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Abstract

In this paper, we study the optimal design of financial safety nets under limited private credit. We ask when it is optimal to restrict ex ante the set of investors that can receive public liquidity support ex post. When the government can commit, the optimal safety net covers all investors. Introducing a wedge between identical investors is inefficient. Without commitment, an optimally designed financial safety net covers only a subset of investors. Compared to an economy where all investors are protected, this results in more liquid portfolios, better social insurance, and higher ex ante welfare. Our result can rationalize the prevalent limited coverage of safety nets, such as the lender of last resort facilities.

Keywords: Public liquidity provision, bailouts, safety nets, time inconsistency
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1 Introduction

Safety nets are a central pillar of the current financial architecture. By granting liquidity support to a collection of institutions, a safety net can relieve the liquidity strains of financially distressed entities. A long-standing concern about safety nets, reinvigorated after the financial crisis, is that they can lead to excessive risk taking.[1] Accordingly, a key question regarding the design of safety nets is: How should the stability gained from a financial safety net be balanced against the moral hazard problem? Despite extensive discussions, the literature lacks a theoretical framework that can be used to address this question.

In this paper, we tackle the design of financial safety nets using a stylized model of liquidity demand under limited private credit. As in Holmström and Tirole (1998), the government can relax credit constraints by providing public liquidity. The question we address is whether the government should restrict ex ante the set of investors to whom it provides liquidity support ex post. In a nutshell, how wide should the financial safety net be?

The model features investors that save in short-term and long-term assets. These investors are subject to private idiosyncratic liquidity shocks, which occur before the long-term asset's payoffs are realized, as in Diamond and Dybvig (1983). Private contracts are not enforceable—which limits borrowing between investors to smooth liquidity shocks—and investors can anonymously trade bonds that the government issues to finance liquidity facilities. The new feature we introduce in this model is a government's choice about the share of investors that are eligible for public liquidity support. Specifically, we consider a government that chooses at time 0 the share of investors that will be eligible for liquidity support in the interim period, but without commitment about the magnitude of this liquidity support. This intends to capture the fact that only a subset of financial institutions are granted access to facilities such as the discount window or deposit insurance, which is a cornerstone of the current financial architecture. We label the set of investors eligible for ex post public support the *protected sphere* and the set of investors excluded from it the *unprotected sphere*.

Our analysis of financial safety nets delivers several results, both positive and normative. We first show that if the government can commit to a future liquidity provision policy, the optimal safety net covers the entire set of investors. With commitment, the government can provide an amount of public liquidity that induces the efficient amount of investment in long-

[1] Greenspan (2001) notably warned that policymakers must be "very cautious about purposefully or inadvertently extending the scope and reach of the safety net." Following the US financial crisis, there have been discussions surrounding Dodd-Frank on whether shadow banks should have access to lender of last resort facilities (see, e.g., Fischer, 2016).

term assets and thereby leads to the efficient allocation.[2] Offering a differential treatment to identical investors is inefficient if the government has commitment. In this case, the optimal size of the unprotected sphere is zero.

We then consider the optimal safety net when the government lacks commitment. Specifically, we study a time-consistent equilibrium in which the government chooses without commitment the liquidity support in the interim period. We can characterize in closed form three distinct regions depending on how wide the safety net is (i.e., how large is the protected sphere chosen by the government at time 0). When there is a large protected sphere, protected investors invest large amounts in the long-term asset and the economy features high production efficiency. As it turns out, all investors achieve the same level of welfare regardless of whether they are in the safety net, because interest rates on short-term and long-term assets are equalized in equilibrium. When there is a midsize protected sphere, protected investors invest only in long term assets and obtain higher welfare than unprotected investors. Interestingly, the welfare benefits of protected agents are decreasing in the share of investors that is eligible for liquidity support. The opposite happens for the unprotected investors. Finally, when there is a small protected sphere, public liquidity provision exclusively benefits the protected sphere and there is a large welfare gap between protected and unprotected investors.

Our main normative result is that in a time-consistent equilibrium, the optimal ex ante government's choice implies a *midsize protected sphere*. Unlike the solution under commmitment, it is optimal to leave a fraction of investors, strictly between 0 and 1, without liquidity support. A safety net covering all investors is undesirable because, under lack of commitment, the government provides too much liquidity support to protected investors ex post. Anticipating the access to public liquidity facilities, protected investors free ride on others' investment in short-term assets and choose excessively illiquid portfolios. In order to finance the liquidity facilities, the government needs to issue a large amount of public debt. This in turn yields an interest rate on government bonds that is too high from a social point of view. A high interest rate redistributes resources away from investors that have liquidity shortfalls and hurts ex ante welfare. Because of incomplete markets, the costs of this higher interest rate for borrowing investors that have a shortfall of liquidity outweigh the benefits to lending investors that have a surplus of liquidity. In addition, a midsize protected sphere also dominates a small protected sphere because it features less socially costly liquidity hoarding. A safety net with a midsize protected sphere is thus desirable from an ex ante welfare perspective.

[2]As in Yared (2013), the amount of liquidity provision under commitment induces a wedge between the technological rate of return on the long asset and the rate of return of government bonds.

Related literature This paper is related to a vast literature on public liquidity provision. Woodford (1990) and Holmström and Tirole (1998) are classic papers showing how public liquidity provision may relax private borrowing limits.In our model, the government also has a special role as a liquidity provider, but we address a distinct issue—the design of financial safety nets. In particular, we show that our model rationalizes a key feature of prevailing safety nets, where some financial institutions have access to a discount window while other institutions performing essentially the same activities do not. Our environment is closer to Yared (2013). He shows that the optimal liquidity provision under commitment entails a wedge between the technological rate of return on the long asset and the rate of return of government bonds. By promising a low return on bonds, the government reduces underinvestment in liquid assets and improves risk sharing.[3] We study instead optimal policy when the government lacks commitment. In particular, we show that the government provides too much liquidity ex post for investors within the safety net, and hence the optimal liquidity provision plan under commitment is not implementable. We characterize investment efficiency and risk sharing as a function of the safety net and show that it is strictly optimal to leave a share of investors outside the safety net.

A related literature highlights how bailouts can increase financial fragility when the government lacks commitment. Farhi and Tirole (2012) show that discretionary interest rate policy makes private leverage decisions strategic complements and generates multiple equilibria. Lack of commitment also plays an important role in the analysis of bailouts by Acharya and Yorulmazer (2007), Diamond and Rajan (2012), Chari and Kehoe (20016). Nosal and Ordoñez (2016) show that a government's uncertainty about whether failed institutions were affected by idiosyncratic or systemic shocks creates strategic restraint in leverage decisions and supports government commitment. Freixas (1999) shows that randomizing between bailing out banks in distress or not can create "constructive ambiguity" and reduce risk taking. Keister (2016) presents an environment in which a commitment to a no bailout policy is undesirable because it can increase the likelihood of bank runs, and Keister and Narasiman (2016) show that these policy conclusions emerge regardless of whether bank runs are driven by expectations or fundamentals. Bianchi (2016) finds that bailouts are desirable despite the moral hazard effects if conducted only during systemic crises. None of these papers, however, study the design of financial safety nets.

Our paper also relates to an growing literature on shadow banking. Existing work emphasizes regulatory arbitrage as the *raison d'être* of shadow banks (see, for instance, Acharya et al. (2013); Gorton and Metrick (2012); Pozsar et al. (2010)). In this spirit, Plantin (2015) devel-

[3] In a different environment, Yared (2015) and Bhandari, Evans, Golosov and Sargent (2015) study the effects of government debt on inequality.

ops a model in which capital requirements lead banks to bypass regulation through a shadow banking sector. Grochulski and Zhang (2015) introduce shadow banks in the model of Farhi et al. (2009) and study the implications for financial regulation. Ordoñez (2013) shows that the bluntness of capital requirements can make shadow banks desirable as a way to build reputation and better align the interests of banks and bondholders. In contrast, our analysis shows that the very existence of these institutions could be the result of the optimal plan of a government that is subject to a classic time-inconsistency problem.

The paper proceeds as follows. Section 2 describes the model. Section 3 describes the main results, and Section 4 concludes. The Appendix includes all of the proofs.

2 The Model

2.1 Technology and Preferences

The environment is based on the Diamond and Dybvig (1983) model of consumer liquidity demand that has been a workhorse in the study of financial intermediation. It is closest to the model presented by Yared (2013). The economy lasts for three dates: $t = 0, 1, 2$. There is a single consumption good and there are two technologies, which we label the short asset and the long asset. The short asset pays one unit of the good at $t+1$ for each unit invested at t. The long asset pays $\hat{R} > 1$ units at date 2 for each unit invested at date 0. For simplicity, we assume that the long asset cannot be liquidated at date 1.[4]

The economy is populated by a unit continuum of ex ante identical agents. These agents are endowed with e units of the good at $t = 0$. The type space has two dimensions. At date 0, each individual draws the first dimension of his type: $s = \{P, U\}$. A fraction $\gamma \in [0, 1]$ of individuals is of type $s = P$, and the complementary fraction $1 - \gamma$ is of type $s = U$. P stands for *protected*, while U stands for *unprotected*. As we will explain below, protected agents have access to public liquidity and unprotected agents do not. The type dimension s is public information, and the parameter γ is a policy choice on which we elaborate more in Section 2.2. At date 1, an agent draws the second dimension of his type, $\theta = \{0, 1\}$, which determines the preference for early consumption. With probability $\pi \in (0, 1)$, an individual is of type $\theta = 0$, while with probability $1-\pi$, he is of type $\theta = 1$. The distribution parameter π is a fundamental of the economy. Agents have Diamond-Dybvig preferences: the utility of an individual of type (s, θ) is given by

$$U\left(c_1^s, c_2^s, \theta\right) = (1-\theta)u\left(c_1^s\right) + \theta\rho u\left(c_1^s + c_2^s\right), \tag{1}$$

[4] All results carry through qualitatively as long as the date 1 liquidation value is strictly smaller than one.

where c_1^s and c_2^s represent the respective date 1 and date 2 consumption levels. The utility function $u(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly concave, and satisfies the Inada conditions $\lim_{c\to 0} u'(c) = \infty$ and $\lim_{c\to\infty} u'(c) = 0$.

