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# Online Appendix for: Two Illustrations of the Quantity Theory of Money Reloaded

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# Online Appendix for

## *Two Illustrations of the Quantity Theory of Money Reloaded*

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### A Algebra

The Bellman equation describing the decision problem is

$$V(\omega) = \max_{x,n,m,b} U(x) + \beta E \left[ V\left(\frac{m + b(1+i) + (1-\theta\nu n)z - x}{1 + \pi(s')} + \tau(s')\right) \right] - \varepsilon [m + b - \omega] - \delta [x - mn],$$

where, for simplicity, we omitted the dependence of current variables on the state of the economy  $s$ . The first order conditions are

$$x : U'(x) = \beta E \left[ \frac{V'(\omega')}{1 + \pi(s')} \right] + \delta \tag{A1}$$

$$n : \delta m = \beta E \left[ \frac{V'(\omega')}{1 + \pi(s')} \right] \theta \nu z \tag{A2}$$

$$m : \delta n + \beta E \left[ \frac{V'(\omega')}{1 + \pi(s')} \right] = \varepsilon \tag{A3}$$

$$b : \beta E \left[ \frac{V'(\omega')}{1 + \pi(s')} \right] (1 + i) = \varepsilon, \tag{A4}$$

and the envelope condition is

$$V'(\omega) = \varepsilon. \tag{A5}$$

In what follows, we focus the analysis on circumstances in which the nominal interest rate is bounded away from zero, so the cash-in-advance constraint (2) is binding. Note that (A3) and (A4) imply

$$\delta n + \beta E \left[ \frac{V'(\omega')}{1 + \pi(s')} \right] = \beta E \left[ \frac{V'(\omega')}{1 + \pi(s')} \right] (1 + i),$$

which, combined with (A2), yields

$$\frac{m}{n} i = \theta \nu z.$$

Replacing the equilibrium conditions (2) and (3), we obtain

$$i = n^2 \frac{\theta \nu}{(1 - \theta \nu n)}.$$

Note that  $\theta\nu n$  represents the welfare cost of inflation. Estimates of this cost for relatively low values of the nominal interest rates, like the ones we will consider in the empirical section, are relatively small, on the order of less than 2% of output. That means  $\gamma n$  ranges between 0 and 0.02. We then approximate the solution by

$$\sqrt{\frac{i}{\theta\nu}} \simeq n,$$

which is the celebrated squared root formula derived by [Baumol \(1952\)](#) and [Tobin \(1956\)](#). We can once again use the cash-in-advance constraint (2) to replace the variable  $n$  in the last equation and obtain

$$\frac{m}{x} = \sqrt{\frac{\theta\nu}{i}},$$

which delivers a relationship between real money balances as a proportion of output and the nominal interest rate in bonds.

In addition, we can use (A4) and (A5) to obtain

$$E \left[ \frac{\beta V'(\omega')}{V'(\omega)} \frac{1}{\pi(s')} \right] (1+i) = 1,$$

which can be written as

$$E \left[ \frac{(1+i)}{1+r(s')} \frac{1}{\pi(s')} \right] = 1,$$

where  $r(s')$  is a measure of the real interest rate. This last expression is the well known Fisher equation relating the nominal interest rate with the real interest rate and the inflation rate. This real interest rate is measured in terms of marginal utilities of real wealth, using the indirect utility function. In order to obtain a real interest rate in terms of the utility function, which is the usual way to measure it, note that (A3) and (A4) imply

$$\delta n = \beta E \left[ \frac{V'(\omega')}{1+\pi(s')} \right] i.$$

Replacing it in (A1) delivers

$$U'(x) = \beta E \left[ \frac{V'(\omega')}{1+\pi(s')} \right] \left(1 + \frac{i}{n}\right).$$

Using (A4), we can write it as

$$\frac{U'(x)}{\left(1 + \frac{i}{n}\right)} = \frac{\varepsilon}{i}.$$

But (A4), together with the envelope conditions, implies

$$\beta E \left[ \frac{\varepsilon'}{1+\pi(s')} \right] (1+i) = \varepsilon.$$

So, using the previous equation, we obtain

$$E \left[ \left[ \frac{\beta U'(x')}{U'(x)} \frac{\frac{1+i'}{(1+\frac{i'}{n})}}{\frac{1+i}{(1+\frac{i}{n})}} \right] \left( \frac{1+i}{1+\pi(s')} \right) \right] = 1,$$

which implies that the expectation of the inverse of the real interest rate times the ratio of the nominal interest rate divided by the inflation rate must be equal to 1.

## B Data

### B.1 The United States

The series of nominal GDP, the three-month Treasury bill rate, currency in circulation, and “standard” M1 are collected from FRED.<sup>27</sup> Currency and the three-month T-bill rate are used as the measures of cash and the interest rate associated with it.

**NewM1** The construction of NewM1 follows [Lucas and Nicolini \(2015\)](#):

$$\text{NewM1} = \text{M1} + \text{MMDAs}.$$

Money Market Demand Accounts (MMDAs) series are constructed by aggregating term RCON6810 under Schedule RC-E from individual banks’ call reports. The original data are publicly available at the Central Data Repository Public Data Distribution website of Federal Financial Institutions Examination Council.<sup>28</sup>

The MMDAs series have been issued since 1982Q3, but the data are available only after 1984Q2. We apply a linear interpolation of money growth rates for the periods in between. [Figure A1](#) depicts the money growth rates of cash, the “standard” M1, and the New M1 series since 1960.

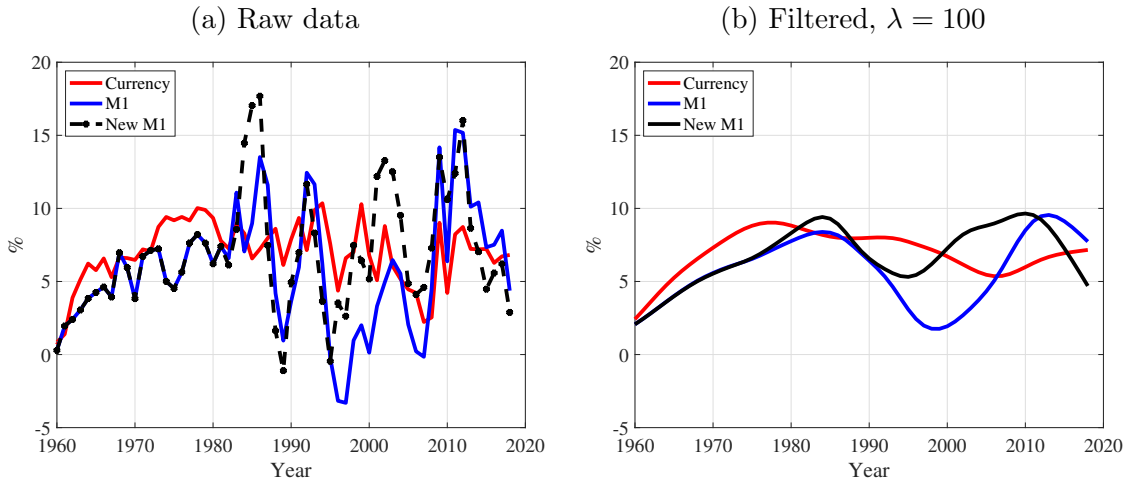


Figure A1: Money growth in the United States

<sup>27</sup>FRED: <https://fred.stlouisfed.org/>.

<sup>28</sup>FFIEC: <https://cdr.ffiec.gov/public/>.

**Imputed interest rate** We impute the interest rate associated with the New M1 by subtracting the fraction of interests paid by deposits and by MMDAs from the three-month T-bill rate; that is,

$$\tilde{r} = r^{3m} - s_d i^d - s_a i^a,$$

where  $s_d$  and  $s_a$  are the ratio of deposits to NewM1 and the ratio of MMDAs to NewM1, and  $i^{3m}$ ,  $i^d$ , and  $i^a$  are the interest rates on three-month T-bills, deposits, and MMDAs, respectively.

**Real interest rates** The real interest rate is constructed by subtracting the three-month T-bill rate by inflation. In view of the lack of real interest rates for other countries, we use the real rates of the United States as the proxy of real rates in other countries for the quantitative illustration of Fisher equation. Figure A2(a) plots the constructed raw series of US real interest rates since 1960 and the HP-filtered series using smoothing parameter 100. Figure A2(b) compares the imputed real interest rates with interest rates on Treasury Inflation-Indexed Securities (TIPS) at the five- and ten-year maturities. As can be seen from Figure A2(b), the difference between our imputed real interest rates and interest rates on long-term TIPS is very stable over time.

