{b}(t) = F(t)$. The government also chooses whether to impose PA, the only alternative being LF.

Choice Problems and Equilibrium Conditions: Permanent Laissez-Faire

We describe the conditions for a perfect foresight, competitive equilibrium in terms of time t markets for claims on time t+1 good in "state" $x(t+1) = x_j$. The members of generation t in their role as consumers deal only in such claims. "Firms," operated by members of generation t in their role as "producers," supply such claims by storing time t good, currency, and newly issued bonds. These firms end up earning zero profits.

As a consumer, member h of generation t is assumed to maximize

$$\sum_j \theta_j u_t^h[c_t^h(t), c_t^h(t+1,j)]$$

subject to

$$(2) \qquad c_t^h(t) + \sum_j s_t(t+1,j)c_t^h(t+1,j) \leq w_t^h(t) + w_t^h(t+1)\sum_j s(t+1,j)$$

by choice of nonnegative $c_t^h(t)$ and $c_t^h(t+1,j)$; $j = 1, 2, \ldots, J$ where $c_t^h(t+1,j)$ is consumption of time t+1 good in state $x(t+1) = x_j$ and $s_t(t+1,j)$ is the price of one unit of this good in units of time t good. Letting $s_t(t+1)$ be the J-element vector of these prices, the solution to this maximization problem is a set of demand functions, $c_t^h(t+1,j) = a_j^h[s_t(t+1)]$; $j = 1, 2, \ldots, J$. We let $A_j[s_t(t+1)] \equiv \sum_h a_j^h[s_t(t+1)]$ be the set of aggregate demand functions, the summation being over the members of generation t.

In their role as producers, members of generation t may store the consumption good, currency, or bonds. Any producer maximizes profit as a price taker with regard to $s_t(t+1)$ and the time t and time t+1 prices of currency, which are taken to be state independent.

Profit in terms of time t good from storing $k \geq K$ units of the consumption good is $k\sum_j x_j s_t(t+1,j) - k$. Since this is linear in k, the condition that storage be finite in any equilibrium implies as an equilibrium condition

$$(3) \qquad \sum_j x_j s_t(t+1,j) \leq 1,$$

a condition that must hold with equality if total storage is as large as K.

Profit in terms of time t good from storing $m \geq 0$ units of currency is $mp(t+1)\sum_j s_t(t+1,j) - p(t)m$. Since this is linear in m, finiteness of the currency supply implies that prices in any competitive equilibrium satisfy

$$(4) \qquad p(t+1)\sum_j s_t(t+1,j) \leq p(t),$$

a condition that must hold with equality if firms store currency.

Profit in terms of time t good from storing bonds with nominal face value b such that $p(t)P_b(t)b \geq F$ is $bp(t+1)\sum_j s_t(t+1,j) - p(t)P_b(t)b$. Since this is linear in b, for b satisfying the constraint we must have

$$(5) \qquad p(t+1)\sum_j s_t(t+1,j) - p(t)P_b(t) \leq 0$$

and with equality if $b > 0$.

Notice that if both bonds and currency are held, then, by equations (4) and (5), $P_b(t) = 1$, a zero nominal interest rate.

We can now define an equilibrium (perfect foresight, competitive) under LF.

Given $\{G(t)\}$, $\{F(t)\}$, $\{\gamma(t)\}$, and $M(0) + B(0)$, an LF equilibrium consists of positive $\{s_t(t+1)\}$ and nonnegative $\{p(t)\}$, $\{K(t)\}$, where $K(t)$ is total storage of time t good and $K(t) = 0$ or $K(t) \geq K$, $\{M(t)\}$, and $\{B(t)\}$ such that for all $t \geq 1$

$$(6) \qquad A_j[s_t(t+1)] = \sum_h w_t^h(t+1) + x_j K(t) + p(t+1)[M(t) + B(t)]$$

for $j = 1, 2, \ldots, J$ and such that equations (1) and (3)-(5) (with their provisos) are satisfied. (The symbol $\{\cdot(t)\}$ is to be interpreted as a sequence defined for all $t \geq 1$.)

The Choice Problem and Equilibrium Conditions: Portfolio Autarky

Under PA, each member h of generation t again maximizes expected utility, but by choosing nonnegative consumption, nonnegative currency $[m^h(t)]$, bonds $[b^h(t)]$, and storage $[k^h(t)]$ subject to

$$(7) \qquad c_t^h(t) + p(t)m^h(t) + p(t)P_b(t)b^h(t) + k^h(t) \leq w_t^h(t),$$

$$(8) \qquad c_t^h(t+1,j) \leq w_t^h(t+1) + p(t+1)m^h(t) + p(t+1)b^h(t) + x_j k^h(t),$$

$$(9) \qquad p(t)P_b(t)b^h(t) \geq F(t) \text{ or } b^h(t) = 0, \text{ and } k^h(t) \geq K \text{ or } K^h(t) = 0.$$

It is convenient to redefine the currency and bond choice variables in real terms. Thus, let $q_1^h(t) = p(t)m^h(t)$ and $q_2^h(t) = p(t)P_b(t)b^h(t)$. Then, for $p(t) > 0$ and $P_b(t) > 0$, we may rewrite the above constraints as

$$(10) \qquad c_t^h(t) + q_1^h(t) + q_2^h(t) + k^h(t) \leq w_t^h(t),$$

(11) $\qquad c_t^h(t+1,j) \leq w_t^h(t+1) + R_1(t)q_1^h(t) + R_2(t)q_2^h(t) + x_j k^h(t),$

(12) $\qquad q_2^h(t) \geq F(t)$ or $q_2^h(t) = 0$, and $k^h(t) \geq K$ or $k^h(t) = 0$,

where $R_1(t) \equiv p(t+1)/p(t)$ and $R_2(t) \equiv p(t+1)/p(t)P_b(t)$. The $R_i(t)$'s are real gross rates of return, which we will hereafter refer to simply as rates of return. Figure 1 depicts the upper boundary in consumption space implied by equations (10)-(12) for the case $x_j \equiv 0$ (no storage), $R_2(t) > R_1(t)$, and $0 < F(t) < w_t^h(t)$.

Note that it is PA that prevents any individual from earning $R_2(t)$ on saving of less than $w_t^h(t) - F(t)$. In other words, under PA, two or more persons cannot share a bond. One person would have to buy the bond and issue IOU's to the others; however, such intermediation is ruled out by assumption under PA.

The solution to this maximization problem consists in part of demand functions (possibly correspondences) $q_i^h(t) = d_i^h[R_1(t),R_2(t),F(t)]$; $i = 1, 2$. We define a PA equilibrium in terms of aggregate demand functions (correspondences) $D_i[R_1(t), R_2(t),F(t)] \equiv \sum_h d_i^h[R_1(t),R_2(t),F(t)]$, $i = 1, 2$.

Given $\{G(t)\}$, $\{F(t)\}$, $\{\gamma(t)\}$, and $M(0) + B(0) > 0$, a PA monetary equilibrium consists of positive $\{p(t)\}$ and $\{P_b(t)\}$ and of nonnegative $\{M(t)\}$ and $\{B(t)\}$ such that for all $t \geq 1$,

(13) $\qquad D_1[R_1(t),R_2(t),F(t)] = p(t)M(t)$

(14) $\qquad D_2[R_1(t),R_2(t),F(t)] = p(t)P_b(t)B(t)$

and such that equation (1) holds, it being understood that the $R_i(t)$ are defined, as above, in terms of currency and bond prices.

STATIONARY POLICIES AND EQUILIBRIA UNDER PORTFOLIO AUTARKY

In the next two sections, we present examples that suggest the range of possibilities that can occur under PA. These examples present the stationary or constant inflation rate and bond yield equilibria for various specifications of the physical environment (tastes, endowments, and storage technologies) and for various constant values of $G(t)$, $\gamma(t)$, and $F(t)$, denoted, respectively, G, γ, and F. Our view is that G is given and that (monetary) policy under PA involves choosing γ and F.