The type dimension θ refers to liquidity shocks. Agents of type $\theta = 0$ are hit by liquidity shocks and only value consumption at date 1, whereas agents of type $\theta = 1$ are not hit by liquidity shocks and are indifferent between consumption at date 1 and date 2. As is standard in the literature, we assume that the type dimension θ is private and cannot be observed by other agents (including the government). We will often refer to agents hit by a liquidity shock as *impatient* agents and to agents not hit by a liquidity shock as *patient* agents.

The type dimension s determines the eligibility for public support at $t = 1$. Agents of type $s = U$ are unprotected and are not entitled to public liquidity provision, whereas agents of type $s = P$ are protected and can receive public liquidity at date 1. Unlike the type dimension θ, the type dimension s is public. In what follows, we will denote an allocation of consumption across consumers by $\{c_1^s(\theta), c_2^s(\theta)\}_{\theta\in\{0,1\},s\in\{U,P\}}$.

We define $\ell^s \in [0, 1]$ as the fraction of the date 0 endowment invested in the short asset by a type s individual. Accordingly, we denote by $L^s \in [0, 1]$ the aggregate choice of type s individuals. In equilibrium, consistency will require that aggregate and individual choices coincide, that is, $L^s = \ell^s$ for $s \in \{U, P\}$.

We make some parametric assumptions to ensure that the equilibria we consider fall within economically interesting regions.

Assumption 1. *As in Diamond and Dybvig (1983), the relative risk aversion is less than or equal to 1:*

$$-\frac{u'(c)}{cu''(c)} \leq 1 \quad \textit{for all} \quad c > 0, \tag{2}$$

and $\hat{R}^{-1} < \rho < 1$.

Assumption 1 notably implies that efficient risk sharing requires impatient agents to consume more than e and patient agents to consume less than $\hat{R}e$.

Assumption 2. *The probability of being hit by a liquidity shock is not too small:*

$$\pi \geq \frac{\rho(\hat{R}-1)}{1+\rho(\hat{R}-1)}.$$

This assumption ensures that in all equilibria we consider, agents make investment choices such that they never consume positive amounts at date 2 when they turn out to be impatient.[5]

2.2 Public Liquidity Provision and Markets

We assume that private contracts are not enforceable. The assumption of unobservability of liquidity shocks implies that contracts cannot be written at date 0 contingent upon their realization at date 1, and the lack of enforceability implies that agents cannot borrow privately at date 1.[6] The assumption of imperfect private contract enforceability motivates the analysis of public liquidity provision and the design of an optimal safety net.

The government makes two distinct choices at date 0 and date 1. At date 0, it sets the share of protected agents γ.[7] At date 1, it provides a liquidity facility to protected agents, and finances it by issuing public debt. We hereafter provide details on the government's policy at date 1 and postpone our exposition of the government's date 0 safety net decision to Section 3.

At date 1, the government issues public debt and extends credit to protected agents. At date 2, it uses the proceeds from protected private investors' repayments to pay back public debt holders. In the background, we assume that the government has a superior technology to enforce repayment.[8] We assume that the credit facility is contingent on protected agents' portfolio position.[9] Because the liquidity shock realization is private information, the credit facility cannot be made contingent on θ.

We denote the quantity of credit extended by the government to agents with short asset position ℓ by $B(\ell)$ and the aggregate amount of public debt by $\mathcal{B}$. The government demands the same interest rate $1/q$ on the credit it extends to protected agents as the one it pays on its own public debt, it has zero initial public debt, and it does not finance any public expenditures.

[5] Agents who turn out to be impatient do not value date 2 consumption, but if liquidity shocks occur with a sufficiently low probability, they might find it optimal to make investment decisions at date 0 that result in an ex post wasting of date 2 resources in the contingency where they are hit by these shocks. Assumption 2 rules out this case.

[6] All of our results continue to hold with a finite borrowing limit as long as the borrowing constraint binds for all unprotected agents who turn out to be impatient at date 1.

[7] In other words, it sets the respective probabilities with which an agent draws a type $s = U$ or $s = P$ at date 0.

[8] This is a standard assumption in the literature on public liquidity provision. As we will show below, this access to a better enforcement technology allows the government to reach the efficient allocations under commitment, but not under discretion.

[9] By making the government liquidity provision contingent on individual variables as opposed to aggregate variables, we are able to abstract from issues of multiplicity that would arise in this model when we turn to the optimal time-consistent equilibria (see e.g. Farhi and Tirole, 2012). In ongoing work, we show that the model with credit contingent on aggregate variables displays similar results for our analysis of safety nets for a range of equilibrium selection mechanisms.

Its budget constraints thus require that

$$\int_0^\gamma B(\ell_j)dj = \mathcal{B}. \tag{3}$$

We denote by $b^s(\theta)$ the individual holdings of government bonds and assume that government bonds cannot be shortened, i.e., ($b^s(\theta) \geq 0$). The dynamic budget constraints of an unprotected agent are represented by

$$\begin{aligned}
\ell^U &\in [0,1], & (4)\\
c_1^U(\theta) &= \ell^U e - qb^U(\theta), & (5)\\
c_2^U(\theta) &= \hat{R}(1-\ell^U)e + b^U(\theta), & (6)
\end{aligned}$$

while those of a protected agent are represented by

$$\begin{aligned}
\ell^P &\in [0,1], & (7)\\
c_1^P(\theta) &= \ell^P e - qb^P(\theta) + qB(\ell^P), & (8)\\
c_2^P(\theta) &= \hat{R}(1-\ell^P)e + b^P(\theta) - B(\ell^P), & (9)
\end{aligned}$$

for $\theta \in \{0,1\}$. We have used in (5) and (8) that in an equilibrium with $q \leq 1$, agents weakly prefer to save using government bonds between date 1 and date 2 rather than use the short asset.[10] Combining the government's budget constraint (3) with (5), (6), (8), (9) and the public debt market clearing condition

$$\mathcal{B} = (1-\gamma)[\pi b^U(0) + (1-\pi)b^U(1)] + \gamma[\pi b^P(0) + (1-\pi)b^P(1)] \tag{10}$$

we obtain that a feasible allocation needs to satisfy the economy's resource constraint

$$\begin{aligned}
&\pi\left[(1-\gamma)\left(c_1^U(0) + \frac{c_2^U(0)}{\hat{R}}\right) + \gamma\left(c_1^P(0) + \frac{c_2^P(0)}{\hat{R}}\right)\right]\\
&+(1-\pi)\left[(1-\gamma)\left(c_1^U(1) + \frac{c_2^U(1)}{\hat{R}}\right) + \gamma\left(c_1^P(1) + \frac{c_2^P(1)}{\hat{R}}\right)\right] = e. \qquad (11)
\end{aligned}$$

An alternative representation of the agents' constraint set is given by

$$\begin{aligned}
c_1^s(\theta) + qc_2^s(\theta) &= \ell^s e + q\hat{R}(1-\ell^s)e\\
c_1^s(\theta) &\leq \ell^s e + \mathbb{I}_{\{s=P\}} qB(\ell^s).
\end{aligned}$$

[10] An equilibrium with $q > 1$ implies $b^s(\theta) = 0$. That is, if the return on government bonds is lower than the return on short-term assets, government bonds would be strictly dominated assets.

These substitutions show that the protected agent's problem induced by a government debt policy is equivalent to that of an agent that faces an exogenous borrowing limit $b^P(\theta) \geq -B(\ell^P)$ in the absence of public liquidity provision. On the other hand, because they do not benefit from public liquidity provision, unprotected agents always face an effective borrowing limit $b^U(\theta) \geq 0$. The government's date 0 choice about the size of the protected sphere will determine the respective fractions of agents facing a relaxed borrowing limit $-B(\ell)$ and of those facing an unrelaxed limit at 0. One might think that the government would like to maximize the fraction of agents who face a relaxed borrowing limit ex post, but as our analysis of Section 3 reveals, this not the case when the government cannot commit ex ante about its debt issuance policy. The timeline is summarized in Figure 1.

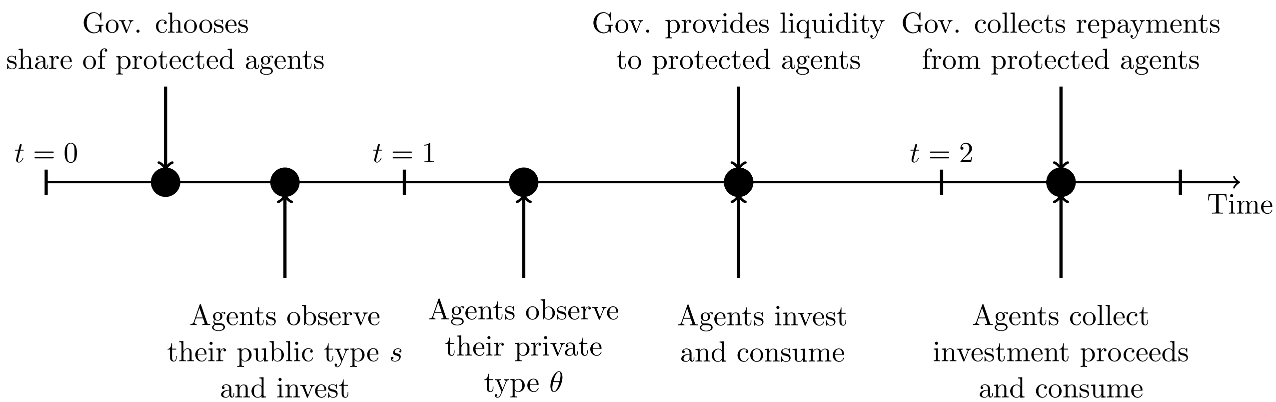

Figure 1: Timeline of the model.