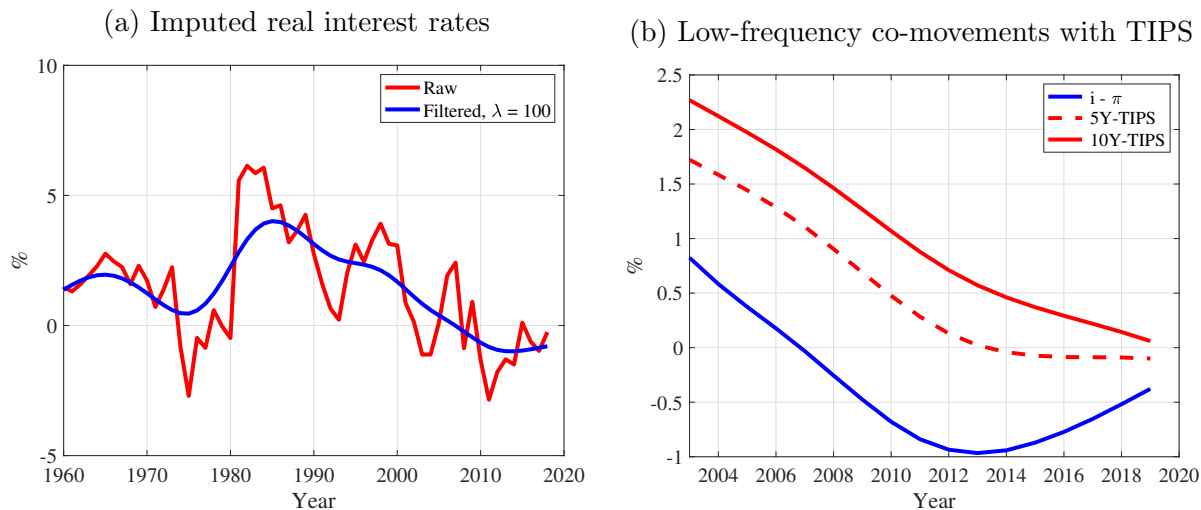


Figure A2: Imputation of US real interest rates

## B.2 Other OECD countries

We need data for prices, money stock M1, GDP, and interest rates for each country. In view of the lack of real GDP, we collect data of nominal GDP in local currency and impute real GDP with prices. The main source for nominal interest rates and M1 is the OECD data website, and the main source for nominal GDP is the International Financial Statistics (IFS) of the International Monetary Fund (IMF).<sup>29</sup> We collect data for all countries starting

<sup>29</sup>OECD data: <https://data.oecd.org/>; IFS data: <https://data.imf.org/>.

from 1960 as long as there is availability. For countries with missing values up till 1960, we splice the series from the OECD and the IFS with data constructed in [Benati et al. \(2021\)](#).<sup>30</sup> Money data for countries in the eurozone (Germany, Italy, Netherlands, Portugal, and Spain) are available only up till 1998.

Finally, we have data for 16 OECD countries other than the United States and break them into two groups based on the similarity of inflation movements:

1. USA, Australia, Canada, Denmark, Germany, Japan, New Zealand, and the UK;
2. Italy, Netherlands, Portugal, South Korea, Spain, Colombia, Chile, Mexico, and Turkey.

The following list details special issues in the construction of the dataset.

**Australia** Interest rates in 1960–1967 and M1 in 1960 are spliced with [Benati et al. \(2021\)](#).

**Canada** Nominal GDP in 1960 is spliced using [Benati et al. \(2021\)](#). Between 1982 and 2005, M1 in the OECD dataset has faster growth at the beginning and lower growth in later years compared with the M1 data in [Benati et al. \(2021\)](#), which results in a similar cumulative growth across these two sources.

**Chile** We use data for Chile after 1985, because in the 1970s, Chile had several years of hyperinflation over 100%. Interest rates in 1985–1997 are spliced with [Benati et al. \(2021\)](#).

**Colombia** The OECD provides nominal interest rates only after 1986. Interest rates in [Benati et al. \(2021\)](#) and the OECD behave similarly after 1995 but are significantly higher in the OECD than in [Benati et al. \(2021\)](#). For consistency, we use [Benati et al. \(2021\)](#) for interest rates in all periods .

**Denmark** Interest rates between 1960 and 1986 are spliced using [Benati et al. \(2021\)](#).

**Germany** The IFS provides nominal GDP only after 1992. For consistency, we use [Benati et al. \(2021\)](#) for nominal GDP in all periods.

**Italy** Interest rates before 1979 are spliced using [Benati et al. \(2021\)](#). The IFS provide nominal GDP only after 1995, and the OECD does not have data for M1. We use [Benati et al. \(2021\)](#) for nominal GDP and M1 in all periods.

**Japan** Interest rates before 2003 are spliced using [Benati et al. \(2021\)](#).

**Mexico** Interest rates before 1997, prices before 1969, and M1 before 1977 are spliced using [Benati et al. \(2021\)](#). Nominal GDP for all years is taken from [Benati et al. \(2021\)](#).

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<sup>30</sup>See [Benati et al. \(2019\)](#) for more details about the original data sources.

**Netherlands** Interest rates before 1982 and nominal GDP before 1995 are spliced using [Benati et al. \(2021\)](#). M1 for all years is taken from [Benati et al. \(2021\)](#).

**New Zealand** Interest rates before 1974, nominal GDP before 1970, and M1 before 1978 are spliced using [Benati et al. \(2021\)](#).

**Portugal** Interest rates before 1986 and nominal GDP before 1995 are spliced using [Benati et al. \(2021\)](#). M1 for all years is taken from [Benati et al. \(2021\)](#).

**South Korea** Interest rates are taken from [Benati et al. \(2021\)](#).

**Spain** Interest rates before 1976 are spliced using [Benati et al. \(2021\)](#). Note that interest rates between 1977 and 1981 in the OECD dataset are higher than those in [Benati et al. \(2021\)](#). The IFS provide nominal GDP since 1995. We use [Benati et al. \(2021\)](#) for nominal GDP in all periods for consistency.

**Turkey** Data for Turkey are available from 1969 onwards. Nominal GDP before 1987 is spliced using [Benati et al. \(2021\)](#). Interest rates for all years are taken from [Benati et al. \(2021\)](#).

**The UK** We use all variables for all years from [Benati et al. \(2021\)](#).

[Table A1](#) provides the summary statistics of mean and standard deviation of inflation  $\pi$ , nominal interest rate  $i$ , money growth  $\mu$ , and real GDP growth  $g$  by country.

### **B.3 Additional results**

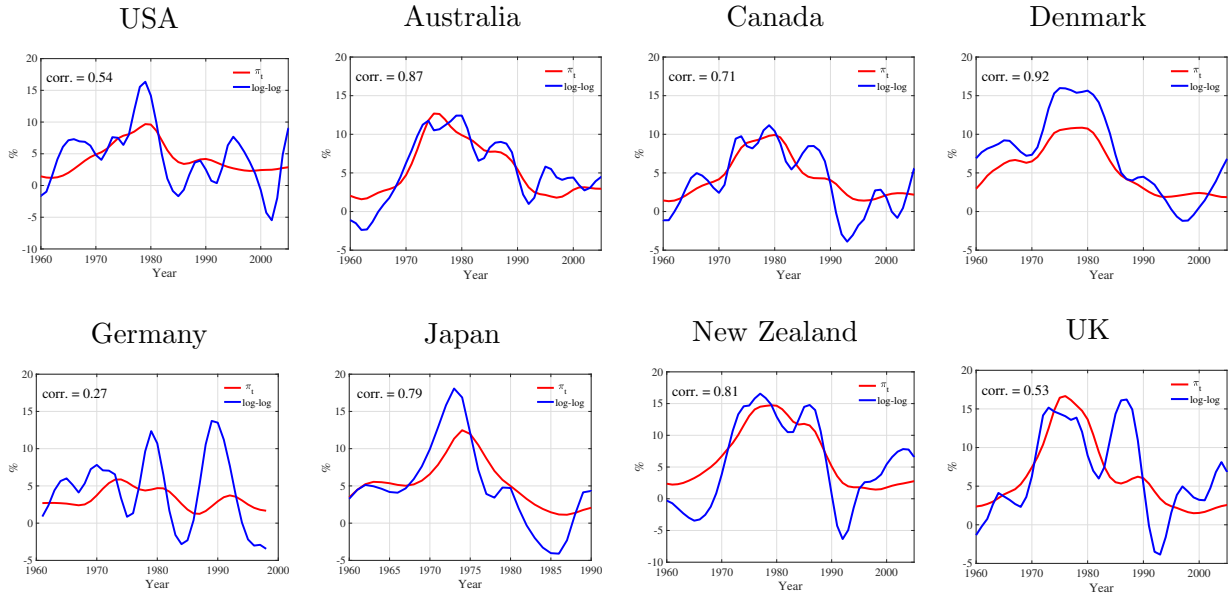
In [Figure A3](#) and [Figure A4](#), we report the two illustrations when we filter series using smoothing parameter  $\lambda = 6.5$ .