We find it convenient to describe such equilibria in the following way. Letting R_i denote a constant value of $R_i(t)$, it follows from equation (1) for $t \geq 2$ and from (13) and (14) that an equilibrium (R_1,R_2) must satisfy $G = (1-R_1)D_1(R_1,R_2,F) + (1-R_2)D_2(R_1,R_2,F)$, where $1-R_1$ should be interpreted as the tax rate on currency holdings and $(1-R_2)$ as the tax rate on bond holdings. Moreover, to be a monetary equilibrium, it must also satisfy $R_2 \geq R_1 > 0$ and $D_i(R_1,R_2,F) > 0$ for at least one value of i. To allow a symbol to represent the set of (R_1,R_2)'s that satisfies these conditions and its dependence on G and F, we let

$$(15) \qquad S(G,F) = \{(R_1,R_2):(1-R_1)D_1(R_1,R_2,F) + (1-R_2)D_2(R_1,R_2F) = G,\ R_2 \geq R_1 > 0 \text{ and } D_i(R_1,R_2F) > 0 \text{ for at least one value of } i\}.$$

To go from a given G and F and a pair (R_1,R_2) in S(G,F) to equilibrium price sequences for currency and bonds, we need an associated initial price of currency, p(1). Using equations (13) and (14) for t = 1 and an initial condition for M(0) + B(0), we find an associated p(1) from equation (1) for t = 1; namely,

$$(16) \qquad G = D_1(R_1,R_2,F) + D_2(R_1,R_2,F) - p(1)[M(0) + B(0)].$$

Thus, given G, F, and M(0) + B(0), a monetary equilibrium is any (R_1,R_2) in S(G,F), an associated solution for p(1) from equation (16), and the associated paths of nominal supplies of currency and bonds given by equations (13) and (14), respectively. Thus, we can first study the set S(G,F) and then find the nominal asset supplies that "support" various elements of S(G,F) as stationary monetary equilibria. Notice that since $1/P_b(t)$, the gross nominal yield on bonds, is $R_2(t)/R_1(t)$, it follows from equations (13) and (14) that the currency and bond sequences imply a constant ratio of currency to bonds, or equivalently, a constant $\gamma(t)$.[8/]

Our last task before turning to examples is to relate solutions for p(1) to features of the S(G,F) sets. We are interested in p(1) because it determines the effects of alternative policies on the current old; p(1) determines the value of the given initial nominal wealth of the current old, M(0) + B(0). The following proposition says that if an interest-bearing bond solution has as low an inflation rate as a bond that does not bear interest (or money only) solution, then it has a lower initial price level.

Proposition 1: For given G and $M(0) + B(0) > 0$, if $(R^*,R^*) \varepsilon S(G,F^*)$, $(\tilde{R}_1,\tilde{R}_2) \varepsilon S(G,\tilde{F})$, $D_2(\tilde{R}_1,\tilde{R}_2,\tilde{F}) > 0$, and $\tilde{R}_2 > \tilde{R}_1 > R^*$, then $\tilde{p}(1) > p^*(1)$, where $\tilde{p}(1)$ is the $p(1)$ solution to equation (16) for $(\tilde{R}_1,\tilde{R}_2,\tilde{F})$ and $p^*(1)$ is that for (R^*,R^*,F^*).

Proof. In view of equation (16), we need only show that $D_1^* + D_2^* < \tilde{D}_1 + \tilde{D}_2$ where $D_i^* \equiv D_i(R^*,R^*,F^*)$ and $\tilde{D}_i \equiv D_i(\tilde{R}_1,\tilde{R}_2,\tilde{F})$. Since (R^*,R^*) ε $S(G,F^*)$ and $(\tilde{R}_1,\tilde{R}_2) \varepsilon S(G,\tilde{F})$, we have $(1-R^*)(D_1^* + D_2^*) = (1-\tilde{R}_1)\tilde{D}_1 + (1-\tilde{R}_2)\tilde{D}_2 < (1-\tilde{R}_1)(\tilde{D}_1 + \tilde{D}_2)$, where the inequality follows from $\tilde{R}_2 > \tilde{R}_1$. But then $\tilde{R}_1 > R^*$ implies $D_1^* + D_2^* < \tilde{D}_1 + \tilde{D}_2$.$\Delta$

Note that the "~" solution is a PA one in which bonds bear interest, while the "*" solution is one in which bonds, if they exist, sell at par.

NO DIVERSITY AND TWO-PART PRICING

We begin with an example that emphasizes the two-part pricing possibility inherent in our PA setup. We first assume that storage is not possible ($x_i \equiv 0$) and that all members of generation t have the same endowments and preferences. It turns out that under PA and some monetary policies, all individuals in equilibrium face a budget set like that shown in Figure 1. Moreover, with all of them alike, if $\gamma \varepsilon (0,1)$ and if bonds bear interest, some of them ("money holders") must be situated at a point like A (see Figure 2), while the others ("bondholders") must be situated at a point like B.

Our discussion of this no-diversity setup is built around Proposition 2.

Before stating the proposition, some notation and explanation is needed. Let N be the size of each generation, let $q(R) = d_1^h(R,R,0) + d_2^h(R,R,0)$ [where $d_i^h(R_1,R_2,F)$ is the PA demand correspondence defined above], let $S(G,0) = \{R: N(1-R)q(R) = G\}$, and let $\underline{R}_1 = \min S(G,0)$ and $\overline{R}_1 = \max S(G,0)$. Moreover, let $(\underline{c}_1,\underline{c}_2)$ be the unique solution to the following three conditions: $\underline{c}_1 + \underline{c}_2 = w_1 + w_2 - G/N$, where $(w_1,w_2) = [w_t^h(t),w_t^h(t+1)]$; $u_t^h(\underline{c}_1,\underline{c}_2) = u_t^h[w_1 - q(\underline{R}_1),w_2 + \underline{R}_1 q(\underline{R}_1)]$; and $\underline{c}_1 < w_1 - q(\underline{R}_1)$. (Hereafter, we drop the subscript and superscript on u.) And, finally, let $(\overline{c}_1,\overline{c}_2)$ be the unique solution to $\overline{c}_1 + \overline{c}_2 = w_1 + w_2 - G/N$; $u(\overline{c}_1,\overline{c}_2) = u[w_1 - q(\overline{R}_1),w_2 + \overline{R}_1 q(\overline{R}_1)]$; and $\overline{c}_1 < w_1 - q(\overline{R}_1)$.

Note that q(R) is per capita saving when all assets bear the rate-of-return R. Equivalently, it is per capita, desired, real money holding when money bears the rate-of-return R and there are no other assets. In Figure 3, we depict the function $(1-R)q(R)$, which is the real per capita revenue obtained by the government when R is the return on money and holding money is the only option. For any G, S(G,0) is the set of values of R that satisfies $N(1-R)q(R) = G$. Elements of the set S(G,0) can be interpreted as alternative money-only ($\gamma=0$) equilibria under PA and as alternative equilibria under LF. Of course, if S(G,0) is not empty, there are, in general, at least two elements in it.

Figure 4 depicts the allocations corresponding to the minimal and maximal elements of S(G,0) and the corresponding indifference curves, labeled $\underline{u}$ and $\overline{u}$, respectively. The 45-degree line represents consumption bundles, which if common to everyone in every generation t, $t > 1$, are consistent with the government consuming G in every period. Moreover, if some of the consumption bundle profiles that are identical across all generation $t > 1$ are outside the 45-degree line depicted, then, in order that the government consumes exactly G, some other bundles must be inside the line. Thus, for example, in a stationary equilibrium in which bondholders end up outside the line, money holders must end up inside it, and vice versa. Finally, cases 1 and 2 in Figure 4 refer to first-period bondholder consumption implied by different ranges for F.