We let (s, ℓ, θ) denote an agent's individual state and $X \equiv (\gamma, L^U, L^P)$ denote the aggregate state. We also let $B(\ell)$ denote a public liquidity provision policy, $q(X)$ denote the bond pricing function, and $\mathcal{C}_1(s, \ell, \theta, X), \mathcal{C}_2(s, \ell, \theta, X)$ represent the date 1 decision rules of an agent, whose problem is

$$\mathcal{V}_1(s, \ell, \theta, X) = \max_{c_1, c_2} U\left(c_1, c_2, \theta\right) \tag{12}$$

subject to

$$c_1 + q(X)c_2 = \ell e + q(X)\hat{R}(1-\ell)e, \qquad \text{and} \qquad c_1 \leq \ell e + \kappa(s, \ell, X),$$

where

$$\kappa(s, \ell, X) \equiv \begin{cases} 0 & \text{for } s = U \\ q(X)B(\ell) & \text{for } s = P \end{cases} \tag{13}$$

is a type- and agent-specific effective borrowing limit. This problem is defined for any policy

$B(\ell)$. We can then define a date 1 continuation equilibrium.

Definition 1 (Continuation equilibrium)**.** *Given a government policy $B(\ell)$, a continuation equilibrium is a value function $\mathcal{V}_1(s,\ell,\theta,X)$ with associated decision rules $\mathcal{C}_1(s,\ell,\theta,X), \mathcal{C}_2(s,\ell,\theta,X)$, and a bond price function $q(X)$ such that*

1. *given $q(X)$ and $B(\ell)$, $\mathcal{V}_1(s,\ell,\theta,X)$ solves the agent's date 1 problem (12), and*

2. *the markets for date 1 and 2 consumption clear:*[11]

$$\sum_s \sum_\theta \gamma_s \pi_\theta \mathcal{C}_1(s, L^s, \theta, X) \leq \sum_s \gamma_s L^s e, \tag{14}$$

$$\begin{aligned}\sum_s \sum_\theta \gamma_s \pi_\theta \mathcal{C}_2(s, L^s, \theta, X) &= \sum_s \gamma_s \hat{R}(1-L^s)e \\ &+ \left[\sum_s \gamma_s L^s e - \sum_s \sum_\theta \gamma_s \pi_\theta \mathcal{C}_1(s, L^s, \theta, X)\right].\end{aligned} \tag{15}$$

This is a standard definition of a competitive equilibrium, adapted to accommodate the dependence of the government's liquidity provision policy upon the ex-ante choices of agents. Condition (14) requires that aggregate date 1 consumption does not exceed the aggregate payoff of the short asset at date 1. Condition (15) requires that aggregate date 2 consumption does not exceed the aggregate payoff of the long asset, plus the payoff of the short asset invested in between date 1 and date 2. By Walras' law, the market clearing condition on government bonds is satisfied. The following lemma characterizes a continuation equilibrium, which will be useful when we turn to analyze the optimal government policy and highlight the role of commitment.

Lemma 1 (Continuation equilibrium)**.** *A continuation equilibrium features*[12]

1. *a bond price function satisfying*

$$q(X) = \min\left\{ e\frac{1-\pi}{\pi} \frac{\gamma L^P + (1-\gamma)L^U}{\gamma \min\{\hat{R}(1-L^P)e, B(L^P)\}}, 1 \right\} \tag{16}$$

[11] To simplify notation, we define $\pi_0 \equiv \pi$ and $\pi_1 \equiv 1-\pi$, as well as $\gamma_P \equiv \gamma$ and $\gamma_U \equiv 1-\gamma$.

[12] When an equilibrium features $q=1$, any other allocation such that $c_1 + c_2 = \hat{R}(1-\ell)e + \ell e$ (and $c_1, c_2 \geq 0$), together with the price $q=1$, also constitutes an equilibrium, but we can focus without loss of generality on the one featuring $c_1 = 0$ and $c_2 = \hat{R}(1-\ell)e + \ell e$.

2. consumption allocations satisfying

$$\begin{aligned} \mathcal{C}_1(s,\ell,0,X) &= \ell e + \min\left\{q(X)\hat{R}(1-\ell)e, \kappa(s,\ell,X)\right\}, && (17) \\ \mathcal{C}_2(s,\ell,0,X) &= \max\left\{\hat{R}(1-\ell)e - \frac{\kappa(s,\ell,X)}{q(X)}, 0\right\}, && (18) \\ \mathcal{C}_1(s,\ell,1,X) &= 0, && (19) \\ \mathcal{C}_2(s,\ell,1,X) &= \hat{R}(1-\ell)e + \frac{\ell e}{q(X)}. && (20) \end{aligned}$$

Proof. See Appendix A.1.

According to this lemma, in the absence of public liquidity provision, all impatient agents consume the proceeds of their short asset at date 1 and consume the payoff of their long asset at date 2. The latter is wasteful, because these agents do not value date 2 consumption, but they have no choice since credit constraints prevent them from transferring resources from date 2 to date 1. By relaxing protected agents' effective credit constraint, the extension of public liquidity allows this set of agents to transfer some or all of their date 2 illiquid wealth stemming from the payoff of their long asset back into date 1. Patient agents, on the other hand, consume only at date 2. These agents are natural savers at date 1, and therefore public debt issuance does not generate an asymmetry between the protected and unprotected patient agents' consumption. However, these agents (at least weakly) benefit from a higher level of public debt to the extent that it (weakly) increases the interest rate they earn on date 1 bond purchases (the bond price q is weakly decreasing in public debt issuance $\mathcal{B}$ since the demand for government bonds by patient agents is decreasing in the price).

For a given liquidity provision policy $B(\ell)$, an agent's date 0 problem can be represented as

$$\mathcal{V}_0(s,X) = \max_{\ell\in[0,1]} \pi\mathcal{V}_1(s,\ell,0,X) + (1-\pi)\mathcal{V}_1(s,\ell,1,X) \tag{21}$$

Given this date 0 problem and Definition 1 of a continuation equilibrium, we have the following definition of a competitive equilibrium

Definition 2 (Competitive equilibrium). *Given government policies $\gamma, B(\ell), \mathcal{B}$ a competitive equilibrium is a vector of aggregate variables X, a date 0 value function $\mathcal{V}_0(s,X)$ with associated policy function $\ell(s,X)$, a date 1 value function $\mathcal{V}_1(s,\ell,\theta,X)$ with associated decision rules $\mathcal{C}_1(s,\ell,\theta,X), \mathcal{C}_2(s,\ell,\theta,X)$, and a bond price function $q(X)$ such that*

1. *$\mathcal{V}_1(s,\ell,\theta,X), \mathcal{C}_1(s,\ell,\theta,X), \mathcal{C}_2(s,\ell,\theta,X)$, and $q(X)$ are induced by a continuation equilibrium according to Definition 1,*

2. *$\mathcal{V}_0(s,X), \ell(s,X)$ solve the agent's problem (21),*

3. aggregate variables are consistent with individual choices: $X = (\gamma, \ell(P, X), \ell(U, X))$,

4. the government's budget constraint (3) is satisfied.

3 Optimal Policy Analysis

3.1 Efficient Allocation

We start by characterizing the efficient allocation. This allocation will serve as a benchmark for our normative analysis. In presenting this allocation, we abstract from the type dimension s of agents, since it is unrelated to their preferences. The efficient allocation solves the following problem:

$$\max_{c_1(0),c_2(0),c_1(1),c_1(2)} \pi U\left(c_1(0), c_2(0), 0\right) + (1-\pi)U\left(c_1(1), c_2(1), 1\right) \tag{22}$$

subject to

$$\pi\left[c_1(0) + \frac{c_2(0)}{\hat{R}}\right] + (1-\pi)\left[c_1(1) + \frac{c_2(1)}{\hat{R}}\right] \leq e,$$

and $c_1(0), c_2(0), c_1(1), c_2(1) \geq 0$.

The solution to this problem is described by the lemma below.

Lemma 2 (Efficient Allocation)**.** *The solution to the planning problem satisfies* $e < c_1^*(0) < c_2^*(1) < \hat{R}e$ *and* $c_2^*(0) = c_1^*(1) = 0$*, with* $u'(c_1^*(0)) = \rho\hat{R}u'(c_2^*(1))$.

Proof. See Appendix A.2.

Thus, as is standard under Diamond-Dybvig preferences, the allocation features zero date 2 consumption of impatient agents, zero date 1 consumption of patient agents, and risk sharing between patient and impatient agents that is consistent with an equalization of the social marginal rate of transformation $1/\hat{R}$ and the marginal rate of substitution $\rho u'(c_2^*(1))/u'(c_1^*(0))$.

3.2 Optimal Safety Net under Commitment

We now turn to analyzing decentralized equilibria. We start by assuming that the government can commit at date 0 about its date 1 liquidity provision policy. This will be important to highlight the role of the government's ability to commit in our analysis of the design of the optimal safety net.

In this case, after the government set γ and credibly announce a future liquidity provision policy $B^c(\ell)$ at date 0, private agents make investment choices. Recall that agents know whether they are protected at the time of making their date 0 investment choice.

Under commitment, the government chooses the policy $\gamma^c, B^c(\ell), \mathcal{B}^c$ to attain the competitive equilibrium with highest time 0 utility.

Proposition 1 (Optimal policy under commitment)**.** *A safety net architecture covering all agents ($\gamma^c = 1$), together with a commitment to provide an amount of public liquidity $B^c(\ell) = B^c = (1-\pi)c_2^*(1)$, achieves the efficient allocation described in Lemma 2.*

Proof. See Appendix A.3.