Table A1: Mean and standard deviation of main variables

Country	Periods	$\pi$	$i$	$\mu$	$g$
USA - Currency	1960–2005	4.26	6.16	7.25	2.94
		(2.91)	(3.13)	(2.34)	(2.43)
USA - Standard M1	1960–2005	4.26	6.16	5.14	2.94
		(2.91)	(3.13)	(3.71)	(2.43)
USA - New M1 - Interp1	1960–2005	4.26	4.97	7.32	2.94
		(2.91)	(2.57)	(6.16)	(2.43)
USA - New M1 - Interp2	1960–2005	4.26	4.97	6.50	2.94
		(2.91)	(2.57)	(4.11)	(2.43)
Australia	1960–2005	5.48	8.28	9.01	3.73
		(4.03)	(4.06)	(6.21)	(2.74)
Canada	1960–2005	4.36	7.18	8.06	3.70
		(3.18)	(3.50)	(4.58)	(2.95)
Denmark	1960–2005	5.45	9.89	10.79	2.73
		(3.55)	(4.39)	(6.26)	(2.39)
Germany	1961–2005	3.00	5.61	8.18	3.16
		(1.80)	(2.53)	(3.51)	(2.95)
Japan	1960–2005	3.85	4.22	11.36	4.51
		(4.37)	(2.60)	(7.07)	(4.94)
New Zealand	1960–2005	6.56	9.65	8.82	2.88
		(5.38)	(4.50)	(7.62)	( 2.35)
The UK	1960–2005	6.38	8.35	9.72	2.75
		(5.44)	(3.57)	(6.13)	(2.04)
Italy	1960–2005	7.14	6.56	13.14	4.26
		(5.63)	(3.64)	(6.58)	(2.79)
Netherlands	1962–2005	3.96	5.29	7.72	3.42
		(2.59)	(1.96)	(4.98)	(2.88)
Portugal	1960–2005	10.28	8.35	12.99	4.87
		(8.13)	(6.62)	(6.64)	(3.32)
South Korea	1962–2005	9.73	10.06	24.47	9.88
		(7.57)	(7.18)	(12.39)	(6.21)
Spain	1960–2005	7.92	8.85	12.99	4.87
		(5.57)	(5.08)	(6.64)	(3.32)
Chile	1980–2005	13.39	26.24	22.01	5.03
		(9.70)	(18.00)	(14.04)	(5.84)
Colombia	1960–2005	17.59	9.20	21.70	4.15
		(7.65)	(6.11 )	(7.66)	(2.15)
Mexico	1960–2005	24.43	24.32	28.22	5.02
		(31.02)	(21.68)	(24.00)	(5.10)
Turkey	1969–2005	46.80	36.52	48.36	4.75
		(27.75)	(20.98)	(22.46 )	(7.14)



## Illustration 1



## Illustration 2

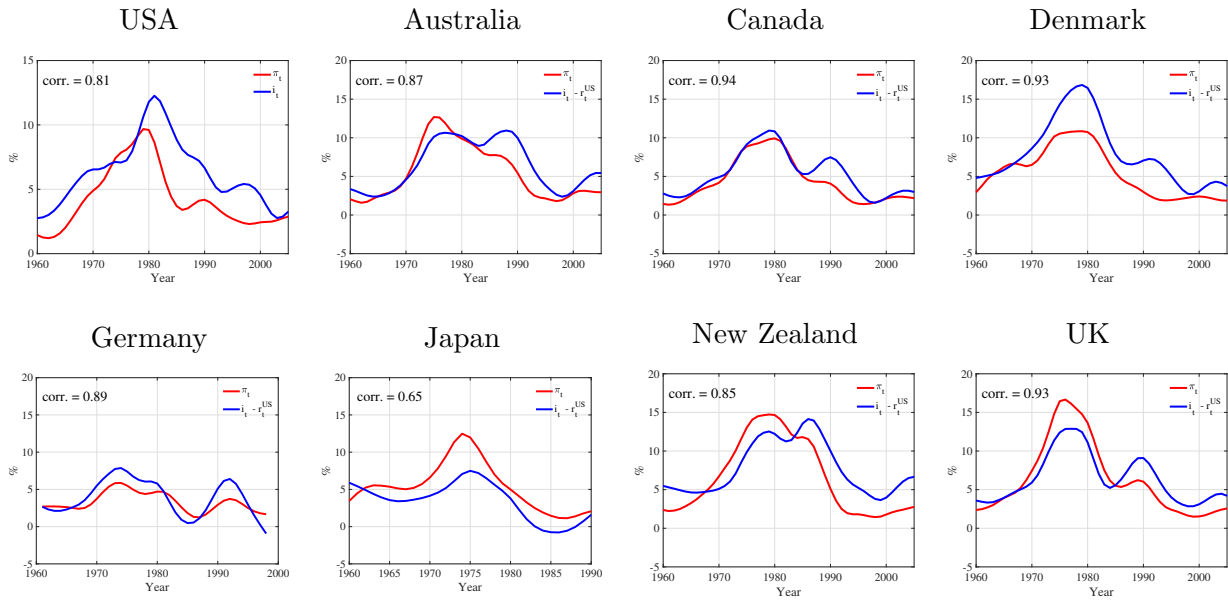
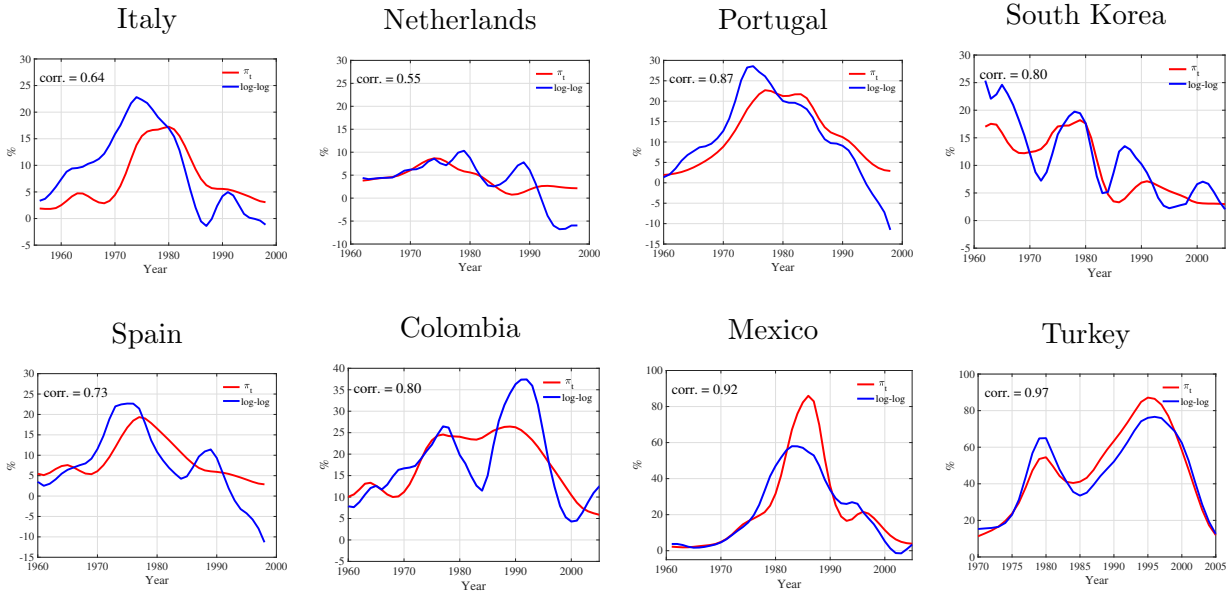


