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A Ramsey Theory of Financial Distortions*

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Abstract

The return on government debt is lower than that of asset with similar payoffs. We study optimal debt management and taxation when the government cannot directly redistribute towards the agents in need of liquidity but otherwise has access to a complete set of linear tax instruments. Optimal government debt provision calls for gradually closing the wedge between the returns as much as possible, but tax policy may work as a countervailing force: as long as financial frictions bind, it can be optimal to *tax* capital even if this magnifies the discrepancy in returns.

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1 Introduction

How should governments finance expenditures in the least costly way when capital is present? This question has attracted much interest. Judd (1985), Chamley (1986), and a large literature that followed their work have all argued that the interest rate on government debt, which is a perfect substitute for capital, should not be distorted, and that taxing capital in the long run is a bad idea.[1] Furthermore,

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[1]More recently, Lansing (1999), Bassetto and Benhabib (2006), and Straub and Werning (2020) show examples of economies where the Chamley-Judd result does not apply, and taxes on capital remain high in the limit. The economy that we study does not fall in this category; in the absence of financial frictions, the Chamley-Judd result would apply.

Chari et al. (1994) show there is no basis to distort capital markets to lower interest rates in periods of high government spending.

The main insight from this line of studies is that distorting intertemporal saving/investment decisions is not ideal as long as the tax system is complete (with respect to choices of private agents). The result will be changed when the tax system is restricted, and previous research have shown that taxing capital is optimal when the tax system is incomplete. For example, Conesa et al. (2009) obtain optimal positive capital tax because the government cannot levy age-dependent taxes. Chari et al. (2020) point out that many studies that obtain optimal distortion on the intertemporal decisions implicitly assume some form of incomplete tax system.

In this paper, we revisit the issue of optimal capital-income taxation when financial frictions generate imperfect substitution between assets and redistribution to financially constrained agents is not possible, but the tax system is otherwise complete. We uncover a tight connection between financial frictions and capital taxes, which is at work both in the short run and even more so in the eventual long-run limit.

Our starting point is a standard neoclassical growth model, in which the government aims to achieve an exogenous stream of expenditures that is financed with taxes on income from labor and capital and by issuing debt. Our key point of departure is that investment is undertaken by entrepreneurs whose net worth affects their ability to access external sources of finance. In the model, private agents face idiosyncratic investment opportunities, as in Kiyotaki and Moore (2019). Some of them have investment projects, while others do not. When private agents have investment projects, they seek outside financing. But, because of asset liquidity frictions, only part of their claims to future investment or existing capital can be pledged. In contrast, government bonds are fully liquid and therefore can better finance any investment opportunity that arises. For this reason, private agents have a precautionary motive to buy them.

We first illustrate the optimal policy in a simple two-period deterministic model in which entrepreneurs finance their investment by selling up to a fixed fraction of their investment, as well as their entire endowment of liquid government debt.[2] When entrepreneurs start with scarce liquidity, financial constraints drive a wedge between the rate of return accruing to buyers of capital and that perceived by the constrained entrepreneurs; the constraints reduce the elasticity of the supply of capital to its after-tax return. Liquid entrepreneurial net worth plays a similar role as a factor in production: it expands the economy's ability to produce capital. The net worth as a fixed factor implies that the government may have an incentive to tax the associated "rents", which can be done through capital-income taxes.

In the limiting case of a perfectly inelastic supply of capital, increasing capital-income taxes has

[2] An alternative, equivalent interpretation is that entrepreneurs borrow and pledge as collateral up to a fraction of their investment and all of their government bonds. In practice, government debt's haircut ranges from 0.5% to 4%, whereas privately issued assets can have haircuts of more than 25%, according to the U.S. Securities and Exchange Commission.

no effect on investment and is simply a way of extracting a rent that entrepreneurs receive on their inframarginal units of investment. However, when financial frictions are such that investment can react to Tobin's q, a countervailing force emerges: by *subsidizing* capital, the government can push up the asset price (Tobin's q) and alleviate underinvestment. Which of these forces dominates is a quantitative question, except when the government starts with enough assets: when the need to raise distortionary taxes is or close to zero, optimal policy calls for undoing the financial distortions by subsidizing capital. Conversely, when the government is desperate for funds, its labor-income tax policy may depress the labor supply so much that investment drops to the point where financial constraints cease to bind, in which case the "Chamley-Judd" result reemerges and the optimal capital-income tax is zero. Positive capital taxation can emerge in an intermediate range where the government finds it optimal to raise funds by exploiting the low elasticity of the capital supply arising from financial frictions.

We then extend the analysis to an infinite-horizon economy and one in which the liquidity (or pledgeability) of capital can itself be endogenously determined from primitive assumptions about the intermediation technology, and we study the long-run optimal allocation. A stark result emerges. If the government is able to issue enough debt to completely eliminate financial frictions, it will choose to do so and set capital-income taxes to zero in the limit. However, if this level of debt cannot be sustained by raising enough labor-income tax revenues, so that the economy converges to a steady state with binding financing constraints, generically the optimal long-run tax on capital is different from zero, and we provide sufficient conditions for it to be *strictly positive*. In this case, even though capital is underprovided relative to an economy with no financial frictions, it is still optimal for the government to *tax* it; this policy magnifies the wedge between the return on government debt and that of capital, which is implied by the different degrees of liquidity.

When investment is inelastically supplied as constraints bind, the planner always has an incentive to equalize the returns on government debt and capital by taxing the latter to the point at which constraints stop binding: this tax raises revenue without introducing any new distortions. In order to have rate of return differentials, it is important that investment react to Tobin's q. The interplay between Tobin's q and rate of return differentials connects our theory to the corporate/banking finance view of public finance, in which other policies related to financial distortions are introduced, such as capital requirements, capital controls, liquidity coverage ratios, and other instruments that drive a wedge between rates of return of assets in different classes, thereby lowering the interest rate on government debt.

Finally, we explore the quantitative implications of our model. The forces that we highlight can drive capital-income taxes to values significantly different from zero, and they typically entail positive taxation. It is optimal for the government to design policy so that the interest rate on government debt is lower in periods of high spending than it would be in the absence of spending movements, thereby financing part of the additional spending through capital market distortions.

Related Literature. Our paper builds on a large literature that introduced financial frictions in the form of imperfect asset liquidity. In addition to Kiyotaki and Moore (2019), similar economic environments appear in Shi (2015), Ajello (2016), and Del Negro et al. (2017), among many others. In particular, Cui and Radde (2016, 2020) and Cui (2016) propose a framework in which asset liquidity is determined by search frictions and the supply of government debt can affect the participation in asset markets.[3] Search frictions exist in many markets, such as those for corporate bonds, IPOs, and acquisitions. They can also capture many aspects of frictional financial markets with endogenous market participation (see, e.g., Rocheteau and Weill, 2011), while still keeping the simple structure of the neoclassical growth model. These frictions imply an endogenous link between policy and asset liquidity. Furthermore, they carry the benefit of smoothing some of the kinks inherent in the financing constraints, thereby improving tractability and intuition.

We analyze optimal liquidity policy related to a recent literature that links government policy and corporate finance. There, firms are also subject to idiosyncratic investment opportunities, financed internally with money (or government bonds) and externally through intermediaries in frictional capital markets (see, e.g., Rocheteau et al., 2018; Bethune et al., 2022). Our discussion highlights the effect of a complete set of distortionary tax system and the implied government debt dynamics.

The presence of liquidity constraints opens the possibility of government bonds or fiat money circulating to improve efficiency, as in Holmström and Tirole (1998).[4] In our paper, government debt provides liquidity and has a "crowding-in" effect, similar to the one in Woodford (1990). This feature is in contrast with Aiyagari and McGrattan (1998), in which government debt is a perfect substitute for capital. In their model, government debt relaxes agents' borrowing constraints but also crowds out capital accumulation. At the same time, the need to raise distortionary taxes limits the government's ability to flood the market with liquidity so that an optimal supply of public liquidity emerges. This crowd-in effect is related to Collard et al. (2020), who also study the optimal provision of public liquidity.[5] In their environment, an interior optimum amount of liquidity is found, as the government trades off the benefits of a lower interest rate for the costs of distorting intertemporal choices. While this trade-off is also present in our paper, we highlight capital-income taxes as an additional instrument that can be used to balance the competing forces. This separates the role of interest-rate distortions as a way of indirectly taxing capital (whose production is facilitated by debt due to the financial frictions) from their germane role as a manipulation of relative intertemporal prices.[6] In addition,

[3] Lagos and Rocheteau (2008), Rocheteau (2011), and Cao and Shi (2023) also use search models to endogenize liquidity and asset prices, but they do not study the individual trade-offs that agents face between asset liquidity and prices. This channel gives rise to different degrees of liquidity constraints and risk sharing.

[4] Changing the portfolio compositions of the two assets can potentially affect the real economy. More recent papers enriched the basic structure by explicitly introducing financial intermediaries that are subject to independent frictions. See, for example, Gertler and Karadi (2011) and Gertler and Kiyotaki (2010).

[5] A similar setup is used in Cao (2014) to analyze inflation as a shock absorber in the government budget constraint.

[6] Capital appears only in the appendix of Collard et al. (2020). In the paper itself, the untaxed good is the "morning" good, and government debt serves a liquidity role in its consumption, rather than in investment.

the completeness of the tax system implies that our results would extend to implementations that use other tax instruments, for example, a consumption tax or an investment credit.

A related recent literature, for example, Bassetto and Cui (2018); Blanchard (2019); Brunnermeier et al. (2020) and Reis (2021), focuses on government debt dynamics when the interest rate is low. Our paper contributes to this body of work by analyzing optimal policy and highlighting how the government budget constraint affects financing constraints of private agents.

While taxes impinge on all of the intratemporal and intertemporal margins of households' choices, the timing we assume rules out the possibility of the government's directly sending differential payments to agents when they need liquidity. In this respect, our paper is different from Itskhoki and Moll (2019), who study the mix of labor- and capital-income taxes as a way of redistribution along the development path of an economy with two classes of agents and financial constraints. Redistribution across different agents also plays the dominant role in Azzimonti and Yared (2017, 2019), who consider the optimal supply of public liquidity with lump-sum taxes when agents differ in their income. Their framework also generates an incentive for the government to manipulate debt prices, keeping interest rates low and some agents liquidity constrained. Finally, redistribution also takes center stage in Chien and Wen (2018, 2020) and Le Grand and Ragot (2022), who revisit capital-income taxation and debt in incomplete-markets models à la Bewley. Our paper complements theirs. Although the frictions are substantially different, as capital tends to be *over*provided in Bewley models, while it is *under*provided in models of financial constraints on capital, a common theme is that the government is pushed to move away from tax smoothing towards increasing debt to relax financial constraints if possible, and it resorts to distorting capital accumulation through taxes only when this avenue is exhausted. In contrast, the specific nature of optimal tax distortions is different in the two settings and has to be tailored to the friction that impinges on capital accumulation. For this reason, our framework features a non-trivial interest-rate spread between capital and government bonds.

A link between government debt and capital-income taxation emerges also in Gottardi et al. (2015), in which labor income is the result of investment in human capital subject to uninsurable idiosyncratic shocks. Issuing government debt partly backed by capital-income tax revenues is an optimal way of indirectly providing insurance against this risk. Farhi et al. (2009) analyze a different motive for manipulating interest rates. This distortion is introduced to alleviate the impossibility of signing exclusive contracts with financial intermediaries in the presence of private information.

The paper has the following structure. Section 2 studies a simple and stylized two-period economy where the key forces at work emerge transparently. In this section, the fraction of capital that entrepreneurs can pledge is exogenous. Section 3 extends the analysis to an infinite-horizon economy, more general preferences, and a richer specification of the intermediation technology. We show that our conclusions are robust to this more general environment and study the properties of the limiting allocation. Section 4 provides a quantitative assessment of the theory, showing that capital-income tax rates are not simply different from zero, but can also be quantitatively significant. Section 5 concludes.

2 A Simple Two-Period Framework

In this section, we analyze how liquidity frictions affect the choice of distorting the intertemporal margin and how this choice depends on the government's fiscal constraints. For simplicity, both the provision of public liquidity and private assets' degree of illiquidity are exogenous. Throughout the paper, we use lowercase variables for individual choices and uppercase ones for aggregate allocations, except for prices and taxes.

2.1 The Environment

In period 1, a continuum of firms can produce output by using total labor L_1 and a constant-returns technology, with one unit of labor normalized to produce one unit of output. In period 2, the firms have a technology $F(K_1, L_2) = K_1^{\alpha} L_2^{1-\alpha}$, where K_1 and L_2 are capital and labor utilized in period 2 and $\alpha \in (0, 1)$ is the capital share. Firms hire labor and rent capital in competitive markets at the wage rates (w_1, w_2) and the rental rate r_2 from a continuum of identical households, each of which has a continuum of agents. A government collects taxes on the wage and capital incomes to finance some legacy debt B_0.

In period 1, a fraction χ of agents from each household start as entrepreneurs, and the remainder $1-\chi$ are workers. Entrepreneurs and workers of each household are separated at the beginning of the period, and they trade with members of other households. Entrepreneurs have B_0^e units of government bonds, whereas workers have B_0^w units, and we define total per-capita bonds to be $B_0 := B_0^e + B_0^w$. Each entrepreneur has initial bonds B_0^e/χ, and each worker has initial bonds $B_0^w/(1-\chi)$.[7]

Households - A representative household's preferences are represented by

$$\sum_{t=1}^{2} \beta^{t-1} \left[c_t - v((1-\chi)\ell_t) \right], \tag{1}$$

where c_t is the household's consumption in period t, ℓ_t is a worker's labor supply, $v(\ell) := \mu \frac{\ell^{1+\nu}}{1+\nu}$, $\mu > 0$, and $\nu > 0$. These preferences are convenient because they abstract from the usual incentive to distort intertemporal prices and devalue the households' initial claims, as emphasized by Armenter (2008). Without financing constraints, they imply that the optimal tax on capital income is zero not just in the long run but in every period with capital. In addition, linear preferences in consumption

[7] In multi-period versions below, the identity will not be known ex ante and $B_0^e/\chi = B_0^w/(1-\chi)$. Here, we separate the two initial conditions in order to study how the problem changes as a function of the entrepreneurs' initial net worth.

avoid any incentive for the government to distort interest rates, and we can thus focus on intertemporal distortions that arise from the interplay of policy and financial frictions. We move to more general preferences when we study the infinite-horizon economy.

In **period 1**, workers supply labor to the firms, while entrepreneurs do not supply labor. Entrepreneurs can turn one unit of the firms' output into one unit of new capital to be used in period 2. This ability will be used only in the first period, since the economy ends after period 2. The amount that each entrepreneur invests is k_1^e, the amount of capital available at the beginning of period 2.

Entrepreneurs cannot sell the capital directly, but they can sell claims to the capital that they have produced in a frictional competitive market, in the amount s_1^e:

$$s_1^e \leq \phi_1 k_1^e, \tag{2}$$

where the parameter ϕ_1 is asset liquidity. While privately issued assets are partially liquid, government bonds are fully liquid, so entrepreneurs can raise further financing by selling them. In practice, government bonds are typically considered risk free and are traded in deep markets featuring tight bid/ask spreads. They are also preferred assets for collateralized borrowing. In the model, an entrepreneur has internal funds arising from holdings of government debt, which are equal to $b_0^e = B_0^e/\chi$ and can be fully pledged. The entrepreneur's financing constraint is

$$b_0^e + q_1 s_1^e - k_1^e \geq 0. \tag{3}$$

Entrepreneurs can "borrow" only by selling claims to capital at the market price q_1. The left-hand side of (3) represents leftover funds (after investment has taken place) brought back to the household for consumption and purchase of new government bonds. If constraint (3) is binding, entrepreneurs use all of their available funds to undertake new investment.

Workers receive income from labor and have internal funds from their own holdings of government debt $b_0^w = B_0^w/(1-\chi)$ that they can use to buy new new claims to capital from the entrepreneurs of other households.[8]. They return the remaining funds to the household.[9] Let $s_1^w \geq 0$ be the end-of-period private claims on capital that they purchase, ℓ_t their labor supply, and τ_t^ℓ the tax rate on labor income. The funds that a worker returns to the household for consumption and purchase of new government bonds are thus

$$(1-\tau_1^\ell) w_1 \ell_1 + b_0^w - q_1 s_1^w. \tag{4}$$

At the end of the first period, entrepreneurs and workers rejoin their household, pool their claims

[8]We assume that entrepreneurs do not buy claims to capital from the entrepreneurs of other households. This is without loss of generality when financial constraints are not binding and it is the optimal choice when entrepreneurs are financing constrained.