This proposition shows that it is optimal to cover all agents, and that the appropriate amount of liquidity provision achieves the efficient allocations. The latter result is related to Yared (2013), who established that under a weaker version of Assumption 2, a fiscal policy scheme equivalent to our credit facility can achieve the efficient allocations when the government has commitment.[13] Proposition 1 indicates that if the government were able to commit to a future liquidity provision policy, it would not leave any agent outside the safety net. In fact, setting a boundary between protected agents and unprotected agents not only is redundant but also would deliver strictly lower ex ante welfare.[14] Below, we relax the assumption that the government can commit to its liquidity provision policy, and show that having a smaller safety net becomes strictly optimal. To put the results below into perspective, it is important to note that under commitment, the amount of liquidity provision that the government commits to provide puts a lower bound on the amount of short assets that agents choose to invest in. If agents were to invest less in short assets than the level associated with the efficient allocation, and were to end up being impatient, they would become credit constrained. This will contrast with the outcome that prevails when the government lacks commitment. As we show below, under discretion the government will ex post relax agents' credit constraints *unconditionally*, i.e., for any values of their investment choice. Anticipating the reaction of the government, agents will invest too little in short assets in the initial period. The inability of the government to commit to the extent of an ex post public liquidity provision will create a rationale for optimal management of the safety net.

3.3 Optimal Safety Net under Discretion

To analyze optimal policy under discretion, we proceed by backward induction. We start by solving for the government's optimal liquidity provision policy at date 1 when it is not

[13]Without this assumption, Yared (2013) finds that the government, despite not reaching the efficient allocation, should still restrict public debt issuances to prevent underinvestment in liquid assets.

[14]To see this, note that an unprotected impatient agent's consumption cannot exceed e, which is strictly lower than the impatient agents' consumption in the efficient allocation.

bound by past commitments. We then move back to date 0 choices and characterize time-consistent equilibria, for a given safety net architecture represented by the value of γ. Finally, we characterize the optimal ex ante choice of γ.

3.3.1 Ex post Policy: Liquidity Provision

After date 0 choices have been made, the government chooses the liquidity provision policy rule $B^d(\ell)$ to maximize the average welfare of unprotected and protected agents subject to the private sector's date 1 response to its actions. The government solves

$$\begin{aligned}\max_{\{B_j\}_{j\in[0,\gamma]}} \quad & \int_0^\gamma \left[\pi \mathcal{V}_1\left(P,\ell_i,0,X\right)+(1-\pi)\,\mathcal{V}_1\left(P,\ell_i,1,X\right)\right]di \\ & +\int_\gamma^1 \left[\pi \mathcal{V}_1\left(U,\ell_i,0,X\right)+(1-\pi)\,\mathcal{V}_1\left(U,\ell_i,1,X\right)\right]di \end{aligned} \tag{23}$$

where $\mathcal{V}_1(\cdot)$ is given by our definition of a continuation equilibrium.

The following proposition establishes that an optimal ex post policy always features a full relaxation of impatient protected agents' effective borrowing constraints at date 1.

Proposition 2 (Optimal ex post bailout)**.** *An equilibrium with an optimal ex post policy features a full relaxation of impatient protected agents' effective credit constraints,* $B^d(\ell) = \hat{R}(1-\ell)e$*. Further, the equilibrium bond price is given by*

$$q(X)=\min\left\{\frac{1}{\hat{R}}\frac{1-\pi}{\pi}\frac{\gamma L^P+(1-\gamma)L^U}{\gamma(1-L^P)},1\right\}, \tag{24}$$

and the equilibrium consumption of protected agents is given by

$$\mathcal{C}_1(P,\ell,0,X) \;=\; \ell e + q(X)\hat{R}(1-\ell)e, \tag{25}$$

$$\mathcal{C}_2(P,\ell,1,X) \;=\; \hat{R}(1-\ell)e+\frac{\ell e}{q(X)}. \tag{26}$$

Proof. See Appendix A.4.

Proposition 2 establishes that it is always optimal for the government to provide an amount of public liquidity that makes a protected agent unconstrained in a date 1 continuation equilibrium. The intuition for the ex post optimality of fully relaxing constraints is as follows. For $B_i < \hat{R}(1-\ell_i)e$, increasing B_i always increases the equilibrium consumption of some agent without decreasing the equilibrium consumption of another agent. To see this, we distinguish the situations in which $q = 1$ from the ones in which $q < 1$. When $q = 1$, an increase in B_i increases the equilibrium consumption of the protected impatient agent i while leaving the equilibrium consumption of other agents unchanged.The increase in agent i's consumption is the result of a borrowing constraint relaxation at a locally unchanged interest rate. When $q < 1$, on the other hand, an increase in B_i increases the equilibrium consumption of all patient agents while leaving the equilibrium consumption of impatient agents unchanged. The increase in patient agents' consumption results from the upward pressure on the return on government debt from date 1 to date 2 (i.e., q is decreasing in B_i). It follows that the government's value function is strictly increasing in B_i for $B_i < \hat{R}(1-\ell_i)e$. For $B_i \geq \hat{R}(1-\ell_i)e$, on the other hand, equilibrium consumption allocations do not locally depend on B_i. It follows that the debt issuance policy $B^d(\ell) = \hat{R}(1-\ell)e$ is optimal.

A higher level of public liquidity provision is thus always desirable ex post up to the point where protected agents' effective credit constraints are fully relaxed. This is true *for any level of private investment*. In the absence of commitment, an optimal public liquidity provision policy hence works as insurance provided freely to protected agents. This contrasts with the optimal policy under commitment, which offers a limited amount of insurance. This extra layer of insurance present under discretion will distort ex ante incentives.

As we will see below, the moral hazard costs induced by discretion in public liquidity provision depend on the size of the protected sphere. In the next sections, we provide a sharp analytical characterization of this relationship and analyze the key trade-offs involved in the optimal setting of the size of the safety net.

3.3.2 Time-Consistent Equilibrium

After the government has set γ at date 0, private agents make investment choices. Agents know γ and forecast aggregate actions L^P, L^U to form beliefs about $q(X)$. They also rationally anticipate the ex post public liquidity provision policy rule $B^d(\ell)$. We can define a time-consistent equilibrium as follows:

Definition 3 (Time-consistent equilibrium for given safety net γ)**.** *For a given γ, a time-consistent equilibrium is a liquidity provision policy $B^d(\ell)$, a bond price $q(X)$, consumption policies $\mathcal{C}_1(s,\ell,\theta,X)$, $\mathcal{C}_2(s,\ell,\theta,X)$ and investment portfolio ℓ such that:*

1. *$B^d(\ell)$ solves (23)*
2. *ℓ solves (21)*
3. *$\mathcal{V}_1(s,\ell,\theta,X)$, $\mathcal{C}_1(s,\ell,\theta,X)$, $\mathcal{C}_2(s,\ell,\theta,X)$, $q(X)$, and $B^d(\ell)$ are a continuation equilibrium according to Definition 1.*

Using that $B^d(\ell) = \hat{R}(1-\ell)e$ as established in Proposition 2, the time-consistent equilibrium for given γ can be conveniently solved for in closed form, as summarized in the following proposition.

Proposition 3 (Characterization of time-consistent equilibria for given safety net γ)**.** *In a time-consistent equilibrium, unprotected agents always invest all of their endowment into the short asset: $L^U = 1$. For other variables, we can distinguish between three mutually exclusive regions, characterized by boundaries $0 < \underline{\gamma} < \overline{\gamma} < 1$, with $\underline{\gamma} \equiv \frac{1-\pi}{1-\pi+\hat{R}\pi}$ and $\overline{\gamma} \equiv 1 - \pi$:*

- *Region I ($0 \leq \gamma < \underline{\gamma}$): protected agents invest $L^P = 0$, the date 1 bond price is $q = 1$, and the consumption allocations are $c^U(0) = c^U(1) = e$ and $c^P(0) = c^P(1) = \hat{R}e$.*
- *Region II ($\underline{\gamma} \leq \gamma \leq \overline{\gamma}$): protected agents invest $L^P = 0$, the date 1 bond price is $q = \frac{1}{\hat{R}} \frac{\pi\gamma}{(1-\pi)(1-\gamma)}$, and the consumption allocations are $c^U(0) = e$, $c^U(1) = \hat{R}\frac{(1-\pi)(1-\gamma)}{\pi\gamma}e$, $c^P(0) = \frac{\pi\gamma}{(1-\pi)(1-\gamma)}e$, and $c^P(1) = \hat{R}e$.*
- *Region III ($\overline{\gamma} < \gamma \leq 1$): protected agents invest $L^P = \frac{\pi+\gamma-1}{\gamma}$, the date 1 bond price is $q = 1/\hat{R}$, and the consumption allocations are $c^U(0) = c^P(0) = e$ and $c^U(1) = c^P(1) = \hat{R}e$.*

Proof. See Appendix A.5.

The equilibrium of the model takes different forms, depending on the value of the size of the protected sphere γ. In all of the cases that arise, unprotected agents always invest all of their endowment in the short asset at date 0. We note that in the laissez-faire benchmark where all agents are unprotected ($\gamma = 0$), everyone invests his entire endowment into the short asset ($L^U = 1$) and consumes an amount equal to that endowment whether hit by a liquidity shock or not at date 1 ($c^U(0) = c^U(1) = e$). Thus, the laissez-faire benchmark features an extreme form of self-insurance that results in clear efficiency losses relative to the efficient allocation. We now discuss equilibrium properties in the different regions.