Figure A3: Countries in Group 1,  $\lambda = 6.5$

## Illustration 1



## Illustration 2

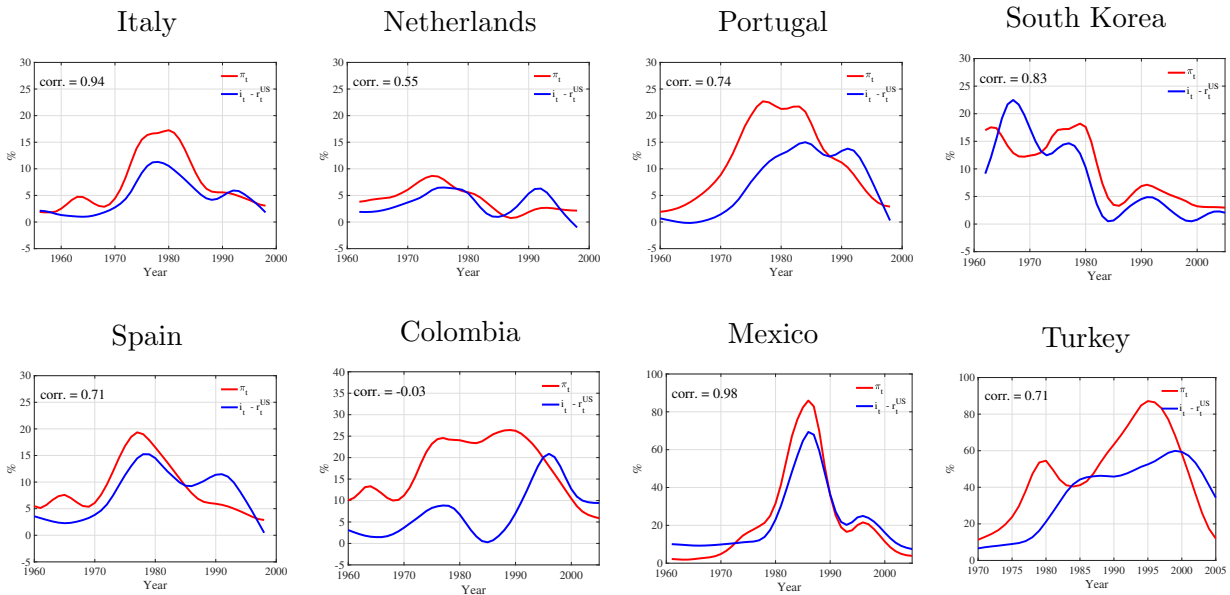


Figure A4: Countries in Group 2,  $\lambda = 6.5$

## C Estimation details

The model of Section 4 is estimated using Bayesian methods. We jointly estimate the structural parameters,  $\vartheta$ , and the dates of regime changes,  $\mathbf{T}$ . We estimate,  $T^{\text{on}}$ ,  $T^{\text{off}}$  and  $T_\kappa$ . The first two correspond to the dates of the high inflation regime for which  $\mathbb{I}^s = 1$ . In the case of  $T^{\text{off}}$ , we sample from a uniform distribution over 1979q4 and 1983q4, which corresponds to the Volcker disinflation;  $T_\kappa$  is the date break for the variance of all structural shocks except for that of the money demand shock. The variance of the remaining structural shocks shifts proportionally at  $T_\kappa$  by a factor of  $\kappa$ , so that the variance covariance matrix shifts from  $\kappa\Omega$  to  $\Omega$ . This specification serves two purposes: First, it helps the model capture the decrease in volatility associated with the Great Moderation. Second, and more important for our purposes, is that it guards against the possibility that the estimation relies on shocks to the inflation target to account for the increased volatility of the 1970s. For the variance of shocks to money demand,  $\sigma_\xi$ , the volatility shifts in 1982q4 to  $\kappa_m\sigma_\xi$ , which, as explained above, lines up with the regime change in the measurement of M1 explained in [Lucas and Nicolini \(2015\)](#).

The model is estimated on real GDP per capita growth; the federal funds rate; core inflation as measured by the CPI, excluding food and energy, the Michigan survey measure of inflation expectations; and money growth.

To construct the likelihood of the model under regime changes, we use the method outlined in [Kulish and Pagan \(2017\)](#). That method deals with a more general case than the application we are considering, so we provide a brief discussion of the case we deal with here.

Let  $t = 1, 2, \dots, T$  index the observations in the sample. From period  $t = 1, 2, \dots, T^{\text{on}} - 1$ , the steady state level of inflation is  $\pi$ . The first-order approximation to the equilibrium conditions around this initial steady state is given by the linear rational expectations system of  $n$  equations that we write as

$$A_0\mathbf{y}_t = C_0 + A_1\mathbf{y}_{t-1} + B_0\mathbb{E}_t\mathbf{y}_{t+1} + D_0\varepsilon_t, \quad (\text{A6})$$

where  $A_0$ ,  $C_0$ ,  $A_1$ ,  $B_0$  and  $D_0$  are the structural matrices of the initial steady-state,  $\mathbf{y}_t$  is a  $n \times 1$  vector of state and jump variables, and  $\varepsilon_t$  is an  $l \times 1$  vector of exogenous *i.i.d* shocks. The unique rational expectations solution to (A6) is

$$\mathbf{y}_t = C + Q\mathbf{y}_{t-1} + G\varepsilon_t. \quad (\text{A7})$$

For  $t = T^{\text{on}}$  until  $T^{\text{off}} - 1$  the steady state level of inflation increases to  $\pi + \Delta\pi$  and  $\mathbb{I} = 1$ , so the structural equations are given by

$$\bar{A}_0\mathbf{y}_t = \bar{C}_0 + \bar{A}_1\mathbf{y}_{t-1} + \bar{B}_0\mathbb{E}_t\mathbf{y}_{t+1} + \bar{D}_0\varepsilon_t, \quad (\text{A8})$$

with solution

$$\mathbf{y}_t = \bar{C} + \bar{Q}\mathbf{y}_{t-1} + \bar{G}\varepsilon_t. \quad (\text{A9})$$

At  $T^{\text{off}}$  the economy reverts to (A6) with steady state  $\pi$ . These structural changes imply

that the reduced form is time-varying over the sample. In general,

$$\mathbf{y}_t = C_t + Q_t \mathbf{y}_{t-1} + G_t \varepsilon_t. \tag{A10}$$

With a sample of data,  $\{y_t^{obs}\}_{t=1}^T$ , where  $y_t^{obs}$  is a  $n_{obs} \times 1$  vector of observable variables that relates to the model's variables through the measurement equation below:

$$y_t^{obs} = H_t \mathbf{y}_t. \tag{A11}$$

Here,  $H_t$  is time varying to account for the fact that the Michigan measure of inflation expectations becomes available only after 1978. The observation equation, Equation (A11), and the state equation, Equation (A10), form a state-space model. The Kalman filter can be used to construct the likelihood function for the sample  $\{y_t^{obs}\}_{t=1}^T$ , given by  $\mathcal{L}(Y|\vartheta, \mathbf{T})$  as outlined in [Kulish and Pagan \(2017\)](#).

Given the joint posterior of the structural parameters and the date breaks,  $p(\vartheta, \mathbf{T}|Y) = \mathcal{L}(Y|\vartheta, \mathbf{T})p(\vartheta)p(\mathbf{T})$ , we simulate from this distribution using the Metropolis-Hastings algorithm as used by [Kulish and Rees \(2017\)](#). As we have continuous and discrete parameters, we separate them into two blocks: one for date breaks and one for structural parameters. The sampler delivers draws from the joint posterior of both sets of parameters.

Below we report results from our baseline estimation. We also estimated the model using cash, rather than M1. We also estimated the slope of the Phillips curve rather than calibrating it. None of this variations altered the main results reported in this section.