[9]Workers are not subject to nonnegativity constraints, although they will return positive amounts in equilibria in which household consumption is positive.

to capital, pay taxes, purchase new government bonds, and consume left-over funds.[10] Combining (3) and (4) and including the cost of purchasing new bonds, household consumption is given by

$$c_1 = (1-\chi)\left[(1-\tau_1^\ell)w_1\ell_1 + b_0^w - q_1 s_1^w\right] + \chi(b_0^e + q_1 s_1^e - k_1^e) - p_1 b_1, \tag{5}$$

where p_1 is the (discounted) price of government bonds between period 1 and period 2. We allow households to sell government bonds short. This is not essential for our results, but it ensures that financial frictions on the entrepreneurs are the only departure from the standard Ramsey framework. At the end of period 1, the household has $\chi(k_1^e - s_1^e) + (1-\chi)s_1^w$ claims to capital and b_1 government bonds in total.

Period 2 is similar to the first, except that no new investment takes place, so entrepreneurs no longer have any role. We can write the joint household budget constraint simply as

$$c_2 = (1-\tau_2^\ell)w_2(1-\chi)\ell_2 + \left[(1-\tau_2^k)r_2\right]\left[\chi(k_1^e - s_1^e) + (1-\chi)s_1^w\right] + b_1, \tag{6}$$

where τ_2^ℓ is the labor-income tax in period 2 and τ_2^k is the capital-income tax in period 2.

Each household takes prices and taxes as given and maximizes its utility (1) with respect to (c_1, c_2, ℓ_1, ℓ_2, s_1^e, s_1^w, k_1^e, b_1), subject to the budget constraints (5) and (6), to the financing constraints that limit the entrepreneurs' access to investment funds, equations (2) and (3), and to nonnegativity constraints on k_1^e, s_1^e, and s_1^w.

In turning each representative household into aggregate quantities, we define aggregates in per-capita terms, so we have $L_t = (1-\chi)\ell_t$ for $t = 1, 2$, $K_1 = \chi k_1^e$, $S_1^w = (1-\chi)s_1^w$, $S_1^e = \chi s_1^e$, and $B_1 = b_1$.

Government - The government has no other expenditures besides the legacy debt B_0 to finance. We add government spending later in Section 3. In period 1, the government's budget constraint ensures that its revenues from labor-income taxation and new borrowing cover debt repayments:

$$B_0 = p_1 B_1 + \tau_1^\ell w_1 L_1. \tag{7}$$

In period 2, the government can tax (or subsidize) both labor and capital. Letting τ_2^k be the tax rate on capital, its budget constraint is

$$B_1 = \tau_2^k r_2 K_1 + \tau_2^\ell w_2 L_2. \tag{8}$$

Our goal is to study how the power of taxing capital is used in the presence of financial frictions.

[10] It is immaterial whether government bonds are purchased at this stage, or by the workers at the earlier stage, since there is no credit constraint on the workers in the interim. While the same is true for *purchases* of capital, so that results would be identical if the purchases occurred at this stage, it would be inconsistent to assume that households purchase capital after the family is reunited while *selling* it in the first stage, when the credit constraints of entrepreneurs may bind.

2.2 Competitive Equilibrium

Definition. A competitive equilibrium is an allocation $\{C_t, L_t, S_1^e, S_1^w\}_{t=1}^2$ and K_1, prices of bonds and claims to capital p_1 and q_1, wage rates $\{w_t\}_{t=1}^2$, a rental rate of capital r_2, taxes $(\tau_1^\ell, \tau_2^\ell, \tau_2^k)$, and an amount of bonds B_1 such that: households maximize their utility subject to (2), (3), (5), (6), and to nonnegativity constraints on k_1^e, s_1^e, and s_1^w, taking prices and taxes as given; firms maximize their profits taking prices and taxes as given; the government budget constraints (7) and (8) hold; and markets clear, that is, $S_1^e = S_1^w$,

$$C_1 + K_1 = L_1, \tag{9}$$

and

$$C_2 = F(K_1, L_2). \tag{10}$$

In order to characterize competitive equilibria, we first study the household problem. Inspecting the budget constraints (5) and (6), each household has four different ways of trading intertemporally to move consumption between periods 1 and 2:

1. Buying government bonds b_1, at a price p_1 with a return $1/p_1$;
2. Buying claims to capital produced by other households s_1^w, at a return $(1-\tau_2^k)r_2/q_1$;
3. Investing the entrepreneurs' own net worth (increasing k_1^e), at a return $(1-\tau_2^k)r_2$; and
4. Investing, selling a fraction ϕ_1 of the investment to other households (that is, increasing k_1^e while at the same time increasing s_1^e by ϕ_1 units for each extra unit of investment).[11] This yields a return $(1-\tau_2^k)r_2(1-\phi_1)/(1-\phi_1 q_1)$ provided $q_1 < 1/\phi_1$, and represents an opportunity for arbitrage otherwise.

Nonnegativity constraints apply to the last three trading strategies. In addition, strategies 3 and 4 are limited by the financing constraints (2) and (3). Each individual household takes bond and capital prices, interest rates, and taxes as given, and so its optimal decision will be at a corner among these trading options for many possible combinations of taxes and prices. With 3 nonnegativity constraints and two inequality constraints, the task of characterizing the solution in all possible cases is somewhat tedious, and we relegate it to online Appendix D.1. However, most of these combinations are incompatible with an equilibrium. Specifically, we can safely rule out all combinations of taxes and prices that lead to the emergence of an arbitrage. We can also rule out all combinations that make it optimal for entrepreneurs not to invest: zero capital cannot be an optimal choice when the marginal productivity of capital is infinite at that point.[12] Finally, we can rule out combinations in which households find it optimal for their entrepreneurs to sell strictly positive amounts of claims to capital, but

[11] Investing and selling a smaller fraction than the maximum ϕ_1 is a combination of strategies 3 and 4.

[12] We have $\tau_2^k < 1$ and $\tau_t^\ell < 1$. It is never optimal for the government to set a confiscatory tax, which would raise no revenue and generate an infinite distortion at the margin.

for the workers not to buy any, which would make market clearing impossible. Lemma 3 in online Appendix D.1 proves that, after ruling out all of these cases, an equilibrium can only exist if prices, taxes, and returns satisfy the following restrictions:

$$\frac{1}{p_1} = \frac{(1-\tau_2^k)r_2}{q_1}; \tag{11}$$

$$1 \leq q_1 < 1/\phi_1. \tag{12}$$

Equation (11) states that the rate of return on government debt and on purchases of capital as evaluated by the workers are equal. Workers are always indifferent between the two investment strategies in a competitive equilibrium, while entrepreneurs are only indifferent if $q_1 = 1$. Equation (12) ensures that the price of capital is high enough to induce entrepreneurs to undertake investment, but not so high that their financial constraints become moot and unbounded profits are possible.

In the main text, we thus study the solution to the household problem when (11) and (12) hold. In addition to these conditions, since we assumed linear preferences in consumption, the household problem would imply an unbounded solution unless $p_1 = \beta$, the level at which households are indifferent between consuming in periods 1 and 2. When (11), (12), and $p_1 = \beta$ hold, the household maximization problem yields the following necessary and sufficient conditions for optimality:

- For $t = 1, 2$,
$$(1-\chi)\ell_t = \left[\frac{(1-\tau_t^\ell)w_t}{\mu}\right]^{1/\nu}; \tag{13}$$

- If $q_1 = 1$, any choice of $k_1^e \geq 0$, $s_1^e \geq 0$, $s_1^w \geq 0$, and b_1 that respects (2) and (3) is optimal, since the return on any investment strategy is equal to the discount factor (and utility is linear in consumption);

- If $q_1 > 1$, (2) and (3) bind, which implies $s_1^e = \phi_1 k_1^e$ and $k_1^e = b_0^e/(1-\phi_1 q_1)$. Any choice of $s_1^w \geq 0$ and b_1 is optimal.

Next, the firms' optimality conditions imply that the following conditions must be met in a competitive equilibrium:

$$w_1 = 1, \quad w_2 = F_L(K_1, L_2), \text{ and } r_2 = F_K(K_1, L_2). \tag{14}$$

As is common in dynamic optimal taxation problems, we take the primal approach and characterize competitive equilibria in terms of sequences of $(C_1, C_2, L_1, L_2, K_1)$ alone, deriving prices and tax rates from the other equations that guarantee optimality for households and firms. Define

$$K^* := B_0^e/(1-\phi_1). \tag{15}$$

K^* is the maximal investment that entrepreneurs can undertake if $q_1 = 1$ (when the financing constraint is not binding). We then have:

Proposition 1. *A vector $(C_1, C_2, L_1, L_2, K_1)$ forms part of a competitive equilibrium if and only if it satisfies the resource constraints (9) and (10) and the implementability constraint*

$$\sum_{t=1}^{2} \beta^{t-1}[C_t - v'(L_t)L_t] = B_0 + \begin{cases} 0 & \text{if } K_1 \leq K^* \\ \left(\frac{1}{\phi_1} - 1\right)(K_1 - K^*) & \text{if } K_1 > K^* \end{cases}. \tag{16}$$

Proof. See online Appendix D.2. □

The implementability constraint has two branches corresponding to the two possible types of equilibria. When $K_1 \leq K^*$, the entrepreneurs can finance enough of the investment with their own funds that (2) remains slack even with $q_1 = 1$. In this case, our economy behaves as a standard neoclassical model such as Judd (1985). When $K_1 > K^*$, entrepreneurs have insufficient funds to finance investment if $q_1 = 1$, so the equilibrium must feature $q_1 > 1$ and binding financial constraints. In this case, entrepreneurs face an intertemporal trade-off different from that faced by workers: entrepreneurs require only one unit of period-1 good to produce one unit of capital, but the price of capital that the workers acquire is $q_1 > 1$. When the present-value budget constraint is evaluated at the trade-off faced by workers, who are the unconstrained agents in the household, capital financed with internal funds appears as an extra source of revenues, and the new term that appears in (16) captures the household's profits from this activity. These profits emerge because entrepreneurial net worth plays the same role as a factor of production: it expands the economy's ability to produce capital.

2.3 The Optimal Policy Problem

We study the Ramsey outcome, that is, the best competitive equilibrium. Given Proposition 1, we can compute this outcome by choosing $(C_1, C_2, L_1, L_2, K_1)$ to maximize (1) subject to (9), (10), and (16). The financing constraint is the only departure from a standard neoclassical model. The optimal policy below highlights the interaction between the entrepreneurs' financing constraint and the government budget constraint.

We highlight two aspects of the policy problem. First, the tax system chosen is complete, subject to the frictions embedded in the environment. This means that introducing any other (linear) tax instrument, such as a consumption tax or an investment tax credit, would not affect the optimal allocation. Second, the financial friction prevents the family from reallocating resources from the workers to the entrepreneurs, and the tax system is constrained to respect this fact: taxes are levied and subsidies are paid when the family has reunited, at which point they cannot contribute to the funds available to the entrepreneurs for investment.[13] While direct reallocation is not possible, the government affects the shadow cost of funds available to the entrepreneurs, and optimal policy exploits this.

Appendix A derives the first-order necessary conditions that characterize a Ramsey plan. Since (16) has a kink at $K_1 = K^*$, we need to take into account that the solution might be at this kink. Define Ψ_1 to be the cost to the planner of starting with an extra unit of debt in period 1. Mathematically, Ψ_1 is the Lagrange multiplier on the implementability constraint (16). In solving the Ramsey problem, the government takes as given the initial level of debt B_0, and Ψ_1 results endogenously from the solution. However, for illustration purposes it is more intuitive to work in reverse, taking Ψ_1 as a primitive, solving for the allocation, prices, and taxes, and backing out of (16) the level of debt that corresponds to Ψ_1. Specifically, we back out the level of debt held by the workers, B_0^w; we hold fixed the level held by entrepreneurs, B_0^e, which governs the level at which financial constraints become binding. By properly rearranging the planner's first-order condition for capital in Appendix A we obtain:

$$\beta\alpha(K_1/L_2)^{\alpha-1} = 1 + \begin{cases} 0 & \text{if } K_1 < K^* \\ \in [0, \frac{\Psi_1(\phi_1^{-1}-1)}{1+\Psi_1}) & \text{if } K_1 = K^* \\ \frac{\Psi_1(\phi_1^{-1}-1)}{1+\Psi_1} & \text{if } K_1 > K^*. \end{cases} \tag{17}$$

It is useful to compare (17) with the competitive-equilibrium condition arising from household and firm optimality, which is

$$\beta\alpha(K_1/L_2)^{\alpha-1}(1-\tau_2^k)/q_1 = \beta r_2(1-\tau_2^k)/q_1 = 1. \tag{18}$$

We can see that the Ramsey plan sets $\tau_2^k = 0$ when financial constraints are not binding ($K_1 \leq K^*$ and $q_1 = 1$), regardless of the tightness of the government budget constraint Ψ_1.[14] In this case, we recover the standard result that it is not optimal to tax capital as an intermediate input. This case can arise either when entrepreneurs have enough wealth to finance investment internally, in which case the private cost of investment is 1 and the social cost is $1 + \Psi_1$, or when they need to sell part of their capital but not to the point at which q needs to exceed 1. In both cases, the private reward in the second period is βr_2, and the social reward is $\beta r_2(1 + \Psi_1)$. Thus, private and social costs are proportional to each other; moreover, in both cases, the trade-off coincides with the marginal rate of transformation coming from technology alone.

When the financial constraints do bind, capital-income taxes will generically not be zero, but their

[13] Of course, if the government could use taxes to directly move resources from workers to entrepreneurs at the investment stage, it could bypass the financing constraint and this would be an optimal course of action, as long as the distortionary costs are not too large.

[14] This result also relies on our assumptions about preferences that rule out distorting intertemporal prices to devalue initial claims or to enhance the present value of taxes on labor. For more general preferences, both of these forces would be in play, as they are in a standard neoclassical growth model, and our effect would appear in addition to those.

sign is determined by competing factors. The presence of $q_1 > 1$ in equation (18) implies that private incentives to invest are depressed by financial constraints, and calls for capital subsidies. On the other hand, the last term in equation (17) represents the fiscal cost of subsidizing capital and/or the fiscal benefit of taxing it when such a tax is a proxy for taxing the rent from the fixed factor (entrepreneurial net worth); mathematically, it arises because changes in the level of investment have an effect on the price of capital in this region, and a higher price of capital tightens the implementability constraint, forcing the government to raise more distortionary taxes.

While we cannot establish in general whether the incentive to tax or subsidize capital dominate, we can do so in some limiting cases.

When $\Psi_1 = 0$, the government has sufficient wealth at the beginning that the shadow cost of resources in the government budget constraint is zero. In this case, the government can undo the effect of financial constraints by subsidizing the return on capital in the second period, thereby raising the price of capital q_1 to a level that replicates the efficient level of investment in the absence of constraints: as $\Psi_1 \to 0$, the three cases of (17) yield the same solution. As the cost of public funds Ψ_1 increases, the fiscal benefits of taxing capital may come to prevail.

The degree by which entrepreneurs can sell their claims to capital also plays an important role in determining whether capital should be taxed or subsidized. Capital subsidies operate through their effect on Tobin's q (the value of q_1 in our case): a larger q_1 relaxes the entrepreneurs' constraint (3), but this effect vanishes if ϕ_1 decreases to zero and entrepreneurs are not allowed to sell claims to their capital. Put it in another way, as $\phi_1 \to 0$, the subsidy required to achieve a given increase in investment grows larger and larger. We can see the consequence of this in equation (17): as $\phi_1 \to 0$, the last term in the equation becomes a stronger and stronger reason not to resort to capital subsidies. In the limit as $\phi_1 \to 0$, capital is in fixed supply once it hits K^*, and optimal policy unambiguously calls for taxing it, independently of Ψ_1.[15]

To further characterize the solution, we substitute the optimality condition for labor L_2 in a Ramsey optimum into (17). With this substitution, (as shown in Appendix A), the top and bottom lines of (17) define two levels of capital $K^u(\Psi_1)$ and $K^c(\Psi_1)$ respectively, that are chosen by the planner when financial constraints are not binding ($K^u(\Psi_1)$) and when they are ($K^c(\Psi_1)$). The overall optimal choice is

$$K_1(\Psi_1) = \min\{K^u(\Psi_1), \max\{K^c(\Psi_1), K^*\}\}. \tag{19}$$

Equation (19) is best understood by looking at Figure 1. At $\Psi_1 = 0$, $K^c(0) = K^u(0)$, since the three cases of (17) coincide: the financial constraint does not affect the allocation because the government is able to subsidize investment so as to achieve the efficient level even if the constraint is

[15]This result would hold even if we generalized the constraint (2) to $\theta(b_0^e + q_1 s_1^e) - k_1^e \geq 0$, which (for $\phi_1 = 0$) becomes the commonly used borrowing-constraint specification $k_1^e \leq \theta b_0^e$.