Region I When the fraction of protected agents is smaller than a threshold $\gamma < \frac{1-\pi}{1-\pi+\hat{R}\pi}$, we say that there is a *small protected sphere*. In this case, the demand for government bonds by patient unprotected agents at date 1 is large enough to push the interest rate down to its lower bound $1/q = 1$. In this region, impatient protected agents benefit from fully relaxed credit constraints *and* a low interest rate at date 1, which allow them to transfer the date 2 proceeds of their long investment back into date 1 one for one. On the other side of the trade, patient unprotected agents are not able to earn a return higher than the storage technology between date 1 and date 2 on the proceeds of their date 0 short investment. Thus, in equilibrium, protected agents always end up consuming $\hat{R}e$, and unprotected agents always end up consuming e, whatever the realization of their liquidity shocks. This region features an extreme form of redistribution between the two spheres. A large unprotected sphere has the same consumption profile as in the laissez-faire benchmark (i.e., when $\gamma = 0$) and implicitly subsidizes a small set of protected agents.

Region II When the fraction of protected agents is between two thresholds $\frac{1-\pi}{1-\pi+\hat{R}\pi} \leq \gamma \leq 1-\pi$, we say that there is a *medium protected sphere.* In this case, the mass of unprotected agents is still large enough for protected agents to completely rely on the short asset investment made by unprotected agents. However, the aggregate amount of debt issued by the government is not high enough relative to the supply of funds to push the date 1 interest rate to $\hat{R}$, so impatient protected agents, whose credit constraints are fully relaxed by the bailout, can enjoy a consumption level higher than e (the date 0 payoff on their long investment, $\hat{R}e$, is worth more than e when discounted into date 1 at the prevailing interest rate). Patient unprotected agents, on the other hand, earn a positive return, albeit lower than $\hat{R}$, between date 1 and date 2 on the proceeds from their date 0 short investment. Their date 2 consumption is therefore higher than the laissez-faire level of e, but it falls short of $\hat{R}e$. This discussion, together with Panel (d) of Figure 2, makes it clear that in this region, government bailouts induce a redistribution of resources from unprotected to protected agents, whose strength decreases with the share of protected agents γ. As γ increases, the gap between the equilibrium consumption of protected and unprotected agents narrows, in both liquidity risk contingencies ($\theta = 0$ and $\theta = 1$). The fact that this gap gap is decreasing in γ reflects the fact that as γ increases, there are fewer and fewer unprotected agents who self-insure by investing in the short asset, which puts an increasing upward pressure on the date 1 interest rate.

Region III When the fraction of protected agents is greater than a threshold $\gamma > 1-\pi$, we say that there is a *large protected sphere.* Protected agents invest only a fraction $L^P = \frac{\pi+\gamma-1}{\gamma}$ of their date 0 endowment in the short asset. This fraction is equal to zero when $\gamma = 1-\pi$, is increasing in γ, and reaches π when $\gamma = 1$. Panel (e) of Figure 2 represents the date 0 investment choice of agents as a function of the size of the protected sphere γ. Protected agents anticipate being bailed out by the government at date 1. This a priori eliminates their incentive to self-insure by investing in the short asset. However, someone needs to invest in the short asset to support the consumption of impatient agents at date 1, and when the number of unprotected agents in the economy is small, protected agents need to do their share of short investment at date 0. Given a full relaxation of credit constraints by the government ex post, for there to be an incentive to invest in the short asset for protected agents, the return on government bonds between date 1 and date 2, $1/q$, needs to equal the return on the long asset

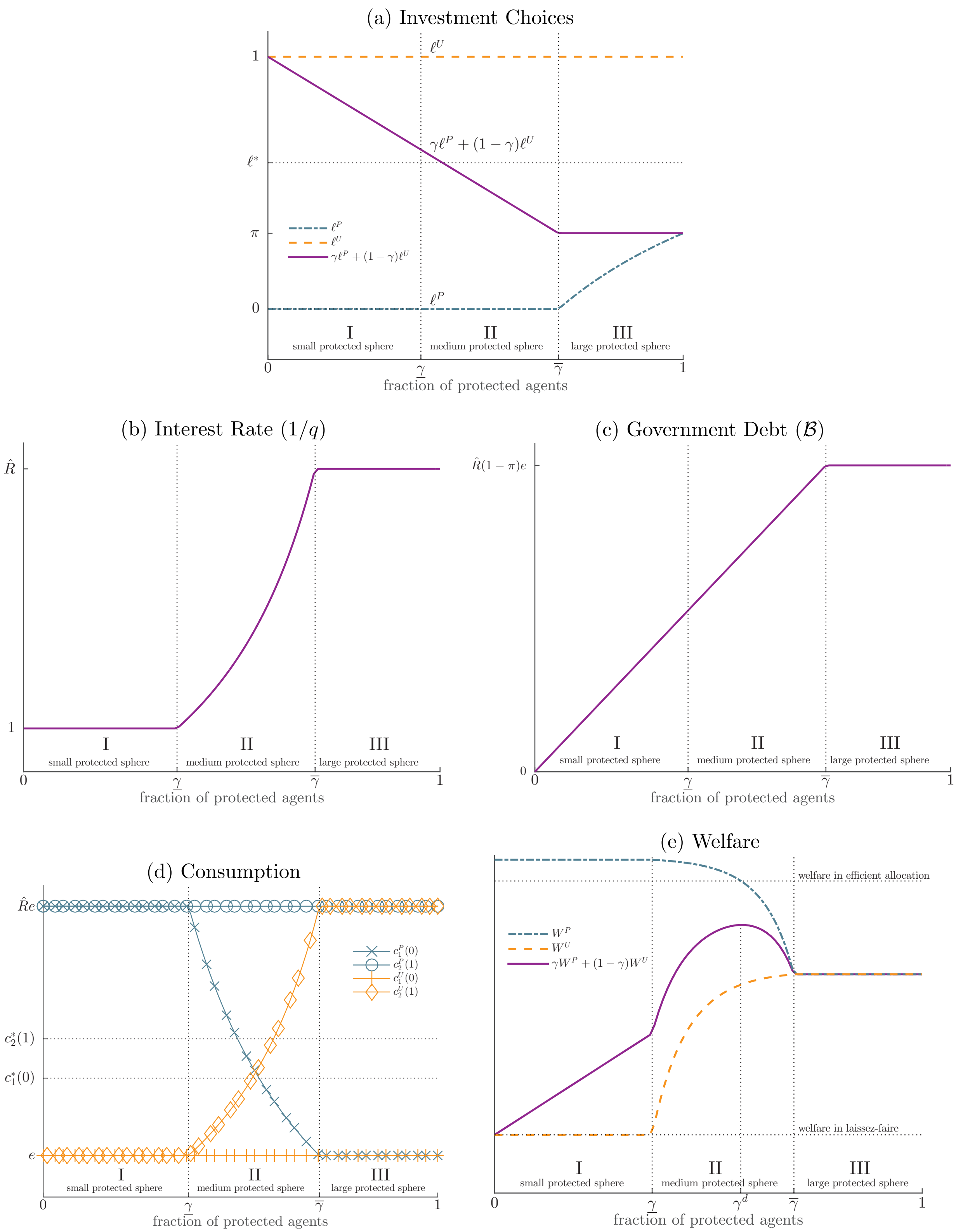

Figure 2: Policies, Welfare, Prices, and the Safety Net

$\hat{R}$. Panel (d) of Figure 2 represents the consumption allocations of agents as a function of the size of the protected sphere γ. In this region, all impatient agents consume e, and all patient agents consume $\hat{R}e$. This consumption allocation strictly dominates the allocation achieved by the benchmark economy without government intervention ($\gamma = 0$). Perhaps surprisingly, protected agents are not better off than unprotected agents in this case, even though the former benefit from a public liquidity provision and the latter do not. Despite not benefiting directly from a public liquidity provision, unprotected agents benefit from it indirectly through the price system. In this region, the government issues aggregate amounts of public debt that are sufficiently high to push the date 1 interest rate on government bonds to its upper bound $\hat{R}$. Unprotected agents who turn out to be patient are thus able to earn a return of $\hat{R}$ between date 1 and date 2 on the proceeds from their date 0 short investment. This enables them to achieve the same equilibrium consumption profile as protected agents. Panel (b) of Figure 2 displays the interest rate as a function of the size of the protected sphere, and Panel (c) represents aggregate public debt issuance.

To see more clearly why the allocations under commitment are not an equilibrium outcome under discretion, consider the date 0 decision of a protected agent in a fully protected economy, when all other agents hypothetically make the same investment choice as under commitment. Note that this collective investment choice leads to an interest rate such that $1/q < \hat{R}$ ex post. And given that the government will always relax the protected individual's credit constraint ex post, it is strictly optimal for this agent to invest all his endowment into the long asset at date 0. By doing so he is better off in any contingency. If he turns out patient, he enjoys a strictly higher date 2 consumption of $\hat{R}e$. If he turns out impatient, he can freely borrow against his date 2 investment income at an interest rate lower than $\hat{R}$. Since there is an incentive for individual deviations, this, of course, cannot constitute an equilibrium. The fundamental problem is that agents free ride on other agents' short investments when the government lacks commitment about its liquidity provision policy.[15]

Now that we have fully characterized time-consistent equilibria conditional on the size of the protected sphere γ, we can finally turn to the analysis of the optimal choice of this parameter by a welfare benevolent government at date 0.

3.3.3 Ex-ante Policy: Size of the Protected Sphere

We now consider the date 0 choice of a welfare benevolent government that sets the size of the protected sphere γ while anticipating the response of agents in a time-consistent equilibrium.[16]

[15]This free rider problem is distinct from the coordination problem typically present in the literature on bailouts (e.g. Farhi and Tirole, 2012, Keister, 2016).

[16]We deliberately abstract from prudential policies that can limit agents' portfolio choices at time 0.

The government solves

$$\mathcal{W}_0 = \max_{\gamma \in [0,1]} \gamma \mathcal{V}_0(P, X) + (1-\gamma)\mathcal{V}_0(U, X) \tag{27}$$

subject to

$$X = (\gamma, \ell(P, X), \ell(U, X)).$$

The following proposition contains our main result.