## C.1 Prior and posteriors of the structural parameters

Table A2: Estimation results – interest rate rule

	Prior distribution			Posterior distribution			
	Shape	Mean	Std Dev.	Mode	Mean	5 %	95 %
<i>Standard Deviations</i>							
$100 \times \sigma_i$	Inv. Gamma	1	2	0.08	0.08	0.06	0.10
$100 \times \sigma_a$	Inv. Gamma	1	2	1.45	1.33	1.17	1.84
$100 \times \sigma_e$	Inv. Gamma	1	2	0.11	0.11	0.09	0.14
$100 \times \sigma_z$	Inv. Gamma	1	2	0.47	0.47	0.42	0.53
$100 \times \sigma_\pi$	Inv. Gamma	1	2	0.10	0.10	0.09	0.12
$100 \times \sigma_\tau^{obs}$	Inv. Gamma	1	2	0.16	0.15	0.12	0.20
$100 \times \sigma_\xi$	Inv. Gamma	2	3	1.25	1.25	1.11	1.41
$\kappa$	Normal	2	0.3	2.00	1.99	1.82	2.20
$\kappa_m$	Normal	2	0.3	1.37	1.32	1.14	1.61
<i>Structural parameters</i>							
$\rho_i$	Beta	0.5	0.2	0.90	0.90	0.86	0.93
$\phi_\pi$	Normal	2	0.5	2.06	2.12	1.40	2.74
$\phi_x$	Normal	0.125	0.05	0.37	0.37	0.31	0.42
$10 \times \omega$	Normal	0.5	0.1	0.52	0.53	0.37	0.69
$\eta$	Normal	0.5	0.05	0.46	0.46	0.38	0.55
$\rho_m$	Beta	0.5	0.2	0.95	0.96	0.91	0.98
$\rho_a$	Beta	0.5	0.2	0.88	0.88	0.84	0.91
$\rho_e$	Beta	0.5	0.2	0.46	0.47	0.34	0.56
$\rho_\tau$	Beta	0.5	0.2	0.79	0.80	0.68	0.89
$\rho_\pi$	Beta	0.5	0.2	0.97	0.98	0.97	0.98
$100 \times \Delta_\pi$	Uniform	[-2 , 6 ]		1.03	0.98	0.18	1.95
$\rho_\xi$	Beta	0.5	0.2	0.60	0.59	0.46	0.74

## C.2 Posteriors of date breaks

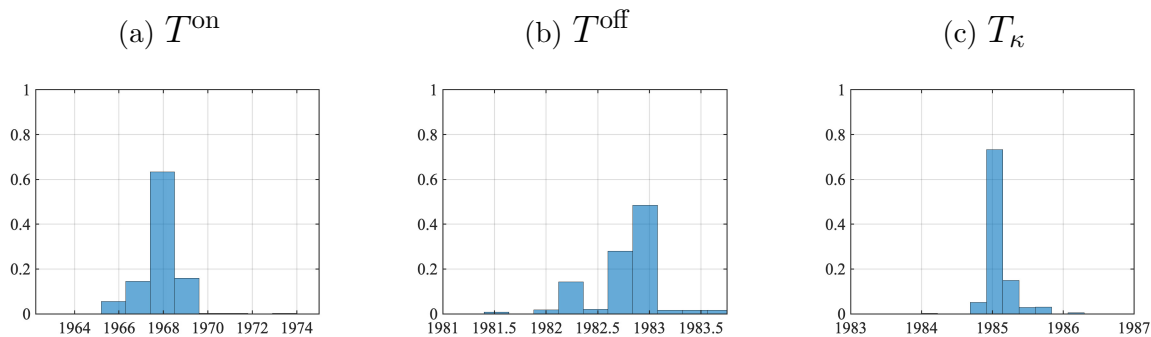


Figure A5: Posterior distributions of regime switching dates – interest rate rule

### C.3 Additional model fitness

Figure A6 and Figure A7 report model fitness of interest rates and money growth rates.

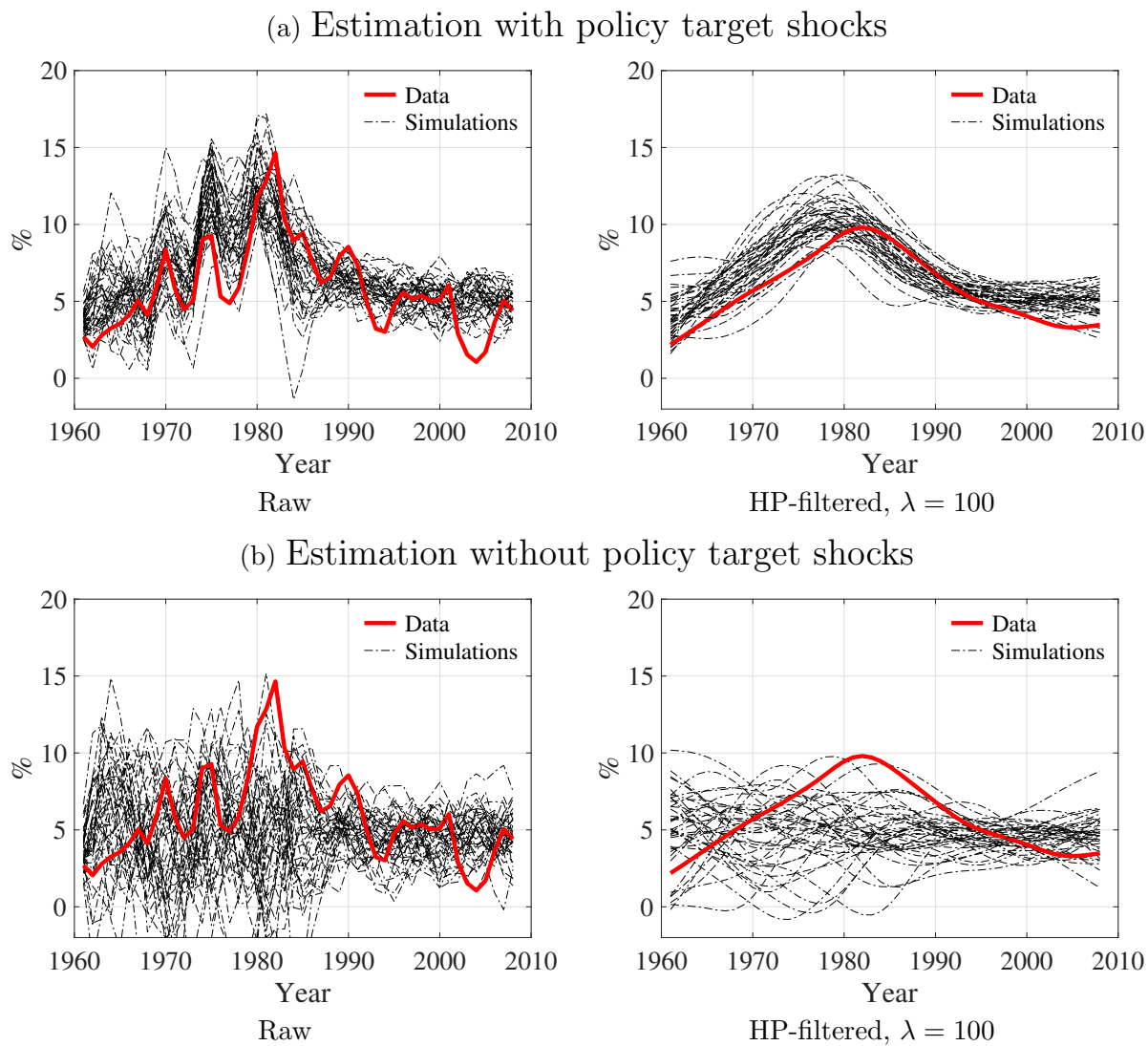
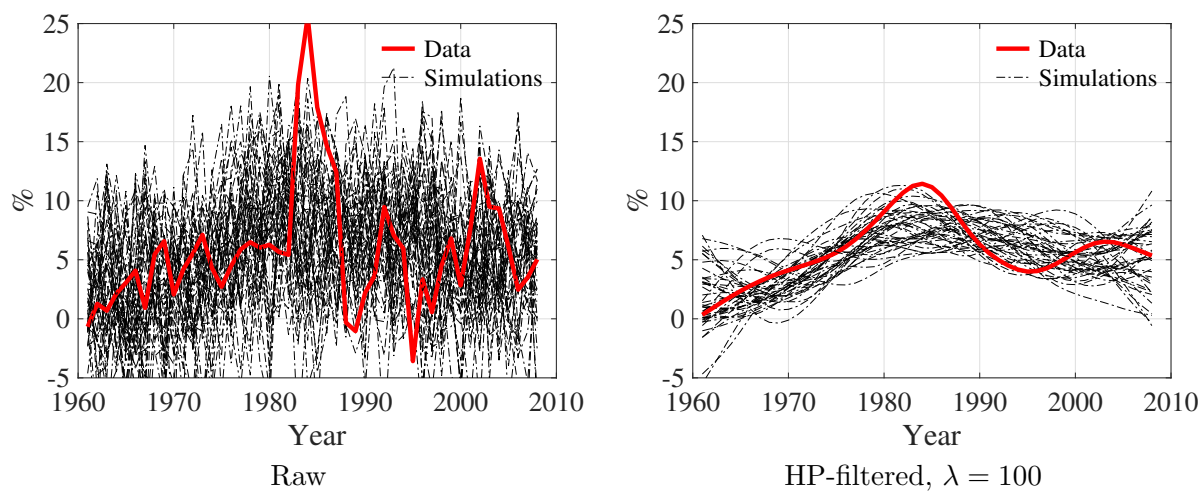