Proposition 2: If $S(G,0)$ is not empty, $F \in (q(\bar{R}_1), w_1-\underline{c}_1)$, and $w_1 > 0$ and $w_2 > 0$, then for any number of bonds $n \in \{1,2, . . ., N\}$ there exists an equilibrium with a constant inflation rate, with positive nominal interest on bonds, and with N-n money holders and n bondholders. Moreover, if $F \in (q(\bar{R}_1), w_1-\bar{c}_1]$ (case 1), then $R_2 < 1$ and every member of generation t, $t > 1$, is on an indifference curve at least as high as $\bar{u}$; while if $F \in (w_1-\bar{c}_1, w_1-\underline{c}_1)$ (case 2), then every member of generation t, $t > 1$, is on an indifference curve lower than u.

The proof of Proposition 2 is given in the Appendix. We now discuss some consequences of Proposition 2 and of well-known optimality results for overlapping generations models.

Corollary 2.1: If $0 < n < N$, then any Proposition 2 equilibrium is not Pareto optimal.

Proof: This is clear from Figure 2. With N-n members of generation t, $t > 1$, having allocation A and with n members of the same generation having allocation B, there exists a rearrangement of these that gives everyone in the generation a preferred allocation.

Corollary 2.2: Let (c_1^*, c_2^*) be the preferred point on the 45-degree line of Figure 4 [that is, $c_1^* + c_2^* = w_1 + w_2 - G/N$ and $u_1(c_1^*, c_2^*)/u_2(c_1^*, c_2^*) = 1$]. If $n = N$ and $F \geq w_1 - c_1^*$, then any Proposition 2 equilibrium is Pareto optimal.

Proof: With $n = N$, all members of generation t, $t \geq 1$, are bondholders; therefore, the kind of within-generation misallocation that occurs when there are both money holders and bondholders is absent. Moreover, with $n = N$, the common consumption of every member of every generation $t \geq 1$ is on the 45-degree line of Figure 4. The lower bound on F ensures that this bundle is either the most preferred point on that line or is southeast of the most preferred point. Conditional on the government getting G per period, it is well known that all such allocations are Pareto optimal. (If the bundle is southeast of the most preferred bundle, then it is easily shown that no allocation improves the well-being of any member of generation t for any $t \geq 1$ without hurting the current old. [See, for example, the proof of Proposition 5 in the appendix in Wallace 1980.])Δ

Corollary 2.3: If $G > 0$, then case 1 is not empty and any Proposition 2, case 1 equilibrium is Pareto superior to any LF (or $n = 0$ PA) stationary equilibrium.

Proof. Under the hypotheses of Proposition 2, nonemptiness of case 1 is obvious if $G > 0$. Under the current setup, Pareto superiority of any Proposition 2, case 1 equilibrium follows if we can establish that any such equilibrium satisfies the hypotheses of Proposition 1.

The stationary LF equilibrium that puts all members of generation t, $t \geq 1$, on the $\bar{u}$ indifference curve is Pareto superior to any other LF stationary equilibrium (see Figure 4). But Proposition 2 says that there exists a case 1 equilibrium that puts all the members of generation t, $t \geq 1$, on an indifference curve at least that high and that has rates of return on money and bonds, (R_1, R_2), that satisfy $R_2 > R_1 \geq \bar{R}_1$. These satisfy the conditions of Proposition 1 and imply, therefore, that the initial price level in the case 1 equilibrium is lower than in the best LF equilibrium. Note, by the way, that $R_2 < 1$ in any case 1 equilibrium; although bonds bear a positive nominal interest rate, they bear a negative real interest rate in any case 1 equilibrium.Δ

Corollary 2.3 describes our first instance in which the imposition of PA and the use of bonds help in an unambiguous way. The general idea is familiar from public finance or second-best theory. With $G > 0$, it is well known that LF gives rise to a nonoptimal equilibrium. (See, for example, Proposition 7 of Wallace [1980].) It gives rise to an equilibrium with a uniform, distorting excise tax on second-period consumption. PA allows for the imposition of nonlinear taxes through the use of bonds. It is no surprise, then, that better allocations are possible under this broader set of possible tax schemes.

As this discussion suggests, it should not be possible to produce Pareto superior allocations with PA and bonds if $G = 0$. This is so. With $G = 0$, it is evident that case 1 is empty; $\bar{u}$ is tangent to the 45-degree line of Figure 4. Thus, if $G = 0$, only case 2 exists; in any case 2 equilibrium, the members of generation t, $t > 1$, are worse off than under the best LF equilibrium. We have not been able to establish whether the current old are necessarily better off in a case 2 equilibrium than under LF. In other words, we have not been able to establish whether the initial price level is necessarily lower in a case 2 equilibrium than it is under LF.

We now briefly turn to this setup with a storage technology for goods. If storage of the good is possible--if $x_j \neq 0$ and if K is not so large as to rule out storage of the good under PA--then $\underline{u}$ in Figure 4 must be replaced by the maximum of $\underline{u}$ and the level of utility implied by maximization of utility given only the option of storing the good. Subject to this reinterpretation of $\underline{u}$, Proposition 2 and the corollaries listed hold.

DISCRIMINATION BETWEEN GROUPS

Here we assume a simple kind of within-generation diversity. Each generation is composed of two groups, the "poor" and the "rich." Each member h of the poor group has an endowment $[w_t^h(t), w_t^h(t+1)] = (w_1^p, w_2^p)$, while each member h of the rich group has an endowment $[w_t^h(t), w_t^h(t+1)] = (w_1^r, w_2^r) = \lambda(w_1^p, w_2^p)$ for some $\lambda > 1$. Moreover, since we dealt with "corner" solutions in the last section, we now deal only with setups that are consistent with "interior" solutions for members of both groups.

We first assume that storage is not possible ($x_j \equiv 0$) and contrast two situations: one in which preferences are so similar that beneficial between-group discrimination is not possible (Proposition 3) and one in which preferences are different enough so that beneficial price discrimination is possible (example 1).

Systematic between-group differences in preferences seem, however, to be a farfetched basis for profitable price discrimination. Groups that differ with regard to their demands for government liabilities seem to do so primarily because they differ with regard to their access to alternative assets. Moreover, most legal restrictions on private intermediation that have the effect of limiting access to government bonds also limit access to other assets. The other setups of this section utilize these notions. In them, there is a real asset that under PA is accessible only to the rich. Even with identical preferences, this implies very different demand functions for government liabilities for the poor and for the rich.

As promised, our first result says that if the rich and the poor are sufficiently alike, then there cannot be an interior price discrimination solution that is Pareto superior to the best LF solution.

<u>Proposition 3</u>: If $x_j \equiv 0$, if the common utility function is homothetic, if $(w_1^r, w_2^r) = \lambda(w_1^p, w_2^p)$, $\lambda > 0$, and if (G,F) is such that $(R_1, R_2) \in S(G,F)$, $R_2 > R_1$, and yields internal solutions for the poor at R_1 and the rich at R_2, then there exists R such that $(R,R) \in S(G,0)$ and $R > R_1$.

<u>Proof</u>: Let $D^p(R)$ and $D^r(R)$ be the aggregate saving functions of the poor and rich, respectively, when members of each group are faced with the single rate-of-return R. It follows from the preference and endowment assumptions that $D^r(R) = \lambda^* D^p(R)$ for

some $\lambda^* > 0$. If the proposition is not true, then for all $R \varepsilon (R_1,R_2]$, $(1-R)D^p(R) + (1-R)\lambda^*D^p(R) < G$. But this implies $(1-R_2)D^p(R_2) < G/(1+\lambda^*)$ and $(1-R_1)D^p(R_1) < G/(1+\lambda^*)$. These inequalities, in turn, imply $(1-R_1)D^p(R_1) + (1-R_2)\lambda^*D^p(R_2) < G$, which contradicts $(R_1,R_2) \varepsilon S(G,F)$.$\Delta$

We now display a numerical example that shows that for nonhomothetic utility, there can exist interest-bearing bond solutions that are Pareto superior to the best LF equilibrium.