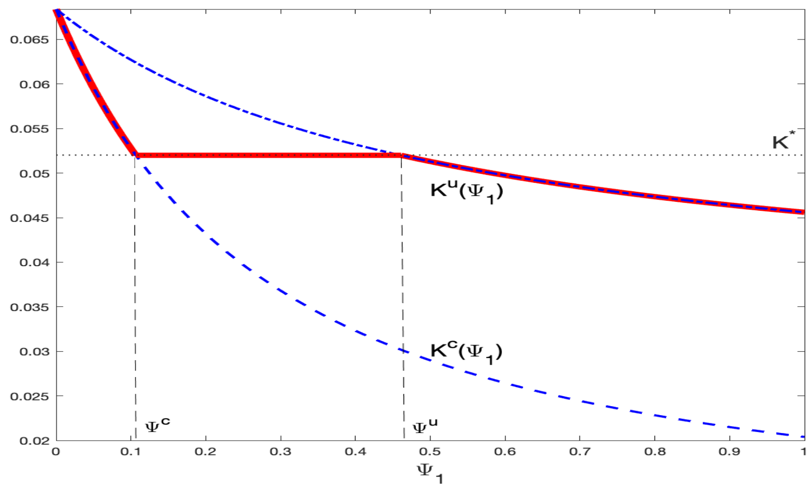

Figure 1: $K^u(\Psi_1)$ **(dashed blue line),** $K^c(\Psi_1)$ **(dash-dotted blue line), and optimal choice (solid red line).** Note: We use $\beta = 0.96$ (discount factor), $\alpha = 0.33$ (capital share), $\mu = 1$ (disutility parameter of labor), $\nu = 1$ (labor supply elasticity), and $\phi = 0.5$ (asset liquidity). Ψ_1 measures the tightness of implementability constraint (16) (or government budget constraint in period 1 goods), which reflects the level of legacy debt B_0. K^* is a function of the initial wealth of the entrepreneurs as defined in equation (15) and it is drawn for $B_0^e = 0.026$.

binding. As Ψ_1 increases, the government is forced to distort labor to raise revenues, which reduces the optimal level of investment as well. As long as $K^c(\Psi_1)$ exceeds K^*, we are in the first case of equation (17), with the constraint binding. The capital subsidy decreases as we increase Ψ_1, and it eventually turns into a tax as the benefit of taxing rents comes to exceed the downward investment distortion. At $\Psi_1 = \Psi^c$, $K^c(\Psi_1) = K^*$ and we hit the kink in equation (17) at which the price of capital is 1 and rents from entrepreneurial net worth are exhausted. From this point to Ψ^u, the planner keeps capital at K^*, gradually reducing the capital-income tax as Ψ_1 increases, to compensate for the lower and lower rents that can be appropriated as the labor supply decreases. Eventually, at $\Psi_1 = \Psi^u$, the price of capital is 1 even with a zero capital-income tax; from that point onward, the government relies only on labor-income taxes, that are sufficiently high so as to make the financial constraint not binding for the desired investment K^u (the third branch of equation (17)), and the solution coincides with that of a standard Chamley-Judd economy.

While for the specific parameter values in Figure 1 K^* intersects both the K^u and K^c line, changing the initial net worth of the entrepreneurs B_0^e (and thus shifting K^* up or down) leads to missing intersections, in which case not all of the three regions that we described arise.[16]

Proposition 2 goes through the four possible cases based on the level of K^* and summarizes the resulting interplay between entrepreneurial wealth B_0^e and the tightness of the government budget constraint.

[16] The functions K^u and K^c are independent of B_0^e and remain the same. However, the value of *total* government debt B_0 that corresponds to a given multiplier Ψ_1 changes as we change B_0^e.

Proposition 2. *The Ramsey allocation can be characterized as follows:*

1. *If entrepreneurs have insufficient liquid assets B_0^e and asset market liquidity ϕ_1 is low so that $K^* \in (0, K^c(\infty)]$, then the economy is financially constrained for any $\Psi_1 \geq 0$ and capital is given by $K_1 = K^c(\Psi_1)$. In this case, optimal policy calls for $\tau_2^k < 0$ if Ψ_1 is close to 0 and $\tau_2^k > 0$ if Ψ_1 is sufficiently large.*

2. *For higher values of B_0^e, we have $K^* \in (K^c(\infty), K^u(\infty)]$. When $0 \leq \Psi_1 < \Psi^c$, the economy is in the interior of the financially constrained region, capital is given by $K_1 = K^c(\Psi_1)$, $\tau_2^k < 0$ for Ψ_1 close to 0 and $\tau_2^k > 0$ for Ψ_1 close to Ψ^c. When $\Psi_1 \geq \Psi^c$, the economy has capital exactly at the kink K^*, and $\tau_2^k > 0$.*

3. *For even higher values of B_0^e, $K^* \in (K^u(\infty), K^u(0))$, the Ramsey plan puts capital in the interior of the financially constrained region $0 \leq \Psi_1 < \Psi^c$, at the kink when $\Psi^c \leq \Psi_1 \leq \Psi^u$, and in the interior of the unconstrained region for $\Psi_1 > \Psi^u$. We have $\tau_2^k < 0$ for Ψ_1 close to 0, $\tau_2^k > 0$ for $\Psi_1 < \Psi^c$ but sufficiently close to Ψ^c, $\tau_2^k > 0$ for $\Psi^c \leq \Psi_1 < \Psi^u$, and $\tau_2^k = 0$ when $\Psi_1 \geq \Psi^u$.*

4. *Finally, for the highest range of values for B_0^e, $K^* \geq K^c(0) = K^u(0)$, and financial constraints never bind in the Ramsey plan, regardless of Ψ_1. In this case, $\tau_k^2 = 0$.*

Proof. See Appendix A. □

Figure 2 plots other salient variables of the Ramsey plan as a function of Ψ_1 (and, implicitly, of B_0, represented by the last panel). The dashed red line uses the same parameter values as in Figure 1, while the solid blue line changes the initial assets of the entrepreneurs to $B_0^e = K^c(\infty) = 0.0043$, in which case capital accumulation is constrained by financial frictions regardless of Ψ_1. The efficient level of capital that would prevail in the absence of tax distortions and financial frictions is the same in the two economies. To achieve it, the planner needs greater subsidies when entrepreneurial net worth is smaller as in the blue line. To pay for those extra subsidies, while retaining the same marginal value of government funds Ψ_1, a greater asset position is needed against the workers, as shown in the value of B_0. For the higher value of entrepreneurial net worth (red line) the capital-income tax starts negative, increases and turns positive up to the point at which K_1 hits K^*, and decreases to zero from then on, while it is monotonically increasing when financial constraints are tighter as in the solid blue line.

To close this section, notice that assuming linear preferences implies $p_1 = \beta$; the government's choice of taxes or subsidies has no effect on the rate of return on government debt. A further channel at work when preferences are not linear is that a capital-income tax reduces the after-tax return on capital and hence further favors government debt, which is an extra beneficial force in the case of a constrained government. This effect will be in play in the full model in the next section.

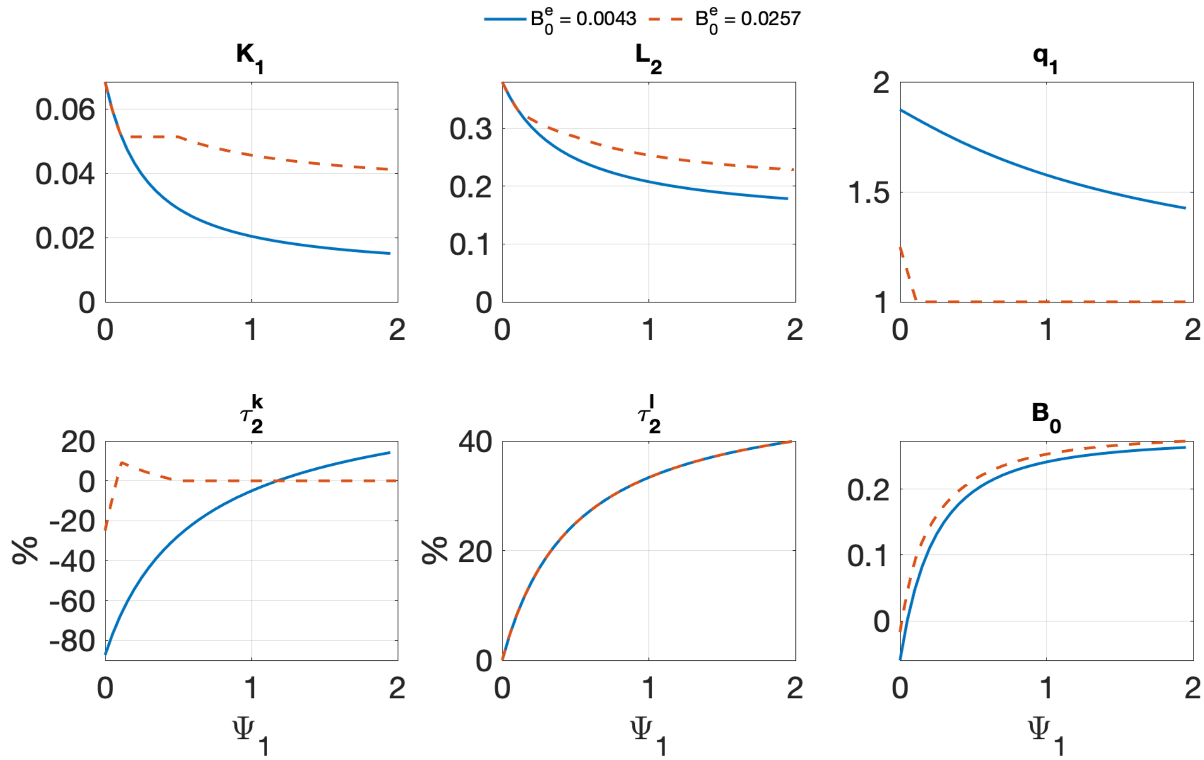

Figure 2: **Two cases: always-binding and sometimes-binding financing constraint.** Note: We use the same parameter values as in Figure 1 for the red dashed line, and vary only B_0^e as above for the blue solid line.

3 Infinite-Horizon Economy with Endogenous Asset Liquidity

We now extend the model to an infinite horizon ($t = 0, 1, 2, ...$), and we endogenize the partial liquidity of private claims. Infinite horizon brings to the table two new features: first, government debt becomes an endogenous state, since bonds accumulated each period can be used to finance investment next period. Second, since capital does not fully depreciate, entrepreneurs can rely on sales of claims to existing capital as well as claims to their new investment as a source of financing. For simplicity, we assume a symmetric friction across the new and existing claims.

3.1 Costly Intermediation

Endogenizing ϕ brings three benefits. First, it generates a new margin by which government policy interacts with asset liquidity ϕ and asset prices q. When an endogenous trade-off is present between a higher ϕ and a higher q, equilibrium liquidity responds to government policies such as the supply of government debt or capital-income taxes. Second, Cui and Radde (2020) showed that endogenizing asset liquidity acts as an amplifier of financial shocks, generating a positive co-movement of ϕ and q.[17] Since optimal capital-income taxation relies on manipulating the tightness of financial constraints and

[17] See Shi (2015) for a critique of models relying on exogenous financial shocks.

the rents manifested in Tobin's q, this is a potentially important determinant of optimal policy. Finally, endogenous liquidity allows us to smooth one of the kinks that we uncovered in the previous section, thereby providing clearer first-order conditions and additional tractability in computing numerical examples, without affecting the economic intuition developed in the previous section.

In the main text, we model the relationship between market tightness and bid-ask spreads by assuming that there are competitive financial intermediaries with the ability to take a fraction $\phi \in [0, 1]$ of an entrepreneur's capital and resell it to other households at a cost $\eta(\phi)$, where η is strictly convex, twice continuously differentiable, and $\eta(0) = \eta'(0) = 0$. This last assumption ensures that there is no kink at the point at which entrepreneurs stop selling capital, since at that point selling is costless at the margin. Online Appendix E provides deeper microfoundations for this technology based on search frictions as in Cui and Radde (2020) and Cui (2016), which in turn build on the wider costly intermediation and OTC markets, including Duffie et al. (2005), Weill (2007), Lagos and Rocheteau (2009), Atkeson et al. (2015), and many others. Our results do not rely on these specific microfoundations; any alternative that generates a smooth trade-off between ϕ and q would imply similar results. As an example, instead of assuming that ϕ_t represents the limits to the ability to pledge capital as a collateral, we could assume that it represents the fraction of meetings in which a buyer is able to recognize the value of capital and accept it, as in Lagos (2010).[18] This margin could then be made endogenous by assuming that intermediaries can invest resources to be able to recognize capital in a greater fraction of meetings. Similar results could also be obtained with informational frictions, where the cost takes place in the form of a delay in selling, as in Guerrieri and Shimer (2014) and Cho and Matsui (2018).

Private financial claims to capital in our framework capture what in the real world are both equity and debt securities. As an example of the intermediation costs involved, consider initial public offerings (IPOs), that raised \$488 billion in the US in 2001, about 1/4 of aggregate investment spending. The gross spread paid to underwriters (intermediaries for IPOs) is sizeable.[19] Search for appropriate investors is crucial in this process. Underwriters typically seek to avoid placing a large number of shares with investors who are likely to flip them (i.e., wait until the price spikes upon the opening trade and then immediately dump the position). Instead, they prefer a balance of different types of investors, such as long-term, short-term, domestic, and foreign. Other types of financing also involve similar costly intermediation.[20]

As in our previous section, our key retained assumption is that the government cannot directly

[18] This alternative is also able to generate comovement of ϕ and q. See endnote 3 in Venkateswaran and Wright (2014) for a discussion of the differences between the two approaches.

[19] From the summary of Cui and Radde (2020), the spread of a deal is around 10%, and more than 90% of the deals up to \$250 million have a spread at or above 7%.

[20] As further examples, seasonal public offerings (SEOs) have comparable volume as IPOs and face similar underwriting costs. Corporate bonds for financing capital investment are also intermediated through underwriters. The gross spread can be as low as 1%–2% of a deal, but the variation is much higher than in the case of stocks.

intervene in this market to undo the financial friction. This may be due to the fact that intermediation happens in decentralized over-the-counter (OTC) markets that the government cannot monitor, or more generally because the government is unable to distinguish between intertemporal trades that are subject to credit frictions and other types of loans that in the real world might arise for a number of alternative reasons.

Given their technology, competitive intermediaries break even and stand ready to participate in any market that satisfies the following condition:

$$q_t(\phi) = q_t^w - \eta(\phi), \tag{20}$$

where q_t^w is the price paid by purchasers of claims to capital and $q_t(\phi)$ is the price received by entrepreneurs selling a fraction ϕ of their holdings. q_t^w must be the same in all active markets, since purchasers have no constraints and would always choose the market with the lowest purchase price.

3.2 Firms

Competitive firms produce a general consumption goods in period t with a constant-returns-to-scale technology $F(K_{t-1}, L_t)$ employing capital and labor, and capital depreciates at the rate δ. Firms hire labor and rent capital in competitive markets at a wage rate w_t and a rental rate r_t respectively. Firm's optimality requires

$$w_t = F_L(K_{t-1}, L_t); \quad r_t = F_K(K_{t-1}, L_t). \tag{21}$$

3.3 The Government

In each period t, the government imposes taxes on labor at a rate τ_t^ℓ and capital at τ_t^k, spends an exogenous amount G_t, and issues bonds in the amount B_t. Its budget constraint is:

$$p_t B_t = B_{t-1} + G_t - \tau_t^k (r_t - \delta) K_{t-1} - \tau_t^\ell w_t L_t, \tag{22}$$

and B_{-1} and K_{-1} are exogenous initial conditions. We assume that a depreciation allowance applies to taxable capital income; this does not change the Ramsey allocation and is only relevant to get its proper magnitude in our quantitative section.

3.4 Households

We adopt the same notation as in the previous section for all variables in common. The family's utility in (1) is now

$$\sum_{t=0}^{\infty} \beta^t \left[u(c_t) - v((1-\chi)\ell_t) \right]. \tag{23}$$

where u and v are strictly increasing and continuously differentiable functions, u is weakly concave, and v is strictly convex.