Figure A6: Model fitness of nominal interest rates – interest rate rule

(a) Estimation with policy target shocks



(b) Estimation without policy target shocks

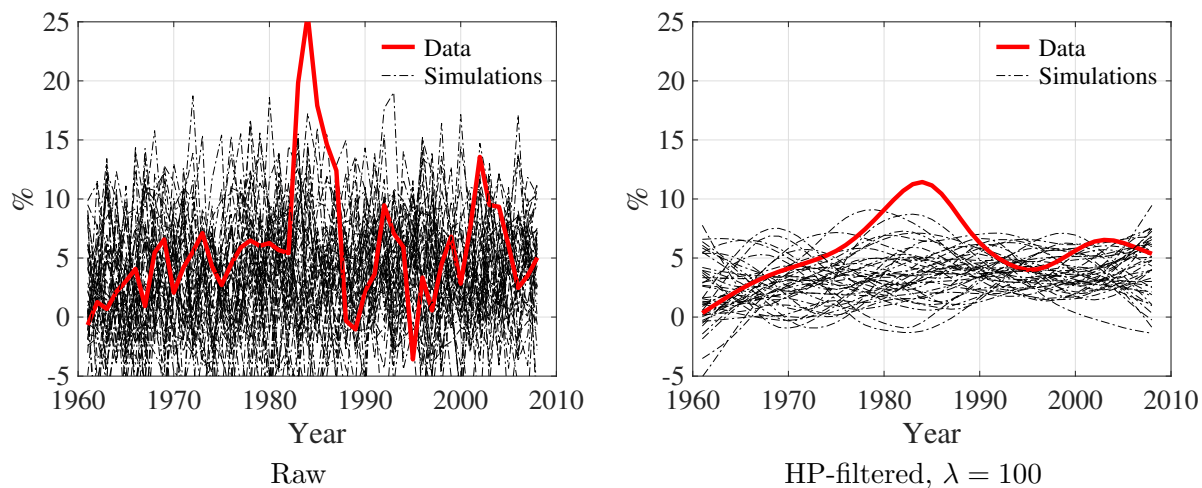


Figure A7: Model fitness of money growth rates – interest rate rule

## D Model with money rule

### D.1 Estimation results

We estimate the model using the same observable variables that we used for the case of interest rate rule. We treat money stock between 1980Q1 and 1984Q2 as unobservables. This quantitatively makes no impact on our estimation of the structural parameters.

Table A3: Estimation results – model with money rule

	Prior distribution			Posterior distribution			
	Shape	Mean	Std Dev.	Mode	Mean	5 %	95 %
<i>Standard Deviations</i>							
$100 \times \sigma_{mu}$	Inv. Gamma	1	2	0.90	0.86	0.77	1.05
$100 \times \sigma_a$	Inv. Gamma	2	3	2.60	2.57	1.17	1.84
$100 \times \sigma_e$	Inv. Gamma	1	2	0.10	0.09	0.08	0.13
$100 \times \sigma_z$	Inv. Gamma	1	2	0.50	0.50	0.424	0.57
$100 \times \sigma_{\mu^*}$	Inv. Gamma	1	2	0.10	0.10	0.09	0.12
$100 \times \sigma_\tau$	Inv. Gamma	1	2	0.13	0.14	0.10	0.17
$100 \times \sigma_\xi$	Inv. Gamma	2	3	1.36	1.35	1.25	1.48
$\kappa$	Normal	2	0.3	2.02	2.00	1.75	2.21
<i>Structural parameters</i>							
$\rho_\mu$	Beta	0.5	0.2	0.30	0.27	0.21	0.38
$\theta_\pi$	Normal	1	0.2	1.25	1.23	0.90	1.60
$\theta_x$	Normal	4	0.5	4.16	4.13	3.44	4.80
$10 \times \omega$	Normal	0.5	0.1	0.49	0.42	0.29	0.68
$\eta$	Normal	0.15	0.025	0.13	0.14	0.11	0.15
$\rho_m$	Beta	0.5	0.2	0.19	0.19	0.12	0.28
$\rho_a$	Beta	0.5	0.2	0.94	0.94	0.91	0.96
$\rho_e$	Beta	0.5	0.2	0.45	0.39	0.35	0.56
$\rho_\tau$	Beta	0.5	0.2	0.80	0.81	0.73	0.88
$\rho_\pi$	Beta	0.5	0.2	0.98	0.98	0.97	0.98
$100 \times \Delta_\pi$	Uniform	[-2 , 6 ]		1.83	2.00	1.37	2.23
$\rho_\xi$	Beta	0.5	0.2	0.99	0.99	0.99	1.00

Table A3 reports the estimated parameters and Figure A8 reports distributions of regime switching dates. Our estimation again detects that the monetary policy regime switches between the mid-1960s and early 1980s.



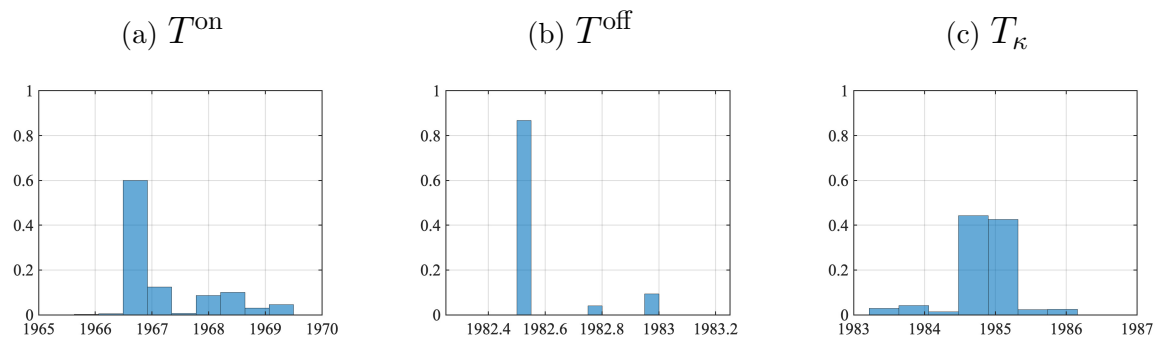
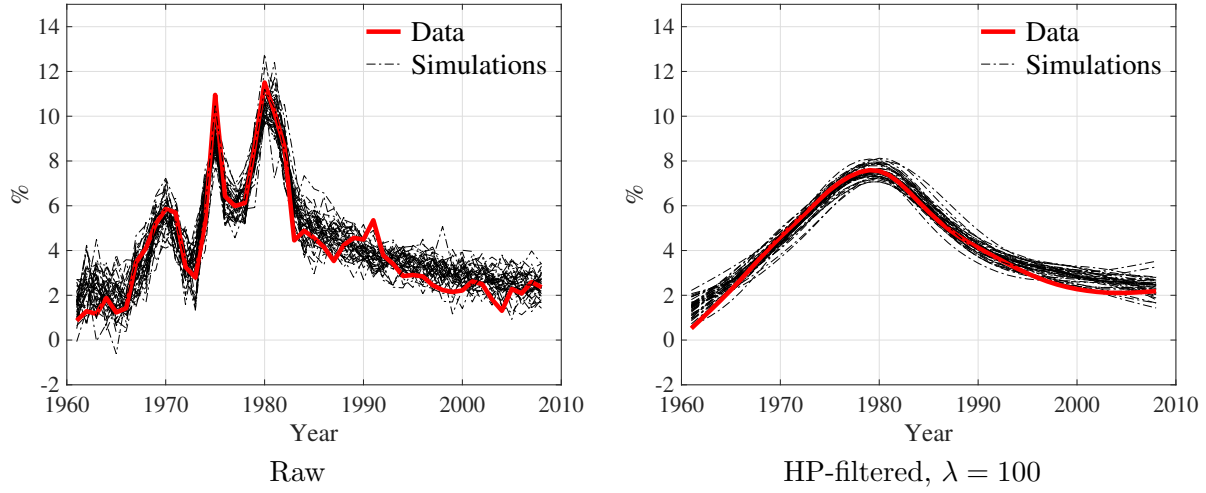