Example 1

Common (nonhomothetic) utility function: $u(c_1,c_2) = z(c_1) + z(c_2)$ with $z(c) = c^{.875} + \ln c$.

Endowments: 10×10^6 poor with $(w_1^p,w_2^p) = (.01,0)$; 100 rich with $(w_1^r,w_2^r) = (1000,0)$.

Storage technology: $x_j \equiv 0$.

Government policy: $G = 25{,}000$, $F = 1.0$, PA.

Note that we have imposed endowments such that in the relevant ranges, $z(c)$ for the poor is approximately $\ln c$, while $z(c)$ for the rich is approximately $c^{.875}$. Letting $d^p(R)$ and $d^r(R)$ be individual saving functions of poor and rich, respectively, Table 1 is generated by solving $(10 \times 10^6)(1-R_1)d^p(R_1) + 100(1-R_2)d^r(R_2) = 25{,}000$ for R_1, given various selected values of R_2. We know that there exists an equilibrium for each such (R_1,R_2) pair satisfying $R_2 > R_1$.

Table 1
Some Alternative Equilibria for Example 1

R_2	R_1	$d^p(R_1)$	$d^r(R_2)$
0.523	.523	.00499	25.24
0.550	.526	.00499	30.35
0.600	.534	.00499	43.57
0.650	.544	.00499	63.40
0.700	.554	.00499	92.33
0.750	.566	.00499	132.76
0.800	.573	.00499	186.32
0.850	.575	.00499	252.98
0.900	.565	.00499	330.51
0.950	.541	.00499	414.65
1.000	.499	.00499	500.00

The first row of Table 1 is the LF solution. Both poor and rich face a single rate-of-return (.523), an inflation rate of almost 100 percent. While each poor person saves almost half of w_1^p (each would save exactly half, or .005, if the utility function was exactly $\ln c_1 + \ln c_2$), each rich person saves only about 2.5 percent of w_1^r. Note that the movement of the initial price level across rows is implied by the movement of the sum $(10 \times 10^6)d^p + (100)d^r$ by way of equation (16); the higher this sum is, the lower the initial price level.

Each of the rows for $R_2 = .55$ to $R_2 = .95$ depicts a discriminating solution that is Pareto superior to LF. In each case, both the poor and the rich face higher rates of return than under LF, and the value of the asset holdings of the current old is also higher than under LF. At $R_2 = 1.00$ (and at values of R_2 sufficiently close to 1.00), the PA solution is not Pareto superior to LF; although the rich and the current old are better off than under LF, the poor are worse off, which is to say that the inflation rate is higher.9/

We now turn to setups with a storage technology for goods. We begin with a nonstochastic technology--namely, one with $x_j = x > 0$ for all j. Thus, if $k \geq K$ units of time t goods are stored, output of time t + 1 good is xk with certainty. We assume in the rest of this section that F and K are such that neither is binding for the rich and both are binding on the poor under PA in a way that limits the poor to holding currency.

Propositions 4a and 4b compare PA with no bonds ($\gamma = 0$) to PA with some bonds ($\gamma > 0$).

Proposition 4a: If $x > 1$ and if $(R_1,R_2) \in S(G,F)$ and $\gamma > 0$ (bonds are outstanding), then there exists a stationary monetary equilibrium with $\gamma = 0$ and a lower inflation rate.

Proof: Since only the rich hold bonds, we have $R_2 \geq x > 1$ and, hence, $(1-R_2)D_2(R_1,R_2,F) < 0$. Therefore, $(1-R_1)D_1(R_1,R_2,F) > G$, $R_1 < 1$, and $D_1(R_1,R_2,F) = D^p(R_1)$, where $D^p(R)$ is the aggregate saving function of the poor, as defined in the proof of Proposition 3. Then, since $(1-R)D^p(R) > G \geq 0$ for $R = R_1 < 1$ and $(1-R)D^p(R) = 0$ for $R = 1$, continuity of $(1-R)D^p(R)$ implies the existence of an $R \in (R_1,1]$, say R^*, with $(1-R^*)D^p(R^*) = G$. This is a $\gamma = 0$ equilibrium because, since $x > R^*$, the rich are content to have their saving entirely in the form of storage of the good.Δ

Proposition 4b: If $x < 1$ and if $(R_1,x) \varepsilon S(G,F)$ with $R_1 < x$ and with $D_2(R_1,R_2,F) < D^r(R_2)$ [a PA equilibrium with saving of the rich, $D^r(R_2)$, composed partly or totally of storage of the good], then there exists a Pareto superior PA equilibrium (with more bonds and less storage).

Proof: We are given $(1-R_1)D^p(R_1) + (1-x)D_2(R_1,x,F) = G$ with $D_2(R_1,x,F) < D^r(x)$. It follows from the continuity of $(1-R)D^p(R)$ that there exist values of $B \varepsilon [D_2(R_1,x,F),D^r(x)]$ such that the R that satisfies $(1-R)D^p(R) + (1-x)B = G$, denoted $R(B)$, satisfies $x > R(B) \geq R_1$, where equality arises if and only if $x = 1$. It is evident that for any such B, $[R(B),x] \varepsilon S(G,F)$ and can be supported as an equilibrium. Pareto superiority follows by showing that the initial price level is lower for any such $[R(B),x]$ equilibrium than it is for the (R_1,x) equilibrium. From $(1-R_1)D^p(R_1) + (1-x)D_2(R_1,x,F) = [1-R(B)]D^p[R(B)] + (1-x)B$ and $x > R(B) \geq R_1$, we get $D^p(R_1) + D_2(R_1,x,F) < D^p[R(B)] + B$. Our conclusion for the price level follows from equation (16).Δ

Although, as these propositions show, bond issue has different effects on the inflation rate depending on the value of x, in some other respects the value of x is not so critical. So long as $R_1 < x$, there is a range over which the demand for bonds is perfectly elastic at $R_2 = x$. Over this range, higher γ almost certainly implies a lower initial price level.[10] Moreover, for small and positive γ, these economies are ones for which the nominal interest rate on bonds and the inflation rate satisfy the Fisherian relationship: unity plus the nominal interest rate equals a constant, x, times unity plus the inflation rate.

Note that the economy of Proposition 4a is one in which there cannot exist a monetary equilibrium under LF with $G > 0$. In that economy, remove the legal restriction and nominal interest rates do not "go to" zero; instead, money "disappears." The economy of Proposition 4b is quite different. There, for small enough G's and some endowment patterns, LF is consistent with the existence of a stationary equilibrium with valued fiat currency and a zero nominal interest rate. In other words, remove the legal restriction in that economy and it is possible that nominal interest rates "go to" zero.

We now describe our last example, one with a stochastic storage technology. It implies a smooth demand function for government bonds on the part of the rich, one that is not perfectly elastic over some range.

Example 2

Common utility function: $u(c_1,c_2) = \ln c_1 + \ln c_2$.

Endowments: 10×10^6 poor with $(w_1^p,w_2^p) = (.01,0)$; 100 rich with $(w_1^r,w_2^r) = (1000,0)$.

Storage technology: $K = 1$, $j = 2$, $x_1 = 2.0$, $x_2 = .35$, $\theta_1 = \theta_2 = .5$.

Government policy: $G = 25{,}000$, $F = 1.0$, PA.