All households start with some initially given claims to capital K_{-1} and bonds B_{-1}. In our two-period economy, we distinguished between the bonds issued by the government and those held by entrepreneurs, so that we could independently discuss the consequences of tightening government finances (by increasing B_0) and loosening financing constraints (by increasing B_0^e). Now, in each period, each member of a family has an i.i.d. chance χ of being an entrepreneur and a chance $1 - \chi$ of being a worker. This opportunity is realized *after* the family has distributed the bonds to its members, so $b_t^w = b_t^e = B_t$. Similarly, each member of a family will start period t with $k_{t-1} = K_{t-1}$ units of claims to capital. The i.i.d. assumption makes it impossible for the family to target its assets to entrepreneurs. In addition, the government is unable to target its taxes and transfers to the entrepreneurs as it interacts with families only after investment has taken place. What is essential for our results is that there is some uncertainty in each agent's future investment opportunities at the moment in which resources are allocated within and across families.

Each entrepreneur can finance new investment by selling her government bonds as well as claims to capital:[21]

$$b_{t-1} + q_t(\phi_t)s_t^e - k_t^e \geq 0 \text{ where } s_t^e \leq \phi_t \left[k_t^e + (1-\delta)k_{t-1}\right]. \tag{24}$$

If $b_{t-1} + q_t(\phi_t)s_t^e - k_t^e > 0$, remaining funds are returned to the household. We assume that entrepreneurs can only sell ex-dividend claims to (undepreciated) existing capital, with the rental rate accruing to the household after it is reunited. This simplifies the algebra as it allows us to treat new and existing capital symmetrically, but it has no bearing on the results. The liquidity of claims to capital is a choice for the entrepreneurs, but markets with different liquidity are characterized by different prices, as described above.

Each worker in the household supplies labor ℓ_t and buys claims to capital sold by entrepreneurs of other households at a price q_t^w, returning to the household any unspent funds in the amount[22]

$$(1-\tau_t^\ell)w_t\ell_t + b_{t-1} - q_t^w s_t^w. \tag{25}$$

After production and trades in capital have taken place, workers and entrepreneurs rejoin the family, collect the rent on capital, and divide left-over funds between consumption and purchases of

[21] An equivalent interpretation is that government bonds and claims to capital are used as collateral for loans, with no haircut on bonds and a haircut $1 - \phi_t$ on claims to capital. For simplicity, we write equations for the case in which gross investment is positive, that is $k_t^e \geq 0$. All of our results continue to apply when $k_t^e < 0$, with the entrepreneur's financing constraint being $s_t^e = 0$ and not binding in that case. Our main result in this section concerns the long-run steady state, in which we must have $K_t^e \geq 0$. In our numerical section, $K_t^e < 0$ happens in the initial periods of our transition, and we take this into account.

[22] Unlike entrepreneurs, that are subject to a credit constraint, workers can return a negative balance to the households, that will be covered by borrowing, if needed.

government bonds:[23]

$$c_t+p_tb_t+\chi\left[k_t^e-q_t(\phi_t)s_t^e\right]=(1-\tau_t^\ell)w_t(1-\chi)\ell_t+b_{t-1}+[r_t(1-\tau_t^k)+\delta\tau_t^k]k_{t-1}-(1-\chi)q_t^ws_t^w. \quad (26)$$

The household capital position evolves according to

$$k_t=(1-\delta)k_{t-1}+(1-\chi)s_t^w+\chi(k_t^e-s_t^e). \quad (27)$$

Each household maximizes (23) with respect to $(c_t,\ell_t,s_t^w,s_t^e,k_t^e,k_t,b_t,\phi_t)_{t=0}^\infty$ subject to the financing constraint (24), the budget constraint (26), and the evolution of capital (27), taking as given $(\tau_t^\ell,\tau_t^k,q_t^w,q_t(\phi),w_t,r_t,p_t)_{t=0}^\infty$.[24] In the aggregate, we have $C_t=c_t$, $K_t=k_t$, $B_t=b_t$, $L_t=(1-\chi)\ell_t$, $S_t^e=\chi s_t^e$, $S_t^w=(1-\chi)s_t^w$, and $K_t^e=\chi k_t^e$.

3.5 Competitive Equilibrium

Definition. A competitive equilibrium is an allocation $\{C_t,L_t,S_t^e,S_t^w,K_t,K_t^e,B_t,\phi_t\}_{t=0}^\infty$, prices of bonds and claims to capital $\{p_t,q_t^w,q_t(\phi)\}$, factor prices $\{w_t,r_t\}_{t=1}^\infty$ and taxes $\{\tau_t^\ell,\tau_t^k\}$, such that: the allocation solves the household problem of maximizing (23) subject to (24), (26), and (27); financial intermediaries break even on all possible trades, that is (20) holds for all ϕ (not just ϕ_t); firms maximize their profits taking prices and taxes as given; the government budget constraint (22) holds; and markets clear, that is, $S_t^e=S_t^w$,

$$C_t+K_t+G_t+S_t^e\eta(\phi_t)=F(K_{t-1},L_t)+(1-\delta)K_{t-1}, \quad (28)$$

and

$$K_t=K_t^e+(1-\delta)K_{t-1}. \quad (29)$$

We next characterize the competitive equilibrium. Following the same proof as in Lemma 3, point 1, we must have $q_t(\phi)\phi<1$ for all $\phi\in[0,1]$ for the household problem to be well defined, otherwise an arbitrage would be present.

As in the two-period economy, we first focus on the household choice of investment. In this case, through the entrepreneurs the household can choose among a continuum of strategies, depending on the leverage ϕ. The following lemma shows how the choice of ϕ emerges from a cost-minimization

[23] As in the two-period economy, households are allowed to sell bonds short, so as to keep this part of the problem identical to the standard Ramsey problem in a neoclassical economy.

[24] In principle, the household could choose to trade at different values of ϕ_t, and get different prices $q_t(\phi)$ on different trades: as an example, it could choose to finance 50% of its investment exclusively with its own funds, and 50% by using funds from a market that allows sales of a fraction ϕ_t. Formally, this would be equivalent to letting the household trade at the upper envelope of the function q_t. Given our intermediation technology, q_t is always strictly concave in equilibrium and a split investment strategy as above would never be optimal, so this generality would simply add notation with no change in the results.

subproblem of the overall household utility maximization. This is convenient because the optimal solution for ϕ is independent of the other choices of the household, so we can establish the optimal choice of ϕ_t first and take it as given when deriving the other optimality conditions.

Lemma 1. *Let (k_t^e, s_t^e, ϕ_t) be part of the optimal allocation chosen by the household. Then the following is true:*

-
$$(k_t^e, s_t^e, \phi_t) = \arg\min_{(\hat{k}, \hat{s}, \hat{\phi})} \hat{k} - q_t(\hat{\phi})\hat{s} \tag{30}$$

  subject to[25]
$$\hat{s} \leq \hat{\phi}[\hat{k} + (1-\delta)k_{t-1}]; \tag{31}$$
$$\hat{k} + (1-\delta)k_{t-1} - \hat{s} \geq k_t^e + (1-\delta)k_{t-1} - s_{t-1}^e. \tag{32}$$

- *Let (20) hold. A necessary condition for (k_t^e, s_t^e, ϕ_t) to solve the problem above is that (31) holds as an equality when evaluated at (k_t^e, s_t^e, ϕ_t) and*
$$\phi_t = \arg\min_{\hat{\phi}} \frac{1 - \hat{\phi}q(\hat{\phi})}{1 - \hat{\phi}} \tag{33}$$

Proof. See online Appendix F.1. □

Using equation (20) and taking the first-order condition of the problem (33), the optimal choice of ϕ by the household is $\phi_t = 0$ if $q_t^w < 1$ and is otherwise characterized by the following relationship between q_t^w and ϕ_t:
$$q_t^w = 1 + \eta(\phi_t) + (1-\phi_t)\phi_t\eta'(\phi_t). \tag{34}$$
We denote as q_t^* the value of q_t that prevails in period t at the optimal choice of ϕ_t. We have $q_t^* = q_t^w$ if $q_t^w \leq 1$ and otherwise
$$q_t^* = 1 + (1-\phi_t)\phi_t\eta'(\phi_t), \tag{35}$$
which implies $q_t^* > q_t^w$ when $q_t^w > 1$.

Having pinned down which market is optimally chosen by entrepreneurs in selling their claims to capital (if they sell any), we next study how the household chooses between the two alternatives for acquiring capital, namely investment by entrepreneurs and purchases of claims by the workers. Lemma 2 proves that the relevant marginal condition in the households' intertemporal condition is always the worker's choice to purchase an extra unit of capital, with the entrepreneur's choice being

[25] If the allocation is chosen optimally, k_{t-1} is also part of the optimal allocation, except for k_{-1}, which is an exogenous initial condition. However, the proof works by choosing that the household can improve upon the allocation whenever (k_t^e, s_t^e, ϕ_t) do not solve the cost minimization problem, independently of the optimality of k_{t-1}.

either indeterminate (when $q_t^w = q_t^* = 1$ and worker purchases and entrepreneur investment are equivalent) or set at the maximum allowed by (24) and (35).

Lemma 2. *Let (20) hold.*

- *If $q_t^w < 1$, either the household find it optimal to buy capital but not to sell it, or the solution to its problem is the same as would prevail if all other prices are the same except $q_t^w = q_t^* = 1$. Since the first option cannot arise in an equilibrium, we restrict our attention to $q_t^w \geq 1$ without loss of generality.*

- *If $q_t^w = 1$, the marginal value to the household of an extra unit of investment or an extra unit of purchases of claims is equal.*

- *If $q_t^w > 1$, the marginal value to the household of an extra unit of investment exceeds that of purchases claims to capital, so either (24) is binding, or the household would optimally sell claims to capital and not buy any (which would never happen for equilibrium prices).*

Proof. See online Appendix F.2. □

Using (24), (27), and Lemmas 1 and 2, we can substitute out k_t^e, s_t^w, and s_t^e and consolidate all of the constraints of the household problem into the following single budget constraint:[26]

$$\begin{aligned} & c_t + p_t b_t + q_t^w k_t = b_{t-1}\left(1 + \chi \rho_t\right) \\ +k_{t-1} & \left[(1-\tau_t^k) r_t + \delta \tau_t^k + q_t^w (1-\delta) \left(1 + \frac{\chi \phi_t (q_t^* - 1)}{1 - \phi_t q_t^*} \right) \right] + (1-\chi)(1-\tau_t^\ell) w_t \ell_t, \end{aligned} \tag{36}$$

where

$$\rho_t := \frac{q_t^w - 1 - \phi_t (q_t^w - q_t^*)}{1 - \phi_t q_t^*}. \tag{37}$$

ρ_t is a measure of the tightness the financing constraint of the entrepreneurs: it is the shadow value of transferring of an extra unit of pledgeable resources in the hands of an entrepreneur.

We use the remaining first-order conditions of the household problem to derive further restrictions that must hold in an equilibrium. The first-order condition for the labor supply yields[27]

$$(1 - \tau_t^\ell) w_t u'(C_t) = v'(L_t). \tag{38}$$

[26] In the problem that follows, we neglect the nonnegativity constraints on s_t^w, s_t^e, and k_t^e. Following the reasoning in the lemmas, the nonnegativity constraints on s_t^w and s_t^e cannot be binding in an equilibrium if $q_t^w > 1$, or if $q_t^w = 1$ and $k_t^e \geq 0$.

[27] Throughout these conditions, we use the first-order condition for consumption to substitute out the Lagrange multiplier of the budget constraint.

The first-order condition for government bonds b_t implies

$$p_t = \frac{\beta u'(C_{t+1})}{u'(C_t)} \left(1 + \chi \rho_{t+1}\right). \tag{39}$$

In equation (39), the term $\chi\rho_{t+1}$ represents the liquidity service that government bonds offer, arising from the fact that bonds can be liquidated with no intermediation costs by the fraction χ of family members who turn out to be entrepreneurs in period $t+1$. This liquidity service pushes up the bond price p_t and pushes down the (gross) interest rate $1/p_t$, giving rise to a corresponding liquidity premium. The first-order condition for capital k_t implies

$$q_t^w = \frac{\beta u'(C_{t+1})}{u'(C_t)} \left\{(1-\tau_{t+1}^k)r_{t+1} + \delta\tau_{t+1}^k + (1-\delta)q_{t+1}^w + \chi\phi_{t+1}(1-\delta)[q_{t+1}^*(1+\rho_{t+1}) - q_{t+1}^w]\right\}. \tag{40}$$

The cost for a worker to acquire one unit of (claims to) capital is represented by q_t^w.[28] In the next period, the family receives a payoff $(1-\tau_{t+1}^k)r_{t+1} + \delta\tau_{t+1}^k + (1-\delta)q_{t+1}^w$ from the investment. In addition, the fraction $\chi\phi_{t+1}$ of undepreciated capital held by entrepreneurs represents extra net worth that they can pledge, with an extra liquidity value captured by $q_{t+1}^*(1+\rho_{t+1}) - q_{t+1}^w$.[29]

After we substitute $S_t^e = \phi_t[K_t^e + \chi(1-\delta)K_{t-1}]$ into (28), the market clearing condition for goods is

$$C_t + G_t + [1 + \phi_t\eta(\phi_t)]\, K_t = F(K_{t-1}, L_t) + [1 + (1-\chi)\phi_t\eta(\phi_t)]\,(1-\delta)K_{t-1}. \tag{41}$$

We are now ready to substitute out prices and taxes and derive the primal representation of a competitive equilibrium, that is, the set of allocations that can be implemented by a competitive equilibrium. In a frictionless economy, this set is characterized by a sequence of feasibility constraints such as (41) and a single present-value implementability condition, in which only the initial values of government debt B_{-1} and capital K_{-1} matter. In our case, the value of government debt B_t in each period t matters for the tightness of financial constraints and is thus needed for the characterization; as a consequence, we also have a sequence of implementability constraints. Also, while an individual household is able to acquire more than $b_{t-1} + q_t^* s_t^e$ units of newly produced capital by letting workers purchase claims, in the aggregate $S_t^e = S_t^w$, which imposes a further constraint on the maximum capital accumulation. Finally, rather than working with the amount of debt to be repaid in period $t+1$ (which is B_t), at time $t \geq 0$ it is more convenient to work with $\tilde{B}_t := p_t B_t$, the value of debt issued in period t. For period 0, B_{-1} and τ_0^k remain as initial conditions.

Proposition 3. *An allocation* $\{C_t, L_t, K_t, \tilde{B}_t, \phi_t\}_{t=0}^{\infty}$ *forms part of a competitive equilibrium if and*

[28] When $q_t^w = 1$, an individual family is indifferent between whether to purchase an extra unit in the market or to increase its own entrepreneurs' investment. Hence, q_t^w remains the correct shadow cost of acquiring an extra unit of capital. This is true, even though in the aggregate, we must have $\phi_t = 0$; hence, no trade in capital claims takes place.

[29] This term shows the difference between the price at which entrepreneurs sell their capital, adjusted for the shadow value of liquidity, and the price at which workers can buy the capital back.

only if it satisfies the resource constraint (41), the implementability constraints for $t > 0$

$$u'(C_t)(C_t + \tilde{B}_t + q_t^w K_t) = v'(L_t)L_t + u'(C_{t-1})(\tilde{B}_{t-1} + q_{t-1}^w K_{t-1})/\beta, \tag{42}$$

the implementability constraint for time 0

$$\begin{aligned} u'(C_0)\left[C_0 + \tilde{B}_0 + q_0^w K_0\right] &= v'(L_0)L_0 + u'(C_0)B_{-1}(1+\chi\rho_0) \\ +u'(C_0)K_{-1}\left[(1-\tau_0^k)F_K(K_{-1}, L_0) + \delta\tau_0^k + q_0^w(1-\delta)\left(1 + \frac{\chi\phi_0(q_0^* - 1)}{1-\phi_0 q_0^*}\right)\right]&, \end{aligned} \tag{43}$$

the financial constraint at $t > 0$

$$u'(C_t)(1-\phi_t q_t^*)K_t \leq \frac{\chi u'(C_{t-1})\tilde{B}_{t-1}}{\beta(1+\chi\rho_t)} + u'(C_t)[1-(1-\chi)\phi_t q_t^*](1-\delta)K_{t-1}, \tag{44}$$

with equality if $\phi_t > 0$, and the financial constraint for time 0

$$(1-\phi_0 q_0^*)K_0 \leq \chi B_{-1} + [1-(1-\chi)\phi_0 q_0^*](1-\delta)K_{-1}, \tag{45}$$

with equality if $\phi_0 > 0$. B_{-1}, K_{-1}, and τ_0^k are exogenously given. q_t^*, q_t^w and ρ_t are functions of ϕ_t only, given by (34), (35), and (37).