Proposition 4 (Optimal ex ante size of protected sphere)**.** *The optimal size of the protected sphere is interior, satisfying* $\underline{\gamma} < \gamma^d < \overline{\gamma}$.

Proof. See Appendix A.6.

Proposition 4 establishes that the optimal size of the protected sphere is not a corner solution. The optimal safety net architecture from an ex-ante perspective features positive masses of protected and unprotected agents. The intuition for this result can be most easily built by considering how welfare depends on γ within each of the three regions defined in section 3.3.2.

Panel (e) of Figure 2 represents ex ante average welfare as a function of the size of the protected sphere γ. We first note that ex ante average welfare is continuous in γ since all equilibrium consumption allocations are continuous in γ. In region I, protected agents always consume $\hat{R}e$, whereas unprotected agents always consume e. Hence in that region, the welfare of protected agents is strictly higher than that of unprotected agents. It follows that ex ante average welfare is strictly increasing in γ in that region, with a maximum of $\underline{\gamma}u(e)+(1-\underline{\gamma})u(\hat{R}e)$ at the upper bound $\gamma = \underline{\gamma}$. Safety net architectures with small protected sectors strictly dominate the laissez-faire benchmark ($\gamma = 0$) because protected agents are strictly better off than in the laissez-faire benchmark and unprotected agents are no worse off. In region III, protected and unprotected agents consume the same amounts in equilibrium in the contingency in which they are patient. Likewise, they consume the same in the contingency in which they are impatient. It follows that within this region, ex ante average welfare is constant with respect to γ and given by $\pi u(e) + (1-\pi)\rho u(\hat{R}e)$. We also note that since $\underline{\gamma} < 1 - \pi$, ex ante average welfare is strictly higher in region III than in region I. By the continuity of ex ante average welfare with respect to γ, it must therefore be that the optimal size of the protected sphere falls in region II. But a key feature of Proposition 4 is that the optimal size of the protected sphere lies in the interior of region II rather than at its right boundary, so that the optimal safety net architecture strictly dominates a fully protected economy.

The ex ante optimality of restricting the scope of protection in the economy can be drawn from the fact that the left derivative of ex ante average welfare is strictly negative at $\gamma = \overline{\gamma} =$

$1-\pi$. The intuition has to do with the improvement in risk sharing induced by the splitting of agents between a protected and an unprotected sphere. In region II, a marginal decrease in γ causes a decrease in the equilibrium date 1 interest rate. Since in this region impatient protected agents borrow in equilibrium from patient unprotected agents, this decrease in the interest rate redistributes wealth from the latter to the former.[17] At $\gamma = \overline{\gamma} = 1-\pi$, such a wealth redistribution is necessarily socially desirable given that (i) the masses of patient unprotected and impatient protected agents are equal (i.e., to $\pi(1-\pi)$), and (ii) relative to the efficient allocation, impatient agents consume too little and patient agents consume too much (a consequence of Assumption 1). Hence, a marginal decrease in the share of protected agents from $\gamma = 1-\pi$ unambiguously increases the ex ante average welfare criterion. It directly follows that the optimal size of the protected sphere γ^d must lie strictly between $\underline{\gamma}$ and $\overline{\gamma}$. This intuition is illustrated in Panel (e) of Figure 2. There, it is apparent that directly to the left of $\gamma = \overline{\gamma} = 1-\pi$, the welfare increase for protected agents more than offsets the welfare decrease for unprotected agents, so that ex ante average welfare is strictly decreasing in γ.

4 Conclusion

In this paper, we studied the optimal design of financial safety nets. In a workhorse model of liquidity provision under limited private credit, we obtain the following results. First, if the government has commitment, the optimal financial safety net covers all agents. Second, when the government lacks commitment, the government provides excessive liquidity to agents protected by the safety net. Third, in the absence of commitment, the optimal financial safety net includes only a subset of agents. Compared with an economy where all agents are protected, this results in more liquid asset portfolios, lower interest rates and higher social insurance.

Our analysis points to the importance of the institutional design of central banks' framework for liquidity provision. Contrary to the view that the financial safety net should be expanded, our model suggests that this would lead to underinvestment in liquid assets and too little risk sharing. When the government lacks commitment to a liquidity provision policy, it should limit ex ante the set of agents that are eligible for support. Indeed, the optimal safety net we derived resembles the current framework where only depositary institutions can access lender of last resort facilities.

[17] The effect of this redistribution on equilibrium consumption can be inferred from Panel (d) of Figure 2.

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Appendix

A Proofs

A.1 Proof of Lemma 1

Step 1 : Date 1 consumption choice

A type s agent solves the following date 1 problem:

$$\mathcal{V}_1(s, \ell, \theta, X) = \max_{c_1, c_2} U(c_1, c_2, \theta) \tag{A.1}$$

$$\textit{subject to}$$

$$c_1 + q(X)c_2 = \ell e + q(X)\hat{R}(1-\ell)e \tag{A.2}$$

$$c_1 \leq \ell e + \kappa(s, \ell, X) \tag{A.3}$$

$$c_1, c_2 \geq 0.$$

Note that c_2 cannot be negative. Thus, for an agent who turns out to be impatient (type $\theta = 0$) at date 1, it is optimal to maximize c_1 and minimize c_2. It must therefore be that

$$\mathcal{C}_1(s, \ell, 0, X) = \ell e + \min\left\{q(X)\hat{R}(1-\ell)e, \kappa(s, \ell, X)\right\}, \tag{A.4}$$

and

$$\mathcal{C}_2(s, \ell, 0, X) = \max\{\hat{R}(1-\ell)e - \frac{\kappa(s, \ell, X)}{q(X)}, 0\}. \tag{A.5}$$

For an agent who turns out to be patient (type $\theta = 1$) at date 1, it is weakly (strongly if $q < 1$) optimal to set $c_1 = 0$ and[18]

$$\mathcal{C}_2(s, \ell, 1, X) = \hat{R}(1-\ell)e + \frac{\ell e}{q(X)}. \tag{A.6}$$

Step 2 : Bonds price

First, note that the opportunity to invest in the short asset at date 1 requires that $q \leq 1$. We now show that conditional on the aggregate state X, if $q(X) < 1$, then it must satisfy

$$q(X) = \frac{\mathcal{C}_1(s, L^s, 0, X)}{\mathcal{C}_2(s, L^s, 1, X) - \mathcal{C}_2(s, L^s, 0, X)} = e\frac{1-\pi}{\pi}\frac{\gamma L^P + (1-\gamma)L^U}{\gamma \min\{\hat{R}(1-L^P)e, B(L^P)\}}. \tag{A.7}$$

To establish this, we use the fact that in equilibrium, we must have $\ell = L^s$ for the agents of

[18] When $q \geq 1$, any plan such that $c_1 + c_2 = \hat{R}(1-\ell)e + \ell e$ (and $c_1, c_2 \geq 0$) is also optimal for a patient agent, but we can focus without loss of generality on the one featuring $c_1 = 0$ and $c_2 = \hat{R}(1-\ell)e + \ell e$.

type s (consistency). From (A.2) and $\mathcal{C}_1(s, L^s, 1, X) = 0$, we have

$$\begin{aligned}\mathcal{C}_1(s, L^s, 0, X) + q(X)\mathcal{C}_2(s, L^s, 0, X) &= L^s e + q\hat{R}(1 - L^s) \\ q(X)\mathcal{C}_2(s, L^s, 1, X) &= L^s e + q\hat{R}(1 - L^s).\end{aligned}$$

Combining these two equations allows us to obtain the first equality in (A.7):

$$q(X) = \frac{\mathcal{C}_1(s, L^s, 0, X)}{\mathcal{C}_2(s, L^s, 1, X) - \mathcal{C}_2(s, L^s, 0, X)} \quad \text{for } s \in \{U, P\}, \tag{A.8}$$

which itself implies

$$q(X) = \frac{\gamma\mathcal{C}_1(P, L^P, 0, X) + (1-\gamma)\mathcal{C}_1(U, L^U, 0, X)}{\gamma[\mathcal{C}_2(P, L^P, 1, X) - \mathcal{C}_2(P, L^P, 0, X)] + (1-\gamma)[\mathcal{C}_2(U, L^U, 1, X) - (\mathcal{C}_2(U, L^U, 0, X)]}. \tag{A.9}$$

Now, as $q < 1$, agents do not invest in short assets between date 1 and date 2, since they could otherwise make themselves strictly better off by saving in public bonds. Thus, the market clearing condition for date 1 consumption holds with equality:

$$\sum_s \sum_\theta \gamma_s \pi_\theta \mathcal{C}_1(s, L^s, \theta, X) = \sum_s \gamma_s L^s e.$$

Along with the fact that $\mathcal{C}_1(s, L^s, 1, X) = 0$ for $s \in \{P, U\}$, this implies

$$\gamma\mathcal{C}_1(P, L^P, 0, X) + (1-\gamma)\mathcal{C}_1(U, L^U, 0, X) = \frac{e}{\pi}(\gamma L^P + (1-\gamma)L^U). \tag{A.10}$$

Using (A.5) (with (13)) and (A.6), the denominator in (A.9) is given by

$$\begin{aligned}\gamma[\mathcal{C}_2(P, L^P, 1, X) - \mathcal{C}_2(P, L^P, 0, X)] + (1-\gamma)[\mathcal{C}_2(U, L^U, 1, X) - \mathcal{C}_2(U, L^U, 0, X)] \\ = \frac{\gamma L^P + (1-\gamma)L^U}{q(X)} e + \gamma \min\left\{\hat{R}(1 - L^P)e, B(L^P)\right\}.\end{aligned} \tag{A.11}$$

Substituting (A.10) and (A.11) into (A.9) yields, after some algebraic manipulation, to the second equality in (A.7):

$$q(X) = e\frac{1-\pi}{\pi}\frac{\gamma L^P + (1-\gamma)L^U}{\gamma \min\left\{\hat{R}(1 - L^P)e, B(L^P)\right\}}.$$

Since the opportunity to invest in the short asset at date 1 requires that $q \leq 1$, the general bond price expression in a continuation equilibrium is given by

$$q(X) = \min\left\{e\frac{1-\pi}{\pi}\frac{\gamma L^P + (1-\gamma)L^U}{\gamma\min\{\hat{R}(1-L^P)e, B(L^P)\}}, 1\right\}. \tag{A.12}$$

A.2 Proof of Lemma 2

We start by establishing that $c_2^*(0) = c_1^*(1) = 0$. First, $c_2(0) > 0$ cannot be optimal, since impatient agents do not value consumption at date 2. Second, if it were the case that $c_1(1) > 0$, then the planner could decrease $c_1(1)$ by some $\epsilon > 0$ arbitrarily small while increasing $c_2(1)$ by $\epsilon\hat{R}$ and while still satisfying the resource constraint (11) and strictly increasing welfare.