Table 2
Some Alternative Equilibria for Example 2

R_2	R_1	d_g^p	d_g^r	d_k^p	d_k^r
0.5000	.5000	.005	0	0	500
0.5957	.5000	.005	0	0	500
0.6000	.5057	.005	7	0	493
0.6500	.5551	.005	79	0	421
0.7000	.5808	.005	135	0	365
0.7500	.5906	.005	181	0	319
0.8000	.5889	.005	222	0	278
0.8500	.5779	.005	240	0	260
0.9000	.5590	.005	295	0	205
0.9500	.5330	.005	330	0	170
1.0000	.5000	.005	365	0	135

Table 2 describes PA equilibria for selected values of R_2. (For this example, an LF equilibrium does not exist.) In Table 2, the subscript g stands for government liabilities and the subscript k for the real asset. Note that the rich hold no government liabilities if bonds do not bear interest. Note also that every solution displayed is Pareto superior to the PA solution with $\gamma = 0$, the first (and second) rows of the table.

Finally, note that there is high substitutability between bonds and real investment (indeed, perfect crowding out) and no substitutability between bonds and currency. This happens despite the fact that currency and bonds have certain rates of return while real investment in our example has a very risky return distribution. On the basis of these rate-of-return distributions, one might expect that bonds and currency would be substitutable and that bonds and real investment would not be highly substitutable (see Tobin 1963b). In this example and in the Proposition 4a and 4b setups, the restriction that allows bonds to dominate currency in terms of rate of return also gives rise to high substitutability between bonds and storage of the good.

CONCLUDING REMARKS

The examples presented above show that models that appeal solely to legal restrictions to explain rate-of-return dominance can be used to address questions about monetary policy and can be made to imply rate-of-return patterns, or kinds of substitutability among assets, like those we sometimes observe. Moreover, our examples show that legal restrictions can in some circumstances make sense.[11/]

Although we do not want the legal restriction theory of rate-of-return dominance to be judged by whether every private sector intermediation restriction ever proposed or put into effect was motivated by price discrimination, we do find it reassuring that this seems sometimes to have been the case. Consider, for example, the following statement by the U.S. secretary of the treasury that was circulated in 1920:

> It has been brought to my attention that numbers of merchants throughout the country are offering to take Liberty Loan Bonds at par, or even in some cases at a premium, in exchange for merchandise. While I have no doubt that these merchants are actuated by patriotic motives, I am sure that they have failed to consider the effect which the acceptance of their offers would have

> upon the situation. We are making the strongest effort to have these Government Bonds purchased for permanent investment by the people at large, to be paid for out of the past or future savings of those who buy them. Purchases thus made not only result in providing funds for the use of the Government, but they also effect a conservation of labor and material. When the bonds are exchanged for merchandise, it defeats the primary object of their sale, it discourages thrift and increases expenditures. . . .

This statement also supports the view that there is a fine line between currency and default-free bonds, a line that would largely disappear without legal restrictions on private intermediation.[12/]

Finally, we should emphasize that our model and examples are motivated by positive rather than normative considerations. We view ourselves as providing an explanation for a seemingly paradoxical observation--namely, legal restrictions on private intermediation and a multiplicity of government liabilities. Of course, if one accepts the theory that underlies our analysis, then a kind of normative analysis is suggested. But a serious normative analysis should treat inflation taxes simultaneously with other taxes and, perhaps, with expenditures and should be concerned with the difficulty of implementing and enforcing various kinds of legal restrictions on intermediation.

Appendix: Proof of Proposition 2

Let $\overline{g}$ be the unique maximum of $(1-R)q(R)$ and let $\underline{R}$ be the unique value of R such that $q(R) = 0$ (see Figure 3). Also, let

$$S_{[a,b]}(R_2) = \{R \mid (N-n)(1-R)q(R) + n(1-R_2)F = G \text{ and } R \varepsilon [a,b]\}.$$

The crucial fact we use, which is implied by the continuity of $(1-R)q(R)$ in R, is as follows: if $[a,b] \quad [\underline{R},1]$, then $\{R_2 S_{[a,b]}(R_2)\}$ is a continuous curve in $\{1-G/nF, 1-[G-(N-n)\overline{g}]/nF\} \times [a,b]$.

Now, let $u^m(R) = u[w_1-q(R), w_2+Rq(R)]$ and let $u^b(R) = u(w_1-F, w_2+RF)$, where u^m is to be interpreted as money-holder utility and u^b as bondholder utility. It follows that $\{R_2, u^m[S_{[a,b]}(R_2)]\}$ is a continuous curve in $\{1-G/nF, 1-[G-(N-n)\overline{g}]/nF\} \times [u^m(a), u^m(b)]$ and that $\Omega = [(R_2,u) \mid R_2 > -w_2/F \text{ and } u < u^b(R_2)]$ is a convex set in $(-\infty,\infty) \times (-\infty,\infty)$.

We now show that there exist points of $\{R_2, u^m[S_{[a,b]}(R_2)]\}$ both outside of and in Ω, and we thereby establish that there exist one or more points of the former that are on the boundary of Ω. We also show that there is an equilibrium corresponding to any such point.

Let $\hat{R}_2$ be such that $(1-\hat{R}_2)F = G/N$. [Note that $\hat{R}_2$ is such that $(w_1-F, w_2+\hat{R}_2F)$ is on the 45-degree line of Figure 4.] We consider separately two cases that correspond to cases 1 and 2 in Figure 4.

<u>Case 1</u>: $u^b(\hat{R}_2) > \overline{u}$. Here we let $[a,b] = [\overline{R}_1, 1]$.

Since $(1-\overline{R}_1)q(\overline{R}_1) = G/N$ and $(1-\hat{R}_2)F = G/N$, $\overline{R}_1 \in S_{[\overline{R}_1,1]}(\hat{R}_2)$. And since $u^b(\hat{R}_2) \geq \overline{u} = u^m(\overline{R}_1)$, $[\hat{R}_2, u^m(\overline{R}_1)]$ is a point of $\{R_2, u^m[S_{[\overline{R}_1,1]}(R_2)]\}$ that is in Ω.

We now show that $[1-G/nF, u^m(1)]$ is a point of $\{R_2, u^m[S_{[\overline{R}_1,1]}(R_2)]\}$ that is not in Ω. First, $1 \in S_{[\overline{R}_1,1]}(1-G/nF)$, which implies that $[1-G/nF, u^m(1)]$ is a point of $\{R_2, u^m[S_{[\overline{R}_1,1]}(R_2)]\}$. Now, if $1 - G/nF \leq -w_2/F$, then, by definition, $[1-G/nF, u^m(1)]$ is not in Ω. If $1 - G/nF > -w_2/F$, it follows from $1 - G/nF \leq 1$ and $F > q(\overline{R}_1)$ that $u^m(1) > u^b(1-G/nF)$. This also implies that $[1-G/nF, u^m(1)]$ is not in Ω.

Having shown that there are points of $\{R_2, u^m[S_{[\overline{R}_1,1]}(R_2)]\}$ both in and outside of Ω, it follows that there is a point on the former that is on the boundary of Ω. Let us denote by (R_2^*, R_1^*) the associated point on the curve $[R_2, S_{[\overline{R}_1,1]}(R_2)]$.

We have shown that (i) $u^m(R_1^*) = u^b(R_2^*)$; (ii) $(N-n)(1-R_1^*)q(R_1^*) + n(1-R_2^*)F = G$; and (iii) $R_1^* \geq \overline{R}_1$. To establish that (R_2^*, R_1^*) is an equilibrium, we will show that $b = F$ maximizes $u(w_1-b, w_2+R_2^*b)$ subject to $b \geq F$. This follows from (i) if we can show that $F > q(R_1^*)$.

Suppose $F \leq q(R_1^*)$. If so, then since $R_2^* \geq R_1^*$ (which follows from [i]), (ii) implies $(1-R_1^*)q(R_1^*) \geq G/N$. But by definition of $\overline{R}_1$, this implies $R_1^* \leq \overline{R}_1$. From (iii) we conclude that $R_1^* = \overline{R}_1$ or that $F \leq q(\overline{R}_1)$, a violation of our hypothesis on F.