Proof. See online Appendix F.3. □

3.6 The Ramsey Plan

We compute the Ramsey plan by maximizing the representative household's utility

$$\sum_{t=0}^{\infty} \beta^t \left[u(C_t) - v(L_t)\right] \tag{46}$$

subject to (41), (42), (43), (44), and (45). Assuming that the best competitive equilibrium is interior, we can derive some of its properties by studying the first-order conditions.

Our main result concerns the long-run properties of the allocation. The main intuition for this result stems from the first-order condition with respect to government bonds $\tilde{B}_t$. Letting $\beta^t\Psi_t$ and $\beta^t\gamma_t$ be the Lagrange multipliers attached to implementability constraints and the financing constraints, the first-order condition is

$$\Psi_{t+1} = \Psi_t + \chi\frac{\gamma_{t+1}}{1+\chi\rho_{t+1}}. \tag{47}$$

An additional unit of debt issuance relaxes the current government budget (or implementability constraint), with a benefit Ψ_t. If the financing constraint is slack (or without financial frictions) in period

$t+1$, at the optimum this would be exactly offset by a tighter budget constraint in period $t+1$, leading to $\Psi_t = \Psi_{t+1}$ that corresponds to the standard tax-smoothing principle. If instead the financing constraint is binding, the additional liquidity benefit of government debt creates an incentive to postpone taxation at the margin so that the tightness of the implementability constraint increases over time. This effect is stronger, the tighter is the financial constraint, as measured by its Lagrange multiplier.

We then have the following result:

Proposition 4. *Assume that the economy converges to a steady state with finite allocations (finite C, K, L, and $\tilde{B}$, given finite G). There are two possibilities:*

1. *The government issues enough debt to fully relax the financing constraints in the limit. In this case, Ψ_t converges to a constant; in the limit, capital-income taxes are zero and the interest rate on government debt is β^{-1}.*

2. *The marginal cost of raising tax revenues Ψ_t grows without bounds and the economy converges to a dynamic equivalent of the top of the Laffer curve. The interest rate on government debt is smaller than β^{-1} in the limit. In addition, if the shadow cost of relaxing the financing constraint is sufficiently low in the limit, then the limiting tax rate on capital is strictly positive, $\lim_{t\to\infty} \tau_t^k > 0$.*

Proof. See Appendix C. □

The first case applies if the government finds it feasible to flood the economy with public liquidity.[30] The second case occurs if the amount of debt that fully relaxes financing constraints exceeds the fiscal capacity. In this case, generically the tax rate on capital is not zero. For the infinite-horizon economy, we cannot obtain analytical expressions even with quasi-linear utility. Moreover, for general preferences, local comparative statics may not apply if the solution "jumps" in the presence of nonconvexities. Nonetheless, when such jumps do not occur, Proposition 4 provides a similar result to what we obtained in Proposition 2 for the two-period environment: in a neighborhood of the point at which the financing constraint ceases to be binding, taxes on capital become unambiguously positive.[31] In that neighborhood financial distortions only have a second-order welfare effect, while taxing rents accruing to capital yields first-order welfare gains.

Proposition 4 differs from the results in Collard et al. (2020), where a steady state with interior debt can be attained. They do not consider capital taxes, and the quantity of government debt determines

[30] This result is reminiscent of Albanesi and Armenter (2012). However, the conditions of their general theorem are not satisfied here. Without financial frictions, it is possible (but not optimal) for the government to attain an undistorted long-run steady state, which implies that the optimal steady state will involve no distortions. The presence of financial frictions implies that it is impossible for the government to attain an undistorted steady state featuring enough liquidity and no taxes: in order to avoid taxation in the long run, the government would need to accumulate assets, but in this case it would not provide entrepreneurs with the liquidity that they need to overcome financial frictions.

[31] There are several parameters that can be adjusted for this comparative statics exercise. The depreciation rate of capital is one.

at the same time intertemporal prices and the asset price. In our model, the government has an additional instrument to lower the debt servicing cost: it can levy a tax on capital, which favors government bonds and acts as an instrument of financial repression. With this extra instrument, the need to maintain intertemporal distortions in the limit disappears, unless fiscal space is insufficient: starting from an allocation in which the economy converges to a steady state with an interior level of public debt and insufficient liquidity, the government would always have an incentive to increase debt and raise some extra revenues to make up for the eventually higher interest rates using capital-income taxes along the transition. Only when fiscal space is exhausted can intertemporal distortions survive.

While we follow the standard Ramsey literature and take as given the tax rate on capital in period 0 (setting it to 0 in our quantitative experiments), it is worth noting that our Ramsey plan would in general not feature full expropriation of initial capital even in the absence of this constraint. Government debt and ample unpledged capital in the hand of entrepreneurs play a productive role in our economy, by allowing entrepreneurs to economize on intermediation costs and permitting larger investment. It is thus not optimal for the government to accumulate a large asset position.

4 Quantitative Analysis

In this section, we evaluate the plausible magnitude of the capital-income taxes that are generated by our model. Financial frictions have the potential to justify significant tax rates, and we find these rates to be typically positive (that is, for plausible degrees of the distortions from labor-income taxes, it is optimal to tax rather than subsidize capital).

4.1 Parameterization

We assume that preferences are given by

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}; \quad v(\ell) = \mu \frac{(\ell_t)^{1+\nu}}{1+\nu}$$

and that the technology is $F(K, L) = K^{\alpha} L^{1-\alpha}$. We set capital share $\alpha = 1/3$. We will discuss σ and set it separately in the following subsections. Our main calibration, that we use in analyzing transition dynamics, sets $\sigma = 1$. We choose $\delta = 0.097$ and $\beta = 0.97$ so that steady-state investment and capital are 25% of and 2.6 times output, respectively. These are in line with standard parameters for a yearly calibration for a macroeconomic model. We set $\nu = 2/3$, which is in line with macroeconomic labor supply elasticity, and μ is chosen so that labor supply is 1/3 units of time (just a normalization) in the steady state. Government spending is pinned down by targeting 20% G/Y (US post-war data).

$\chi = 0.16$ corresponds to the fraction of firms adjusting capital stock each year. This parameter is discussed in Shi (2015), who uses the empirical estimates in Doms and Dunne (1998) and Cooper et al. (1999). We set

$$\eta(\phi) = \omega_0 \phi^{\omega_1},$$

where $\omega_1 = 2$, which results from a matching function where the elasticity of matches to buy orders and saleable assets is the same and it is costly to process the buy orders (see online Appendix E). Lastly, $\omega_0 = 0.45$ is picked so that the liquidity premium of government debt is about 1% at the time of the fiscal shock experiment to be discussed later. A popular measure of the government debt liquidity premium is the difference between yields on AAA corporate bonds and those on government bonds with similar maturity. From 1984 to 2018, the difference is about 1%.[32]

4.2 Comparative Statics in the Long Run

When $\sigma = 1$, the steady state turns out to be always unconstrained. No matter how high public debt needs to be to satiate the demand for liquidity, the government is able to sustain it by a suitable choice of taxes. This is a standard result: when the intertemporal elasticity of substitution is low, agents in each period are so desperate to consume that the government is able to extract even the entire GDP in taxes. For our transition experiments, this is not an issue, since the constraint can be binding even as the economy converges to the eventual unconstrained steady state itself. In this subsection, we study comparative statics of the constrained steady state to better illustrate the economic forces at work. To this end, we choose $\sigma = 0.2$ so that the economy features an upper bound on sustainable debt in the limit (a maximum of the "dynamic Laffer curve.")

Our first comparative-statics exercise analyzes the effect of changing government spending (Table 1). We keep other parameters unchanged, except for G, which is used to vary the position of the top of the Laffer curve. When the economy converges to a steady state with a binding implementability constraint, the Lagrange multiplier on the implementability constraint Ψ_t grows at a constant rate in the limit, as shown in Appendix C. We pick values of G such that it grows at 1% a year, 2% a year, or 3% a year (recall that it is constant, i.e., 0% growth, in the limit for the baseline economy).

As G increases, the maximum sustainable steady-state debt level *decreases*. The government is forced to cut back on public liquidity. With smaller amounts of public liquidity, entrepreneurs increasingly rely on financial intermediaries to sell their assets and fund their investment; the fraction ϕ of capital that is intermediated increases. From our theoretical results, it is ambiguous whether capital-income taxes become positive or negative. In this numerical example, the incentive to tax quasi-rents dominates and $\tau^k > 0$ and it is economically significant, while the tax on labor income drops somewhat. Government debt commands a liquidity premium, and its interest rate drops as it becomes scarcer because the increase of G requires a greater liquidity premium, equivalent to more-severe financial distortions, to finance the government budget.

[32]See, for example, Krishnamurthy and Vissing-Jorgensen (2012) , Del Negro et al. (2017), and Cui and Radde (2020).

Table 1: Steady state of the Ramsey allocation for different government expenditures

	$\frac{\Psi_t}{\Psi_{t-1}} = 1.00$	$\frac{\Psi_t}{\Psi_{t-1}} = 1.01$	$\frac{\Psi_t}{\Psi_{t-1}} = 1.02$	$\frac{\Psi_t}{\Psi_{t-1}} = 1.03$
$G/Y(\%)$	30.15	33.67	34.66	34.85
Capital K (%)	100.00	90.90	86.32	84.52
Capital tax τ^k (%)	0.00	10.00	15.73	19.52
Labor tax τ^l(%)	52.00	51.10	50.36	49.72
Interest rate(%)	3.09	2.12	1.20	0.23
Debt-to-output $\tilde{B}/Y(\%)$	156.20	67.72	33.33	6.73
Asset liquidity ϕ	0.00	0.21	0.29	0.36

Note: Capital is normalized to 1 in the first column.

Next, we explore the role of financial intermediation costs. Specifically, we increase ω_0 in three steps of 10% each (Table 2). At the baseline steady state, this would be irrelevant, since no intermediation takes place. We thus use government spending from the second column of Table 1. We experimented with different values, and the results are robust.

Table 2: Steady state of the Ramsey allocation for different financial frictions

Intermediation technology	ω_0	$1.1\omega_0$	$1.2\omega_0$	$1.3\omega_0$
$G/Y(\%)$	33.67	33.87	34.09	34.36
Capital K (%)	100.00	98.38	96.56	94.43
Capital tax τ^k (%)	10.00	11.78	13.83	16.27
Labor tax τ^l(%)	51.10	50.93	50.73	50.46
Interest rate(%)	2.12	1.97	1.79	1.55
Debt-to-output $\tilde{B}/Y(\%)$	67.72	64.83	61.20	56.12
Asset liquidity ϕ	0.209	0.214	0.221	0.231

Note: Capital is normalized to 1 in the first column.

The interest rate falls from 2.12% to 1.55% when financial frictions are tighter, since agents have more incentive to hold liquid government debt. Perhaps surprisingly, when intermediation is more costly it is used *more*, relative to government debt in the limit. The reason is that the fiscal capacity of the economy contracts (e.g., the capital stock falls about 5.5% when ω_0 increases by 30%), so the government is less able to issue debt. Greater financial frictions are associated with greater rents from entrepreneurial net worth; this factor dominates the comparative statics for the tax rate on capital, which increases.

4.3 Transition Dynamics

For our main experiments we take a more standard value of $\sigma = 1$. Qualitatively, the results are similar with a lower σ, but our goal here is to provide a more plausible calibration that allows us to evaluate the correct magnitude of the forces at work. With the parameters shown in Section 4.1, the unconstrained steady state features a debt-to-output ratio of 156.2%, a 0% capital tax, and a 36.9% labor tax. While in this case the long-run steady state has a slack financing constraint and interest rates equal to $1/\beta$, this does not need to happen during the transition.[33]

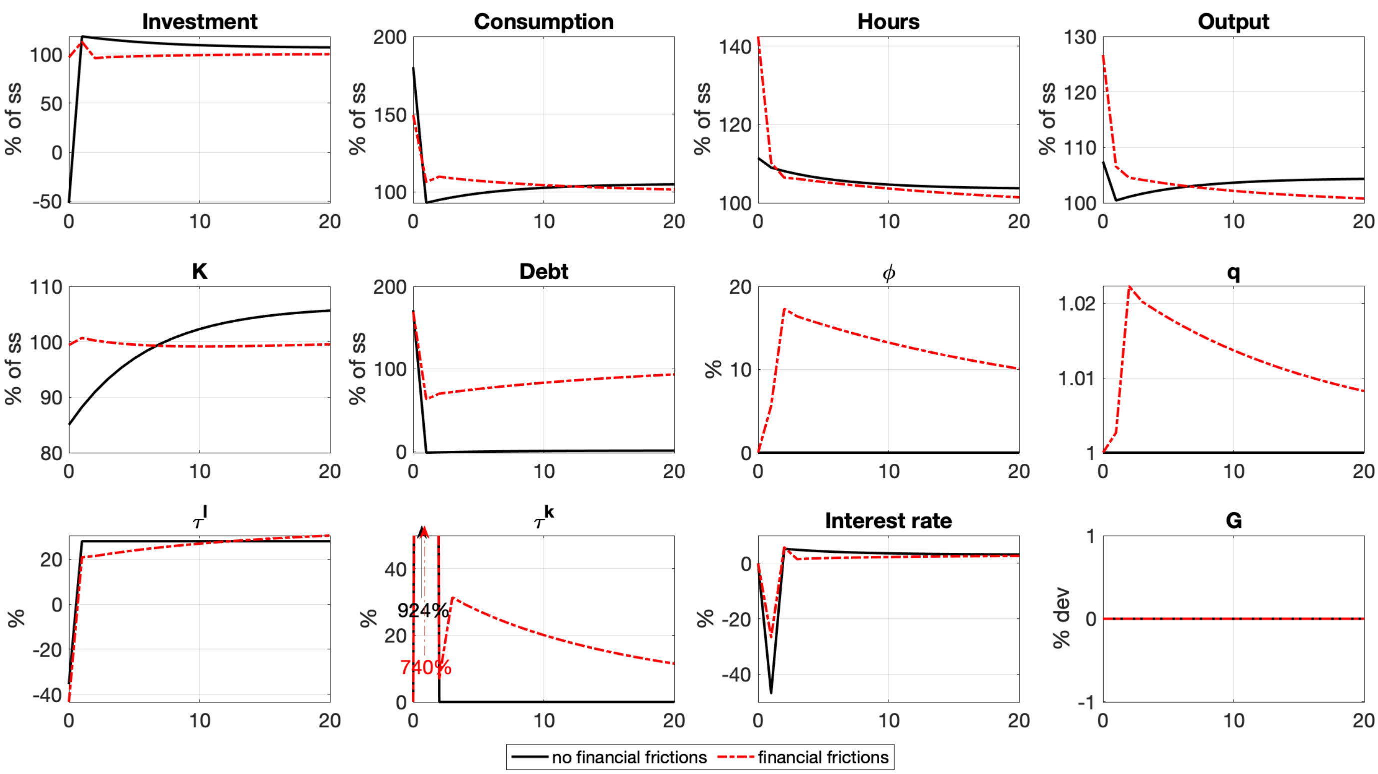

Figure 3: **Transition dynamics with constant government spending.** Note: Allocation variables are plotted as percentages of steady-state levels in the economy with financial frictions. "% of ss" means percentage of steady-state levels. "% dev" means percentage deviation.

We start the economy at the steady-state levels of debt and capital. We impose an upper bound on initial capital-income tax rate, which is set to zero. Our results below are similar with different choices, but using zero is convenient to compare with the Ramsey plan without financial distortions. In our case, we would obtain an interior solution even without an initial upper bound on capital-income taxes: a confiscation of initial capital deprives entrepreneurs of the net worth that they need to invest. While the Ramsey plan eventually converges back to these values, in the short run the government

[33] We use the platform AMPL and the solver KNITRO to compute the transition path, assuming that for a large enough period $T \geq 200$, the economy converges to the (unconstrained) steady state.

has an incentive to deviate, run a surplus, and tax capital (from period 1); the financing constraint is binding along the transition. Figure 3 displays the Ramsey plan when government spending is constant (red dashed lines) and compares it to the evolution of an economy without financial frictions (i.e., $\omega_0 = 0$) but that otherwise has the same parameters and initial conditions.

In the economy without financial frictions, taxes on capital are positive only in period 1 (they would be positive in period 0 as well, if we allowed that); Proposition 3 in Chari et al. (1994) proves this for a class of preferences that includes ours. The large fiscal surplus in period 1 is used to permanently withdraw government debt, and the economy settles to a permanently lower level of debt, which is almost zero in our numerical simulation.[34]

In the early periods, optimal policy under financial constraints is qualitatively similar to the economy without frictions, but quantitatively very different. While the dominant factor early on is still the desire to enact a surprise tax on initial capital to the extent possible, this is tempered by the fact that government debt will be needed for liquidity purposes in the future. Government debt in period 1 drops to 65% (instead of 0) of the steady-state level. As a consequence, the surplus that the government runs in period 1 is much smaller, and so is the capital tax that generates the surplus (740% compared to 924% in the frictionless economy).