Figure A8: Posterior distributions of regime switching dates - money rule

## D.2 Model performance

(a) Simulation with policy target shocks



(b) Simulation without policy target shocks

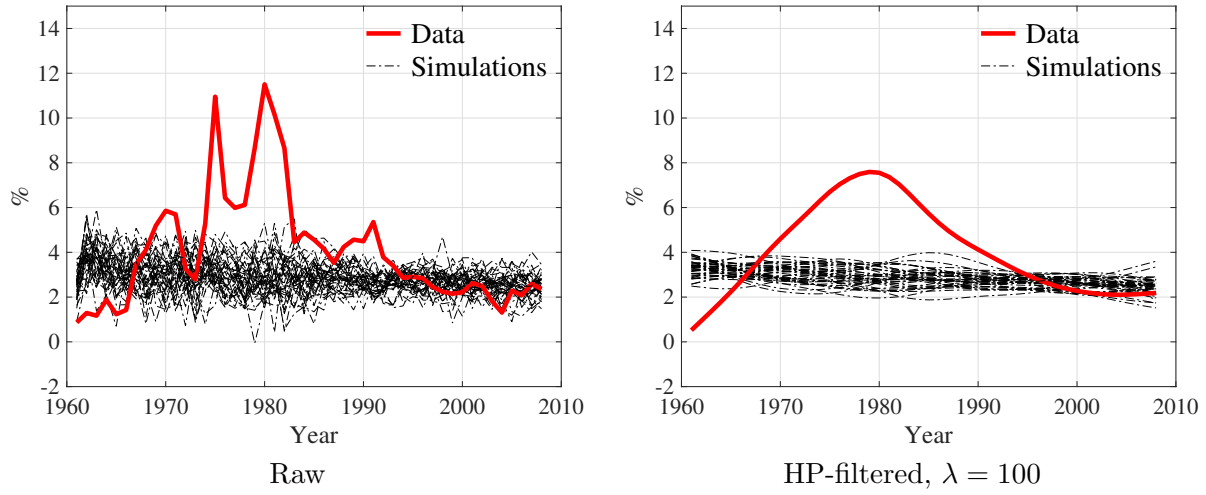
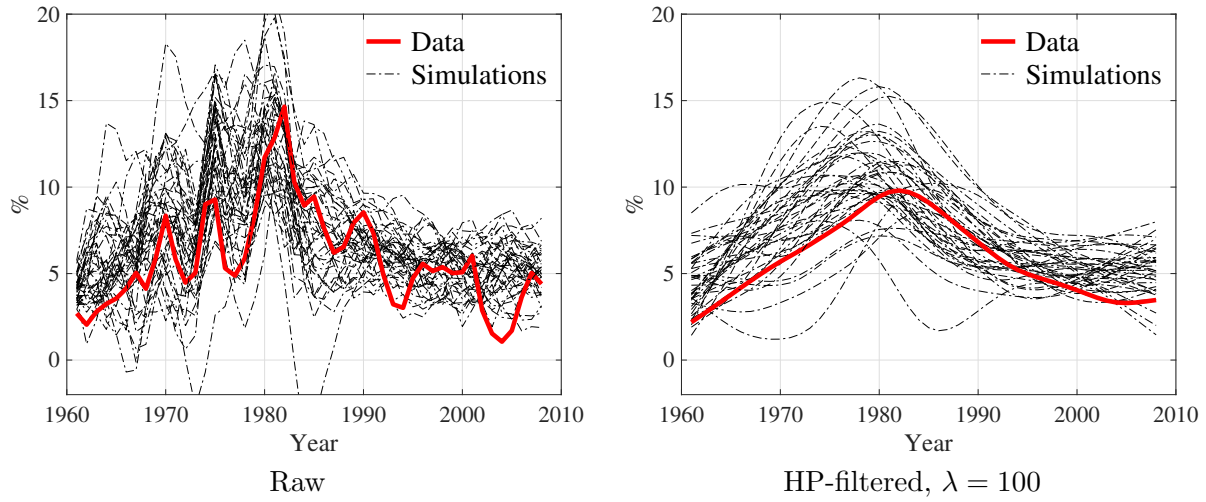


Figure A9: Model fitness of inflation rates – money rule

(a) Simulation with policy target shocks



(b) Simulation without policy target shocks

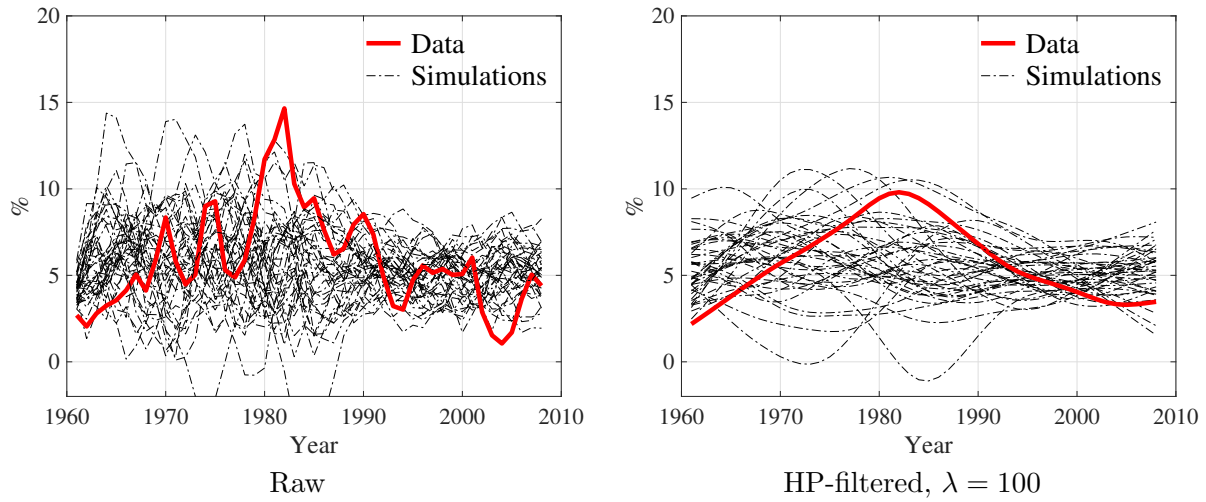
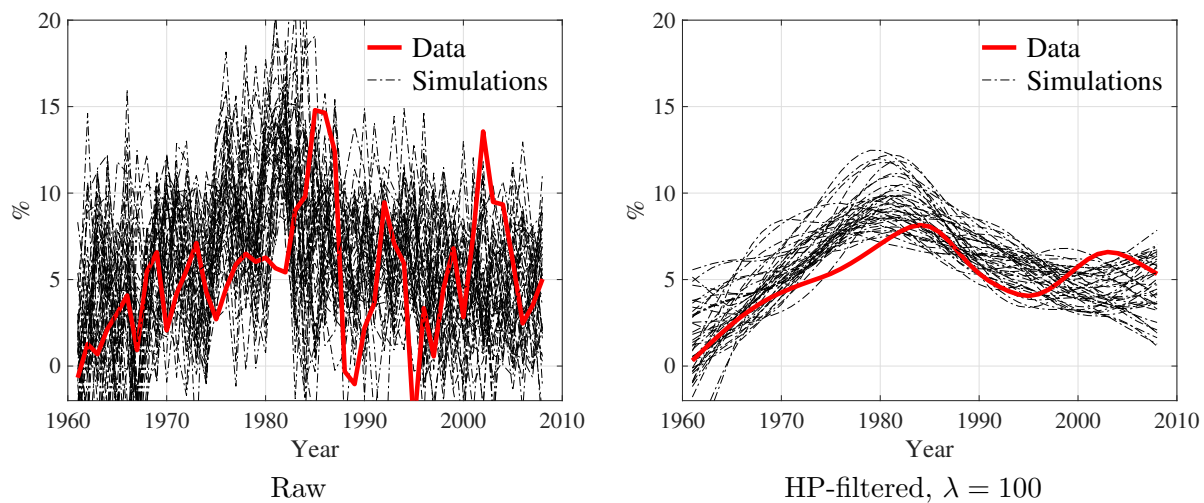