Our last task for case 1 is to show that $R_2^* \leq 1$. Since $R_1^* \geq \overline{R}_1$, $(1-R_1^*)q(R_1^*) \leq G/N$. This and (ii) imply $(1-R_2^*)F \geq G/N$ and, therefore, $R_2^* \leq \hat{R}_2$. Since $\hat{R}_2 \leq 1$, we have $R_2^* \leq 1$.

<u>Case 2</u>: $u^b(\hat{R}_2) < \overline{u}$. Here we let $[a,b] = [\underline{R}_1, \overline{R}_1]$.

Clearly, $\underline{R}_1 \in S_{[\underline{R}_1,\overline{R}_1]}(\hat{R}_2)$. Also, $u^m(\underline{R}_1) < u^b(\hat{R}_2)$. It follows that $[\hat{R}_2, u^m(\underline{R}_1)]$ is a point of $\{R_2, u^m[S_{[\underline{R}_1,\overline{R}_1]}(R_2)]\}$ that is in Ω. Since $\overline{R}_1 \in S_{[\underline{R}_1,\overline{R}_1]}(\hat{R}_2)$ and $u^b(\hat{R}_2) < u^m(\overline{R}_1)$ by assumption, it follows that $[\hat{R}_2, u^m(\overline{R}_1)]$ is a point of $\{R_2 u^m[S_{[\underline{R}_1,\overline{R}_1]}(R_2)]\}$ that is not in Ω. Therefore, as in case 1, there is a point of the former that is on the boundary of Ω. Let us again denote by (R_2^*, R_1^*) the corresponding point of the curve $[R_2, S_{[\underline{R}_1,\overline{R}_1]}(R_2)]$.

We now show that $u^b(R_2^*) < \overline{u}$. Suppose to the contrary that $u^b(R_2^*) \geq \overline{u}$. Because $u^b(\hat{R}_2) < \overline{u}$, $R_2^* > \hat{R}_2$ and, therefore, $(1-R_2^*)F < G/N$. This implies $(1-R_1^*)q(R_1^*) > G/N$. And since $u^m(R_1^*) = u^b(R_2^*) \geq \overline{u}$, we also have $R_1^* \geq \overline{R}_1$. But this and $(1-R_1^*)q(R_1^*) > G/N$ contradict the assumption that $\overline{R}_1$ is the largest value of R satisfying $(1-R)q(R) = G/N$.

For case 2 we have now shown that (i) $u^m(R_1^*) = u^b(R_2^*) < \overline{u}$ and (ii) $(N-n)(1-R_1^*)q(R_1^*) + n(1-R_2^*)F = G$. To complete the argument we must, as for case 1, show that $F > q(R_1^*)$.

Suppose instead that $F \leq q(R_1^*)$. Then since $R_2^* \geq R_1^*$ by the first equality of (i), (ii) implies $(1-R_1^*)q(R_1^*) \geq G/N$ and $(1-R_2^*)F \leq G/N$. In words, money holders are on or inside the 45-degree line of Figure 4 and bondholders are on or outside it. This and (i) and $F \leq q(R_1^*)$ imply $F < q(\overline{R}_1)$, a violation of our hypothesis on F.

NOTES

*This paper was written while John Bryant was on the staff of the Federal Reserve Bank of Minneapolis. We are indebted for financial support to the Bank and to the National Science Foundation under grant SOC 77-22743 awarded to the University of Minnesota.

1/However, Tobin (1963a) and Fama (1980) express somewhat similar views.

2/In Bryant and Wallace 1979a, we showed that an intermediation technology characterized by total real resource costs that are proportional to the real value of the assets intermediated implies an upper bound on the difference between nominal yields on intermediary assets and liabilities, the kind of spread often observed when there is free entry into intermediation. Another source of information about the costs of note-issue intermediation is provided by estimates of the cost to the U.S. government (cost to the Treasury and Federal Reserve combined) of maintaining the stock of U.S. currency. For all but the smallest denominations, the annual costs seem to be a small fraction of 1 percent of the outstanding stock.

3/Some readers of earlier drafts of this paper have questioned our emphasis on currency on the ground that demand deposits have long been dominated in rate of return and that there is approximately free entry into the provision of demand deposits. Many observers, however, have explained the pricing of demand deposit services in the United States in the past 20 years or so as arising primarily from (i) reserve requirements, (ii) interest ceilings, and (iii) zero marginal-cost check clearing provided by the Federal Reserve. Their prediction is that absent (i)-(iii), interest would be paid on deposits at the rate paid on default-free securities, with a per unit charge levied on each check written. Under such pricing of demand deposits, the main rate-of-return paradox to be explained would be that between currency and other assets. We will, by the way, soon get a test of this pricing prediction. Almost at the time we are writing this, the Federal Reserve is implementing a per unit charge on check clearing and has plans to eliminate interest ceilings. Note, by the way, that under such pricing of demand deposits services, the "cash" of the inventory models of money demand cannot be identified as including demand deposits.

4/ See Samuelson's (1947, pp. 123-24) discussion of these possibilities.

5/ The model is similar to those used in Bryant and Wallace 1980 and Wallace 1981.

6/ If government consumption of time t good, G(t), affects individual welfare, it is assumed to do so in a separable way. That is, if $V_t[c_t^h(t), c_t^h(t+1), G(t), G(t+1)]$ is the utility function of member h of generation t, then $V_t^h(\quad) = U_t^h\{u_t^h[c_t^h(t), c_t^h(t+1)], v_t^h[G(t), G(t+1)]\}$, where U_t^h is increasing in its first argument.

7/ Note that equation (1) implies that explicit taxes are not levied and, in particular, are not levied to cover interest on debt. One interpretation of this is that the government has exhausted the possible use of explicit taxes and that G(t) represents government consumption in excess of that financed by explicit taxes.

8/ That stationary solutions for real variables are supported by constant ratios of currency to bonds is not surprising. Basically, it follows from a well-known neutrality result: neutrality holds for once-for-all proportional changes in both bonds and currency (see, for example, Patinkin 1961). Stationary solutions are not in general supported by an arbitrary, constant rate of growth of currency. In many settings, a given rate of growth of currency implies a highly nonstationary path for the ratio of currency to bonds and, therefore, is a policy that is inconsistent with the existence of any simple kind of equilibrium. See Bryant and Wallace 1979b for a discussion of this point in a setting with a stochastic deficit.

9/ One can construct examples in which the initial price level is not decreasing in R_2. One such example is the following. Common utility function: $u(c_1,c_2) = c_1^{1/2} + c_2^{1/2}$; endowments: 1,000 poor with $(w_1^p,w_2^p) = (1.0,0)$ and 100 rich with $(w_1^r,w_2^r) = (10,0)$; storage technology: $x_j \equiv 0$; government policy: $G = 0$, $F = 1.0$, PA. The reader can verify numerically that for $R_2 \varepsilon [1.0,1.1]$ and $(R_1,R_2) \varepsilon S(G,F)$, $1000d^p(R_1) + 100d^r(R_2)$ is decreasing in R_2 and, hence, that the initial price level is increasing in R_2.

10/ It is easy to produce examples in which higher γ implies a lower initial price level and a higher inflation rate. In such instances, casual observers could mistake the once-for-all price level effect for a favorable inflation rate effect.

11/ Although we have emphasized situations in which legal restrictions and bonds can generate Pareto superior outcomes, those situations are very special. With more diversity within generations--in particular, with both savers (lenders) and dis-savers (borrowers) in the same generation--our policies would tend to produce noncomparable outcomes. See Sargent and Wallace 1982 for an analysis that emphasizes the different impacts of various monetary policies on borrowers and lenders.