Going forward, new investment is constrained by the smaller availability of government bonds, which reduces the interest rate on government debt and further benefits government finances. In this exercise, the race between taxing quasi-rents arising from the financial frictions and the benefits of subsidizing further investment is won by the former, and the government optimally continues to tax capital substantially. Eventually, the economy reverts to the original steady state: as in Section 3, there is an incentive to move away from tax smoothing and slowly reaccumulate debt to provide the private sector with greater liquidity. This process stops when the liquidity constraint is fully relaxed, which happens (by our assumptions) at the initial steady state with 151.7% debt-to-output ratio.

The presence of financial frictions has large implications not only for the optimal policy but for the allocation as well. In the absence of financial frictions, investment collapses in period 0 in anticipation of the large capital-income tax that will occur in period 1, and it jumps above steady state from period 1 onwards. Eventually, capital settles at a higher steady-state value because of the smaller tax distortions needed when government debt is lower. In contrast, the investment recovery is hampered by financial frictions, and investment is persistently *below* the steady-state level when financing constraints are present. This gives agents greater incentives to invest earlier on, before government debt is lowered; these incentives are supplemented by the capital-income tax, which does not jump as high in period 1, and by the period-0 labor subsidy, which is comparable in magnitude with and without financial frictions. The resulting path for capital is much smoother, reflecting the lower elasticity of capital supply, coupled with the policies tailored to this lower elasticity.

[34]In period 0, the government runs a large deficit. In an attempt to limit the consequences of the coming large capital tax, the government subsidizes labor as a way of subsidizing initial investment.

4.4 How to Finance Government Spending Surges

We now compare the baseline transition dynamics, in which government spending stays constant, with another path, in which exogenous government spending is increased by 10% between periods 10 and periods 19. We choose to have an anticipated movement as our main experiment because the initial periods of a Ramsey plan are very special.[35] For simplicity, we refer to the time-varying path as the effect of a "spending shock," but both economies are deterministic.[36]

When the government spending rises in period 10, our calibration generates a 1% liquidity premium for government debt the interest rate is 2.09%. Figure 4 shows the consequences of this time-varying path in the presence and absence of financial frictions. We represent these consequences as the difference between the optimal path when it is known at time 0 that spending will increase and what would otherwise be optimal (i.e., the path of Figure 3). In this way, we isolate the effects of the shock from those of the transition.

In anticipation of the jump in spending, investment ramps up at the expense of consumption. However, the extent to which this is the case is more than twice as large for the economy without financial frictions: financing constraints limit the entrepreneurs' ability to produce new capital, reducing the capital supply elasticity. This raises Tobin's q and in turn spurs the entrepreneurs to rely more on (costly) financial intermediation, as the increase in ϕ attests. Once the shock hits, the comparison flips: investment falls further when financial frictions are not present, cushioning the drop in private consumption, whereas the increased debt that arises from government deficits alleviates financial frictions when they are present and thus limits the drop in investment.

On the policy front, in anticipation of the rise in G, the government reduces its debt more in the baseline case without financial frictions; with financial frictions, the debt reduction is limited, since retiring further government debt would drain even more liquidity from the market and force entrepreneurs to spend additional resources in intermediation. Similar to the real allocation, this reduction is also reversed in the periods of the shock, when bigger deficits are run by the government when financial frictions are not present. For the preferences that we assumed, capital-income taxes without financial frictions are unaffected by the presence of the shock. In contrast, when financing constraints are present, capital-income taxes are desirable because the constraints make the capital supply less elastic for the same reasons as in Section 2.

The timing of taxes is particularly striking. The government modestly increases τ^k in the periods leading to the spending jump, reserving the punch for the last two periods of high investment (in which they are 15% and 7% higher, respectively), when credit constraints are tightest. The difference between τ^k with and without shocks is actually greatest in period 9 (when capital supply elasticity is

[35]The case in which high spending starts in period 0 is available upon request. The economics are similar, but now the forces that lead to initial taxation confound those that lead to capital-market distortions in the longer run.

[36]This is commonly referred to as an "MIT shock." Notice, however, that the surprise is at time 0, not at the time at which spending jumps.

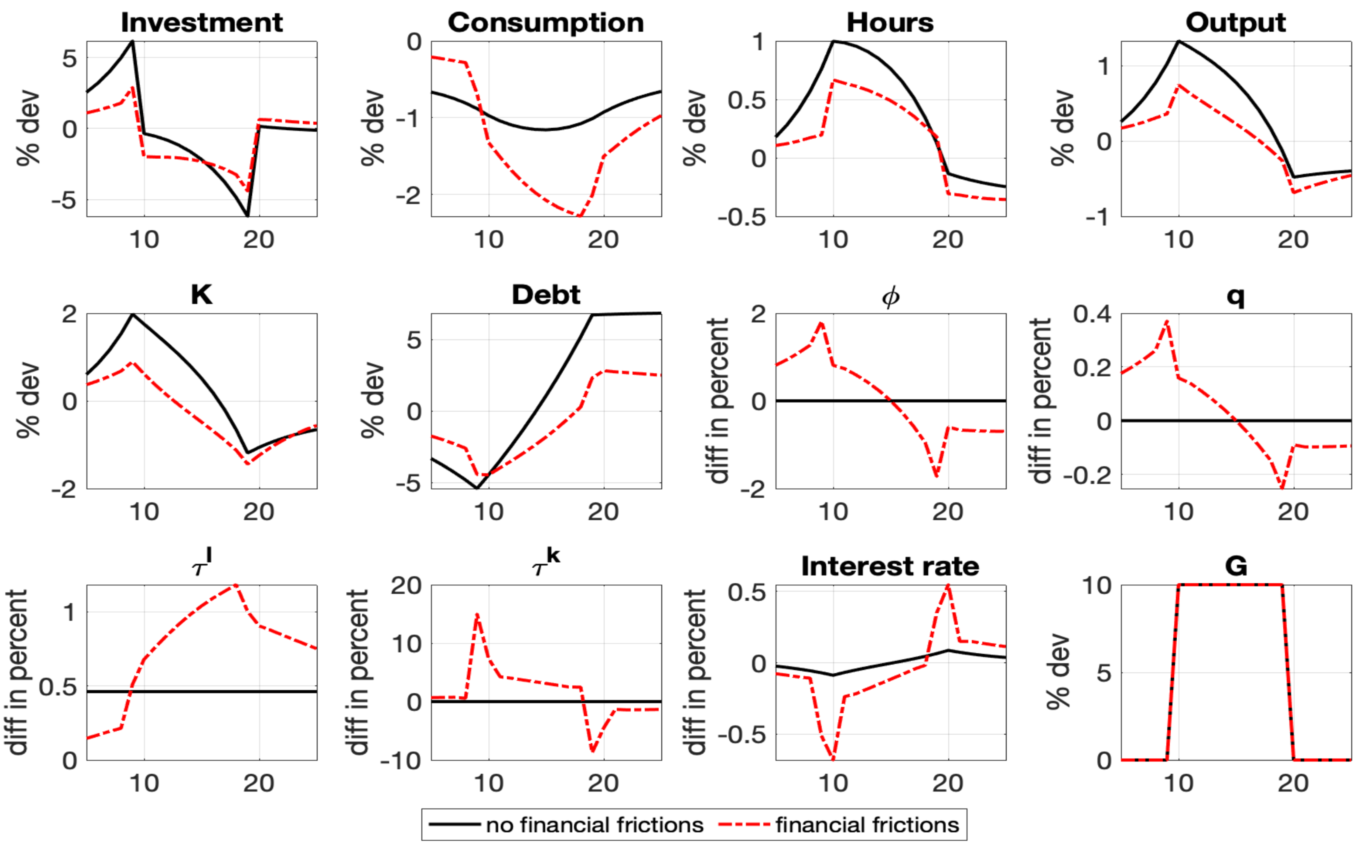

Figure 4: **The effect of a fiscal shock: deviations from the transition paths of Figure 3.** Note: allocation variables are normalized by steady-state levels in the economy with financial frictions. "% dev" means percentage deviation. "diff in percent" means difference in percentage terms.

the lowest), affecting the Euler equation between periods 8 and 9, rather than in period 10.

Because of the countervailing forces that we previously identified, the optimal capital-income tax is quantitatively affected by movements in the interest rates. Compared with the interest rate between periods 8 and 9, the real interest rate between periods 9 and 10 jumps down by about 50 to 60 basis points even in the absence of taxes, since families anticipate lower consumption when government spending ramps up in period 10.[37] A symmetrical effect is in play at the end of the shock period.

Note that the bunching of capital-income taxes is because a tax in period t affects the rewards from investing in many periods beforehand. This is less true for labor-income taxes, whose direct effect is to distort an intratemporal margin. Capital-income taxes remain elevated for the duration of the shock. Interest rates are also quite different in the two economies. Without financial frictions, interest rates' movements in response to the shock are minor and do not account for much of the evolution of government debt. In contrast, when financial frictions are present, the optimal policy distorts capital

[37] If government bonds are nominal with a fixed nominal interest rate, this could be translated to an inflation rise of 0.5-0.6 percentage points (for the 10% increase in government spending).

accumulation and leads to significantly lower rates on government debt for the duration of the shock (65 basis points below steady state at the onset of the shock). Along with the direct effect of revenues from capital-income taxes, this indirect price effect finances a significant fraction of the spending shock, and debt increases much less than in the frictionless case.

While the shock has a permanent effect in the absence of financial frictions, a feature associated with optimal policy in a standard model, our economy reverts to the initial (unique) steady state.

From this experiment, we conclude that financial constraints provide a justification for policies of financial repression during periods of public budget stress: our optimal solution features both positive capital-income taxes and low interest rates on public debt in periods of high spending.

5 Conclusion

Within the context of a Ramsey model of capital taxation, we identified a force that operates as in Sargent and Wallace (1982) and pushes the government to increase its indebtedness to mitigate frictions in private asset markets. We showed that when it is impossible to completely undo those frictions in the long run, it is optimal to tax capital, even though its provision is already inefficiently low. This outcome happens because the frictions that prevent efficient investment also alter the elasticity of the supply of capital. In this case, a wedge between the returns on capital and bonds is also optimal. This paper considered an economy with no aggregate risk, in which no force countervails the upward drift in government debt. In a stochastic economy with non-contingent debt, Aiyagari et al. (2002) identify an opposite force, which induces the government to accumulate assets for self-insurance. In our next step, we plan to study how capital-income taxes and government debt are optimally chosen when both of these forces are present.

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A The Two-period Planner's Problem and Proof of Proposition 2

Let $\beta^{t-1}\lambda_t$ be the Lagrange multiplier on the resource constraint, for $t = 1, 2$. Let Ψ_1 be the Lagrange multiplier on the implementability constraint. The planner's first-order necessary conditions (FOCs) for consumption C_1 and C_2 are

$$1 + \Psi_1 - \lambda_1 = 0 \text{ and } 1 + \Psi_1 - \lambda_2 = 0.$$

So, $\lambda_1 = \lambda_2 = 1 + \Psi_1$. The planner's first-order conditions for labor supply L_1 and L_2 are

$$v'(L_1)(1 + \Psi_1) + \Psi_1 v''(L_1)L_1 = \lambda_1; \tag{48}$$

$$v'(L_2)(1 + \Psi_1) + \Psi_1 v''(L_2)L_2 = \lambda_2 F_L(K_1, L_2). \tag{49}$$

The first-order condition for capital K_1 is

$$-\lambda_1 + \beta\lambda_2 F_K(K_1, L_2) \begin{cases} = 0 & \text{if } K_1 < K^* \\ \in (0, \Psi_1(1/\phi_1 - 1)) & \text{if } K_1 = K^* \\ = \Psi_1(1/\phi_1 - 1) & \text{if } K_1 > K^* \end{cases}. \tag{50}$$

After we use $\lambda_1 = \lambda_2 = 1 + \Psi_1$, we have the planner choice of capital (17) in the main text. L_1 is simply a function of Ψ_1 after we use $\lambda_1 = 1 + \Psi_1$ in (48), and we can solve L_2 from (49):

$$\mu L_2^{\nu} \frac{1 + \Psi_1(1 + \nu)}{1 + \Psi_1} = (1 - \alpha)\left(\frac{K_1}{L_2}\right)^{\alpha} \Longrightarrow L_2 = \left[\frac{(1 - \alpha)}{\mu} \frac{1 + \Psi_1}{1 + (1 + \nu)\Psi_1}\right]^{\frac{1}{\alpha+\nu}} K_1^{\frac{\alpha}{\alpha+\nu}}. \tag{51}$$

Combining (51) and (17) we obtain (19), with

$$K^u(\Psi_1) := \left[\frac{1 + \Psi_1}{1 + (1 + \nu)\Psi_1} \frac{(1 - \alpha)}{\mu}\right]^{\frac{1}{\nu}} \left[\frac{1}{\alpha\beta}\right]^{\frac{\alpha+\nu}{(\alpha-1)\nu}} \quad \text{and}$$

$$K^c(\Psi_1) := \left[\frac{1 + \Psi_1}{1 + (1 + \nu)\Psi_1} \frac{(1 - \alpha)}{\mu}\right]^{\frac{1}{\nu}} \left[\frac{\phi_1 + \Psi_1}{\alpha\beta\phi_1(1 + \Psi_1)}\right]^{\frac{\alpha+\nu}{(\alpha-1)\nu}}.$$

Once we have K_1 (with a given Ψ_1) as explained in the main text, we can obtain L_2. Consumption C_1 and C_2 can be derived from the resource constraints. The solution to these necessary conditions yields continuous functions of Ψ_1, and it is well defined even as $\Psi_1 \to \infty$: this is because (given the assumed preferences) there is a maximal amount of sustainable initial debt B_0, to which the solution converges as $\Psi_1 \to \infty$.

The statements about the sign of τ_2^k are proven by using equations (17) and (18):

- As $\Psi_1 \to 0$, if $q_1 > 1$, we get $1 = q_1/(1 - \tau_2^k)$, which implies $\tau_2^k < 0$.
- As $\Psi_1 \to \infty$, if $q_1 > 1$, we get $1/\phi_1 = q_1/(1 - \tau_2^k)$, which implies $\tau_2^k > 0$, since $\phi_1 q_1 < 1$.
- When the optimal Ramsey plan features $K_1 > K^*$ for $\Psi_1 = 0$ and $K_1 = K^*$ for some higher values (that is, when there is a finite point Ψ^c as in Figure 1), the following equation applies at

the supremum of the value of Ψ_1 for which $K_1 > K^*$:

$$1 + \frac{\Psi^c(\phi^{-1} - 1)}{1 + \Psi^c} = \frac{q_1}{1 - \tau_2^k} = \frac{1}{1 - \tau_2^k}.$$

That $q_1 = 1$ at Ψ^c follows from the definition of K^* and the fact that both (2) and (3) must hold as an equality by continuity. With $\Psi^c > 0$, it then follows that we must have $\tau_2^k > 0$.

B First-Order Conditions of the Infinite-Horizon Ramsey Problem

To derive first-order conditions, it is convenient to relax the problem and impose constraints (42) and (43) as (weak) inequalities even when $\phi_t > 0$. Online Appendix F.4 shows that they are necessarily binding for the planner if $\phi_t > 0$ anyway, so the solution to the relaxed problem coincides with the solution of the Ramsey problem. This happens because, in the absence of a financial constraint, setting $\phi_t > 0$ wastes resources and distorts an intermediate input, an undesirable outcome. In contrast, when financial constraints are binding, a meaningful interaction between the quantity of capital, its price, and interest rates emerges.