Next, we characterize $c_1^*(0)$ and $c_2^*(1)$. The planner's first-order condition with respect to $c_1(0)$ and $c_2(1)$ is given by

$$u'(c_1^*(0)) = \rho\hat{R}u'(c_2^*(1)).$$

Since $\rho\hat{R} > 1$ by Assumption 1, this implies that $c_1^*(0) < c_2^*(1)$. As shown in Diamond and Dybvig (1983) (footnote 3), condition (2) on the relative risk aversion in Assumption 1 further implies that $u'(e) > \rho\hat{R}u'(\hat{R}e)$, and therefore that $c_1^*(0) > e$ and $c_2^*(1) < \hat{R}e$.

A.3 Proof of Proposition 1

We show that $\gamma = 1$ and $B = (1-\pi)c_2^*(1)$ achieve the efficient allocation described in Lemma 2. Since this safety net architecture only features protected agents, we ignore unprotected agents in what follows.

The protected agents' date 0 problem is

$$\max_{\ell\in[0,1]} \pi u\left(\ell e + q\min\left\{\hat{R}\left(1-\ell\right)e, B\right\}\right) + \left(1-\pi\right)\rho u\left(\hat{R}\left(1-\ell\right)e + \frac{\ell e}{q}\right)$$

We consider separately the agent's problem in the two intervals $[0, 1-B/(\hat{R}e)]$ and $[1-B/(\hat{R}e), 1]$. In the first interval, the problem is

$$\max_{\ell\in\left[0,1-\frac{B}{\hat{R}e}\right]} \pi u\left(\ell e + qB\right) + \left(1-\pi\right)\rho u\left(\hat{R}\left(1-\ell\right)e + \frac{\ell e}{q}\right)$$

The first-order condition is given by

$$\Psi\left(\ell\right) \equiv e\pi u'\left(\ell e + qB\right) - e\left(1-\pi\right)\rho\left(\hat{R} - \frac{1}{q}\right)u'\left(\hat{R}\left(1-\ell\right)e + \frac{\ell e}{q}\right) \lesseqqgtr 0$$

with "$\leq$" if $\ell = 0$, "≥ 0" if $\ell = 1 - B/(\hat{R}e)$, and "$=$" if $\ell \in (0, 1-B/(\hat{R}e))$. Note that the agent's objective function is strictly concave in ℓ, as $\Psi'(\cdot) < 0$ for $\ell \in [0, 1 - B/(\hat{R}e)]$. In the second interval, the problem is

$$\max_{\ell \in \left[1-\frac{B}{\hat{R}e}, 1\right]} \pi u\left(\ell e + q\hat{R}(1-\ell)e\right) + (1-\pi)\rho u\left(\hat{R}(1-\ell)e + \frac{\ell e}{q}\right).$$

We conjecture that the policy consisting of $\gamma = 1$ and $B = (1-\pi)c_2^*(1)$ leads to an equilibrium collective choice of $L^P = 1 - B/(\hat{R}e) = 1 - (1-\pi)c_2^*(1)/(\hat{R}e) = \pi c_1^*(0)/e$, and verify this conjecture. Under the conjecture, the bond price is given by

$$q = \min\left\{e\frac{1-\pi}{\pi}\frac{L^P}{\hat{R}(1-L^P)e}, 1\right\} = \frac{1-\pi}{\pi}\frac{L^P}{\hat{R}(1-L^P)} = \frac{c_1^*(0)}{c_2^*(1)} < 1.$$

Starting with the first interval, we have

$$\begin{aligned}
\Psi\left(1 - \frac{B}{\hat{R}e}\right) &\equiv e\pi u'\left(c_1^*(0)\right) - (1-\pi)e\left(\hat{R} - \frac{1}{q}\right)\rho u'\left(c_2^*(1)\right) \\
&> e\pi u'\left(c_1^*(0)\right) - (1-\pi)e\left(\hat{R} - 1\right)\rho u'\left(c_2^*(1)\right) \\
&> e\left[\pi - \rho(1-\pi)e\left(\hat{R} - 1\right)\right]u'\left(c_1^*(0)\right) > 0,
\end{aligned}$$

where the last inequality follows from Assumption 2. The fact that $\Psi(1 - B/(\hat{R}e)) > 0$ implies that within this first interval, $\ell = 1 - B/(\hat{R}e)$ is optimal. Since the conjecture induces a bond price such that $q = c_1^*(0)/c_2^*(1) > 1/\hat{R}$, in the second interval the agent's objective function is strictly decreasing in ℓ, so $\ell = 1 - B/(\hat{R}e)$ is optimal. We have thus verified that the privately optimal investment choice over $\ell \in [0, 1]$ is consistent with the conjectured aggregate investment choice above. The fact that the policy in question achieves the efficient allocation follows from simple algebra.

A.4 Proof of Proposition 2

At date 1, the government chooses a debt issuance policy $B(\ell)$ to maximize the average welfare of agents, subject to the private sector's date 1 response to its action. The government solves

$$\max_{\{B_j\}_{j\in[0,\gamma]}} \int_0^\gamma \left[\pi \mathcal{V}_1(P, \ell_j, 0, X) + (1-\pi)\mathcal{V}_1(P, \ell_j, 1, X)\right] di + \int_\gamma^1 \left[\pi \mathcal{V}_1(U, \ell_j, 0, X) + (1-\pi)\mathcal{V}_1(U, \ell_j, 1, X)\right] dj$$

Using Lemma 1, this problem can be written as

$$\max_{\{B_j\}_{j\in[0,\gamma]}} \quad \int_0^{\gamma}\left[\pi u\left(\ell_j e+q\left(X\right)\min\left\{\hat{R}\left(1-\ell_j\right)e,B_j\right\}\right)+\left(1-\pi\right)u\left(\hat{R}\left(1-\ell_j\right)e+\frac{\ell_j e}{q}\right)\right]dj$$
$$+\int_{\gamma}^{1}\left[\pi u\left(\ell_j e\right)+\left(1-\pi\right)u\left(\hat{R}\left(1-\ell_j\right)e+\frac{\ell_j e}{q}\right)\right]dj$$

subject to[19]

$$q=\min\left\{e\frac{1-\pi}{\pi}\frac{\int_0^1 \ell_j dj}{\int_0^{\gamma}\min\left\{\hat{R}\left(1-\ell_j\right)e,B_j\right\}dj},1\right\}.$$

The first-order condition for B_i is

$$\begin{aligned}
&\mathbb{I}_{\left\{B_i<\hat{R}(1-\ell_i)e\right\}}\pi u'\left(\ell_i e+qB_i\right)q+\\
\mathbb{I}_{\left\{e\frac{1-\pi}{\pi}\frac{\int_0^1 \ell_j dj}{\int_0^{\gamma}\min\left\{\hat{R}\left(1-\ell_j\right)e,B_j\right\}dj}<1\right\}}\times\mathbb{I}_{\left\{B_i<\hat{R}(1-\ell_i)e\right\}}&\left[-\frac{q}{\int_0^{\gamma}\min\left\{\hat{R}\left(1-\ell_j\right)e,B_j\right\}dj}\right]\\
\times\left[\pi\int_0^{\gamma}u'\left(\ell_j e+q\min\left\{\hat{R}\left(1-\ell_j\right)e,B_j\right\}\right)\right.&\min\left\{\hat{R}\left(1-\ell_j\right)e,B_j\right\}dj\\
\left.+\left(1-\pi\right)\int_0^1 u'\left(\hat{R}\left(1-\ell_j\right)e+\frac{\ell_j e}{q}\right)\left(-\frac{\ell_j e}{q^2}\right)dj\right] &= 0.
\end{aligned}$$

To show that the optimal bailout rule satisfies $B\left(\ell\right)\geq\hat{R}\left(1-\ell\right)e$, suppose otherwise, seeking a contradiction. Then the first-order condition, evaluated at symmetric date 0 investment choices, becomes

$$\begin{aligned}
&\pi u'\left(\ell_i e+qB_i\right)q\\
+\mathbb{I}_{\left\{e\frac{1-\pi}{\pi}\frac{\int_0^1 \ell_j dj}{\gamma B_i}<1\right\}}\left[-\frac{q}{\gamma B_i}\right]\times\left[\pi\gamma u'\left(\ell_i e+qB_i\right)B_i+\left(1-\pi\right)u'\left(\hat{R}\left(1-\ell_i\right)e+\frac{\ell_i e}{q}\right)\left(-\frac{\ell_i e}{q^2}\right)\right] &= 0.
\end{aligned}$$

If $e\frac{1-\pi}{\pi}\frac{\int_0^1 \ell_j dj}{\gamma B_i}\geq 1$, we are left with $\pi u'\left(\ell_i e+qB_i\right)q=0$, which is a contradiction. If $e\frac{1-\pi}{\pi}\frac{\int_0^1 \ell_j dj}{\gamma B_i}<1$, we have

$$\left(1-\pi\right)u'\left(\hat{R}\left(1-\ell_i\right)e+\frac{\ell_i e}{q}\right)\frac{\ell_i e}{q\gamma B_i}=0,$$

which is also a contradiction. It follows that the optimal rule satisfies $B\left(\ell\right)\geq\hat{R}\left(1-\ell\right)e$. Note that any such rule trivially satisfies the first-order condition, since in that case the indicator variable $\mathbb{I}_{\left\{B_i<\hat{R}(1-\ell_i)e\right\}}$ is zero. Without loss of generality, the solution is thus $B\left(\ell\right)=\hat{R}\left(1-\ell\right)e$.