Figure A10: Model fitness of nominal interest rates – money rule

(a) Simulation with policy target shocks



(b) Simulation without policy target shocks

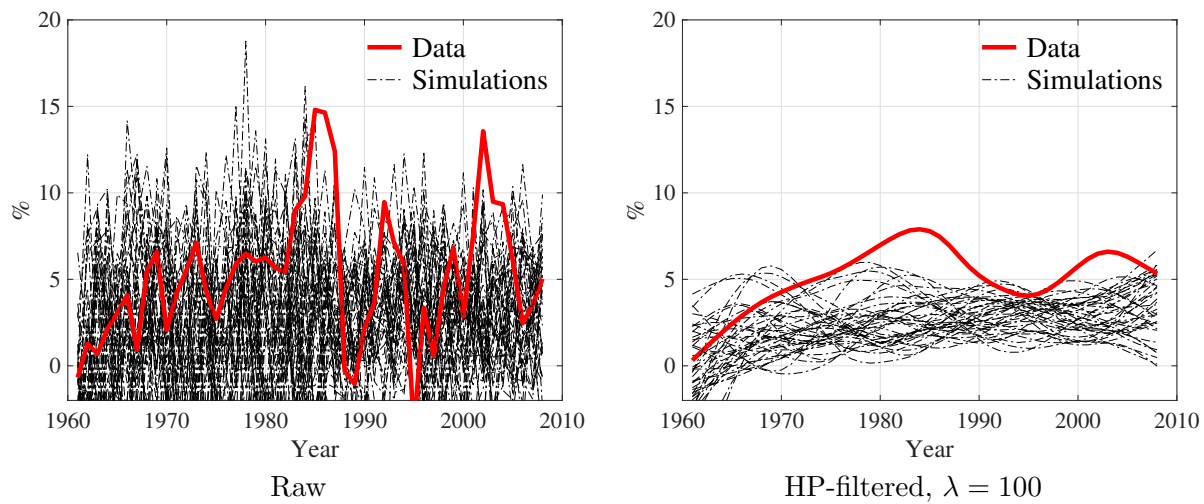
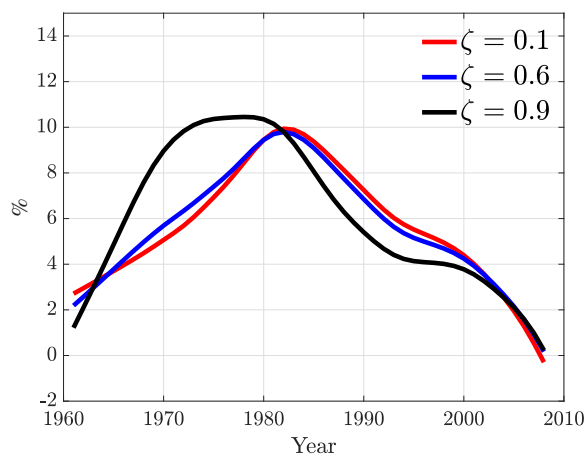
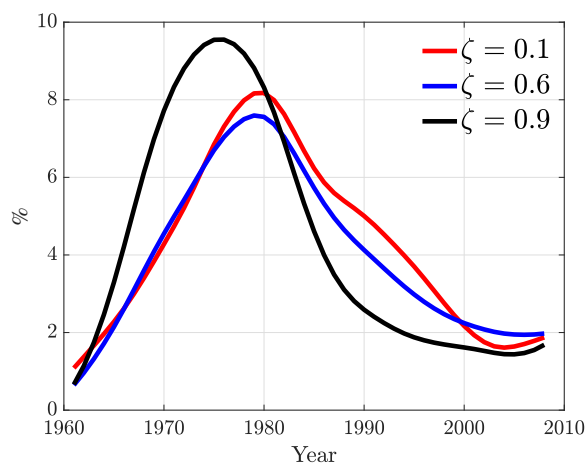


Figure A11: Model fitness of money growth rates – money rule

(a) Simulation with policy target shocks



(b) Simulation without policy target shocks

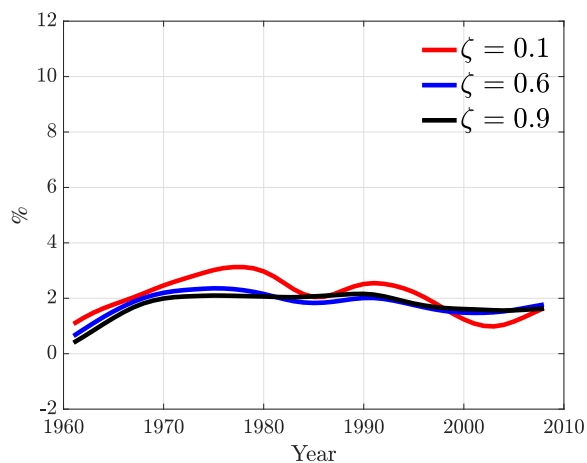
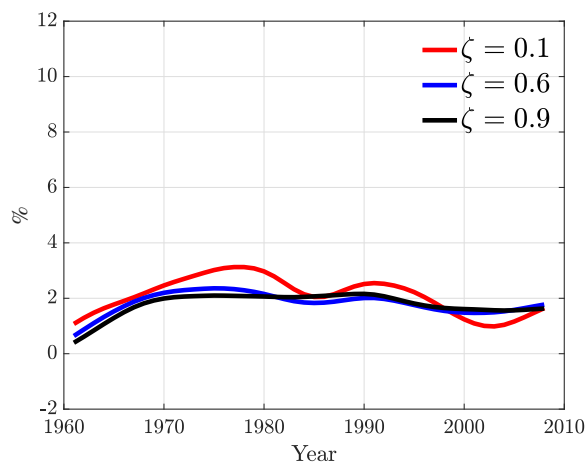
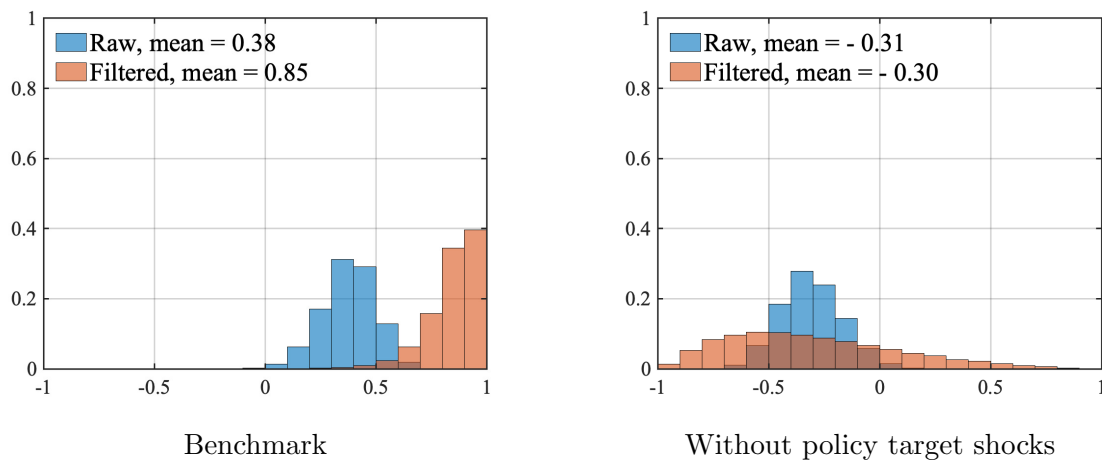


Figure A12: Sensitivity to price stickiness - money rule

(a) Illustration 1



(b) Illustration 2

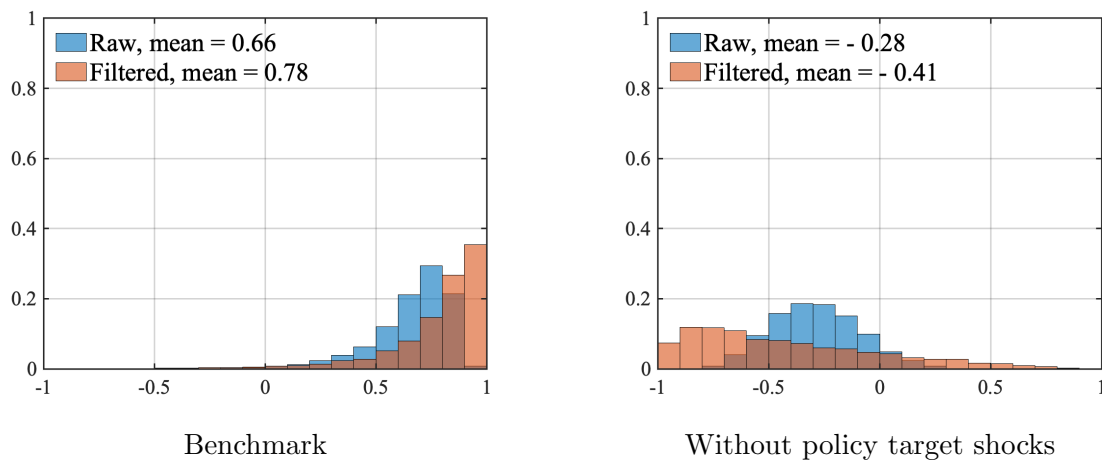


Figure A13: Correlations of series in simulated data – money rule