12/This statement came into our hands quite by accident. It was sent by John R. Shuman, president of the Commonwealth Club of California, to Lindley Clark of the Wall Street Journal with a copy to Preston Miller of the Federal Reserve Bank of Minneapolis. Here is the complete text of Mr. Shuman's October 10, 1980, letter to Mr. Clark: "Your October 7 article expressing Mr. Miller's views that the securities the Treasury sells are themselves 'money' was of particular interest to me in light of this 1920 letter which we found when recently cleaning out our Club files. It appears that even as long as 60 years ago the distinction between Treasury securities and money was sustainable only by exhortations from the Department." The letter Mr. Shuman refers to was written in September 1920 by Treasury officials to members of the Commonwealth Club and referred to the statement of the secretary of the treasury quoted in the text.

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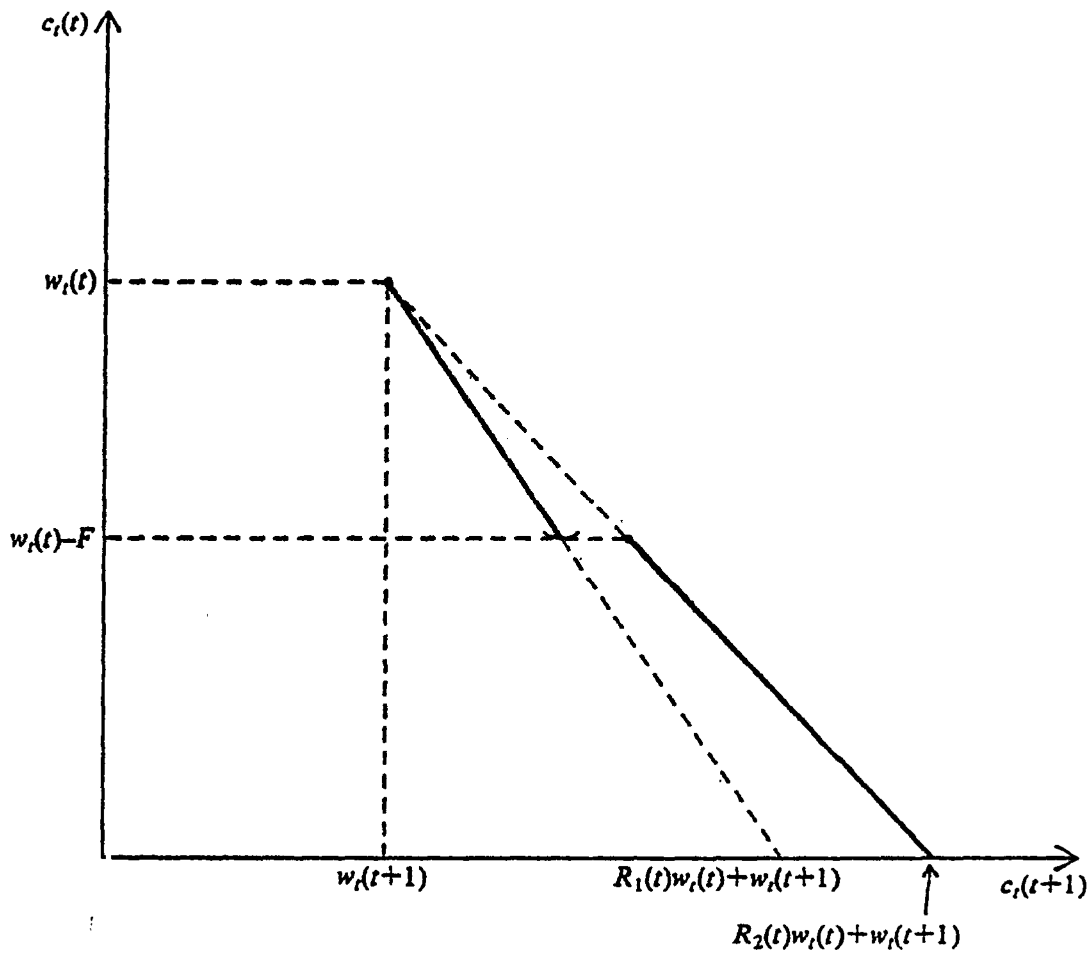