[19] This equilibrium price expression is obtained by a procedure analogous to that of Lemma 1, but without (yet) imposing the symmetry of date 0 investment choices.

A.5 Proof of Proposition 3

Step 1 : Date 0 short asset choice

An agent at date 0 faces this problem:

$$\mathcal{V}_0(s,X) = \max_{\ell\in[0,1]} \pi u\left(\mathcal{C}_1(s,\ell,0,X)\right) + (1-\pi)\rho u\left(\mathcal{C}_1(s,\ell,1,X) + \mathcal{C}_2(s,\ell,1,X)\right) \tag{A.13}$$

subject to (A.4) with $B(\ell) = \hat{R}(1-\ell)e$, (A.6), and (24).

First, it is useful to prove that $1/\hat{R} \leq q(X) \leq 1$. We have already argued that the presence of the short asset at date 1 requires $q(X) \leq 1$. We now show that $1/\hat{R} \leq q(X)$. Seeking a contradiction, we suppose that $q(X) < 1/\hat{R}$. In this case, from the perspective of date 0, investing in the short asset strictly dominates investing in the long asset. As a result, all agents invest only in the short asset at date 0, resulting in $L^R = L^U = 1$, and, according to (24), in $q(X) = \min\{\infty, 1\} = 1$, a contradiction. It follows that $1/\hat{R} \leq q(X) \leq 1$.

Next, we specialize the problem (A.13) for an unprotected agent as

$$\max_{\ell\in[0,1]} \pi u\left(\ell e\right) + (1-\pi)\rho u\left(\hat{R}(1-\ell)e + \frac{\ell e}{q(X)}\right). \tag{A.14}$$

The first-order condition is

$$\psi(\ell) \equiv e\pi u'\left(\ell e\right) - e(1-\pi)(\hat{R} - \frac{1}{q(X)})\rho u'\left(\hat{R}(1-\ell)e + \frac{\ell e}{q(X)}\right) = 0.$$

Note that the agent's objective function is strictly concave in ℓ, as $\psi'(\cdot) < 0$ for $\ell \in [0,1]$. Seeking a contradiction, suppose that $\ell = 1$ is not optimal. It must thus be that $\psi(1) < 0$, or

$$\pi u'(e) < (1-\pi)\left(\hat{R} - \frac{1}{q(X)}\right)\rho u'\left(\frac{e}{q(X)}\right) \leq (1-\pi)(\hat{R}-1)\rho u'\left(\frac{e}{q(X)}\right) \leq (1-\pi)(\hat{R}-1)\rho u'(e),$$

which requires $\pi < \frac{\rho(R-1)}{1+\rho(R-1)}$. This contradicts Assumption (2). The solution to (A.14) must thus feature $\ell = 1$.

Problem (A.13) specialized for a protected agent is given by

$$\max_{\ell\in[0,1]} \pi u\left(\ell e + q(X)\hat{R}(1-\ell)e\right) + (1-\pi)\rho u\left(\hat{R}(1-\ell)e + \frac{\ell e}{q(X)}\right). \tag{A.15}$$

We distinguish two cases: $q(X) = 1/\hat{R}$ and $q(X) > 1/\hat{R}$. When $q(X) = 1/\hat{R}$, date 1 and 2 consumption does not depend on ℓ, and therefore protected agents are then indifferent across all levels of $\ell \in [0,1]$. When $q(X) > 1/\hat{R}$, agents optimally choose $\ell = 0$, since in that case the objective function is strictly decreasing in ℓ.

Step 2 : Time-consistent equilibrium (as a function of γ)
The investment decision of unprotected agents always leads to $\ell = L^U = 1$. Regarding protected agents, we consider several cases. When $q = 1/\hat{R}$, protected agents are indifferent across any short-term investment level. Therefore, we can have $L^P \in [0, 1]$, but consistency with the equilibrium price expression (24) requires

$$L^P = \frac{\pi + \gamma - 1}{\gamma}.$$

And since $L^P \geq 0$, this constellation only prevails when $\gamma \geq 1 - \pi$. The equilibrium consumption allocations are then given by $c_2^s(0) = c_1^s(1) = 0$ for $s \in \{U, P\}$ and

$$c_1^U(0) = c_2^P(0) = e, \quad \text{and} \quad c_2^U(1) = c_2^P(1) = \hat{R}e. \tag{A.16}$$

When $q > 1/\hat{R}$, protected agents' short asset decision at date 0 leads to $L^P = 0$. Substituting $L^P = 0$ and $L^U = 1$ into the equilibrium price expression (24), we obtain

$$q = \min\left\{\frac{1}{\hat{R}}\frac{1-\pi}{\pi}\frac{1-\gamma}{\gamma}, 1\right\}.$$

Consistency thus requires $\gamma < 1 - \pi$, and the equilibrium consumption allocations are given by

$$c_1^U(0) = e, \quad c_2^U(1) = \frac{1}{q}e, \quad c_1^P(0) = q\hat{R}e, \quad \text{and} \quad c_2^P(1) = \hat{R}e. \tag{A.17}$$

A.6 Proof of Proposition 4

Let us define $\underline{\gamma} \equiv \frac{1-\pi}{1-\pi+\hat{R}\pi}$ and $\overline{\gamma} \equiv 1 - \pi$. The government chooses γ to maximize the average indirect utility function of private agents. It solves

$$\mathcal{W}_0 = \max_{\gamma \in [0,1]} \gamma \mathcal{V}_0\big(P, (\gamma, L^P(\gamma), 1)\big) + (1-\gamma)\mathcal{V}_0\big(U, (\gamma, L^P(\gamma), 1)\big). \tag{A.18}$$

To characterize the solution to this problem, it is convenient to separately consider the optimal choice of γ in the three intervals $[0, \underline{\gamma}]$, $[\underline{\gamma}, \overline{\gamma}]$, and $[\overline{\gamma}, 1]$. We note that the objective function is continuous in γ.

First, for $\gamma \in [\overline{\gamma}, 1]$, the problem reduces to

$$\max_{\gamma \in [\overline{\gamma}, 1]} \pi u(e) + (1-\pi)\rho u(\hat{R}e).$$

The objective function is constant with respect to γ, and therefore any $\gamma \in [\overline{\gamma}, 1]$ is optimal.

Next, for $\gamma \in [0, \underline{\gamma}]$, the problem is

$$\max_{\gamma \in [0,\underline{\gamma}]} [\pi + (1-\pi)\rho] \left[\gamma u\left(\hat{R}e\right) + (1-\gamma) u\left(e\right)\right].$$

The objective function is strictly increasing in γ, so the optimal choice is given by $\gamma = \underline{\gamma}$.

Finally, for $\gamma \in [\underline{\gamma}, \overline{\gamma}]$, the problem is given by

$$\max_{\gamma \in [\underline{\gamma},\overline{\gamma}]} \gamma \left[\pi u\left(e\frac{1-\pi}{\pi}\frac{1-\gamma}{\gamma}\right) + (1-\pi)\rho u(\hat{R}e)\right] + (1-\gamma)\left[\pi u(e) + (1-\pi)\rho u\left(\hat{R}e\frac{\pi}{1-\pi}\frac{\gamma}{1-\gamma}\right)\right]. \tag{A.19}$$

Since the overall objective function in (A.18) is strictly increasing over $[0, \underline{\gamma}]$ and constant over $[\overline{\gamma}, 1]$, it must be that if (A.19) admits a strictly interior solution, then it will be the global solution of (A.18).

The first-order condition for problem (A.19) is

$$\begin{aligned} \phi(\gamma) \equiv \; & \left[\pi u\left(e\frac{1-\pi}{\pi}\frac{1-\gamma}{\gamma}\right) + (1-\pi)\rho u(\hat{R}e)\right] - \left[\pi u(e) + (1-\pi)\rho u\left(\hat{R}e\frac{\pi}{1-\pi}\frac{\gamma}{1-\gamma}\right)\right] \\ & - e\frac{1-\pi}{\gamma}u'\left(e\frac{1-\pi}{\pi}\frac{1-\gamma}{\gamma}\right) + e\frac{\pi}{1-\gamma}\rho\hat{R}u'\left(\hat{R}e\frac{\pi}{1-\pi}\frac{\gamma}{1-\gamma}\right) \lesseqqgtr 0 \end{aligned}$$

with "$\leq$" if $\gamma^d = \underline{\gamma}$, "$\geq$" if $\gamma^d = \overline{\gamma}$, and "$=$" if $\gamma^d \in (\underline{\gamma}, \overline{\gamma})$. Evaluating $\phi(\cdot)$ at the bounds $\underline{\gamma}$ and $\overline{\gamma}$, we have

$$\begin{aligned} \phi(\underline{\gamma}) &= [\pi + (1-\pi)\rho]\left[u\left(e\hat{R}\right) - u\left(e\right)\right] + e\left(1-\pi+\hat{R}\pi\right)\rho\left[u'\left(e\right) - u'\left(e\hat{R}\right)\right] > 0 \\ \phi(\overline{\gamma}) &= -e\left[u'\left(e\right) - \rho\hat{R}u'\left(\hat{R}e\right)\right] < 0. \end{aligned}$$

The global optimum is therefore strictly interior: $\gamma^d \in (\underline{\gamma}, \overline{\gamma})$.