Following the notation that we already introduced, let $\beta^t \lambda_t$, $\beta^t \Psi_t$, Ψ_0, $\beta^t \gamma_t$, and γ_0 be the Lagrange multipliers on constraints (41), (42), (43), (44), and (45). The necessary first-order conditions for a Ramsey outcome are the following:

Consumption and leisure FOCs in period $t \geq 1$ are (see online Appendix F.5 for $t = 0$):

$$\begin{aligned}
&(1 + \Psi_t)u'(C_t) + \Psi_t u''(C_t)(C_t + \tilde{B}_t + q_t^w K_t) \\
&+\gamma_t u''(C_t)\left[\left[1 - (1 - \chi)\phi_t q_t^*\right](1 - \delta)K_{t-1} - (1 - \phi_t q_t^*)K_t\right] - \lambda_t \\
&= -\gamma_{t+1} u''(C_t)\chi \tilde{B}_t (1 + \chi\rho_{t+1})^{-1} + \Psi_{t+1} u''(C_t)(q_t^w K_t + \tilde{B}_t);
\end{aligned} \tag{52}$$

$$v'(L_t)(1 + \Psi_t) + \Psi_t v''(L_t)L_t = \lambda_t F_L(K_{t-1}, L_t); \tag{53}$$

The first-order condition for capital is:

$$\begin{aligned}
&\lambda_t (1 + \phi_t \eta_t) - \Psi_t u'(C_t) q_t^w + \gamma_t u'(C_t)(1 - \phi_t q_t^*) \\
=&\beta \lambda_{t+1} \left\{F_K(K_t, L_{t+1}) + \left[1 + (1 - \chi)\phi_{t+1}\eta_{t+1}\right](1 - \delta)\right\} - \Psi_{t+1} u'(C_t) q_t^w \\
&+\beta \gamma_{t+1} u'(C_{t+1})\left[1 - (1 - \chi)\phi_{t+1} q_{t+1}^*\right](1 - \delta);
\end{aligned} \tag{54}$$

The first-order condition for bonds is equation (47). Finally, let η_t', $(q_t^w)'$, $(q_t^*)'$, and ρ_t' denote the derivatives of each (previously defined) function of ϕ_t. The first-order condition for liquidity in period $t \geq 1$ is (see online Appendix F.5 for $t = 0$):

$$\begin{aligned}
&\Psi_t u'(C_t) K_t (q_t^w)' - \gamma_t u'(C_{t-1}) \frac{\chi \tilde{B}_{t-1}}{\beta} \frac{\chi \rho_t'}{(1 + \chi\rho_t)^2} \\
&+\gamma_t u'(C_t)\left[K_t - (1 - \chi)(1 - \delta)K_{t-1}\right](q_t^* + \phi_t (q_t^*)') \\
&+\lambda_t \left[(1 - \chi)(1 - \delta)K_{t-1} - K_t\right](\eta_t + \phi_t \eta_t') = \Psi_{t+1} u'(C_t) K_t (q_t^w)'.
\end{aligned} \tag{55}$$

C Proof of Proposition 4

We denote steady-state allocations by removing the time subscript of each variable. From the first-order condition for bonds, equation (47), Ψ_t is weakly increasing. Moreover, it is constant iff $\gamma_{t+1} = 0$, which happens iff the financing constraint is slack.We have two possibilities.

Case 1: Ψ_t converges to a finite constant $\Psi > 0$.[38] The Lagrange multiplier of the financing constraint converges to zero in the limit, and so does the financial-market trading in (claims to) capital; that is, $\phi_t \to 0$. The limiting first-order conditions look like those of a standard neoclassical growth model. In particular, the limit of the planner's first-order condition with respect to capital becomes

$$\beta[F_K(K, L) + 1 - \delta] = 1,$$

which coincides with the first-order condition for capital of the families with $\tau^k = 0$.[39] With $\rho = 0$, the families' first-order condition for bonds evaluated at steady state implies $1/p_b = 1/\beta$.

Case 2: Ψ_t diverges to infinity. In this case, we use equations (47) and (53) to substitute out λ_t and γ_t in equations (52), (54), and (55). If the Ramsey allocation converges to a steady state, these three equations in the limit turn into linear second-order difference equations in Ψ_t. These equations are generically distinct. In order for the system to have a solution, the five variables $(C, L, K, \tilde{B}, \phi)$ must be such that equations (41), (42), and (44) (the resources, implementability, and financing constraints, respectively) are satisfied in the steady state and the three difference equations share at least one root. This gives us five (nonlinear) conditions to solve for the five variables. In addition, Ψ_{t+1}/Ψ_t must converge to a constant ζ.[40] In addition, for the first-order conditions to be optimal, Ψ_t cannot grow at rate larger than $1/\beta$ (the transversality condition); that is, $\zeta < \beta^{-1}$. The economy can be captured by finite levels of K, $\tilde{B}$, C, L, ϕ, ζ, $\tilde{\gamma} := \lim_{t\to\infty} \gamma_t/\Psi_t$, and $\tilde{\lambda} := \lim_{t\to\infty} \lambda_t/\Psi_t$. We can thus write the limiting conditions that hold in steady state as follows. The financing constraint becomes

$$\chi\tilde{B}[\beta(1 + \chi\rho)]^{-1} + [[1 - (1 - \chi)\phi q] (1 - \delta) - (1 - \phi q)] K = 0; \tag{56}$$

The implementability condition becomes

$$C - v'(L)L/u'(C) + (\tilde{B} + Kq^w)(1 - \beta^{-1}) = 0; \tag{57}$$

The FOC for consumption (after we use the financing constraint) becomes

$$\frac{u'(C)}{u''(C)} + C + \tilde{B} + q^w K - \tilde{\gamma}\frac{\chi\tilde{B}}{1 + \chi\rho}\frac{1 - \beta\zeta}{\beta} = \frac{\tilde{\lambda}}{u''(C)} + \zeta\left(q^w K + \tilde{B}\right); \tag{58}$$

[38] If $\Psi_t = 0$, $\forall t$, it is straightforward to show that $\Psi_t = 0$ in all periods and that the Ramsey solution attains the first best. In this case, capital is subsidized if the financing constraint is binding as discussed in the two-period example.

[39] That λ_t converges to a constant follows from the first-order conditions with respect to consumption or labor.

[40] Expressing the 2nd order difference equations as two-equation systems of the 1st order difference equations for the vector (Ψ_{t+1}, Ψ_t), the constant ζ corresponds to Ψ_{t+1}/Ψ_t in the eigenvector associated with the common eigenvalue across the three systems. This eigenvalue must be real; if the systems had complex eigenvalues, matching eigenvalues would imply two additional constraints, giving us seven conditions for five variables and implying that generically there would be no solution.

The first-order conditions for capital, bonds, and liquidity become

$$\begin{aligned}&\tilde{\lambda}\left(1+\phi\eta\right)+u'(C)q^{w}(\zeta-1)+\tilde{\gamma}u'(C)(1-\phi q^{*})\\=&\beta\tilde{\lambda}\zeta\left[F_{K}(K,L)+\left[1+(1-\chi)\phi\eta\right](1-\delta)\right]+\beta\tilde{\gamma}\zeta u'(C)\left[1-(1-\chi)\phi q^{*}\right](1-\delta),\end{aligned} \tag{59}$$

$$\chi\zeta\tilde{\gamma}=(1+\chi\rho)(\zeta-1), \quad \text{and} \tag{60}$$

$$\begin{aligned}&-\tilde{\gamma}\frac{\chi\tilde{B}}{\beta}\frac{\chi\rho'}{(1+\chi\rho)^{2}}+\tilde{\gamma}K\left[1-(1-\chi)(1-\delta)\right](q^{*}+\phi(q^{*})')\\&-\frac{\tilde{\lambda}K}{u'(C)}\left[1-(1-\chi)(1-\delta)\right](\eta+\phi\eta')=(\zeta-1)K(q^{w})'.\end{aligned} \tag{61}$$

If $\zeta > 1$, equation (60) implies $\tilde{\gamma} > 0$, so that the financial constraint binds for the planner. Equation (61) implies then that the financial constraint is also binding from the perspective of each individual household, with $\phi_t > 0$, $\rho_t > 0$, $1/p < 1/\beta$, and $q_t^w > q_t^* > 1$.

Next, we establish a sufficient condition for $\tau^k > 0$. To this end, we study a first-order Taylor expansion around the knife-edge point at which the financial constraint holds as an equality when $\Psi_t \to \infty$, but it is not binding, so that $\zeta = 1$ and $\gamma = 0$ at steady state. We use δ as the source of the perturbation, but the same proof can be adapted to varying other parameters, since it relies only on the way the perturbation affects the incentives for the planner to set bonds and capital vs. the privately optimal capital accumulation equation. Using the symbol $\check{}$ to denote first-order deviations from steady state, equation (60) yields

$$\chi\check{\gamma}=\check{\zeta} \tag{62}$$

After deleting terms that sum to zero due to the steady-state properties, equation (59) becomes

$$u'(C)(\check{\zeta}+\check{\gamma})=\beta\tilde{\lambda}(\check{F}_{K}(K,L)-\check{\delta})+\beta\check{\gamma}u'(C)(1-\delta) \tag{63}$$

From these two equations, we get

$$\check{F}_{K}(K,L)-\check{\delta}=\frac{u'(C)(1-\beta(1-\delta)+\chi)}{\beta\tilde{\lambda}}\check{\gamma} \tag{64}$$

Taking a similar first-order expansion of the private first-order condition for capital (40) we obtain

$$0=\beta u'(C)[-\check{\tau}^{k}(F_{K}(K,L)-\delta)+\check{F}_{K}(K,L)-\check{\delta}] \tag{65}$$

Equations (64) and (65) show that $\check{\tau}^k$ is positive for perturbations of δ that make the financial constraint binding ($\check{\gamma} > 0$).

Online Appendix – Not for Publication

In this part, we provide omitted details of the models and omitted proofs in the main text.

D Proofs and Further Results for the 2-period Model

D.1 Lemma 3

Lemma 3. *1. If $q_1 \geq 1/\phi_1$ an arbitrage is possible;*

2. If $q_1 < 1$, optimal investment by the households is zero;

3. If $1/p_1 < (1-\tau_2^k)r_2/q_1$ an arbitrage is possible;

4. If $1 \leq q_1 \leq 1/\phi_1$ and $1/p_1 > (1-\tau_2^k)r_2(1-\phi_1)/(1-\phi_1 q_1)$, optimal investment by the households is zero;

5. If $q_1 \leq 1/\phi_1$ and $(1-\tau_2^k)r_2/q_1 < 1/p_1 < (1-\tau_2^k)r_2(1-\phi_1)/(1-\phi_1 q_1)$ (implying $q_1 > 1$), optimal sales of claims to capital by the household are strictly positive, but optimal purchases are zero;

6. If $q_1 \leq 1/\phi_1$ and $(1-\tau_2^k)r_2/q_1 < 1/p_1 = (1-\tau_2^k)r_2(1-\phi_1)/(1-\phi_1 q_1)$ (implying $q_1 > 1$), optimal purchases of claims to capital by the household are zero, and households are only willing to undertake investment if they can sell a fraction ϕ_1 to other households.

Proof. In this proof, we label trading strategies as in the main text.

1. Suppose an entrepreneur increases her investment by 1 unit. This comes at a unit resource cost. She can sell a fraction ϕ_1 fetching revenues $\phi_1 q_1 \geq 1$, so the resources available for consumption in period 1 by the household are weakly increased and the constraint (3) is relaxed. Furthermore, the household retains the right to $(1-\phi_1)$ units of capital, which increases resources available for consumption in the second period by $(1-\tau_2^k)r_2(1-\phi_1) > 0$: so each unit of new investment weakly increases consumption in period 1, strictly increases consumption in period 2, and it increases the entrepreneurs' funds available for investment.

2. Trading strategy 2 has a strictly higher return than trading strategies 3 and 4, so investing is strictly dominated by purchasing claims to capital produced by other households. It is therefore optimal to set $k_1^e = 0$.

3. The arbitrage relies on the fact that households are not prevented from shorting government bonds. A household can instruct its workers to purchase claims to capital while returning a negative balance to the household, which the household can in turn cover using borrowed funds (shorting government bonds); each unit of capital costs q_1 and the resulting loan requires a payment of q_1/p_1 in period 2, whereas the purchased capital pays $(1-\tau_2^k)r_2 > q_1/p_1$ in the same period.

4. First note that (conditional on $q_1 < 1/\phi_1$)

$$q_1 > 1 \iff \frac{1}{q_1} < 1 < \frac{1-\phi_1}{1-\phi_1 q_1}:$$

whenever $1 < q_1 < 1/\phi_1$, trading strategy 4 has a strictly higher return than trading strategy 3, which in turn has a strictly higher return than trading strategy 2. In this case, trading strategy 1 has a strictly higher return than strategies 2, 3, and 4. As a consequence, the nonnegativity constraint is binding for the latter three trading strategies, and optimal investment is zero.

5. Trading strategy 1 has a strictly higher return than strategy 2, so optimal purchases of claims to capital by the household are zero. Trading strategy 4 has a strictly higher return than strategy 1, so optimal investment is positive and as high as permitted by constraints (2) and (3). Finally, since $q_1 > 1$, trading strategy 4 has a strictly higher return than trading strategy 3, so the household finds it optimal to sell as much capital as allowed by (2): with strictly positive investment, this implies strictly positive sales are optimal.

6. This case is very similar to the previous one, except that trading strategies 1 and 4 have the same rate of return. A household is indifferent between undertaking levered investment or investing in government bonds; however, it strictly prefers buying government bonds to investing unless investment is financed by outside funds as much as allowed by (2), and it strictly prefers buying government bonds to purchasing claims to capital. Optimal purchases of capital are zero; either investment is zero, or, if it strictly positive, then optimal sales of claims to capital are strictly positive as well.

□

Collecting all of the cases that are ruled out by Lemma 3, the set of prices, taxes, and interest rates that are left are those described by (11) and (12) in the main text.

D.2 Proof of Proposition 1

Suppose that the vector $(C_1, C_2, L_1, L_2, K_1)$ satisfies (9), (10), and (16). This allocation is optimal for the firms if w_1, w_2, and r_2 are set according to (14). Substituting factor prices and $L_t = (1-\chi)\ell_t$ into equation (13), this equation can be made to hold for a suitable choice of τ_t^ℓ.[41] We set $p_1 = \beta$, as we proved that this is necessary for an equilibrium, and τ_2^k so that (11) holds. With these choices, a household is indifferent on the timing of consumption between periods 1 and 2, as long as its budget constraint is exhausted.

Next, we proceed separately for the two cases: $K_1 \leq K^*$ and $K_1 > K^*$.

Suppose first that $K_1 \leq K^*$. If we set $q_1 = 1$, households are indifferent on their investment level, so K_1 is weakly optimal, provided it satisfies (2) and (3). With $q_1 = 1$, any choice of sales and purchases of claims to capital is also weakly optimal, as long as they satisfy the same equations. One possible solution is $s_1^e = \max\{0, k_1^e - b_0^e\} = S_1^e/\chi = S_1^w/\chi = s_1^w(1-\chi)/\chi$, which makes (3) hold with equality and implies that (2) holds as well. This solution satisfies market clearing for claims to capital. Generically, the solution is not unique, since households are indifferent at the margin between selling capital, buying capital, or investing their own funds in capital produced by their entrepreneurs.[42]

Finally, we need to verify that the budget constraints of the households or those of the government are satisfied (Walras' law implies that the government budget constraints are satisfied if those of the

[41] Note that both sides of the equation are positive, so the solution implies $\tau_t^\ell < 1$; it is possible that it features $\tau_t^\ell < 0$, which corresponds to labor subsidization.

[42] $q_1 > 1$ is impossible in this case, as long as $K_1 > 0$: with $q_1 > 1$, households would optimally sell a fraction ϕ_1 of the capital that they produce, but not buy any of the capital produced by the entrepreneurs of other households, so market clearing would be impossible.

households are, and vice versa). We substitute the allocation and the prices and taxes that we derived above into the household budget constraint in period 1 and we obtain

$$C_1 = v'(L_1)L_1 + B_0^w - \beta B_1 + B_0^e - K_1. \tag{66}$$

We solve equation (66) for B_1, thereby ensuring that it holds. Substituting this value of B_1 along with prices and taxes into (6) we obtain the first case of (16), thereby verifying that the budget constraints hold at the given prices.

Second, suppose that $K_1 > K_1^*$. Set $S_1^e = \phi_1 K_1$ and $q_1 = (K_1 - B_0^e)/(\phi_1 K_1)$, so that (2) and (3) hold with equality. When $K_1 > K^*$, the resulting value for q_1 is strictly greater than 1, so the household finds it optimal to invest and sell as much of the capital produced by its entrepreneurs as possible, which is consistent with (2) and (3) binding. Market clearing requires $S_1^w = S_1^e$; this choice is weakly optimal for the household given (11).[43] Repeating the steps for the case $K_1 \leq K^*$ we compute prices and taxes, and we substitute them into the household budget constraint in period 1, obtaining (66) again. This can solved for B_1 as in the case of $K_1 < K^*$. Substitution of the resulting value of B_1 into (6) yields the second case of (16). This concludes the proof that any vector that satisfies (9), (10), and (16) is part of a competitive equilibrium.

To proceed in reverse, any allocation that does not satisfy (9) or (10) is not part of a competitive equilibrium, since those conditions are necessary. Consider any allocation that does not satisfy (16). We can repeat the steps that we used before to deduce prices, taxes, and B_1 from the necessary conditions for a competitive equilibrium, and substitute them into the budget constraint of the households. If (16) fails, then at the given prices, taxes, and B_1, the budget constraint (6) will also fail. Specifically, if the left-hand side of (16) is larger than the right-hand side, the resulting allocation, prices, and taxes, violate the household budget constraint. If instead the left-hand side is smaller, they violate the government budget constraint.