Figure 1

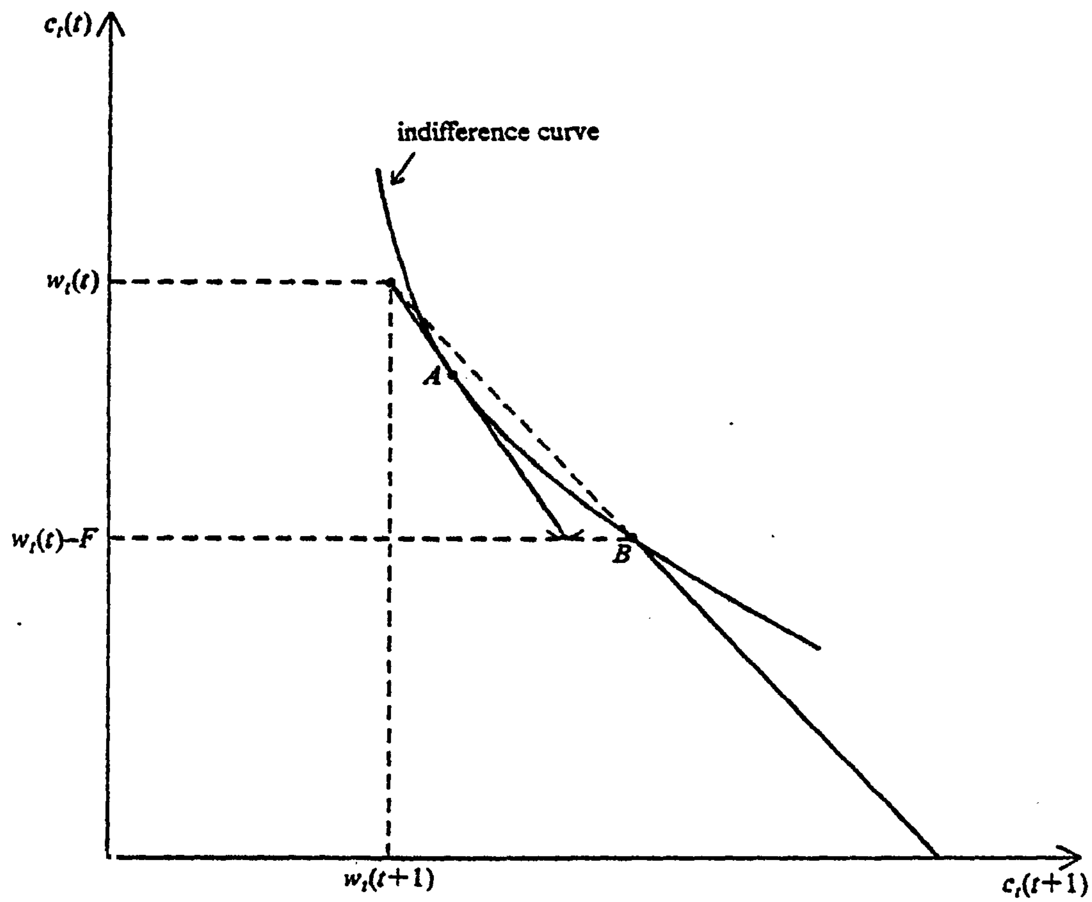

Figure 2

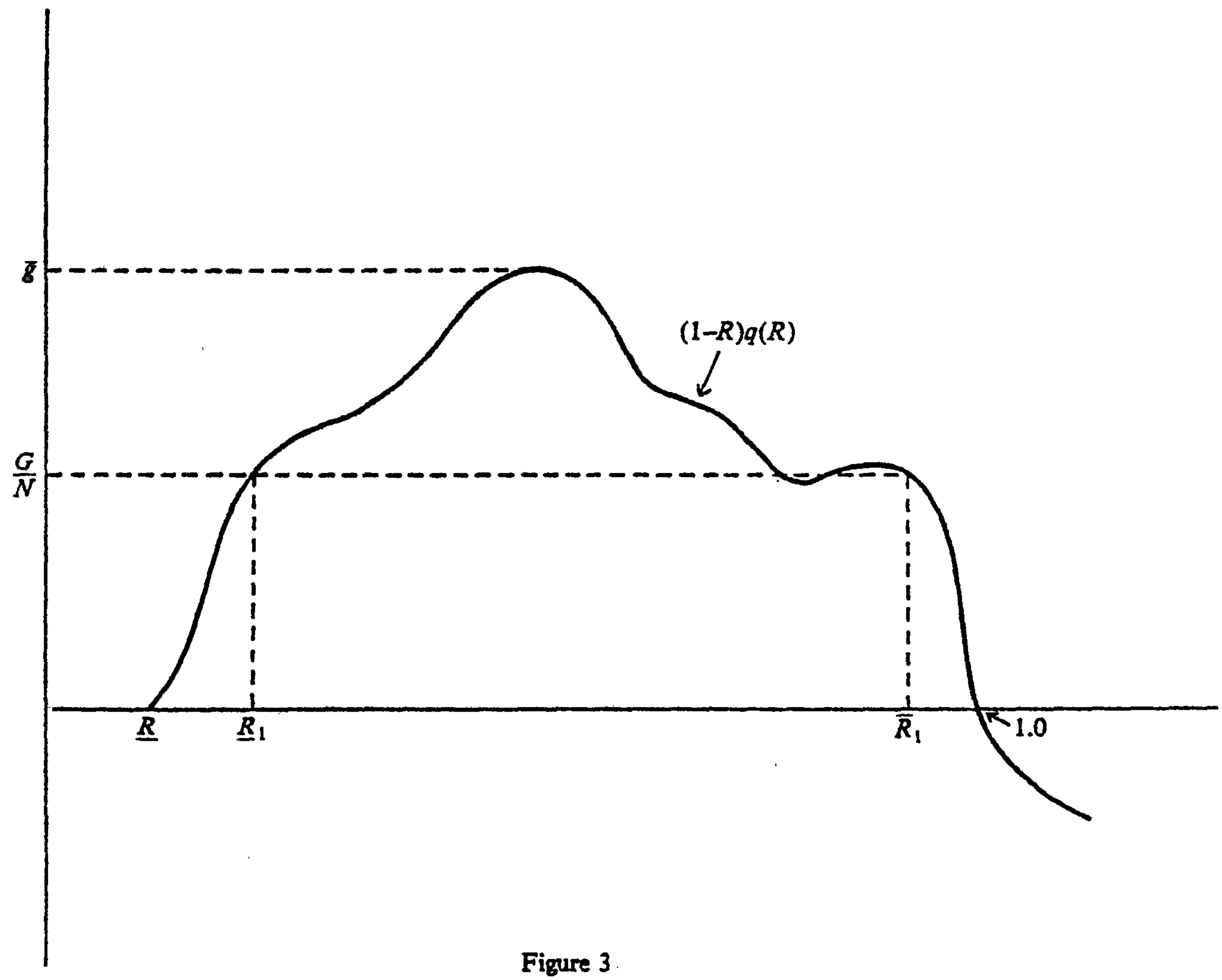

Figure 3

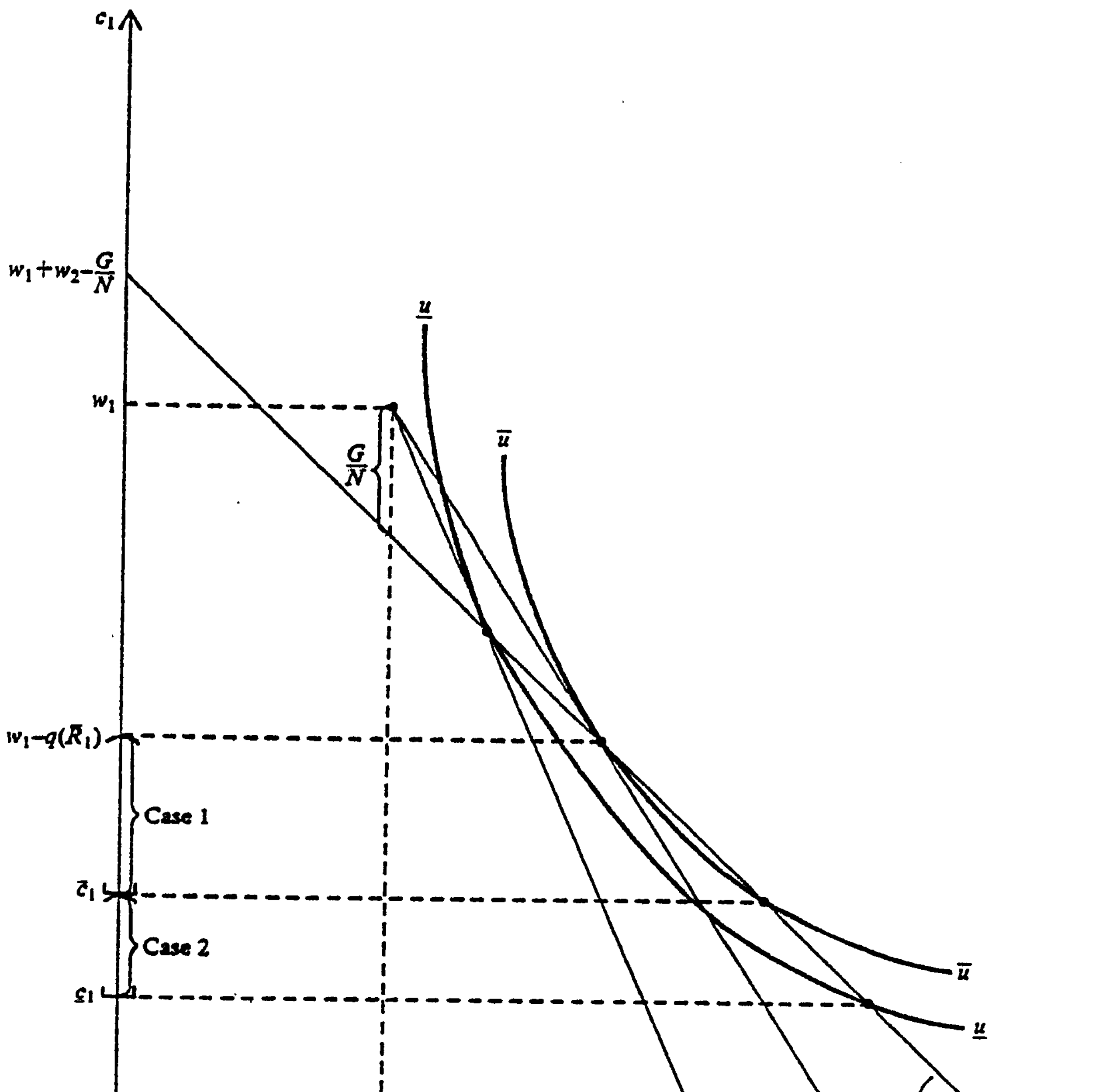
c_1
$w_1+w_2-\frac{G}{N}$
w_1
$\frac{G}{N}$
$\underline{u}$
$\overline{u}$
$w_1-q(\overline{R}_1)$
Case 1
$\overline{c}_1$
Case 2
$\underline{c}_1$
$\overline{u}$
$\underline{u}$
45°
w_2
$w_2+\underline{R}_1w_1$
$w_2+\overline{R}_1w_1$
c_2

Figure 4