E Endogenous Asset Liquidity: a Microfoundation

The intermediation techology follows mostly Cui and Radde (2020) and Cui (2016). There are capital submarkets, denoted by superscripts $m = 1, 2, 3, ...$ As we shall see, the number of submarkets is not important. On each submarket, entrepreneurs and workers post U^m units of sell orders and V^m units of buy orders, respectively. If matched, intermediaries ensure that buyers have enough resources to fill buy orders; sell orders U^m need to be backed by private claims, i.e., each entrepreneur cannot post more than the sum of new and old capital for sale.[44]

There is a continuum of competitive financial intermediaries. Each chooses on which submarket to collect and match quotes at per-quote costs of κ units of consumption goods. The probability of filling a buy quote is f^m, while the probability of filling a sell order (or asset saleability) is ϕ^m.

On each submarket m, financial intermediaries' gross profit amounts to the difference between the competitive buy price $q^{w,m}$ collected from workers and the sell price q^m paid to entrepreneurs on the fraction of successfully matched quotes. Notice that workers direct their quotes to the submarket with the lowest purchase price $q^{w,m} = q^w$, which is taken as given by intermediaries.

[43] In this case, setting $q_1 = 1$ would not be compatible with an equilibrium, since either (2) or (3) would be violated by any choice of S_1^e.

[44] This assumption is for the existence of binding financing contraints. If entrepreneurs can freely post sale orders, they will post the number of orders (give the probability of matching ϕ) to undo financing constraints. We coulld relax the assumption and allow entrepreneurs to post a fraction $x > 1$ of new and used capital, as long as x is not too large.

Since financial intermediaries operate in a competitive environment, they earn zero (net) profit from each transaction, i.e., $\kappa/f^m = q^{w,m} - q^m$. In light of this zero-profit condition, intermediaries are indifferent between all submarkets and we can omit the superscript m:

$$\frac{\kappa}{f} = q^w - q \tag{67}$$

The corresponding $\eta(\phi)$ function in the main text is the same as κ/f, an increasing function of ϕ.

The matching probabilities depend on intermediaries' matching technology. This technology is characterized by a matching function

$$M(U,V) = \xi U^\gamma V^{1-\gamma}$$

where ξ captures matching efficiency and γ is the matching elasticity with respect to sell orders U. Then, asset saleability and the probability of filling buy orders are

$$\phi \equiv \frac{M(U,V)}{U}, \quad f \equiv \frac{M(U,V)}{V} = \xi^{\frac{1}{1-\gamma}} \phi^{\frac{\gamma}{\gamma-1}} \tag{68}$$

Defining market tightness θ as the ratio of buy orders to sell orders, that is, $\theta \equiv V/U$, asset liquidity ϕ has a one-to-one mapping relationship with θ.

Entrepreneurs post orders amounting to $U = K^e + (1-\delta)\chi K_{-1}$, of which a fraction $\phi U = M$ is sold. In this sense, ϕ indeed captures asset saleability. Their optimal choice of which market to choose for their sales is dictated by Lemma 1. In equilibrium, financial intermediaries operate only in the market that minimizes (30) subject to (31) and (32), since the price in other markets would not attract any entrepreneurs (or would not allow intermediaries to break even). Similarly, workers post total orders $V = f^{-1}[S^w]$ and they have enough resources to fill matched buy orders (as they are not financing constrained).

If we set $\gamma = 1/2$, then κ/f in (67) becomes $\kappa\xi^{-2}\phi^2$. Therefore, the cost function $\eta(\phi) = \omega_0\phi^{\omega_1}$ used in the main text can be obtained if we set $\omega_0 = \kappa\xi^{-2}$ and $\omega_1 = 2$.

F Proofs and Other Results for Section 3

F.1 Proof of Lemma 1

- By contradiction, suppose that (k_t^e, s_t^e, ϕ_t) do not solve the given problem. Let $(\tilde{k}, \tilde{s}, \tilde{\phi})$ be an alternative triple that achieves a strictly lower cost while respecting the constraints (31) and (32). If we replace (k_t^e, s_t^e, ϕ_t) by $(\tilde{k}, \tilde{s}, \tilde{\phi})$ into (27), equation (32) implies that capital accumulation by the household is no smaller than before. The second inequality in (24) holds for the alternative allocation since it is precisely (31), and the first inequality is strictly relaxed, since the contradiction assumption implies $q_t(\phi_t)s_t^e - k_t^e > \hat{q}(\hat{\phi})\hat{s} - \hat{k}$. This in turn implies that the budget constraint (26) is also strictly relaxed, since entrepreneurs bring more resources for consumption at the end of the period, and the household could improve upon its allocation by increasing period-t consumption without ever having to increase consumption or the labor supply in any other period; this would then imply that (k_t^e, s_t^e, ϕ_t) is not optimal.

- From (20), q is a strictly decreasing (and concave) function of ϕ. This implies that (31) must hold as an equality at the optimum, for otherwise the household could choose a lower value of $\hat{\phi}$ and the same value for k_t^e and s_t^e and still satisfy (31) and (32), while lowering the cost of this investment (which implies increasing resources available for consumption).[45] After using (31) to substitute out s_t^e, the problem becomes

$$\min_{(\hat{k},\hat{\phi})}[\hat{k} + (1-\delta)k_{t-1}][1 - \hat{\phi}q_t(\hat{\phi})] - (1-\delta)k_{t-1}$$

subject to

$$[\hat{k} + (1-\delta)k_{t-1}][1 - \hat{\phi}] \geq k_t^e + (1-\delta)k_{t-1} - s_t^e. \tag{69}$$

We now see that (69) must also be binding, and use it to substitute for $\hat{k}$, obtaining

$$\min_{\hat{\phi}}[k_t^e + (1-\delta)k_{t-1} - s_t^e]\frac{1 - \hat{\phi}q_t(\hat{\phi})}{1 - \hat{\phi}} + (1-\delta)k_{t-1},$$

completing the proof.

F.2 Proof of Lemma 2

- If $q_t^w < 1$, we proved that the optimal choice of ϕ_t is zero (and thus $q_t^* = q_t^w$), so no claims to capital are sold. If the optimal household choice is $s_t^w > 0$, the given prices and taxes cannot form part of an equilibrium, since market clearing requires $S_t^w = S_t^e$. If instead the optimal choice is s_t^w, suppose we now raise the price q_t^w to 1, which implies q_t^* also is raised to 1. On the selling side, $\phi_t = 0$ remains optimal and thus so is $s_t^e = 0$. The choice of investment k_t^e is unaffected by q_t when $\phi_t = 0$. $s_t^w = 0$ is a fortiori optimal at the new higher price (and the constraint $s_t^w \geq 0$ must be binding at the new price). Hence, the same allocation remains optimal for the household. This price change has no effect on the firm or government problem, and intermediaries still break even at the new price schedule implied by (20) with $q_t^w = 1$.

- As in the previous point, we have $\phi_t = 0$ and hence $s_t^e = 0$. Looking at the household budget constraint (28) and capital evolution equation (29), a unit increase in χk_t^e or a unit increase in $(1-\chi)s_t^w$ decrease resources available for consumption in period t by the same amount, and increase capital holdings in period $t+1$ (k_t) also by the same amount.

- If $q_t^w > 1$, we have $\phi_t > 0$, $q_t^* < q_t^w$, and from Lemma 1 equation (31) holds as an equality at an optimal choice by the household, which implies $s_t^e > 0$. Substituting this equation, the household budget constraint becomes

$$c_t + p_t b_t + \chi k_t^e(1 - q_t^*\phi_t) = (1-\tau_t^\ell)w_t(1-\chi)\ell_t + b_{t-1} + [r_t(1-\tau_t^k) + \delta\tau_t^k + \chi\phi_t q_t^*(1-\delta)]k_{t-1} - (1-\chi)q_t^w s_t^w$$

the capital evolution equation is

$$k_t = (1-\delta)k_{t-1} + (1-\chi)s_t^w + \chi(1-\phi_t)k_t^e - \chi\phi_t(1-\delta)k_{t-1},$$

[45] Once ϕ hits zero, the household cannot lower it any further, but at that point it must also be that $s_t^e = 0$ and (31) has to hold as an equality nonetheless.

and the financial constraint is

$$k_t^e(1-\phi_t q_t^*) \leq b_{t-1} + \phi_t q_t^*(1-\delta)k_{t-1}. \tag{70}$$

If $s_t^w = 0$ is optimal, then at the given prices and taxes the household finds it optimal to sell some claims to capital (possibly just undepreciated capital from the previous period), but not to buy any. Suppose instead by contradiction that $s_t^w > 0$ is optimal and that (70) does not bind. Then the household could consider the following perturbation: decrease s_t^w by $\epsilon/(1-\chi)$ and increase k_t^e by $\epsilon/[\chi(1-\phi_t)]$. This perturbation leaves capital k_t unaffected, respects (70) for ϵ sufficiently small, and increases resources available for consumption in period t by

$$-\frac{1-\phi_t q_t^*}{1-\phi_t} + q_t^w = \frac{q_t^w - 1 - \phi_t(q_t^w - q_t^*)}{1-\phi_t} > 0,$$

thereby contradicting the assumption that the original allocation is optimal.

F.3 Proof of Proposition 3

Proof. Consider first an allocation that satisfies the conditions in the proposition. We can infer the equilibrium value of K_t^e from equation (29). From Lemma 1, the values of S_t^e must satisfy

$$S_t^e = \phi_t[K_t - (1-\chi)(1-\delta)K_{t-1}]. \tag{71}$$

and q_t^w is set according to equation (34). The schedule $q_t(\phi)$ is then given by (20). Market clearing requires $S_t^w = S_t^e$. Factor prices w_t and r_t are pinned down by the firms' optimality conditions (21). We can infer the tax rate on labor from (38), and the price of bonds from (39) and (37), where $q_t^* = q_t(\phi_t)$ as defined in the main text. We recover $B_t = \tilde{B}_t/p_t$, and the tax rate on capital (except for the exogenously given τ_0^k) from equation (40). This constraint is equivalent to (44) when expressed in terms of aggregate variables. Finally, after we substitute the appropriate values of p_t, B_t, q_t^w, ρ_t (as defined in (37)), $q_t^* = q_t(\phi_t)$, r_t, τ_t^k, τ_t^ℓ, and w_t, equations (36) and (42) are equivalent for period $t > 0$, and so are equations (36) and (43) for time 0, (44) and (70) at $t > 0$, and (45) and (70) at $t = 0$.

Conversely, suppose that an allocation does not satisfy the conditions in the proposition. From necessary conditions for a competitive equilibrium we could derive $K_t^e, S_t^e, S_t^w, q_t^w, q_t(.), w_t, r_t, \tau_t^\ell, \tau_t^k, p_t, B_t$, and τ_t^k as above. If (42) does not hold for some period t, then the same substitutions as in the previous step (but in reverse) would imply that can (36) does not hold either. Similarly, if (43) fails, then (36) fails at time 0. If (44) fails, then so does (70); for time 0, if (45) fails, then so does (70). In all of these cases, the allocation would not be part of a competitive equilibrium. □

F.4 Relationship between the relaxed and the original Ramsey problem

In the characterization of the set of competitive-equilibrium allocations, equation (44) has to hold as an equality if $\phi_t > 0$. We show here that any interior solution to the problem remains the same if we impose it as a weak inequality for all values of $\phi_t \in [0, 1]$. Suppose that the solution of maximizing (46) subject to (41), (42), (43), and subject to (44) treated as a weak inequality even for $\phi_t > 0$ is interior, and suppose that the financial constraint (44) is not binding; Letting $\beta^t\lambda_t$ be the Lagrange multiplier on constraint (41), the first-order effect of ϕ_t (with (44) slack) on the Lagrangean for $t > 0$

is given by

$$-\lambda_t[K_t - (1-\chi)(1-\delta)K_{t-1}][\eta_t + \phi_t\eta_t'] + (q_t^w)'(\Psi_t - \Psi_{t+1})u'(C_t)K_t.$$

The first-order condition for $\tilde{B}_t$ implies $\Psi_{t+1} \geq \Psi_t$, so the expression above is strictly negative for all $\phi_t \in (0,1)$. The proof for time 0 is similar; in this case, the first-order effect is

$$\begin{aligned}
&-\lambda_0[K_0 - (1-\chi)(1-\delta)K_{-1}][\eta_0 + \phi_0\eta_0'] \\
&+u'(C_0)\left[(q_0^w)'(\Psi_0 - \Psi_1)K_0 - \Psi_0(q_0^w)'(1-\delta)K_{-1}\left(1 + \frac{\chi\phi_0(q_0^* - 1)}{1-\phi_0 q_0^*}\right)\right] \\
&-u'(C_0)\left[\Psi_0\left(q_0^* - 1 + (1-\phi)(q_0^*)'\right)\frac{(1-\delta)\chi K_{-1}q_0^w}{(1-\phi_0 q_0^*)^2} + \Psi_0 B_{-1}\chi\rho'\right],
\end{aligned}$$

which is also strictly negative for all $\phi_0 \in (0,1)$.[46] This proves that the relaxed planner's problem always features $\phi_t = 0$ whenever the financial constraint is not binding in period t, implying then that the constraint binds whenever the relaxed problem has $\phi_t > 0$: the solution of the relaxed problem thus coincides with the Ramsey plan.

F.5 First-order conditions of the Infinite-Horizon Planner's Problem at $t = 0$

- consumption in period 0:

$$\begin{aligned}
&(1+\Psi_0)u'(C_0) + \Psi_0 u''(C_0)[C_0 + \tilde{B}_0 + q_0^w K_0] \\
&-\Psi_0 u''(C_0)\left\{B_{-1}(1+\chi\rho_0) + \left[(1-\tau_0^k)F_K(K_{-1},L_0) + \delta\tau_0^k + q_0^w(1-\delta)\left(1 + \frac{\chi\phi_0(q_0^*-1)}{1-\phi_0 q_0^*}\right)\right]K_{-1}\right\} \\
&-\lambda_0 = -\gamma_1 u''(C_0)\frac{\chi\tilde{B}_0}{\beta(1+\chi\rho_1)};
\end{aligned}$$

- leisure in period 0:

$$v'(L_0)(1+\Psi_0) + \Psi_0 v''(L_0)L_0 - \lambda_0 F_L(K_{-1},L_0) + \Psi_0 u'(C_0)(1-\tau_0^k)F_{KL}(K_{-1},L_0)K_{-1} = 0;$$

- liquidity in period 0:

$$\begin{aligned}
&-\lambda_0[K_0 - (1-\chi)(1-\delta)K_{-1}][\eta_0 + \phi_0\eta_0'] \\
&+u'(C_0)\left[(q_0^w)'(\Psi_0 - \Psi_1)K_0 - \Psi_0(q_0^w)'(1-\delta)K_{-1}\left(1 + \frac{\chi\phi_0(q_0^*-1)}{1-\phi_0 q_0^*}\right)\right] \\
&-u'(C_0)\left[\Psi_0\left[q_0^* - 1 + \phi_0(1-\phi_0)(q_0^*)'\right]\frac{(1-\delta)\chi K_{-1}q_0^w}{(1-\phi_0 q_0^*)^2} - \Psi_0 B_{-1}\chi\rho_0'\right] \\
&+\gamma_0[K_0 - (1-\chi)(1-\delta)K_{-1}]\left[q_0^* + \phi_0\left(q_0^*\right)'\right] = 0.
\end{aligned}$$

[46] To get positive signs for the Lagrange multipliers, the right-hand side of the resource constraint (41) must be weakly larger than the left-hand side, reflecting the fact that the social value of extra production is positive, and the left-hand side of the implementability constraints (42) and (43) must be weakly bigger than the right-hand side, which is the way these constraints would appear if we allowed the planner to use lump-sum transfers but not lump-sum taxes.

- capital in period 0:

$$
\begin{aligned}
&\lambda_0 \left(1 + \phi_0 \eta_0\right) - \Psi_0 u'(C_0) q_0^w + \gamma_0 (1 - \phi_0 q_0^*) \\
=&\beta \lambda_1 \left\{ F_K(K_0, L_1) + \left[1 + (1-\chi)\phi_1 \eta_1\right](1-\delta) \right\} - \Psi_1 u'(C_0) q_0^w \\
+&\beta \gamma_1 u'(C_1) \left[1 - (1-\chi)\phi_1 q_1^*\right](1-\delta);
\end{aligned}
$$